DISMATH
Discrete Mathematics
Logic, Sets, and Functions

De La Salle University

September 2014

Overview

- Introduction to Logic

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- Logical Operators

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- Proposition/Logical Equivalences

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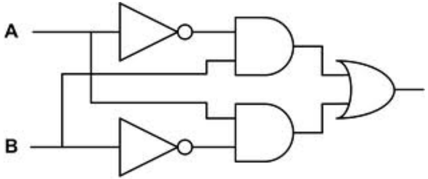
- Functions

Introduction to Logic

1. SOFTWARE:
Understand logic operators in computer programming (`!p,p&&q,p||q,p?q:true`)

2. HARDWARE:

Design digital logic circuits



A xor B = A'B + AB'

Propositional Logic

PROPOSITION

Declarative statement with a truth value (either true (`T`) or false (`F`), but not both.

NOT PROPOSITION

Commands (Imperative), Questions (Interrogative), and statements that are neither true nor false

Logical Operators/ Connectives

- Negation - $\neg p$ is true iff p is false

Truth Table

p	$\neg p$
F	T
T	F

- Application: hardware and software

Logic Diagram

- $Y = \overline{A}$

```
>> p=true

p =  1

>> not_p=~p

not_p =  0
```

Logical Operators/ Connectives

- Conjunction - $p \wedge q$ is true iff both p and q are true

Truth Table

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

- Application: hardware and software

Logic Diagram


```
>> p=[0 0 1 1]; q=[0 1 0 1];
>> p&q

ans =  0     0     0     1
```

Logical Operators/ Connectives

- Disjunction - $p \vee q$ is true iff p is true or q is true

Truth Table

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

- Application: hardware and software

Logic Diagram


```
>>  p=[0 0 1 1]; q=[0 1 0 1];
>> p|q

ans =  0     1     1     1
```

Logical Operators/ Connectives

- Exclusive Or - $p \oplus q$ is true iff exactly one of p and q is true

Truth Table

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

- Application: hardware and software

Exclusive OR

```
>> p=[0 0 1 1]; q=[0 1 0 1];
>> xor(p,q)
  ans =  0      1      1      0
```

Logical Operators/ Connectives

- Conditional - $p \to q$ is true if both p and q are true, and when p is false

Truth Table

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Logical Operators/ Connectives

- Biconditional - $p \leftrightarrow q$ is true iff p and q have the same truth values

Truth Table

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Review Question

When is the statement "if today is Monday, then $1 + 2 = 12$" false?

A. "when it is Monday"

B. "when it is Friday"

C. when "$1 + 2 = 12$" is true

D. All of the Above

Review Question

When is the statement "if today is Monday, then $1 + 2 = 12$" false?

A. "when it is Monday" ✓

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Review Question

Which is equivalent to the statement "the home team wins whenever it is raining"?

A. "If the home team does not win, then it is not raining."

B. "If the home team wins, then it is raining."

C. "If it is not raining, then the home team does not win."

D. All of the above

Review Question

Which is equivalent to the statement "the home team wins whenever it is raining"?

A. "If the home team does not win, then it is not raining." ✓

B. "If the home team wins, then it is raining."

C. "If it is not raining, then the home team does not win."

D. All of the above

Various Expressions of Conditional Statement

- "if p, then q"
- "if p, q"
- "p is sufficient for q"
- "q if p"
- "q when p"
- "a necessary condition for p is q"
- "q unless $\neg p$"

- "p implies q"
- "p only if q"
- "a sufficient condition for q is p"
- "q whenever p"
- "q is necessary for p"
- "q follows from p"

Contrapositive, Converse, and Inverse

Inverse of $p \rightarrow q$:

$$\neg p \rightarrow \neg q$$

Converse of $p \rightarrow q$:

$$q \rightarrow p$$

Contrapositive of $p \rightarrow q$:

$$\neg q \rightarrow \neg p$$

Compound Proposition

DEFINITION:

Propositions combined by using logical operators/ connectives

“ Although both Mel and Vin are not young, Mel has a better chance of winning the next chess tournament, despite Vin’s considerable experience ”

Answer

Let:

p = “ Mel is young ”

q = “ Vin is young ”

r = “ Mel has a better chance of winning the next chess tournament ”

s = “ Vin has considerable experience in chess ”

Formalisation:

$$\boxed{(\neg p) \wedge (\neg q) \wedge r \wedge s}$$

Compound Proposition

How can this English sentence be translated into a logical expression?

