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Multilevel Modelling of Country Effects: A Cautionary Tale

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Abstract

Country effects on outcomes for individuals are often analysed using multilevel (hierarchical) models applied to harmonized multi-country data sets such as ESS, EU-SILC, EVS, ISSP, and SHARE. We point out problems with the assessment of country effects that appear not to be widely appreciated, and develop our arguments using Monte Carlo simulation analysis of multilevel linear and logit models. With large sample sizes of individuals within each country but only a small number of countries, analysts can reliably estimate individual-level effects but estimates of parameters summarizing country effects are likely to be unreliable. Multilevel modelling methods are no panacea.

Introduction

Researchers often wish to estimate 'country effects' on socio-economic outcomes of individuals. The most popular quantitative approach is regression analysis of harmonized data from multiple countries in which individual-level outcomes are modelled as a function of both individual-level and country-level characteristics (observed and unobserved). In this article, we argue that the small number of countries in most multi-country data sets limits the ability of multilevel regression models to provide robust conclusions about 'country effects'.

Some of the multi-country data sets that are commonly used in contemporary social science research are listed in Table 1. In each of them, there is a natural hierarchy with observations at the individual level nested within a higher level (countries). The data sets typically contain thousands of individuals per country but the number of countries is small, at most around 30. The number of countries used in analysis is often fewer, nearer 20 and sometimes less, because of missing data or analytical focus.

Multi-country data sets are attractive because they offer a means of quantifying the extent to which differences in outcomes reflect differences in the effects of country-specific features of demographic structure, labour markets, and other socio-economic institutions such as tax-benefit systems, which are distinct from the differences in outcomes associated with variations in the characteristics of the individuals themselves. That is, multi-country data sets potentially provide information about 'country effects' as well as 'individual effects', and also about interactions between them ('cross-level effects').

The popularity of regression analysis of multilevel country data is illustrated by the *European Sociological Review*. Of the 340 articles published between 2005 and 2012, approximately 75 exploit multilevel data sets with individual respondents within countries. Multilevel models, also known as hierarchical models or mixed

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Table 1. Multilevel country data sets: selected examples

Data sources (in alphabetical order)	Number of countries per data round
Eurobarometer	27
European Community Household Panel (ECHP)	15
European Quality of Life Survey (EQLS)	31
European Social Survey (ESS)	30
European Union Statistics on Income and Living Conditions (EU-SILC)	27
European Values Study (EVS)	45
International Social Survey Program (ISSP)	36
Luxembourg Income Study (LIS)	32
Survey of Health, Ageing and Retirement in Europe (SHARE)	14

Note: the number of countries per data round is indicative only, as the number of countries can vary from round to round. The number of countries in data sets used by researchers is usually smaller than the maximum available, and often around 25 or fewer.

models, are used in 43 of the 75 articles (57 per cent; or 13 per cent of all 340 articles). There are articles based on regression analysis of multilevel country data in other social science journals as well (e.g. 14 of the 111 articles published in the *Journal of European Social Policy* between 2005 and 2009). The topics addressed vary widely, ranging from labour force participation and wages to political and civic participation rates, and social and political attitudes.

We believe that many researchers do not appreciate the problems that can arise when the number of countries in a multi-country data set is small. This article analyses the number-of-countries issue in detail, considering when multilevel model estimates of individual- and country-level effects and their standard errors (SEs) can be trusted, with the exposition intended to be accessible to applied researchers without specialist statistical knowledge.

The intuition underlying our arguments is relatively straightforward, however. The derivation of model parameter estimates with desirable properties is contingent on sample sizes being ‘large’. In particular, a large number of countries is needed to estimate country effects reliably.

In section 2, we review regression approaches to modelling individual and country effects from multilevel country data, including multilevel modelling. The literature on the performance of multilevel estimators and sample size is reviewed in section 3. Because most of this literature does not cover the data structure of interest here, we present our own Monte Carlo simulation analysis of how multilevel estimator performance varies as the number of countries varies, for both linear and binary logit models, drawing out some rules of thumb (section 4). In section 5, we summarize our findings and make recommendations. At a number of points in the article, we direct readers to the article’s Supplementary Material for further discussion and additional estimates.

Multilevel Regression Analysis of Multilevel Country Data

To facilitate discussion, we refer to a *basic linear model* for a metric outcome:

$$y_{ic} = \boldsymbol{X}_{ic}\boldsymbol{\beta} + \boldsymbol{Z}_{c}\gamma + u_c + \varepsilon_{ic}, \quad \text{with } i = 1, \ldots, N_c; \quad c = 1, \ldots, C. \tag{1}$$

Outcome y_{ic} for each person i in country c is assumed to depend on observed predictors and unobserved factors. $\boldsymbol{X}_{ic}$ contains variables that summarize individual-level characteristics such as age, education, or marital status; $\boldsymbol{Z}_c$ contains variables summarizing country-level features such as socio-economic institutions or labour markets. There are also unobserved individual effects (ε_{ic}) and country effects (u_c), each assumed to be normally distributed and uncorrelated with $\boldsymbol{X}_{ic}$ and $\boldsymbol{Z}_c$. We suppose the researcher has available a data set with a large number of individuals for each country (N_c is typically in the thousands) sampled from each of a small number of countries (C is typically less than 30). The parameters associated with the observed predictors $\boldsymbol{\beta}$ and γ are sometimes called ‘fixed’ regression parameters to distinguish them from the parameters characterizing the joint distribution of the random terms ε_{ic} and u_c (their variances σ^2_ε and σ^2_u). These variances are often referred to as ‘random effect’ parameters or ‘variance components’.

Four main modelling approaches have been used in this context, and in principle each can provide estimates of the individual-level fixed effects ($\boldsymbol{\beta}$) with desirable properties: see Table 2 for a summary. Our discussion is limited to the classical ‘frequentist’ statistical

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Table 2. Four regression modelling approaches commonly applied to multilevel country data sets

	Approach	Remarks about specification
1.	Common model for all countries, pooled data, country-specific clustered SEs	Country effects controlled for, not modelled
2.	Separate model fitted to the data for each country	Country effects not separately identified (absorbed into the intercept of each country's model). Every model parameter is country-specific
3.	Common model applied to pooled data (as in approach 1), except that model has country fixed effects	All country-level factors are absorbed into the country fixed effect; estimates refer to specific sample of countries
4.	Common model applied to pooled data (as in approach 1), except that model has country random effects (multilevel model)	Country effects can be specified in terms of a country error variance and fixed effects of country-level predictors; 'exchangeable' estimates

Notes: The remarks refer to how the models specify country effects (cf. equations (1) and (2) in the text). See also the discussion in the text about a fifth approach (based on a two-step estimation method) and the distinction between population-averaged and cluster-specific effects in the case of multilevel logit models. The performance of estimators of country effects, and how this performance varies with the number of countries in the multi-country data set, is discussed in sections 3 and 4.

framework, which is the one most commonly used by applied social science researchers. (We remark on the Bayesian approach later.) The focus here is on the fourth, random effects (RE), approach—multilevel modelling.

Country effects induce correlations across observations that need to be addressed by any regression approach or else SE estimates are downwardly biased (Moulton, 1986), and all four approaches cited in Table 2 take account of this issue though in different ways. The first approach simply controls for the problem using country-cluster-robust SEs but country effects are not explicitly modelled. (The procedure also relies on the number of countries not being small: see e.g. Angrist and Pischke, 2009.) The second and third approaches represent country-specific differences through country-specific intercept terms. In the second case, any country effect (u_c) is absorbed into, and cannot be identified separately from, the intercept term in each country's regression model (an element of $\boldsymbol{\beta}$). All model parameters are country-specific. In the third case, the fixed effects approach, the data from the country surveys are pooled but the model specification includes distinct country intercepts (modelled as fixed effects but not to be confused with the fixed effects $\boldsymbol{\beta}$ and γ also included in the other approaches). Again, each country intercept represents the effect of unobserved factors that are shared within each country.

The multilevel modelling approach also pools the data but, rather than treating country effects as distinct values to be estimated, they are modelled as random draws from a distribution with mean zero and variance which is estimated. In this case, equation (1) characterizes a 'random effects' or 'random intercepts' model. A key parameter is the intra-class correlation $\rho = \sigma_u^2/(\sigma_\varepsilon^2 + \sigma_u^2)$, where σ_ε^2 and σ_u^2 are the variances of the individual and country random effects, respectively. (Individual random effect (ε_{ic}) and country random effect (u_c) are assumed to be uncorrelated with $\boldsymbol{X}_{ic}$ and $\boldsymbol{Z}_c$ and with each other.) The intra-class correlation summarizes the extent to which unobserved factors within each country are shared by individuals ($\rho \to 0$ as $\sigma_u^2 \to 0$). Assuming that the correlation structure of the random effects has a particular form leads to more efficient estimates of the individual-level effects represented by $\boldsymbol{\beta}$, i.e. estimates with SEs smaller than the cluster-robust ones (approach 1).

