Lab Notes
Measurement Techniques

Sadegh Ebrahimkhani (sadegh.ebrahimkhani@vub.be)
Antoine Marchal (antoine.marchal@vub.be)

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Contents

Lab 1

Generating multisines and random signals in Matlab

Objectives

The goal of this lab is to get acquainted with the use of Matlab to generate multisines and random noise signals, to be used in later labs as excitation signals for dynamic systems. You will learn

- how the DFT is defined (in Matlab in particular) and how it can be used to analyse periodic signals,
- how to generate multisines in the time domain and in the frequency domain,
- how you can construct an excitation signal in a given frequency band, and with a given frequency resolution.

1.1 Discrete Fourier Transform (DFT)

Consider a discrete-time signal $x(n)$, in the time window $n = 0, \ldots, N-1$. Recall the definition of the DFT of $x(n)$, and of the inverse DFT, in Matlab (the functions `fft` and `ifft` respectively):

DFT	iDFT	
$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}},$ for $k = 0, 1, \ldots, N-1$	$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$ for $n = 0, 1, \ldots, N-1$	(1.1)

An interpretation is that the DFT decomposes the time domain signal $x(n)$ into a linear combination of cosines and sines – or complex exponentials – of which the (complex) amplitudes are given by $X(k)$. The frequency axis k is expressed in *bin*. At bin k, the complex exponential $e^{\frac{j2\pi kn}{N}}$ is periodic in n, and has a period which fits exactly k times in the time interval of N points. This is shown graphically in Figure 1.1, keeping in mind that $e^{j\phi} = \cos(\phi) + j\sin(\phi)$.

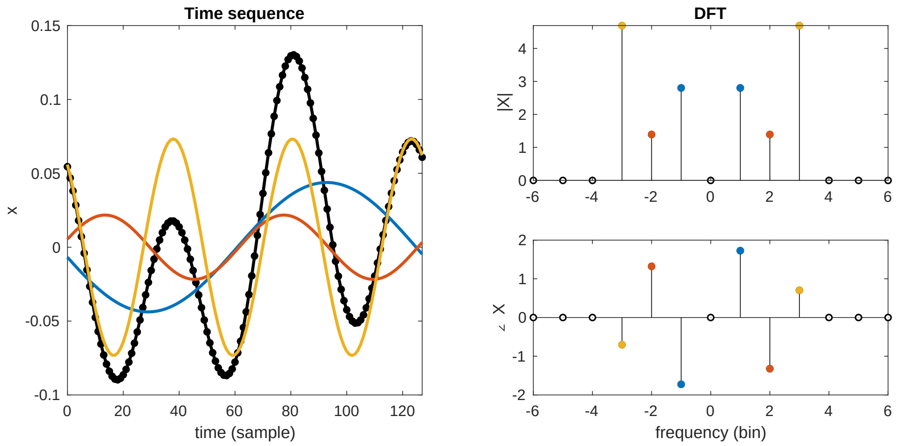


Figure 1.1: Left: time domain signal $x(n)$ (black), and its (co)sine components (coloured). Right: DFT of the signal: amplitude (top) and phase (bottom) of individual components $X(k)$.

If the discrete-time signal $x(n)$ represents a sampled continuous time signal, with a sampling frequency of f_s, then the iDFT can be rewritten as

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\omega_k n T_\mathrm{s}} \tag{1.2}$$

where $T_\mathrm{s} = \frac{1}{f_\mathrm{s}}$ is the sample time, and ω_k is the angular frequency at bin k.

TASK 1.1.1. Frequency axis in $\mathrm{rad/s}$**.** Prove that

$$\omega_k = \frac{2\pi}{T}k \tag{1.3}$$

(where $T = NT_\mathrm{s}$ is the window length expressed in seconds). (Hint: compare (1.1) and (1.2).) This will allow you to construct the frequency axis of the DFT of a sampled signal: ω_k for $k = 0, 1, \ldots, N/2$.

In fact, the DFT discretises the frequency axis. The resolution of this discretisation increases when the measurement time T increases – i.e. there are more 'bins per Hz' when measuring longer.

TASK 1.1.2. Fundamental frequency (1 bin). Prove that the fundamental frequency ω_1 of the DFT (which corresponds to one bin) is equal to

$$\omega_1 = \frac{2\pi}{T} = 2\pi\frac{f_\mathrm{s}}{N}. \tag{1.4}$$

TASK 1.1.3. Conjugate symmetric DFT. Prove that

$$X(N-k) = X(-k) = X^*(k). \tag{1.5}$$

(Hint: use $e^{j2\pi n} = 1$ for $n \in \mathbb{Z}$, and $x(n) \in \mathbb{R}$.)

This demonstrates that the DFT of a real signal is conjugate symmetric around the origin. Thus, the negative frequencies are obtained from the upper half of the DFT: $X(-k) = X(N-k)$.

1.2 DFT of a (co)sine

TASK 1.2.1. DFT of 3 periods of a cosine. Generate a cosine sequence in Matlab with a randomly selected phase, and with a period that fits exactly 3 times in a data sequence of $N = 1000$ samples. Make a plot of the DFT of this sequence (amplitude and phase).

TASK 1.2.2. Perfect reconstruction. From the DFT plot, check that the condition for perfect reconstruction is satisfied. Is there any leakage visible?

TASK 1.2.3. Interpretation of the frequency axis. At which indices of the DFT do you obtain non-zero values? Explain. (Keep in mind that Matlab indices start at 1, not at 0.)

TASK 1.2.4. Frequency axis in bins. Construct the frequency axis for the plots, expressed in bins.

TASK 1.2.5. Frequency axis in Hz. Consider that the sample frequency is $f_s = 100\,\text{Hz}$. Construct the frequency axis for the plots, expressed in Hz. (Hint: use the results from **Task 1.1.1**.)

1.3 Time domain construction of a multisine

Recall that a multisine is a sum of cosines, with frequencies that satisfy the condition for perfect reconstruction:

$$x(n) = \sum_{m=1}^{K} A_m \cos(\omega_m n T_s + \varphi_m) = \sum_{m=1}^{K} A_m \cos\left(\frac{2\pi m}{N} n + \varphi_m\right) \tag{1.6}$$

$$\text{with } n = 0, 1 \ldots, N-1 \tag{1.7}$$

The frequencies ω_m for which the amplitudes A_m are non-zero are called the **excited frequencies**.

TASK 1.3.1. Time domain random phase multisine. Generate a multisine in the time domain, by implementing (1.6), with $N = 1000$ samples and $K = 10$ excited frequencies. Set the amplitudes $A_m = 1$, and choose the phases φ_m randomly between 0 and 2π (i.e. a *random phase* multisine). Check that this multisine satisfies the condition for perfect reconstruction by plotting its DFT. Include the frequency axis, expressed in bin.

TASK 1.3.2. Frequency axis in Hz. For the multisine generated in **Task 1.3.1**, consider that the sampling frequency is $f_s = 100\,\text{Hz}$. Include the frequency axis expressed in Hz in the DFT plot, and the time axis expressed in seconds for the time domain plot.

TASK 1.3.3. Excite specific frequency lines. Generate a random phase multisine with a sampling frequency of 200 Hz, with excited frequencies

$$[4, 8, 12, 16, 20, 24]\ \text{Hz}. \tag{1.8}$$

Plot the time and frequency domain results, with appropriate axes.

1.4 Frequency domain construction of a multisine

A multisine can easily be generated by immediately specifying the amplitudes and phases of the components. This comes down to constructing $X(k)$ directly (that is, in the frequency domain). One difficulty is that $X(k)$ must be constructed both for the positive and the negative frequencies. However, a trick can be used such that only the positive frequencies must be constructed, as in the following task.

