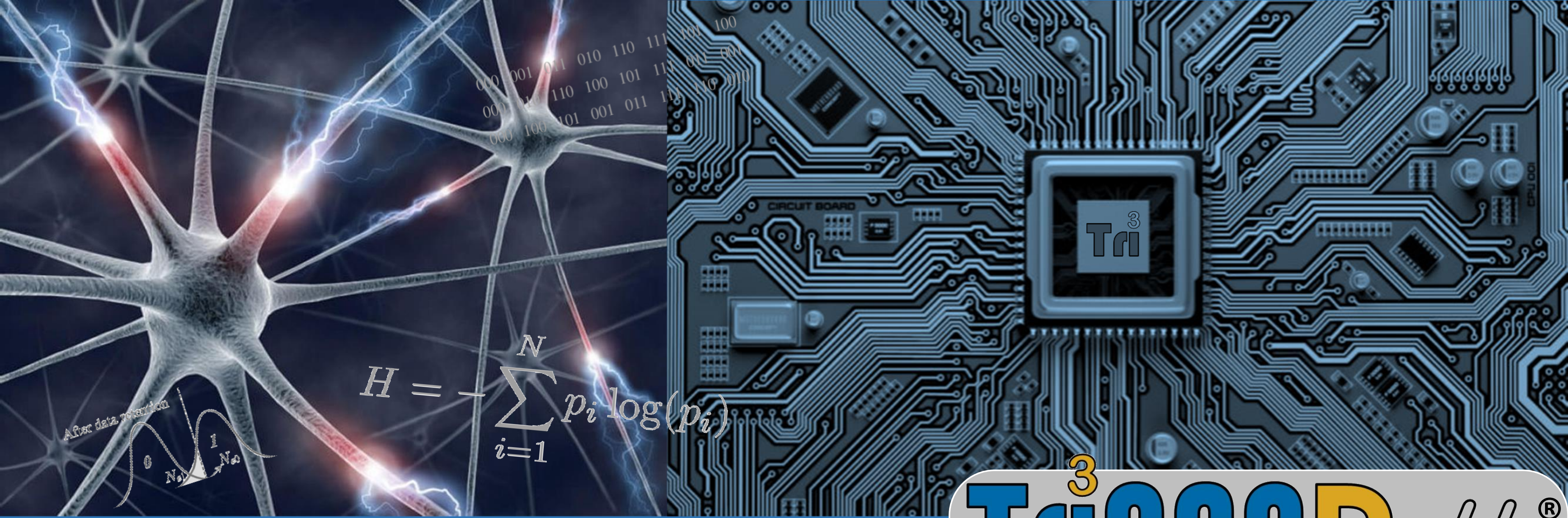


LDPC codes for storage applications

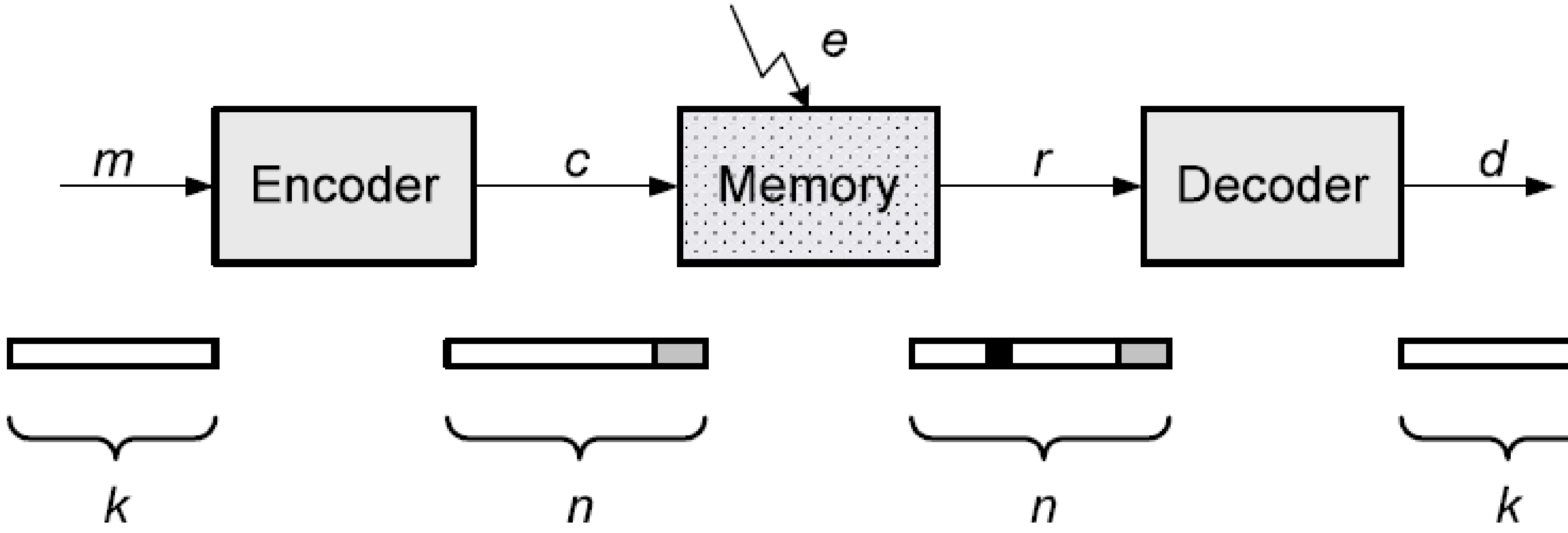
Valentina De Cian -- TriNANDable

November, 2021

Introduction

- LDPC (Low-Density Parity-Check) codes are a class of linear block codes, characterized by a very sparse parity-check matrix.
- LDPC codes were invented in 1960 by R. G. Gallager, then they were almost ignored for 35 years (except for the fundamental