# Developing Innovative Frameworks for Efficient Code-based Signatures

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# IN THIS TALK

- Introduction
- Traditional Approach
- Zero-Knowledge Protocols
- New Frameworks
- Conclusions

# Part I

## Introduction

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Main areas of research:

- Lattice-based cryptography.
- Hash-based cryptography.
- Code-based cryptography (McEliece, Niederreiter).
- Multivariate cryptography.
- Isogeny-based cryptography.

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Can we fix this?

### $[n, k]$ linear code over $\mathbb{F}_q$

A subspace of dimension $k$ of $\mathbb{F}_q^n$. Value $n$ is called length.

# Error-Correcting Codes

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## Hamming metric

$wt(x) = |\{i : x_i \neq 0, 1 \leq i \leq n\}|$, $d(x, y) = wt(x - y)$.

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$w$-error correcting: $\exists$ algorithm that corrects up to $w$ errors.

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Very well-studied, solid security understanding (ISD).

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Choose a code family with efficient decoding algorithm associated to description $\Delta$ and hide the structure.

# Part II

## Traditional Approach

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For code-based, trapdoor is decoding: CFS scheme.
(Courtois, Finiasz, Sendrier, 2001)

# The CFS Scheme

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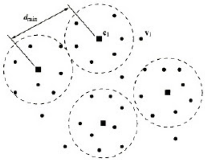
## Verify

- Compute $y' = H\sigma^T$.
- Accept if $y' = \mathbf{H}(msg)$, otherwise reject.

# ABOUT CFS

Decoding a random word is not always possible!



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CFS parameters:

| $q$ | $m$ | $n$ | $w$ | PK (kB) | Sig (bits) | Security |
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Also, signing is very slow: in the order of seconds.

# Considerations

Additional security concerns: very high rate leads to distinguishers.

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Wave parameters:

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| 3 | 8492 | 3558 | 2047 | 7980 | 3.2 | 1.6 | 128 |

# Part III

## Zero-Knowledge Protocols

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Use random codes and exploit hardness of finding low-weight words.
(Stern, 1993)

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### PROVER | VERIFIER

Choose $y \in \mathbb{F}_2^n$ and permutation $\pi$.
Set $c_1 = \mathbf{H}(\pi, Hy^T), c_2 = \mathbf{H}(\pi(y))$
$c_3 = \mathbf{H}(\pi(y + e))$

$$\xrightarrow{c_1, c_2, c_3}$$

$$\xleftarrow{b}$$ Select random $b \in \{0, 1, 2\}$.

If $b = 0$ set $rsp = (y, \pi)$ — Verify $c_1, c_2$.

If $b = 1$ set $rsp = (y + e, \pi)$ $\xrightarrow{rsp}$ — Verify $c_1, c_3$.

If $b = 2$ set $rsp = (\pi(y), \pi(e))$ — Verify $c_2, c_3$ and $wt(\pi(e)) = w$.

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For instance: choose random $y$ and $\pi$, then $x \in \mathbb{F}_2^n$ with $Hx^T = Hy^T + s$. Build $c_1$ and $c_2$ normally and $c_3 = \mathbf{H}(\pi(x))$ . Then $rsp = (y, \pi)$ and $rsp = (x, \pi)$ pass verification for $b = 0$ and $b = 1$ (strategy fails for $b = 2$). Similarly for other combinations.

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Stern's ZKID parameters:

| $q$ | $n$ | $w$ | $\tau$ | PK (bits) | Sig (kB) | Security | Auth. |
|---|---|---|---|---|---|---|---|
| 2 | 512 | 56 | 35 | 256 | 5 | 60 | 20 |
| 2 | 620 | 68 | 137 | 310 | 93.3 | 80 | 80 |
| 2 | 1024 | 112 | 219 | 512 | 245 | 128 | 128 |

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### Key Generation

- Choose random $q$-ary code $\mathcal{C}$, given by parity-check matrix $H$.
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## Prover / Verifier

| Prover | | Verifier |
|---|---|---|
| Choose $y \in \mathbb{F}_q^n$ and monomial $\mu$. Set $c_1 = \mathbf{H}(\mu, Hy^T)$, $c_2 = \mathbf{H}(\mu(y), \mu(e))$ | $\xrightarrow{c_1, c_2}$ | |
| | $\xleftarrow{c}$ | Select random $c \in \mathbb{F}_q^*$. |
| $z = \mu(y + ce)$ | $\xrightarrow{z}$ | |
| | $\xleftarrow{b}$ | Select random $b \in \{0, 1\}$. |
| If $b = 0$ set $rsp = \mu$ | | Verify $c_1 = \mathbf{H}(\mu, H\mu^{-1}(z))^T - cs)$. |
| If $b = 1$ set $rsp = \mu(e)$ | $\xrightarrow{rsp}$ | Verify $c_2 = \mathbf{H}(z - c\mu(e), \mu(e))$ and $wt(\mu(e)) = w$. |

# Part IV

## New Frameworks

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"Opening" of a subset does not compromise security...

