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Bilateral Bargaining in the Interbank Market

by

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Abstract

This thesis analyses unsecured overnight interbank market lending activities as bilateral bargaining games between borrowers and lenders, with interbank rates as bargaining outcomes. Banks regard the central bank as the outside option and may face asymmetric information when bargaining. The efficiency of the bargaining outcomes depends on information completeness. A strategic bargaining model is introduced to explain the formation of interbank trades and various deal rates between different banks. In particular, banks' information set and beliefs effect their strategies and consequently the bargaining results. The parameters of the model are calibrated using money market data reported by EU agents.

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Contents

1 Introduction

During the global financial crisis of 2007-2008, liquidity risk turned out to be the key risk driver. The money market, the core of how liquidity demand and supply is matched, all but froze. Central banks became the intermediary for large parts of the money market. What activity remained was moving from unsecured to secured funding sources. This experience has stimulated the debate about the organisational structure of interbank markets, in particular how bouts of market uncertainty can lead to sudden shifts from one part of the market (e.g. unsecured) to other parts (e.g. secured or synthetic leverage through derivative positions). To assess what drives banks choices for different funding channels, one needs to gauge the extent to which private information about counterparties affects the liquidity allocation in money markets.

In this thesis, two different funding channels for banks are considered: the central bank and the unsecured interbank market. Banks constantly face liquidity shocks, and they choose among different funding channels from either the central bank or interbank market in order to meet the reserve requirement set by the central bank or for speculative reasons. The central bank provides unlimited access for banks, with a high lending rate and a low deposit rate and funding secured by collateral. Alternatively, banks can trade with other banks in the unsecured market, with an interest rate within the central banks interest rate corridor. Banks with liquidity deficit can choose to obtain secured funding from central banks or borrow from other banks in the unsecured market, and banks with liquidity surplus can either deposit at the central bank or lend unsecured fund to other banks.

In the unsecured interbank market, banks approach their counterparties and bilaterally bargain over the interbank rates. A transaction is made once an agreement is reached, otherwise banks part ways and search for other counterparties for bargaining and trading.

Several papers have focused on the formation of unsecured interbank market and how interbank rates are determined. In Vollmer and Wiese (2018) [1], interbank lending activities are modelled based on Diamond-Dybvig model ([2]). The lending rates are regarded as outcomes of bilateral bargaining under symmetric information, and Nash bargaining solution is applied. It shows that the success of interbank lending depends on the profitability of borrowers' investment projects. The model in Heider et al. (2009) [3] is another extension of the Diamond-Dybvig model, where banks' private information about assets and counterparty risk are taken into account. It explains the functioning and breakdowns in interbank market. The equilibrium conditions in [3] are derived from optimization problems under liquidity constraints. In Blasques et al. (2018) [4], the interbank rate is determined by a generalized Nash bargaining solution and a structural micro-founded dynamic stochastic network model is introduced to explain important dynamic features of the interbank market network. In particular, monitoring of counterparty risk and directed search are shown to be key factors in the formation of interbank trading relationships that are associated with improved credit conditions. Then the model was used to filter the optimal search and monitoring expenditures in the network and to analyse optimal network responses to changes in the policy of central banks. Afonso and Lagos in [5] model the fed fund market dynamically while taking into account counterparty search and bilateral negotiation. The model explains how fed funds rates are determined and market allocation of funds. The negotiation outcomes between banks are considered as Nash bargaining solutions and the counterparty risk is not taken into account.

One problem with the mentioned literature above is the assumption of symmetric information in the bargaining procedure. Banks' credit conditions are not completely transparent to each other, so banks bargain under asymmetric information over the interbank lending rate in the interbank lending market. This informational asymmetry is considered as a common explanation for interbank lending breakdowns. However, this asymmetric feature for bargaining is not taken into account in [1] and [4] when modelling bargaining in the interbank market, and thus the assumption of bargaining is problematic.

In this thesis, a mathematically rigorous description of the bilateral bargaining problem is studied under information asymmetry conditions. A screening bargaining procedure is presented for analysing the formation of interbank lending relationships under asymmetric information, with the interbank rates as bargaining outcomes. Then this bargaining model is analysed using hypothetical data based on actual money markets.It is shown that under this model, banks' bargaining positions are effected by the information obtained from trading history and new information on relative default risk between borrowers and lenders. Finally, the estimated model is used to study the formation of interbank lending market.

To better understand the factors that determines the formation of unsecured market and to avoid skewed results, the maturities of interbank loans are taken as control variables in the following chapters. In particular, only overnight loans in unsecured interbank market are considered. Compared with interest rates in unsecured term money market, unsecured overnight interbank rates are less influenced by funding liquidity risk, as is discussed by Tapking and Eisenschmidt in [6].

The outline of the thesis is as follows. Chapter 2 introduces prerequisites in bargaining theory relevant to research methods in later chapters, including basic bargaining frameworks and fundamental concepts. In particular, bargaining games under two different categories are discussed, namely, (i) bargaining under complete information and (ii) bargaining under incomplete information. In Chapter 3, I begin analysing the interbank bargaining problem with addressing the normative question: can the bargaining outcome can be efficient? It is shown that the completeness of information effects the efficiency of a bargaining game. An increasing uncertainty between players in a bargaining game can decrease the probability of inducing an efficient outcome. In Chapter 4, a sequential bargaining model is used for analysing the interbank bargaining game. The parameters of the model are estimated by calibration using the hypothetical data. The estimated bargaining model explains how the unsecured overnight interbank rates are determined in the over-the-counter market. In particular, the unsecured overnight interbank rates are influenced by private information, the borrower's and the lender's beliefs on their counterparties' bargaining power. The formation of these beliefs are greatly effected by banks' mutual trust based on their transaction in the past and new market information.

2 Bargaining Theory

Consider bilateral bargaining in the unsecured overnight interbank market between a borrower B and a lender L. The borrower suffers from liquidity deficit and searches for potential lenders with liquidity surplus, and promises to repay the loan on the next day. Due to the over-the-counter nature of the unsecured interbank market, the interbank rate between the borrower and the lender is determined through bargaining.

A general description of bargaining can be found in [7] and [8], which refers to a process through which the players try to reach an agreement on their own. In such situations:

1. Each player has the possibility of concluding a mutually beneficial agreement;
2. There is a conflict of interests about which agreement to conclude;
3. No agreement may be imposed on any player without his approval.

In the unsecured interbank market, banks can contact their potential counterparties for bargaining if both sides of bargaining games think it is mutually beneficial. When bargaining over the interbank rates, the borrower and the lender have conflicting interests over all possible values of interbank rates. For example, an increase in the interbank rate will increase the lender's expected profit but decrease the borrower's. Under a given setting, there are various possible bargaining procedures that induce different outcomes. The bargaining process typically includes offers and counteroffers made by players to each other, and ends either an agreement is struck or at least one player quits.

Bargaining is a situation that exists in many fields such as economics and politics where decisions are made through human interactions, so it is necessary to study bargaining under different settings. The main focus of bargaining theory is on the efficiency and distribution of the bargaining outcome. Efficiency relates to the possibility that the bargaining outcome is Pareto efficient[1]. Distribution relates to how the total gain of the bargaining game is divided among players, which is determined by players' relative bargaining power.

This chapter is a review of bargaining theory based on [7] and [8]. In particular, I focus on bilateral bargaining situations. All the bargaining theories discussed in this chapter assume that the players are rational, with either complete or incomplete information. That is, whenever a player has to choose from several alternatives, he chooses the one that yields a most preferred outcome. Furthermore, the difference between players' bargaining skills is abstracted from bargaining theories.

[1] An outcome of a game is Pareto efficient iff there is no other outcome that would make all players better off.(See [9] page 97)

2.1 Bargaining under Complete Information

In this section, I introduce two bargaining models: Nash's axiomatic model and Rubinstein's strategic model, where players have complete information about each other. In particular, I restrict my attention to bilateral bargaining situations where only two players are involved. As the name suggest, in Nash's axiomatic model the outcome is determined by four axioms that the outcome is required to satisfy and the bargaining solution is defined as a simple formula. In Rubinstein's strategic model, the solution is an equilibrium of an explicit model of a sequential bargaining process.

2.1.1 Nash Bargaining Solution

Consider the bargaining framework in [10], where no bargaining procedure is explicitly described and the solution is derived axiomatically.

Denote the set of players by $N = \{1, 2\}$. At the end of a bargaining game, the players either reach an agreement $a \in A$ or a disagreement D. Assume $\mathcal{M}$ is a convex subset of the set of all probability measures on the measurable space $(A \cup \{D\}, \sigma(A \cup \{D\}))$, then any probability measure in $\mathcal{M}$ is a *lottery* (see [11] Chapter 3). Each player $i \in N$ has a preference ordering $\succeq_i$ defined on the set of lotteries over $A \cup \{D\}$. Assume each player's preference ordering allows for a Von Neumann-Morgenstern representation, then for each player $i \in N$ there exists a utility function $u_i : A \cup \{D\} \to \mathbb{R}$ that is unique up to affine transformation and characterizes the preference ordering. In other words, a lottery is preferred to others if and only if the corresponding expected utility of this lottery is greater than its alternatives. Therefore, to describe the preferences of players in a bargaining game, it is sufficient to consider all utility pairs that can be the bargaining outcome. Let $S := \{(u_1(a), u_2(a)) | a \in A\}$ be the set of all possible utility pairs if an agreement is struck in the bargaining game, and $d = (u_1(D), u_2(D))$ be the utility pairs of the disagreement event D.

A *bargaining problem* is a pair (S, d) where $S \subset \mathbb{R}^2$ is the set of possible utility pairs obtained from agreement and is compact and convex, $d \in S$ is the utility pair obtained from disagreement, and there exists $s \in S$ such that $s_i > d_i$ for $i = 1, 2$. The set of all bargaining problems is denoted by $\mathcal{B}$. A *bargaining solution* is a function $f : \mathcal{B} \to \mathbb{R}^2$ that assigns a unique element of S to any bargaining problem $(S, d) \in \mathcal{B}$.

Instead of studying bargaining procedures explicitly, the bargaining solution f of Nash bargaining model is axiomatic and unique if it satisfies four axioms that seem natural for a bargaining solution. Next I explain each of these axioms in details.

The first axiom states that the players' preferences, not the utility functions, are basic. To be more specific, if the bargaining problem (S', d') is obtained from (S, d) by affine transformation $d'_i = \alpha_i d_i + \beta_i$ and $S' = \{(\alpha_1 s_1 + \beta_1, \alpha_2 s_2 + \beta_2) \in \mathbb{R}^2 | (s_1, s_2) \in S\}$ (where $i \in N$), then (S, d), (S', d') represents the same situation and any outcome corresponding to the solution of (S, d) also corresponds to the solutions of (S', d'). Essentially, it is the preferences of players, not the explicit form of utility functions, that determine the bargaining outcomes. Formally,

Axiom 2.1. *(Invariance to Equivalent Utility Representations; INV)*
If (S', d') and (S, d) are defined as above, then $f_i(S', d') = \alpha_i f_i(S, d) + \beta_i$ for $i = 1, 2$.

The second axiom requires the bargaining solutions assign the same utility for both players if the players are interchangeable. Formally,

Axiom 2.2. *(Symmetry; SYM)*
The bargaining problem (S, d) is symmetric if $d_1 = d_2$ and $(s_1, s_2) \in S$ if and only if $(s_2, s_1) \in S$. If (S, d) is symmetric, then $f_1(S, d) = f_2(S, d)$.

The third axiom requires the solution should not depend on irrelevant alternatives. In other words, if the players agree on some $s \in S \subset T$ when all elements in T are available alternatives, then the players should agree on the same outcome s when only the subset S is available.

Axiom 2.3. *(Independence of Irrelevant Alternatives; IIA)*
If (S, d) and (T, d) are bargaining games with $S \subset T$ and $f(T, d) \in S$, then $f(S, d) = f(T, d)$.

The fourth axiom requires the bargaining outcome to be *Pareto efficient.* That is, the bargaining process continues until no one can be better off without harming his counterparty's benefits. Formally,

Axiom 2.4. *(Pareto Efficiency; PAR)*
For a bargaining game (S, d), if $s, t \in S$ and $t_i > s_i$ for $i = 1, 2$, then $f(S, d) \neq s$.

Nash ([10]) in 1950 showed that there is a unique bargaining solution satisfying the four axioms above.

Theorem 2.5. *(Nash bargaining solution)*
There is a unique bargaining solution $f^N : \mathcal{B} \to \mathbb{R}^2$ satisfying the axioms INV, SYM, IIA and PAR and is given by

$$f^N(S, d) = \underset{(d_1,d_2) \leq (s_1,s_2) \in S}{\arg\max} (s_1 - d_1)(s_2 - d_2). \tag{2.1}$$

The theorem can be proved by first checking the function f^N is well defined and satisfies the four axioms, and then showing its uniqueness by contradiction. I omit the details and refer to [8].

Nash bargaining solution (2.1) depends only on the set of possible utility pairs S and the disagreement point d. However, in some cases, the bargaining outcomes also depend on other factors, such as the player's discount rates, that cannot be incorporated into (2.1). A generalization of Nash bargaining solution can be applied in such cases.

Definition 2.6. For $\theta \in (0, 1)$, a *generalized Nash bargaining solution* is a function $f^N_\theta : \mathcal{B} \to \mathbb{R}^2$ that satisfies the axioms INV, IIA and PAR. It is unique and given by

$$f^N_\theta(S, d) = \underset{(d_1,d_2) \leq (s_1,s_2) \in S}{\arg\max} (s_1 - d_1)^\theta (s_2 - d_2)^{1-\theta}. \tag{2.2}$$

Therefore the influences from factors excluded from Nash bargaining solution are taken into account in θ. Note that if $\theta = \frac{1}{2}$, then the generalized Nash solution (2.2) is identical to the Nash solution (2.1).

2.1.2 Rubinstein's Bargaining Solution

Unlike Nash's axiomatic approach, Rubinstein's model embodies an explicit description of a bargaining procedure with intuitive appeal, and provides a basic framework for bargaining models later developed.

Consider two players A and B who bargain over the partition of a good with fixed size π following an alternating offer procedure. At time 0, A offers a proposal of a partition of the good to B. If B accepts this offer, then the agreement is struck and players divide the good according to the agreed offer. If B rejects this offer, then at time Δ B makes a counteroffer. If this counteroffer is accepted by A, then the agreement is struck at time Δ, otherwise A makes a counter-counteroffer at time 2δ. The process continues until one player accepts an offer. If the players agree on a partition (x_A, x_B) at time $n\Delta$, then the player i's (i=A,B) payoff is $\delta_i x_i$, where $\delta_i = \exp(-r_i\Delta)$ is the player i's discount factor and r_i is the player i's discount rate. If the players perpetually disagree, then both players have zero payoff.

