Symmetric-Key Encryption Schemes for Multi-Party Computation (MPC) Application

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Motivation: Research of New Designs

Motivated by progress in practical applications of

- secure multi-party computation (MPC)
- fully homomorphic encryption (FHE)
- zero-knowledge proofs (ZK)
- ...

where

- *primitives from symmetric cryptography instantiated in* $(\mathbb{F}_{2^n})^t$ *and/or* $(\mathbb{F}_p)^t$ *are needed*;
- *performance of symmetric-key algorithms influences the protocols efficiency.*

Multi-Party Computation (MPC)

Jointly evaluate a function on private inputs s.t. *no party can learn anything more than the output of the function*:

- *input:* parties P_i with (private) input x_i;
- *output:* jointly compute a (known) function $y = f(x_1, ..., x_n)$ s.t. *correctness* and *privacy* are guaranteed.

Roughly speaking:

$$f(x_1, ..., x_n) \text{ “}\equiv\text{” } \mathrm{Dec}\Big(f'(\mathrm{Enc}(x_1), ..., \mathrm{Enc}(x_n))\Big)$$

where $\mathrm{Enc}(x)$ “$\equiv$” $(E'_{pk}(k), E''_k(x))$.

Roughly Speaking:

- Linear/Affine functions: *almost free*
- Non-linear functions: *expensive*

MPC (joint evaluation of a function in individually known but globally secret inputs):

- shared data are (often) elements of a finite field $(\mathbb{F}_p)^t$ for **large** p (e.g., $p \approx 2^{64}, 2^{128}$);
- multiplications require communications between the parties $\Rightarrow$ *total number of multiplications is a good estimate of the complexity of an MPC protocol*;
- additions for free, but other metrics influence the cost (namely, number of offline & online communication rounds).

Linearly Sharing MPC Scheme: Cost Metrics

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“New” Schemes: Which Differences?

In “traditional” Ciphers/Hash Functions (e.g., AES, Keccak, ...), there is a good balance between the number of linear and non-linear operations (since they have approximately the same cost in Hardware/Software implementations).

In these new schemes:

- the number of non-linear operations is usually much smaller than the number of linear operations;
- the size of the S-Box does “not” influence the performance $\rightarrow$ “huge” S-Box (e.g., over $\mathbb{F}_{2^n}$ or $\mathbb{F}_p$ for $n \approx 128$ or $p \approx 2^{128}$);
- *simple algebraic representation*: “new” algebraic attacks become much more powerful than “traditional” statistical attacks.

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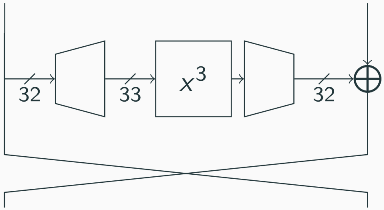
MiMC

An old design: KN cipher

Knudsen-Nyberg cipher [NK95]:

- 64-bit block cipher using Feistel mode of operation



- Broken with Interpolation Attack (algebraic) [JK97]
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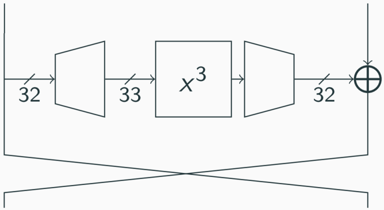

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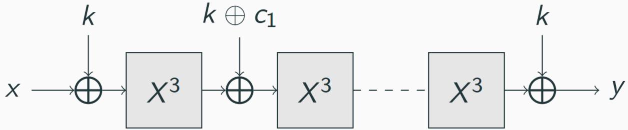
MiMC block cipher [AGR+16]: MiMC-n/n and MiMC-p/p



$(x \mapsto x^3$ is a permutation **iff** $n = 2n' + 1$ odd and $p \equiv_3 2)$

MiMC block cipher: Number of Rounds

Large number of rounds:

$$\lceil n \cdot \log_3 2 \rceil \approx 0.64 \cdot n \qquad \text{or} \qquad \lceil \log_3 p \rceil$$

(where $p \approx 2^n$)

E.g., for $p \approx 2^{128}$:

- AES: 10 rounds and ≈ 960 (MPC) multiplications (no look-up table in MPC!!!);
- MiMC: 81 rounds and 162 (MPC) multiplications.