introduction of Tanner graphs in 1981). In 1990s they were independently discovered by MacKay, Luby and others.
- Nowadays LDPC codes are widely used in telecommunication standards (https://www.ldpc-decoder.com/) and storage.

ECC role in storage applications

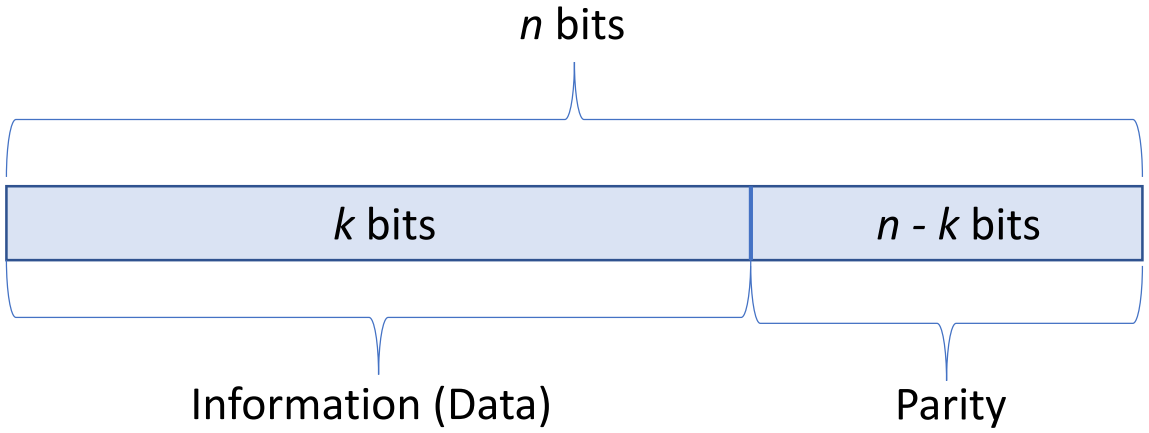


m : *message*
c : *encoded message* (*codeword*)
e : error
r : *read codeword*
d : *decoded codeword* (hopefully equal to *message*)

