Systemic Risk in Financial Systems
Eisenberg and Noe (2001)

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Overview

- Motivation
- Overview of Eisenberg & Noe framework
- Summary of earlier literature
- Details of Eisenberg & Noe model
- Extensions of Eisenberg & Noe model
- Other extensions: impact of information, connectivity and initial shock size

Motivation

- Structure of financial systems:
    - Consists of a network of financial obligations
    - A firm's value depends on payoffs received from claims on other firms
    - The value of claims in turn depends on financial health of other firms in the system
- Cyclical interdependence:
    - Example: Firm A has a claim on B; C has a claim on B; and A has a claim on C
    - Feedback effects: A default by A $\rightarrow$ default by B $\rightarrow$ default by C $\rightarrow$ feedback effects on A $\rightarrow \cdots$
- General research question:
    - How do shocks to particular institutions or assets propagate through the network?

Framework of Eisenberg & Noe

- A set of n nodes:
  - Representing financial entities: e.g. banks, broker-dealers, insurance companies, e.t.c
- A nominal liability matrix:
  - Representing nominal liability claims (promised payments) between financial entities.
- Exogenous cash flows:
  - A vector representing total payments from nonfinancial to financial entities
  - A vector representing total payments from financial to nonfinancial entities

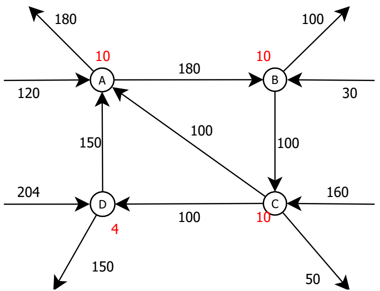
  Negative shocks:
  - The outside assets (exogenous cash-inflow) suffer a negative shock causing a default to some financial entities
  - Pro rata payments to the claimants in case of default

Research question in Eisenberg & Noe framework

- Research questions:
  - Conditional on the initial shock, what is the consistent set of payments?
    - A vector of consistent set of payments is also called the *clearing payment vector*
  - When is the clearing payment vector unique?
- Challenges:
  - Cyclical interdependence: the payout by defaulting node to claimants depends on the payout from her debtors, which depends on payout from her debtor's debtors, and so on$\cdots$
  - Developing analytical and numerical solutions for the clearing vector can thus be challenging

Demonstrative example

Figure 1: Taken from Glasserman and Young (2016)



- Numbers on directed edges represent payment obligation
- Numbers in red represent node's net worth = Assets - Liabilities
- Question: what is the impact of an exogenous shock that causes households to default on their payment to bank C's so that they pay 40 instead of 160?

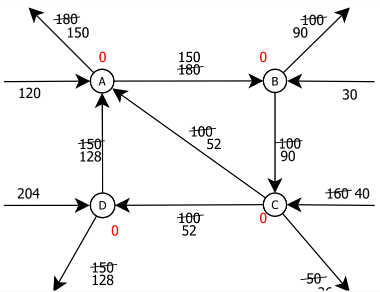
Demonstrative example cont'd

- Step 1: bank C defaults
  - C's assets $= 140$; Liabilities $= 250$; net worth (after default)$= 0$
  - Pro rata payout allocation: C pays $\frac{100}{250} \times 140 = 56$ to D and to A; and 28 to outside depositors
- Step 2: bank D defaults:
  - D's assets $= 260$; Liabilities $= 300$; net worth (after default)$= 0$
  - Pro rata payout allocation: D pays $\frac{150}{300} \times 260 = 130$ to A and outside depositors
- Step 3: bank A defaults:
  - A's assets $= 306$; Liabilities $= 360$; net worth (after default)$= 0$
  - Pro rata payout allocation: A pays 153 to B and outside depositors
- Step 4: bank B defaults:
  - B's assets $= 183$; Liabilities $= 200$; net worth (after default)$= 0$
  - Pro rata payout allocation: B pays 91.5 to C and outside depositors
- Hence, bank C's assets are worth less than 140; the iterative process continues until a consistent clearing vector is reached

Demonstrative example cont'd

- After multiple steps of iteration, the clearing set of payments is:

Figure 2: Taken from Glasserman and Young (2016)



- Eisenberg and Noe provide a framework and an algorithm for computing the clearing vector.