TASK 1.4.1. Trick for frequency domain multisine construction. Consider the vector $\tilde{X}(k)$, such that

$$\tilde{X}(k) = A_k e^{j\varphi_k} \qquad \text{for } 1 \leqslant k \leqslant K \tag{1.9}$$

$$\tilde{X}(k) = 0 \qquad \text{otherwise} \tag{1.10}$$

Prove that

$$x(n) = N\Re\left\{\texttt{iDFT}\big(\tilde{X}(k)\big)\right\} = \sum_{k=1}^{K} A_k \cos\left(\frac{2\pi k}{N}n + \varphi_k\right) \tag{1.11}$$

where $\Re$ denotes the real part. (Hint: use the definition of `iDFT` in (1.1).)

TASK 1.4.2. Frequency domain multisine. Use the frequency domain approach to construct a random phase multisine, by using the trick from **Task 1.4.1**. Let $N = 1000$, and excite the first 30 bins. Make time and frequency domain plots (frequency axis expressed in bins).

TASK 1.4.3. Specified excited frequency band and frequency resolution. Construct a random phase multisine in the frequency domain, which excites the frequency band $[5, 15]$ Hz at 30 equidistantly spaced frequencies. Choose an appropriate sampling frequency. Make time domain and frequency domain plots (time axis in seconds, frequency axis in Hz). How long is one period of this multisine (expressed in seconds)?

1.5 Influence of the phase of the multisine

Until now, we choose the phases of the components of the multisine to be random. The choice of the phase has an important impact on the time domain properties of the multisine. More specifically, it will influence its crest factor (CF), defined as:

$$\text{Crest Factor} = \frac{\max(|x|)}{\text{RMS}(x)} \qquad \text{with RMS}(x) = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} x^2(n)}. \tag{1.12}$$

TASK 1.5.1. Crest Factor. Construct a multisine, with $N = 500$, with the first $K = 60$ bins excited, and with the following phases:

- Random phase: chosen randomly in $[0, 2\pi]$ (uniform distribution),
- Schroeder phase: $\varphi_m = \frac{m(m+1)\pi}{K}$,
- Linear phase: $\varphi_m = m\pi$.

Make time and frequency domain plots (in samples and bins), and compute the Crest Factors. Describe, qualitatively, the relationship between the time domain plot and the crest factor. What is the advantage of a low/high crest factor?

1.6 Random noise signals

A popular excitation signal in the literature is random white noise. This is a signal which excites all frequencies, and which is stochastic, both in the time and in the frequency domain.

TASK 1.6.1. White Gaussian random noise. Generate a normally distributed (Gaussian), random, white noise sequence of $N = 1000$ samples, by using the Matlab function `randn`. Make time and frequency domain plots (axes in samples and bins). Observe that all the bins are excited, with random amplitudes and phases.

Note that this approach generates a signal which excites the full available frequency band. This is often not desired, for multiple reasons. For instance, most practical systems are only active in a limited frequency band. Thus, the input energy outside of that frequency band is typically wasted. Also, exciting the full frequency band is prone to cause alias errors, because harmonics created by the system under test will lie beyond the Nyquist frequency. For these reasons, it is better to limit the excitation frequency band, for instance by filtering.

TASK 1.6.2. Filtered random noise. Generate a filtered random noise sequence with $N = 1000$, sampling frequency 100 Hz, from a Gaussian white noise sequence (use `randn`). Do this by using the function `cheby1` to create a lowpass digital Chebyshev filter of order 5, ripple 2 dB, and such that the passband edge lies at 5 Hz. Filter the sequence by using the function `filter`. Make time and frequency domain plots (axes in seconds and Hz), and check that the excited frequency band is as expected. What do you observe in the stop-band of the filter? Is it equal to 0? Explain.

TASK 1.6.3. Periodic band-limited random noise. Generate a Gaussian random noise sequence (use `randn`), with $N = 1000$ and sampling frequency 100 Hz. Compute the DFT, and set the DFT at all frequencies beyond 5 Hz to zero:

$$\tilde{X}(k) = 0 \qquad \text{for } \omega_k > 2\pi 5\,\mathrm{rad/s}, \tag{1.13}$$

$$\tilde{X}(k) = X(k) \qquad \text{otherwise}, \tag{1.14}$$

and use the expression

$$x(n) = 2\Re\left\{\mathtt{iDFT}\left(\tilde{X}(k)\right)\right\} \tag{1.15}$$

to obtain the time sequence. Make time and frequency domain plots (axes in seconds and Hz).

If you repeat this time domain sequence $x(n)$ (by putting multiple copies of the sequence after each other), and computing the DFT of the result, no leakage should occur. Check this, and explain why this is the case.

In fact, the signal generated in **Task 1.6.3** can be interpreted as a multisine, with random phase *and* random amplitude.

1.7 Set the Root-Mean-Square of the signal

For all the signals generated above, the Root-Mean-Square (RMS) of the signal was not specified. The actual RMS value depends on the number of excited frequencies and on the choice of the amplitudes A_k.

TASK 1.7.1. RMS value. Set the RMS value of your favourite signal from the previous tasks to $\mathrm{RMS}_{\mathrm{des}} = 3$:

$$x_{\mathrm{des}}(n) = x(n)\frac{\mathrm{RMS}_{\mathrm{des}}}{\mathrm{RMS}(x)}. \tag{1.16}$$

Lab 2

Measuring transfer functions

The transfer function of a Linear Time Invariant (LTI) system completely describes the systems' behavior. It is a system property, that can be measured using:

$$H(j\omega) = \frac{Y(j\omega)}{U(j\omega)} \tag{2.1}$$

with $Y(j\omega)$, $U(j\omega)$ the Fourier spectra as shown in Figure 2.1.

In this lab assignment we will investigate the influence of the choice of $U(j\omega)$ on the quality of the measured transfer function. As an example, the transfer function of a filter will be measured.

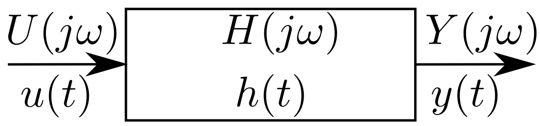


Figure 2.1: Representation of a Linear Time Invariant (LTI) system, where $U(j\omega), Y(j\omega)$ represent the input and the output spectra respectively.

2.1 Choosing an excitation signal

The choice of $u(t)$ can be used to optimize a number of criteria such as the control on the amount of excitation power, the measured frequency band, the minimization of the time needed to perform the experiment, the minimization of the cost of the experiment, the maximization of the accuracy of the results, etc.

The outcome of such an optimization is highly dependent on the considered system. Hence, we will study the effect of the spectrum $U(j\omega)$ on the accuracy of the measured transfer function.

To generate arbitrary analog signals in continuous time and continuous amplitude, we use an Arbitrary Waveform Generator (AWG). This Digital to Analog Converter (DAC) based device converts a data stream with a predefined spectrum into the analog signal with the same spectrum that is needed to excite the system. To obtain the expected analog excitation spectrum, the data sequence must obey the theoretical rules (Nyquist, leakage avoidance, ...)

2.2 Discretization of the measured signals

Analog signals cannot be measured directly. Instead, it is mandatory to transform them into a data sequence again to obtain a stream of digital numbers. This process discretizes the signals both in time and in amplitude. In general, these discretizations always result in a loss of information, hereby degrading the measurement significantly. Information can only be preserved if all the theoretical rules are obeyed very carefully.

2.2.1 Quantization of the voltage

The loss of information due to the discretisation of the voltage depends on the resolution of the ADC in the measurement channel. This is determined by the number of bits used to encode the measured samples. ADCs nowadays have a resolution in excess of 10 bit. This means that 1024 different voltage levels can be measured.

The voltage swing associated to the Least Significant Bit (LSB) is the voltage difference (in Volt) between two successive discretization levels of the ADC. The dynamic range of the ADC is then the ratio between the maximum voltage that can be represented and 1 LSB.