❝ You can access the Internet from campus only if you are a computer science major or you are not a freshman ❞

Simple Example

How can this English sentence be translated into a logical expression?

“ You can access the Internet from campus only if you know the password ”

Simple Example

How can this English sentence be translated into a logical expression?

“ You can access the Internet from campus only if you know the password ” Let:
p = “ You can access the Internet from campus ”
q = “ You know the password ”
Formalisation:

$$\boxed{p \rightarrow q \equiv \neg q \rightarrow \neg p}$$

Simple Example Equivalent

" If you do know the password then you cannot access the Internet from campus "

Let:

p = " You can access the Internet from campus "

q = " You know the password "

Formalisation:

$$\boxed{\neg q \rightarrow \neg p \equiv p \rightarrow q}$$

Review Question

Whichof the following are other ways of expressing $p \leftrightarrow q$?

A. "p is necessary and sufficient for q."

B. "if p then q, and conversely."

C. "p iff q."

D. All of the above

Review Question

Whichof the following are other ways of expressing $p \leftrightarrow q$?

A. "p is necessary and sufficient for q."

B. "if p then q, and conversely."

C. "p iff q."

D. All of the above ✓

Propositional Equivalences

Tautology

A compound proposition that is always true, no matter what the truth values of the propositions that occur in it.

$$\text{Ex. } p \vee \neg p = \mathrm{T}$$

Contradiction

A compound proposition that is always false, no matter what the truth values of the propositions that occur in it.

$$\text{Ex. } p \wedge \neg p = \mathrm{F}$$

Tautologies and Contradictions in Programming

“ Tautologies and contradictions in source code usually correspond to poor programming. ”

Ex.

```
while (a > 2 || a <= 2)
   a++;
```

Logical Equivalences

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.

- Propositions p and q are logically equivalent ($p \equiv q$) if $p \leftrightarrow q$ is a tautology.

Logical Equivalences

Name	Logical Equivalence
Identity laws	$p \vee \mathbf{F} \equiv p$ $p \wedge \mathbf{T} \equiv p$
Domination laws	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$
Negation laws	$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$
Double Negation law	$\neg(\neg p) \equiv p$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Absorption laws	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$

Drill

Express the implication $p \to q$ using basic connectives $\neg$, $\vee$, $\wedge$, or its combination.

Example

Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

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Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

Using Truth Table:

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
F	F	T	_	_
F	T	T	_	_
T	F	F	_	_
T	T	F	_	_

Example

Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

Using Truth Table:

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

Possible Quiz Question

- In general, how many rows are required if a compound proposition involves n propositional variables?

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- Answer: 2^n

Drill

Show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

Drill

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

Drill

Evaluate $(p \wedge q) \rightarrow (p \vee q)$.

Propositional and Predicate Logic

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  area of logic that deals with propositions

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  area of logic that deals with propositions
- Predicate Logic
  concerned not only with logic relations between sentences or propositions as wholes, but also their internal structure in terms of subject and predicate.

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Predicate Logic is the area of logic that deals with predicates and quantifiers.

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Predicate refers to a property that the subject of the statement can have.

Ex. In the statement "x is greater than 12", the variable 'x' is the subject and "is greater than 12" is the predicate.

Quantifiers

It indicates the generality of the open sentence in which a variable occurs.

- Existential quantifier $(\exists x)$
  “ there Exist ”
- Universal quantifier $(\forall x)$
  “ for All ”

Universal Quantifier

Many mathematical statements assert that a property is true for all values of a variable in a particular domain.

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“ If x is a triangle then the sum of its internal angles is 180° ”

Predicate logic:

$$\boxed{\forall x P(x)}$$

“ For every x such that x is a triangle, the sum of the internal angles of x is 180° ”

Possible Quiz Question

- The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. An element for which $P(x)$ is false is called

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- The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. An element for which $P(x)$ is false is called
- Answer: Counterexample of $\forall x P(x)$.

Existential Quantifier

It is true if and only if $P(x)$ is true for at least one value of x in the domain.

“ x is an integer and $x^2 = 25$ ”

Predicate logic:

$$\boxed{\exists x P(x)}$$

“ There exists x such that x is an integer and $x^2 = 25$. ”

Quantifiers

Statement	***When True?***
$\forall x P(x)$	$P(x)$ is true for every x.
$\exists x P(x)$	There is an x for which $P(x)$ is true.

