Code-based Signatures: New Approaches and Research Directions

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IN THIS TALK

- Code-based Cryptography
- Traditional Approach: Hash-and-sign
- Stern's Scheme and Zero-Knowledge
- Code Equivalence and LESS

Part I

CODE-BASED CRYPTOGRAPHY

Error-Correcting Codes

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t-error correcting: $\exists$ algorithm that corrects up to t errors.

Example: Goppa Codes

Select $g(X) \in \mathbb{F}_{q^m}[X]$ and non-zero $\alpha_1, \dots, \alpha_n \in \mathbb{F}_{q^m}$ with $g(\alpha_i) \neq 0$.

Parity-check given by $H = \{H_{ij}\} = \{\alpha_j^{i-1}/g(\alpha_j)\}$. Codewords over $\mathbb{F}_q$.

Let noisy codeword be $y = x + e$, $x \in \mathcal{C}, wt(e) \leq t$.

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To decode:

1. Compute syndrome $s = Hy^T = (s_0, \ldots, s_{r-1})$.
2. Obtain *error locator* poly $\sigma(X)$ and *error evaluator* poly $\omega(X)$ by solving *key equation*
$$\frac{\omega(X)}{\sigma(X)} \equiv s(X) \mod X^r.$$
3. Find roots; error positions are reciprocals (values from $\omega(X)$).

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Gilbert-Varshamov (GV) Bound

The largest integer d_0 such that

$$|\mathcal{B}(0, d_0 - 1)| \leq q^{n-k}.$$

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Hardness of assumption depends on chosen code family.

Choose a code family with efficient decoding algorithm associated to description Δ and hide the structure.

Code-Based Cryptosystems

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Niederreiter: “dual”/equivalent version (1985), based on SDP.

Chosen code family: Generalized Reed-Solomon (GRS) codes.

KeyGen chooses parity-check matrix H and forms public key as SHP.

Plaintext is encrypted as syndrome (scheme is deterministic).

Part II

Traditional Approach: Hash-and-sign

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For code-based, trapdoor is decoding: CFS scheme
(Courtois, Finiasz, Sendrier, 2001).

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Verify

- Compute $y' = H\sigma^T$.
- Accept if $y' = \mathbf{H}(\mu)$, otherwise reject.

About CFS

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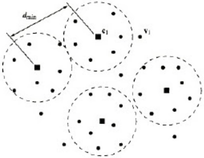

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CFS parameters:

q	m	n	t	PK Size (KB)	Sig Size (bits)	Security
2	16	65536	9	1152	144	80

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Still completely impractical: implementations show GB of public key, and several seconds to sign.
(Landais, Sendrier, 2012; Bernstein, Chou, Schwabe, 2013).

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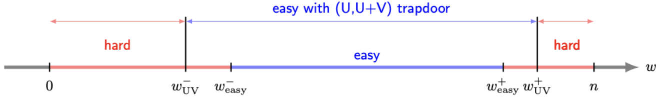

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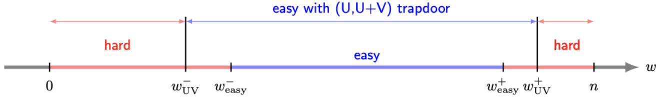

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"Manifacture" high-weight syndrome: rejection sampling.

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Wave parameters:

q	n	k_u	k_v	t	PK (MB)	Sig (kB)	Security
3	8492	3558	2047	7980	3.2	1.6	128

Part III

STERN'S SCHEME AND ZERO-KNOWLEDGE

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For code-based, exploit hardness of finding low-weight words (Stern, 1993).

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Prover | Verifier

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Choose $y \in \mathbb{F}_2^n$ and permutation π. Set $c_1 = \mathbf{H}(\pi, Hy^T), c_2 = \mathbf{H}(\pi(y))$ $c_3 = \mathbf{H}(\pi(y+e))$		
	$\xrightarrow{c_1, c_2, c_3}$	
	$\xleftarrow{b}$	Select random $b \in \{0, 1, 2\}$.
If $b = 0$ set $Rsp = (y, \pi)$		Verify c_1, c_2.
If $b = 1$ set $Rsp = (y + e, \pi)$	$\xrightarrow{Rsp}$	Verify c_1, c_3.
If $b = 2$ set $Rsp = (\pi(y), \pi(e))$		Verify c_2, c_3 and $wt(\pi(e)) = t$.

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For instance: choose random y and π, then $x \in \mathbb{F}_2^n$ with $Hx^T = Hy^T + s$. Build c_1 and c_2 normally and $c_3 = \mathbf{H}(\pi(x))$. Then $Rsp = (y, \pi)$ and $Rsp = (x, \pi)$ pass verification for $b = 0$ and $b = 1$ (strategy fails for $b = 2$). Similarly for other combinations.

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Stern's ZKID parameters:

q	n	t	ℓ	PK (bits)	Sig (KB)	Security	Auth.
2	512	56	35	256	5	60	20
2	620	68	137	310	93.3	80	80
2	1024	112	219	512	245	128	128

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...yet, sizes remain very large.

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	$\xleftarrow{c}$	Select random $c \in \mathbb{F}_q^*$.
$z = \tau(y + ce)$	$\xrightarrow{z}$	
	$\xleftarrow{b}$	Select random $b \in \{0, 1\}$.
If $b = 0$ set $Rsp = \tau$		Verify $c_1 = \mathbf{H}(\tau, H\tau^{-1}(z))^T - cs)$.
If $b = 1$ set $Rsp = \tau(e)$	$\xrightarrow{Rsp}$	Verify $c_2 = \mathbf{H}(z - c\tau(e), \tau(e))$ and $wt(\tau(e)) = t$.