QUESTION 2.1. Dynamic range. What is the dynamic range of the ADC used in the ELVIS II board? (refer to the datasheet found in `http://www.ni.com/pdf/manuals/372590b.pdf`)

2.2.2 Discretisation of the time

To convert the continuous time signals to a data sequence requires to know their value at a discrete set of time instants only. This discretisation in time is bound to hard constraints if the conversion is to happen without loss of information: the sampling theorem or theorem of Nyquist is to be obeyed.

- What is the minimum sampling frequency required to perfectly reconstruct a sinusoidal signal with a frequency of 10 Hz?

2.3 Influencing measurement accuracy for noisy measurements

The random errors on the FRF will depend on the input power that is present in the excitation signal, as more power means a higher Signal-to-Noise Ratio (SNR). To influence the power that is present in an excitation signal of a fixed time domain amplitude, we can tune the crest factor and the power spectrum of the signal.

Crest factor

If the ratio between the peak value of the signal in the time domain and the RMS value of the signal is very high, the power that can be put in the signal decreases for a fixed maximal signal amplitude. This ratio is called the crest factor and is defined as:

$$\text{Crest factor} = \frac{\text{Peak value}\, x(t)}{\text{RMS value}\, x\,(t)} = \frac{\max_{t\in[0,T]}\left(|x\,(t)|\right)}{\sqrt{\frac{1}{T}\int_0^T x^2\,(t)\,dt}} \tag{2.2}$$

This quantity somehow quantifies to what extent the signal concentrates its energy in the time.

The crest factor depends on the shape of the signal in the time domain. Consider all the signals with a given power spectrum $S(j\omega)$. The shape of the associated time waveform is determined by the phase spectrum of the signal. The accuracy of the FRF is therefore also affected by the phase spectrum of $U(j\omega)$.

QUESTION 2.2. Accuracy FRF. Why is this the case?

To obtain signals with a fixed power spectrum and different crest factors, we associate a random phase spectrum with a fixed power spectrum and calculate the associated time waveform.

Power spectrum

The shape of the power spectrum $S_{UU}(j\omega) = U(j\omega)U^*(j\omega)$ can also be used to influence the measurement variability.

It is clear that it makes no sense to excite the DUT (Device Under Test) in a frequency band where $H(j\omega)$ is of no interest. Since most systems require that the amplitude of the excitation remains bounded, it is an advantage to concentrate the allowable excitation power in the bandwidth that is to be characterized. This will increase the signal to noise ratio in that band, and hence decrease the variability of the measurement. For the characterization of a linear time invariant system, an increase of the excitation power level in the band of interest results in a reduction of the variability of the measured FRF.

Measurement errors

Up to now, we have implicitly assumed that measurement errors result from random perturbations of the measured spectra. In general, measurement errors can either be stochastic or systematic. Only stochastic errors can be influenced by a changing signal to noise ratio.

Systematic errors or bias errors are more insidious. They often result in a smooth shift of the measured characteristics, that is very hard to detect. To influence them, additional knowledge is mandatory. Systematic errors are removed by calibration. The idea here is to compare the measured and the a priori known FRF of a reference system. In this lab, we will concentrate on the stochastic measurement errors only.

2.4 Comparing FRF measurements obtained with different excitation signals

The idea is to compare the FRF of a system that is obtained using different excitation signals. These signals are all normalized to have a constant RMS value in time domain as explained in Section 1.7. The variability of the measurements is obtained by repeating the same measurement and calculating the measurement's sample variance. The signals that will be used in this lab assignment are shortly described below.

2.4.1 The multisine excitation

A multisine is a sum of harmonically related sine waves with a well chosen amplitude and phase spectrum,

$$x(t) = \sum_{i=1}^{K} X_i \sin(2\pi\nu_i f_0 t + \psi_i) \tag{2.3}$$

with $\nu_i \in \mathbb{N}$. The excited frequencies $\nu_i f_0$ are always an integer multiple of the fundamental frequency f_0 (the frequencies are therefore called commensurate). The number of excited frequency lines K, the amplitude of the excited lines X_i and their respective phase ψ_i can all be chosen freely to match the requirements of the application.

Figure 2.4.1 shows the time domain waveform of a multisine that consists of 1024 samples and contains $K = 100$ spectral components of equal amplitude. The phase spectrum has been calculated using Schroeder's equation to obtain a signal with a low crest factor immediately.

The amplitude spectrum is shown in Figure 2.4.1. Note that since the multisine is a periodic signal, its spectrum is discrete.

Now that we have taken a fast look at this signal, it is time for some action!

QUESTION 2.3. Generation of multisine signals. Use MATLAB to calculate the data sequence of a multisine that consists of 4096 time samples and contains 100 excited spectral lines, located at the low end of the band (from line 1 to 100). Calculate (and show the code in the report) the time waveform for a multisine with:

(a) a constant phase spectrum,

(b) a random phase spectrum with phases drawn from a uniform distribution with $0 \leqslant \psi_i < 2\pi$,

(c) a Schroeder phase spectrum. Choose the phases according to $\psi_k = \frac{k(k+1)\pi}{K}$, where k is the line number (= the frequency of the line expressed in bins) and K is the number of excited lines in the signal.

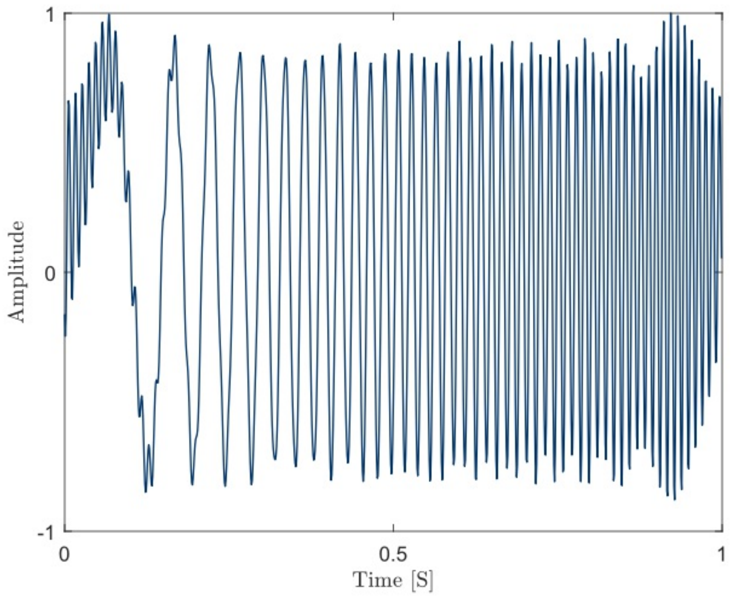
QUESTION 2.4. Crest factor. Calculate the crest factor of these signals and explain the differences.

QUESTION 2.5. Plotting and interpretation. Visualize the signals both in the time and the frequency domain then discuss differences and similarities.

2.4.2 Noise excitation

Noise is a very popular choice for an excitation signal. Be aware that there can be a difference between noise and noise in the literature. The noise that we con-

[Time domain representation]



[Amplitude spectrum]

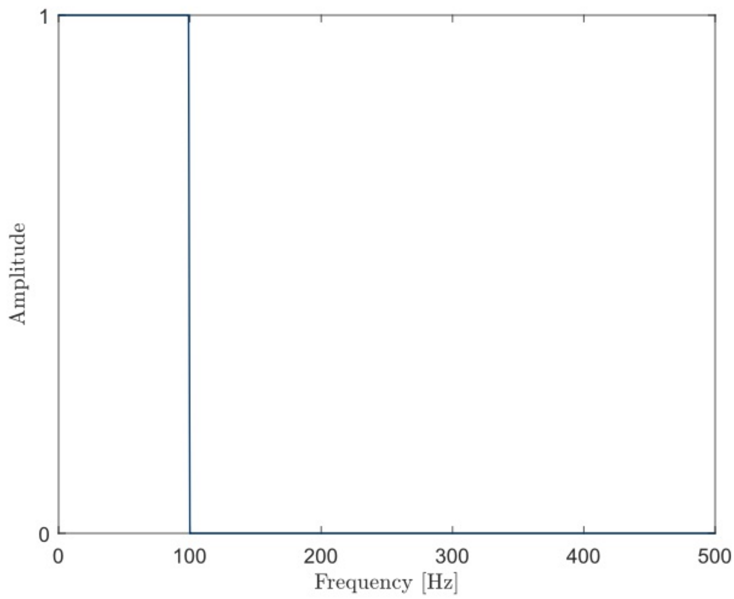


Figure 2.2: Example multisine with Schroeder phase spectrum.