For each time $m\Delta$ ($m \in \mathbb{N}_0$), define a *subgame* as the alternating offer bargaining game starting at $m\Delta$, and the *subgame perfect equilibrium* as a Nash equilibrium of this subgame which can be employed to characterize the bargaining outcome. It has been shown in [7] that there is a unique subgame perfect equilibrium of the alternating offer game. The main steps and conclusions are discussed below. First, consider a subgame perfect equilibrium satisfying the following properties:

Property 1: No Delay Whenever a player makes an offer, his equilibrium offer is accepted by the other player.

Property 2: Stationarity In equilibrium, a player makes the same offer whenever he has to make an offer.

Given Property 2, I denote the equilibrium offer that the player i makes by x_i^*. Consider an arbitrary starting time $m\Delta$ of a subgame where A makes an offer x_A to B. The player B's payoff is $(\pi - x_A)$ if he accepts the offer. If the offer is rejected, then by the two properties above, B proposes the equilibrium offer x_B^* which is accepted by A at time $(m+1)\Delta$ and obtains payoff $\delta_B x_B^*$. Comparing the two different payoffs, player B accepts the offer x_A if and only if $\pi - x_A \geq \delta_i x_B^*$. Then by Property 1, it holds that $\pi - x_A^* \geq x_B^*$. Note that if $\pi - x_A^* > x_B^*$, then there exists some $x'_A > x_A^*$ such that $\pi - x_A^* > \pi - x'_A > \delta_B x_B^*$, which implies that (x_A^*, x_B^*) is not Pareto efficient. This inefficiency contradicts the PAR assumption of Nash bargaining solution, so

$$\pi - x_A^* = x_B^*. \tag{2.3}$$

Likewise, the same analysis can be applied for player A which gives

$$\pi - x_B^* = x_A^*. \tag{2.4}$$

(2.3) and (2.4) give a unique solution for the subgame perfect equilibrium that satisfies

Property 1 and Property 2, namely

$$x_A^* = \frac{1-\delta_B}{1-\delta_A\delta_B}\pi, \quad x_B^* = \frac{1-\delta_A}{1-\delta_A\delta_B}\pi. \tag{2.5}$$

The uniqueness of solution implies there is at most one subgame perfect equilibrium satisfying Property 1 and Property 2. In this equilibrium, the player A always offers x_A^* and accepts B's offer x_B if and only if $x_B \leq \pi - x_A^*$, and the player B always offers x_B^* and accepts A's offer x_A if and only if $x_A \leq \pi - x_B^*$. From [7] Theorem 3.1, this subgame perfect equilibrium is the unique subgame perfect equilibrium of the alternating offer game, which implies that the subgame perfect equilibrium of an alternating offer game must satisfy Property 1 and Property 2.

Note that if the interval time Δ of alternating offers is close to 0, then by L'Hospital's rule, (2.5) becomes

$$\begin{aligned} x_A^* &= \frac{1-\delta_B}{1-\delta_A\delta_B}\pi = \frac{r_B}{r_A+r_B}\pi, \\ x_B^* &= \frac{1-\delta_A}{1-\delta_A\delta_B}\pi = \frac{r_A}{r_A+r_B}\pi, \end{aligned}$$

which is consistent with the general Nash bargaining solution (2.2) when $\theta = \frac{r_B}{r_A+r_B}$. If $\Delta \to 0$ and $r_A = r_B$, then the solution is the same as Nash bargaining solution (2.1).

2.2 Bargaining under Asymmetric Information

In some bargaining games, players may have private information that is relevant to the bargaining outcome. For example, in the unsecured interbank market, borrowers and lenders do not have complete information about their counterparties' risks, liquidity conditions, etc. Incomplete information in bargaining games raises new questions for analysis, because a Nash bargaining solution and Rubinstein's strategic approach are no longer applicable. In a bargaining game with incomplete information, the bargaining procedure combines both the players' preference (as in the complete information bargaining game) and available information.

Bargaining under incomplete information is the main focus of this thesis. To avoid repetition, I introduce some fundamental and prerequisite concepts that will appear in the following chapters. More technical details can be found in Chapters 3 and 4.

Similar to the Nash equilibrium and subgame perfect equilibrium for bargaining under complete information, the concepts Bayesian Nash equilbrium and perfect Bayesian equilibrium are essential for understanding bargaining under incomplete information. A *Bayesian Nash equilibrium* is an action or a strategy that maximizes the expected payoff for each player of any possible type, given their beliefs and the strategies played by the other player. In other words, in a Bayesian Nash equilibrium each player would maximize his own expected utility when he knows his own information but only has a belief on the other players' private information.

When players move sequentially under incomplete information, some Bayesian Nash equilibria may involve strategies that are not sequentially rational. Examples can be found in [12]. Under the *perfect Bayesian equilibrium*, the strategies and beliefs are required to be *sequentially rational* and *consistent*. Sequential rationality guarantees the strategies be optimal in conditional expectations given beliefs. The strategies determine how players behave given their current information set. Consistency requires player's beliefs be updated using Bayes' rule in any game path that can be reached with positive probability. The belief of a player consists of his information about himself and his belief about the other player's private information. A *perfect Bayesian equilibrium* combines strategies and conditional beliefs that players have about the other player's types for any information set. More discussion can be found in Chapter 4.

A natural extension of Pareto efficiency is ex post efficient. A bargaining outcome is *ex post efficient* if and only if the player's payoffs related to this bargaining outcomes are Pareto efficient after all of the information is revealed ([7], p.252). For any bargaining problem, if there exists a bargaining procedure that induces a game with a Bayesian Nash equilibrium with an ex post efficient outcome, then the bargaining outcome can be ex post efficient.

One problem when analysing the possibility of an ex post efficient outcome is that all possible procedures need to be considered. Fortunately, the Revelation Principle allows the analysis to focus only on a special set of bargaining procedures - direct revelation procedures. In a direct revelation procedure, there exists a coordinator to whom both players report their own information. The coordinator then determines how the good is divided to each player. Without loss of generality, it is sufficient to restrict attention to direct revelation procedures that satisfy the following two conditions. First, both players have no incentive to lie about their true information when reporting to the coordinator. In other words, the expected payoff of a player when reporting truth is no less than that when lying to the coordinator. This condition is called *incentive compatibility*. Second, the expected payoff of each player is no less than the payoff from disagreement if the player reports truth, which is called *individual rationality*.

As the end of this section, I state the Revelation Principle in [7] that simplifies the analysis of bargaining under incomplete information.

The Revelation Principle Fix an arbitrary bargaining situation with asymmetric information and an arbitrary bargaining procedure. For any Bayesian Nash equilibrium outcome of the induced bargaining game, there exists an incentive-compatible and individually rational direct revelation procedure that implements this Bayesian Nash equilibrium outcome.

3 Bargaining Efficiency under Incomplete Information

In this chapter, I analyse the normative question about whether the bargaining result in the interbank market under incomplete information can be ex post efficient. To analyse how information completeness effects the bargaining procedures and results, in the following sections I first consider bargaining games under complete information, then I study bargaining games when banks are subject to one-sided or two-sided incomplete information. It will be shown that an increasing level of information completeness will increase the possibility of obtaining an efficient bargaining outcome.

The methods in this chapter are introduced in Chapter 9 of [7] and in [14] where bargaining games between a buyer and a seller is studied. I extend the analysis to interbank markets, which is different from the “buyer and seller” problem setup and is not mentioned in other literature. Compared with [14], the setup in Section 3.2.3 can better describe the bargaining problems in the interbank market.

Due to the over-the-counter nature of unsecured interbank market, the interbank rates can be regarded as bilateral bargaining outcomes between borrowers and lenders. Assume the counterparty default risk can be inferred from available information such as credit default swap (CDS) costs, that is, q_B is public information. However, banks may have incomplete information about their counterparties' outside options. These outside options reflect the maximum and minimum rates that the borrower and the lender can accept in the bargaining game.

To be more precise, consider a bilateral bargaining game between a fixed pair of banks: the borrower B and the lender L. They contact each other in interbank market and bargain over the unsecured overnight interbank rate. Let $r_B \in [\underline{r}, \bar{r}]$ be the borrower B's outside alternative rate that the maximum interbank rate that B can accept when bargaining with L. If the proposed rate is greater than r_B, then the borrower does not expect itself to benefit from the interbank trade with L. I call r_B the *reservation rate* of the borrower B. Similarly, let v_L be the minimum amount that the lender L expects to obtain by trading with B per unit loan. I call v_L the *reservation value* of L.

Let q_B and q_L be the probability that B and L survive (i.e. do not default), then the reservation value of the borrower B is $q_B(1+r_B)$,and the expected profits per unit from the interbank trade between L and B are

$$\begin{aligned} &v_B - q_B(1+r) = q_B(r_B - r), \quad \text{for the borrower } B, \\ &q_L\left[q_B(1+r) - v_L\right], \quad \text{for the lender } L, \end{aligned} \tag{3.1}$$

where r is the interbank rate between B and L determined through bilateral bargaining. If banks have access to central bank's standing facilities that offers deposit rate $\underline{r}$ and lending rate $\bar{r}$, then in the interbank market the borrowers' expect to accept rates lower than $\bar{r}$ because otherwise it can resort to the central banks and obtain higher profits. Likewise, the lenders' reservation value in interbank market is higher than the expected repayment $1 + \underline{r}$ using central banks' facilities, because otherwise it has no incentive to bargain in OTC market. Therefore $v_B \leq q_B(1 + \bar{r})$ and $v_L \geq 1 + \underline{r}$ for any borrower B and lender L, and the interbank rate $r \in [\frac{1+\underline{r}}{q_B} - 1, \bar{r}]$.

3.1 Complete Information

I first consider the situation when all the parameters mentioned at the beginning of this chapter (q_B, q_L, r_B, v_L) are public to both the borrower B and the lender L. It will be shown that the bargaining result under complete information can be Pareto efficient.

The interbank trade happens if and only if $r_B \geq \frac{v_L}{q_B} - 1$, that is, $v_B \geq v_L$. If not, then by (3.1) either the borrower or the lender expects a negative profit from the interbank trade and thus has no incentive to participate in bargaining.

Under complete information, the interbank rate r can be determined using Nash bargaining solution where the rate is the maximizer of the product of both sides' expected profits, namely,

$$r^* \in \underset{r \in [\frac{1+\underline{r}}{q_B} - 1, \bar{r}]}{\arg\max} \; q_B(r_B - r) \cdot q_L \left[q_B(1 + r) - v_L \right],$$

which implies $r^* = \frac{1}{2}\left(\frac{v_L}{q_B} - 1 + r_B\right)$. By (3.1), the corresponding expected profits from the trade with rate r^* are

$$q_B(r_B - r^*) = \frac{1}{2}(v_B - v_L), \text{ for the borrower B;}$$
$$q_L \left[q_B(1 + r^*) - v_L \right] = \frac{q_L}{2}(v_B - v_L), \text{ for the lender L.}$$

This implies that neither the borrower nor the lender can obtain more profit by changing the deal rate and without hurting its counterparty, and thus the bargaining result is Pareto efficient, which is consistent with the result in Chapter 2.

A generalization of the Nash solution above is to consider the borrower and the lender have different bargaining power, denoted by $\theta \in [0, 1]$. Then the bargaining result r_θ^* satisfies

$$r_\theta^* \in \underset{r \in [\frac{1+\underline{r}}{q_B} - 1, \bar{r}]}{\arg\max} \; \left[q_B(r_B - r) \right]^\theta \left[q_L \left[q_B(1 + r) - v_L \right] \right]^{1-\theta},$$

which gives

$$r_\theta^* = \theta \left(\frac{v_L}{q_B} - 1 \right) + (1 - \theta) r_B. \tag{3.2}$$

By (3.1), the expected profits from the interbank trade with rate r_θ^* are

$$q_B(r_B - r_\theta^*) = \theta q_B \left[r_B - \left(\frac{v_L}{q_B} - 1 \right) \right], \text{ for the borrower B}$$

$$q_L \left[q_B(1 + r_\theta^*) - v_L \right] = (1-\theta) q_L q_B \left[r_B - \left(\frac{v_L}{q_B} - 1 \right) \right], \text{ for the lender L.}$$

The bargaining result is also Pareto efficient, as is mentioned in Chapter 2. An increase in θ implies an increase in the borrower's bargaining power relative to the lender which results in a lower r_θ^* and a higher profit for the borrower B. The bargaining power θ can be estimated given perfect information. In Chapter 4, I estimated bargaining powers by calibration.

3.2 Incomplete information

The bargaining game under incomplete information has been analysed with different approaches in literature. An axiomatic approach for analysing bargaining outcomes under incomplete information is the neutral bargaining solution proposed by Myerson in [13], which is a generalization of Nash bargaining solution. The neutral bargaining solution captures the idea of *inscrutable compromise*, which can be explained as follows.

For agents with private information in the bargaining game, their optimal mechanisms depend on their private information. Following their optimal mechanisms, these agents may reveal private information, which can be unfavorable to their bargaining position. Therefore, agents should maintain an inscrutable facade during the bargaining. To achieve this inscrutability, agents make some compromise between their real optimal choices and optimal choices under other possible values of their private information.

One problem of implementing the neutral bargaining solution is computational tractability. Kim [15] illustrates neutral bargaining solutions numerically for two investors in an OTC market by selecting the parameters with discrete values. Similar numerical analysis cannot be done in general cases, and the computational complexity increases with information incompleteness. In the next chapter, the bargaining problem in the interbank market under incomplete information is studied following a strategic approach. For now, I study the question about whether the bargaining outcomes can be ex post efficient under incomplete information.

In the following sections, three possible situations in the market are considered: when the aggregate liquidity is sufficient, when the aggregate liquidity is insufficient, and the general case. Here sufficiency means the extreme case when there are much more lenders with abundant liquidity compared with the number of borrowers in the market, so it is difficult for lenders to find counterparties in the interbank market. Similarly, insufficiency means that there is more demand than supply for liquidity and it is thus difficult for borrowers to find a lender. As is shown below, when there is extreme constraint on aggregate liquidity, the bilateral bargain is under asymmetric information with one-sided uncertainty. In the general case, the bargaining game is under incomplete information with two-sided uncertainty.

3.2.1 When the aggregate liquidity is sufficient

When the aggregate liquidity is sufficient, the liquidity supply is far greater than demand. This can happen in current market where the central bank provides ample liquidity through quantitative easing (QE)[1]. It then becomes difficult for the lender to find a borrower in the interbank market. Therefore I assume the reservation value v_L of the lender to be its lowest value $1+\underline{r}$, and the lender has to deposit in the central bank once the current bargaining fails to reach an agreement. In this case, the bilateral bargaining is under asymmetric information, where the borrower B has private information r_B and the lender's reservation value v_L is public.

If the reservation values v_B and v_L are independent, then it can be shown in the next proposition that the bargaining outcome can be ex post efficient.

