Cryptographic primitives from elliptic curve isogenies

An overview and some preliminary results

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Breaking the scheme $\Rightarrow$ solving the difficult mathematical problem

The threat of quantum computers

Hard mathematical problems:

- **integer factorisation: RSA, …**
- **discrete logarithm problems over finite fields: DSA, …**
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  Elliptic Curve Cryptography (since 1985)

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Shor (1994): ***those problems are easy for quantum computers!***

A new class of hard mathematical problems

Problems supposed hard also for quantum computers:

- Bounded decoding problem → Code-based Cryptography
- Second pre-image problem for hash functions → Hash-based Cryptography
- Solving systems of multivariate polynomial equations (over finite fields) → Multivariate Cryptography
- Computational lattice problems → Lattice-based Cryptography
- Isogeny problems → Isogeny-based Cryptograpy

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which studies algorithms that:

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NIST (National Institute of Standards and Technology) has initiated a process to

solicit, evaluate, and standardise one or more quantum-resistant public-key cryptographic algorithms

in December 2016 (first round's results published at the end of Jan 2019).

Isogeny-based Cryptography

Isogeny problem: given a prime *p* and two supersingular isogenous curves,

$$E_1 \text{ and } E_2 \text{ over } \mathbb{F}_{p^2}\text{, compute an isogeny}$$

$$\varphi : E_1 \to E_2$$

of a given degree *d*.

SIKE: isogeny-based key-exchange protocol submitted to NIST process

Outline of the talk

1. Isogenies and their properties

2. The isogeny assumptions

3. Cryptographic protocols

4. What it is missing and open problems

5. A new path for Isogeny-based Cryptography

What is an isogeny?

Let E_1, E_2 be two elliptic curves defined over a finite field $\mathbb{F}_{p^n}$:

$$E_i : y^2 = x^3 + A_i x + B_i \qquad A_i, B_i \in \mathbb{F}_{p^n} \qquad i = 1, 2$$

The sets

$$E_i(\overline{\mathbb{F}_{p^n}}) = \{P = (x, y) \in \overline{\mathbb{F}_{p^n}} \times \overline{\mathbb{F}_{p^n}} \mid y^2 = x^3 + A_i x + B_i\} \cup \{\infty\}$$

are abelian groups with respect to an addition having ∞ as identity.

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$E_i(\mathbb{F}_{p^n})$ is the subgroup of rational points.

An isogeny from E_1 to E_2 is a non-constant morphism

$$\varphi : E_1 \to E_2$$

$$(x, y) \mapsto \left(\frac{f_1(x, y)}{g_1(x, y)}, \frac{f_2(x, y)}{g_2(x, y)} \right) \qquad f_1, f_2, g_1, g_2 \in \mathbb{F}_{p^{kn}}[x, y]$$

sending ∞ in ∞. We will say that φ is defined over the extension $\mathbb{F}_{p^{kn}}$.

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- φ admits a dual isogeny $\hat{\varphi} : E_2 \to E_1$ having the same degree d and such that

$$\hat{\varphi} \circ \varphi = \varphi \circ \hat{\varphi} = [d]$$

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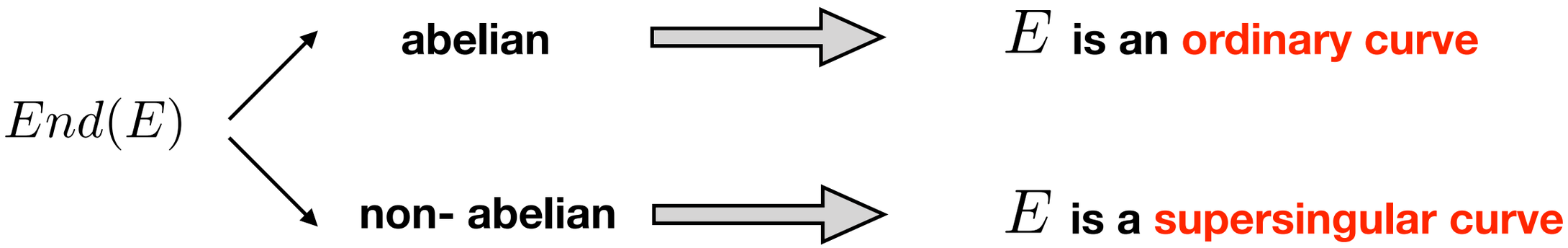

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Given an elliptic curve $E : y^2 = x^3 + Ax + B$ defined over $\mathbb{F}_{p^n}$, we define its j-invariant as:

$$j(E) = 1728\frac{4A^3}{4A^3 + 27B^2}$$

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Theorem: two elliptic curves defined over $\mathbb{F}_{p^n}$ are isomorphic (on some extension field) iff they have the same *j*-invariant.