(Remember: AES works over $(\mathbb{F}_{2^8})^{16}$ so conversion from/to $\mathbb{F}_p$ takes place!)

Interpolation Attack [JK97]

Goal: construct a polynomial corresponding to the encryption function without knowledge of the secret key. E.g., given plaintexts and ciphertexts (x_i, y_i), use Lagrange's Formula:

$$P(x) = \sum_{i=0}^{d} y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$$

Such polynomial can then be used for a forgery attack or/and a key-recovery attack.

If the degree is "maximum" (as in the case of a random permutation), then cost of the attack $\approx$ cost of brute force attack:

- the degree of 1-round MiMC is 3: hence, 3^r after r rounds;
- for a security level of $\log_2 p$ bits: $3^r \approx p$ implies $r \approx \log_3(p)$.

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Experimental Results – MiMC in MPC Applications

Table: Two-party performance of different PRFs in a Local Area Network (*LAN*) – "op(s)" $\equiv$ operation(s):

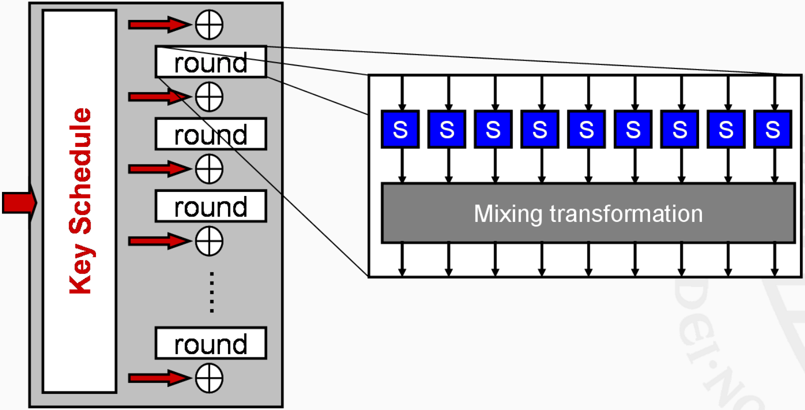
PRF	**Latency** (ms/op)	**Throughput** (ops/s)	**Preproc** (ops/ms)
AES [DR02]	7.713	530	5.097
LowMC [ARS+15]	4.302	591	2.562
MiMC	5.889	*6388*	33.575

where

- latency: the *best running time of a single cipher evaluation* (by running sequential single-threaded executions of it);
- throughput: the encryption rate given in the *number of field elements that can be encrypted in parallel per second* (by running multiple executions using different threads).

From SPN to Hades

SPN Ciphers


Key Schedule
round
round
round
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S
S
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S
Mixing transformation

Partial-SPN Ciphers

Move from a full S-Box layer

$$\mathcal{S} : x = [x_1 \| x_2 \| ... \| x_t] \in \mathbb{F}^t \to \mathcal{S}(x) = [S(x_1) \| S(x_2) \| ... \| S(x_t)]$$

to a Partial S-Box layer, e.g.

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Question:

can we guarantee security and at the same time reduce the total number of non-linear operations w.r.t. a SPN cipher?

Note: we do "not" care about the number of linear operations (which obviously increases by increasing the number of rounds!)

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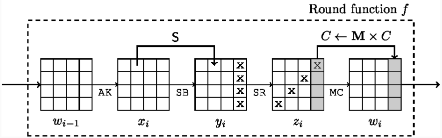
Zorro

Zorro [GGN+13] (proposed for Masking):

- 24-round AES: only 4 S-Boxes (in the first row) are applied in each round;



- Less S-Boxes than for AES: $24 \cdot 4 = 96 < 160 = 16 \cdot 10$;
- Broken by statistical attacks:
  (1) "wide-trail' design strategy [DR01] does not apply any-more: ad-hoc security argument by the designers;
  (2) using the same (AES) MixLayer in each round introduces weakness (in P-SPN)!