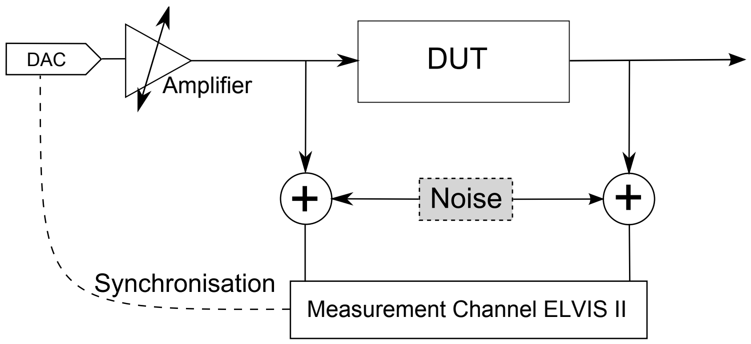


Figure 2.3: Measurement setup Lab 1. Is the noise generator needed in this lab?

sider here is the purely random noise. In the literature, many signals are referred to as periodic noise. The difference is quite subtle, but results in a very different behavior.

For 'periodic' noise the AWG is loaded with one noise record that is repeated periodically and is used for all the experiments. In fact, this boils down to the creation of a periodic excitation. Periodic noise is a kind of multisine whose amplitude and phase spectra are both selected in a random way. Clearly, this is not what we call a random excitation in the context of this lab.

A purely random noise excitation requires that the signal generator is loaded with a fresh realization of the random signal for each experiment that is performed. Therefore, there is no periodicity at all and the signal will behave as predicted by the theoretical analysis for a noise excitation.

As mentioned in the theory, two complications arise when working with arbitrary excitations. First, the system never reaches a steady state, hence a transient term is present. Secondly, the input spectra can be very small or even zero at certain frequencies, leading to problems in the calculation of the resulting FRF. To address these problems, the signals can be multiplied by a window (e.g. a Hann window), prior to computing the DFTs. This will reduce the influence of the transient error.

2.5 Experimental setup

Figure 2.3 shows the block diagram of the measurement setup. The excitation signal is generated by an Arbitrary Waveform Generator (AWG). The voltages that are present at the input and the output of the DUT are both measured by the ELVIS II acquisition channels.

The internal operation of the acquisition channel is shown in Figure 2.4. It uses a multiplexer (MUX) followed by an Analog to Digital Converter (ADC). The multiplexer circuit ensures simultaneous sampling of the signals, even if their conversion is performed sequentially. The signals that are fed to the anti-alias (AA) filter are conditioned to increase the dynamic range of the setup.

To ease the operation, the AWG and the acquisition are integrated in the ELVIS II hardware. They use the same sampling clock. The software drivers that are needed to load and acquire signals are all provided in the LabVIEW environment.

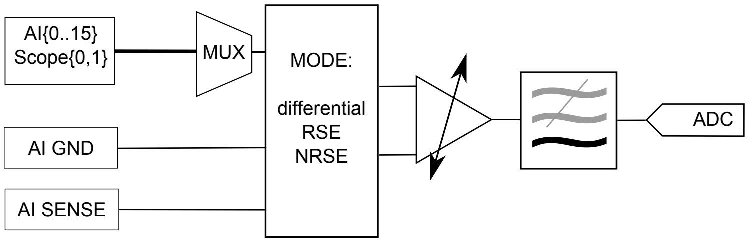


Figure 2.4: Measurement channel of ELVIS II. (More information can be found in the manual of ELVIS II.)

QUESTION 2.6. Trigger signal What is the purpose of a trigger signal? A number of processing methods were discussed in the theory to improve the SNR of the FRF by averaging of measurements. Which methods require a trigger signal to operate properly? Explain what happens if the trigger is absent while needed.

QUESTION 2.7. Synchronization If the clocks of the generator and the acquisition are not synchronized, which effect can you expect? Is this equally important for low and high frequencies?

2.6 Now, it's up to you!

2.6.1 Get to know your DUT

The DUT that is to be characterized is unknown a priori. To obtain maximal information, the FRF is to be measured in a well chosen frequency band.

We will select this frequency band first using a 'get-to-know-your-system' measurement that covers a wide frequency band. In the lab, it is known that the DUT has a bandwidth which is smaller than 500 Hz. The first experiment is executed as follows:

- Configure the instrument to use a sampling frequency $f_\mathrm{s} = 8\,\mathrm{kHz}$.
- Construct a multisine with a Schroeder phase such that
  - The excited lines are present at frequencies between 1 Hz and 500 Hz
  - A frequency resolution of 1 Hz is obtained
  - The RMS value of the input signal is $V_\mathrm{RMS} = 100\,\mathrm{mV}$.
- Perform the measurement. Measure a few (5 to 10) periods of the input and output signals.

QUESTION 2.8. Differences between periods Compare the measurements of the different periods of the signal in the time domain and calculate the FRF for each period of the signals separately. Explain what you see and decide which periods to select or leave out and why.

QUESTION 2.9. Frequency resolution Visualize the spectra of input and output signals. What does this measurement show? Focus on the bandwidth where all the important features of the system are included and modify the frequency resolution to improve the representation of these features. Fix the frequency resolution for the remainder of this lab.

2.6.2 Compare the FRFs obtained with different excitation signals

To quantify the influence of the choice of the excitation signal on the quality of the measured FRF, the experiment is repeated for the different signals given below. A distinction is made between periodic and aperiodic signals.

With the procedure explained in the previous section, perform the measurements for the following signals:

Periodic

(a) The Schroeder multisine designed in the previous section.

(b) A multisine with constant phase and excitation in the specified analysis band only.

(c) A multisine with an arbitrary (random) phase and excitation in the specified analysis band only.

(d) A periodic noise signal.

Aperiodic

(e) An aperiodic noise signal. Remember that here measuring P 'periods' requires to load P different signals in the AWG.

Window the aperiodic noise signal using a Hann window and observe the results. How are the results in comparison to the signal without the window?

- **Hint:** You can generate a Hann window using the Matlab command as follows: `[Hann] = hanning(N,'periodic')`

QUESTION 2.10. FRF For each measurement, compute the FRF and provide a plot in the report. Which differences do you observe?

