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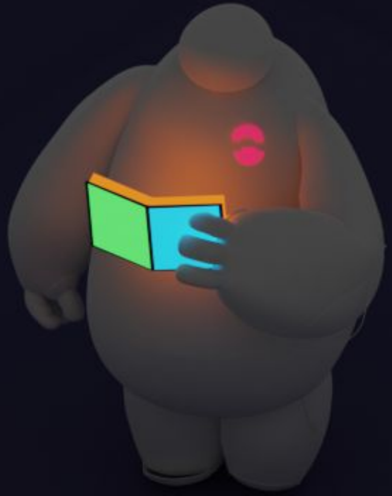
MACHINE LEARNING

Learning from Data



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IIM LÉONARD DE VINCI | INSTITUT DE L'INTERNET ET DU MULTIMEDIA PARIS-LA DÉFENSE

EMLV LÉONARD DE VINCI | ÉCOLE DE MANAGEMENT PARIS-LA DÉFENSE

|00 INTRODUCTION

Pattern Recognition

Study **Correlations** Between **Data Domains**

Tune a **Parametric Model** with **Data**

|00 INTRODUCTION

SUPERVISED

Learn parameters from
Labeled Data

Regression   Classification

UNSUPERVISED

Learn parameters from
Unlabeled Data

Clustering   Latent Space

REINFORCEMENT

Autonomous Agent Learning
from **Experience**

Q-Learning

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01|

LINEAR ALGEBRA

The Last Space Bender

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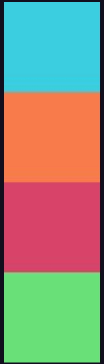
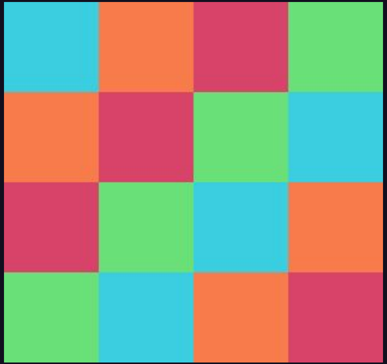
|01 LINEAR ALGEBRA

VECTOR



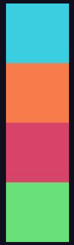
MATRIX



TENSOR

|01 LINEAR ALGEBRA

VECTOR



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

LENGTH

$$||\vec{v}|| = \sqrt{v_x^2 + v_y^2}$$

|01 LINEAR ALGEBRA

VECTOR

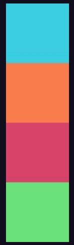
ADDITION / SUBTRACTION

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

$$\vec{v} + \vec{u} = \begin{pmatrix} v_x + u_x \\ v_y + u_y \end{pmatrix}$$

VECTOR



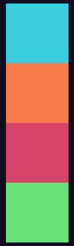
$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

SCALAR (DOT) PRODUCT

$$\vec{v}.\vec{u} = v_x u_x + v_y u_y = |v|\,|u|\,cos\theta$$

VECTOR



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

CROSS PRODUCT

$$\vec{v} \times \vec{u} = \begin{pmatrix} v_y u_z - v_z u_y \\ v_z u_x - v_x u_z \\ v_x u_y - v_y u_x \end{pmatrix}$$

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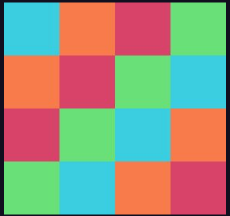
MATRIX



ADDITION / SUBTRACTION

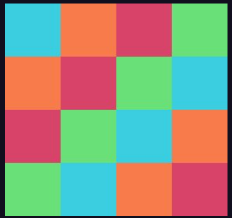
$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A + B = \begin{pmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



HADAMARD PRODUCT

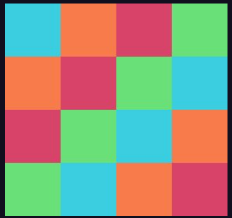
$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \odot B = \begin{pmatrix} a_{1,1}b_{1,1} & a_{1,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,2}b_{2,2} \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



