Code-based Cryptography and Post-Quantum Standardization

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In This Talk

- Motivation
- Intro: a bit of Background
- Conservative Code-Based Cryptography
- Structured Codes
- Sparse-Matrix Codes
- Conclusions

Part I

MOTIVATION

Revolutionary approach (Rivest, Shamir, Adleman 1978, Diffie-Hellmann 1976)

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- Encryption
- Signatures
- Key Exchange
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Main areas of research:

- Lattice-based cryptography.
- Hash-based cryptography.
- Code-based cryptography (McEliece, Niederreiter).
- Multivariate cryptography.
- Isogeny-based cryptography.

Part II

Intro: a bit of Background

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Goal: find a word $e \in \mathbb{F}_q^n$ with $wt(e) \leq w$ such that $He^T = y$.

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GV Bound

For a given finite field $\mathbb{F}_q$ and integers n, k, the Gilbert-Varshamov (GV) distance is the largest integer d_0 such that

$$|\mathcal{B}(0, d_0 - 1)| \leq q^{n-k}$$

where $\mathcal{B}(x, r) = \{y \in \mathbb{F}_q^n \mid d(x, y) \leq r\}$ is the n-dimensional ball of radius r centered in x.

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A subspace of dimension k of $\mathbb{F}_q^n$.
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$H \in \mathbb{F}_q^{(n-k) \times n}$ defines the code as follows: $x \in \mathcal{C}_H \iff Hx^T = 0$.
Systematic form: $(M^T | I_{n-k})$.

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Problem (General Decoding)

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Use ISD as a tool to assess security level.

Part III

CONSERVATIVE CODE-BASED CRYPTOGRAPHY

Overall Strategy

1. Code family.

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$\rightarrow$ More practical to use Niederreiter.

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Key Encapsulation Mechanism (KEM)

- KeyGen: generates private key SK and public key PK.
- $\mathrm{Enc}^{KEM}(PK)$: produces a symmetric key K and a ciphertext c_0.
- $\mathrm{Dec}^{KEM}(SK, c_0)$: returns the symmetric key K (or $\perp$).

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Data Encapsulation Mechanism (DEM)

- $\mathrm{Enc}^{DEM}(K, m)$: produces the ciphertext c_1.
- $\mathrm{Dec}^{DEM}(K, c_1)$: returns the plaintext m (or $\perp$).

HYBRID ENCRYPTION SCHEME

- KeyGen: generates private key SK and public key PK.
- $\mathrm{Enc}^{HY}(PK, m)$:
  - Run $\mathrm{Enc}^{KEM}(PK)$ and get (K, c_0).
  - Run $\mathrm{Enc}^{DEM}(K, m)$ and get c_1.
  - Final ciphertext $c = (c_0, c_1)$.
- $\mathrm{Dec}^{HY}(SK, c)$:
  - Run $\mathrm{Dec}^{KEM}(SK, c_0)$ and get K.
  - Run $\mathrm{Dec}^{DEM}(K, c_1)$ and recover m.

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"Asymmetric" structure key exchange suitable for different scenarios.

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- Sample a word $e \in \mathbb{F}_2^n$ of weight w.
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Decapsulation

- Set $e' = Decode(c_0)$.
- $c' = (c'_0, c'_1)$ where $c'_0 = He'^T$, $c'_1 = \mathbf{H}(e')$.
- Return $K = \mathbf{K}(c', s)$ if decoding fails or $c \neq c'$.
- Else return $K = \mathbf{K}(c', e')$.

About Classic McEliece

Classic McEliece parameters (bytes):

m	n	w	PK Size	SK Size	Ciph Size	Security
13	8,192	128	1,357,824	14,080	240	5
13	6,960	119	1,046,739	13,908	226	5
13	6,688	128	1,044,992	13,892	240	5
13	4,608	96	524,160	13,568	188	3
12	3,488	64	261,120	6,452	128	1

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Classic McEliece parameters (bytes):

m	n	w	PK Size	SK Size	Ciph Size	Security
13	8,192	128	1,357,824	14,080	240	5
13	6,960	119	1,046,739	13,908	226	5
13	6,688	128	1,044,992	13,892	240	5
13	4,608	96	524,160	13,568	188	3
12	3,488	64	261,120	6,452	128	1

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Inherent Aspects of Conservative CBC

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Very large key and slow key generation.

Part IV

STRUCTURED CODES

Traditional approach at current security levels produces very large keys: several Kb to $\approx$ 1Mb.

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Need families of codes with particular automorphism group.