2.7 The LabVIEW-interface corner

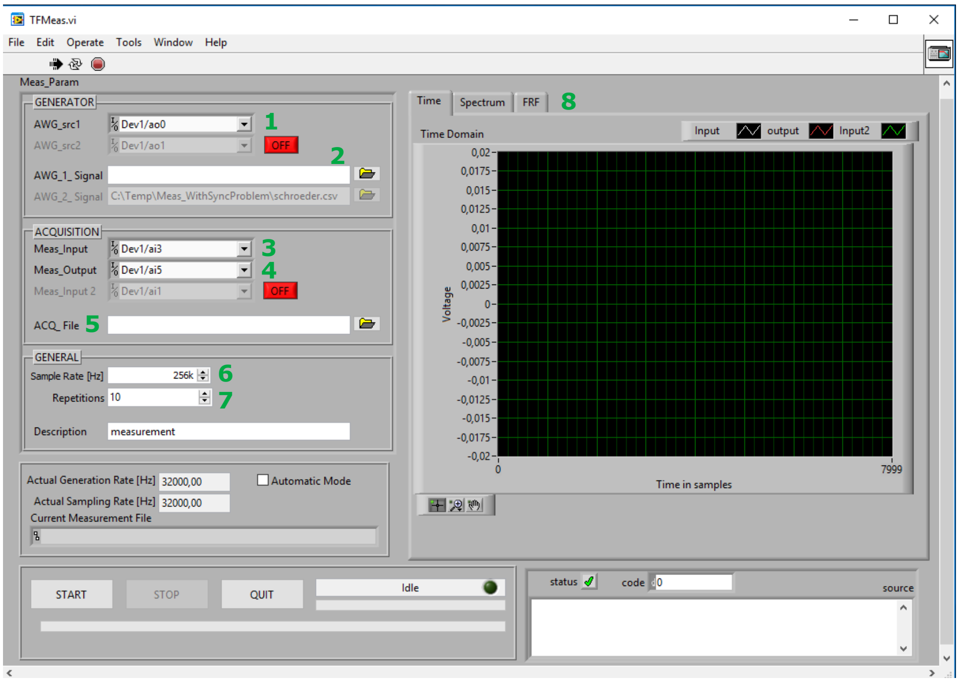


Figure 2.5: LabView-interface.

Figure 2.5 shows the interface of the LabVIEW Virtual Instrument (VI) that is used in the lab. The main parts are numbered on the figure and are explained below:

1. The connection that is used to connect a signal to the generator.
2. The path to the file of the input signal. The file must be in `.mat` format.
3. The connection that is used to connect a signal to the measured input.
4. The connection that is used to connect a signal to the measured output.
5. The path to save the measured signals. The file must be in `.mat` format.
6. The sampling rate of the generator and the acquisition channels.
7. The integer number of periods of the signal that are to be acquired.
8. The measured signals (input & output) are shown here in the time domain (first tab) and the frequency domain (second tab). The third tab shows the Frequency Response Function (FRF).

Obtaining a correct measurement requires that all the settings are filled out properly.

Next, the `start` button is used to fire up the measurement. A message that shows the actual status is shown during the measurement and the progress bar indicates the ratio of the measurements that are already done over the total count of requested measurements. The measurement can be stopped by using the `stop` button and the measurement program can be totally interrupted using the `quit` button.

Lab 3

FRF Measurements in the presence of noise

Unlike simulations, real life experiments will always be influenced by noise. This noise can lead to a biased estimate and/or high variability of your model if not taken into account. A solution to this problem is averaging over multiple repetitions to obtain the Frequency Response Function (FRF). In this lab assignment we study different methods to measure/estimate the FRF. The results obtained will be compared and evaluated.

3.1 Different averaging methods to calculate the Frequency Response Function

Consider that the measured input and output data records $u(nT_s)$ and $y(nT_s)$ have been obtained. Herein,

$$\begin{aligned} &n = 0, 1, \ldots, N-1 \\ &T_s = 1/f_\mathrm{s} \end{aligned} \tag{3.1}$$

N is the number of measured samples and f_s is the sampling frequency at which the samples are taken.

Define $U(k)$ and $Y(k)$ as the discrete Fourier transform (DFT) of the measured time records:

$$U(k) = \mathrm{DFT}\left\{u(nT_s)\right\} \tag{3.2}$$

$$Y(k) = \mathrm{DFT}\left\{y(nT_s)\right\} \tag{3.3}$$

$$\tag{3.4}$$

with $k = 0, 1, \ldots, N-1$. In the ideal periodic case, the FRF $H(jk\omega_0)$ is simply the division of the DFT spectra.

$$H(jk\omega_0) = \frac{Y(k)}{U(k)} \tag{3.5}$$

where the angular frequency $\omega_0 = 2\pi f_\mathrm{s}/N$.

Under practical conditions, the measurements will always be distorted by measurement noise. As a result, the calculated FRF will only approximate the ideal one with a finite accuracy. To improve the accuracy, one can measure the data record more than once (spending more time and money). Those multiple repetitions $u_i(nT_s)$ and $y_i(nT_s)$, $i = 1, 2, \ldots, M$, can then be averaged to improve the accuracy and precision. This averaging can be performed in different ways, resulting in different results, even when fed by the same data. We will now look at the different methods and determine their advantages and disadvantages.

1. Averaging time domain data records:

$$u(nT_s) = \frac{1}{M}\sum_{i=1}^{M} u_i(nT_s) \tag{3.6}$$

$$y(nT_s) = \frac{1}{M}\sum_{i=1}^{M} y_i(nT_s) \tag{3.7}$$

$$H(jk\omega_0) = \frac{\mathrm{DFT}\left\{y(nT_s)\right\}}{\mathrm{DFT}\left\{u(nT_s)\right\}} \tag{3.8}$$

2. Averaging the DFT spectra:

$$U(k) = \frac{1}{M}\sum_{i=1}^{M} U_i(k) \tag{3.9}$$

$$Y(k) = \frac{1}{M}\sum_{i=1}^{M} Y_i(k) \tag{3.10}$$

$$H(jk\omega_0) = \frac{Y(k)}{U(k)} \tag{3.11}$$

3. Averaging the FRF:

$$H_i(jk\omega_0) = \frac{Y_i(k)}{U_i(k)} \tag{3.12}$$

$$H(jk\omega_0) = \frac{1}{M}\sum_{i=1}^{M} H_i(jk\omega_0) \tag{3.13}$$

4. Averaging the auto-power of the input signal and the cross-power:

$$S_{YU}(k) = \frac{1}{M}\sum_{i=1}^{M} Y_i(k)U_i^*(k) \tag{3.14}$$

$$S_{UU}(k) = \frac{1}{M}\sum_{i=1}^{M} U_i(k)U_i^*(k) \tag{3.15}$$

$$H_1(jk\omega_0) = \frac{S_{YU}(k)}{S_{UU}(k)} \tag{3.16}$$

5. Averaging the auto-power of the output signal and the cross-power results in similar equations:

$$H_2(jk\omega_0) = \frac{S_{YY}(k)}{S_{UY}(k)} \tag{3.17}$$

For the first two methods to work properly the data repetitions have to be identical up to the noise contribution. This calls for a periodic excitation signal and a triggering signal that ensures that each measurement starts at a fixed point in the period of the excitation. The last two methods are also applicable to the measurements obtained with noise as an excitation signal (i.e. with *arbitrary* excitation). They do not at all require a triggering of the records but come at the cost of a bias. Fortunately, this bias is proportional to the signal to noise ratio of the signal. Therefore, it can often safely be neglected. However, when the signal to noise ratio is small or the required accuracy is high, it can be important. Remember that it can be removed completely without an increase in the measurement time if a different (periodic) excitation would be selected! The bias can be proven to be such that the true value of the FRF is bound by the H_1 and the H_2 estimate,

$$|H_1| \leqslant |H| \leqslant |H_2| \tag{3.18}$$

For a fast conversion of the signals between the time and the frequency domain the use of the FFT is preferred.

QUESTION 3.1. Length data records. Note that this algorithm works faster when the length of the data records, N, is a power of 2. Do you know why?

3.2 Excitation signals

Two types of signals will be used in this lab:

- Aperiodic noise signals
- Multisine (Schroeder or random phase)

These signals have been generated and used in **Lab 2**. However, if you have generated your multisine in the time domain you might have noted that it was quite tricky and time-consuming. Therefore in this lab we suggest that you generate the multisine from the frequency domain using the following hints:

Hint 1: Define the DFT spectrum $U(k)$ for $k = 0, 1, 2, ..., \frac{N}{2}$, and $U(k) = 0$ for $k > \frac{N}{2}$, calculate `u = ifft(U)`. Apply a scale factor $N/2$ to compensate for internal scaling factor of the FFT. Notice that the FFT coefficient of a sinusoid with zero phase is $e^{(-j\frac{\pi}{2})}$ for the positive frequency. See also **Task 1.4.1** in **Lab 1**.