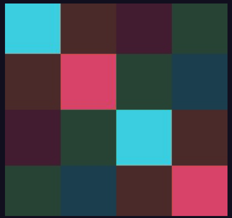
MATRIX MULTIPLICATION

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$

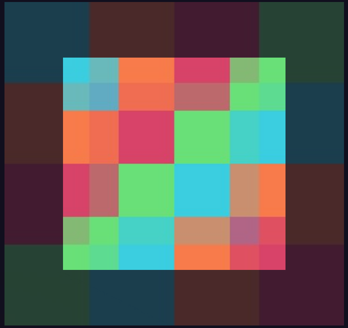
MATRIX



EYE

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

MATRIX

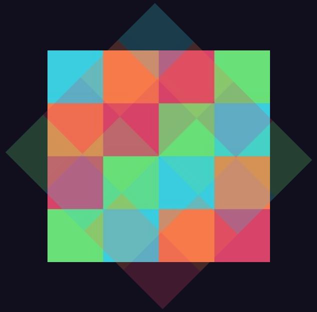


SCALE

$$\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|01 LINEAR ALGEBRA

MATRIX



ROTATION

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ cos(\theta) & -sin(\theta) & 0 & 0 \\ sin(\theta) & cos(\theta) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 & 0 \\ sin(\theta) & cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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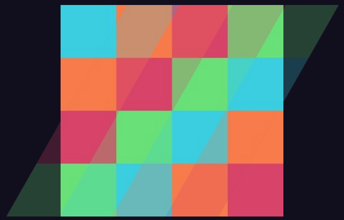
MATRIX



SHEAR

$$\begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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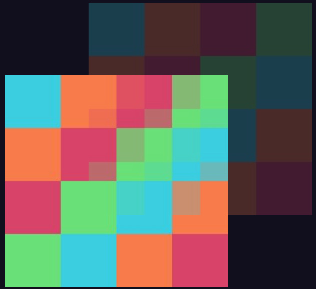
|01 LINEAR ALGEBRA

MATRIX

REFLECTION

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

MATRIX



TRANSLATION

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

|02

PROBABILITIES

Roll the Dice

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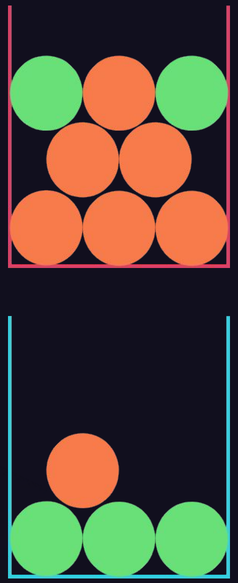
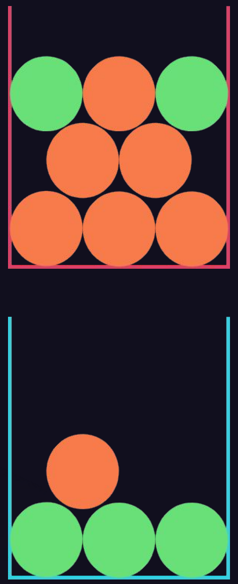
RANDOM VARIABLES



$$B = \begin{cases} r & \text{if Box is red} \\ b & \text{if Box is blue} \end{cases}$$

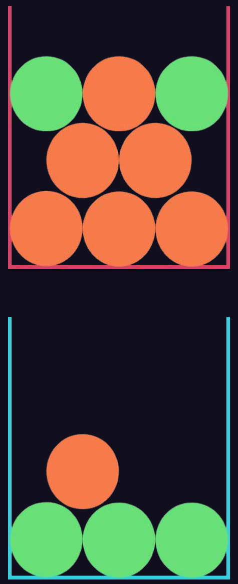
$$F = \begin{cases} a & \text{if Fruit is an apple} \\ o & \text{if Fruit is an orange} \end{cases}$$



PROBABILITIES

$$p(B = r) = \frac{\text{\# red boxes picked}}{\text{\# total boxes picked}} \in [0; 1]$$

$$p(B = r) + p(B = b) = 1$$



JOINT & CONDITIONAL

Sum Rule

Joint probabilties $p(B = r, F = a)$

Marginal probabilty $p(B = r) = p(B = r, F = a) + p(B = r, F = o)$

Product Rule

Conditional probabilties $p(B = r|F = o)$

$$p(B = r, F = o) = p(F = o|B = r)\ p(B = r)$$

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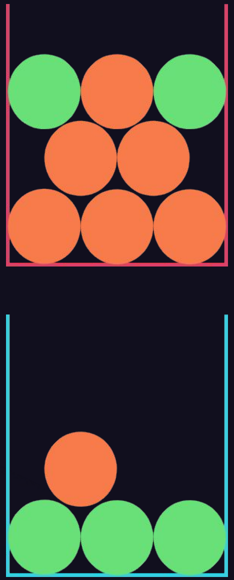
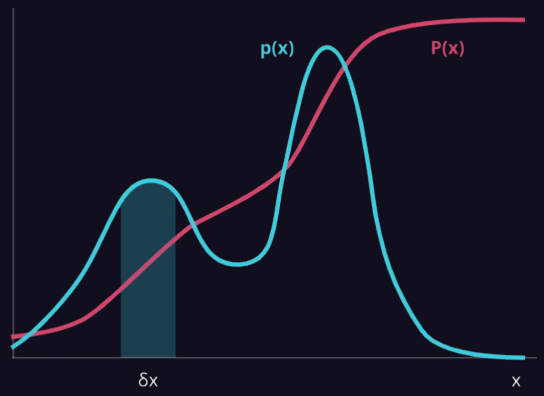


