Leakage Resilient Non-Malleable Secret Sharing

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Part 2

State of the art

Tampering model		Leakage model	Reference	Notes
1-time	independent tampering	/	[GK18]	
		/	[SV18]	
		Bounded-leakage	[KMS18]	
1-time	joint tampering	/	[GK18]	$\mathcal{B}$ partition of $\mathcal{T}$
1-time	cover-free tampering	/	[GSZ20]	
p-time	independent tampering	/	[BS18]	NAT
		/	[ADN+20]	NAT, NACR
p-time	joint tampering	Bounded-leakage	[BFOSV20]	$\mathcal{B}$ partition of $\mathcal{T}$
		/	[BFOSV20]	Semi-adaptive partitioning
continuous	independent tampering	Noisy-leakage*	[FV19]	Non-standard leakage model, ramp
		Noisy-leakage*	[BFV19]	Non-standard leakage model
continuous	joint tampering	Bounded-leakage	[BFV19]	CRS model
/	/	Bounded-leakage	[KMZ20]	$O(t/\log(t))$-sized partitioning
/	/	Bounded-leakage	[CGGL20]	$(0.99n)$-sized partitioning, n-out-of-n

A common technique: NMC, then Share [GK18]

Building blocks

[GK18] "Non-Malleable Secret Sharing", *Vipul Goyal, Ashutosh Kumar*, 50th STOC 2018

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Building blocks

- A one-time ε-non-malleable code $(\mathsf{NMEnc}, \mathsf{NMDec})$ that encodes a message $\mu \in \mathcal{M}$ in two shares in $\mathcal{L} \times \mathcal{R}$ and is also $(\log(|\mathcal{L}|) + \log(1/\varepsilon))$-leakage-resilient on the right share.

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- A t-out-of-n Shamir Secret Sharing scheme $(\mathsf{Share}_n^t, \mathsf{Rec}_n^t)$ taking as input values in $\mathcal{L}$.
- A k-out-of-n Shamir Secret Sharing scheme $(\mathsf{Share}_n^k, \mathsf{Rec}_n^k)$ taking as input values in $\mathcal{R}$, where $k = 1 + \lfloor t/2 \rfloor$.

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  - compute $(\sigma_{\mathsf{L}}, \sigma_{\mathsf{R}}) \leftarrow\!\!\$\ \mathsf{NMEnc}(\mu)$;

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- **Sharing algorithm** NMShare**:** upon input a message $\mu \in \mathcal{M}$,
  - compute $(\sigma_{\mathsf{L}}, \sigma_{\mathsf{R}}) \leftarrow\!\!\$\ \mathsf{NMEnc}(\mu)$;
  - compute $(\sigma_{\mathsf{L},1}, \ldots, \sigma_{\mathsf{L},n}) \leftarrow\!\!\$\ \mathsf{Share}_n^t(\sigma_{\mathsf{L}})$ and $(\sigma_{\mathsf{R},1}, \ldots, \sigma_{\mathsf{R},n}) \leftarrow\!\!\$\ \mathsf{Share}_n^k(\sigma_{\mathsf{R}})$;

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  - compute $(\sigma_{\mathsf{L}}, \sigma_{\mathsf{R}}) \leftarrow\$ \mathsf{NMEnc}(\mu)$;
  - compute $(\sigma_{\mathsf{L},1}, \ldots, \sigma_{\mathsf{L},n}) \leftarrow\$ \mathsf{Share}_n^t(\sigma_{\mathsf{L}})$ and $(\sigma_{\mathsf{R},1}, \ldots, \sigma_{\mathsf{R},n}) \leftarrow\$ \mathsf{Share}_n^k(\sigma_{\mathsf{R}})$;
  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.