Hint 2: Make use of the fact that $U(k) = U^*(N-k)$ and $xy + x^*y^* = 2\,\mathrm{real}\,(xy)$.

QUESTION 3.2. Multisine frequency domain. In your report, show the Matlab code in which you generate the multisine signal (either with Schroeder or random phase). Make sure that the code is sufficiently commented to improve its readability.

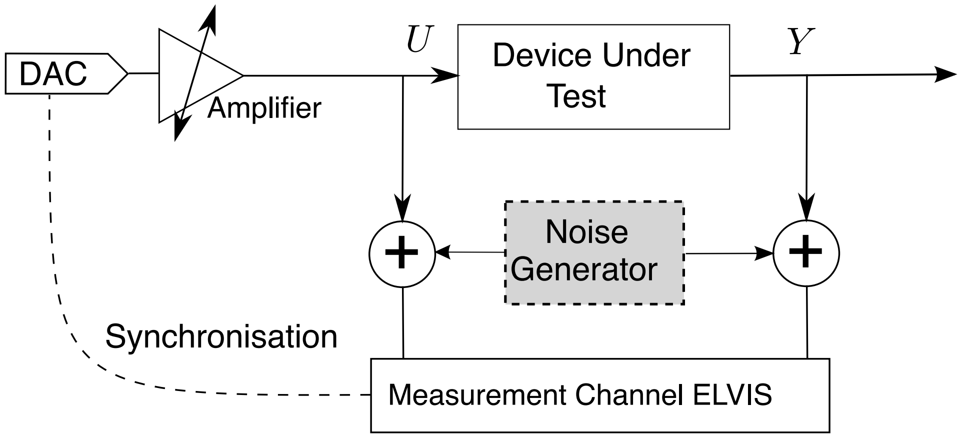


Figure 3.1: Measurement Setup

Let's go practical!

The measurement setup used in this lab is the same as in **Lab 2**, with the addition of a noise generator interposed between the Elvis and the DUT (see Figure 3.1). The Noise Generator adds noise to the measured input and output signals. We use it in this lab so that we can study the effects of noise while computing the FRF of the system. The noise level that is added to the measurements can be tuned manually.

Periodic multisine signal

- Construct a multisine with random or Schroeder phase such that
  - The sampling frequency is 16 kHz
  - The excited lines are present at frequencies between 4 Hz and 1000 Hz
  - The RMS value of the input signal is $V_{\mathrm{RMS}} = 500\,\mathrm{mV}$.
  - The signal contains $N_1 = 4000$ samples, and this corresponds to one period of the multisine.
- Perform the measurement on the DUT. Measure 40 repetitions of the input and output signals.

Aperiodic noise signal

- Construct an aperiodic noise signal, such that
  - The sampling frequency is 16 kHz.
  - The RMS value of the input signal is $V_{\mathrm{RMS}} = 500\,\mathrm{mV}$.
  - The signal contains $N_2 = 40 \cdot N_1 = 160000$ samples.
- Perform the measurement on the DUT. Measure a single repetition.

3.2.1 Prepare the data

Before applying the different averaging techniques, you need to 'prepare the data' to make sure that they can easily be analyzed. For each measurement, we are going to discard the first 8 repetitions, and you need to create an input matrix 'umat' and and output matrix 'ymat' of size $N_1 \times 32$. For the measurement with the multisine, you can use the provided function `ReadDataLab2`, while for the aperiodic noise you can use the Matlab function `reshape`.

3.2.2 Averaging the measurements

Apply the techniques that were described in Section 3.1 to the measured signals[1]. The lab files contain a function called `TransferFunc` that calculates the FRF and its standard deviation when fed with time records. This function does all the pre-processing and even calculates the standard deviation. However, there is a small part that is missing: the actual calculation of the FRF requires that the user provides a MATLAB function. The explanation of the technicalities that are needed are given in Section 3.3 on page 25. The idea is that you write the MATLAB functions that calculate the FRF using the proposed averaging methods when fed with repeated time domain records.

QUESTION 3.3. Which techniques are applicable and which are not? What is the reason for this?

QUESTION 3.4. Provide relevant plots of the estimated FRF, with the different methods, signals and number of averaged records.

QUESTION 3.5. Determine the effect of the number of averaged records on the variability of the averaged result. Compare the standard deviation.

QUESTION 3.6. Discuss the differences and explain according to you, which will deliver the best result. Discuss the pros and cons of each excitation signal.

- *Good to know*: the Standard Deviation (STD) is obtained as the sample variance of the FRFs that were calculated for each repetition of the measurement. For example, if 32 repetitions are available and one wants to obtain the variance based on the processing of 4 averaged data records per FRF calculation, only 8 repeated FRF measurements can be obtained. One of these FRFs is considered to be the result, while the standard deviation is evaluated using the 8 obtained FRFs. Note that as only 32 repetitions of the measurements are present, there is only 1 FRF that can be calculated by taking the 32 repeated experiments into account and therefore the provided processing routine can not calculate the sample standard deviation of this FRF.

[1] For the cross- and auto-spectra methods, calculating either H_1 or H_2 is sufficient.

3.3 MATLAB self-help corner

3.3.1 `ReadDataLab2`

`function [umat,ymat]=ReadDataLab2(N,Nrep,Drep,FileName)`
Starting from a measurement with multiple repetitions stored in FilaName, it combines all the measured input signals into one matrix, and all the measured output signals into another matrix.

Usage:

- `[umat,ymat] = ReadDataLab2(N,NRep,DRep,Filename)`

Input arguments:

`N` Length of one repetition (Ex: length(Su))

`Nrep` How many repetitions were measured

`Drep` How many repetitions you want to retain

`FileName` Name of the matfile where the measurement is stored. Ex: 'Groupx_Outputk.mat'

Output arguments:

`umat,ymat` Matrices with the selected repetitions of the input/output. Size:[NxDrep]

3.3.2 `TransferFunc`

`function [H, stdH] = TransferFunc(umat, ymat, Avgs, HFunction)`

`TransferFunc` calculates the FRF and its standard deviation, starting from a series of time domain measurements.

Usage:

- `[H, stdH] = TransferFunc(umat, ymat, Avgs, @HFunction)`

Input arguments:

`umat,ymat` Matrices containing the time domain measurements of the input and output respectively. Consecutive experiments are stored column-wise.

`Avgs` Specifies how many repetitions must be averaged for the calculation of the FRF. If this is an array, all calculations are repeated for each element of this array and different FRFs are returned, one corresponding to each element.

`HFunction` MATLAB string containing the name of the function or function handle to be used for the actual calculation of the FRF. You must write such a function for every averaging method. See later for the requirements on these functions.

Output arguments:

`H` FRF of the system. If `Avgs` was an array, there is one FRF for each element of that array, and they are stored column-wise.

`stdH` Standard deviation on the different calculated FRFs, also stored column-wise.

3.3.3 `HFunction`

```
function [H] = HFunction(u, y)
```

For each averaging technique, you should write yourself the corresponding averaging function `HFunction`. You should name the function differently for each of the averaging technique it is performing.
Given an input matrix u of size [N_1xNrep] and an output matrix of the same size, the function should output the estimated transfer function H, of size [N_1x1], obtained using the respective averaging technique.

Lab 4

Distortion measurements

The goal of this lab assignment is to characterize the (nonlinearities) in the response of a system. In this lab we study the response of a nonlinear system using a single sinewave as input.