Proposition 3.1. *If the borrower's reservation value $v_B = q_B(1+r_B)$ is private information (more specifically, r_B is private and q_B is public) and the lender's reservation value $v_L = 1+\underline{r}$ is public information, then the bargaining outcome can be ex post efficient.*

Proof. To show the bargaining outcome can be ex post efficient, it is sufficient to find a bargaining procedure that induces a game with an ex post efficient Bayesian Nash equilibrium.

Consider a bargaining procedure where the borrower proposes a rate to the lender in one-shot. If the lender agrees, then the bargaining game stops and the trade happens. Otherwise the bargaining game stops and no trade happens. Consider the following strategy:

- The borrower proposes $r = \min\{r_B, \frac{1+\underline{r}}{q_B} - 1\}$
- The lender accepts if and only if $r \geq \frac{1+\underline{r}}{q_B} - 1$

It can be proved as below that this strategy induces a game with ex post efficient Bayesian Nash equilibrium. By the definition of ex post efficiency in Chapter 1, the bargaining outcome is ex post efficient if and only if

- Trade happens if $q_B(1+r_B) \geq v_L$
- Trade does not happen if $q_B(1+r_B) < v_L$

which is satisfied using the above strategy.

Let the pair (x_B, x_L) be the profits from the interbank trade for the borrower and the lender, respectively. Denote the borrower's and the lender's preference by $\preceq_B$ and $\preceq_L$, where $x \preceq_B y$ means the borrower weakly prefers y to x or the borrower wants y at least as much as x. Similarly, $x \preceq_L y$ for the lender means the lender weakly prefers y to x or the lender wants y at least as much as x. It can be shown as follows that the borrower will not be better off if it changes its strategy while the lender does not.

[1]Quantitative easing (QE) is a monetary policy whereby a central bank buys predetermined amounts of government bonds or other financial assets in order to inject liquidity directly into the economy. See `https://www.bankofengland.co.uk/faq` for details.

(a) If $q_B(1 + r_B) \geq 1 + \underline{r}$ and the borrower follows the strategy above, then the trade happens and the profits are $x^* = (q_B(1 + r_B) - v_L, 0)$ for the borrower and the lender, respectively. However, if the borrower does not follow this strategy and proposes some r' such that $r' \neq \frac{1+\underline{r}}{q_B} - 1$, then

- If $r' \geq r_B$, then the lender agrees, the trade happens with profits $(q_B(r_B - r'), q_B(1 + r') - (1 + \underline{r}))$. Note that $(q_B(r_B - r'), q_B(1 + r') - (1 + \underline{r})) \preceq_B x^*$ because $q_B(r_B - r') \leq 0$ and $q_B(1 + r_B) - (1 + \underline{r}) \geq 0$. Therefore the borrower cannot obtain more profit by deviating from the strategy.
- If $r' < \frac{1+\underline{r}}{q_B} - 1$, then the lender disagrees and the trade does not happen. The profits for both sides become $(0, 0)$ and thus the borrower's profit becomes less if it deviates from the strategy.
- If $r' \in (\frac{1+\underline{r}}{q_B} - 1, r_B)$, then the trade happens with profits $(q_B(r_B - r'), q_B(1 + r') - (1 + \underline{r}))$. Note that $(q_B(r_B - r'), q_B(1 + r') - (1 + \underline{r})) \preceq_B x^*$, so the borrower cannot obtain more profit by changing its strategy.

(b) If $v_B < 1 + \underline{r}$, the trade does not happen following the strategy and the profits become $d^* = (0, 0)$. Using the same analysis as part (a), I conclude that the borrower cannot obtain more profit by deviating from the strategy.

Likewise, it can be shown that the lender will not be better off by changing its strategy while the borrower does not. If the borrower proposes $r = \frac{1+\underline{r}}{q_B} - 1$ but the lender rejects, then the resulting profits are $(0, 0)$ and $(0, 0) \sim_L x^*$. If the borrower proposes $r = r_B$ but the lender accepts, then the resulting profits $(0, q_B(1 + r_B) - (1 + \underline{r})) \preceq_L d^*$ since the borrower proposes r_B if and only if $q_B(1 + r_B) - (1 + \underline{r}) < 0$. Therefore the bargaining game under the proposed strategy has an ex post efficient Bayesian Nash equilibrium, and thus the bargaining outcome can be ex post efficient. □

The proposition above shows that if only borrowers have reservation values as their private information, then the bargaining outcome can be ex post efficient. That is, the profits obtained by the players through bargaining can be Pareto efficient if the private information is revealed. A similar conclusion can be drawn when only the lenders have reservation values as their private information, as is shown in Section 3.2.2.

3.2.2 When the aggregate liquidity is insufficient

Now consider the situation when there is shortage of the aggregate liquidity in the market and the liquidity supply is far less than demand. This happens during the "normal" times before financial crisis, when banks is under the monetary policy of the central bank. Without loss of generality, I exclude the cases where banks receive central bank money through Emergency Liquidity Assistance (ELA)[2].

[2]Euro area credit institutions can receive central bank credit not only through monetary policy operations but exceptionally also through emergency liquidity assistance (ELA). ELA aims to provide central bank money to solvent financial institutions that are facing temporary liquidity prob-

Assume that once the bilateral bargaining ends without trade happening, the borrower's outside option r_B achieves its highest value, i.e. $r_B = \bar{r}$. This is reasonable since it becomes difficult for borrowers to find another counterparty in the interbank market when the aggregate liquidity is insufficient. In this case, the bilateral bargaining is under asymmetric information, with the lender L holding private information v_L and the borrower's reservation value $v_B = q_B(1 + \bar{r})$ being public.

If the reservation values v_B and v_L are independent, then a similar claim to Proposition 3.1 can be proved.

Proposition 3.2. *If the lender's reservation value v_L is private information and the borrower's reservation value $v_B = q_B(1 + \bar{r})$ is public information, then the bargaining outcome can be ex post efficient.*

Proof. Similar to the proof of Claim 1, it is sufficient to find a bargaining procedure that induces a game with an ex post efficient Bayesian Nash equilibrium.

Consider a bargaining procedure where the lender proposes the rate to the borrower. If the borrower agrees, then the trade happens, otherwise the bargaining stops and no trade happens. Consider the following strategy:

- The lender proposes $r = \max\{\bar{r}, \frac{v_L}{q_B} - 1\}$
- The lender accepts if and only if $r \leq \bar{r}$

It can be proved as follows that this strategy induces a game with ex post efficient Bayesian Nash equilibrium. By definition, the bargaining outcome is ex post efficient if and only if trade happens when $q_B(1+\bar{r}) \geq v_L$ and does not happen when $q_B(1+\bar{r}) < v_L$. This condition is satisfied if both sides use the above strategy. Therefore, under this strategy, the outcome is ex post efficient and

- Trade happens if $q_B(1 + \bar{r}) \geq v_L$, when the lender propose $\bar{r}$ and the borrower accepts, and the profits are $y^* = (0, q_B(1+\bar{r}) - v_L)$ for the borrower and the lender respectively.
- Trade does not happen if $q_B(1 + \bar{r}) > v_L$, when the lender propose $\frac{v_L}{q_B} - 1$ and the borrower rejects, and the profits are $(0, 0)$.

It can be shown below that the borrower will not be better off if it changes its strategy while the lender does not.

(a) If $q_B(1 + r_B) \geq v_L$ but the borrower rejects the lender's offer, then the profits $(0, 0) \preceq_B y^*$.

(b) If $q_B(1 + r_B) < v_L$ but the borrower accepts the lender's offer $\frac{v_L}{q_B} - 1$, then the borrower's profit is negative. Thus the borrower becomes worse off if changing strategy.

lems, outside of normal Eurosystem monetary policy operations. See `https://www.ecb.europa.eu/mopo/ela/html/index.en.html` for details.

The lender will not be better off by changing its strategy while the borrower does not. This can be proved using the same methods as in the proof of Claim 1. If the lender proposes some $r' \neq \bar{r}$ when $q_B(1+r_B) \geq v_L$, or propose $r' \neq \frac{v_L}{q_B} - 1$ when $q_B(1+r_B) < v_L$, then the lender will not be better off with the borrower's strategy unchanged. Hence the claim is proved. $\square$

The proposition above shows that if only lenders have reservation values as their private information, then the bargaining outcome can be ex post efficient. Therefore it can be concluded that if banks bargain under one-sided private information, i.e. either v_B or v_L is private, then the bargaining outcomes can be ex post efficient. However, when both the borrower and the lender hold private information in a bargaining game, there may not exist an ex post efficient bargaining outcome, as is shown in Section 3.2.3.

3.2.3 General case

Consider the general case when the liquidity condition in the market is not as extreme as those in previous sections. Banks contact each other and bargain over the price for interbank trade. If the bilateral bargaining fails, banks part ways and search for other counterparties. Since there are many other potential counterparties for borrowers and lenders, banks' outside options are not limited to the central bank's standing facilities. In this case, the profit from outside options depends not only on the central bank's rates, but also the banks' own conditions and the market conditions. In this section I assume the reservation values v_B and v_L are the borrower's and the lender's private information, but q_B and q_L can be inferred from CDS data, then the bargaining between the lender L and the borrower B is under two-sided uncertainty.

Assume the private information V_B of the borrower and V_L of the lender are random variables with probability density function f_B and f_L and cumulative distribution function F_B and F_L on $[a_B, b_B]$ and $[a_L, b_L]$, respectively. The distribution functions F_B and F_L are known to both sides of the bargaining game, but the realization v_B (v_L) of the random variable V_B (V_L) is private information of the borrower (lender) when bargaining.

Myerson in [14] analysed a similar bargaining game, and showed the impossibility of an ex post efficient mechanism that is incentive compatible and individually rational. The main difference between [14] and this thesis is whether there is payment when bargaining fails. In [14], the borrower has to pay even if the trade does not occur, which is not the case in this thesis. However, using the same method, a similar conclusion can be drawn. In the interbank market, let v_B be the private information of the borrower and v_L be private for the lender. Then if the ranges of v_B and v_L intersect, then there does not exist a direct revelation procedure that is incentive compatible, individually rational and ex post efficient. By the revelation principle, the bargaining outcome cannot be ex post efficient under two-sided private information.

Using the same setup as discussed at the beginning of this chapter, the expected profit for the borrower B when borrowing one unit of loan from the lender L is $v_B - q_B(1+r)$, and for the lender is $q_L\left[q_B(1+r) - v_L\right]$, where r is the deal rate. The goal of the following analysis is to characterize a feasible bargaining mechanism that determines whether the interbank trade happens and the interbank rate under two-sided uncertainty with private information.

A bargaining mechanism is feasible if it is *incentive compatible* and *individually rational.* A mechanism is *incentive compatible* if and only if it is a Bayesian Nash equilibrium for both players to announce their beliefs honestly. Under such mechanisms, neither of the players can obtain more profit by unilaterally lying about its belief, and each player maximizes its expected profit by being honest. A mechanism is *individually rational* if and only if the expected profit gained from the trade is not less than the payoff from disagreement.

By the *revelation principle*, for any Bayesian Nash equilibrium (BNE) of any bargaining procedure, there is an equivalent incentive compatible direct revelation mechanism that yields the same outcome (see Chapter 2). Therefore, when analysing the ex-post efficiency of bargaining outcomes, it is sufficient to consider only the direct revelation procedures rather than all possible bargaining procedures.

The set of direct revelation procedures is a subset of all bargaining procedures. A direct revelation procedure is characterized by a pair of functions

$$(p, r) : [a_L, b_L] \times [a_B, b_B] \to [0, 1] \times [\frac{1+\underline{r}}{q_B} - 1, \bar{r}].$$

In a direct revelation procedure, the borrower and the lender simultaneously announce to a coordinator their perceived probabilities that the borrower will not default when the payment is due. The coordinator then determines whether the trade happens and the interbank rate based on the reported values. If the announced values from the borrower and the lender are v_B and v_L, then the interbank trade happens with probability $p(v_B, v_L)$ and rate $r(v_B, v_L)$. With probability $1-p(v_B, v_L)$, the interbank trade will not take place.

Under the direct revelation procedure, the expected profit per unit from interbank trade for the borrower is

$$U_B(v_B) = \int_{a_L}^{b_L} p(v_B, t_L) \left[v_B - q_B(1 + r(v_B, t_L))\right] f_L(t_L) dt_L, \tag{3.3}$$

and the expected profit per unit loan for the lender is

$$U_L(v_L) = q_L \int_{a_B}^{b_B} p(t_B, v_L) \left[q_B(1 + r(t_B, v_L)) - v_L\right] f_B(t_B) dt_B. \tag{3.4}$$

The direct revelation procedure is incentive compatible if and only if

$$\begin{aligned} U_B(v_B) &= \int_{a_L}^{b_L} p(v_B, t_L) \left[v_B - q_B(1 + r(v_B, t_L))\right] f_L(t_L) dt_L \\ &\geq \int_{a_L}^{b_L} p(\tilde{v}_B, t_L) \left[v_B - q_B(1 + r(\tilde{v}_B, t_L))\right] f_L(t_L) dt_L, \ \forall v_B, \tilde{v}_B \in [a_B, b_B]; \end{aligned} \tag{3.5}$$

$$\begin{aligned} U_L(v_L) &= q_L \int_{a_B}^{b_B} p(t_B, v_L) \left[q_B(1 + r(t_B, v_L)) - v_L \right] f_B(t_B) dt_B \\ &\geq q_L \int_{a_B}^{b_B} p(t_B, \tilde{v}_L) \left[q_B(1 + r(t_B, \tilde{v}_L)) - v_L \right] f_B(t_B) dt_B, \ \forall v_L, \tilde{v}_L \in [a_L, b_L]. \end{aligned} \tag{3.6}$$

By symmetry, under incentive compatibility we also have

$$\begin{aligned} U_B(\tilde{v}_B) &= \int_{a_L}^{b_L} p(\tilde{v}_B, t_L) \left[\tilde{v}_B - q_B(1 + r(\tilde{v}_B, t_L)) \right] f_L(t_L) dt_L \\ &\geq \int_{a_L}^{b_L} p(v_B, t_L) \left[\tilde{v}_B - q_B(1 + r(v_B, t_L)) \right] f_L(t_L) dt_L, \ \forall v_B, \tilde{v}_B \in [a_B, b_B]; \end{aligned} \tag{3.7}$$

$$\begin{aligned} U_L(\tilde{v}_L) &= q_L \int_{a_B}^{b_B} p(t_B, \tilde{v}_L) \left[q_B(1 + r(t_B, \tilde{v}_L)) - \tilde{v}_L \right] f_B(t_B) dt_B \\ &\geq q_L \int_{a_B}^{b_B} p(t_B, v_L) \left[q_B(1 + r(t_B, v_L)) - \tilde{v}_L \right] f_B(t_B) dt_B, \ \forall v_L, \tilde{v}_L \in [a_L, b_L]. \end{aligned} \tag{3.8}$$