...but allows for much larger challenge space.

# GPS Protocol

KeyGen: as in CVE, using a commitment scheme **Com**.

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## Helper

- Generate random $y, \tilde{e} \in \mathbb{F}_q^n$, with $\tilde{e}$ of weight $w$, from seed.
- Compute $aux = \{\mathbf{Com}(y + c\tilde{e})\}_{c \in \mathbb{F}_q}$.
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| Determine $\mu$ s.t. $e = \mu(\tilde{e})$ | | |
| $\alpha = \mathbf{Com}(\mu, H(\mu(y))^T)$ | $\xrightarrow{\alpha}$ | |
| | $\xleftarrow{c}$ | Select random $c \in \mathbb{F}_q$. |
| $z = y + c\tilde{e}$ | $\xrightarrow{z}$ | |
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Here the soundness error is $1/q$.

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GPS scheme parameters ($\lambda = 128$, sizes in kB):

| $M$ | $\tau$ | $q$ | $n$ | $k$ | $w$ | PK | Sig |
|---|---|---|---|---|---|---|---|
| 512 | 23 | 128 | 220 | 101 | 90 | 0.10 | 27.06 |
| 1024 | 19 | 256 | 207 | 93 | 90 | 0.11 | 23.98 |
| 2048 | 16 | 512 | 196 | 92 | 84 | 0.11 | 21.22 |
| 4096 | 14 | 1024 | 187 | 90 | 80 | 0.12 | 19.76 |

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FJR scheme parameters ($\lambda = 128$, sizes in kB):

| $M$ | $\tau$ | $q$ | $n$ | $k$ | $w$ | PK | Sig |
|---|---|---|---|---|---|---|---|
| 389 | 28 | 2 | 1280 | 640 | 132 | 0.96 | 16.34 |

# Proving Hamming Weight via Polynomials

Observation: if $H = (H'|I_{n-k})$ write $e = (e_A, e_B)$, so $s = H(e_A, e_B)^T$.
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This transforms SDP into a polynomial problem and completely avoids the need for an isometry.

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Create some shares $e_A = \sum_{j=1}^{M} e_A^{(j)}$ and hence $e = \sum_{j=1}^{M} e^{(j)}$.

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This is done directly on shares $Q^{(j)}(r_l)$, $S^{(j)}(r_l)$ and $(P \cdot F)^{(j)}(r_l)$, via standard MPC techniques to verify multiplication triple.

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| 256 | 17 | 2 | 1280 | 640 | 132 | $2^{11}$ | $2^{22}$ | 0.96 | 11.2 |
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$q$-ary parameters can be refined, leading to improved performance (e.g. Sig $\approx$ 7 kB).

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| $M$ | $\tau$ | $q$ | $n$ | $k$ | $w$ | $\mathbb{F}_{\text{poly}}$ | $\mathbb{F}_{\text{points}}$ | PK | Sig |
|---|---|---|---|---|---|---|---|---|---|
| 256 | 17 | 2 | 1280 | 640 | 132 | $2^{11}$ | $2^{22}$ | 0.96 | 11.2 |
| 256 | 17 | $2^8$ | 256 | 128 | 80 | $2^8$ | $2^{24}$ | 0.15 | 8.5 |

Possible improvement using linear complexity to avoid interpolation.
(P., Randrianarisoa, 2022)

*q*-ary parameters can be refined, leading to improved performance (e.g. Sig $\approx$ 7 kB).

Optimized implementation underway, NIST submission on the horizon.

# Part V

## Conclusions

Finally, viable solutions for code-based signatures begin to appear.

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Investigate applicability to several advanced frameworks (e.g. ring sigs, identity-based sigs, threshold sigs, multi-sigs...)

Grazie per l'attenzione!