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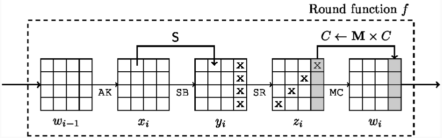

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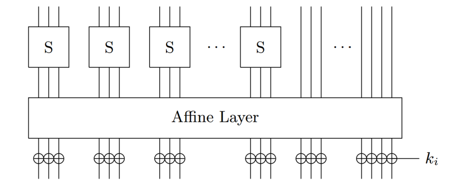

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  (1) proposed solution could be quite expensive, both computationally and memory-wise;
  (2) security analysis could become more complicated
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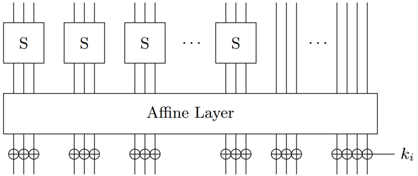

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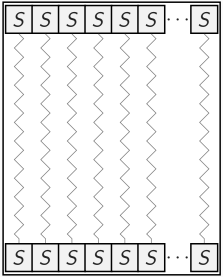
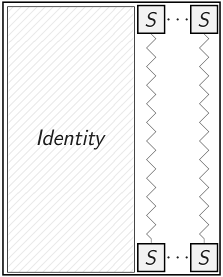
"Hades" Strategy

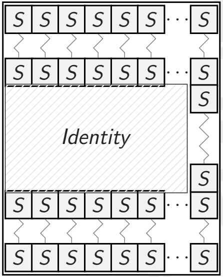
How to reduce number of non-linear operations & guarantee security with simple/elegant argument?



(a) SPN



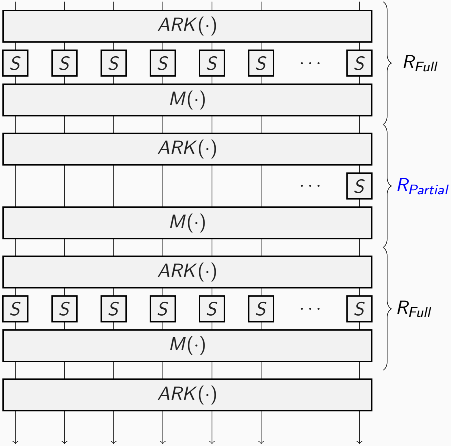
(b) P-SPN



(c) *"Hades" strategy*

HadesMiMC

From SHARK [RDP+96] to HadesMiMC



HadesMiMC defined over $(\mathbb{F}_p)^t$ (similar for $(\mathbb{F}_{2^n})^t$):

- Cube S-Box: $S(x) = x^3$ – invertible iff $\gcd(p-1, 3) = 1$;
- MixLayer: multiplication via MDS matrix (e.g., Cauchy matrix – assuming $t + 1 < p$);
- Affine key schedule: $k_i = M^i \cdot k + c_i$;
- *Efficient Implementation*: only for rounds with partial S-Box layer, MixLayer implemented via an equivalent matrix of the form

$$\begin{bmatrix} x_0 & y_1 & y_2 & \dots & y_{t-1} \\ z_1 & 1 & 0 & \dots & 0 \\ z_2 & 0 & 1 & \dots & 0 \\ \vdots & & & \ddots & \\ z_{t-1} & 0 & 0 & \dots & 1 \end{bmatrix}$$

Number of Rounds & Security Analysis

Number of rounds $R_F + R_P = 2 \cdot R_f + R_P$: depends both on p, t and on the security level:

- exploit rounds with full S-Box layer (together with Wide-Trail design strategy [DR01]) to guarantee security against statistical attacks;
- exploit rounds with partial S-Box layer in order to increase the degree;
- security against algebraic attacks (in particular, Grobner basis attacks) depend both on the rounds with full and partial S-Box layer!

Number of Rounds

Find the best ratio between R_F and R_P that guarantees security and minimizes the metric cost!

Text Size $\log_2 p \times t$	**Security** κ	**Word Size** ($\log_2 p$)	**# Words** (t)	**Rounds** $\mathrm{R_F}$ (Full S-Box)	**Rounds** $\mathrm{R_P}$ (Partial S-Box)
128	128	8	16	10	4
128	128	16	8	8	10
256	128	128	2	6	71
256	256	128	2	12	76
1 024	128	128	8	6	71
1 024	1 024	128	8	16	72
1 024	1 024	128	8	14	79

Experimental Results – MPC

Two-party performance of CTR-MiMC [AGR+16], HadesMiMC and Rescue [AAB+19] over a *LAN* over $t = 2, 4$ and 32 blocks (*total size* $\approx 128 \times t$ bits):