Quasi-Cyclic Codes (Berger, Cayrel, Gaborit, Otmani '09).

$$G = \left[\begin{array}{cc|c} 1 & & \boxed{g} \\ & 1 & \end{array}\right]$$

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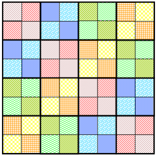
Quasi-Dyadic Codes (Misoczki, Barreto '09).



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Original QC/QD Goppa proposals severely broken, but QD-GS perform better.

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Improved resistance against FOPT led to NIST submission DAGS.

DAGS: A QD-GS BASED KEM

Select hash functions $\mathbf{G}, \mathbf{H}, \mathbf{K}$.

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- Generate a QD-GS code $\mathcal{C}$.
- SK: description $(\bar{u}, \bar{v}, \bar{z})$ for $\mathcal{C}$ (in alternant form) over $\mathbb{F}_{q^m}$.
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ENCAPSULATION

- Choose random word $\mu \in \mathbb{F}_q^k$.
- Compute $(\rho \parallel \sigma) = \mathbf{G}(\mu)$, $d = \mathbf{H}(\mu)$.
- Generate $e \in \mathbb{F}_2^n$ of weight w from seed σ.
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DECAPSULATION

- Set $(\rho' \parallel \mu'), e' = Decode(c_0)$.
- Recompute $\mathbf{G}(\mu')$, $d = \mathbf{H}(\mu')$ and e, then compare.
- Return $\perp$ if decoding fails or any check fails.
- Else return $K = \mathbf{K}(\mu')$.

About DAGS

DAGS parameters (bytes):

q	m	n	w	PK Size	SK Size	Ciph Size	Security
2^8	2	1,600	176	19,712	6,400	1,632	5
2^8	2	1,216	176	11,264	4,864	1,248	3
2^8	2	704	88	7,744	2,816	736	1

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...after a few years of fixes and new attacks: keys getting bigger, confidence/interest getting smaller.

Part V

Sparse-Matrix Codes

Family of codes characterized by very sparse parity-check matrix.

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Decodable with very efficient probabilistic "bit flipping" algorithm (Gallager, '63), small decoding failure rate (DFR).

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Still decodable, gain in security makes up for degradation.

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Matrices formed by circulant blocks

$$\begin{bmatrix} a_0 & a_1 & \dots & a_{r-1} \\ a_{r-1} & a_0 & \dots & a_{r-2} \\ \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & \dots & a_0 \end{bmatrix}$$

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QC property alone does not provide a structural attack.

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Key Generation

- Choose h_0, h_1 in $\mathcal{R}$ of combined weight ω.
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- Take message $\mu \in \mathcal{R}$.
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Decryption

- Set $(e_0, e_1) = Decode_{BitFlipping}(c)$.
- Return $\perp$ if decoding fails.
- Else recover μ (truncate).

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Suite of KEM schemes based on the bit-flipping decoder and QC-MDPC codes.

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In Round 3, narrowed scope to BIKE-2 as most promising.

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IND-CCA conversions require perfect correctness or at least trivial DFR ($\approx 2^{-128}$).

Solution: security is formally restricted to IND-CPA security.

Challenge: design an IND-CCA secure variant with static keys.

BIKE

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- Sample message $\mu \in \{0, 1\}^*$.
- Compute $(e_0, e_1) = \mathbf{H}(\mu)$ in $\mathcal{R}$ of combined weight t.
- Output $c = (c_0, c_1) = (e_0 + e_1 h, \mu \oplus \mathbf{L}(e_0, e_1))$ and $K = \mathbf{K}(\mu, c)$.

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Decapsulation

- Set $(e'_0, e'_1) = \mathit{Decode}_{\mathit{BitFlipping}}(c_0 h_0)$, or $(e'_0, e'_1) = (0, 0)$ if $\perp$.
- Compute $\mu' = c_1 \oplus \mathbf{L}(e'_0, e'_1)$.
- If $(e'_0, e'_1) = \mathbf{H}(\mu')$ then $K = \mathbf{K}(\mu, c)$, else $K = \mathbf{K}(\sigma, c)$.

BIKE parameters (bytes):

r	w	t	PK Size	SK Size	Ciph Size	Security
40,973	274	264	5,122	580	5,154	5
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Scheme is "CCA-ready" but without a formal claim and not recommending static keys.

Part VI

CONCLUSIONS

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Todo: signatures!

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Out of scope of this presentation (but happy to discuss!).

Follow the NIST Competition

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Grazie per l'attenzione!