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- Velu's formulas allow to compute both E/G and φ with a complexity linear in $\#G$.

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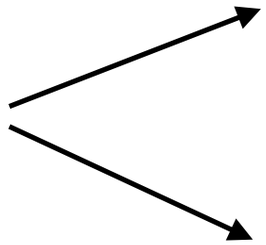

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<u>Non standard assumptions</u>: variations of the above problem, that are assumed as hard as the original problem.

First cryptographic protocols

- **CLG Hash function** (Charles, Lauter, Goren - 2009)
- **SIDH Key Exchange** (Jao, De Feo - 2011)
- **Encryption Scheme** (Jao, De Feo, Plût - 2014)
- **Zero-knowledge Proof** (Jao, De Feo, Plût - 2014)

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finding pre-images implies the resolution of the isogeny problem.

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Given an integer ℓ, the set

$$E_0[\ell] = \{P \in E_0(\overline{\mathbb{F}_{p^2}}) \mid \ell P = \infty\}$$

is isomorphic to

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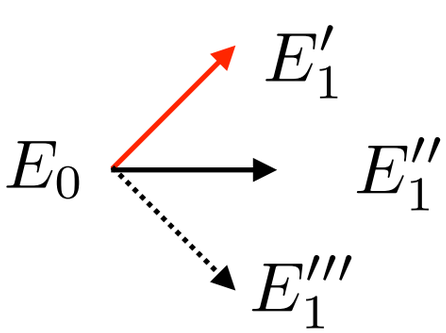

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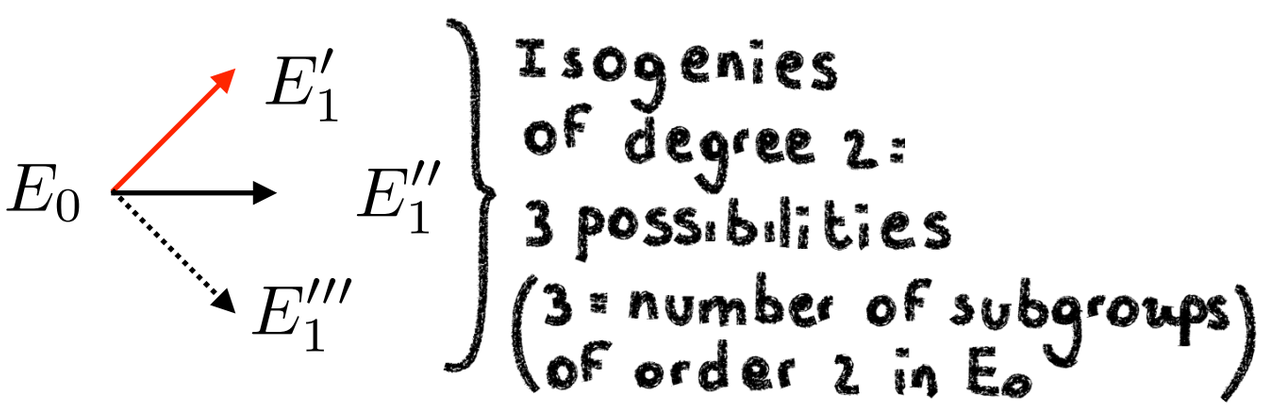

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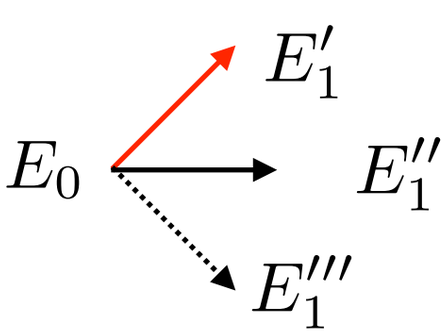