Main results in Eisenberg & Noe

- For every financial system, there exists at least one clearing payment vector

- The clearing payment vector is unique if:
    - The vector of total payments from the nonfinancial to the financial entities is nonzero, with at least one strictly positive element.
    - The network of payment obligations is strongly connected

- Cyclic interdependencies lead to amplification effects that lower the total value (debt plus equity) of nodes in the system (see example above)

Earlier literature on systemic risk

- Rochet & Tirole (1996) develop a theoretical model to study the incentive and monitoring impact of interbank loans
  - Beyond the efficient liquidity allocation that can be equally provided by the central bank, decentralized interbank lending can facilitate effective peer monitoring
- Angelini et al. (1996) develop an empirical model of intercorporate defaults for the Italian netting system:
  - The 'domino effect' from a participants failure depends on volume of funds flowing through the system
- Elimam (1997) develop a linear programming model for identifying insolvent traders (and payout vector) during the Kuwait's stock market crash of 1982.
- Eisenberg & Noe provide the first general model of intercorporate cash flows in financial systems with cyclical interdependencies

Model details: Basic elements

- A set of n nodes denoted by $N = \{1, 2, \cdots, n\}$
- An $n \times n$ liability matrix $L = (L_{ij})$, where $L_{ij} \geq 0$ represents payment due from i to j; $L_{ii} = 0$ for every i
- Outside assets: represented by a vector $e = (e_1, e_2, \cdots, e_n)$, where $e_i \geq 0$ is the payment due from nonfinancial entities to node i
- Outside liabilities: represented by a vector $b = (b_1, b_2, \cdots, b_n)$, where $b_i \geq 0$ is the payment due from node i to nonfinancial entities
- For a node i:
    - Total assets $= e_i + \sum_{j \neq i} L_{ji}$
    - Total liabilities, $\bar{p}_i = b_i + \sum_{j \neq i} L_{ij}$
    - Net worth, $w_i = e_i + \sum_{j \neq i} L_{ji} - \bar{p}_i$

Model details: shocks and defaults

- A shock realization: represented by a vector $x = (x_1, x_2, \cdots, x_n)$, where $0 \leq x_i \leq e_i \geq 0$ for every i

- Default: the shock directly reduces the network of a node, i.e.

$$w_i(x) = e_i - x_i + \sum_{j \neq i} L_{ji} - \bar{p}_i$$

- Node i defaults if $w_i(x)$ i negative, that is

$$x_i > w_i = e_i + \sum_{j \neq i} L_{ji} - \bar{p}_i$$

Model details: Assumption 1

- Define the relative liability matrix, $\Pi = (\Pi_{ij})$ as

$$\Pi_{ij} = \frac{L_{ij}}{\bar{p}_i} \quad \text{if } \bar{p}_i > 0; \qquad \Pi_{ij} = 0 \quad \text{if } \bar{p}_i = 0$$

  - Each Π_{ij} is i's obligation to j as a proportion of i's total liabilities

- Let $p(x) = (p_1(x), p_2(x), \cdots, p_n(x))$ (different from $\bar{p}$) be a vector of total payments made by nodes

- Assumption 1: Limited liability $\Rightarrow$ total payments made by a node must not exceed cash flows available

$$p_i(x) \leq (e_i - x_i) + \sum_{j \neq i} \Pi_{ji} p_j(x)$$

Model details: Assumption 2

- Assumption 2: debt obligations have equal priority and assets are distributed pro rata $\rightarrow$ either obligations are paid in full, i.e., $p_i(x) = \bar{p}_i$, or all value is paid to creditors, i.e.

$$p_i(x) = (e_i - x_i) + \sum_{j \neq i} \Pi_{ji} p_j(x)$$

Results: existence of clearing payment vector

- A clearing payment vector p^*: a set of consistent payments satisfying Assumptions 1 & 2; that is, for every i

$$p_i^*(x) = \min\left[(e_i - x_i) + \sum_{j \neq i} \Pi_{ji} p_j^*(x), \bar{p}_i\right]$$

Theorem

(Eisenberg & Noe, Theorem 1) Corresponding to every financial system, there exists a greatest and least clearing payment vectors

- The proof uses a general fixed-point theorem on lattices due to Tarski (1955)

Results: uniqueness of clearing payment vector

- Regularity condition: at least one node has positive equity value, and the network of liability flows is strongly connected
  - A network of liabilities is strongly connected if from every node i there exists a chain of positive obligations to every other node $j \in N$
  - Positive equity and strong connectedness ensure that all nodes have positive operating cash flows

Theorem

(Eisenberg & Noe, Theorem 2) If the financial system is regular, the greatest and least clearing payment vectors are the same, implying that the clearing vector is unique

Results: computing clearing payment vector

- Fictitious default algorithm:
  - For a given shock realization x, let $p = p(x)$; and define the map $\Phi : \mathbb{R}^n_+ \to \mathbb{R}^n_+$ as

$$\Phi_i(p) = \min \left[(e_i - x_i) + \sum_{j \neq i} \Pi_{ji} p_j, \bar{p}_i \right] \quad \forall\, i \in N$$

  - Starting with $p^0 = \bar{p}$, let

$$p^1 = \Phi(p^0), \quad p^2 = \Phi(p^1), \quad \cdots \tag{1}$$

  - The iterations in (1) yield a monotone decreasing sequence $p^0 \geq p^1 \geq p^2 \geq \cdots$
  - This sequence is bounded below by the zero vector; hence it has a limit $p^* = p^*(x)$, which is a clearing payment vector

Limitations of Eisenberg & Noe model

- The Eisenberg & Noe model is oversimplified in several respects:
  - Pro rata distribution ignores further impairments to asset value and other potential costs
  - Equal priority claim on debt obligations ignores a variety of claims that banks have on one another
  - There are many other sources of financial crises other than a reduction in payments (shocks on outside assets): e.g. funding run, fire sales, loss of creditworthiness, e.t.c.

