



DE VINCI
INNOVATION
CENTER

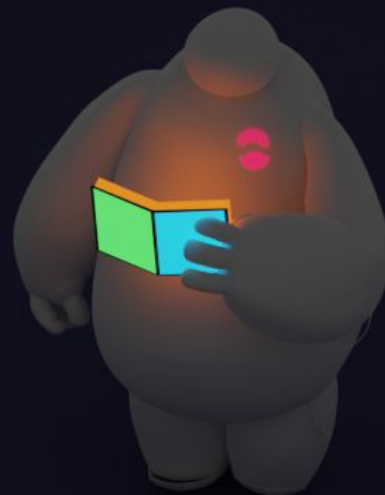
MACHINE LEARNING

Learning from Data

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**LÉONARD
DE VINCI**
PARIS-LA DÉFENSE



ÉCOLE
D'INGÉNIEURS
PARIS-LA DÉFENSE



INSTITUT DE
L'INTERNET
ET DU MULTIMÉDIA
PARIS-LA DÉFENSE



ÉCOLE DE
MANAGEMENT
PARIS-LA DÉFENSE

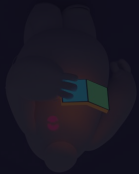
|00 INTRODUCTION



Pattern Recognition

Study **Correlations** Between **Data Domains**

Tune a Parametric Model with **Data**



|00 INTRODUCTION



SUPERVISED

Learn parameters from
Labeled Data

Regression

Classification

UNSUPERVISED

Learn parameters from
Unlabeled Data

Clustering

Latent Space

REINFORCEMENT

Autonomous Agent Learning
from **Experience**

Q-Learning



01|

LINEAR ALGEBRA

The Last Space Bender

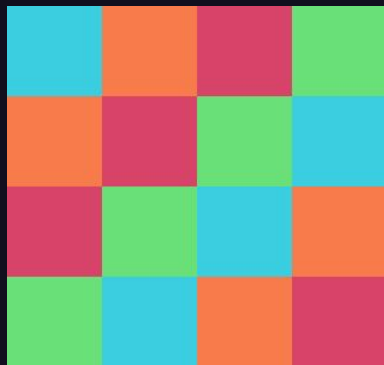


|01 LINEAR ALGEBRA

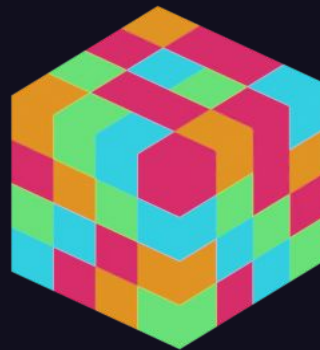
VECTOR



MATRIX



TENSOR



|01 LINEAR ALGEBRA

VECTOR

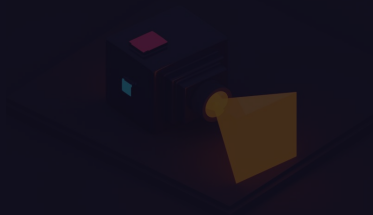


$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

LENGTH

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

|01 LINEAR ALGEBRA



VECTOR

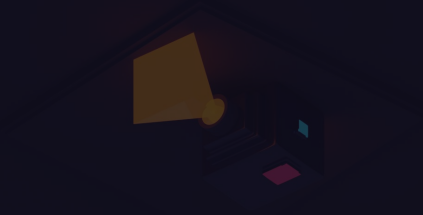


$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

ADDITION / SUBTRACTION

$$\vec{v} + \vec{u} = \begin{pmatrix} v_x + u_x \\ v_y + u_y \end{pmatrix}$$



|01 LINEAR ALGEBRA

VECTOR



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

SCALAR (DOT) PRODUCT

$$\vec{v} \cdot \vec{u} = v_x u_x + v_y u_y = |v| |u| \cos \theta$$

|01 LINEAR ALGEBRA

VECTOR



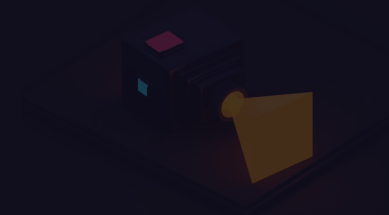
$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