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- **Sharing algorithm** $\mathsf{NMShare}$**:** upon input a message $\mu \in \mathcal{M}$,
  - compute $(\sigma_\mathsf{L}, \sigma_\mathsf{R}) \leftarrow\$ \mathsf{NMEnc}(\mu)$;
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  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.
- **Reconstruction algorithm** NMRec**:** upon input a set of t shares $(\sigma_i^*)_{i \in \mathcal{I}}$,

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- **Sharing algorithm** NMShare**:** upon input a message $\mu \in \mathcal{M}$,
  - compute $(\sigma_{\mathsf{L}}, \sigma_{\mathsf{R}}) \leftarrow\$ \mathsf{NMEnc}(\mu)$;
  - compute $(\sigma_{\mathsf{L},1}, \ldots, \sigma_{\mathsf{L},n}) \leftarrow\$ \mathsf{Share}_n^t(\sigma_{\mathsf{L}})$ and $(\sigma_{\mathsf{R},1}, \ldots, \sigma_{\mathsf{R},n}) \leftarrow\$ \mathsf{Share}_n^k(\sigma_{\mathsf{R}})$;
  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.
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  - compute $(\sigma_\mathsf{L}, \sigma_\mathsf{R}) \leftarrow\$ \mathsf{NMEnc}(\mu)$;
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  - parse, for all $i \in \mathcal{I}$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$;
  - verify if all the shares $(\sigma_{\mathsf{R},i})_{i\in\mathcal{I}}$ are consistent under k-out-of-n Shamir Secret Sharing, and output $\perp$ if not;

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  - compute $(\sigma_\mathsf{L}, \sigma_\mathsf{R}) \leftarrow\$ \mathsf{NMEnc}(\mu)$;
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  - verify if all the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{I}}$ are consistent under k-out-of-n Shamir Secret Sharing, and output $\perp$ if not;
  - reconstruct $\sigma_\mathsf{L} = \mathsf{Rec}_n^t((\sigma_{\mathsf{L},i})_{i \in \mathcal{I}})$ and $\sigma_\mathsf{R} = \mathsf{Rec}_n^k((\sigma_{\mathsf{R},i})_{i \in \mathcal{I}})$;

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  - parse, for all $i \in \mathcal{I}$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$;
  - verify if all the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{I}}$ are consistent under k-out-of-n Shamir Secret Sharing, and output $\perp$ if not;
  - reconstruct $\sigma_\mathsf{L} = \mathsf{Rec}_n^t((\sigma_{\mathsf{L},i})_{i \in \mathcal{I}})$ and $\sigma_\mathsf{R} = \mathsf{Rec}_n^k((\sigma_{\mathsf{R},i})_{i \in \mathcal{I}})$;
  - decode $\mu = \mathsf{NMDec}(\sigma_\mathsf{L}, \sigma_\mathsf{R})$ and output μ.

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  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.
- **Reconstruction algorithm** NMRec**:** upon input a set of t shares $(\sigma_i^*)_{i \in \mathcal{I}}$,
  - parse, for all $i \in \mathcal{I}$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$;
  - verify if all the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{I}}$ are consistent under k-out-of-n Shamir Secret Sharing, and output $\perp$ if not;
  - reconstruct $\sigma_\mathsf{L} = \mathsf{Rec}_n^t((\sigma_{\mathsf{L},i})_{i \in \mathcal{I}})$ and $\sigma_\mathsf{R} = \mathsf{Rec}_n^k((\sigma_{\mathsf{R},i})_{i \in \mathcal{I}})$;
  - decode $\mu = \mathsf{NMDec}(\sigma_\mathsf{L}, \sigma_\mathsf{R})$ and output μ.

The above scheme is a $(t-1)$-joint* t-out-of-n one-time 2ε-non-malleable secret sharing scheme.

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A common technique: NMC, then Share [GK18] — Proof

- By reduction to the underlyng leakage-resilient non-malleable code. Fix any set $\mathcal{T} \subset [n]$ such that $|\mathcal{T}| = t$ and any partition $(\mathcal{B}_1, \mathcal{B}_2)$ of $\mathcal{T}$ such that $|\mathcal{B}_1| \geq k > |\mathcal{B}_2|$ (since $k = 1 + \lfloor t/2 \rfloor$). Let (f_1, f_2) be the tampering query.

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- **Setup:** the reduction $\hat{\mathsf{A}}$ samples random strings ρ, ρ_1, ρ_2 and random shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$ and $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$.