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Works well for lattices using vectors of small norm (Euclidean)
(Lyubashevsky, 2009).

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Unlikely to overcome the metric* limitations.

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"MPC-in-the-head" approach used e.g. in Picnic.

(Ishai, Kushilevitz, Ostrovsky, Sahai, 2007; Katz, Kolesnikov, Wang, 2018)

GPS's Protocol

KeyGen: as in CVE, using a commitment scheme **Com**.

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- Generate random $y, \tilde{e} \in \mathbb{F}_q^n$, with $\tilde{e}$ of weight t, from seed.
- Compute $aux = \{\mathbf{Com}(y + c\tilde{e})\}_{c \in \mathbb{F}_q}$.
- Send seed to prover and aux to verifier.

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Prover / Verifier

Prover		Verifier
Regenerate $y, \tilde{e}$ from seed.		
Determine τ s.t. $e = \tau(\tilde{e})$		
$\alpha = \mathbf{Com}(\tau, H(\tau(y))^T)$	$\xrightarrow{\alpha}$	
	$\xleftarrow{c}$	Select random $c \in \mathbb{F}_q$.
$z = y + c\tilde{e}$	$\xrightarrow{z}$	
		Verify $\alpha = \mathbf{Com}(\tau, H(\tau(z))^T - cs)$.
		Verify $\mathbf{Com}(z)$ with corresponding value from *aux*.

Use “cut-and-choose” technique to remove preprocessing.

Producing a Signature Scheme

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Theoretical work available (ePrint 2021/1020), implementation is being developed.

Part IV

Code Equivalence and LESS

Isometries in the Hamming Metric

Three types:

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2. Monomials: permutations + scaling factors: $\tau = (v; \pi)$, with $v \in {\mathbb{F}_q^*}^n$

$$\tau\big((a_1, a_2, \dots, a_n)\big) = \big(v_1 \cdot a_{\pi(1)}, v_2 \cdot a_{\pi(2)}, \dots, v_n \cdot a_{\pi(n)}\big)$$

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Two codes are equivalent if they are connected by an isometry.

PERMUTATION EQUIVALENCE PROBLEM (PEP)

Given $\mathcal{C}, \mathcal{C}' \subseteq \mathbb{F}_q^n$, find a permutation π such that $\pi(\mathcal{C}) = \mathcal{C}'$.
Equivalently, given generators $G, G' \in \mathbb{F}_q^{k\times n}$, find $S \in \mathrm{GL}_k$, permutation matrix P such that

$$SGP = G'$$

If two codes are permutation equivalent, we write $\mathcal{C} \sim_{PE} \mathcal{C}'$.

Computational Problems

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Linear Equivalence Problem (LEP)

Given $\mathcal{C}, \mathcal{C}' \subseteq \mathbb{F}_q^n$, find a monomial τ such that $\tau(\mathcal{C}) = \mathcal{C}'$. Equivalently, given generators $G, G' \in \mathbb{F}_q^{k \times n}$, find $S \in \mathrm{GL}_k$, monomial matrix Q such that

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If two codes are monomially equivalent, we write $\mathcal{C} \sim_{LE} \mathcal{C}'$.

General Considerations

PEP is a special case of LEP, which in turn is a special case of Semi-Linear Equivalence (monomial + field automorphism).

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There very efficient solvers for certain specific cases.

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Hard-to-solve Instances

Weakly self-dual codes for PEP.
Random codes over $\mathbb{F}_q$, with $q \geq 5$, for LEP.

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Applications in Cryptography

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This can be then transformed into a full-fledged signature scheme via Fiat-Shamir.

Protocol can be tweaked to increase efficiency (e.g. multiple public keys, fixed-weight challenges) (Barenghi, Biasse, P., Santini, 2021)

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Public data: hash function **H**, code $\mathcal{C}$ with generator G

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- SK: invertible matrix S and monomial matrix Q.
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Prover's Computation

- Choose random monomial matrix $\tilde{Q}$.
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  (After receiving challenge bit b).
- If $b = 0$ respond with $\tau = \tilde{Q}$.
- If $b = 1$ respond with $\tau = Q^{-1}\tilde{Q}$.

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Verifier's Computation

- If $b = 0$ verify that $\mathbf{H}(SystForm(G\tau)) = h$.
- If $b = 1$ verify that $\mathbf{H}(SystForm(G'\tau)) = h$.

The State-of-the-art in Code-based Signatures

Scheme	Security Level	Public Data	Public Key	Sig.	PK + Sig.	Security Assumption
Stern	80	18.43	0.048	113.57	113.62	Decoding with low Hamming
Veron	80	18.43	0.096	109.06	109.16	
CVE	80	5.18	0.072	66.44	66.54	
Wave	128	-	3205	1.04	3206.04	Decoding with high Hamming
cRVDC	125	0.050	0.15	22.48	22.63	Decoding with low rank
Durandal - I	128	307.31	15.24	4.06	19.3	
Durandal - II	128	419.78	18.60	5.01	23.61	
GPS - I	128	9.78	0.11	24.60	24.71	MPC with q-ary SDP
GPS- IV	128	13.71	0.13	19.50	19.63	
LESS-FM - I	128	9.78	9.78	15.2	24.97	Code Equivalence Problem
LESS-FM - II	128	13.71	205.74	5.25	210.99	
LESS-FM - III	128	11.57	11.57	10.39	21.96	

TABLE: A comparison of public keys and signature sizes for code-based signature schemes. All sizes are in Kilobytes (kB).

Grazie per l'attenzione!