Statement	***When False?***
$\forall x P(x)$	There is an x for which $P(x)$ is false.
$\exists x P(x)$	$P(x)$ is false for every x.

Possible Quiz Question

- Translate Newton's 2nd law of motion in terms of predicate logic.
  In English:
  "for every x of a certain type referred to as an Object, x is stationary, x is in uniform motion, or there is an f of type Force such that x is acted upon by f"

Answer

"for every *x*" "of type called *object*" "or"

$$\forall\, x: \mathit{Object} \bullet \mathit{stationary}(x) \vee \mathit{in\text{-}uniform\text{-}motion}\,(x)$$
$$\vee\, \exists\, f: \mathit{Force} \bullet x\ \mathit{is\text{-}acted\text{-}upon\text{-}by}\, f$$

"there exists an *f*"

Sets

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- The objects in a set are called the <u>elements</u>, or <u>members</u>, of the set.
- Ex. Set V of all vowels in the English alphabet, $V = \{a, e, i, o, u\}$.
- Ex. Set of positive integers less than 100, $\{1, 2, 3, \ldots, 99\}$.

Set builder notation

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- Ex. Express set O of odd positive integers less than 10, $O = \{1, 3, 5, 7, 9\}$ in terms of set builder notation.
- Ans. $O = \{x | x \text{ is an odd positive integer less than } 10\}$
- Or: $O = \{x \in Z^+ | x \text{ is odd and } x < 10\}$.

Important DISMATH Sets

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of **natural numbers**
$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of **integers**
$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of **positive integers**
$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$, the set of **rational numbers**
$\mathbf{R}$, the set of **real numbers**

Equality of Sets

- Two sets are equal if and only if they have the same elements.

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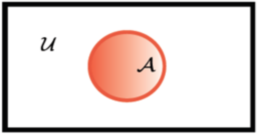
- If A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.
- Ex. The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal.

Possible Quiz Question

- Are sets { 1, 3, 5} and { 1, 3 , 3 , 3 , 5 , 5 , 5 , 5 } equal?

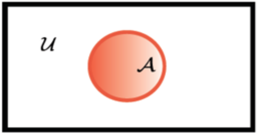
Venn Diagram

- A tool used to graphically illustrate set relationships by representing sets as simple plane areas



Venn Diagram

- A tool used to graphically illustrate set relationships by representing sets as simple plane areas
- Ex. Set $\mathcal{A}$ as a circle and universal set $\mathcal{U}$ as the entire rectangle.



Empty Set

- A special set that has no elements.

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- This set is called the empty set, or null set, and is denoted by $\varnothing$ or$\{\ \}$.

Possible Quiz Question

Which of the following is false?

A. $\varnothing \in \{1, 2, 3\}$

B. $\varnothing \in \{\varnothing, 1, 2, 3\}$

C. $\varnothing \subseteq \{1, 2, 3\}$

D. $\{x\} \subseteq \{x\}$

E. None of the Above

Subset

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- We use the notation $A \subseteq B$.
- $A \subseteq B$ if and only if the quantification

$$\forall x (x \in A \rightarrow x \in B)$$

is true.

Possible Quiz Question

- Are sets $A = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$ and $B = \{x | x \text{ is a subset of the set } \{a, b\}\}$ equal?

Possible Quiz Question

- Are sets $A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $B = \{x | x \text{ is a subset of the set } \{a, b\}\}$ equal?

✍

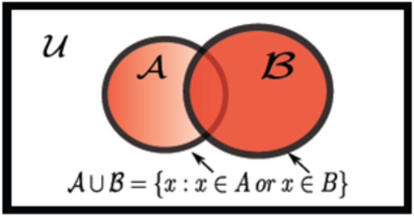
Empty set ($\emptyset$) is a subset to any set.