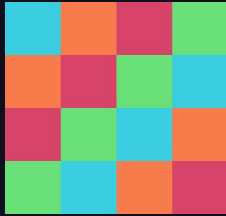
CROSS PRODUCT

$$\vec{v} \times \vec{u} = \begin{pmatrix} v_y u_z - v_z u_y \\ v_z u_x - v_x u_z \\ v_x u_y - v_y u_x \end{pmatrix}$$

|01 LINEAR ALGEBRA



MATRIX

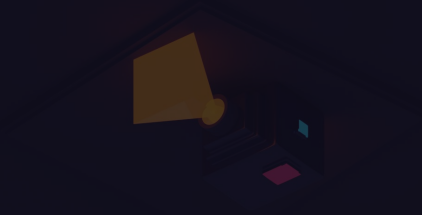


ADDITION / SUBTRACTION

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

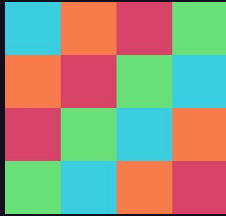
$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A + B = \begin{pmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{pmatrix}$$



|01 LINEAR ALGEBRA

MATRIX



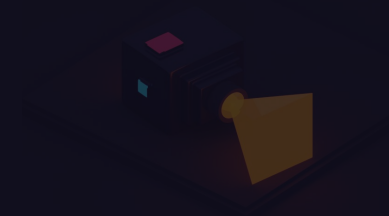
HADAMARD PRODUCT

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

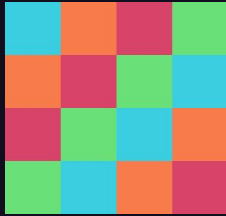
$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \odot B = \begin{pmatrix} a_{1,1}b_{1,1} & a_{1,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,2}b_{2,2} \end{pmatrix}$$

|01 LINEAR ALGEBRA



MATRIX

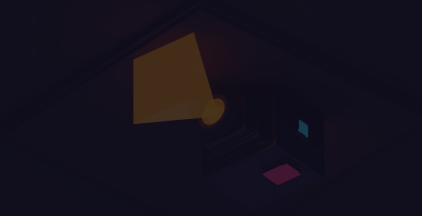


$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

MATRIX MULTIPLICATION

$$A \cdot B = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$



|01 LINEAR ALGEBRA

MATRIX

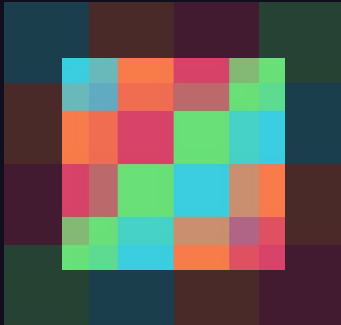


EYE

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



SCALE

$$\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



ROTATION

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



SHEAR

$$\begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX

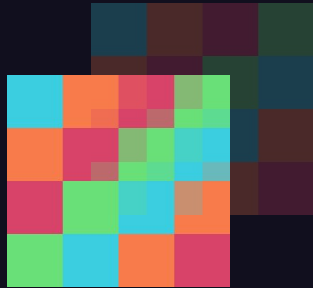


REFLECTION

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



TRANSLATION

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

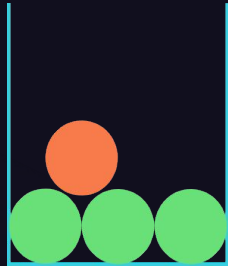
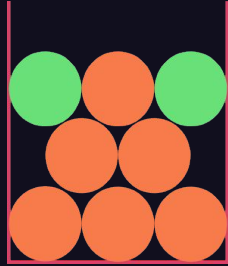
|02

PROBABILITIES

Roll the Dice



|02 PROBABILITIES



RANDOM VARIABLES

$$B = \begin{cases} r & \text{if Box is red} \\ b & \text{if Box is blue} \end{cases}$$

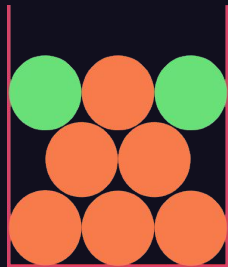
$$F = \begin{cases} a & \text{if Fruit is an apple} \\ o & \text{if Fruit is an orange} \end{cases}$$



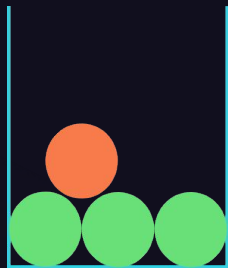
|02 PROBABILITIES



PROBABILITIES



$$p(B = r) = \frac{\# \text{ red boxes picked}}{\# \text{ total boxes picked}} \in [0; 1]$$