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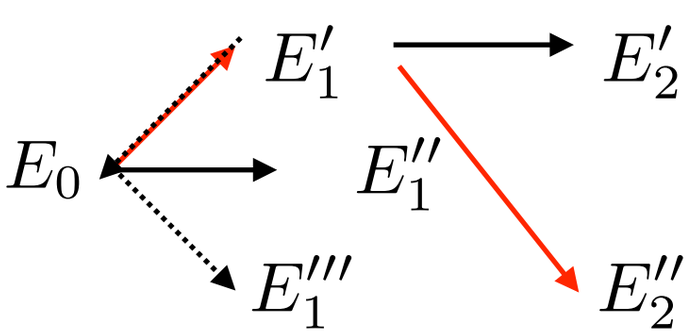

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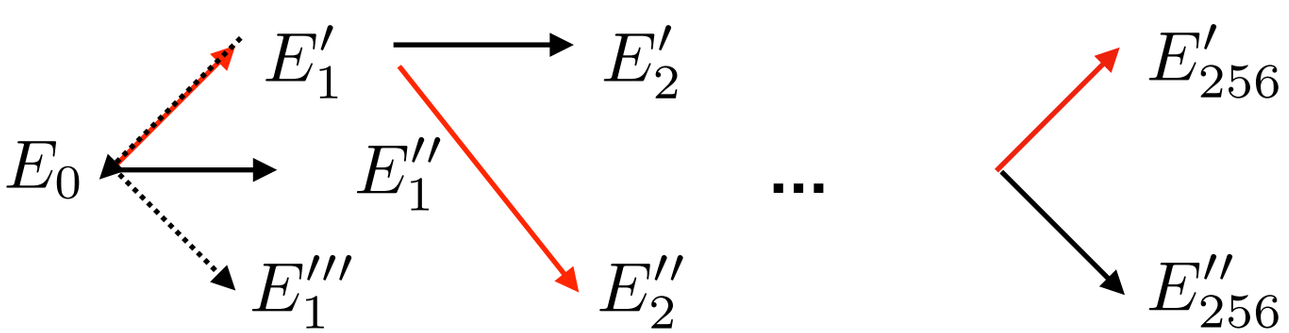

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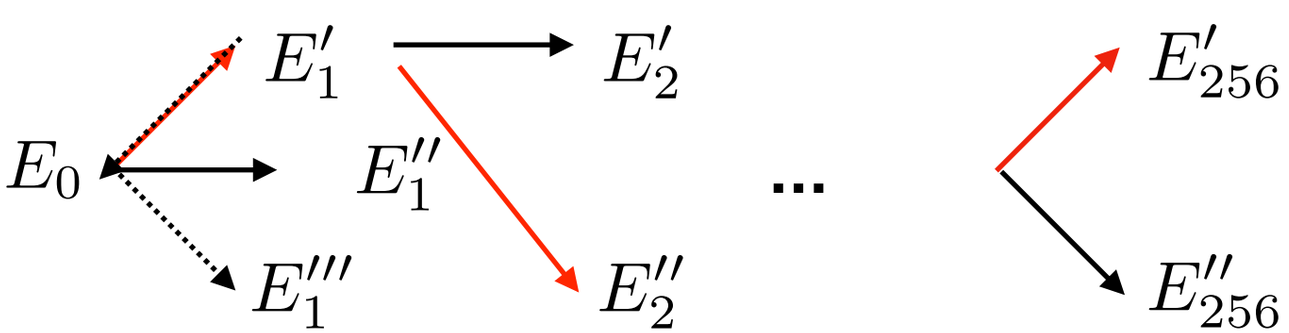

$$H(m) = j(E'_{256})$$

SIDH - Supersingular Isogeny Diffie-Hellman

Public Parameters

- two positive integers, e_2 and e_3, s.t. $2^{e_2} \simeq 3^{e_3}$
- a prime $p = 2^{e_2} 3^{e_3} f \pm 1$ and the finite field $\mathbb{F}_{p^2}$
- a supersingular elliptic curve E_0 over $\mathbb{F}_{p^2}$, s.t. $\#E_0(\mathbb{F}_{p^2}) = (2^{e_2} 3^{e_3} f)^2$
- P_2, Q_2 s.t. $E_0[2^{e_2}] =< P_2, Q_2 >$
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It always exists!