BAYES THEOREM

$$p(B=r|F=o) = \frac{p(F{=}o|B{=}r)p(B{=}r)}{p(F{=}o)}$$

$$p(B=r|F=o) = \frac{p(F{=}o|B{=}r)p(B{=}r)}{p(F{=}o|B{=}r)p(B{=}r)+p(F{=}o|B{=}b)p(B{=}b)}$$

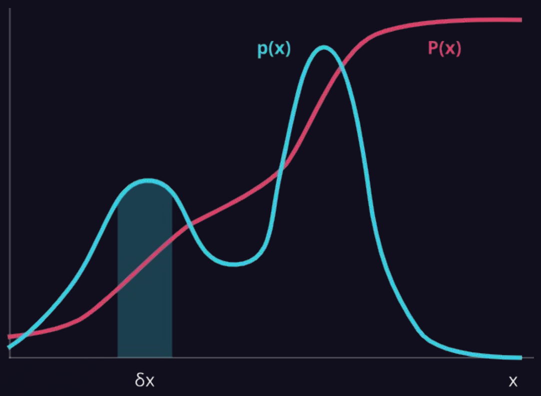


DENSITY

$$p(x \in (a,b)) = \int_a^b p(x)dx$$

$$\int_{-\infty}^{+\infty} p(x)dx = 1$$

$$p(x) \geq 0$$



EXPECTATION & COVARIANCE

$$\mathbb{E}[f] = \int p(x) f(x) dx$$

$$var[f] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$

$$var[f] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

$$cov[x, y] = \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y]$$

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03|

OPTIMIZATION

One Method to Rule them All

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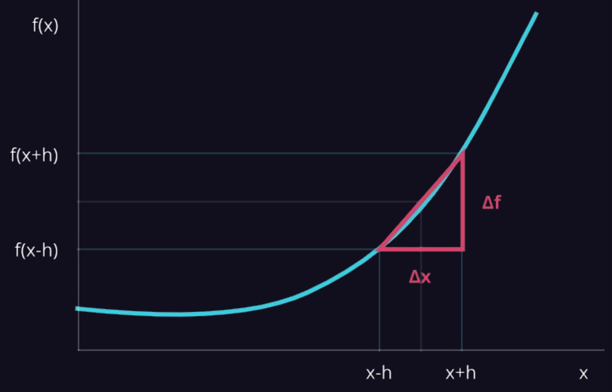
|03 OPTIMIZATION



DERIVATIVES

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

First order Derivative

Direction of the Slope

Second order Derivative

Rate of Changes in the Slope

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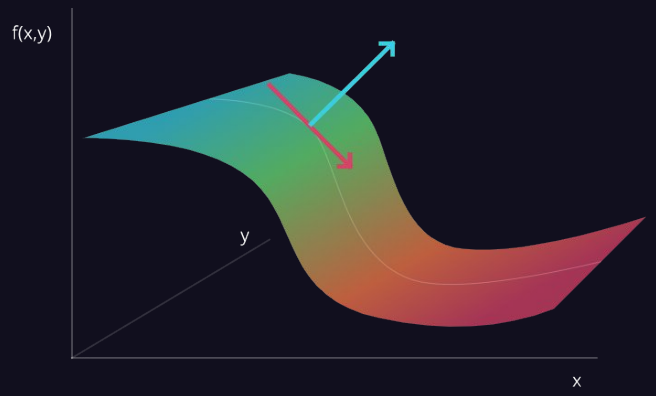
|03 OPTIMIZATION

GRADIENTS



$$\nabla f = \begin{pmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{pmatrix}$$

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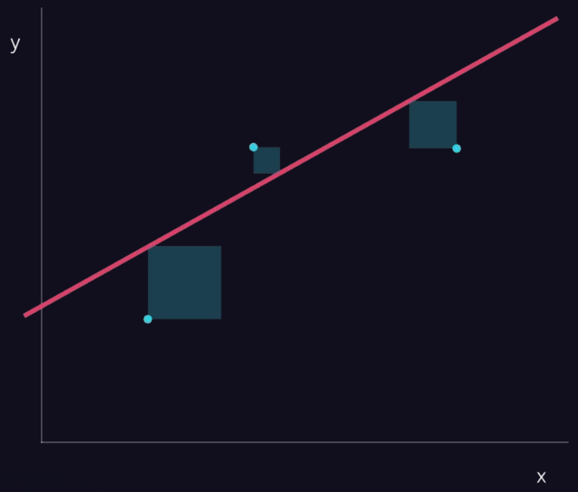
|03 OPTIMIZATION