Combine (3.5) and (3.7),

$$(v_B - \tilde{v}_B)\bar{p}_B(\tilde{v}_B) \leq U_B(v_B) - U_B(\tilde{v}_B) \leq (v_B - \tilde{v}_B)\bar{p}_B(v_B). \tag{3.9}$$

where $\bar{p}_B(v_B) = \int_{a_L}^{b_L} p(v_B, t_L) f_L(t_L) dt_L$. If $v_B > \tilde{v}_B$, then $\bar{p}_B(v_B) \geq \bar{p}_B(\tilde{v}_B)$, so $\bar{p}_B$ is increasing and thus Riemann integrable. Then from (3.9) we have $U_B'(v_B) = \bar{p}_B(v_B)$ for almost every $v_B \in [a_B, b_B]$, and thus

$$U_B(v_B) = U_B(a_B) + \int_{a_B}^{v_B} \bar{p}_B(t_B) dt_B, \ \forall v_B \in [a_B, b_B]. \tag{3.10}$$

Likewise, combining (3.6) and (3.8) gives

$$q_L(\tilde{v}_L - v_L)\bar{p}_L(\tilde{v}_L) \leq U_L(v_L) - U_L(\tilde{v}_L) \leq q_L(\tilde{v}_L - v_L)\bar{p}_L(v_L). \tag{3.11}$$

where $\bar{p}_L(v_L) = \int_{a_B}^{b_B} p(t_B, v_L) f_B(t_B) dt_B$. If $v_L < \tilde{v}_L$, then $\bar{p}_L(v_L) \geq \bar{p}_L(\tilde{v}_L)$, so $\bar{p}_L$ is decreasing and thus Riemann integrable. From (3.11) we have $U_L'(v_L) = -q_L\bar{p}_L(v_L)$ for almost every $v_L \in [a_L, b_L]$, and thus

$$U_L(v_L) = U_L(b_L) + \int_{v_L}^{b_L} q_L\bar{p}_L(t_L) dt_L, \ \forall v_L \in [a_L, b_L]. \tag{3.12}$$

The direct revelation procedure is individually rational if and only if $U_B(v_B) \geq 0$ and $U_L(v_L) \geq 0$ for all $v_B \in [a_B, b_B]$ and $v_L \in [a_L, b_L]$. Moreover, it is ex post efficient if and only if $p(v_B, v_L) = \mathbf{1}_{\{v_B \geq v_L\}}$. Then the following result can be proved:

Theorem 3.3. *If $a_B < b_L$, $a_L < b_B$ and $v_B \perp\!\!\!\perp v_L$, then there does not exists a direct revelation procedure that is incentive compatible, individually rational and ex post efficient, and thus the bargaining outcome cannot be ex post efficient under two-sided private information v_B, v_L.*

Proof. By (3.3), (3.4), (3.10) and (3.12), the following equation holds:

$$
\begin{aligned}
&\int_{a_L}^{b_L}\int_{a_B}^{b_B}(v_B - v_L)f_B(v_B)f_L(v_L)p(v_B, v_L)dv_B dv_L \\
=&\int_{a_B}^{b_B} U_B(v_B)f_B(v_B)dv_B + \frac{1}{q_L}\int_{a_L}^{b_L} U_L(v_L)f_L(v_L)dv_L \\
=&U_B(a_B) + \int_{a_B}^{b_B}\int_{a_B}^{v_B}\bar{p}_B(t_B)f_B(v_B)dv_B + \frac{1}{q_L}\left[U_L(b_L) + \int_{a_L}^{b_L}\int_{v_L}^{b_L} q_L\bar{p}_L(t_L)dt_L f_L(v_L)dv_L\right] \\
=&U_B(a_B) + \frac{1}{q_L}U_L(b_L) + \int_{a_B}^{b_B}\int_{t_B}^{b_B} f_B(v_B)dv_B\bar{p}_B(t_B)dt_B + \int_{a_L}^{b_L}\int_{a_L}^{v_L} f_L(v_L)dv_L\bar{p}_L(t_L)dt_L \\
=&U_B(a_B) + \frac{1}{q_L}U_L(b_L) + \int_{a_B}^{b_B}\int_{a_L}^{b_L}\left[(1 - F_B(t_B))\, f_L(t_L) + F_L(t_L)f_B(t_B)\right]p(t_B, t_L)dt_B dt_L
\end{aligned}
$$

Note that the direct revelation procedure induces an ex post efficient outcome if and only if $p(v_B, v_L) = \mathbf{1}_{\{v_B \geq v_L\}}$. Insert this expression of $p(v_B, v_L)$ into the above equation, we have

$$
\begin{aligned}
&U_B(a_B) + \frac{1}{q_L}U_L(b_L) \\
=&\int_{a_B}^{b_B}\int_{a_L}^{b_L\wedge v_B}\left[v_B - \frac{1 - F_B(v_B)}{f_B(v_B)}\right]f_B(v_B)f_L(v_L)dv_L dv_B \\
&-\int_{a_B}^{b_B}\int_{a_L}^{b_L\wedge v_B}\left[v_L + \frac{F_L(v_L)}{f_L(v_L)}\right]f_B(v_B)f_L(v_L)dv_L dv_B \\
=&\int_{a_B}^{b_B}\int_{a_L}^{b_L\wedge v_B}\left(v_B + F_B(v_B) - 1\right)f_L(v_L)dv_L dv_B - \int_{a_B}^{b_B}\int_{a_L}^{b_L\wedge v_B}\left(v_L f_L(v_L) + F_L(v_L)\right)dv_L dv_B \\
=&\int_{a_B}^{b_B}\left(v_B + F_B(v_B) - 1\right)F_L(v_B)dv_B - \int_{a_B}^{b_B}\left[\int_{a_L}^{b_L\wedge v_B} v_L dF_L(v_L) + \int_{a_L}^{b_L\wedge v_B} F_L(v_L)dv_L\right]f_B(v_B)dv_B \\
=&\int_{a_B}^{b_B}\left(v_B + F_B(v_B) - 1\right)F_L(v_B)dv_B - \int_{a_B}^{b_B}(v_B\wedge b_L)F_L(v_B\wedge b_L)f_B(v_B)dv_B \\
=&-\int_{a_B}^{b_B}\left[1 - F_B(v_B)\right]F_L(v_B)dv_B + \int_{a_B}^{b_B} v_B f_B(v_B)F_L(v_B)dv_B \\
=&-\int_{a_B}^{b_B}\left[1 - F_B(v_B)\right]F_L(v_B)dv_B + \int_{b_L}^{b_B}(v_B - b_L)f_B(v_B)dv_B \\
=&-\int_{a_B}^{b_B}\left[1 - F_B(v_B)\right]F_L(v_B)dv_B + (b_B - b_L) - \int_{b_L}^{b_B} F_B(v_B)dv_B,
\end{aligned}
$$

This can be further simplified as

$$
\begin{aligned}
&-\int_{a_B}^{b_L}\left[1-F_B(v_B)\right]F_L(v_B)dv_B-\int_{b_L}^{b_B}\left[1-F_B(v_B)\right]dv_B-\int_{b_L}^{b_B}(1-F_B(v_B))dv_B\\
=&-\int_{a_B}^{b_L}\left[1-F_B(v_B)\right]F_L(v_B)dv_B<0,
\end{aligned}
$$

which contradicts individual rationality. Hence the theorem is proved. □

The proof of Theorem 3.3 implies that there exists no incentive compatible and individually rational direct revelation procedure that induces a bargaining game with an ex post efficient outcome. By the Revelation Principle, no bargaining procedure under induces Bayesian Nash equilibrium that gives an ex post efficient bargaining outcome.

In this chapter I showed that unlike the complete information setting, when banks bargain under asymmetric or incomplete information, the bargaining outcome are not necessarily Pareto efficient after revealing all information. With one-sided uncertainty, the bargaining outcome can be ex post efficient, no matter who holds the private information. Under two-sided uncertainty, the bargaining outcome cannot be ex post efficient. The more private information between borrowers and lenders, the less possible that the bargaining outcome can be ex post efficient.

4 A Bargaining Procedure under Asymmetric Information

After answering the normative question in Chapter 3 about whether the bargaining outcome can be ex post efficient in the interbank market, in this chapter I take a closer look of the bargaining procedures and strategies. Namely, I study a bargaining procedure under asymmetric information between two agents in the interbank market. The bargaining game proceeds sequentially and consists of several rounds. In each round, the uninformed agent (i.e. the agent without private information) has a prior belief on its counterparty's private information and repeatedly offers interest rates to its counterparty (i.e. the agent with private information). The agent with private information then responds to the offer by either accepting or rejecting it. If the informed agent accepts the current offer rate, then the agreement is struck and the interbank trade happens with the agreed rate. Otherwise the uninformed agent updates its belief according to the response it receives using Bayes' rule, and then proposes another rate in the next round of bargaining based on the updated belief.

I will show in this chapter that under specific conditions, the uninformed agent is able to screen its counterparty's private information if both sides agree to bargain over infinite horizons (or very long finite horizons) using a sequence of offered rates. The uninformed agent cannot be guaranteed to know the exact value of the private information under this mechanism, but its belief on private information becomes closer to the true value as the bargaining proceeds. With a strategic bargaining model, I show that the deal rates depend on banks' beliefs on each other. The beliefs are based on both banks' trading history and new information about the counterparty risk. The result obtained from the model is consistent with that in [17] which implies that the relationship between banks is an important determinant of banks' ability to access interbank market liquidity.

4.1 Setup

Consider the bilateral bargaining between a lender L and a borrower B over the unsecured overnight interbank rate. The definitions of parameters are the same as in Chapter 3. The default risk of each bank can be deduced from the CDS (Credit Default Swap) data, and thus the probability q_B, q_L are regarded as public information. Assume the lender's reservation rate $\underline{r}$ is the same as the deposit rate offered by the central bank. This assumption can be justified by the empirical work done in [4] or [18]. The range of unsecured overnight rates among the reporting agents indicates that most unsecured overnight trade happens under rates as low as $\underline{r}$.

The borrower's reservation rate r_B is its private information, which can be regarded as a random variable with cumulative distribution function F. This distribution function is derived by a bank's own activities (e.g. trading) but also by its clients' action. The borrower knows the realization of this random variable, but the lender does not. The lender takes F as its prior belief on the private information r_B, and updates its belief immediately after the borrower's response in each round. Assume F is continuously differentiable with support $[r_l, r_h]$, where $r_l = \frac{1+\underline{r}}{q_B} - 1$ and $r_h = \bar{r}$ are known to both sides. More precisely, let F be uniform distribution function. Let F_n be the posterior belief of the lender after the n-th round, that is, F_n is the updated posterior after the lender has been rejected n times by the borrower. Denote by s_n the highest possible reservation rate that the lender believes the borrower could have after the n-th round. The lender L offers a rate r_n at time $n\tau$ in Round n, where $n \in \mathbb{N}_0$ and $\tau > 0$. If an offered rate at $n\tau$ is rejected by the borrower and the bargaining continues at $(n+1)\tau$, then the obtainable payoffs for both sides will be discounted with rate r_d. Denote by $\delta = \exp(-r_d\tau)$ the discount factor for each bank.

If both sides agree to trade with rate r_n after the n-th round of bargaining, then the expected payoff for borrower is

$$\delta^n \left[q_B(1 + r_B) - q_B(1 + r_n) \right] = \delta^n q_B(r_B - r_n),$$

and the expected payoff for the lender is

$$\delta^n q_L \left[q_B(1 + r_n) - (1 + \underline{r}) \right].$$

If the borrower rejects the offered rates perpetually, then the expected payoff for each agent is zero.

Consider the following strategies:

- The borrower accepts rate r if and only if $r \leq \alpha\underline{r} + (1 - \alpha)r_B$
- The lender offers rate $r(s) = \beta\underline{r} + (1 - \beta)s$, where s is the highest possible value of r_B that L believes. If the offered rate r is rejected by the borrower, then the lender infers that $r > \alpha\underline{r} + (1 - \alpha)r_B$, that is,

$$r_B < \frac{r - \alpha\underline{r}}{1 - \alpha}.$$

Then the lender updates its belief on the highest possible value of r_B as

$$s^*(r) = \frac{r - \alpha\underline{r}}{1 - \alpha}$$

In the above strategy, the parameters α and β remain fixed for the borrower B and the lender L on the day they bargain. Both the borrower's upper bound for the interest rate and the lender's offer are linear combinations of the lender's reservation rate $\underline{r}$ and the borrower's reservation rate (either true value r_B or the lender's belief s on the maximum value of r_B). The intuition comes from the generalized Nash bargaining solution which is a linear combination of both sides' reservation rates under complete information, and the weights α, β can be regarded as the equivalence of θ in general Nash solution (2.2).

Consider a bilateral bargaining game between the lender L and the borrower B starting at round 0. For notational convenience, let r_n be the lenders offered rate in the n-th round, and $s_n := s^*(r_{n-1})$ be the maximum possible value of r_B that the lender believes the borrower could have before offering in the n-th round. If the agents bargain over the interbank rate following the above strategies, then in Round 0 the lender offers a rate $r_0 := r(s_0) = \beta \underline{r} + (1-\beta)s_0$, where s_0 is the lender's belief on the maximum possible value of r_B at time 0, and its prior belief on the distribution of r_B is

$$F(s) = \frac{s - r_l}{s_0 - r_l}.$$

The borrower accepts r_0 at time 0 if and only if $r_0 \leq \alpha \underline{r} + (1-\alpha)r_B$, that is, $r_B \geq s^*(r_0) =: s_1$. If the borrower rejects r_0, then the lender updates its belief as s_1 and its posterior as

$$F_1(s) = \frac{s - r_l}{s_1 - r_l}.$$

In the next round, the offered rate from the lender is $r_1 := r(s_1) = \beta \underline{r} + (1-\beta)s_1$. The borrower accepts r_1 in Round 1 if and only if $r_1 \leq \alpha \underline{r} + (1-\alpha)r_B < r_0$, which is equivalent to $r_B \in [s_2, s_1)$, where $s_2 := s^*(r_1)$. This procedure continues until either the borrower accepts the offered rate or the lender's belief s_n on the upper bound of r_B is so low that the offered rate r leads to a non-positive expected payoff for the lender, i.e. $r \leq \frac{1+r}{q_B} - 1$.

4.2 Non-Existence of Perfect Bayesian Equilibrium

First, I will show that a perfect Bayesian equilibrium does not exist under this screening procedure. Perfect Bayesian equilibrium is essential for sequential bargaining game under incomplete information, the definition of which is introduced in Chapter 2. In this section I prove that due to counterparty risk there does not exist a perfect Bayesian equilibrium under the strategies introduced in the previous section. To show this, I first assume the existence of a perfect Bayesian equilibrium and deduce the relation between α and β under such equilibrium. Then I show that there is no (α, β) that results in a perfect Bayesian equilibrium.