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SIDH - Supersingular Isogeny Diffie-Hellman

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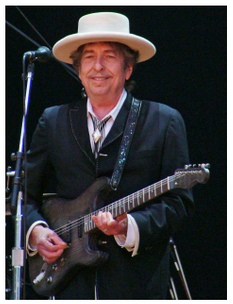

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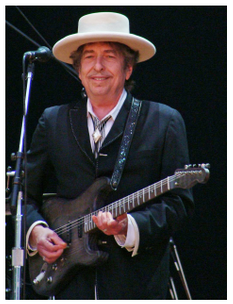

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- Computes $\phi_B : E_0 \to E_B = E_0/G_B$ and $\phi_B(P_3), \phi_B(Q_3)$

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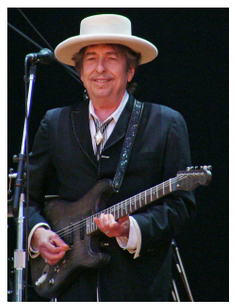

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The two curves obtained by Alice and Bob are isomorphic

THEY HAVE THE SAME *j*-INVARIANT!

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SIDH's security relies on a non standard isogeny problem:

Given a prime *p* and two supersingular isogenous curves,

E_1 **and** E_2 **over** $\mathbb{F}_{p^2}$**, compute an isogeny**

$$\varphi : E_1 \to E_2$$

of given degree $d = 2^{e_2} \simeq \sqrt{p}$

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No counterexamples so far, except for active attacks

The NIST submission: SIKE

3.17 SIKE

SIKE was the only candidate based on arithmetic properties of elliptic curves over finite fields. While quantum computers will break currently deployed elliptic curve cryptosystems, SIKE uses pseudo-random walks on supersingular isogeny graphs of curves, which are not known to be susceptible to quantum attacks. The nature of SIKE allows for a key exchange algorithm which is very similar to the classic Diffie-Hellman. The submission includes a CPA-secure encryption scheme which is converted to a CCA-secure KEM via a standard transformation.

SIKE has the smallest key sizes among all the remaining submissions, with public keys less than 750 bytes even for its level 5 security parameters. Another advantage of SIKE is that it can leverage existing optimized code for elliptic curve operations and can thus be easily combined with traditional elliptic curve cryptography to create a hybrid classical/post-quantum scheme. The SIKE scheme also benefits from much research into protecting elliptic curve operations from side-channel attacks.

The basic security problem upon which SIKE relies, finding isogenies between supersingular elliptic curves, has not been studied as much as some of the security problems associated with other submissions. Another drawback is that the performance of SIKE seems to be an order of magnitude slower than many of the other candidates.

The zero-knowledge proof

It is a $\sum$ - protocol $((P^1, P^2), V)$ which:

- allows to prove knowledge of the secret key sk_B, ϕ_B corresponding to a public key $E_B, \phi_B(P_3), \phi_B(Q_3)$

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Computes and sends $com = P^1(sk_B, E_B)$ $\longrightarrow$

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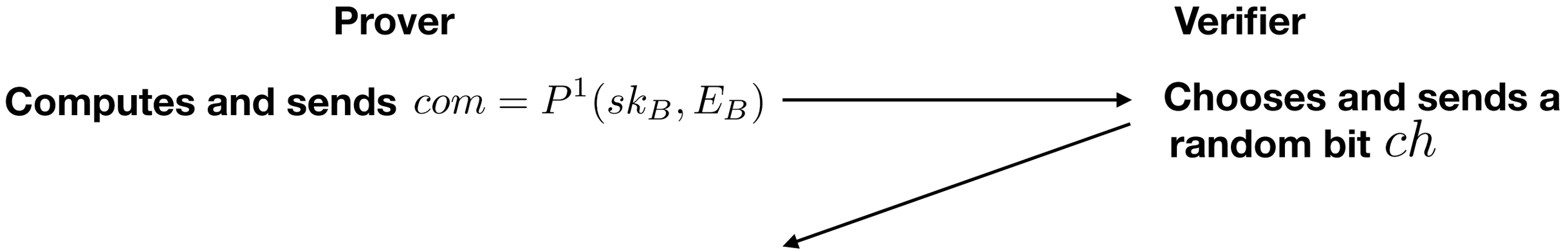