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- **Leakage from σ_{R}:** using σ_{R}, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_1}$; then,

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- **Setup:** the reduction $\hat{\mathrm{A}}$ samples random strings ρ, ρ_1, ρ_2 and random shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$ and $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$.
- **Leakage from σ_{R}:** using σ_{R}, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_1}$; then,
    - apply the tampering function f_1;

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- **Setup:** the reduction $\hat{\mathsf{A}}$ samples random strings ρ, ρ_1, ρ_2 and random shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$ and $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$.
- **Leakage from σ_{R}:** using σ_{R}, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_1}$; then,
  - apply the tampering function f_1;
  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;

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  - apply the tampering function f_1;
  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
  - compute an auxiliary information α that depends on the randomness ρ and the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_2}$ obtained by interpolating the values $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_1}$ (if they are consistent);

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  - apply the tampering function f_1;
  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
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  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.

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- **Setup:** the reduction $\hat{\mathsf{A}}$ samples random strings ρ, ρ_1, ρ_2 and random shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$ and $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$.
- **Leakage from σ_R:** using σ_R, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_1}$; then,
  - apply the tampering function f_1;
  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
  - compute an auxiliary information α that depends on the randomness ρ and the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_2}$ obtained by interpolating the values $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_1}$ (if they are consistent);
  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.
- **Tampering with σ_L:** using σ_L, randomness ρ_2 and the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$, obtain the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_2}$; then,

A common technique: NMC, then Share [GK18] — Proof

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    - apply the tampering function f_2;

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- By reduction to the underlyng leakage-resilient non-malleable code. Fix any set $\mathcal{T} \subset [n]$ such that $|\mathcal{T}| = t$ and any partition $(\mathcal{B}_1, \mathcal{B}_2)$ of $\mathcal{T}$ such that $|\mathcal{B}_1| \geq k > |\mathcal{B}_2|$ (since $k = 1 + \lfloor t/2 \rfloor$). Let (f_1, f_2) be the tampering query.
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  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
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  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.
- **Tampering with σ_{L}:** using σ_{L}, randomness ρ_2 and the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$, obtain the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_2}$; then,
  - apply the tampering function f_2;
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- **Leakage from σ_R:** using σ_R, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i \in \mathcal{B}_1}$; then,
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  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
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  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.
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  - if everything is consistent, use the partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ and the tampered shares $(\tilde{\sigma}_{\mathsf{L},j})_{j \in \mathcal{B}_2}$ to obtain the tampered left share $\tilde{\sigma}_\mathsf{L}$; otherwise, output $\perp$;

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- **Leakage from σ_R:** using σ_R, randomness ρ_1 and the shares $(\sigma_{\mathsf{R},i})_{i\in\mathcal{B}_2}$, obtain the shares $(\sigma_{\mathsf{R},i})_{i\in\mathcal{B}_1}$; then,
  - apply the tampering function f_1;
  - compute a partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ of the left tampered share;
  - compute an auxiliary information α that depends on the randomness ρ and the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j\in\mathcal{B}_2}$ obtained by interpolating the values $(\tilde{\sigma}_{\mathsf{R},j})_{j\in\mathcal{B}_1}$ (if they are consistent);
  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.
- **Tampering with σ_L:** using σ_L, randomness ρ_2 and the shares $(\sigma_{\mathsf{L},i})_{i\in\mathcal{B}_1}$, obtain the shares $(\sigma_{\mathsf{L},i})_{i\in\mathcal{B}_2}$; then,
  - apply the tampering function f_2;
  - use randomness ρ and the auxiliary information α to check that the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j\in\mathcal{B}_2}$ are consistent with the ones computed during the leakage phase;
  - if everything is consistent, use the partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ and the tampered shares $(\tilde{\sigma}_{\mathsf{L},j})_{j\in\mathcal{B}_2}$ to obtain the tampered left share $\tilde{\sigma}_\mathsf{L}$; otherwise, output $\perp$;
  - output $\tilde{\sigma}_\mathsf{L}$.