Claim 4.1. *If the strategy is a perfect Bayesian equilibrium, then $\alpha = \delta\beta$.*

Proof. I first prove that if the strategy is perfect Bayesian equilibrium, then

$$s_1 - r_0 = \delta\,(s_1 - r_1) \tag{4.1}$$

By contradiction, suppose $s_1 - r_0 > \delta\,(s_1 - r_1)$, then there exists $s \in [s_2, s_1)$ such that $s - r_0 > \delta\,(s - r_1)$. If the borrower's reservation rate is s, then it should accept the offer in Round 0 rather than in Round 1 because its expected profit $q_B(s - r_0)$ in Round 0 is higher than its expected profit $\delta q_B(s - r_1)$ in the next round. This contradicts that the borrower accepts the offered rate in Round 1 if and only if the reservation rate is in $[s_2, s_1)$. Likewise, if $s_1 - r_0 < \delta(s_1 - r_1)$, then there exists some $s' > s_1$ such that $s' - r_0 < \delta(s' - r_1)$. The borrower should trade in Round 0 if its reservation rate is s', which leads to contradiction. Therefore (4.1) is proved.

Next I prove $\alpha = \delta\beta$ based on (4.1). Replacing s_1, r_1 in (4.1) with the following expressions

$$s_1 = s^*(r_0) = \frac{r_0 - \alpha \underline{r}}{1 - \alpha},$$
$$r_1 = \beta \underline{r} + (1 - \beta)s_1 = \beta \underline{r} + (1 - \beta)\frac{r_0 - \alpha \underline{r}}{1 - \alpha},$$

gives $\alpha(r_0 - \underline{r}) = \delta\beta(r_0 - \underline{r})$. Hence $\alpha = \delta\beta$. □

Claim 4.2. *If $\alpha = \delta\beta$, then the borrower's strategy is immune to profitable one-shot deviation, which means that the borrower cannot gain more profit by unilaterally deviating from the strategy for one round.*

Proof. To show this, consider two cases:

- If the lender offers rate r_m in the m-th round such that $r_m \le \alpha \underline{r} + (1 - \alpha) r_B$, then the borrower should accept r_m if it follows the strategy with expected profit

  $$\delta^m q_B(r_B - r_m).$$

  If the borrower deviates from the strategy in this round and follow the strategy the next round, then he will accept the offer in the $(m + 1)$-th round since the lender's belief $\{s_n\}_n$ is decreasing as bargaining proceeds. The borrower's expect profit after one-shot deviation is

  $$\delta^{m+1} q_B(r_B - r_{m+1}) = \delta^{m+1} q_B \left(r_B - \beta \underline{r} - (1 - \beta)\frac{r_m - \alpha \underline{r}}{1 - \alpha} \right).$$

  If $\alpha = \delta\beta$, then the additional profit gain $\delta^{m+1} q_B(r_B - r_{m+1}) - \delta^m q_B(r_B - r_m)$ of the borrower's one-shot deviation is non-positive.

- The same method can be applied to the case where the lender offers rate r_m in the m-th round such that $r_m > \alpha \underline{r} + (1 - \alpha) r_B$. If the borrower follows the strategy, then it rejects the offered in this round and its expected profit in the future is at least equal to the expected profit when it accepts the next offer in the $m + 1$-th round. So the borrower's expected profit following the strategy is at least

  $$\delta^{m+1} q_B(r_B - r_{m+1}) = \delta^{m+1} q_B \left(r_B - \beta \underline{r} - (1 - \beta)\frac{r_m - \alpha \underline{r}}{1 - \alpha} \right).$$

  If the borrower deviates from the strategy in the m-th round, then it accepts the offered rate r_m and its expected profit is

  $$q_B(r_B - r_m),$$

which is at most equal to the lower bound of the expected profit following the strategy if $\alpha = \delta\beta$.

Therefore it can be concluded that the borrower's strategy is immune to one-shot deviation only if $\alpha = \delta\beta$. $\square$

Next I will prove that there does not exist a pair (α, β) such that both the borrower's and the lender's strategies are immune to profitable one-shot deviation. This implies that the strategies cannot induce a perfect Bayesian equilibrium, which differs from the case discussed in [7] where without default risk there is a unique perfect Bayesian equilibrium. With the existence of borrowers' default risk, i.e. $q_B < 1$, it can be proved that lenders can deviate from the strategy in order to obtain higher expected profits.

Claim 4.3. *If the discount factor $\delta < 1$ and the borrower's default risk exists, then there does not exists (α, β) such that the lender's strategy is immune to profitable one-shot deviation.*

Proof. Suppose the lender's strategy is immune to one-shot deviation under some (α, β). Let $U(s)$ be the lender's expected profit following the strategy when the lender's belief on r_B is s at the beginning of the current round, and $W(r, s)$ be the lender's expected payoff if it offers rate r and follows the strategy in the next round if r is rejected.

The lender's strategy is immune to profitable one-shot deviation if and only if for all $s \in [\underline{r}, \bar{r}]$

$$U(s) = W(r(s), s) = \max_{r \in [\frac{1+\underline{r}}{q_B} - 1, \bar{r}]} W(r, s) \tag{4.2}$$

Note that if $s \leq \frac{1}{1-\beta}\left(\frac{1+\underline{r}}{q_B} - 1 - \beta\underline{r}\right)$, then $r(s) \leq \frac{1+\underline{r}}{q_B} - 1$ and

$$W(r(s), s) = q_L\left[q_B(1 + r(s)) - (1 + \underline{r})\right] = 0 \geq W(r, s)$$

for any $r \in [\frac{1+\underline{r}}{q_B} - 1, \bar{r}]$. If $s > \frac{1}{1-\beta}\left(\frac{1+\underline{r}}{q_B} - 1 - \beta\underline{r}\right)$, then $r(s) > \frac{1+\underline{r}}{q_B} - 1$, and

- If $r < \frac{1+\underline{r}}{q_B} - 1$, then the lender expects the borrower to accept r in the current round and the lender's expected profit $W(r, s) < 0$.
- If $r > \alpha\underline{r} + (1 - \alpha)s$, then the lender expects the borrower to reject the current offer r, so the lender's expected profit $W(r, s)$ is the expected profit it can obtain if the borrower accepts the offered rate in the next round, that is, $W(r, s) = \delta U(s)$. Note that $W(r, s) \leq U(s)$ if the discount factor $\delta < 1$, and thus the lender has no incentive to offer such r since the expected profit $W(r, s)$ is always below the maximum $W(r(s), s)$.
- If $r \in [\frac{1+\underline{r}}{q_B} - 1, \alpha\underline{r} + (1 - \alpha)s]$, then the lender's expected profit is

$$\begin{aligned} W(r, s) &= G(s)\delta U(s^*(r)) + (1 - G(s))\, q_L\left[q_B(1 + r) - (1 + \underline{r})\right] \\ &= \frac{s^*(r) - \underline{r}}{s - \underline{r}}\delta U(s^*(r)) + \frac{s - s^*(r)}{s - \underline{r}} q_L\left[q_B(1 + r) - (1 + \underline{r})\right], \end{aligned} \tag{4.3}$$

where $G(s^*(r)) = \frac{s^*(r)-r_l}{s-r_l}$ is the lender's posterior probability of $r_B < s^*(r)$. Note that $G(s^*(r)) = \frac{s^*(r)-r_l}{s-r_l}$ is also the lender's posterior probability that the borrower rejects the current offer r, since the borrower accepts r if and only if $r_B \geq \frac{r-\alpha \underline{r}}{1-\alpha} = s^*(r)$. The equation (4.3) can be explained as follows: According to the lender's current belief, With probability $G(s)$ the offered rate r will be rejected and the lender's discounted expected profit is $\delta U(s^*(r))$ if it follows the strategies from the next round; with probability $1 - G(s)$, the rate r is accepted by the borrower and the lender's expected profit is $q_L\left[q_B(1+r) - (1+\underline{r})\right]$.

Consider (4.2) and the above cases, I have

$$\begin{aligned}
U(s) &= \max\left\{0, \max_{r \in [\frac{1+\underline{r}}{q_B}-1, \alpha\underline{r}+(1-\alpha)s]} W(r,s)\right\} \\
&= \max_{r \in [\frac{1+\underline{r}}{q_B}-1, \alpha\underline{r}+(1-\alpha)s]} \left\{\frac{s^*(r)-\underline{r}}{s-\underline{r}}\delta U(s^*(r)) + \frac{s-s^*(r)}{s-\underline{r}} q_L\left[q_B(1+r)-(1+\underline{r})\right]\right\}
\end{aligned} \tag{4.4}$$

For computational convenience, let $V(s) := (s-\underline{r})U(s)$ and r^* be an arbitrary solution to (4.4), then the optimization problem (4.4) becomes

$$\begin{aligned}
V(s) &= \max_{r \in [\frac{1+\underline{r}}{q_B}-1, \alpha\underline{r}+(1-\alpha)s]} \left\{\delta V(s^*(r)) + (s-s^*(r))\, q_L\left[q_B(1+r)-(1+\underline{r})\right]\right\} \\
&= \delta V(s^*(r^*)) + (s-s^*(r^*))\, q_L\left[q_B(1+r^*)-(1+\underline{r})\right] \\
&= \delta V(\frac{r^*-\alpha\underline{r}}{1-\alpha}) + \left(s - \frac{r^*-\alpha\underline{r}}{1-\alpha}\right) q_L\left[q_B(1+r^*)-(1+\underline{r})\right]
\end{aligned} \tag{4.5}$$

and by Envelope Theorem

$$V'(s) = q_L\left[q_B(1+r^*)-(1+\underline{r})\right].$$

The optimizer r^* satisfies the first-order condition

$$\begin{aligned}
0 &= \delta V'(\frac{r^*-\alpha\underline{r}}{1-\alpha})\frac{1}{1-\alpha} + q_L\frac{-1}{1-\alpha}\left[q_B(1+r^*)-(1+\underline{r})\right] + q_L q_B\left(s - \frac{r^*-\alpha\underline{r}}{1-\alpha}\right) \\
&= \delta q_L\left[q_B(1+r^*)-(1+\underline{r})\right]\frac{1}{1-\alpha} + q_L\frac{-1}{1-\alpha}\left[q_B(1+r^*)-(1+\underline{r})\right] + q_L q_B\left(s - \frac{r^*-\alpha\underline{r}}{1-\alpha}\right)
\end{aligned}$$

which holds for any possible value of s. From (4.2) we know that $r(s)$ is a solution to the optimization problem (4.4) and (4.5). Replace r^* with $r(s) = \beta\underline{r} + (1-\beta)s$, the first-order condition becomes

$$0 = q_B\left[(\beta-\alpha) - (1-\delta)(1-\beta)\right]s - q_B\left[(1-\delta)\left(\frac{1+\underline{r}}{q_B} - r - \beta\underline{r}\right) - (\beta-\alpha)\underline{r}\right],$$

which holds for any $s \in [r_l, r_h]$. This implies

$$\begin{cases} \beta - \alpha = (1-\delta)(1-\beta) \\ (1-\delta)\left(\frac{1+r}{q_B} - r - \beta\underline{r}\right) - (\beta-\alpha)\underline{r} = 0 \end{cases} \tag{4.6}$$

The first condition in (4.6) is satisfied if $\alpha = \delta\beta$, which coincides with the condtion in Claim 4.2. Insert the first condition into the second, I get

$$\begin{aligned} &(1-\delta)\left(\frac{1+\underline{r}}{q_B} - r - \beta\underline{r}\right) - (1-\delta)(1-\beta)\underline{r} \\ =&(1-\delta)(1+\underline{r})\left(\frac{1}{q_B} - 1\right) = 0 \end{aligned}$$

which contradicts $\delta < 1$ and $q_B < 1$. Hence such (α, β) does not exists. □

From Claim 4.2 and 4.3, it can be concluded that there does not exists (α, β) such that both agents have no incentive to deviate from the strategy. Therefore the perfect Bayesian equilibrium does not exists under this strategy.

In Chapter 9 of [7], a similar case is discussed, where a buyer and a seller bilaterally bargain over the price of a good under asymmetric information. Following the strategies above, it was shown that there exists a unique pair of (α, β) that makes the strategies perfect Bayesian equilibrium. However, for the interbank market case in this section, the same conclusion cannot be drawn due to the borrower's default risk. If there is no default risk, i.e. $q_B = 1$, then the strategies would be perfect Bayesian equilibrium if and only if $\alpha = 1 - \sqrt{q-\delta}$ and $\beta = \frac{1-\sqrt{1-\delta}}{\delta}$. The proof can be found in [7] page 275-278.

Note that the above analysis is based on the assumption of one-sided uncertainty where the lender's only alternative is the central bank and its reservation rate $\underline{r}$ is public. A natural generalization is bargaining under two-sided uncertainty, where both the borrower's and the lender's reservation rates are private information. To screen their counterparties' private information, both sides of the bargaining game make alternating offers and update their beliefs as soon as their counterparties respond. However, this generalization is not studied in this chapter for the following reasons. First, the data shows relatively low deal rates in unsecured interbank markets that are close to central bank depositing rates, therefore it is reasonable to take $\underline{r}$ as the lenders' reservation rate for computational convenience. Second, more parameters will be introduced in the strategies under two-sided uncertainty. For example, the borrower may use α_1 and α_2 to characterize its offering strategy and responding strategy, respectively. Additional parameters can skew the bargaining outcomes and bring difficulties to the analysis. Third, perfect Bayesian equilibrium cannot be obtained by strategies in two-sided uncertainty cases.

4.3 Bargaining Procedure in the Interbank Market

It has been shown in the previous section that given the borrower and the lender, there does not exists (α, β) that induces a perfect Bayesian equilibrium under the given strategies. This implies that in reality borrowers and lenders can determine their own parameters α and β according to public information, borrower's private information (e.g. borrowers' liquidity and market risks), and lenders' beliefs on borrowers' private information. Moreover, it is possible that borrowers lie about their risks and use a high α while bargaining. In the following analysis, I assume borrowers use the true value of their reservation rate r_B while bargaining but may choose α to hide its private information. In other words, a borrower's attempt to lie is incorporated in α.

In this section I analyse the bargaining game using the same strategies while taking α, β as the bargaining powers of the borrower perceived by the borrower and the lender, respectively. The values of α and β are determined by the borrower and the lender respectively and are made public at the beginning of the bargaining. They remain fixed during the bargaining game on date t, and may vary with the dates and differ among possible pairs of borrowers and lenders. For a fixed lender, its bargaining power not only depends on its own conditions but also its belief on its counterparties. Therefore it is possible that the deal rates of a lender vary due to different counterparties it trades with.