One of the substantial attractions of the RE approach is that a number of different country-level fixed effects can be estimated by using appropriately defined country-level predictors (elements of $\boldsymbol{Z}_c$), though the number of parameters that can be reliably fitted is constrained by the number of countries. Any remaining unobserved country effects (u_c) are treated as being generated by some common mechanism and so are 'exchangeable' between countries (Snijders and Bosker, 2012: pp. 46–47). The regression intercept is a population average (for example, a common European intercept if the data are from EU countries), and deviations from this average are assumed to be uncorrelated with country-level variables included in the model. With these assumptions, the RE results can be generalized from the sample at hand (for instance, to other European countries with different policies and institutions, to pursue the EU example). Clearly, exchangeability is a strong assumption but also potentially unrealistic (depending on research context).

Because the four approaches differ in fundamental ways, one cannot straightforwardly recommend one approach over another. In sections 1 and 2 of this article's Supplementary Material, we explain why this is,

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referring to modelling goals and statistical performance of the various estimators, and also discuss the approaches at greater length. We conclude that analysts primarily interested in the individual-level fixed effects associated with observed predictors ($\boldsymbol{\beta}$) may favour one of the first three approaches. However, multilevel modelling is the natural choice if the interest is in the effects (γ) of country-level predictors or the variance component structure. Non-economist social scientists have tended to favour the multilevel modelling approach, assessing country 'effects' in terms of either country fixed effects or in terms of the proportion of the outcome variance explained by the country variance component(s). See for example Snijder and Bosker (2012: chapter 4). Multilevel modelling approaches are the focus in the rest of the article.

It is also instructive to consider a fifth approach in which estimation is undertaken in two steps (see section 3 of the Supplementary Material for further discussion). In the first step of the two-step approach, one fits equation (1) using ordinary least squares and country-specific fixed effects (as in approach 3). Country-level effects (γ) are ascertained from the second step in which the fitted country intercepts from step 1 are regressed on the country-level predictors ($\mathbf{Z}_c$). This two-step procedure has several advantages. First, it highlights the sources of variation in the data and shows why a small number of countries can affect the reliability of estimates (the sample size in the second step is the number of countries). Second, the estimates are unbiased (with correct SEs) and so can be used to benchmark the other methods. Third, the two-step method leads naturally to a graphical summary of country-level variations in outcomes in which one plots the country intercepts fitted at step 1 against elements of $\mathbf{Z}_c$ (Bowers and Drake, 2005; Kedar and Shively, 2005). We return to this third property in section 5.

Two-step estimation of hierarchical structures dates back to at least Hanushek (1974) and Saxonhouse (1976) among economists, but the method has been periodically rediscovered. Borjas and Sueyoshi (1994) presented a two-step estimator for the probit model, and other proponents include Card (1995), Jusko and Shively (2005), and other papers in a special issue of *Political Analysis* (Kedar and Shively 2005). Donald and Lang (2007) discuss the statistical properties of the two-step estimator (compared with Generalized Least Squares (GLS)) in detail. For textbook discussion, see Wooldridge (2010: chapter 20). The two-step estimator is effectively what is done in meta-analysis in which estimates and SEs from a number of studies are combined to derive an overall effect estimate. For more on these parallels, see Hox (2010: chapter 11).

The various approaches just set out, and our discussion of them, also apply to non-linear models. The *basic logit model* for a binary outcome that is analogous to equation (1) for metric outcomes is of the following form:

$$\log[p_{ic}/(1–p_{ic})] = \mathbf{X}_{ic}\boldsymbol{\beta} + \mathbf{Z}_c\boldsymbol{\gamma} + u_c + \varepsilon_{ic}, \quad \text{with } i = 1, \ldots, N_c; c = 1, \ldots, C, \tag{2}$$

where p_{ic} is the probability of the binary outcome for person i in country c and variance σ_u^2 is normalized to equal $\pi^2/3$. However, there is an important conceptual difference between the logistic and linear multilevel models that is distinct from issues of estimator performance. This concerns the nature of the effects that the researcher is interested in. Given estimates of multilevel logistic model parameters, researchers may be interested in population-averaged ('marginal') effects or cluster-specific ('conditional') effects. In the former case, the interest is in the impact on the outcome probability of a change in an individual- or country-level characteristic which is the average across the distribution of unobserved characteristics (hence the population average label). In latter case, the interest is in the impact on outcome probabilities of a change in an individual- or country-level characteristic for an individual with a specific set of characteristics, observed and unobserved. (In the analogous linear model, the two types of effect coincide.) For more about the distinction between population-averaged and cluster-specific effects, see for example Molenberghs and Verbeke (2005) or Neuhaus, Kalbfleisch and Hauck (1991). In our examination of multilevel logit models, we consider estimation of cluster-specific effects (represented by $\boldsymbol{\beta}$ and γ in equation (2)) as these have been the focus of interest of virtually all the applied social science research in the literature that we cited in the Introduction.

Our Monte Carlo simulation analysis assesses the performance of estimators not only for basic linear and logit models, but also 'extended' linear and logit models in which the specifications set out in equations (1) and (2) are supplemented to allow for individual variation in two individual-level fixed effects ('random slopes') and also the effect of an individual-level predictor varies with the size of a country-level fixed effect (a cross-level interaction). We focus on the results for the basic models here (section 4); detailed results for the extended models are presented in section 7 of the Supplementary Material.

How Many Countries Are Required for Reliable Estimates? Literature Review

In this section, we review theoretical results and Monte Carlo simulation evidence to summarize what is known

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about the statistical properties of standard multilevel model estimators. Unless stated otherwise, the estimators we refer to are Restricted (or Residual) Maximum Likelihood (REML) for mixed linear models and Maximum Likelihood (ML) for mixed logit models. These are the most commonly used estimators nowadays and also have the best properties (among classical statistical estimators): see the review by Hox (2010: chapters 3 and 6), for example. We refer to some other estimators later.

In general, the statistical properties of these estimators are well-defined only when both the number of groups (countries here) and group sizes (numbers of individuals) are large, in which case estimates of parameters and their sampling variability are consistent (they converge to their true values with sufficiently large samples) and are asymptotically normally distributed. That is, with samples that are large in both the individuals and countries dimensions, estimators are *accurate* and there can be *reliable inference* about parameter values (employing the estimates of the parameters and of their SEs). This is the case for both linear and logit mixed models.

What if the number of groups is small? For the *linear mixed model*, estimates of the fixed effects ($\boldsymbol{\beta}$ and γ in equation (1)) are unbiased (Kackar and Harville, 1981, 1984). However, if the number of groups is small, and even if the group sizes are large, estimates of the variance components and of their SEs are imprecise and likely to be biased downwards (Raudenbush and Bryk, 2002: p. 283; Hox, 2010: p. 233). Estimates of the SEs of fixed parameters are also affected by the uncertainty in the variance estimates: they are biased downwards and the distribution of test statistics is unknown (Raudenbush and Bryk, 2002: p. 282). There are some methods available to derive better estimates of the SEs of the fixed effects and hence undertake more reliable inference about them (see below). However, these methods provide little comfort to most applied social scientists because these researchers are also particularly interested in the magnitudes of, and inference about, country-level variance components.

The conclusions about estimator performance cited in the previous paragraph also apply to *logit mixed models*, but with additional stings in the tail. That is, there are no theoretical results available concerning the bias of fixed effect estimators; fixed effect SEs are biased downwards (but there are no convenient bias-correction methods); and variance component estimators and their SEs are biased downwards, often substantially.

The upshot is that, if the number of countries is small, estimates of 'country effects' produced may be unreliable. Although estimates of the effect of a country-level predictor may be unbiased, assessments of the statistical significance of the effect and inference more generally will be unreliable (SEs are too small and confidence intervals (CIs) too narrow). Country variance components will be under-estimated, providing incorrect estimates of intraclass correlations (ICCs), and inference about them is also unreliable.

Specific and consistent guidance to modellers about the number of groups required to avoid the problems cited is difficult to find. The Centre for Multilevel Modelling's FAQ on sample sizes for multilevel modelling states that '[r]ules of thumb such as only doing multilevel modelling with 15 or 30 or 50 level 2 units can be found and are often personal opinions based on personal experience and varying reasons' (Centre for Multilevel Modelling, 2011). Most multilevel modelling textbooks mention the issues and also sometimes cite rules of thumb, recommending anywhere between 10 and 50 groups as a minimum. They stress that the minimum number depends on application-specific factors like the number of group-level predictors (Raudenbush and Bryk, 2002: p. 267) and whether interest is focussed on the coefficients on the fixed regression predictors or the parameters describing random effects such as variance components (Hox, 2010: p. 235). Moreover, advice about sample size is often bound up with considerations of the cost of primary data collection and survey design *ex ante*: see Snijders and Bosker (2012: chapter 11). However, these cost issues are not relevant for secondary analysis of the many multilevel country data sets that are already in existence.

Most discussion of the small group size issue is based on Monte Carlo analysis of simulated data because theory does not provide specific guidance. See for instance the review by Hox (2010: chapter 12), who also summarizes a number of earlier unpublished studies. A number of recent Monte Carlo studies of two-level linear and logit models are listed in Table 3, together with a summary of their principal design features.

