Financial Networks and Contagion

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Table of contents

Basics

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- The environment
- Integration
- Diversification
- Firm failures
- Failure cascade

Questions

Questions

The broad question is: **How do interconnections in the financial markets affect systemic risk?** To this end:

- Study financial networks and analyze:
  - How Integration can impact the probability of a first failure and failure cascade (caused by exogenous shocks to asset values)?
  - Once a failure occurs, how does the extent of diversification and integration affect the *extent* of failure cascades?
- Run simulations based on random networks
- Empirical results set in the market for European debt

Preview of Main Results

Main results

- Non monotonic impact of integration and diversification on the risk of failure cascade (contagion)
  - Integration$\uparrow \Rightarrow$ Prob of first failure $\downarrow$, prob of cascade if first failure$\uparrow$
  - Diversification$\uparrow \Rightarrow$ Interdependence $\uparrow$ (more spread of shock), Risk insurance$\uparrow$ (higher power to absorb shock)
    - Low diversification - Limited cascades
    - Intermediate diversification - Large cascades
    - High diversification - Limited or no cascades
- Highest risk to economy (large cascades) when *intermediate* integration and *intermediate* diversification

Contribution to Literature

Contribution to Literature

No other paper in the literature discusses the joint impact of diversification and integration on shocks to networks and their propagation.

Further differences from other closely papers which also look at shocks in networks and their propagation:

- Acemoglu et al. (2015)
  - Only extremes in shocks.
- Cabrales et al. (2017)
  - Consider only minimally and maximally connected networks
- Gai and Kapadia (2010)
  - Only degree distribution of a network is considered
  - Equilibrium dependencies are not calculated

Model

Model

- n firms, m assets
- Price of asset k is p_k
- D_{ik} denotes share of asset k owned by firm i, and D is the corresponding matrix.
- Similarly C_{ik} denotes share of firm k owned by firm i, and C is the corresponding matrix.
- C_{ii} is assumed to be zero, and $(1 - \sum_j C_{ji})$ represents the part of firm i held by non-firm investors. Denoted by $\hat{C}_{ii}$

Model

- Book value of the firm ($V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j \Rightarrow V = (I - C)^{-1} Dp$) double counts the assets
  - 1 dollar of asset of firm i is counted once for firm i and then partially for other firms which hold stake in firm i.
- So better to use 'market' value (denoted by lower case v) defined by the book value of the firm to non-firm investors. Thus, $v_i = \hat{C}_{ii} V_i \Rightarrow v = \hat{C}(I - C)^{-1} Dp = ADp$
- A = Dependency matrix, where A_{ij} describes the dependence of firm i's market value on firm j's assets.

Model

- Failure - If the market value of firm i goes below a threshold $\underline{v_i}$ then firm i fails and there is a discontinuous drop in its market value by $\beta_i(p)$
- Discontinuous drop is important for 2 reasons:
  - Amplifies shock - A drop in price of asset k affects the market value of firms, if a firm $i's$ value goes below threshold there is further shock of β_i.
  - Cascades - A drop in price of an asset may cause one firm i to fail $\Rightarrow$ further shock of $\beta_i \Rightarrow$ firm j fails$\rightarrow$ shock amplifies further by β_j... *A small shock can have big consequences!*
- Without amplifying shock - one firm's failure has no impact on the failure of other firms. Only initial asset price shock matters.

Model

- Value after including discontinuous shock on failure:
  - Book value: $V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j - \beta_i(p) I_{v_i < \underline{v_i}} \Rightarrow V = (I - C)^{-1}(Dp - \beta(v, p))$
  - Market value: $v = A(Dp - \beta(v, p))$
- After a shock there could be multiple equilibrium values of the v vector. The authors focus on the equilibrium in which the fewest firms fail $\Rightarrow$ Any conditions required for cascades are the necessary conditions for any cascade.