4.1 The concepts of Distortion and Nonlinearity

For a Linear Time Invariant (LTI) system whose transfer function is labelled $H(f)$, it has been proven before that when it is excited by an input signal $u(t) = A_1 \cos(2\pi f_1 \cdot t)$ results in a response that is again a sinewave at the same frequency, but with a different amplitude and a different phase with respect to the input signal:

$$y(t) = |H(f_1)| A \cos(2\pi f_1 \cdot t + \angle H(f_1)) \tag{4.1}$$

Any system that obeys the superposition principle is a linear system per definition. As a consequence, all the systems that do not obey the superposition principle are called nonlinear. This negative definition has the disadvantage that it is too general. Therefore, we will restrict the class towards systems that have a close to linear behaviour.

4.1.1 Static nonlinear system

A STATIC NONLINEAR SYSTEM is a nonlinear system where the relation between the input signal u and the output signal y can be described by a function. We will assume that this function can be written as a polynomial series expansion of the response with respect to the input signal in a given interval:

$$y(t) = \sum_{k=0}^{\infty} K_k u^k(t) = K_0 + K_1 u(t) + K_2 u^2(t) + \ldots \tag{4.2}$$

When nonlinear systems are considered, equation (4.1) is no longer valid. Consider that we have a static nonlinear system containing only a quadratic and a cubic response term. The output response $y(t)$ of this system to an excitation $u(t)$ is then given by:

$$y(t) = u(t) + u^2(t) + u^3(t). \tag{4.3}$$

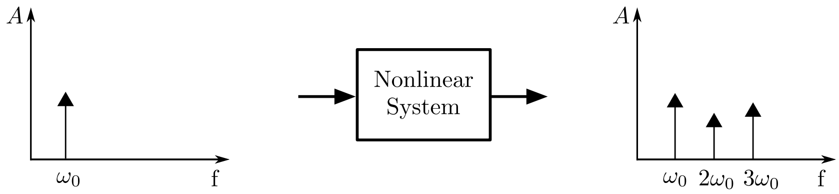


Figure 4.1: Input and output of a nonlinear system excited by a sinewave.

Whenever $u(t) = \cos(\omega_0 t)$, the output $y(t)$ can be rewritten as:

$$y(t) = \sum_{k=1}^{3} \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right]^k = [\cos(\omega_0 t)] + \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t) \right] + \left[\frac{1}{2}\cos(\omega_0 t) + \frac{1}{4}\cos(3\omega_0 t) \right] \tag{4.4}$$

The linear part of the response results in an output at the frequency $\omega_0 = 2\pi f_0$. The quadratic part of the response creates a DC component and a component at the second harmonic $2\omega_0$. Finally, the cubic term creates energy at the fundamental frequency ω_0 and the third harmonic $3\omega_0$ (see Figure 4.1). Hence, the response now contains energy at frequencies that are integer multiples of the basic frequency ω_0. This type of behaviour is called HARMONIC DISTORTION.

If the input signal is a sum of two sinewaves, the response becomes more complex, as $x(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$, results in an output signal as is shown below:

$$\begin{aligned} y(t) &= [\cos(\omega_1 t) + \cos(\omega_2 t)] \ldots \qquad (4.5) \\ &+ \left[1 + \frac{1}{2}\cos(2\omega_1 t) + \frac{1}{2}\cos(2\omega_2 t) + \cos((\omega_1 + \omega_2) t) + \cos((\omega_1 - \omega_2) t) \right] \ldots \\ &+ \left[2\cos(\omega_1 t) + 2\cos(\omega_2 t) + \frac{1}{2}\cos(3\omega_1 t) + \frac{1}{2}\cos(3\omega_2 t) + \frac{3}{4}\cos(\omega_2 t - 2\omega_1 t) \ldots \right. \\ &+ \left. \frac{3}{4}\cos(\omega_2 t + 2\omega_1 t) + \frac{3}{4}\cos(\omega_1 t - 2\omega_2 t) + \frac{3}{4}\cos(\omega_1 t + 2\omega_2 t) \right] \end{aligned}$$

The linear term results now in energy that appears at the frequencies ω_1 and ω_2. The quadratic term will not only produce a contribution at DC and at the frequencies $2\omega_1$ and $2\omega_2$, but also at the sum and difference frequencies $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$. The cubic term creates energy at the frequencies ω_1, ω_2, $3\omega_1$ and $3\omega_2$ as could be expected, but in addition to that, contributions also appear at $\omega_2 - 2\omega_1$, $\omega_2 + 2\omega_1$, $\omega_1 - 2\omega_2$ en $\omega_1 + 2\omega_2$. Besides the harmonic distortion that was expected to be present at multiples of ω_1 and ω_2, a new type of contribution pops up. It is called the intermodulation distortion and appears at frequencies that are a combination of ω_1 and ω_2 simultaneously.

Determining the frequencies where energy is expected to pop up at the output fortunately does not always require to fully work out the equations (4.4) and (4.5). A simple rule of thumb can be used, and it is possible to derive it based on the simple example given above. An important point to be remembered is that a sinewave does not only have a contribution at positive frequencies, but also conveys energy at the negative frequency. Consider again a sinewave input as in (4.4). The lin-

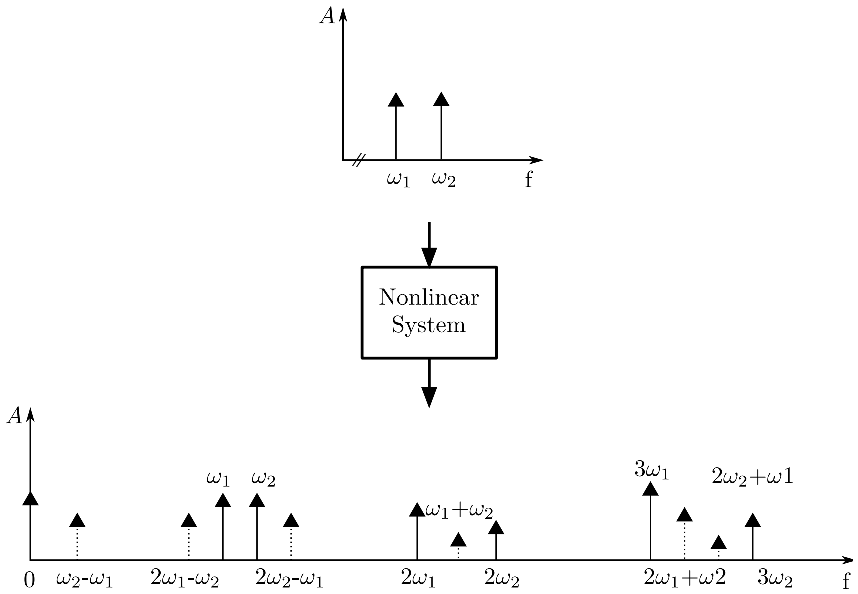


Figure 4.2: Input and output spectrum of a nonlinear system (full line = harmonic distortion, dashed line = intermodulation distortion).

ear term u^1, only creates energy in the output signal at frequencies that match the frequency of the input signal.

Input	Output
f,$-f$	f,$-f$

To evaluate the frequencies at which the quadratic term u^2 creates contributions, we must synthesize all possible outcome of the sum of 2 frequencies that are present in the initial signal,

Input	Output	
$f, -f$	$-f - f$	$-2f$
	$-f + f$	DC
	$f + f$	$2f$

For u^3 we now have to combine 3 frequencies, taking the negative frequencies in consideration.