Most Monte Carlo evidence to date is for linear models. Recent studies include Bell *et al.* (2014), Maas and Hox (2004, 2005), and Stegmueller (2013). The research indicates that estimates of the parameters associated with fixed predictors ($\boldsymbol{\beta}$ and γ) are unbiased, which is to be expected given Kackar and Harville's (1981, 1984) theoretical results. However, estimates of group-level variances under-estimate their true values, and the magnitude of this is larger, the smaller is the number of groups. As stated above, the SEs of both the coefficients on fixed predictors and especially the variance parameters are biased downwards when the number of groups is small.

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Based on their simulation evidence, Maas and Hox's (2004) rules of thumb for multilevel linear models are: 10 groups are sufficient for unbiased estimates of the $\boldsymbol{\beta}$ and γ, at least 30 groups are needed for good variance estimates, and at least 50 groups are required for accurate SE estimates especially for those associated with (co)variance component parameters.

There is less evidence for multilevel logit models than for linear models but the few existing studies suggest broadly similar conclusions: see, for example, Austin (2010), Moineddin, Matheson and Glazier (2007), Paccagnella (2011), and Stegmueller (2013). With a small number of groups, estimates of the fixed effect parameters in binary logit or probit models are generally unbiased—though not always for level-2 fixed effects (Moineddin, Matheson and Glazier, 2007; Paccagnella 2011). Estimates of variance components are biased downwards with the magnitude of the problem depending on the type of estimator used to maximize the likelihood (adaptive quadrature appears to provide the least bad estimates), and the SEs associated with both fixed and variance parameters are too small. Stegmueller (2013) urges caution in using classical ML methods with <10 or 15 groups, especially when the model includes cross-level interactions and random coefficients, whereas Moineddin, Matheson and Glazier (2007) recommend using at least 50 groups.

One caveat regarding all of the Monte Carlo studies is that their conclusions are potentially sensitive to model specification, including choices of parameter values and numbers and types of predictors. Also, studies have typically been based on models with relatively simple specifications: see Table 3. The result is that the sample sizes and data distributions used in these studies are rarely similar to what is used in the hierarchical cross-national data context. For example, Maas and Hox (2004) specify a linear model for a continuous outcome with a random intercept, a single individual-level regressor (with random slope), a single group-level regressor, and an interaction of the two (both regressors are normally distributed). Austin (2010) specifies a logit model but with an even simpler specification, consisting of a random intercept and two (joint normally distributed) individual-level regressors. Few studies investigate estimator performance using data with combinations of group sizes and numbers of groups that correspond to those typical in the hierarchical cross-national data context. Stegmueller (2013) is the exception, but his model specifications and data generation process are relatively simple, and his analysis focuses almost entirely on fixed effects parameters with no discussion of variance parameters (which are also of interest to applied social science researchers). In addition, Stegmueller uses ML estimators for all models including his linear models, for which the recommended estimator is REML (see above).

In our own Monte Carlo analysis in the next section, we use study designs with sample sizes, simulated data distributions, and model specifications that are more like those in real-life multilevel country data. This is particularly important because the number of countries in the multilevel country data sets typically available (Table 1) falls within the range identified by these Monte Carlo studies as providing unreliable estimates.

Unfortunately there are no easy ways to increase the reliability of all the estimates that researchers are interested in using the multilevel model estimation commands in commonly used software. Not all software routinely makes small-sample adjustments to estimates of CIs or test statistics, for instance. One exception is HLM (Raudenbush *et al.*, 2004, cited in Hox, 2010), which uses the t distribution with degrees of freedom based on the number of groups (similar to the second-step estimation outlined above) and which should give better inference for the fixed effect parameters. Some simple adjustment methods for linear models to the same end are discussed by Cameron and Miller (2015). It has also been argued that if there is a small number of groups, specialist bootstrapping methods may reduce bias and improve inference for variance components as well as fixed effects (Carpenter, Goldstein and Rasbash, 2003). These methods are not currently in widespread use among applied social science researchers.

More well known are the small-sample corrections to SEs with associated sample size adjustments that have been developed for the REML estimator (Kenward and Roger, 1997, 2009), and which are available in SAS's PROC MIXED and R's ASREML package. On the one hand, Monte Carlo analysis has shown that these methods work well. On the other hand, the methods provide better inference only for fixed effects in linear mixed models and, as we have argued, applied social science researchers are often interested in estimates of variance component parameters and also in non-linear models.

It is also the case that when the number of groups is small, one has to assume that group-level effects (u_c) are normally distributed in order to apply standard inference methods to the group-level fixed parameters (γ) and the variance parameters (σ_u^2). If the normality assumption cannot be justified, special bootstrapping methods may provide acceptable inference (Carpenter, Goldstein and Rasbash 2003; Cameron, Gelbach and Miller, 2008; Cameron and Miller 2015). Alternatively, and especially if the country effects are considered to be fixed rather than random (Table 2), the option remains

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Table 3. Monte Carlo simulation studies of two-level model estimator performance: selected recent examples

Study	Covariate specification, by level	Number of level-2 units (C)	Number of level-1 units within each level-2 unit (N_C)	ICC	Estimation method
Linear models					
Maas and Hox (2004, 2005)	1 level-1, 1 level-2, with cross-level interaction	30, 50, 100	5, 30, 50	0.1, 0.2, 0.3	REML
Stegmueller (2013)	1 level-1, 1 level-2, with and without cross-level interaction	5(5)30	500	0.05, 0.10, 0.15	ML with adaptive quadrature (and Bayesian methods)
Bell *et al*. (2014)	2 or 3 level-1, 2 or 3 level-2, with and without cross-level interaction(s)	10, 20, 30	Randomly chosen from 5–10, 10–20, 20–40	Varying	REML (with Kenward-Roger adjustments for fixed effects)
Logit models					
Moineddin *et al*. (2007)	1 level-1, 1 level-2, with cross-level interaction	30, 50, 100	5, 30, 50	0.1, 0.2, 0.3	ML with adaptive quadrature
Austin (2010)	2 level-1	5(1)20	5(5)50	Not applicable	ML with adaptive quadrature (and Bayesian methods)
Paccagnella (2011)	2 level-1, 2 level-2	10(20)70, 100, 150, 350	65[a]	0.071, 0.304, 0.655	ML with adaptive quadrature
Stegmueller (2013)[b]	1 level-1, 1 level-2, with and without cross-level interaction	5(5)30	500	0.05, 0.10, 0.15	ML with adaptive quadrature (and Bayesian methods)

[a] Paccagnella (2011) uses unbalanced group sizes, as described further in his article; all other studies use equal-sized groups.

[b] Stegmueller (2013) fits probit models for a binary outcome rather than logit models. ML: Maximum likelihood. REML: Restricted (or Residual) Maximum Likelihood.

Notes: In the hierarchical multi-country data set context, level-1 corresponds to individuals within countries, and level-2 corresponds to countries. All studies use continuous covariates generated by simulation from standard normal distributions. Moineddin *et al*. (2007) also run simulations with uniform and *t* distributed variables, and Bell *et al*. (2014) and Paccagnella (2011) dichotomize some of their covariates. The notation ‘5(5)50’ is an abbreviation for ‘5, 10, 15, 20, 25, 30, 35, 40, 45, 50’, and similarly elsewhere in the table.

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to use graphical methods to describe estimates of cross-country differences from step 1 of a two-stage approach (Bowers and Drake 2005).

How Many Countries are Needed for Reliable Estimates? Monte Carlo Simulation Analysis

We use Monte Carlo simulations to assess how large the number of countries needs to be to derive accurate estimates of model parameters and their SEs from the standard multilevel model estimators. For several reasons, previous analysis does not necessarily translate to typical multi-country data set applications. First, previous studies have mainly been concerned with education and health research contexts that involve moderate numbers of both groups and numbers of observations within groups. Thus they do not usually consider the sample sizes of most relevance to cross-country researchers, i.e. a number of groups often well below 30 and group sizes of many hundreds (at least). Second, previous studies use simple, rather unrealistic, model specifications, typically including only two or three 'well-behaved' (normally distributed) regressors. In contrast, we consider both linear and non-linear models using data structures that are similar to those found in multi-country data sets, we employ a greater range in the number of countries, and we also give greater attention to various aspects of accuracy than previous research—this turns out to be relevant when assessing the properties of estimates of some individual-level and country-level effects (see below). We include binary, categorical, and continuous variables in our simulated data sets, and do not impose normality.

Our simulation results are based on two-level linear and logit models. In this article, we focus on a 'basic' specification with random intercepts corresponding to equation (1). The regressors include a constant (intercept), individual-level fixed effects, a country-level fixed effect, and a random country intercept. (The model also includes an individual-specific error term.) In common with most social science applications, we assume that the random effects are uncorrelated with each other. To make the models more concrete, we refer to the outcome variables for the linear and non-linear models as 'hours' (of work) and (labour force) 'participation', respectively. We shall also refer briefly to our Monte Carlo analysis of 'extended' linear and logit models that include the same regressors but add two cross-level interactions, and two random slopes. Further details of the results for these models and Stata code for running all of the simulations are provided in the Supplementary Material (sections 7 and 9).