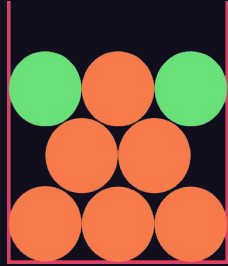
$$p(B = r) + p(B = b) = 1$$



|02 PROBABILITIES



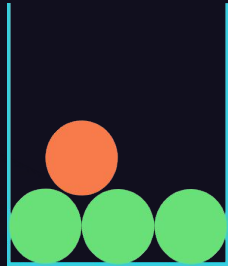
JOINT & CONDITIONAL



Sum Rule

Joint probabilities $p(B = r, F = a)$

Marginal probability $p(B = r) = p(B = r, F = a) + p(B = r, F = o)$



Product Rule

Conditional probabilities $p(B = r|F = o)$

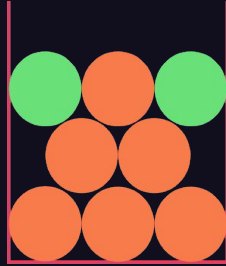
$p(B = r, F = o) = p(F = o|B = r) p(B = r)$



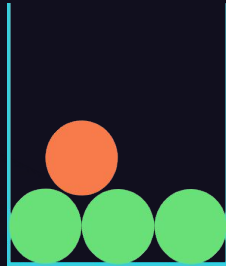
|02 PROBABILITIES



BAYES THEOREM



$$p(B = r | F = o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o)}$$



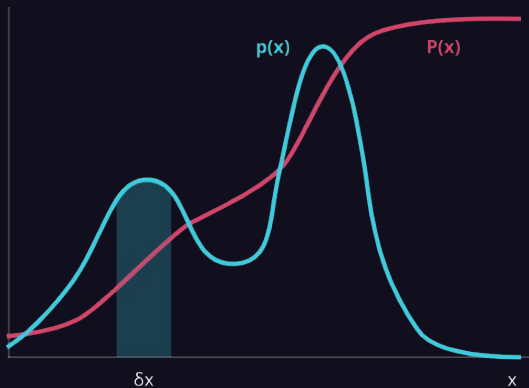
$$p(B = r | F = o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o|B=r)p(B=r) + p(F=o|B=b)p(B=b)}$$



|02 PROBABILITIES



DENSITY



$$p(x \in (a, b)) = \int_a^b p(x) dx$$

$$\int_{-\infty}^{+\infty} p(x) dx = 1$$

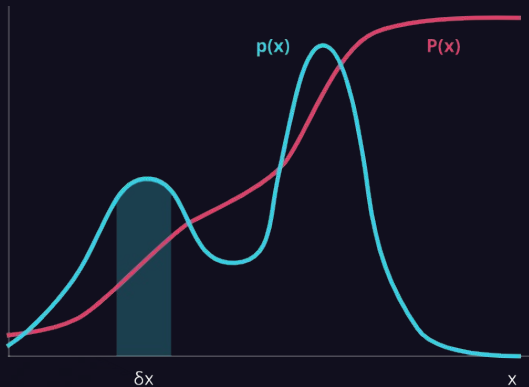
$$p(x) \geq 0$$



|02 PROBABILITIES



EXPECTATION & COVARIANCE



$$\mathbb{E}[f] = \int p(x) f(x) dx$$

$$var[f] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$

$$var[f] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

$$cov[x, y] = \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y]$$





03|

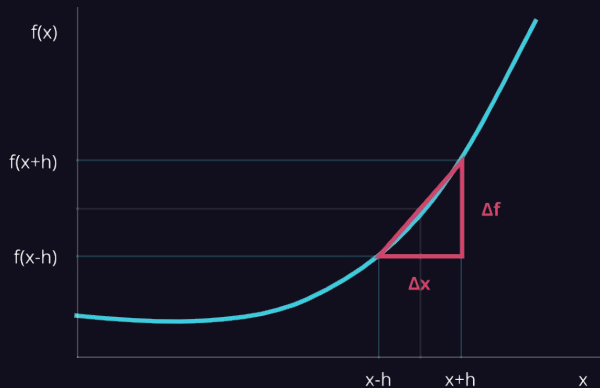
OPTIMIZATION

One Method to Rule them All

|03 OPTIMIZATION



DERIVATIVES



$$f'(x) = \lim_{h \rightarrow 0} \frac{\Delta f}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