Union

the union of two sets $\mathcal{A}$ and $\mathcal{B}$ is the set containing all the elements that belong to $\mathcal{A}$ or $\mathcal{B}$ or both:

$\mathcal{A} \cup \mathcal{B} = \{x : x \in A \text{ or } x \in B\}$

$\mathcal{A} \cup \mathcal{B} = \{x | x \in A \vee x \in B\}$



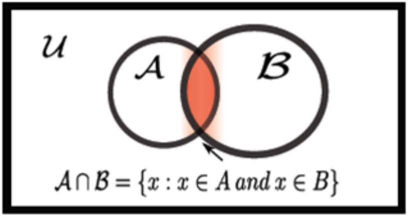
Union of two sets $\mathcal{A}$ and $\mathcal{B}$

Intersection

the intersection of two sets $\mathcal{A}$ and $\mathcal{B}$ is the set containing all the elements common to both $\mathcal{A}$ and $\mathcal{B}$:

$\mathcal{A} \cap \mathcal{B} = \{x : x \in A \; and \; x \in B\}$

$\mathcal{A} \cap \mathcal{B} = \{x | x \in A \wedge x \in B\}$

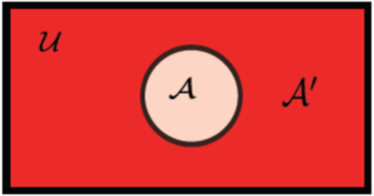


Intersection of two sets $\mathcal{A}$ and $\mathcal{B}$

Complement

the complement of set $\mathcal{A}$ with respect to a universal set $\mathcal{U}$ is the subset of all elements of $\mathcal{U}$ that are not in $\mathcal{A}$:

$\mathcal{A}' = \{x : x \in U\ ,\ x \notin A\}$

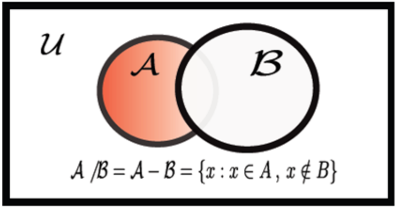


Complement of set $\mathcal{A}$

Set Difference

the set difference of $\mathcal{A}$ and $\mathcal{B}$ is the set of elements that belongs to $\mathcal{A}$ but not to $\mathcal{B}$; also known as Relative Complement:

$$\mathcal{A}\ /\mathcal{B} = \mathcal{A} - \mathcal{B} = \{x : x \in A,\ x \notin B\} = \mathcal{A} \cap \mathcal{B}'$$



Set difference of set $\mathcal{A}$ and $\mathcal{B}$

Possible Quiz Question

- Prove De Morgan's Law $\overline{A \cap B} = \overline{A} \cup \overline{B}$ using set builder notation and logical equivalences.

Set Identities

Identity	*Name*
$A \cup \emptyset = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

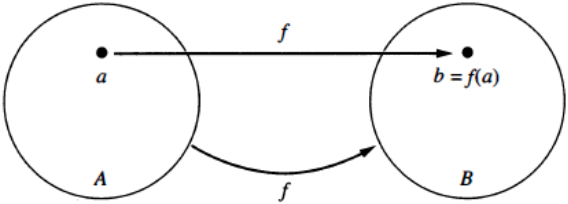
Possible Quiz Question

- Let A, B, and C be sets. Show that

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

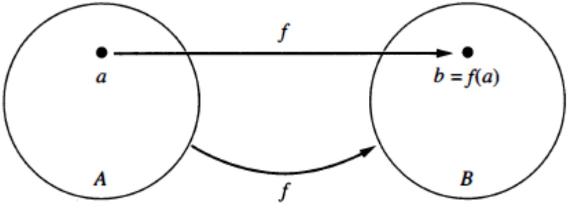
Function

- A function f from set A to set B is an assignment of exactly one element of B to each element of A.



Function

- A function f from set A to set B is an assignment of exactly one element of B to each element of A.
- also called mappings or transformations



✍Remember

- If f is a function from set A to B, we say that A is the domain of f and B is the codomain of f.

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- The range is the set of values a function that actually occurs.

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- If f is a function from set A to B, we say that A is the domain of f and B is the codomain of f.
- The range is the set of values a function that actually occurs.
- Example: What are the domain, codomain, and range of the function that assigns grades to students?

☞Understand

- The domain and codomain of functions are often specified in programming languages. For instance, the Java statement:

  int floor(float real)…

  Give the domain and codomain.