	Online Cost			**(Entire) Runtime**	
	Latency ($ms/\mathbb{F}_p$)	*Throughput* ($\mathbb{F}_p/s$)	*Communication* per $\mathbb{F}_p$	*Throughput* ($\mathbb{F}_p/s$)	*Communication* per $\mathbb{F}_p$
HadesMiMC_2	3.85	**117 358**	**1.90**	**261**	**266**
MiMC_2	**3.53**	79 728	3.50	192	366
$\textit{Rescue}_2$	5.54	23 464	6.10	70	971
HadesMiMC_4	1.90	**185 160**	**1.14**	**526**	**133.2**
MiMC_4	1.69	83 876	3.50	192	366
$\textit{Rescue}_4$	**1.25**	46 890	3.08	136	485
HadesMiMC_{32}	**0.32**	**258 610**	**0.39**	**1 098**	**60.8**
MiMC_{32}	0.34	87 831	3.5	192	366
$\textit{Rescue}_{32}$	0.42	57 695	1.93	274	243

(GMiMC_{erf} [AGP+19] broken – *Rescue* has largest security margin!)

Key-Recovery Attack on Full MiMC-n/n

Given a function $F : \mathbb{F}_{2^N} \to \mathbb{F}_{2^N}$

$$F(x) = \phi_0 \oplus \bigoplus_{i=1}^{d} \phi_i \cdot x^i \qquad \text{(where } \phi_d \neq 0\text{)},$$

it admits an equivalent representation over $\mathbb{F}_2^N$, namely $F \equiv (F_0, ..., F_{N-1})$ where $F_i : \mathbb{F}_2^N \to \mathbb{F}_2$:

$$F_i(x_0, x_1, ..., x_{N-1}) = \bigoplus_{u=(u_0,...,u_{N-1})\in\mathbb{F}_2^N} \varphi(u) \cdot x_0^{u_0} \cdot ... \cdot x_{N-1}^{u_{N-1}}$$

In the following:

- $d \equiv$ degree of F over $\mathbb{F}_{2^N}$
- $\delta \equiv$ *algebraic* degree of F over $\mathbb{F}_2^N$

where $\delta(F) = \max_{0\leq i\leq 2^N-1}\{hw(i)\,|\,\phi_i \neq 0\}$.

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Higher-Order Differential Attack

Given a a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$ under a fixed secret key k, **higher-order differential cryptanalysis** [Knu94] exploits the fact that

for any vector subspace $V \subseteq \mathbb{F}_2^n$ with dimension greater than the algebraic degree of E_k:

$$\dim(V) \geq \deg(E_k) + 1$$

and for any (fixed) element $v \in \mathbb{F}_2^n$:

$$\bigoplus_{x \in V \oplus v} x = \bigoplus_{x \in V \oplus v} E_k(x) = 0.$$

Problem: estimate the algebraic degree of $E_k(\cdot)$!

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Trivial Estimation of the Growth of the Degree

The degree of the composition of two functions $F \circ G(\cdot)$ is always upper bounded by

$$\deg(G \circ F(\cdot)) \leq \deg(F) \cdot \deg(G).$$

Given a SPN cipher $(\mathbb{F}_{2^n})^t \to (\mathbb{F}_{2^n})^t$ with round functions defined as

$$R(\cdot) = k \oplus M \circ [\underbrace{S\|...\|S\|S}_{\equiv t \text{ S-Boxes}}](\cdot)$$

where $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$ has algebraic degree $\delta \geq 2$, then the degree of $E_k(\cdot)$ after R rounds is *upper bounded* by δ^R. Thus, **at least**

$$\log_\delta(n \cdot t - 1) \equiv \log_\delta(N - 1) \textbf{ rounds}$$

are **necessary** to reach maximum degree.