LEAST SQUARES

Objective Function

Minimize Squared Distances
from Expected Value

$$\hat{y} = ax + b$$

$$\sum_i (y_i - \hat{y}_i)^2 = 0$$

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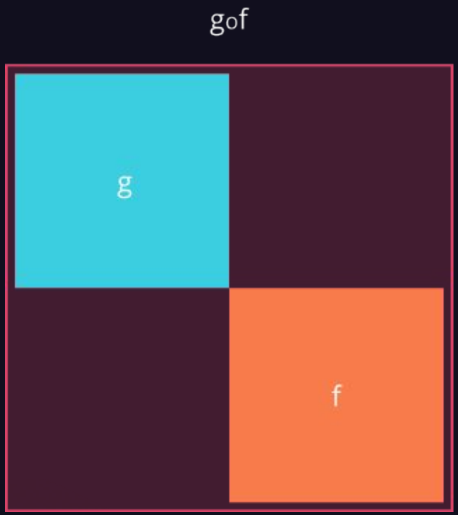
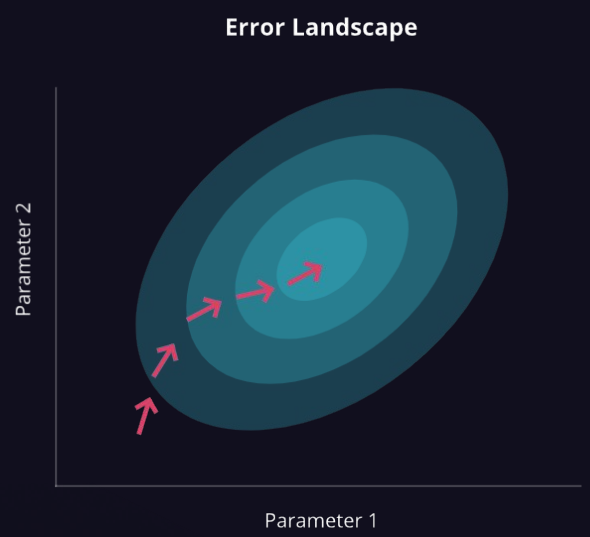


CHAIN RULE

$$h = g \circ f$$

$$h'(x) = g'(f(x))f'(x)$$

|03 **OPTIMIZATION**



GRADIENT DESCENT

Steps

1) **Forward Propagate** Through the Chain
2) Compute Output **Error**
3) **Backpropagate** Error Through the Chain
4) **Update** Weights w/ Learning Rate
5) **Repeat** Until Convergence Threshold

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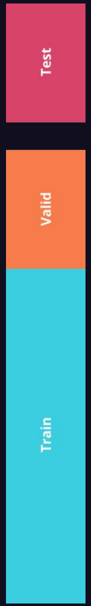
|03 OPTIMIZATION



DATASET SPLIT

Training Set

Samples used to Fit/Train the Model

Validation Set

Samples used to provide an Unbiased Evaluation of the Model Becomes Biased during Training

Testing Set

Samples used to provide an Unbiased Evaluation of the Model after Training

|03 OPTIMIZATION

CROSS-VALIDATION

	Fold 1	Fold 2	Fold 3	Fold 4
Iteration 1	Valid	Train	Train	Train
Iteration 2	Train	Valid	Train	Train
Iteration 3	Train	Train	Valid	Train
Iteration 4	Train	Train	Train	Valid

Method

1) Split Dataset into **k-Folds**
2) **Train on k-1 Folds** and Validate w/ Last
3) **Repeat** k-times
4) Use **Ensemble** Method for **Inference** or **Retrain** on all Dataset