Consider the strategy where the lender offers $r(s) = \beta\underline{r} + (1-\beta)s$ if the lender believes the maximum possible value of r_B is s, and the borrower accepts the offered rate r if and only if $r \leq \alpha\underline{r} + (1-\alpha)r_B$. If the offered rate r is accepted by the borrower, then the agreement is struck and trade happens, otherwise the lender updates its belief of the maximum of r_B as $s^*(r)$. For notational simplicity, suppose the bargaining game starts from round 0. Let s_n be the lender's belief when offering in the n-th round, and r_n be the lender's offer in the n-th round.

The bargaining procedure proceeds as follows: Suppose the lender's prior belief s_0 is the central bank's lending rate $\bar{r}$, because the borrowers can always resort to central bank for loans and thus its optimal outside option offers rate at most equal to $\bar{r}$. In Round $n \in \mathbb{N}_0$, the lender proposes rate $r_n = \beta\underline{r} + (1-\beta)s_n$, which is accepted by the borrower if and only if $r_n < \alpha\underline{r} + (1-\alpha)r_B$. If r_n is rejected, then the lender updates its belief to $s_{n+1} = \frac{r_n - \alpha r}{1-\alpha}$ on which the offer in the next round is based.

It can be shown below that

$$r_n = \underline{r} + \frac{(1-\beta)^{n+1}}{(1-\alpha)^n}\left(\bar{r} - \underline{r}\right). \tag{4.7}$$

By induction, if $n = 0$, then $r_0 = \beta\underline{r} + (1-\beta)\bar{r}$ coincides with (4.7). Now suppose (4.7) holds for an arbitrary $n \geq 0$, then if the bargaining continues to the next round, the lender's new offer becomes

$$\begin{aligned} r_{n+1} &= \beta\underline{r} + (1-\beta)s_{n+1} = \beta\underline{r} + (1-\beta)\frac{r_n - \alpha\underline{r}}{1-\alpha} \\ &= \beta\underline{r} + \frac{1-\beta}{1-\alpha}\left[\underline{r} + \frac{(1+\beta)^{n+1}}{(1+\alpha)^n}(\bar{r} - \underline{r}) - \alpha\underline{r}\right] = \underline{r} + \frac{(1-\beta)^{n+2}}{(1-\alpha)^{n+1}}\left(\bar{r} - \underline{r}\right). \end{aligned}$$

According to the borrower's strategy, the borrower accepts the offered rate in the n^*-th round if and only if n^* is the smallest integer such that $\underline{r} + \frac{(1-\beta)^{n^*+1}}{(1-\alpha)^{n^*}}\left(\bar{r} - \underline{r}\right) \leq \alpha\underline{r} + (1-$

$\alpha)r_B$, which is equivalent to

$$\left(\frac{1-\beta}{1-\alpha}\right)^{n^*+1} \leq \frac{r_B-\underline{r}}{\bar{r}-\underline{r}} < \left(\frac{1-\beta}{1-\alpha}\right)^{n^*}. \tag{4.8}$$

This implies that the trade happens only if $\beta \geq \alpha$, because if $\beta < \alpha$, then n^* does not exist since $\left(\frac{1-\beta}{1-\alpha}\right)^n > 1 \geq \frac{r_B-\underline{r}}{\bar{r}-\underline{r}}$ for any $n \in \mathbb{N}_0$ which means the borrower will never accept any rate the lender offers. The failed interbank trades are not reported, so the case $\alpha > \beta$ cannot be studied given the available dataset. Without loss of generality, I take $1 > \beta > \alpha > 0$ as default and focus only on the transactions that happens.

Taking the logarithm on all terms in (4.8) gives

$$(n^*+1)\log\frac{1-\beta}{1-\alpha} \leq \log\frac{r_B-\underline{r}}{\bar{r}-\underline{r}} < n^*\log\frac{1-\beta}{1-\alpha}.$$

Note that $0 < \alpha < \beta < 1$, then $\log\frac{1-\beta}{1-\alpha} < 0$ and thus

$$\frac{\log\frac{r_B-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta}{1-\alpha}} - 1 \leq n^* < \frac{\log\frac{r_B-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta}{1-\alpha}}.$$

Then the explicit expression for n^* is:

$$n^* = \lfloor\frac{\log\frac{r_B-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta}{1-\alpha}}\rfloor \tag{4.9}$$

where $\lfloor\cdot\rfloor$ is the floor function that gives the greatest integer less or equal to the input. Therefore, if the interbank trade happens, the final deal rate between the given borrower and the lender is given by

$$r = \underline{r} + \frac{(1-\beta)^{n^*+1}}{(1-\alpha)^{n^*}}\left(\bar{r}-\underline{r}\right),$$

from which we have

$$\frac{(1-\beta)^{n^*+1}}{(1-\alpha)^{n^*}} = \frac{r-\underline{r}}{\bar{r}-\underline{r}}. \tag{4.10}$$

Taking logarithm on both sides of (4.10) and replacing n^* with (4.9) gives

$$\log\frac{r-\underline{r}}{\bar{r}-\underline{r}} = \log(1-\beta) + \lfloor\frac{\log\frac{r_B-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta}{1-\alpha}}\rfloor \cdot \log\frac{1-\beta}{1-\alpha} \tag{4.11}$$

Note that (4.11) is an abuse of notation, because here I only consider a transaction with deal rate r between a given pair of borrower B and lender L. This notation will be further clarified when considering all interbank trades in the market where all pairs of borrowers and lenders are considered.

In the analysis below I focus on two factors: the total number $N_{B,L}$ of unsecured trades between the given lender L and borrower B, and the difference between the CDS costs of B and L.

The unsecured interbank trades have been happening since long before the earliest date in the data. Before bargaining games take place, banks have already held beliefs their own bargaining power against their counterparties based on trading history. The total number $N_{B,L}$ of unsecured trades between the lender L and the borrower B implies their preference for each other based on their trading history from the past. If trades happen frequently between a borrower and a lender, then it can be inferred that these banks prefer trading with each other and the mutual trust is reinforced after each successful transaction. On the other hand, a low trading count between a given trading pair implies either banks' lack of information about each other or lack of mutual trust based on their history trading records. Therefore the total number $N_{B,L}$ of unsecured trades between B and L reflects banks' information on counterparties from the past.

A bank's CDS insurance cost is related to its default risk and is updated daily. The higher the CDS costs, the higher the bank's default risk becomes. Define the *relative riskiness* $\Delta C(B, L, t)$ of the borrower B and the lender L as the difference between the CDS costs of B and L on date t, that is,

$$\Delta C(B, L, t) = CDS(B, t) - CDS(L, t).$$

where $CDS(B, t)$ and $CDS(L, t)$ represent the cost of credit default swap of the borrower B and the lender L on date t, respectively. An increase in $\Delta C(B, L, t)$ implies a decrease of the borrower B's current bargaining power relative to the lender L. The value of $\Delta C(B, L, t)$ changes daily and reflects banks' assessment on each other's bargaining power based on new information.

4.4 Data

Note that in the current version of the thesis all the actual data has been replaced with hypothetical data. Any reference below to the real data is inadvertent. For those with adequate access rights, a copy with the actual results can be requested.

In this section, I estimate and calibrate the parameters with MMSR data. Then I use this model as a laboratory to study how players set their perceived bargaining powers α and β, and how α and β effect the bargaining procedures and results.

4.4.1 The MMSR Data

The Money Market Statistical Reporting (MMSR) provides information on the money market activities of large banks and a selection of their counterparties both on the lending and the borrowing side.[1] It is based on transaction-by-transaction data from a sample of EU reporting agents covering the following four segments:

[1]See `https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/money_market/html/index.en.html` for general information

(i) **Secured segment** consists of daily repurchase agreement transactions (borrowing and lending) denominated in euro with a maturity of up to and including one year.[2] Transaction with central banks related to Eurosystem tender operations and marginal lending facilities are exempted;

(ii) **Unsecured segment** consists of daily unsecured transactions covering

- all borrowing denominated in euro with a maturity of up to and including one year from financial corporations (except central banks where the transaction is not for investment purposes), general government as well as from non-financial corporations classified as wholesale under the Basel III LCR framework, using the instruments defined in the MMSR Regulation, in particular unsecured deposits and call accounts, and the issuance of fixed-rate or variable-rate short-term debt securities;
- all lending denominated in euro to other credit institutions with a maturity of up to and including one year via unsecured deposits[3] or call accounts[4], or via the purchase from the issuing credit institutions of fixed-rate or variable-rate short-term debt securities[5] with an initial maturity of up to and including one year;

(iii) **Foreign exchange swaps (FX swaps)** consists of daily foreign exchange swaps transactions with a maturity of up to and including one year, in which euro are bought or sold on a near-term value date against a foreign currency with an agreement to resell the purchased currency on a forward, pre-agreed maturity date. Transaction with central banks related to Eurosystem tender operations are exempted;

(iv) **Overnight index swaps (OIS)** consists of daily euro overnight index swap transactions denominated in euro of any maturity. It is the maturity of the underlying asset that qualifies the OIS as a money market instrument, regardless of the final maturity of the OIS.

The reporting population currently consists of the 50 largest euro area MFIs, based on the size of their total main balance sheet assets relative to the total main balance sheet assets for all euro area MFIs. The 50 reporting agents report on a consolidated basis,

[2] One year is defined as transactions with a maturity date of not more than 397 days after the settlement date.

[3] Unsecured deposits refer to unsecured interest-bearing deposits that are either redeemable at notice or have a maturity of not more than one year, and that are either taken (borrowing) or placed (lending) by the reporting agent.

[4] Call accounts refer to (1) cash accounts with daily changes in the applicable interest rate, giving rise to interest payments or calculations at regular intervals, and a notice period to withdraw money, or (2) saving accounts with a notice period to withdraw money.

[5] Fixed-rate and variable-rate short-term debt refers to borrowing via the issuance of short-term securities listed in Table 1, which are denominated in euro, from the reporting agent to counterparties, or refers to lending via the purchase on the primary market of short-term securities listed in Table 4.1, which are denominated in euro, issued by other credit institutions.

Table 4.1: Short-term securities covered in the daily statistical information reported under the MMSR Regulation

Short-term security identifier	Description
Certificate of deposit	A fixed rate debt instrument, in either a negotiable or non-negotiable form, that is issued by an MFI entitling the holder to a specific fixed rate of interest over a defined fixed term after the settlement date and is either interest-bearing or discounted.
Commercial paper (CP)	An unsecured debt instrument that is issued by an MFI and is either interest-bearing or discounted.
Asset-backed CP	A debt instrument that is issued by an MFI, is either interest-bearing or discounted and is backed by some form of collateral.
Floating rate note (FRN)	A debt instrument in which the periodic interest payments are calculated on the basis of the value, i.e. through fixing of an underlying reference rate, such as EURIBOR, on predefined dates known as fixing dates.
Other short-term debt securities	Unsubordinated securities other than equity, which are instruments that are usually negotiable and traded on secondary markets or which can be offset on the market and which do not grant the holder any ownership rights over the issuing institution. This item includes: (a) securities that give the holder the unconditional right to a fixed or contractually determined income in the form of coupon payments and/or a stated fixed sum on a specific date (or dates) or starting from a date defined at the time of issue; (b) non-negotiable instruments that subsequently become negotiable and are reclassified as debt securities.

including for all their Union and EFTA-located branches daily statistical information relating to money market instruments denominated in euro, as specified in the MMSR Regulation.

The transactions in the data are broken down into borrowing and lending transactions. Borrowing transactions comprises all borrowing of the reporting agent, denominated in euro with a maturity of up to and including one year, using the instruments defined in the MMSR Regulation. With regard to the unsecured segment, the instruments defined in the MMSR Regulation include, in particular, unsecured deposits and call accounts, as well as fixed-rate or variable-rate short-term debt securities issued with a maturity of up to and including one year. Lending transactions comprises all lending of the reporting agent to other monetary financial institutions with the exception of central banks and money market funds (euro area or non-euro area), denominated in euro with a maturity of up to and including one year via unsecured deposits or call accounts, or via the purchase from the issuing credit institutions of fixed-rate or variable-rate short-term debt securities with an initial maturity of up to and including one year.

The tenor of a transaction is the difference between settlement date and maturity date, and is quoted using the codes listed in Table 4.2.

In the next section, I use the Money Market Statistical Reporting (MMSR) data and the credit default swap (CDS) data to study how banks' information on each other effect the borrower's bargaining power α and the borrower's bargaining power perceived by the lender, β. The Money Market Statistical Reporting (MMSR) data is based on transaction-by-transaction data from 54 EU reporting agents. The CDS data contains the basis point cost of insurance for a year on daily basis for 36 out of the 54 reporting agents in MMSR data. In the following analysis, all overnight unsecured loans among the 36 EU reporting agents from July 2016 to May 2019 are considered. Before estimating parameters using unsecured overnight interbank lending data, I exclude transactions with abnormal and extreme values, such as transactions with outlying deal rates or negligible transaction amounts.

Due to the data confidentiality, the actual results are still awaiting clearance. To show what kind of results can be expected, we resort to amongst others [18],Table 2, and generate hypothetical data from multivariate normal distribution with approximately the same mean and variance of a typical interbank network.

Table 4.2: Tenor breakdown in MMSR

Code	Tenor	Description
O/N	Overnight	Transactions for which the settlement date is the trade date and that mature the following business day
T/N	Tomorrow/Next	Transactions for which the settlement date is the business day after the trade date (T+1) and that mature the following business day.
S/N	Spot/Next	Transactions for which the settlement date is two business days after the trade date (T+2) and that mature the following business day.
1W	One week	Transactions for which the settlement date is two business days after the trade date and that mature exactly one week after the settlement date.
1M	One month	Transactions for which the settlement date is two business days after the trade date and that mature exactly one month after the settlement date.
3M	Three months	Transactions for which the settlement date is two business days after the trade date and that mature exactly three months after the settlement date.
6M	Six months	Transactions for which the settlement date is two business days after the trade date and that mature exactly six months after the settlement date.
9M	Nine months	Transactions for which the settlement date is two business days after the trade date and that mature exactly nine months after the settlement date.
12M	Twelve months	Transactions for which the settlement date is two business days after the trade date and that mature exactly twelve months after the settlement date.

4.4.2 Calibration

In this section, I calibrate the values of α and β that characterize the bargaining strategies between the borrower B and the lender L on date t in Section 4.3. The calibration is based on (4.11) and the data discussed in Section 4.4.1.