First order Derivative

Direction of the Slope

Second order Derivative

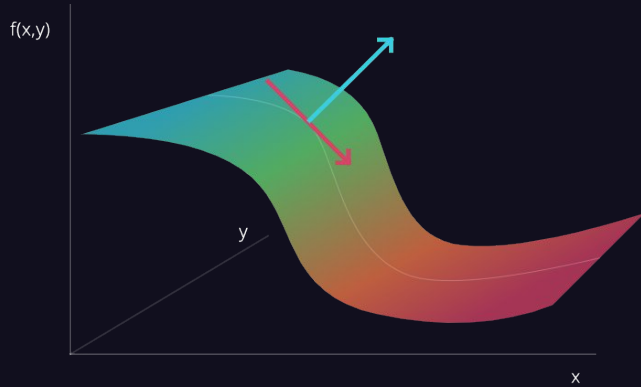
Rate of Changes in the Slope



|03 OPTIMIZATION



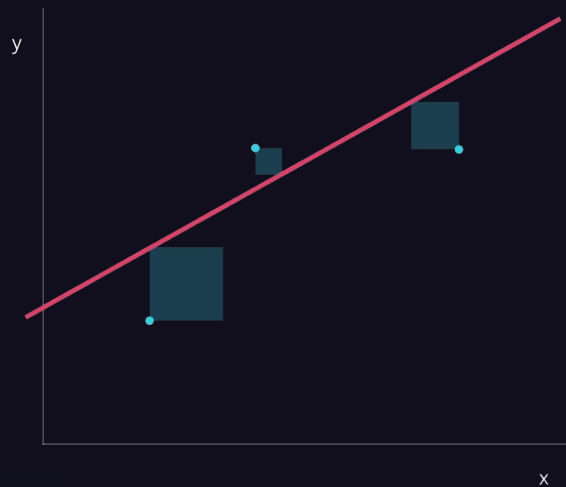
GRADIENTS



$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$



|03 OPTIMIZATION



LEAST SQUARES

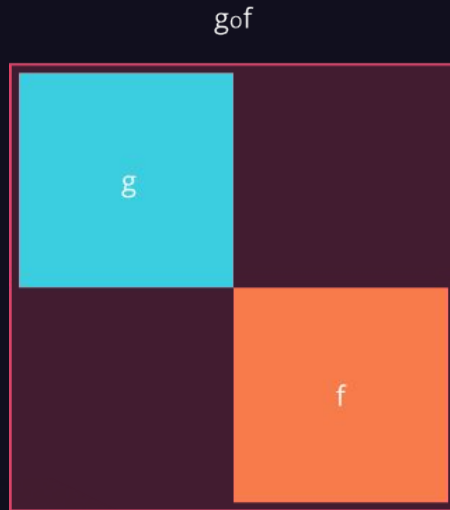
Objective Function

Minimize Squared Distances
from Expected Value

$$\hat{y} = ax + b$$

$$\sum_i (y_i - \hat{y}_i)^2 = 0$$

|03 OPTIMIZATION

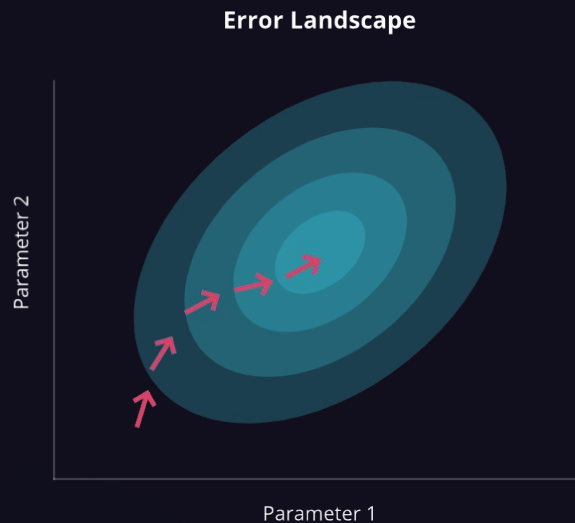


CHAIN RULE

$$h = g \circ f$$

$$h'(x) = g'(f(x))f'(x)$$

|03 OPTIMIZATION



GRADIENT DESCENT

Steps

- 1) **Forward Propagate** Through the Chain
- 2) Compute Output **Error**
- 3) **Backpropagate** Error Through the Chain
- 4) **Update** Weights w/ Learning Rate
- 5) **Repeat** Until Convergence Threshold



|03 OPTIMIZATION



DATASET SPLIT

Training Set

Samples used to Fit/Train the Model

Validation Set

Samples used to provide an Unbiased Evaluation of the Model
Becomes Biased during Training

Testing Set

Samples used to provide an Unbiased Evaluation of the Model after Training

|03 OPTIMIZATION



CROSS-VALIDATION

Iteration 1	Valid	Train	Train	Train
Iteration 2	Train	Valid	Train	Train
Iteration 3	Train	Train	Valid	Train
Iteration 4	Train	Train	Train	Valid
	Fold 1	Fold 2	Fold 3	Fold 4

Method

- 1) Split Dataset into **k-Folds**
- 2) **Train on k-1 Folds** and Validate w/ Last
- 3) **Repeat** k-times
- 4) Use **Ensemble** Method for **Inference** or **Retrain** on all Dataset