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  - compute an auxiliary information α that depends on the randomness ρ and the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_2}$ obtained by interpolating the values $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_1}$ (if they are consistent);
  - output $(\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}, \alpha)$.
- **Tampering with σ_L:** using σ_L, randomness ρ_2 and the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_1}$, obtain the shares $(\sigma_{\mathsf{L},i})_{i \in \mathcal{B}_2}$; then,
  - apply the tampering function f_2;
  - use randomness ρ and the auxiliary information α to check that the tampered shares $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_2}$ are consistent with the ones computed during the leakage phase;
  - if everything is consistent, use the partial reconstruction $\tilde{\sigma}_{\mathsf{L},\mathcal{B}_1}$ and the tampered shares $(\tilde{\sigma}_{\mathsf{L},j})_{j \in \mathcal{B}_2}$ to obtain the tampered left share $\tilde{\sigma}_\mathsf{L}$; otherwise, output $\perp$;
  - output $\tilde{\sigma}_\mathsf{L}$.
- **Tampering with σ_R:** perform the same steps as in the leakage phase, but output the value $\tilde{\sigma}_\mathsf{R}$ if the shares $(\tilde{\sigma}_{\mathsf{R},j})_{j \in \mathcal{B}_1}$ are consistent and $\perp$ otherwise.

Achieving leakage resilience [BFOSV20]

Building blocks

[BFOSV20] “Non-Malleable Secret Sharing against Bounded Joint-Tampering Attacks in the Plain Model”, *Gianluca Brian, Antonio Faonio, Maciej Obremski, Mark Simkin, Daniele Venturi*, CRYPTO 2020

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- A one-time ε_2-non-malleable code $(\mathsf{NMEnc}, \mathsf{NMDec})$ that encodes a message $\mu \in \mathcal{M}$ in two shares in $\mathcal{L} \times \mathcal{R}$.

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- **Sharing algorithm** NMShare**:** upon input a message $\mu \in \mathcal{M}$,
  - compute $(\sigma_{\mathsf{L}}, \sigma_{\mathsf{R}}) \leftarrow\!\$\ \mathsf{NMEnc}(\mu)$;
  - compute $(\sigma_{\mathsf{L},1}, \ldots, \sigma_{\mathsf{L},n}) \leftarrow\!\$\ \mathsf{Share}_{\mathsf{L}}(\sigma_{\mathsf{L}})$ and $(\sigma_{\mathsf{R},1}, \ldots, \sigma_{\mathsf{R},n}) \leftarrow\!\$\ \mathsf{Share}_{\mathsf{R}}(\sigma_{\mathsf{R}})$;
  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.

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- **Sharing algorithm** NMShare**:** upon input a message $\mu \in \mathcal{M}$,
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  - output the shares $(\sigma_1^*, \ldots, \sigma_n^*)$, where, for each $i \in [n]$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$.
- **Reconstruction algorithm** NMRec**:** upon input a set of t shares $(\sigma_i^*)_{i \in \mathcal{I}}$,
  - parse, for all $i \in \mathcal{I}$, $\sigma_i^* = (\sigma_{\mathsf{L},i}, \sigma_{\mathsf{R},i})$;
  - reconstruct $\sigma_\mathsf{L} = \mathsf{Rec}_\mathsf{L}((\sigma_{\mathsf{L},i})_{i \in \mathcal{I}})$ and $\sigma_\mathsf{R} = \mathsf{Rec}_\mathsf{R}((\sigma_{\mathsf{R},i})_{i \in \mathcal{I}_{t_\mathsf{R}}})$;
  - decode $\mu = \mathsf{NMDec}(\sigma_\mathsf{L}, \sigma_\mathsf{R})$ and output μ.

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Achieving leakage resilience [BFOSV20]

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- A one-time ε_2-non-malleable code $(\mathsf{NMEnc}, \mathsf{NMDec})$ that encodes a message $\mu \in \mathcal{M}$ in two shares in $\mathcal{L} \times \mathcal{R}$.
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- **Sharing algorithm** NMShare**:** upon input a message $\mu \in \mathcal{M}$,
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  - reconstruct $\sigma_\mathsf{L} = \mathsf{Rec}_\mathsf{L}((\sigma_{\mathsf{L},i})_{i \in \mathcal{I}})$ and $\sigma_\mathsf{R} = \mathsf{Rec}_\mathsf{R}((\sigma_{\mathsf{R},i})_{i \in \mathcal{I}_{t_\mathsf{R}}})$;
  - decode $\mu = \mathsf{NMDec}(\sigma_\mathsf{L}, \sigma_\mathsf{R})$ and output μ.