Space requirement

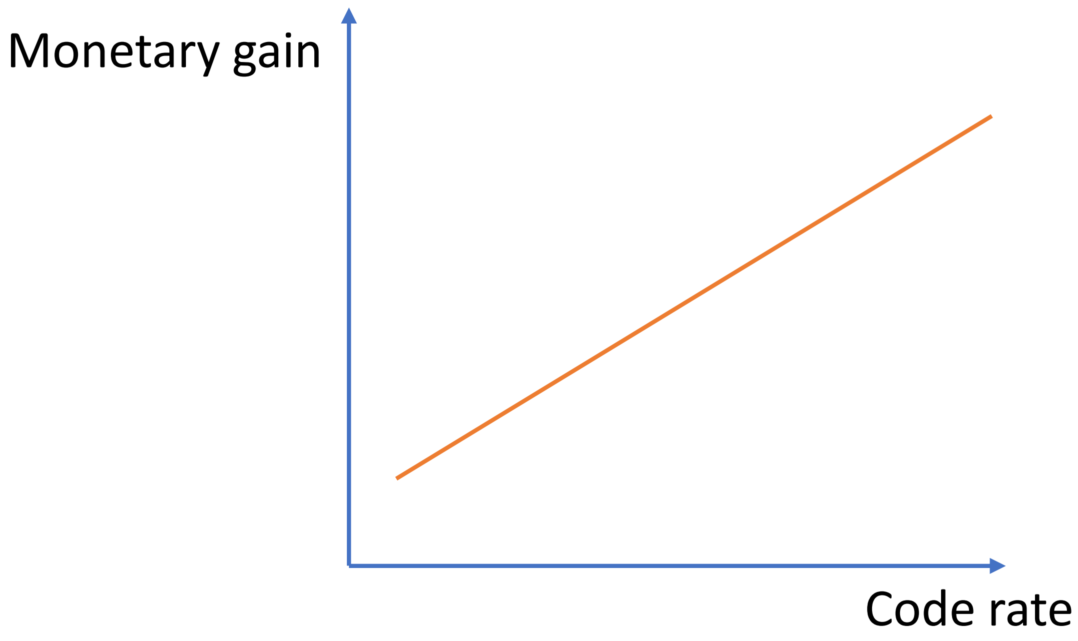
Little space is required for two main reasons:

- ECC must be implemented in the controller
- Parity is stored inside the NANDs → we pay it!



Code Rate: a trade-off between performance and monetary gain

$$\text{Code Rate } (CR) = \frac{\text{Data bits}}{\text{Data bits + Parity bits}} = \frac{k}{n}$$



The lower the code rate, the better the code performance... but the lower the gain!