Definitions

- *Fair trades* - Given the current asset price vector p, a fair trade refers to a reallocation of assets and cross holding amongst the firms such that the market value of the firms remain unchanged. $(C, D) \Rightarrow (C', D')$ represents a fair trade at price vector p iff $v(C, D, p) = v(C', D', p)$
- *Integration* - A financial system under crossholdings C' is more integrated than under C iff $\sum_j C'_{ji} > \sum_j C_{ji}$ i.e. when the aggregate share of non-firm share holders reduce.
- *Diversification* - A financial system under cross-holdings C' is more diversified than under C when the cross-holdings of the initial cross holders reduces, and the number of cross holders increase. Formally:

    1. $C'_{ij} \leq C'_{ij}$ whenever $C_{ij} > 0$, with strict inequality somewhere
    2. $C'_{ij} > C'_{ij}$ for some i, j such that $C_{ij} = 0$

Results

Result on Integration

Proposition

Given a price vector p, suppose a fair trade changes the ownership structure from (C, D) to (C', D') such that the financial system becomes more integrated. Given any initial shock to p:

1. *The set of firms who fail first is the same under (C, D) and (C', D')*
2. *Given any j, if firm j fails in the cascade under (p,C,D) $\Rightarrow$ firm j fails in the cascade under (p,C',D')*

Thus, more integration can only worsen the effect of a cascade, once it starts.

Problem in making general claims on impact of diversification

- Given a level of diversification and integration, there are many ways in which we can increase diversification/integration.
- The impact of a shock may be different given how we choose to increase diversification/integration
- This makes it difficult to make general claims on what happens when diversification/integration goes up?
- To this end, the authors make some assumptions

Assumptions

- Let G denote a $n \times n$ matrix where $G_{ij} = 1$ if firm i has a claim on firm j, else it is zero. (Represents a directed graph)
- Suppose each firm has a fraction c held by other firms collectively
- Suppose that all firms who hold a stake in any given firm hold an equal stake
- Thus, we have that $C_{ij} = \frac{cG_{ij}}{\sum_k G_{kj}}$ if $\sum_k G_{kj} > 0$, else $C_{ij} = 0$
- Let $d_j = \sum_k G_{kj} =$ out-degree of j, $d_j^{in} = \sum_j G_{jk} =$ in-degree of j
- Increasing only c represents increasing integration without increasing diversification, and increasing d_j for any j represents increasing diversification without increasing integration.

Solving the problem with Random Graphs model

- Let G be any graph and let π_{kl} denote the fraction of nodes which have in-degree k and out-degree l. Let $\pi = (\pi_{kl})$ represent the degree distribution of the entire graph.
- Let $G(\pi, n)$ be the set of graphs with n nodes and a degree distribution of π.
- π is said to be feasible for n if $G(\pi, n) \neq \phi$
- A random graph with n nodes and a degree distribution of π is a draw from the set $G(\pi, n)$ when all graphs have equal probability of selection.

Solving the problem with Random Graphs model

- We solve the original problem of making general claims by restricting the space of financial networks we look at to random graphs with n nodes and degree distribution π where:
    1. $C_{ij} = \frac{cG_{ij}}{d_j}$ if $d_j > 0$, else $C_{ij} = 0$
    2. $D = I$ (each firm has complete ownership of one asset)
    3. $p = (1, \ldots, 1)$ (all assets have equal value)
    4. $\beta_i(p) = p_i$ (shocks completely devalue an asset)
    5. $\underline{v_i} = \underline{v} \; \forall \; i$
    6. Shocks are represented by one of the asset prices dropping to zero, where the asset is selected uniformly at random
- Now we can ask questions like - in case of a shock, what is the expected fraction of firms which fail?
    - Given a shock, let $f(\pi, n)$ denote the expected fraction of firms which fail when we apply the shock to a randomly selected financial network where the number of nodes is n, the degree distribution is π, and it satisfies the properties listed above.

Notation and definitions

- Given π, let $\overline{d} = max\{k : \pi_{kl} > 0 \text{ or } \pi_{lk} > 0 \text{ for some l}\}$
- Given π, let $\underline{d} = min\{k : \pi_{kl} > 0 \text{ or } \pi_{lk} > 0 \text{ for some l}\}$
- Let d denote the *average directed degree* which is the expected out-degree of the node at the end of a link chosen uniformly at random from $G(\pi, n)$
- $v_{min} = \frac{(1-c)}{1-\frac{c\underline{d}}{\overline{d}}}$, $v_{max} = \frac{(1-c)}{1-\frac{c\overline{d}}{max\{\underline{d},1\}}}$