The above scheme is a $(t_\mathsf{R} - 1)$-joint* ℓ-bounded leakage resilient one-time non-malleable secret sharing scheme with security $2(\varepsilon_\mathsf{L} + \varepsilon_\mathsf{R}) + \varepsilon_2$ so long as $t_\mathsf{R} = \sqrt{k}$, $\ell_\mathsf{L} = \ell + 1$ and $\ell_\mathsf{R} = \ell + n \cdot \log|\mathcal{S}_{\mathsf{L},i}|$ for all $i \in [n]$.

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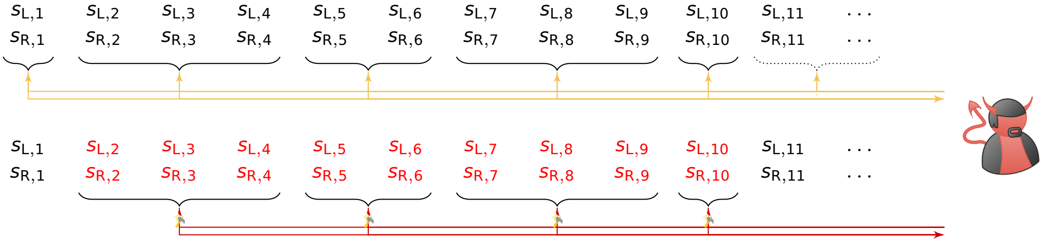
Achieving leakage resilience [BFOSV20] — Proof strategy



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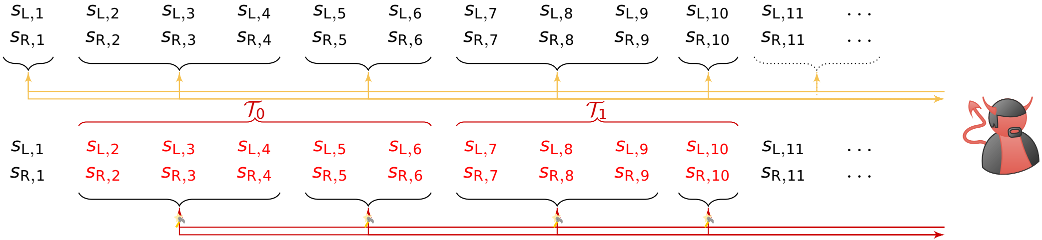

- Split the tampering set into two subsets $\mathcal{T}_0$ and $\mathcal{T}_1$ such that $|\mathcal{T}_0| \geq$ threshold of $\mathsf{Share}_{\mathsf{R}}$.

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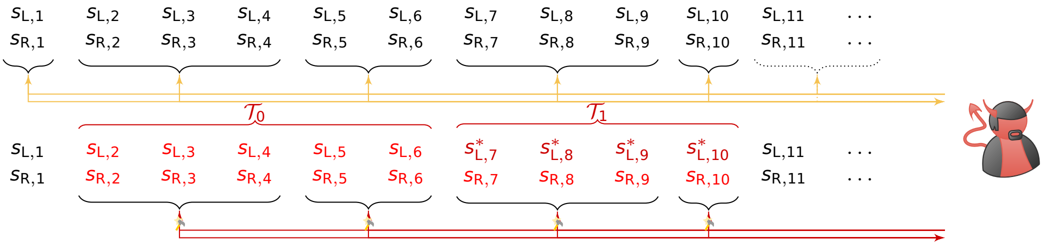

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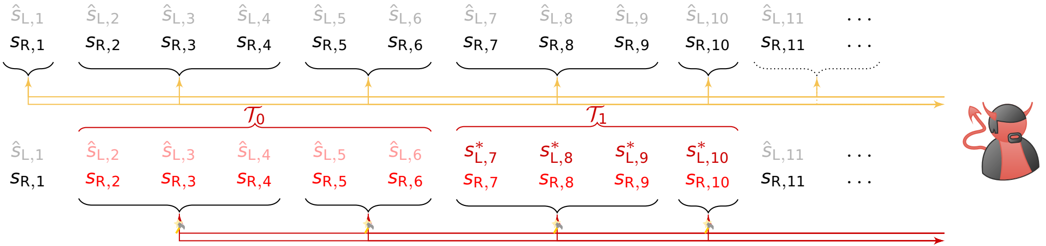