Compared with previous Monte Carlo simulations of multilevel models, our specifications include a greater number and different types of regressors. For example, the model used in the oft-cited Maas and Hox (2005) study included only one individual-level regressor and one country-level regressor (both of which were continuous, normally distributed, variables): see Table 3. By including a more realistic set of regressors, we can be more confident that the performance of the estimators will hold up in practical applications and does not depend on the simplicity of the experimental specification. As Burton *et al*. (2006) have stated,

> *The simulated data sets should have some resemblance to reality for the results to be generalizable to real situations and to have any credibility. A good approach is to use a real data set as the motivating example and hence the data can be simulated to closely represent the structure of this real data set. (Burton* et al. *2006: p. 4283)*

We chose parameters to correspond with those estimated by first fitting models for hours of work and labour force participation probabilities to EU-SILC data for 2007 on women aged 18–64 years from 26 countries: see Table 4 for the parameter values for our basic models. For concreteness, we refer to the individual-level fixed effects as *age* (continuous), *age-squared*, *cohab* (whether married or cohabiting; binary), *nownch* (number of own children; integer), *isced* (educational level; four categories with the lowest excluded from the regressions). The country-level fixed effect is *chexp* (country spending on childcare and pre-primary spending as a percentage of Gross Domestic Product, continuous). The implied ICC values are 0.120 for the basic linear model and 0.012 for the basic logit model. That is they are relatively small—as commonly found with multi-country data. We did not vary them across simulations as previous research suggests that the choice of ICC has little effect on results.

We specified the joint distribution of the regressors by exploiting the fact that each combination of regressor values defines a cell with an associated probability of occurrence. We derived the cell probabilities from the empirical frequency distributions in the 2007 EU-SILC estimation samples cited earlier (separately for the hours and participation models), and then generated data sets reflecting these distributions for each value of C and for each model using a random number generator. (See section 4 of the Supplementary Material for details.) In common with other simulation studies of multilevel

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Table 4. Model specifications for Monte Carlo simulation analysis (basic linear and logit models)

Regressors		Linear model ('hours')		Logit model ('participation')	
		Parameter value	Mean of regressor	Parameter value	Mean of regressor
Fixed effects					
Intercept	*constant*	22	1	−9.1	1
Age	age_{ic}	0.8	41.6	0.5	41.0
Age-squared	$(age_{ic})^2$	−0.01	1832.5	−0.006	1862.4
In cohabiting partnership	$cohab_{ic}$	−1	0.725	0.02	0.658
Number of own children present	$nownch_{ic}$	−1.2	1.110	−0.27	0.911
ISCED category 3	$isced3_{ic}$	0.7	0.446	0.7	0.449
ISCED category 4	$isced4_{ic}$	1.4	0.058	0.9	0.052
ISCED categories 5, 6	$isced56_{ic}$	1.6	0.328	1.4	0.243
Expenditure on children	$chexp_c$	−0.23	0.535	0.98	0.586
Random effect variances					
Individual	σ_e (*sig_e*)	9.5		$\pi/\sqrt{3}$	
Country	σ_u (*sig_u*)	3.5		0.275	
	ICC	0.120		0.022	

Notes: See main text for explanation of the models and regressors. For detailed discussion of how the regressors were simulated, see section 4 of the Supplementary Material (and the Stata code in section 9). The RE are: an individual-specific error $e_{ic} \sim \text{N}(0, \sigma_e^2)$; a random country intercept $u_c \sim \text{N}(0, \sigma_u^2)$. The ICC are implied by the error variance values (see text). The country-level regressor is $chexp_c$. The omitted ISCED (International Standard Classification of Education) category is $isced12_{ic}$. The mean value of the outcome is 35.7 in the linear model and 0.78 in the logit model. All means refer to the data set associated with the case in which $C = 25$. Specifications for the 'extended' linear and logit models are shown in the Supplementary Material.

models, the joint distribution of the regressors is the same across replications for each value of C. The mean values of the regressors and outcome variables from the data set for the $C = 25$ case are shown in Table 4. (Mean values are similar across the data sets corresponding to different C, as expected, given the nature of the data generation process.)

For each model, our simulations hold the number of individuals per country, N_C, fixed at 1,000. We vary the number of countries, C, from 5 to 50 in intervals of 5, and also consider $C = 100$ to have a reference point for a case in which researchers would agree that C is large.

Estimation and simulation were undertaken using Stata (StataCorp, 2011). The linear ('hours') models were fitted by ML using the xtmixed command's REML estimator. The logit ('participation') models were fitted by ML using the xtmelogit command's adaptive Gaussian quadrature procedure with seven integration points (the default). Doubling the number of integration points to 14 led to virtually identical estimates.

The number of replications for each model, R, was chosen to be as large as possible to reduce the impact of simulation variability on assessment of accuracy (Cameron and Trivedi, 2010: section 4.6), while also taking into account estimation time (which is much longer for non-linear models than linear models). We were able to use values of R that are larger than those commonly used. For the basic linear model, $R = 10{,}000$ for the basic linear model and $R = 5{,}000$ for the basic logit model. For the extended models, the corresponding values of R are 5,000 and 1,000.

Our simulations were designed to examine not only bias but also accuracy and coverage of the estimates of model parameters, and hence also the reliability of inference. We report four types of summary measure, defined as follows.

Relative parameter bias This is the percentage difference between each estimated parameter and the corresponding true parameter at each replication, averaged over R replications. Ideally, relative bias equals 0 per cent for each parameter.

The 95 per cent CIs for relative bias statistics (and Root Mean Squared Error) The CIs for each relative parameter bias statistics are calculated using the 'empirical' SE, which is the standard deviation of the estimated statistic calculated from the R replications (Burton *et al.*, 2006: p. 4286). The wider is the CI, the greater is the variability of the estimate. As pointed out by Burton *et al.* (2006),

> *When judging the performance of different methods, there is a trade-off between the amount of bias and the variability. Some argue that having less bias is more crucial than producing a valid estimate of sampling*

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variance ... However, methods that result in an unbiased estimate with large variability or conversely a biased estimate with little variability may be considered of little practical use. (Burton et al., 2006: p. 4286.)

We demonstrate below that the combination of lack of bias but large variability is a feature of country-level fixed effects estimates from multi-country data sets. Researchers sometimes use composite measures of estimator accuracy that combine summaries of bias and variability, the most common of which is the Root Mean Squared Error (RMSE) statistic associated with each parameter (the square root of the sum of absolute bias squared and the empirical SE squared). We have also calculated RMSE statistics for our basic model simulations, and they yield conclusions about accuracy consistent with the discussion below (see section 6 of the Supplementary Material).

Relative SE bias We compare the empirical SE described above with the 'analytical' SE reported by the software and averaged over *R* replications (Greene, 2004). Relative SE bias is the percentage difference between the analytical and empirical SEs, assuming the empirical SE is an accurate estimate of the true SE. Ideally, the relative bias equals 0 per cent for each SE.

Non-coverage rate To assess inference performance overall, we calculate a 95 per cent CI for each estimated parameter assuming normality (Maas and Hox, 2005: p. 89). A non-coverage indicator variable was set equal to zero if this CI included the true parameter and one if it did not. The average over *R* replications of this variable is the non-coverage rate. Ideally, the non-coverage rate for a 95 per cent CI is 0.05. Rates greater than 0.05 indicate that the software-estimated CI is too narrow and significance tests on parameters will be anticonservative.

Most simulation studies of multilevel model performance report parameter bias and non-coverage rates only, and often interpret non-coverage rates as indicating the accuracy of the SEs. However, non-coverage depends on a combination of parameter bias, the distribution of the parameter estimates (usually assumed normal), and the accuracy of the SEs. For example, non-coverage will tend to exceed 0.05 if the parameter estimate is biased even if SEs are accurate, or if bias is not an issue but estimate variability is. By reporting estimates of SE bias in addition to non-coverage rates we provide a fuller picture of the potential sources of unreliability.

The simulation results are summarized for the basic linear models in Figures 1–3 and the basic logit models in Figures 4–6. In each figure, a measure of estimator performance is plotted against the number of 'countries' (C). For brevity, the results for some of the individual-level fixed effects are excluded.

Simulation Results: Linear Models

For the basic linear model, the individual-level variance and all the individual-level fixed effect parameters are unbiased regardless of C. In Figure 1, relative parameter bias for *sig_e*, *cohab*, *nownch*, and *age* is close to zero, and there is also almost no variability (the 95 per cent CIs are very narrow). The bias results are fully consistent with the theoretical results of Kackar and Harville (1981, 1984) cited earlier.

The results for the country-level regressor (*chexp*) stand out, however. The point estimates of relative bias bounce around zero, and there is substantial variability in them even for large values of C. At first glance, the results are inconsistent with the Kackar-Harville results about bias since the 95 per cent CI does not include zero for all values of C (the exceptional case is $C = 20$). This anomaly can be attributed to chance. When we reran all the simulations using a different initial random seed value, we found that 95 per cent CIs for relative bias spanned zero for all C values. We continued to observe substantial variability however. (See section 5 of the Supplementary Material.)