Types of Functions

- One-to-one Function (Injective)

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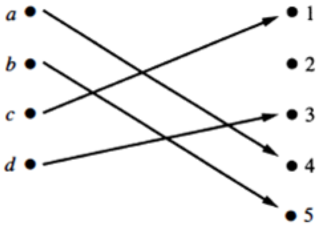
- Onto Function (Surjective)
- One-to-one Correspondence (Bijection)

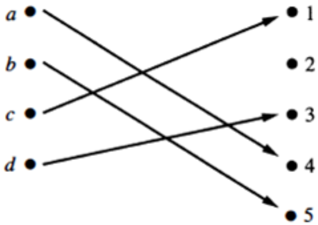
One-to-one Function (Injective)

- are functions that never assign the same value to two different domain elements



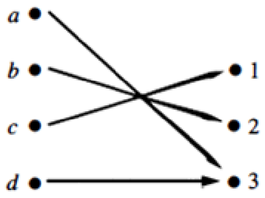
One-to-one Function (Injective)

- are functions that never assign the same value to two different domain elements
- $\forall x \forall y (f(x) = f(y) \rightarrow x = y)$



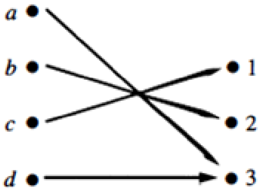
Onto Function (Surjective)

- are functions having equal range and codomain.



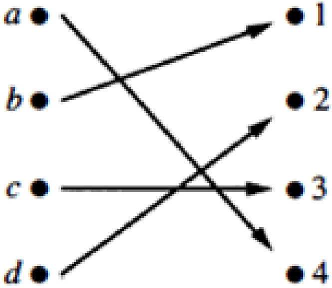
Onto Function (Surjective)

- are functions having equal range and codomain.
- $\forall y \exists x (f(x) = y)$; x - domain, y - codomain



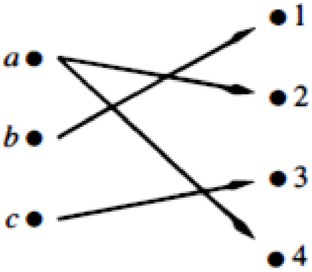
One-to-one Correspondence (Bijective)

- is both one-to-one and onto.



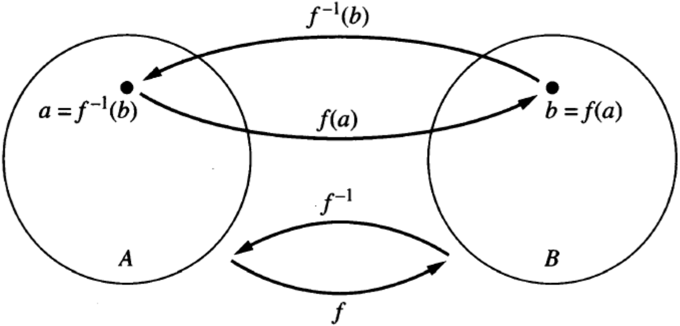
☞ Understand

- What type of correspondence is the following:



Inverse Functions

- function that assigns to an element y belonging to B the unique element x in A such that $f(x) = y$.



☞ Understand

- A one-to-one correspondence is called <u>invertible</u> because we can define an inverse of this function.

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- A one-to-one correspondence is called <u>invertible</u> because we can define an inverse of this function.
- A function is not invertible if it is not a one-to-one correspondence, because the inverse of such a function does not exist.

☞ Understand

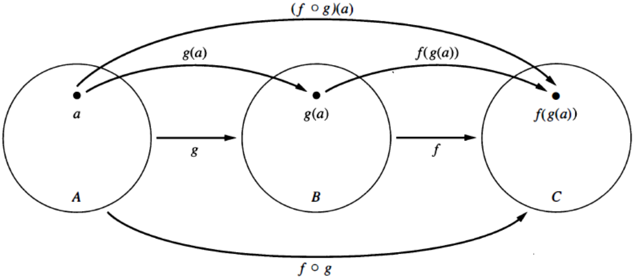
- Let $f : \mathcal{Z} \rightarrow \mathcal{Z}$ be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse?

☞ Understand

- Let f be the function from $\mathcal{R}$ to $\mathcal{R}$ with $f(x) = x^2$. Is f invertible?

Composition of Functions

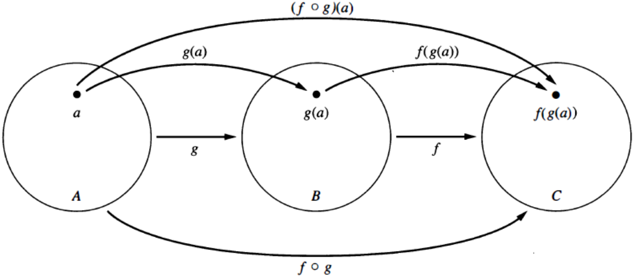
- Let $g : A \to B$ and f: $B \to C$.