For notational clarity, (4.11) can be rewritten as

$$\begin{aligned}\log\frac{r(B,L,t)-\underline{r}}{\bar{r}-\underline{r}} &= \log(1-\beta(B,L,t)) + n^*\cdot\log\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)} \\ &= \log(1-\beta(B,L,t)) + \lfloor\frac{\log\frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}}\rfloor\cdot\log\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}\end{aligned} \tag{4.12}$$

which is satisfied for the unsecured overnight lending between borrower B and lender L on date t. In (4.12), $\bar{r}$ and $\underline{r}$ are the lending rate and deposit rate by the central bank, and $r_{B,L,t}$ is the reservation rate of the borrower B when bargaining with the lender L on date t. The deal rate $r(B,L,t)$ is determined by the bargaining strategies between B and L characterized by parameters $\alpha(B,L,t)$ and $\beta(B,L,t)$. As has been discussed in 4.3, $\alpha(B,L,t)$ is the borrower B's bargaining power when bargaining with L on date t, $\beta(B,L,t)$ is the lender L's belief on B's bargaining power on date t, and n^* is the number of rounds in the bargaining game before the deal rate is finalized. The time trend is not taken into account in this model, because the deal rates from the data stay at a relatively stable level between a given borrower and a given lender, and the changes of deal rates are more driven by other factors.

Note that the borrower's private information $r_{B,L,t}$ is not explicitly reflected in the dataset, so before calibrating $\alpha(B,L,t)$ and $\beta(B,L,t)$ I first estimate the parameter $n^* = \lfloor\frac{\log\frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}}\rfloor$ in (4.12) from the data.

Given the borrower's strategy and (4.7), the deal rate between the borrower B and the lender L on date t satisfies

$$\begin{aligned}r(B,L,t) = r_{n^*}(B,L,t) &= \underline{r} + \frac{(1-\beta(B,L,t))^{n^*+1}}{(1-\alpha(B,L,t))^{n^*}}(\bar{r}-\underline{r}) \\ &\leq \alpha(B,L,t)\underline{r} + (1-\alpha(B,L,t))r_{B,L,t} \\ &< \underline{r} + \frac{(1-\beta(B,L,t))^{n^*}}{(1-\alpha(B,L,t))^{n^*-1}}(\bar{r}-\underline{r}) = r_{n^*-1}(B,L,t),\end{aligned}$$

where $r_n(B,L,t)$ is the offered rate from the lender L to the borrower B in the n-th round of their bargaining game on date t. Then

$$\begin{aligned}r_{B,L,t} \in &\left[\underline{r} + \left(\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}\right)^{n^*+1}(\bar{r}-\underline{r}),\ \underline{r} + \left(\frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}\right)^{n^*}(\bar{r}-\underline{r})\right) \\ &:= [\underline{r}_{B,L,t}, \bar{r}_{B,L,t})\end{aligned} \tag{4.13}$$

which implies that the accuracy of estimation on $r_{B,L,t}$ depends on both α and β. Given the lower and upper bounds of $r_{B,L,t}$ in (4.13), we have the following inequality

$$\lfloor \frac{\log \frac{\bar{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor \leq \lfloor \frac{\log \frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor < \lfloor \frac{\log \frac{\underline{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor, \tag{4.14}$$

where the arguments of the floor function in the first and third term has difference 1, namely,

$$\frac{\log \frac{\underline{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} - \frac{\log \frac{\bar{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} = \frac{\log \frac{\underline{r}_{B,L,t}-\underline{r}}{\bar{r}_{B,L,t}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} = \frac{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} = 1.$$

Then the property of floor function gives

$$\lfloor \frac{\log \frac{\underline{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor - \lfloor \frac{\log \frac{\bar{r}_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor = 1. \tag{4.15}$$

Note that $\underline{r}_{B,L,t} = \frac{r(B,L,t)-\underline{r}}{1-\alpha(B,L,t)}$, where $r(B, L, t)$ is the finalized deal rate available in data. If we replace $\underline{r}_{B,L,t}$ with $\frac{r(B,L,t)-\underline{r}}{1-\alpha(B,L,t)}$ in (4.14) and combine the result with (4.15), the inequality (4.14) becomes

$$\lfloor \frac{\log \frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor \in \left[\lfloor \frac{\log \frac{r(B,L,t)-\underline{r}}{(1-\alpha(B,L,t))(\bar{r}-\underline{r})}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor, \lfloor \frac{\log \frac{r(B,L,t)-\underline{r}}{(1-\alpha(B,L,t))(\bar{r}-\underline{r})}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor + 1 \right).$$

The length of the interval above cannot be narrowed and thus the value of $\lfloor \frac{\log \frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor$ cannot be further clarified given the available information. In the following analysis, the supremum of the interval is taken as an estimate for $\lfloor \frac{\log \frac{r_{B,L,t}-\underline{r}}{\bar{r}-\underline{r}}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor$. Then (4.12) becomes

$$\log \frac{r(B,L,t)-\underline{r}}{\bar{r}-\underline{r}} = \log(1-\beta(B,L,t)) + \left(\lfloor \frac{\log \frac{r(B,L,t)-\underline{r}}{(1-\alpha(B,L,t))(\bar{r}-\underline{r})}}{\log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}} \rfloor + 1 \right) \cdot \log \frac{1-\beta(B,L,t)}{1-\alpha(B,L,t)}. \tag{4.16}$$

Note that $\alpha(B, L, t)$ and $\beta(B, L, t)$ cannot be explicitly presented using (4.16), and the floor function in (4.16) brings difficulty to calculation of derivatives. Moreover, the irregular form of (4.16) makes iterative optimization algorithms sensitive to initial values and step sizes (see Figure 4.1). Therefore I calibrate the values of α and β using brute-force search for each trade in unsecured overnight lending data. To be more specific, for a given pair of borrower B and lender L on date t, $(\alpha(B, L, t), \beta(B, L, t))$ is a point in $\{(a, b)|a \in [0, 1], b \in [0, 1], a \leq b\}$ such that it minimizes the mean squared error with regard to (4.16). Here the constraint $a \leq b$ is obtained from (4.8) which implies that the interbank trade happens only when $\alpha \leq \beta$.

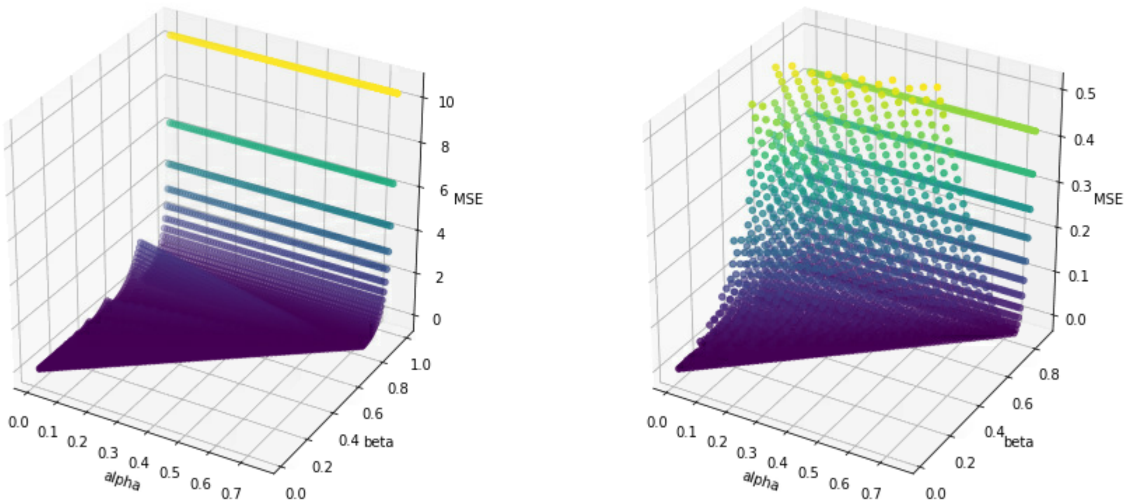

Figure 4.1: Mean Square Error (MSE) for all possible combinations of α and β in $[0.01, 1]$ with step=0.005 where $\alpha < \beta$. The value of mean square errors are computed using model (4.16). Left: the MSE for all possible combinations of α and β. Right: A zoom-in for MSE less than 0.5. The existence of several local minima increases the difficulty of implementing iterative methods for optimization.

Given a borrower and a lender, the difference between bargaining powers α and β characterizes the borrower's relative bargaining power and is influenced by banks' relation. The value of $\beta - \alpha$ varies among different borrowers and lenders. Figure 4.2 shows the plot of borrowers' relative bargaining power $\beta - \alpha$ against the total number of transactions $N(B, L)$ and relative risk $\Delta C(B, L, t)$ between the borrower B and the lender L. The parameters $N(B, L)$ and $\Delta C(B, L, t)$ represent banks' information on each other from the past and CDS data on the day of bargaining. It shows that the difference between α and β is close to zero for most transactions in the hypothetical data. However, the value of $\beta - \alpha$ is more likely to be positive with larger total number of trades $N(B, L)$ in the past and lower relative risk $\Delta C(B, L, t)$. To be more specific, the median of $\beta - \alpha$ becomes greater when only the transactions between players with low relative risk and frequent history transactions are considered (as is shown in the first row of Table 4.3). This implies the lender hold more confidence in the borrower when they have low relative risk and frequent history transactions, and is thus willing to offer a lower deal rate to the borrower. Therefore the difference $\beta - \alpha$ measures the relative relation between the borrower and the lender during the bargaining process. The larger $\beta - \alpha$ is, the more advantages the borrower has.

In the next section, I continue with analysing the relation between α and β and show how their values effect the formation of unsecured overnight interbank lending. Moreover, the difference between α and β can influence the efficiency of a bargaining game.

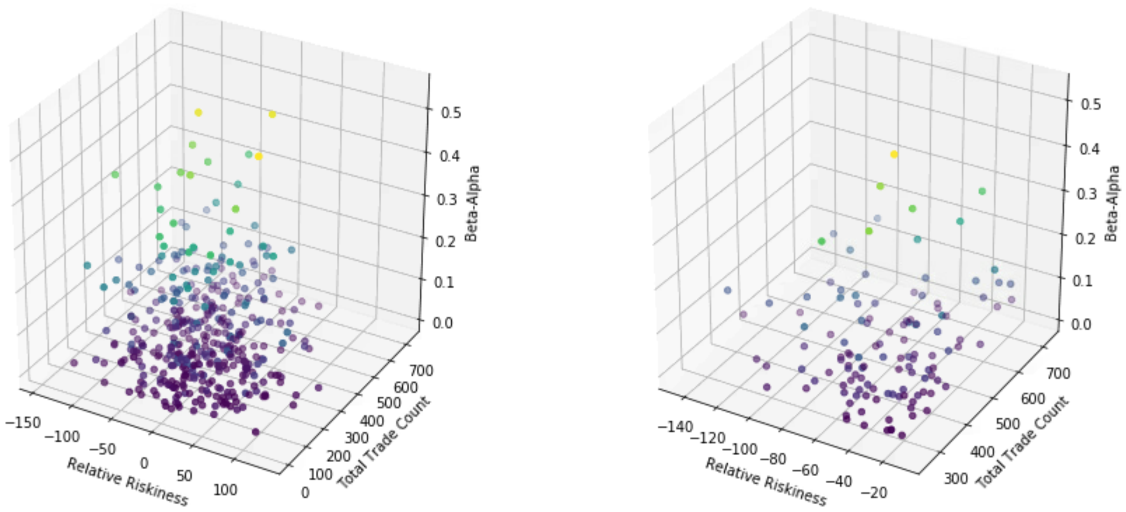

Figure 4.2: Scatter plot of $\beta - \alpha$ against $N(B, L)$ and $\Delta C(B, L, t)$. Each point represents a sampled point from the hypothetical data. The value of each α (β) is calibrated using brute-force method.

	All transactions	Transactions with low relative risk and frequent history transactions
All Data	0.040	0.050
Transactions with Large Volume	0.035	0.055
Transactions during Margin Period	0.040	0.020

Table 4.3: Median of $\beta - \alpha$ in different data sets.

4.4.3 Formation of Unsecured Overnight Interbank Lending

It has been shown in (4.8) that under the bargaining strategies in this chapter, the bargaining game between the borrower B and the lender L on date t fails to reach an agreement if $\alpha(B, L, t) > \beta(B, L, t)$. On the other hand, since the bargaining game is assumed to start at Round 0, the inequality (4.8) must hold for some $n^* \in \mathbb{N}_0$ if $\alpha(B, L, t) \leq \beta(B, L, t)$. Therefore it can be concluded that the bargaining game succeeds if and only if

$$\alpha(B, L, t) \leq \beta(B, L, t) \tag{4.17}$$

Intuitively, if the lender has a lower estimation β on the borrower's bargaining power compared with the borrower's estimation α, then this implies the lender believes itself to have a relatively high bargaining power compared with its counterparty and thus it is reasonable to propose a high deal rate. Likewise, the borrower has a higher estimation on its own bargaining power and believes a lower deal rate is reasonable. As a result, the lender always offers rates higher than the maximum value that the borrower can accept, and thus the bilateral bargaining game will never reach an agreement.

On the other hand, if the lender's estimation of the borrower's bargaining power is higher than the borrower's estimation, then an agreement must be reached within finite number of rounds. Note that there is value discount with factor δ for both sides after every intermediate bargaining round before the final deal rate is determined, so the less bargaining round it takes, the more efficient the procedure is.

The value of n^* is effected by both α and β as can be seen in (4.9), but the relation between n^*, α and β cannot be explicitly expressed. A qualitative illustration on how α and β influence the number of bargaining rounds is shown in Figure 4.3. When the

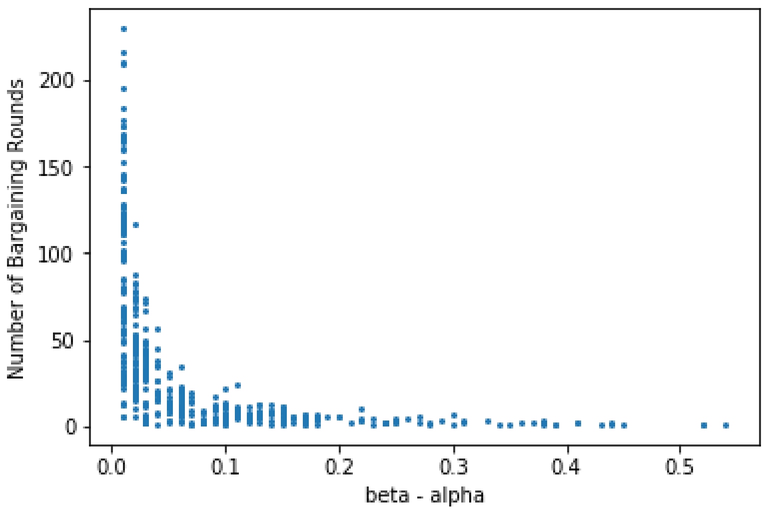

Figure 4.3: Plot of the number of bargaining rounds n^* and $\beta - \alpha$. Each point represents a bilateral transaction where the corresponding n^*, β and α are calibrated using hypothetical data.

difference between β and α is small, i.e. α and β are close, the bargaining will take more rounds before the agreement is struck. This would lead to less value discount during the bargaining process.