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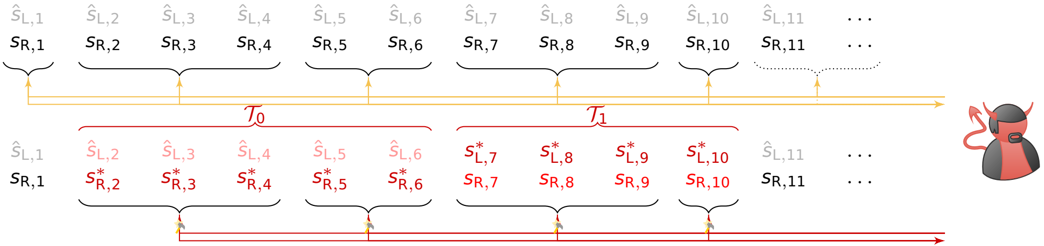

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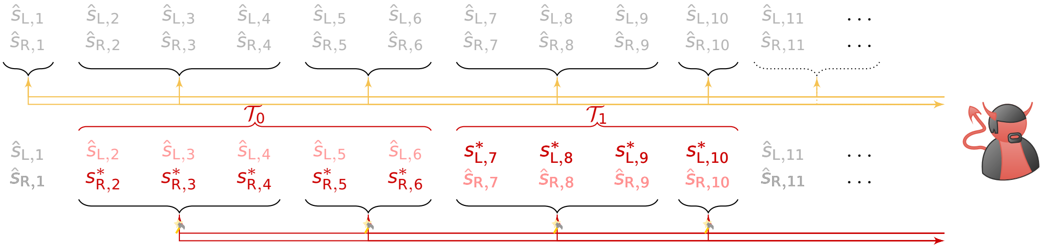

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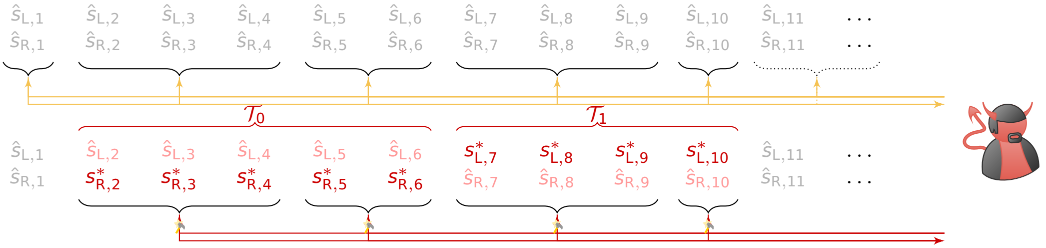

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- Now we can safely reduce to non-malleability of the non-malleable code.

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- The previous construction is secure in a model that is stronger that selective partitioning, called *semi-adaptive partitioning*.

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- Actually, non-malleability against adaptive partitioning is very hard to achieve. Even constructing a 3-out-of-3 secret sharing scheme that is non-malleable against adversaries who perform joint leakage from each of the three subsets $\{1,2\},\{1,3\},\{2,3\}$ and then independent tampering appears to be a challenging task [KMS18].

[KMS18] "Leakage Resilient Secret Sharing", *Ashutosh Kumar, Raghu Meka, Amit Sahai*, IACR Cryptology ePrint Archive, Vol.2018/1138

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- This is because joint leakage leads to loss of independence among the shares, therefore the subsequent tampering queries are not independent anymore.