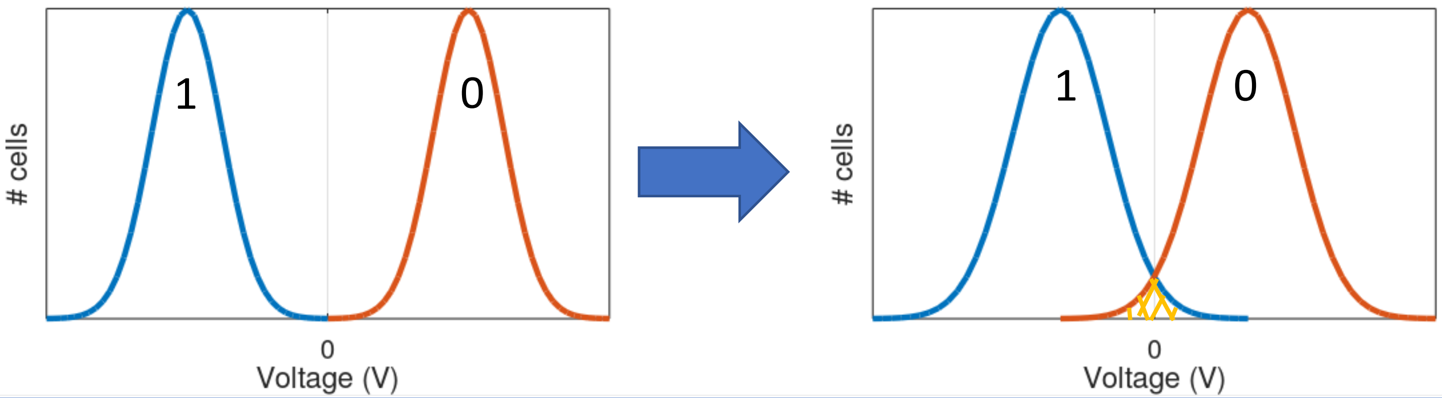
Channel model

The activity of writing and subsequent reading of data in an SSD can be modelled as transmitting data through a BI-AWGN channel, but with the addition of some specific functionality of the storage application.

Thus we have to remember the following.

- We cannot ask for retransmission.
- There is no erasure (we can only read 0 or 1).
- The SSD is subjected to degradation over time:

Initial condition



The condition after some time of use. The yellow area represents the overlapping of the two distributions.

Shannon limit

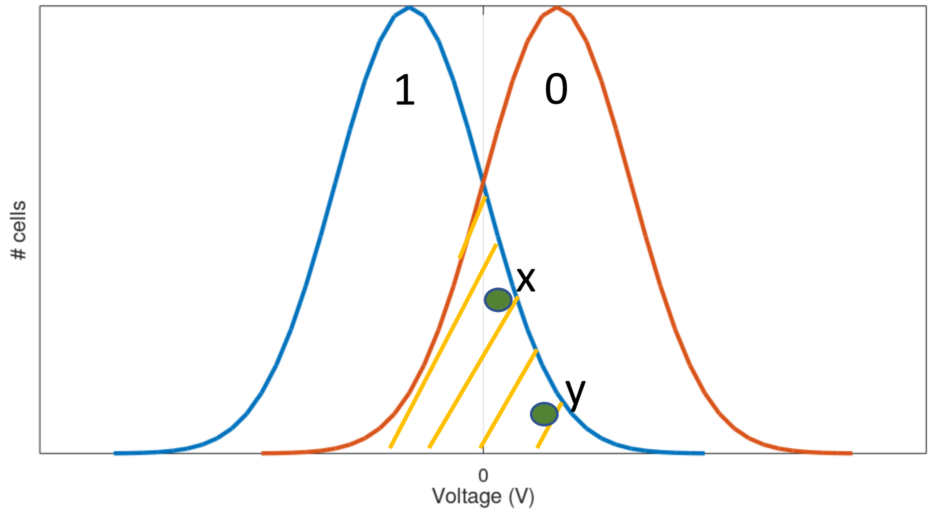
A channel can be characterized by a parameter C, called the *channel capacity*, which is a measure of how much information the channel can convey.

Theorem (Shannon). Codes exist that provide arbitrarily reliable communication provided that the code rate CR satisfies:

$$CR < C.$$

Conversely, if $CR > C$, there exists no code that provides reliable communication.

Hard decision and soft decision



Hard decision:
all positive values are decoded as 0, all negative values are decoded as 1.
If a value lies in the dashed yellow area, we can have an error when reading.
In our figure, *x* and *y* are both decoded as 0.

Soft decision:
Knowing the exact position of the bit, we can exploit some additional information, called soft information. It is expressed by the log likelihood ratio (LLR): $log\,\frac{P(input = 0\,|\,read)}{P(input = 1\,|\,read)}$.
In practice, this method tells us *how much* a bit is in error.
It is more accurate than hard decision!

Degradation over time

In order to withstand performance degradation over time we can choose one of the following methods.