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Estimation from [BCD11]

Theorem (C. Boura, A. Canteaut, C. De Cannière – FSE'11)
Let F be a function from $\mathbb{F}_2^N$ to $\mathbb{F}_2^N$ corresponding to the concatenation of t smaller S-Boxes $S_1, \ldots, S_t$ defined over $\mathbb{F}_2^n$. Then, for any function G from $\mathbb{F}_2^N$ to $\mathbb{F}_2^N$, we have

$$\deg(G \circ F(\cdot)) \leq \min\left\{\deg(F) \cdot \deg(G), N - \frac{N - \deg(G)}{\gamma}\right\},$$

where

$$\gamma = \max_{i=1,\ldots,n-1} \frac{n-i}{n-\delta_i} \leq n-2$$

where δ_i is the maximum degree of the product of any i coordinates of any of the smaller S-Boxes

Comparison btw [BCD11] and Trivial Estimation

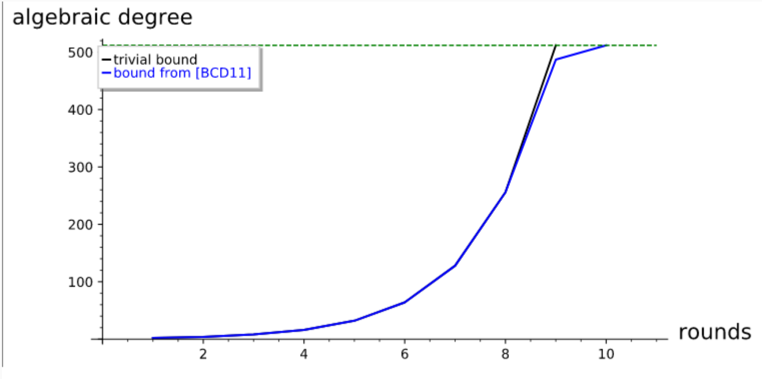

Figure: Different *upper bounds* of the growth of the algebraic degree of a typical SPN cipher (with cubic S-Box) over $(\mathbb{F}_{2^{19}})^{27}$

Theorem ([EGL+20])
Consider an iterated Even-Mansour cipher $EM_k^r(\cdot) : \mathbb{F}_{2^N} \to \mathbb{F}_{2^N}$

$$EM_k^r(\cdot) := k^r \oplus (...R(k^1 \oplus R(k^0 \oplus \cdot))...)$$

of $r \geq 1$ *rounds, where* $R(\cdot)$ *is a polynomial of degree* $d \geq 3$*:*

$$R(x) = \rho_0 \oplus \bigoplus_{i=1}^{d} \rho_i \cdot x^i \qquad \text{(where } \rho_d \neq 0\text{).}$$

The algebraic degree (= degree over $\mathbb{F}_2^N$*) after* r *rounds is upper bounded by*

$$\lfloor \log_2(d^r + 1) \rfloor.$$

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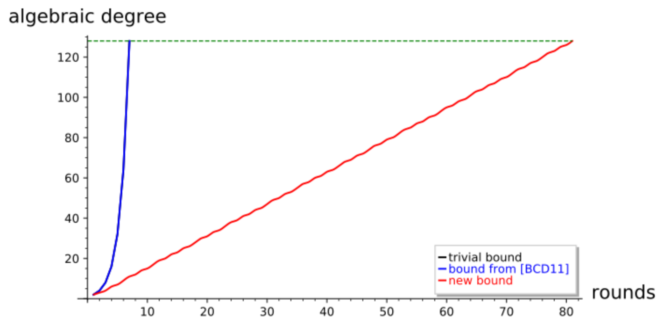
of $r \geq 1$ rounds, where – as before – $R(\cdot)$ is a polynomial of degree $d \geq 3$.

The minimum number of rounds **necessary** to prevent a (secret-key) high-order differential distinguisher is given by

$$\left\lceil \log_d \left(2^{N-1} - 1\right) \right\rceil \approx (N-1) \cdot \log_d(2).$$

Concrete Results on MiMC-N/N



First secret-key zero-sum distinguisher for $\lceil \log_3(2^{N-1} - 1) \rceil$ rounds (out of $\lceil N \cdot \log_3(2) \rceil$):

- **security margin: 1 or 2 rounds (depending on N)**

Theoretical & Practical Results for MiMC

n	$\mathcal{R}$ (our estimation)	$\mathcal{R}^{[BCD11]}$	Practical $\mathcal{R}$
5	3	3	4
7	4	3	5
9	6	4	6
11	7	4	7
13	8	4	9
15	9	4	10
17	11	5	11
33	21	6	21
65	41	7	-
129	81	8	-
257	162	9	-

$R \equiv$ necessary number of rounds to prevent zero-sum .