The important lesson regarding the country-level coefficient is that inaccuracy in the form of variability is the issue rather than bias. Observe that, even for $C = 50$, the CI ranges from -15 to $+14$ per cent and is not much smaller for $C = 100$. Thus, there is substantial uncertainty associated with the estimation of the fixed country effect, a problem that stems from the relatively small number of countries underlying the estimates. Relative parameter bias for the country-level coefficient is greater than reported by Stegmueller (2013: Figure 2) for most values of C, presumably because we use a more complicated (and more realistic) data generation process.

The other parameter of particular interest is the country-level variance (*sig_u*). Here the accuracy issues appear to relate more to bias than to variability. The variance is under-estimated (as expected) but the bias falls rapidly with the number of countries, from 8 per cent for $C = 5$ to around 1 per cent or less for $C \geq 20$. This is consistent with Maas and Hox (2004: p. 135) who report a bias of 25 per cent with 10 groups but negligible bias for 30 or more groups (though using a design with much smaller group sizes).

The relative bias of the SEs for the basic linear model is shown in Figure 2. For *chexp*, the SE is underestimated by 8 per cent for $C = 5$ but the bias declines to

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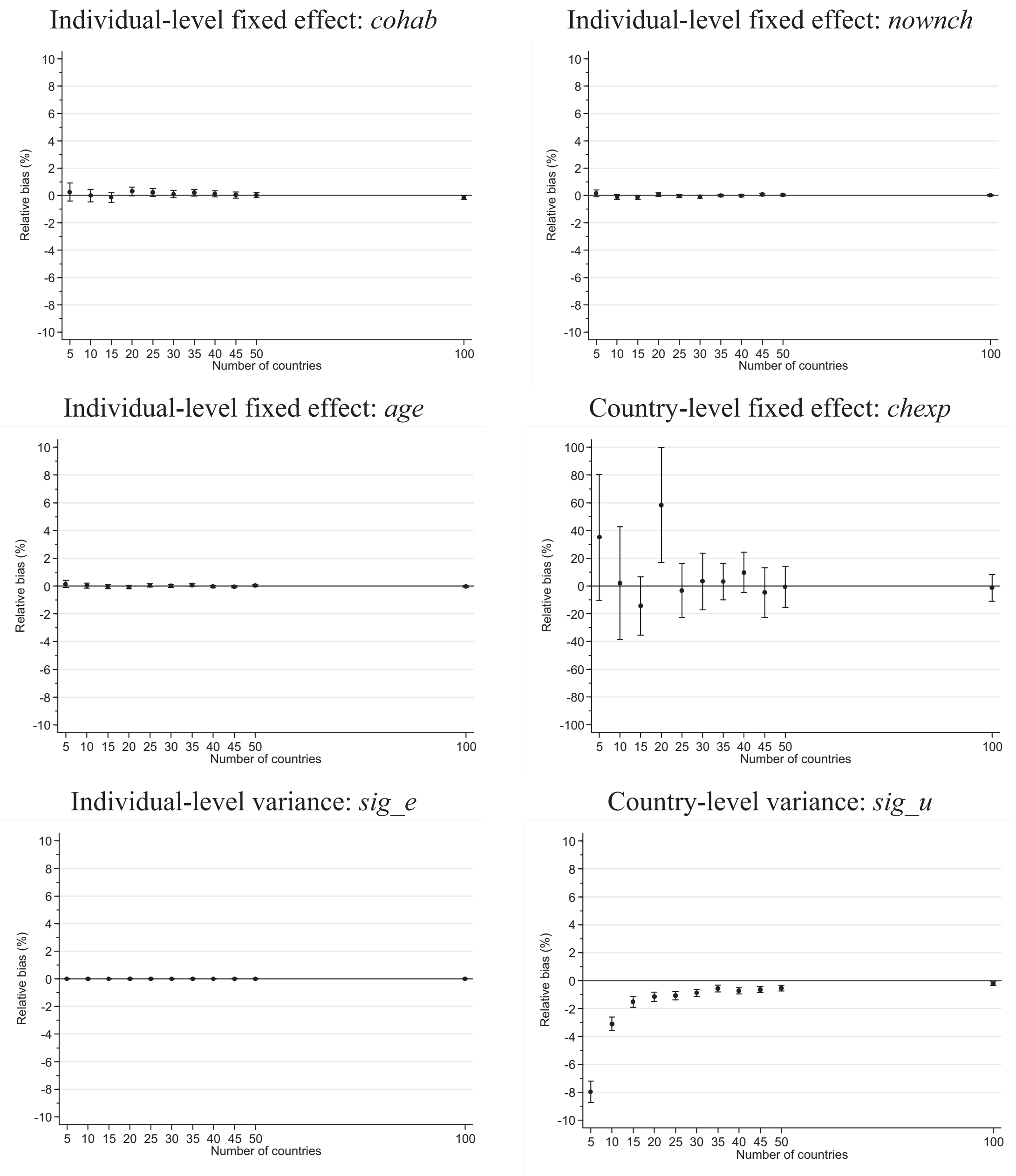

Individual-level fixed effect: *cohab*

Individual-level fixed effect: *nownch*

Individual-level fixed effect: *age*

Country-level fixed effect: *chexp*

Individual-level variance: *sig_e*

Country-level variance: *sig_u*

Figure 1. Relative parameter bias (per cent): linear model with random intercept and country-level regressor (basic model for 'hours'), selected parameters

Notes: The filled circles show estimates of relative parameter bias, and the vertical bars show their 95 per cent CIs (see main text for definitions). The parameters and their labels are defined in Table 4 and the main text. Observe the different vertical scale in the graph for country-level fixed effect *chexp*. Number of Monte Carlo replications, $R = 10{,}000$

under 2 per cent for $C \geq 15$. For the country-level variance, there appears to be negligible bias in the SEs for almost all values of C. Even for $C = 5$, the SEs are downward biased by only 3 per cent.

The corresponding non-coverage rates are shown in Figure 3. Rates are estimated to be close to the nominal rate of 0.05 at all values of C, for the individual-level variance and for all the fixed effect coefficients

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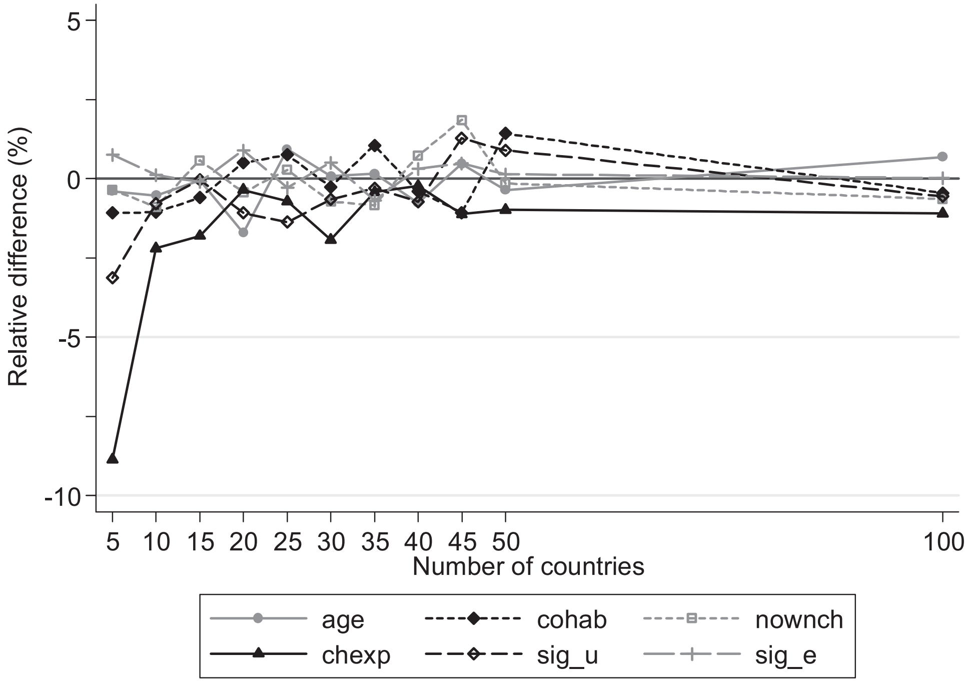

Figure 2. Relative SE bias (per cent): linear model with random intercept and country-level regressor (basic model for 'hours'), selected parameters

Notes: Relative SE bias is defined in the main text. The parameters and their labels are defined in Table 4 and the main text. Number of Monte Carlo replications, $R = 10{,}000$

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except the one associated with the country-level variable. As expected from the under-estimated SEs, non-coverage rates for *chexp* are markedly greater than 0.05 when C is very small, but they reach around 0.06 for $C \geq 20$. Rates diverge to a greater extent for the country-level variance. It is only for $C > 35$ that the non-coverage rate is within one percentage point of 0.05. Since the SEs are unbiased for *sig_u*, the high non-coverage rates at small *C* stem from parameter bias (Figure 1) or from a non-normal distribution of parameter estimates.