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- The previous construction is secure in a model that is stronger that selective partitioning, called *semi-adaptive partitioning*.
  - Intuitively, this model combines the adaptive partitioning for the leakage queries with the selective partitioning for the tampering query.
  - In particular, the admissible adversary is defined so that, at the end of the experiment, the leakage performed inside the reconstruction set $\mathcal{T}$ of the tampering query is leakage under selective partitioning, while the leakage performed outside $\mathcal{T}$ is leakage under adaptive partitioning.
- Actually, non-malleability against adaptive partitioning is very hard to achieve. Even constructing a 3-out-of-3 secret sharing scheme that is non-malleable against adversaries who perform joint leakage from each of the three subsets $\{1,2\},\{1,3\},\{2,3\}$ and then independent tampering appears to be a challenging task [KMS18].
- This is because joint leakage leads to loss of independence among the shares, therefore the subsequent tampering queries are not independent anymore.
- **Cover-free tampering [GSZ20]:** let $\mathcal{T}_1,\ldots,\mathcal{T}_n \subseteq [n]$. $(\mathcal{T}_1,\ldots,\mathcal{T}_n)$ is a *k-cover-free* family of subsets if, for all $i \in [n]$, the union of all $\mathcal{T}_j \ni i$ has at most $k-1$ elements.

[KMS18] "Leakage Resilient Secret Sharing", *Ashutosh Kumar, Raghu Meka, Amit Sahai*, IACR Cryptology ePrint Archive, Vol.2018/1138
[GSZ20] "Multi-Source Non-Malleable Extractors and Applications", *Vipul Goyal, Akshayaram Srinivasan, Chenzhi Zhu*, IACR Cryptology ePrint Archive, Vol.2020/157

Example: an actual scheme achieving non-malleability against cover-free tampering

Building blocks

[GSZ20] “Multi-Source Non-Malleable Extractors and Applications”, *Vipul Goyal, Akshayaram Srinivasan, Chenzhi Zhu*, IACR Cryptology ePrint Archive, Vol.2020/157

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  - for all $i \in \mathcal{I}$, reconstruct $\sigma_{\mathsf{R},i} = \mathsf{Rec}((\sigma^{(j)}_{\mathsf{R},i})_{j\in\mathcal{I}})$;

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  - reconstruct $\mu = \mathsf{Rec}((\sigma_i)_{i\in\mathcal{I}})$ and output μ.

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  - reconstruct $\mu||\rho = \mathsf{Rec}((\sigma_i)_{i \in \mathcal{I}})$;
  - if $\gamma = \mathsf{Commit}(\mu; \rho)$, output μ, otherwise output $\perp$.

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  - reconstruct $\mu || \rho = \mathsf{Rec}((\sigma_i)_{i \in \mathcal{I}})$;
  - if $\gamma = \mathsf{Commit}(\mu; \rho)$, output μ, otherwise output $\perp$.

- If Π is ℓ-bounded leakage-resilient against selective/semi-adaptive partitioning, then the above scheme is p-time non-malleable against selective/semi-adaptive partitioning as long as $\ell = p \cdot (|\gamma| + n) + 1$.

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- If Π is ℓ-bounded leakage-resilient against selective/semi-adaptive partitioning, then the above scheme is p-time non-malleable against selective/semi-adaptive partitioning as long as $\ell = p \cdot (|\gamma| + n) + 1$.
- If Π is ℓ-noisy* leakage-resilient against independent leakage and tampering, then the above scheme is ℓ'-noisy* leakage-resilient continuously non-malleable as long as $\ell = \ell' + |\gamma| + 1 + O(\log(\lambda))$.

Achieving multiple tampering queries [BFOSV20] — Proof

Hybrid argument

- **Original game:**
  - sample random coins ρ and compute $\gamma = \mathsf{Commit}(\mu; \rho)$;
  - compute $(\sigma_1, \ldots, \sigma_n) \leftarrow\!\!\$\ \mathsf{Share}(\mu || \rho)$;
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Achieving multiple tampering queries [BFOSV20] — Proof

Hybrid argument

- **Original game:**
  - sample random coins ρ and compute $\gamma = \mathsf{Commit}(\mu; \rho)$;
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  - Using another leakage query, the reduction obtains a bit for each share in $\mathcal{T}$ telling if the corresponding commitment equals $\tilde{\gamma}_{i^*}$ or not; in the latter case, return $\perp$.
  - Then, the reduction forwards the tampering query to the oracle;

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  - Using another leakage query, the reduction obtains a bit for each share in $\mathcal{T}$ telling if the corresponding commitment equals $\tilde{\gamma}_{i*}$ or not; in the latter case, return $\perp$.
  - Then, the reduction forwards the tampering query to the oracle;
  - Finally, the reduction checks that $\tilde{\gamma}_{i*}$ is a valid commitment for the outcome of the tampering query and returns either the result of the tampering or $\perp$.