Theorem ([BC13])
Let f be a permutation over $\mathbb{F}_2^N$. Then, $\deg(f^{-1}) = N - 1$ *if and only if* $\deg(f) = N - 1$.

Chosen-Ciphertext Key-Recovery Attack:

$$\text{plaintexts} \xrightarrow[\text{Key-Recovery}]{R(\cdot) \text{ or } R^2(\cdot)} \textit{zero-sum} \xleftarrow[\text{Distinguisher}]{R^{-r}(\cdot)} \text{ciphertexts}$$

- set up a system of (low-degree) algebraic equations for the first 1/2 round(s);
- solve them to find the key.

Total cost of the attack: 2^{n-1} chosen ciphertexts & $\approx 2^{n-\log_2(n)+1}$ encryptions.

Theorem ([BC13])
Let f be a permutation over $\mathbb{F}_2^N$. Then, $\deg(f^{-1}) = N - 1$ if and only if $\deg(f) = N - 1$.

Chosen-Ciphertext Key-Recovery Attack:

$$\text{plaintexts} \xrightarrow[\text{Key-Recovery}]{R(\cdot) \text{ or } R^2(\cdot)} \textit{zero-sum} \xleftarrow[\text{Distinguisher}]{R^{-r}(\cdot)} \text{ciphertexts}$$

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Key-Recovery Attack: New Number of Rounds

In order to provide security, new number of rounds for MiMC over $\mathbb{F}_{2^n}$:

$$\lceil n \cdot \log_3(2) \rceil + \underbrace{\lceil \log_3(2n \cdot \log_3(2)) \rceil}_{\text{new term!}}$$

(e.g., for $n = 129$: 5 more rounds – from 82 to 87).

- No change for the prime case! (the previous attack works only over a binary field)
- Cryptanalysis is never finished: We can only guarantee security against **KNOWN** attacks!!! It is always possible that new attacks are discovered and a scheme (including AES & SHA-3) is broken!!

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Open Problems

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As every new construction, more cryptanalysis is necessary:

- improve attacks based on higher-order differentials over $\mathbb{F}_{2^n}$: is it possible to estimate the growth of the degree for generic SPN/Feistel schemes with big S-Boxes?
- what about other attacks that work better/differently over $\mathbb{F}_p$ than over $\mathbb{F}_{2^n}$? How does the value of p influence the possibility to set up an attack (e.g., is there any attack that performs better for $p \approx 2^n \pm \varepsilon$ or not)?

Is it possible to design a scheme with better performances w.r.t. the current ones present in the literature?

Thanks for your attention!

Questions?

Comments?

Proof (1/2)

Let $\mathfrak{D}_r \equiv \mathfrak{D}$ be the degree of $EM_k^r(\cdot) = \bigoplus_{i=0}^{\mathfrak{D}} \varepsilon_i \cdot x^i$ after r rounds. Given a subspace $\mathcal{V} \subseteq \mathbb{F}_{2^N}$ of dimension $N-1$, then

$$\bigoplus_{x \in \mathcal{V} \oplus v} E_k(x) = \bigoplus_{x \in \mathcal{V} \oplus v} \left(\bigoplus_{i=0}^{\mathfrak{D}} \varepsilon_i \cdot x^i \right) = \bigoplus_{i=0}^{\mathfrak{D}} \varepsilon_i \left(\bigoplus_{x \in \mathcal{V} \oplus v} x^i \right) = 0$$

if $deg(x \mapsto x^i) \equiv hw(i) \leq N-2$ for each $i = 0, ..., d$.

Necessary condition to prevent a (secret-key) high-order differential distinguisher:

$E_k(\cdot)$ must contain at least one monomial x^i with $hw(i) \geq N-1$.

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Proof (2/2)

Since

- the smallest i s.t. $hw(i) \geq N - 1$ is $i = 2^{N-1} - 1$
- the degree of $EM_k^r(\cdot)$ is upper bounded by $\mathfrak{D}_r \leq d^r$

it follows that the minimum number of rounds $\mathcal{R}$ to prevent such attack must satisfy

$$d^{\mathcal{R}} \geq 2^{N-1} - 1 \quad \Longrightarrow \quad \mathcal{R} \geq \log_d(2^{N-1} - 1).$$

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