Results for the extended linear model for hours (including a cross-level interaction and two random slopes) are summarized in section 7 of the Supplementary Material. Compared with the results about bias for the basic linear model, the main change compared to Figure 1 is the greater prevalence of variability in estimates of bias for the fixed parameters with the exception of that for age. (Having a relatively small number of countries now has implications for estimates of cross-level interaction effects, as well as for the country-level effect itself.) Nonetheless, relative bias is less than 2 per cent for values of $C > 10$, and the 95 per cent CI is −2 to +2 per cent for all but one of the cross-level interaction effects for $C > 30$. The random slope and country-level variances are all under-estimated, but the downward bias is less than 2 per cent as long as $C \geq 25$.

Non-coverage rates for the extended linear model are generally too large for all parameters except the age effect. Compared with the simpler linear model, this is apparent for more of the fixed parameters. As before, the explanation is that having a relatively small number of countries has implications for the SE estimates of effects in addition to those for the country-level intercept, transmitted via the cross-level interactions or random slopes. Non-coverage rates generally decrease as the number of countries increases, dropping sharply between $C = 5$ and $C = 20$, for both fixed parameters and random effect variances.

Simulation Results: Logit Models

We summarize results for the basic logit model in Figures 4–6. Because the small-sample properties of this model are less well-known than for the linear model, the simulations are of particular relevance. As it happens,

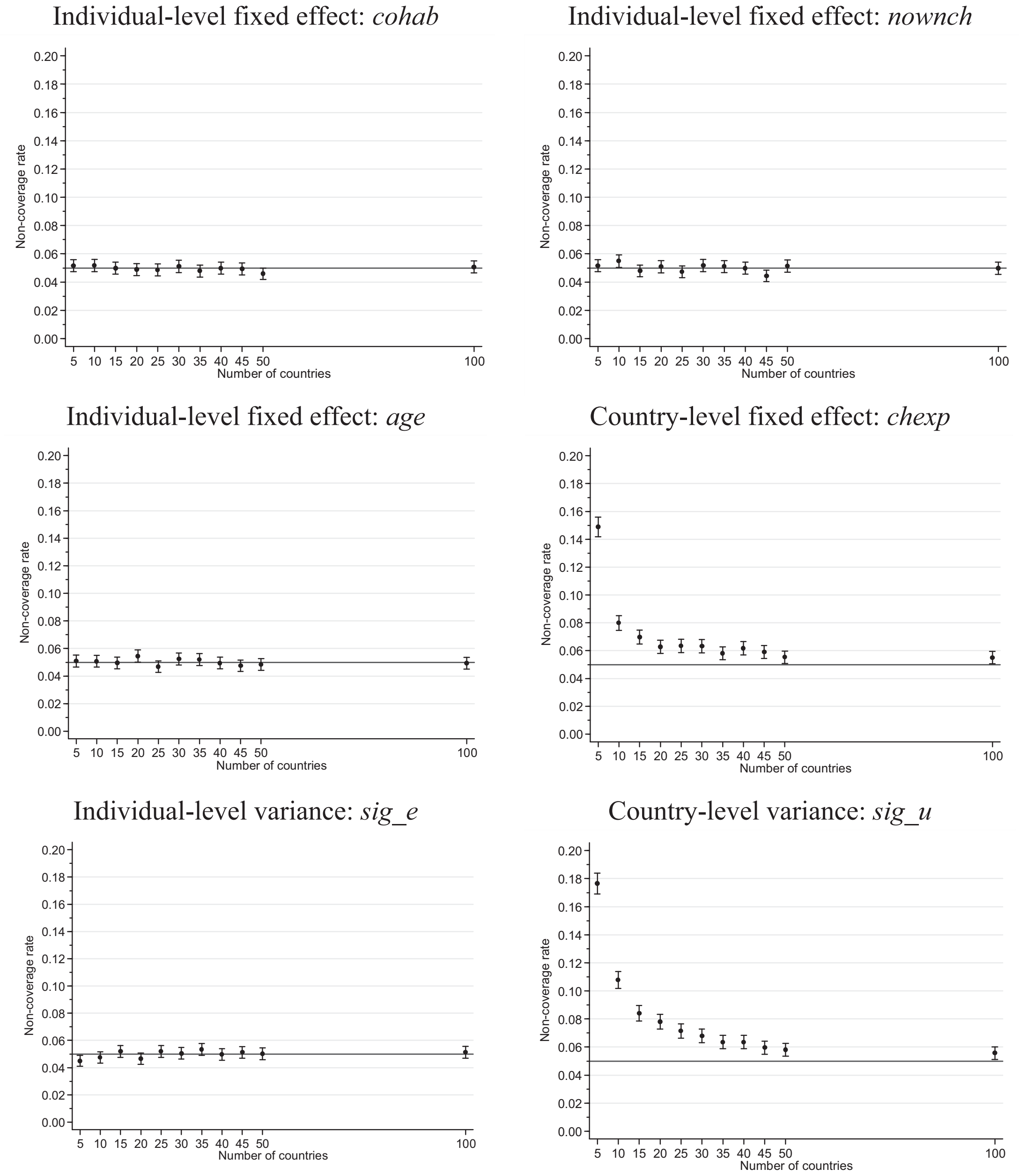

Figure 3. Non-coverage rate: linear model with random intercept and country-level regressor (basic model for 'hours'), selected parameters

Notes: The filled circles show estimates of non-coverage rates (as defined in the main text) and the vertical bars show their 95 per cent CIs. The parameters and their labels are defined in Table 4 and the main text. Number of Monte Carlo replications, $R = 10,000$

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there are some similarities with the results for the corresponding linear model.

Figure 4 shows that the relative bias in the fixed effect is near zero for almost all values of *C*. The main difference from the linear model (Figure 1) is that there is now relatively little variability in the country-level effect. Instead there is substantial variability in the estimate of bias in the effect of *cohab*: there is marked downward

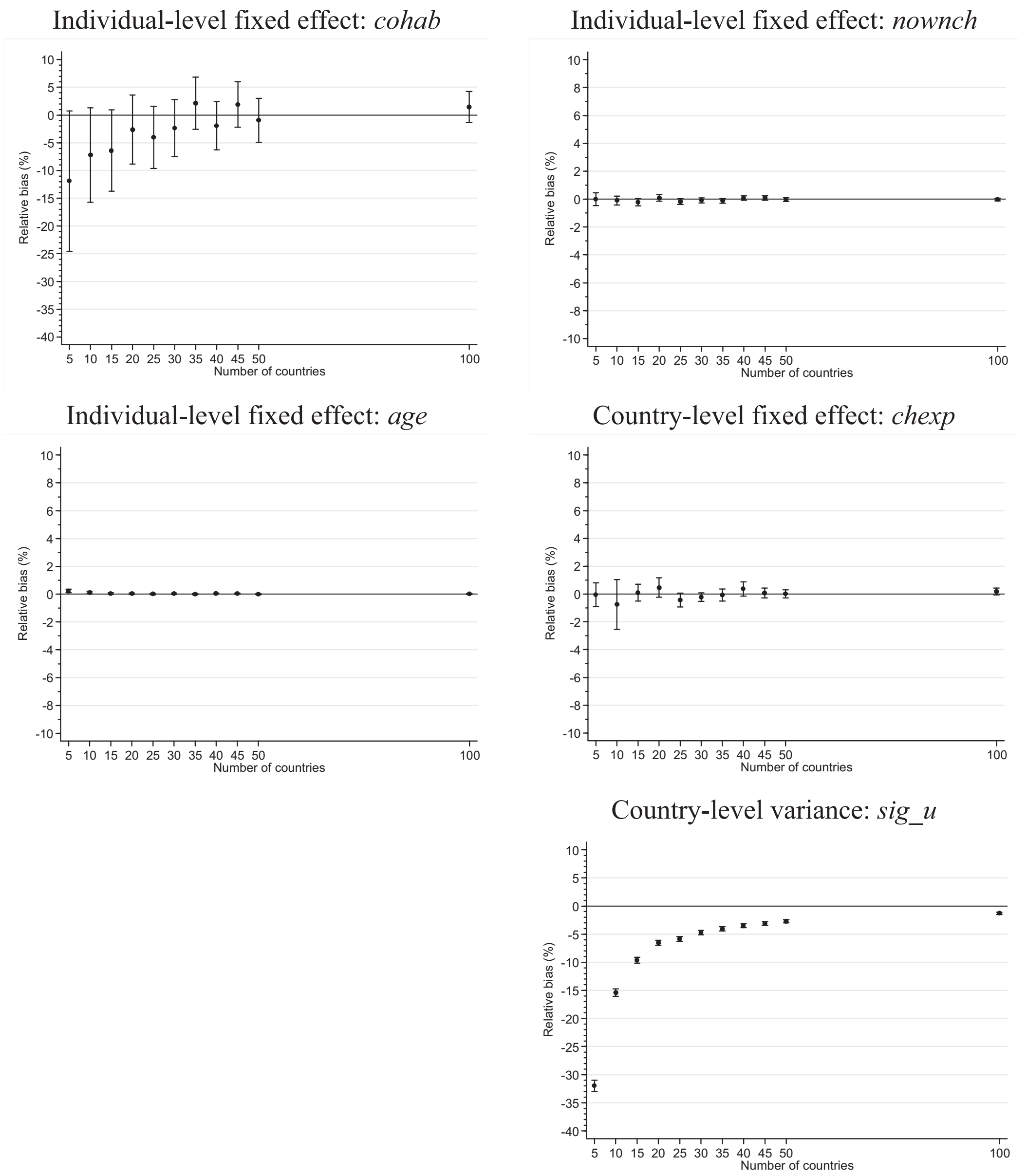

Figure 4. Relative parameter bias (per cent): binary logit model with random intercept and country-level regressor (basic model for 'participation'), selected parameters