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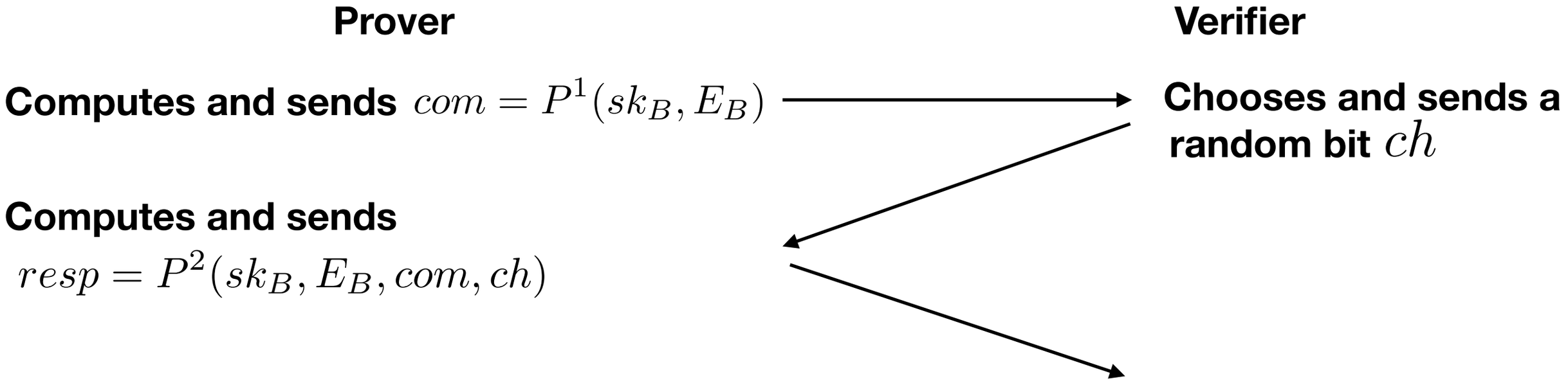

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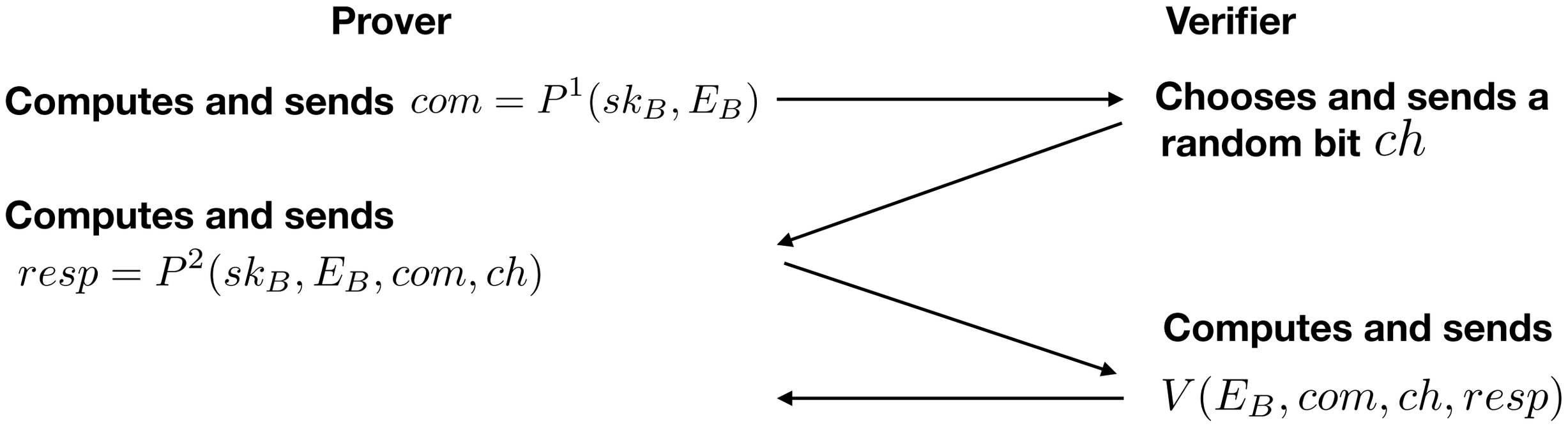