Result on Diversification and Integration

Proposition

Given a degree distribution π and a sequence of natural numbers n_k such that π is feasible for each n_k:

1. $f(\pi, n) \to 0$ *if either of the following hold:*
   - $d < 1$ *(low diversification)*
   - $\underline{d} > \frac{c(1-c)}{v_{min} - \underline{v}}$ *(diversification is very high, or integration is either too high or too low)*
2. $f(\pi, n) \to > 0$ *if both the following hold:*
   - $d > 1$ *(diversification high enough)*
   - $\overline{d} < \frac{c(1-c)}{v_{max} - \underline{v}}$ *(diversification has an upper bound, and integration is intermediate)*

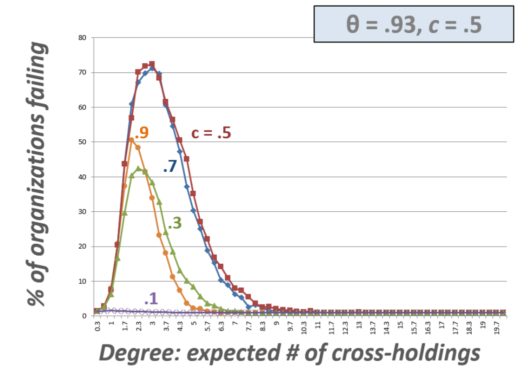
Thus, if either integration or diversification is extreme, the shock doesn't cause much damage. Contagion occurs only when both are intermediate.

Intuition

- Integration:
  - c too Low - Firms don't hold enough of each other for failure cascades to occur
  - c too high - Firms are reliant so little on their own assets that a first failure becomes improbable
- Diversification:
  - d too low - firms are not connected enough for one firm's proprietary asset failure to influence many others
  - d too high - Firms are connected to so many others that no one firm bears a high fraction of the failure cost of any one firm
- These results make use of 'large network' properties i.e. they work when n is large enough. Impact on small size networks are difficult to generalize because it is difficult to predict properties that all small networks will satisfy.

Simulations

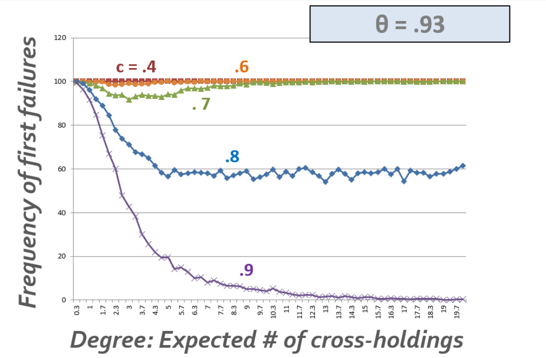
Figure 1: Impact of levels of integration and diversification on fraction of firms which fail after a proprietary shock



- x axis - d (diversification), n=100, $\underline{v} = 0.93v$, Averaged over 1000 simulations, c represents integration level.
- *Source* - Slides on Ben Golub's Harvard page

Simulations

Figure 2: Impact of levels of integration on fraction of simulations in which at least one firm fails after a proprietary shock



- x axis - d (diversification), n=100, $\underline{v} = 0.93v$, Averaged over 1000 simulations, c represents integration level.
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Proof of Concept - Data from European Debt Cross-Holdings

- 2011 data from the Bureau of International Settlements
- Cross holdings - Claims of foreign banks on the debt obligations of a country
- Proprietary assets - nation's own fiscal stream valued at some proportion of GDP $\Rightarrow p$ is also proportional to GDPs.
- Failure threshold is some fraction of country's 2008 value
- If a country fails, its 'market value' dips by half
- Failure cascade looks like - Greece$\rightarrow$Portugal$\rightarrow$Spain$\rightarrow$France and Germany$\rightarrow$ Italy

Conclusion

Conclusion

- The model helps us understand the impact of shocks on financial networks
    - Useful property of the model - it is not very difficult to take to data
- Diversification and Integration affect the systemic risk in different ways, and their impact is non-monotonic
- Intermediate levels of integration and diversification can be the greatest threat to systemic risk
    - Developing countries most at risk?
- The paper can guide us on bailouts, and on prudential policy

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