Notes: The filled circles show estimates of relative parameter bias, and the vertical bars show their 95 per cent CIs (see main text for definitions). The parameters and their labels are defined in Table 4 and the main text. Observe the different vertical scale for the country-level variance *sig_u*. Number of Monte Carlo replications, $R = 5{,}000$.

bias at values of $C < 20$, though also observe that the CIs for relative bias include zero at all C values. As with the linear model, the accuracy issues for fixed effects are more to do with variability than bias.

The country variance (*sig_u*) is downwardly biased, also as before, but now to a much greater extent than in the basic linear model. At $C = 5$, *sig_u* is underestimated by over 30 per cent (compared with 8 per cent for the linear model) and it is only for $C \geq 30$ that the bias is less than 5 per cent.

The estimated bias of the SEs is summarized in Figure 5. There is little SE bias for the fixed effect

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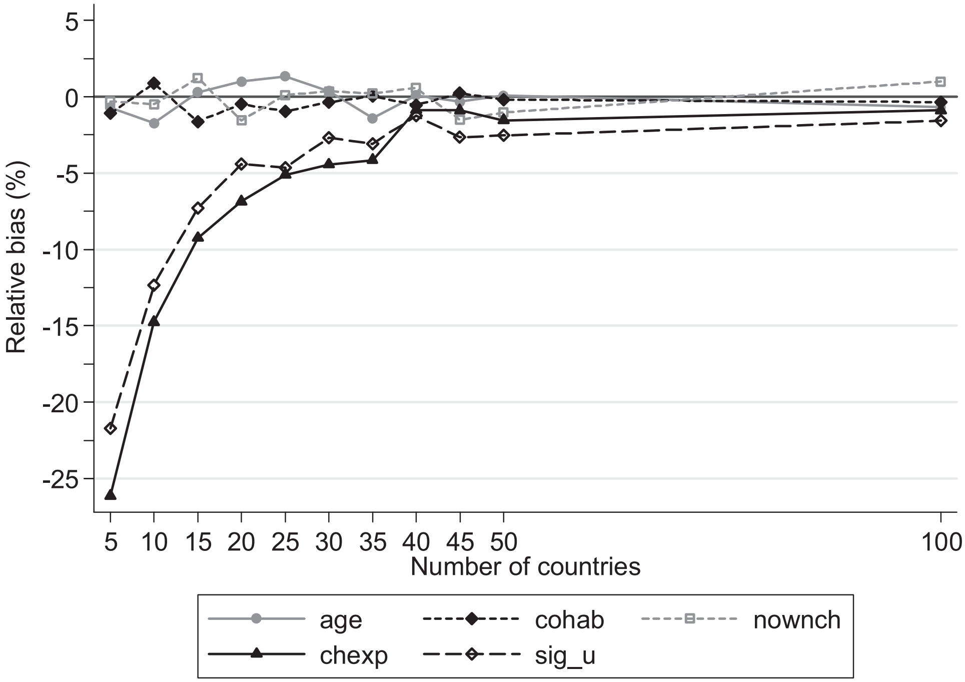

Figure 5. Relative SE bias (per cent): binary logit model with random intercept and country-level regressor (basic model for 'participation'), selected parameters

Notes: Relative SE bias is defined in the main text. The parameters (and their labels) are defined in Table 4 and the main text. Number of Monte Carlo replications, $R = 5{,}000$.

associated with individual-level predictors. However the SEs of the country-level fixed effect, *chexp*, and of the country-level random intercept variance, *sig_u*, are substantially under-estimated for small values of C. These biases exceed those of the linear model (Figure 2). Only for $C \geq 25$ does the bias fall below 5 per cent for *chexp* ($C \geq 20$ for *sig_u*).

Non-coverage rates for the basic logit model are shown in Figure 6. As for the basic linear model (Figure 3) and mirroring the negligible SE bias results, non-coverage rates are close to 0.05 for the fixed effects of individual-level predictors. Again, the exceptions are the fixed country-level effect and the country-level intercept variance. For *chexp*, non-coverage rates are greater than in the linear model case. Only for $C = 40$ does the non-coverage rate for *chexp* get to within 1 percentage point of 0.05. But if one were prepared to tolerate a non-coverage rate of 0.08, then having $C > 20$ would suffice. Similarly, the non-coverage rate for the country-level variance is also much too large for most C values, and by a greater amount than in the corresponding linear model case (note the vertical axis scale in this case). For $C = 30$, the non-coverage rate is around 0.10, i.e. twice the nominal rate of 0.05. Even when $C = 100$, the non-coverage rate is around 0.07.

The results for the extended logit specification parallel those for the corresponding linear model and, again, the accuracy of corresponding estimates is less, for both parameters and SEs. (See section 7 of the Supplementary Material.) Variability is relatively large for all of the estimates of bias in the fixed parameters. Again, however, virtually every CI for these estimates includes zero, and for all C. And, for all fixed parameters except that for *cohab*, the relative bias estimate itself is no more than 2 per cent as long as $C \geq 20$. (By contrast, the estimated relative bias for *cohab* is around −7 per cent when $C = 100$.) The random slope and intercept variances are substantially under-estimated when the number of countries is small. For example, the random slope variances are around half the true value for $C = 5$, though 'only' 90 per cent of their true value for $C = 20$. Relative bias falls to 5 per cent or less only if C is around 40. For the country variance, this degree of bias is achieved if $C \geq 30$.

Non-coverage rates tend to be larger than for the corresponding extended linear model, especially at small

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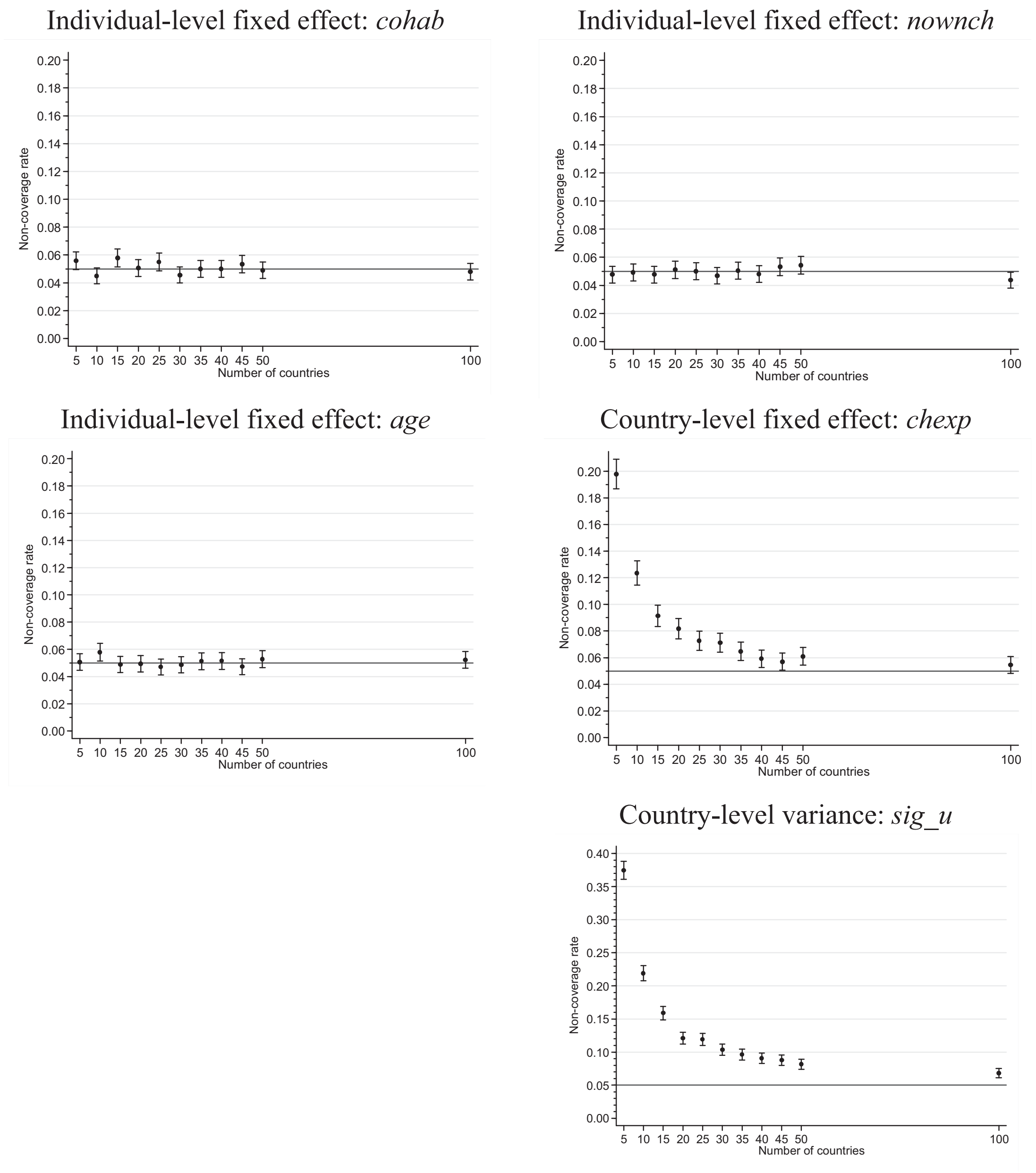
Individual-level fixed effect: *cohab*