- Store several matrices with various CRs and choose the most suitable, decreasing the CR over time (but this needs a lot of space!)
- Increase the number of soft bits over time (but we have seen that it is very expensive in terms of readings!)
- Store only one matrix and vary the CR through shortening and puncturing over time

Other codes, such as BCH codes, do not allow the latter strategy, as puncturing is much more complicated.

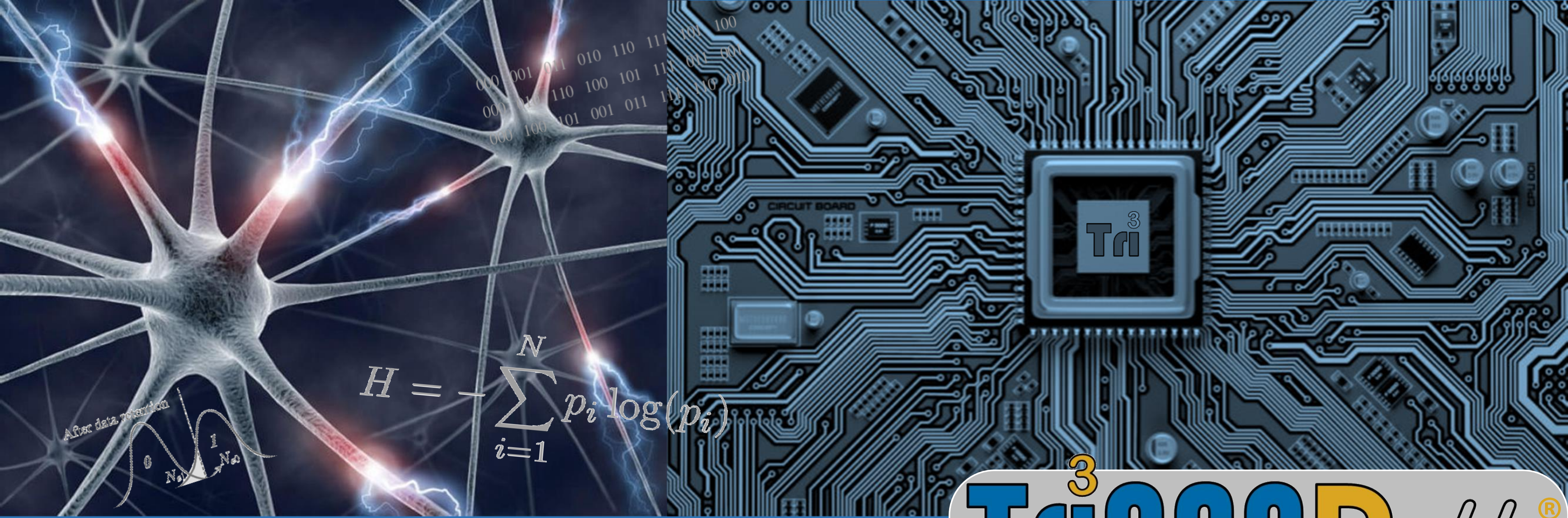
Conclusions: pros and cons of LDPC codes for storage

- Performance close to Shannon limit (especially with soft decoding)
- QC LDPC for saving space and changing CR easily

- Unavoidable error floor
- There is no closed form describing the performance curve (it takes tons of simulations)

Bibliography

- Error floor of LDPC codes; *Richardson*; 2003
- Quasi-Cyclic Low-Density Parity-Check Codes from Circulant Permutation Matrices; *Marc P.C. Fossorier*; 2004
- A Method of Counting the Number of Cycles in LDPC Codes; *Jun Fan, Yang Xiao*; 2006
- Channel Codes, Classical and Modern; *William E. Ryan, Shu Lin*; 2009
- Inside NAND Flash Memories; *Rino Micheloni, Luca Crippa, Alessia Marelli*; 2010
- 3D Flash memories; *Rino Micheloni & al.*, 2016



TriNANDable®

Thank you for your attention!