Composition of Functions

- Let $g : A \to B$ and f: $B \to C$.
- The composition of the functions f and g, denoted by $f \circ g$, is

$$(f \circ g)(a) = f(g(a))$$



☞ Example

- Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$. What is the composition of f and g, and what is the composition of g and f?

☞ Example

- Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f?

Graphs of Functions

- Let $f : A \to B$. The graph of the function f is the set of ordered pairs $\{(a, b) | a \in A\}$ and $\{f(a) = b\}$.

Graphs of Functions

- Let $f : A \rightarrow B$. The graph of the function f is the set of ordered pairs $\{(a, b) | a \in A\}$ and $\{f(a) = b\}$.
- Graph $f(n) = 2n + 1$ from $\mathcal{Z}$ to $\mathcal{Z}$.

Graphs of Functions

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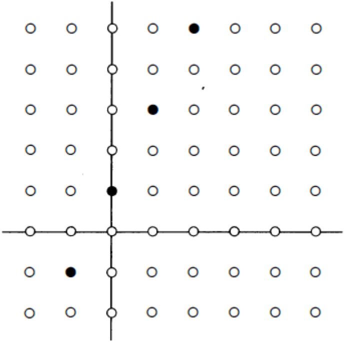

Graphs of Functions

- Graph $f(x) = x^2$ from $\mathcal{Z}$ to $\mathcal{Z}$.

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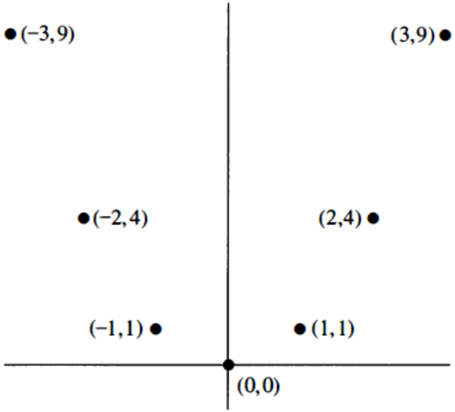
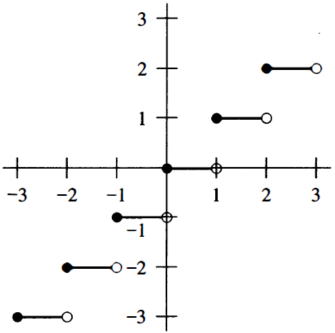


Floor Function

- assigns to the real number x the largest integer that is less than or equal to x.

Floor Function

- assigns to the real number x the largest integer that is less than or equal to x.
- $\lfloor x \rfloor = n$ if and only if $n \leq x < n + 1$.

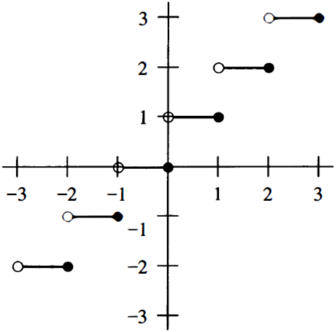


Ceiling Function

- assigns to the real number x the smallest integer that is greater than or equal to x.

Ceiling Function

- assigns to the real number x the smallest integer that is greater than or equal to x.
- $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$.



Examples

$$\lfloor \tfrac{1}{2} \rfloor = 0,\ \lceil \tfrac{1}{2} \rceil = 1,$$

$$\lfloor -\tfrac{1}{2} \rfloor = -1,\ \lceil -\tfrac{1}{2} \rceil = 0,$$

$$\lfloor 3.1 \rfloor = 3,\ \lceil 3.1 \rceil = 4,$$

$$\lfloor 7 \rfloor = 7,\ \lceil 7 \rceil = 7$$

☞ Example

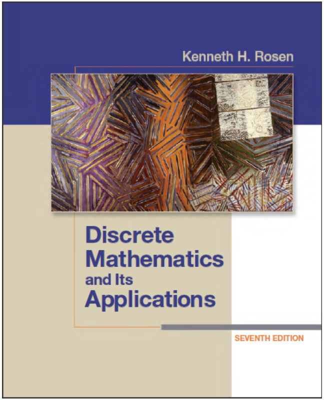
- Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

Drill

- In asynchronous transfer mode (ATM) (a communications protocol used on backbone networks), data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second?

Reference:

Rosen, K.H. Discrete Mathematics and Its Applications (7 ed.), New York, McGraw-Hill



Thank you for your attention!