Input	Output	
$f, -f$	$-f - f - f$	$-3f$
	$-f - f + f$	$-f$
	$f + f - f$	f
	$f + f + f$	$3f$

To obtain the output contributions for a two-tone input signal, the reasoning is completely similar. The linear term does only create contributions at the excited

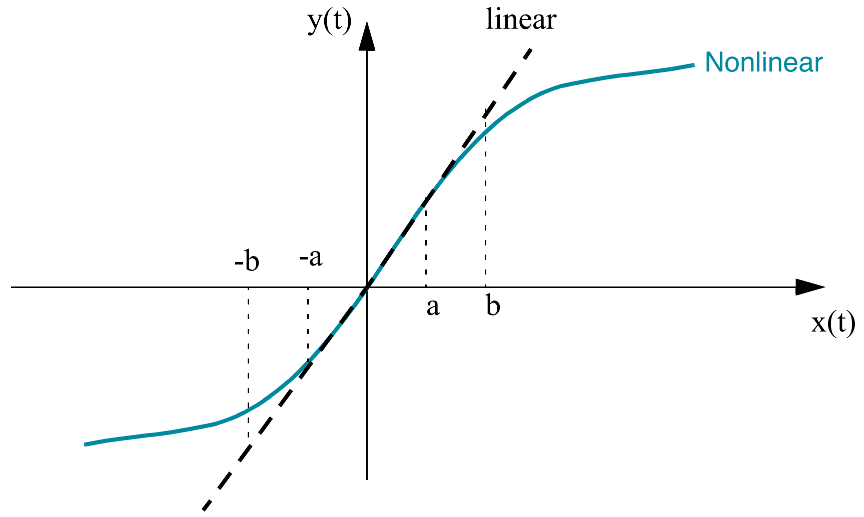


Figure 4.3: A static nonlinear response.

frequencies, the quadratic term creates energy at all frequencies that can be reached by the sum of 2 frequencies that are present in the input signal and the cubic term reaches all frequencies that can be written as the sum of 3 input frequencies.

Figure 4.3 clearly illustrates that for an amplitude that lies between a and $-a$ the nonlinear curve is very close to its linear component, while for a range of amplitudes between b and $-b$ the difference with the linear term is way bigger. Stated differently, one sees that the nonlinearity is way less excited by the first range of excitations and hence the harmonic contributions will remain smaller.

4.1.2 1 dB expansion or compression points

When a linear system is excited with an input signal whose amplitude is gradually increased starting from zero, it can be expected that there will be very little influence of the nonlinearity for the very low amplitudes. The key issue here is to know what a low amplitude is for a certain considered system. To quantify the amplitude level at which the system becomes significantly nonlinear, a number of figures of merit have been defined. Here we will take a look at the 1 dB compression point and the intercept points.

For the linear time invariant systems it is quite easy to understand that the output power and the input power are linearly related considering equation (4.2), the following relation holds:

$$P_y = K_1^2 P_u = \tilde{K}_1 P_u \tag{4.6}$$

For a nonlinear system the response will gradually start to deviate from this linear law when the power of the input signal is increased. When the output power increases slower than predicted by the linear law (4.6), COMPRESSION occurs. If the output power increases faster than the predicted law, the nonlinear behaviour is called EXPANSION.

Since the compression or expansion behaviour increases gradually, a certain level of deviation has to be set in order to condense the characteristic to a single

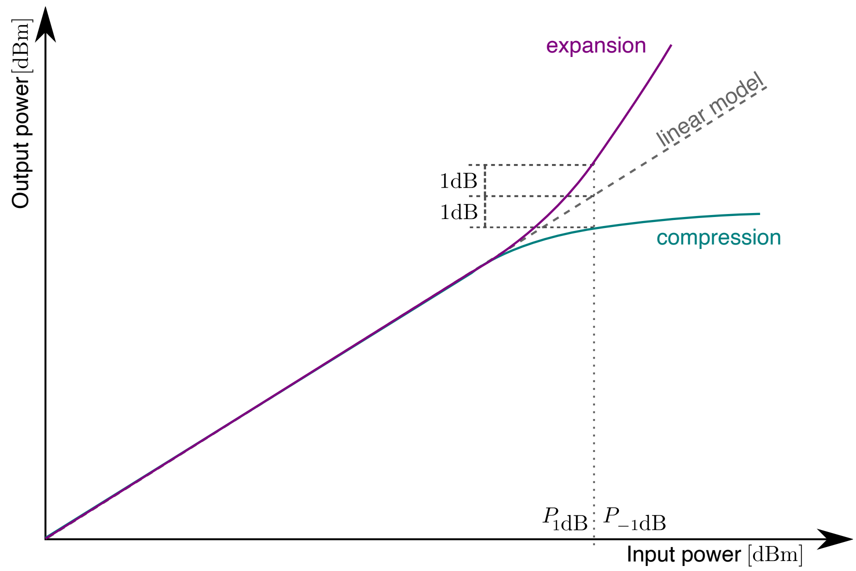


Figure 4.4: Compression and expansion with the 1dB points.

number. Often, a 1 dB level is selected and the corresponding points are called the 1 dB compression point ($P_{-1\,\mathrm{dB}}$) for a system that compresses or the 1 dB expansion point ($P_{+1\,\mathrm{dB}}$) for a system that expands the amplitude.

In Figure 4.4 the linear law as taken from (4.6) is represented by a dashed line. Around it, a system that exhibits compression and one that exhibits expansion are shown on log-log scale, input amplitude versus output amplitude. The 1 dB compression and expansion points are then easily obtained as the input powers where the difference between the nonlinear and the linear characteristic reaches 1 dB (dotted line).

4.1.3 Evaluating the linear and the nonlinear response

We know that most real systems will exhibit harmonic distortions. When excited with a pure sinewave at frequency f_0 the output will contain energy at the harmonics of this frequency, namely at (0, f_0, $2f_0$, $3f_0$, ...).

QUESTION 4.1. Consider the nonlinear static system whose response is given by:

$$y = x - \frac{1}{2}x^3 - \frac{1}{4}x^4. \tag{4.7}$$

What are the frequencies at which energy will appear at the output when the input is excited by the sum of 2 sinewaves, one at frequency 4 Hz and one at frequency 11 Hz? Include the answer in the Lab report.

4.2 Time to go practical!

In this lab assignment, all measurements need to be taken at a sampling frequency of 10 kHz and records that contain 4096 samples are acquired. Remember that real systems are dynamic, hence they are bound to transient behaviour and this has to be removed before the processing happens. Therefore, the measurements are repeated several times (with the generator left on) to allow that the transients damp. Check that the transients are indeed gone before you start the data processing!

QUESTION 4.2. Design and calculate an excitation sinewave with a frequency of 100 Hz and an amplitude of 1 V. After obtaining the output signal from the DUT, plot the input signal and the output signal of the DUT in the frequency domain.
What do you observe? How can you remove the unexpected behaviour? Give a list of possible solutions. Design experiments to test your solutions and mention the experiment that shows that the proposed solution does indeed work. Mention all of your solutions in your report.

QUESTION 4.3. Change the excitation frequency to a frequency in the neighborhood of 100 Hz such as to solve the problem in the previous step. Your signal must contain several periods of the single sine. Plot the DFT of only one period.
What do you observe this time? Is it what you expect? Is it possible to use a single period for distortion detection? What is your solution? Explain them and use one of them for the measurement.

QUESTION 4.4. Change the excitation frequency to a frequency in the neighborhood of 100 Hz again. Use approximately 10 different amplitudes that are spaced properly to allow a determination of the compression and intercept points (from 100 mV to 1.1 V amplitude, **logarithmic spacing** [`logspace` command in MATLAB]). Do not forget to save the values of these amplitudes, as you will need them afterwards. Then concatenate all of the signals in a row as your input. Plot the DFT of the output for a single period for each amplitude.
What are the frequencies at which you expect that distortion will appear? Are they all present in reality? Explain them in detail in your report.

QUESTION 4.5. Plot the measurements on an amplitude in – amplitude out plot. Do you see compression or expansion? From this plot, estimate the 1 dB compression/expansion point.
Check your estimate. Generate a single sine wave with the determined amplitude and see how accurate your guess is.