Extensions: Bankruptcy costs and recovery rates

- Bankruptcy costs: result from delays in paying creditors, and legal and administrative costs
- Rogers & Veraart (2013) extend Eisenberg & Noe model to include bankruptcy costs
  - Recovery function $r(\alpha, \bar{p})$: represents the amount paid to creditors; its a function of a bank's assets α and obligations $\bar{p}$, where

$$0 \leq r(\alpha, \bar{p}) \leq \alpha \quad \text{if } \alpha < \bar{p}; \quad r(\alpha, \bar{p}) = \bar{p} \quad \text{if } \alpha \geq \bar{p}$$

  - The clearing condition is then of the form

$$p_i^*(x) = r\left((e_i - x_i) + \sum_{j \neq i} \Pi_{ji} p_j^*(x), \bar{p}_i\right)$$

  - Conclusions from the model:
    - Bankruptcy costs amplify the impact of initial shocks
    - In the presence of bankruptcy costs, there is incentive for solvent banks to rescue failing banks

Extensions: Claims of different seniority

- Claims may have different seniority e.g. when banks hold stakes on each other's equity
- Elsinger (2009) and Gourieroux et al. (2013) extend Eisenberg & Noe to include cross-holdings of equity:
    - Let $\beta_{ij} \in (0,1)$ be a fraction of bank i equity owned by bank j
        - β_{ij} is thus j's claim on i's net worth provided it is positive
- For a given payment vector $p = p(x)$, the "interim" net worth of each node i is

$$w_i(p) = \left[e_i + \sum_{j \neq i} \Pi_{ji} p_j + \sum_{j \neq i} \beta_{ji} \left(\max[w_j(p), 0] \right) \right] - \bar{p}_i \quad \forall i \in N \tag{2}$$

- Elsinger (2009) show that for every $p \in [0, \bar{p}]$, there exists a vector of net worths $w(p)$ satisfying (2)

Extensions: Claims of different seniority cont'd

- To close model, Elsinger (2009) solve for a vector of payments $p \in [0, \bar{p}]$ which is a fixed point of the mapping

$$p = \min\left[w(p) + \bar{p}, \bar{p}\right] \tag{3}$$

- This mapping involves two fixed point, one nested inside the other
- Elsinger (2009) show that a solution to (3) always exists

Extensions: Fire sales

- Fire sales arise from spillover effects resulting from common exposures
  - Occurs when e.g. banks are forced to sell illiquid assets to prop up their balance sheet by increasing cash reserves
  - If other banks are exposed to the same asset, sales put a downward pressure on its price $\Rightarrow$ a negative impact on balance sheets
- Cifuentes et al. (2005) extend Eisenberg & model to incorporate fire sales contagion
  - Assume bank i's assets are of three parts: cash reserves c_i, a quantity of illiquid assets q_i with current price θ, and payment from other banks $\sum_{j \neq i} p_{ji}$; hence assets' total is

$$\theta q_i + c_i + \sum_{j \neq i} p_{ji}$$

  - Liabilities consists of interbank obligations and obligations to depositors

Extensions: Fire sales cont'd

- Liquidity shock: forces bank i to sell a portion of illiquid assets to increase cash reserves
  - Assume banks hold the same illiquid assets and let $q_i' \leq q_i$ be the amount bank i is forced to sell
  - Assume also the price $\theta(q')$ decrease with quantity q'; then bank i's assets become

$$\theta(q')(q_i - q_i') + (c_i + \theta(q')q_i') + \sum_{j \neq i} p_{ji}(q')$$

- Cifuentes et al. (2005) show that there is a level of capital requirement that forces banks to raise more cash the lower the price of illiquid asset
  - Forced sales further depress the price $\Rightarrow$ downward price spiral and possibly outright default by some banks

Other variations of risk contagion models

- Imperfect information:
  - Battiston et al. (2012) extend Eisenberg & Noe model to incorporate imperfect information regarding creditworthiness of debt holders
    - Banks may know which other banks have defaulted, but do not know the exposures towards their counterparties
- The impact of network connectivity:
  - Allen & Gale (2000), Nier et al. (2007), Gai & Kapadia (2010) and Elliot et al. (2014) show that increasing connectivity increases shock transmission and shock absorption
    - However, shock transmission dominates at lower connectivity, and shock absorption dominates at higher connectivity
- The impact of initial shock distribution:
  - Glasserman & Young (2015) and Acemoglu et al. (2015) study the impact of varying the size of initial shocks
    - The ring network always produces the greatest number of defaults; it concentrates the spillover from one node to another
    - A complete network produces the least number fo defaults when shocks are small, and the greatest when shocks are large