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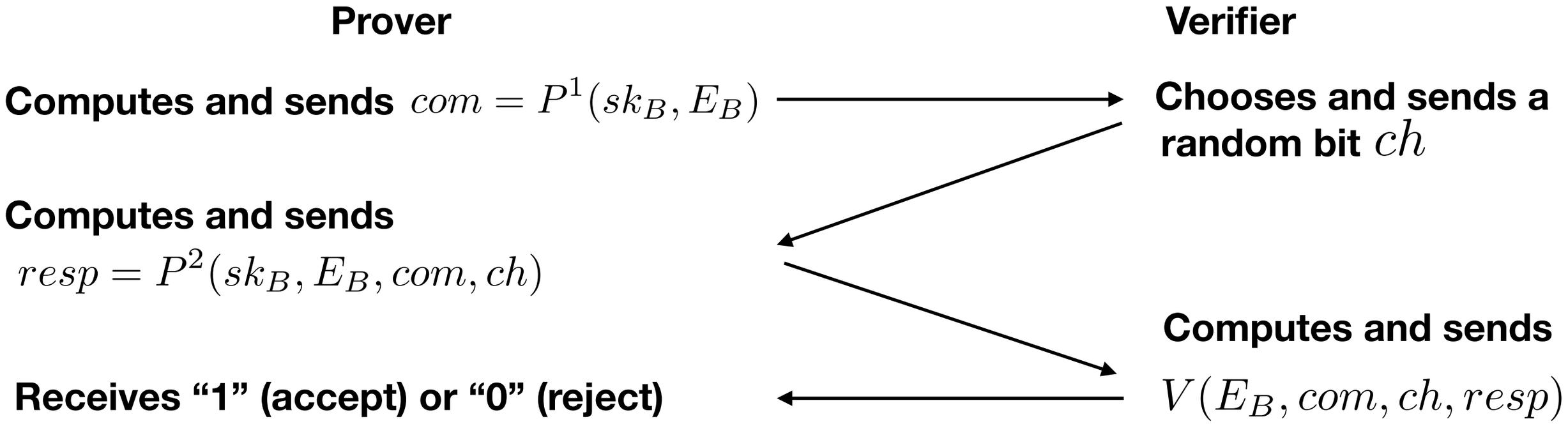

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2. Special-Soundness: if the prover convinces twice the verifier - on the same commitment but different challenges - then the prover must be the legitimate owner.

3. Honest-verifier Zero-Knowledge: the verifier cannot distinguish between a valid interaction and a simulated interaction.

From ZK proofs to Digital Signatures

A $\sum$-protocol can be turned into a non-interactive protocol.

Classically, this is done through the well-known Fiat-Shamir construction which exploits an hash function $H : \{0,1\}^* \to S_{ch}$ to compute the challenge as

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The Fiat-Shamir construction could be not safe in a quantum setting,
where it has been replaced by the Unruh's transform.

What is missing

- **Group key-exchange**
- **Efficient digital signature algorithm**
- **Aggregate multi-signature**
- **Attribute-based encryption**
- **Privacy-preserving digital signatures**

…advanced cryptographic schemes…

Open problems

- **Efficiency**

- **Security of non-standard problems**

- **Development of advanced primitives**

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$Ell_p(\mathbb{Z}[\sqrt{-p}])$ is the set of all supersingular curves E over $\mathbb{F}_p$ s.t.

$$End_p(E) \simeq \mathbb{Z}[\sqrt{-p}]$$

New directions in Isogeny-based Cryptography

The ideal class group $\mathcal{C}\ell(\mathbb{Z}[\sqrt{-p}])$ acts transitively on $Ell_p(\mathbb{Z}[\sqrt{-p}])$:

$$* : \mathcal{C}\ell(\mathbb{Z}[\sqrt{-p}]) \times Ell_p(\mathbb{Z}[\sqrt{-p}]) \to Ell_p(\mathbb{Z}[\sqrt{-p}])$$

$$([I], E) \qquad \mapsto \qquad I * E$$

Given $G_I = \{P \in E | \alpha(P) = \infty \ \ \forall \alpha \in I\}$, the action is defined as $I * E = E/G_I$.