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  - use another leakage query to check if all the tampered commitments correspond, and return $\perp$ if not;
  - find by brute force the opening $(\mu^{(q)}, \rho^{(q)})$ of $\tilde{\gamma}_{i*}$, and return $\perp$ if no such value is found;

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  - after checking that $\mu^{(q)}$ is "good", return $\mu^{(q)}$ to the adversary.

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- Upon input the last tampering query $(\mathcal{T}^{(p+1)}, f^{(p+1)})$:

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- Upon input the last tampering query $(\mathcal{T}^{(p+1)}, f^{(p+1)})$:
  - construct a special leakage query checking that the simulation did not cause any inconsistency so far and outputs a bit b_{ok};

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- Output the same distinguishing bit as the adversary if $b_{\mathsf{ok}} = \mathsf{ok}$ and 0 if $b_{\mathsf{ok}} = \mathsf{error}$.

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Leakage analysis

The total leakage performed by the reduction amounts to $(p+1) \cdot (|\gamma| + n) + 1$.

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Final step

- We proved that the original game and the hybrid game are statistically close.

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- *Proof:* by reduction to the computational hiding property of the commitment scheme.

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Digression on the non-standard noisy-leakage notion

- **Admissible adversaries:** an adversary A is ℓ-admissible if it is allowed to ask as many leakage queries he wants, chosen adaptively, as long as, for all $i \in [n]$,

$$\tilde{\mathbb{H}}_\infty\left(\boldsymbol{\Sigma}_i \mid \boldsymbol{\Lambda}_i\right) \geq \mathbb{H}_\infty\left(\boldsymbol{\Sigma}_i\right) - \ell$$

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Note: the non-standard version is tricky and “dangerous”, since there are many more leakage queries performing 0 bits of noisy leakage, and some of them could even break non-malleability.

Achieving multiple tampering queries [BFV19] — Proof

Inductive step

- By reduction to statistical leakage-resilience one-time non-malleability.

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  - after checking that $\mu^{(q)}$ is "good", return $\mu^{(q)}$ to the adversary.

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- Upon input the last tampering query $(\mathcal{T}^{(p+1)}, f^{(p+1)})$:
  - construct a special leakage query checking that the simulation did not cause any inconsistency so far and outputs a bit b_{ok};

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- Upon input the last tampering query $(\mathcal{T}^{(p+1)}, f^{(p+1)})$:
  - construct a special leakage query checking that the simulation did not cause any inconsistency so far and outputs a bit b_{ok};
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- Output the same distinguishing bit as the adversary if $b_{\mathrm{ok}} = \mathrm{ok}$ and 0 if $b_{\mathrm{ok}} = \mathrm{error}$.

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Achieving multiple tampering queries [BFV19] — Leakage analysis

For all $i \in [n]$,

$$\tilde{\mathbb{H}}\left(\mathbf{\Sigma}_i \mid (\mathbf{\Sigma}_j)_{j \neq i}, \mathbf{\Lambda}_i\right)$$

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$$\tilde{\mathbb{H}}\left(\mathbf{\Sigma}_i \mid (\mathbf{\Sigma}_j)_{j \neq i}, \mathbf{\Lambda}_i\right) \geq \tilde{\mathbb{H}}\left(\mathbf{\Sigma}_i \mid (\mathbf{\Sigma}_j)_{j \neq i}, \tilde{\gamma}_i^{(1)}, \ldots, \tilde{\gamma}_i^{(\mathbf{q}_{\mathsf{sd}})}, \tilde{\gamma}_i^{(p+1)}\right) - \ell' - 1$$

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&\geq \tilde{\mathbb{H}}\left(\boldsymbol{\Sigma}_i \mid (\boldsymbol{\Sigma}_j)_{j\neq i}\right) - \ell' - 1 - |\gamma| - O(\log(\lambda)).
\end{aligned}$$

Therefore, the overall performed leakage by the reduction amounts to $\ell = \ell' + 1 + |\gamma| + O(\log(\lambda))$.

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- p-time/continuously non-malleable secret sharing against joint leakage and tampering.

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