From above plots and analysis, it can be concluded that a decrease in relative riskiness on the day of bargaining and an increase in mutual trust gained from the past transactions (represented by the total number of unsecured trades) between the borrower and the lender increase the probability that the bargaining succeeds. Moreover, if the bargaining game succeeds, then it would take less bargaining rounds before an agreement is reached and thus less value is discounted during the bargaining process.

If a borrower pretends itself to have a higher α than the actual value for more benefits, then it also faces a larger probability of interbank trade failure because the reported α may be greater than the β reported by its potential lender. Even if β is greater than α, an increase in α value would decrease the value of $\beta - \alpha$ and thus leads to more value discount for both players during the bargaining process.

4.4.4 Sensitivity Test

In this section, I analyse the sensitivity of the bargaining model proposed in 4.3 by considering two special subsets of data:

1. Transactions with large trading volume;

2. Transactions during margin periods, i.e. when the total trading volume in the market exceeds the normal level.

The following analysis shows the effect of total number of history trade $N(B, L)$ and relative risk $\Delta C(N, L, t)$ on $\beta - \alpha$ can be different under different conditions. The value of $\beta - \alpha$ is sensitive to the influence from $N(B, L)$ and $\Delta C(N, L, t)$ when the trading volume is large. During the margin period, this influence is insignificant.

Transactions with large trading volume

This part analyses the subset of data consisting of transactions with trading volume greater than the corresponding daily median volume. All parameters from the model are estimated using hypothetical data. Namely, the joint distribution of deal rate, relative risk and total number of trade is normal with the same mean and variance as the subset of the original data which consists of transactions with large trading volume. This normal distribution approximates the real distribution of the original data, and leads to results similar to those obtained from the original data.

Following the same methods as in Section 4.4.2, I plot the relative bargaining power $\beta - \alpha$ against the total number $N(B, L)$ of trade and the relative riskiness $\Delta C(B, L, t)$, as can be seen in Figure 4.4. The pattern in Figure 4.4 is similar to that in Figure 4.2, where β and α are close for most transactions. Furthermore, if the borrower and the lender have a low relative risk and has transacted with each other frequently in the past, then the lender may have a higher estimation on the bargaining power of the

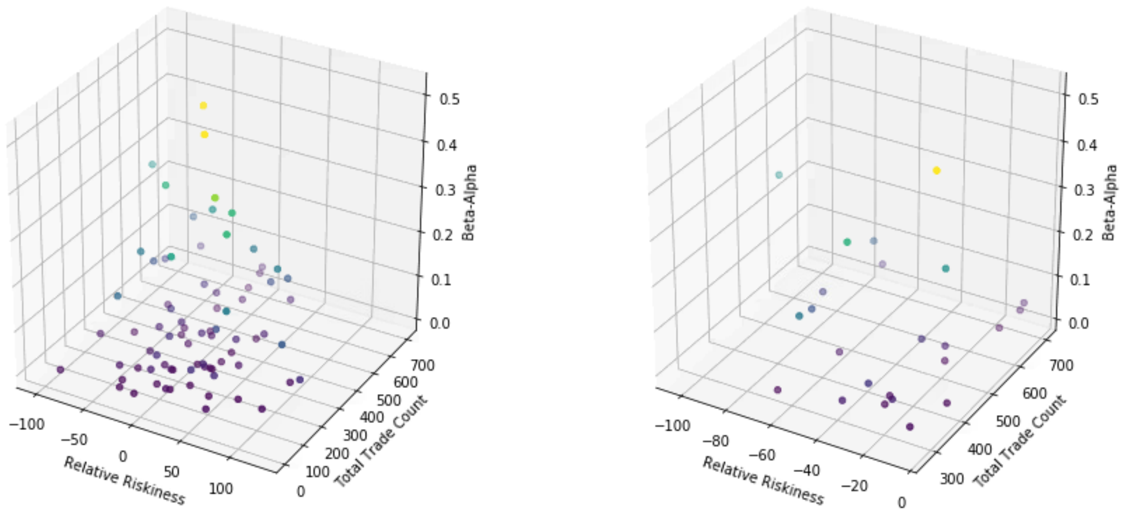

Figure 4.4: Scatter plot of $\beta - \alpha$ against $N(B, L)$ and $\Delta C(B, L, t)$ when only considering transactions with large volume. Each point represents a sampled point from the hypothetical data generated from transactions with large volume. The value of each α (β) is calibrated using brute-force method.

borrower (as is shown in the second row of Table 4.3), and thus offers a low deal rate to the borrower. This result is consistent with that analysed in Section 4.4.2 where all transactions are considered.

Transactions during margin periods

In this part, I show the results when analysing the subset of the data consisting of transactions during margin periods, namely, when the total trading volume of the 36 banks in the unsecured overnight market spikes. To detect the margin periods, I first select the dates with volume 'spikes' when the total volume exceeds the upper Bollinger band (i.e. two standard deviations positively away from the simple moving average of the transactions nominal amount with window=20 days), and then take all nearby dates (± 5 days) of the dates with volume spikes as the margin periods.

To estimate the parameters from the model, I first generate the hypothetical data from a normal distribution with the same mean and variance as this subset of the original data during margin period. The results from the generated data illustrate how the real data perform.

Figure 4.5 shows the relation between $\beta - \alpha$, the total number of trades in the past, and the relative risk between the borrower and the lender. The parameters α and β are calibrated following the methods in Section 4.4.2. The relation between $\beta - \alpha$, relative risk and total number of history transactions analysed in Section 4.4.2 and Section 4.4.4, as has been discussed in the previous section, cannot be concluded during the margin period. From the plot on the right in Figure 4.5, the values of $\beta - \alpha$ are very

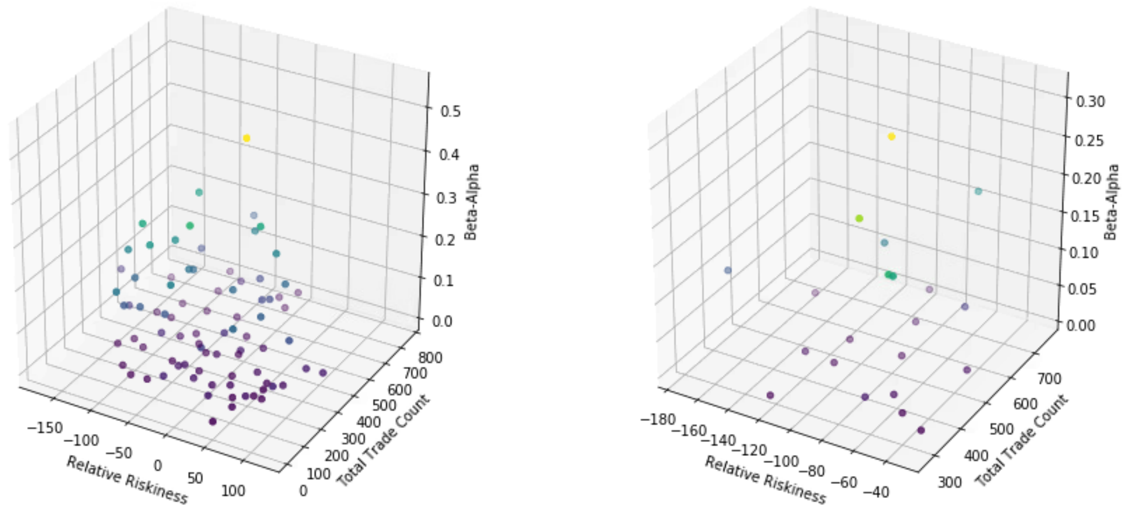

Figure 4.5: Scatter plot of $\beta-\alpha$ against $N(B, L)$ and $\Delta C(B, L, t)$. Each point represents a sampled point from the hypothetical data generated using data in margin period. The value of each α (β) is calibrated using brute-force method.

small compared with the plot on the left. Table 4.3 also shows that the median value of $\beta - \alpha$ during the margin period behaves differently from that when considering all transactions, or transactions with large volume. This implies that the borrower's relative bargaining power is not positively correlated with the joint influence of a low relative risk and frequent trading history anymore when the unsecure overnight interbank market is active in terms of total volume. Comparing the results obtained using different data set, we can see that the value of $\beta - \alpha$ can also be effected by the market condition. When the total volume in the interbank market exceeds the normal level, more lenders may fear the default of their counterparties due to the large amount of loans they lend, and thus there is more other information of the borrowers that needs to be considered when bargaining over the deal rates.

4.4.5 Adverse Selection

The strategies discussed in this chapter can explain adverse selection phenomena where there is asymmetric information between players and the uninformed player fears an unfair trade when bargaining with an informed player. This information asymmetry can lead to the uninformed players transacting more with riskier counterparties. A textbook example of adverse selection can be found in [16] in the context of market with asymmetric information.

Consider the bargaining game between a borrower B and a lender L. From the analysis above we know that there are other factors that determines the value of α and β but are excluded from public information $\Delta(B, L, t)$ and $N(B, L)$. Let $\gamma(B, t)$ be the parameter that determines the value of $\alpha(B, L, t)$ and $\beta(B, L, t)$ and is independent of relative riskiness and total numbers of trades. Moreover, this parameter $\gamma(B,t)$ is private information of the borrower, which can be interpreted as latent risks of the borrower unobservable for the lender. For simplicity, assume $\gamma(B,t)$ takes only two possible values $\{h,l\}$, where h means the borrower's latent risk is high and l means a low latent risk. Furthermore, given public information $\Delta(B,L,t)$ and $N(B,L)$, the lender would transact with the borrower if and only if $\gamma(B,t)=l$ under complete information. Then it can be concluded from (4.17) that

$$\beta(h) < \alpha(h) \leq \alpha(l) \leq \beta(l),$$

where $\alpha(\gamma)$ and $\beta(\gamma)$ (where $\gamma = h,l$) are the borrower's and the lender's beliefs of bargaining power if γ is public.

However, if the true value of $\gamma(B,t)$ is h, then the borrower has the incentive to lie and pretend itself to have a low latent risk. To be more specific, the borrower of type h would pretend to have a bargaining power as high as $\alpha(l)$. On the other hand, the lender is aware of the borrower's incentive to lie, so it sets its parameter β to be strictly lower than $\beta(l)$ and the deal rate proposed by the lender is strictly higher than what the type l borrower could accept. Therefore, the lender may lose some mutually beneficial opportunities of trading with borrowers' of low latent risks.

5 Conclusion

In this thesis I study the unsecured overnight interbank transactions using bargaining theory. The transactions in the over-the-counter interbank market are modelled as bilateral bargaining games. A main focus in this thesis is analysing interbank transactions under asymmetric information where the Nash assumption is not satisfied.

I begin with studying bargaining theory, equilibrium concepts and bargaining solutions. The analysing methods for bargaining under complete information and incomplete information are based on different assumptions, so it is necessary to identify private information in the bargaining game and then select the appropriate bargaining solution.

Then I study the relation between information completeness and the efficiency of bargaining results. Three different kinds of information sets in bargaining are considered: complete information, one-sided and two-sided private information. The information completeness between a borrower and a lender effects the efficiency of the bargaining outcome. To be specific, an increasing level of information completeness is more likely to lead to an efficient bargaining result.

Next, a sequential bargaining model is proposed for the unsecured overnight interbank lending between a borrower and a lender. The strategies for the bargaining are characterized by the borrower's bargaining powers perceived by the borrower and the lender, which are effected by their relationship in the past and new information obtained on the day they bargain. The parameters in the model are estimated using hypothetical data. This model then explains how the deal rates in the unsecured overnight interbank market are determined, the formation of the interbank lending and adverse selection under asymmetric information in the interbank market.

The model is well-suited to describe the bargaining situations when lenders have limited outside options. In consistency with the data, I assume the lender's expected profit per unit loan comes from the difference between the interbank rate and the central bank's deposit rate. The model can be further extended to the two-sided private information case by adjusting the discussed bargaining strategies.

The model has abstracted from the counterparty searching process before bargaining and focused only on the bargaining procedures. In reality, the interbank rates are also effected by the searching process, because banks agree on different rates when facing different counterparties. Therefore, it might be useful to consider the network relations among all banks and incorporate directedness of searching.

Bibliography

[1] U. Vollmer and H. Wiese. *Explaining breakdowns in interbank lending: A bilateral bargaining model*, Finance Research Letters, **11(3)**, 247-253, 2014.

[2] D. Diamond and P. Dybvig. *Bank runs, deposit insurance, and liquidity.* Journal of political economy, **91.3**, 401-419, 1983.

[3] F. Heider, M. Hoerova and C. Holthausen. *Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk.* EBC Discussion Paper, 2009.

[4] F. Blasques, F. Bruning and I. van Lelyveld. *A dynamic stochastic network model of the unsecured interbank lending market*, Journal of Economic Dynamics and Control, **90**, 310-342, 2018.

[5] G. Afonso and R. Lagos. *Trade dynamics in the market for federal funds.* Econometrica, **83(1)**, 263-313, 2015.

[6] J. Tapking and J. Eisenschmidt. *Liquidity risk premia in unsecured interbank money markets.* European Central Bank, No. 1025, 2009.

[7] A. Muthoo. *Bargaining Theory with Applications.* Cambridge University Press, 1999.

[8] M.J. Osborne and A. Rubinstein. *Bargaining and markets.* Academic Press, 1990.

[9] R.B. Myerson. *Game Theory: Analysis of Conflict.* Academic Press, 1997.

[10] J.F. Nash. *The Bargaining Problem.* Econometrica: Journal of the Econometric Society, **18(2)**, 155-162, 1950.

[11] P.J.C. Spreij. *Portfolio Theory.* Lecture notes Department of Mathematics Amsterdam, 2010.

[12] J. Peck. Perfect Bayesian Equilibrium and Refinements of Sequential Equilibrium. https://www.asc.ohio-state.edu/peck.33/gametheory/gameL9.pdf

[13] R.B. Myerson. *Mechanism design by an informed principal.* Econometrica: Journal of the Econometric Society, 1767-1797, 1983.

[14] R.B. Myerson and M.A. Satterthwaite. *Efficient mechanisms for bilateral trading.* Journal of economic theory, **29(2)**, 265-281, 1983.

[15] J.Y. Kim. *Neutral Bargaining in Financial Over-The-Counter Markets.* AEA Papers and Proceedings, **109**, 539-544, 2019.

[16] G.A. Akerlof. *The Market for 'Lemons': Quality Uncertainty and the Market Mechanism*. Quarterly Journal of Economics, The MIT Press, **84(3)**, 488500, 1970.

[17] J.F. Cocco, F.J. Gomes and N.C. Martins. *Lending relationships in the interbank market*. Journal of Financial Intermediation, **18(1)**, 24-48, 2009.

[18] K. Anand, et al. *The missing links: A global study on uncovering financial network structures from partial data*. Journal of Financial Stability, **35**, 107-119, 2018.