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Given $G_I = \{P \in E | \alpha(P) = \infty \ \ \forall \alpha \in I\}$, the action is defined as $I * E = E/G_I$.

For some ideals, this action can be computed efficiently.

New directions in Isogeny-based Cryptography

The ideal class group $\mathcal{C}\ell(\mathbb{Z}[\sqrt{-p}])$ acts transitively on $Ell_p(\mathbb{Z}[\sqrt{-p}])$:

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Most of the ideals I should be factorised as

$$I_1^{e_1} I_2^{e_2} \cdots I_r^{e_r}$$

for suitable integers e_i.

New directions in Isogeny-based Cryptography

The class-group action induces a key-exchange *a la* Diffie-Hellman.

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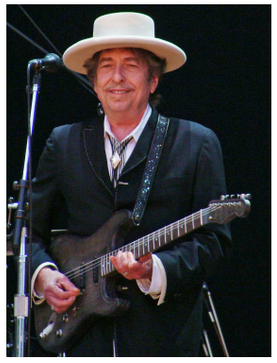

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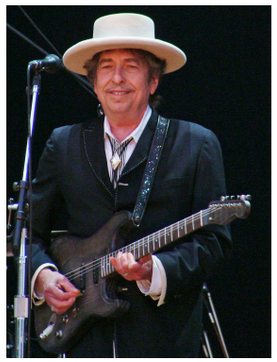

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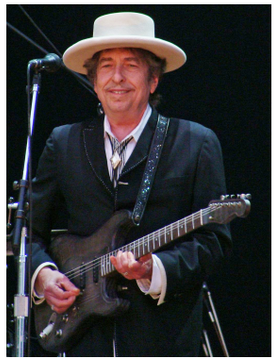

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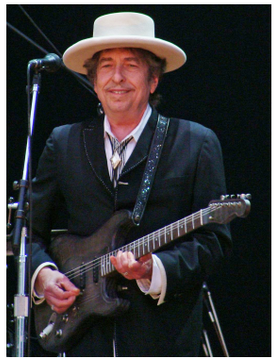

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Computes and sends $I_B * E_0$. ⟶

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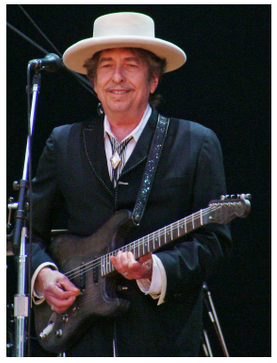

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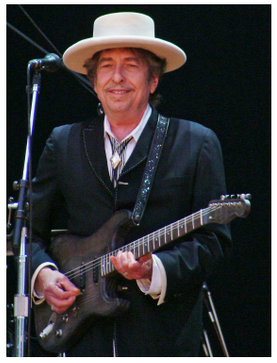

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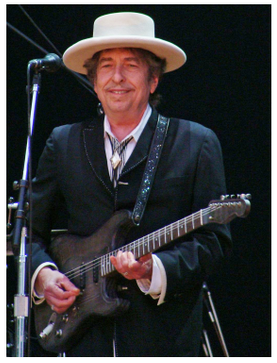

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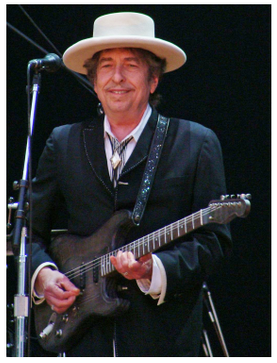

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Security: the best know attack has subexponential complexity.

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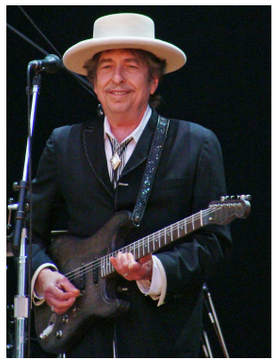

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Security: the best know attack has subexponential complexity.

Efficiency: slower than SIDH, the bottleneck is the isogenies computation.

Thanks for your attention

Questions?

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Trento, 11th February 2019