Individual-level fixed effect: *nownch*

Individual-level fixed effect: *age*

Country-level fixed effect: *chexp*

Country-level variance: *sig_u*

Figure 6. Non-coverage rate: binary logit model with random intercept and country-level regressor (basic model for 'participation'), selected parameters

Notes: The filled circles show estimates of non-coverage rates (defined in the main text), and the vertical bars show their 95 per cent CIs. The parameters and their labels are defined in Table 4 and the main text. Number of Monte Carlo replications, $R = 5{,}000$.

values of C. Even with $C \geq 35$, the non-coverage rate is greater than 0.06 for several fixed parameters. On the other hand, if one is prepared to tolerate a non-coverage rate up to 0.08, the simulations suggest that having at least 25 countries would suffice. To generate the same non-coverage rate for the random coefficient variances appears to require around 30 countries or more, whereas for the country variance, more than 35 are required. The results suggest that to lower the rate further would require a large number of countries: even

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when $C = 100$, the non-coverage rate is greater than 0.06, for all three variances.

Lessons of the Monte Carlo Simulation Analysis

How many countries does one need for multilevel model analysis of multi-country data to provide reliable estimates? One short answer based on our simulation analysis might be: at least 25 countries for linear models and at least 30 countries for logit models. However, there is no simple 'magic' number for researchers to appeal to.

For instance, the critical number of countries depends on a researcher's definition of acceptable accuracy. We have used as reference points a relative bias of 0 per cent and a non-coverage rate of 0.05, but fewer countries might be sufficient if one is content to be merely fairly 'close' to these ideals, or more if the reference points are applied strictly. It is the responsibility of researchers to be clear about what counts as acceptable accuracy.

Crude rules of thumb should not be applied blindly, in any case. We have demonstrated that the minimum number of countries required depends on what model is being estimated and which effects the researcher is primarily interested in.

At one extreme, it is well known that for a linear model, REML produces unbiased estimates of the effects of fixed individual-level covariates and our simulations confirm this. But our simulations also show that unbiasedness may coincide with a substantial degree of estimate inaccuracy owing to variability, particularly for effects associated with country-level factors (country effects and cross-level interaction effects), reflecting the small number of countries relative to the number of individuals per country. Country-level variances are also prone to underestimation and reported SEs for them lead to unreliable inference. In addition, our extended model results suggest that introducing more country effects in the form of cross-level interactions or country-level random slopes can lead to additional reliability problems. There is also the more general point that our results refer to data sets with very large numbers of individuals per country. If there are substantially fewer level-1 observations per level-2 cluster, for example, the critical number of observations is likely to differ.

Put differently, we recommend that researchers fitting relatively complicated models should seek data sets with numbers of countries greater than those cited above if they wish to be confident of having reliable results. By 'relatively complicated models' we mean models with multiple country-level or cross-level fixed effects (the greater the number, the fewer the effective degrees of freedom) and, more generally, models that differ from the 'basic' specifications that we have focused on.

More positively, we have shown that non-coverage rates for fixed effects in linear models are relatively good as long as the number of countries is greater than around 25. With this number of countries, linear model estimates of random effect variances and their SEs also appear to be accurate to an extent that may satisfy many practising researchers.

Our simulation results for the binary logit models regarding relative bias and non-coverage have parallels with those for the corresponding linear models. The primary difference between models is that a greater number of countries are necessary for logit models to generate the same degree of accuracy in parameter estimates and SEs, other things being equal. In particular for random coefficient variances (if specified) and especially the country-level variance, at least 30–35 countries may be required to derive accurate estimates—which is more countries than is usually available (see Table 1). Our recommendations above concerning 'relatively complicated models' have particular force in the case of non-linear models.

An additional warning concerning non-linear multilevel models in general and the binary logit mixed model in particular is that the estimator used for maximization also matters. We have used ML with adaptive Gaussian quadrature. This has been found to produce more accurate estimates than penalized quasi-likelihood both with data structures different from ours (Rodriguez and Goldman, 2001; Callens and Croux, 2005; Pinheiro and Chao, 2006; Austin, 2010) and also with the same data structure as used in this article (Jenkins, 2013). Other researchers have shown that Bayesian estimation methods using Markov chain Monte Carlo methods also perform better than ML when the number of groups ('countries') is small. See Austin (2010), Browne and Draper (2000, 2006), and Stegmueller (2013).

Summary and Conclusions

When there are few countries in a multi-country data set, there is little information with which to estimate country effects, whether these effects refer to the fixed parameters on country-level predictors or the variances of random country intercepts. Multilevel model users need to be cautious in the claims they make about country effects of either type.

Our Monte Carlo simulations suggest that users require 25 countries for linear models and 30 countries for logit models at the very minimum, and most likely more for models with a specification other than a relatively

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basic one. Otherwise, estimates of country-level fixed parameters are likely to be estimated imprecisely and this will not be adequately reflected in test statistics reported by commonly used software: users will conclude too often that a country effect exists when it does not. Country random variances will be biased downwards and have CIs that are too narrow. The only estimates that are unaffected by having a small number of countries are the fixed parameters on individual-level predictors (the number of individuals per country is typically large): provided there is not also a random component attached to the slope, these parameters are estimated without bias and with the correct SEs (and non-coverage rate).

Since the critical number of countries required for reliable estimation of country effects is larger than the number of countries in many existing data sets, what can analysts do in the small-*C* case (in addition to being cautious in their claims)? We recommend three strategies.

One is to supplement regression-based modelling with more descriptive analysis of measured country differences. We referred earlier in the discussion of the two-step approach to exploratory data analysis based on visualization of country differences. On this, see Bowers and Drake (2005) and also the examples in section 8 of this article's Supplementary Material. The two-step approach may reveal features of the data, such as country groupings, that are worthy of further investigation; or could highlight outliers that have an undue influence owing to the small number of countries—perhaps prompting a more systematic examination using jackknifing and influence statistics (Van der Meer, Te Grotenhuis and Pelzer, 2010).

A second strategy is to use methods that are more robust to small numbers of countries, as mentioned in section 3. These include small sample corrections, such as those available in SAS and R for linear models, and bootstrapping. However, we note that some of these techniques require specialized knowledge and are not routinely available in the software packages most commonly used by social scientists, and they are not applicable to all parameters of interest.

A third strategy would be to move beyond the classical ('frequentist') statistical framework used by most applied social science researchers and to make greater use of Bayesian methods of estimation and inference, as there is some evidence that they perform better in the small-*C* case (see section 4). In the Bayesian paradigm, the researcher specifies a 'prior distribution' for the model parameters that embodies a belief about their plausible range of values. Estimation produces a 'posterior distribution' of parameters that reflects both the observed data and the priors. It is because Bayesian inferences account for the uncertainty in the parameters across their plausible range that they are more reliable than classical methods when level-2 (country) sample sizes are small (Raudenbush and Bryk, 2002: chapter 13). However, this superior performance of Bayesian estimation relies on the priors being correctly specified. With a small number of countries, the prior may have a large influence on the posterior and so it is essential to check the sensitivity of results to alternative priors (Seltzer, Wong and Bryk, 1996).

There is a general challenge to a prescription of more widespread use of Bayesian methods for multilevel modelling, and that is that such methods require statistical expertise beyond that of most applied social science researchers, as well as specialist software (or software with which such researchers are unfamiliar). It is apparently not enough that there are already short introductions in general multilevel modelling textbooks (for example, Raudenbush and Bryk, 2002: chapter 13; Brown and Prescott, 2006: section 2.3), review articles such as Draper (2008), and a number of textbooks focusing on Bayesian multilevel modelling such as Gelman *et al.* (2013) or Gill (2008).

In addition to these three strategies for analysis of country effects using hierarchical multi-country data sets, there remains a need for detailed consideration of the workings of national institutions and policies. Multilevel modelling methods are no panacea.

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