|04

DIMENSIONALITY REDUCTION

Small Worlds are Filled with Things to See

Paul Safranek

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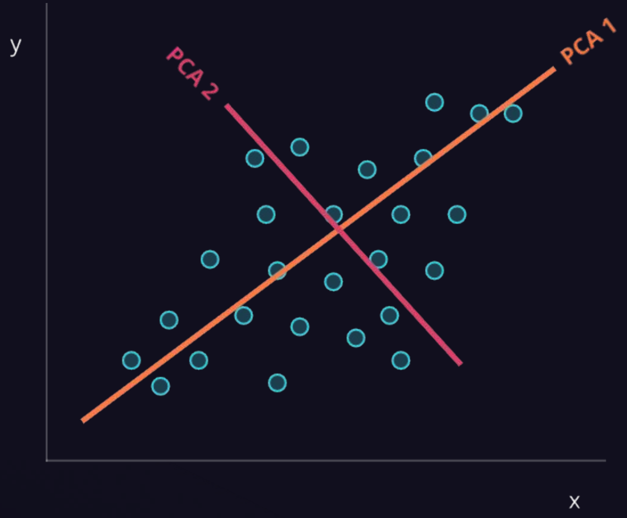
|04 DIMENSIONALITY REDUCTION



PCA

$$C = \begin{pmatrix} Var(x) & Cov(x,y) & Cov(x,z) \\ Cov(y,x) & Var(y) & Cov(y,z) \\ Cov(z,x) & Cov(z,y) & Var(z) \end{pmatrix}$$

$v_1 \quad \lambda_1$ Big

$v_2 \quad \lambda_2$

$v_3 \quad \lambda_3$ Small

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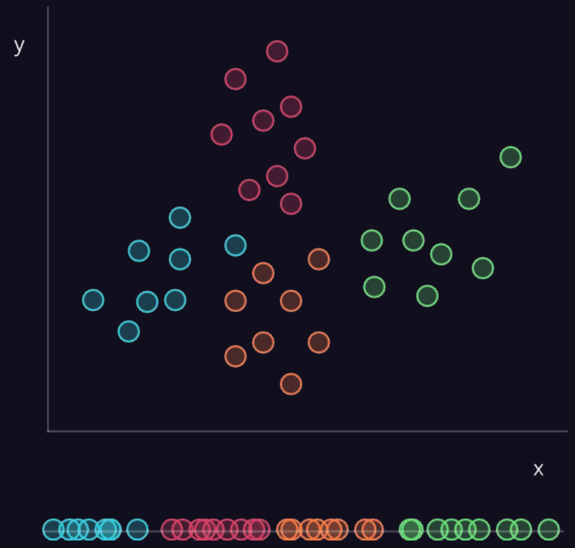

t-SNE

Steps

1) Use **Normal Distribution** to Estimate **Similarity** b/ Data Points
2) Create another Distribution (**t-distribution**) Capturing the **Same Similarity** Between Data Points using **Gradient Descent** on **KL-divergence**

05|

SUPERVISED LEARNING

Keep Calm & Label them All

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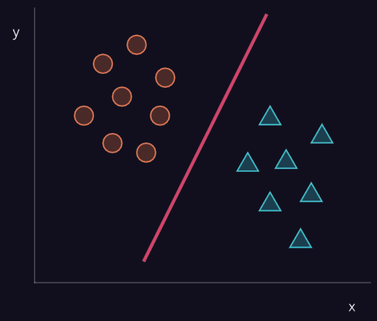
|05 SUPERVISED LEARNING

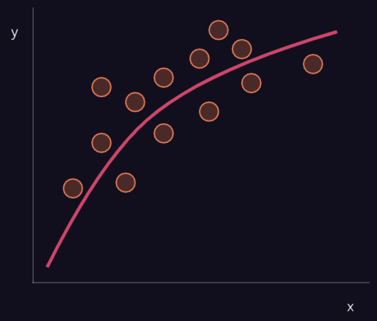
LEARN FROM LABELED DATA

$$Y = f(X)$$

Classification



Regression



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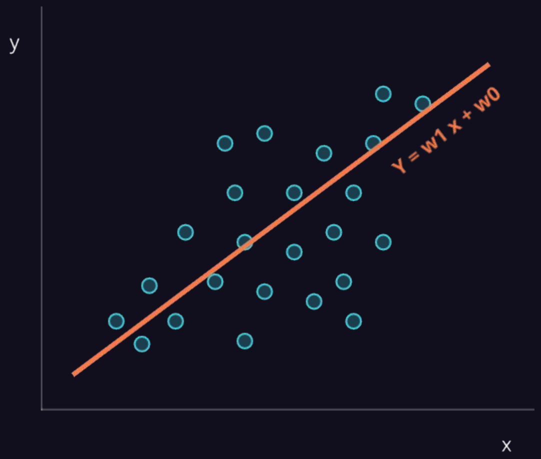
|05 SUPERVISED LEARNING



LINEAR REGRESSION

$$Y = f(W, X) = XW^t + \epsilon$$

$$\mathcal{L} = MSE(\hat{Y}, Y) = \frac{1}{N}\sum_i (w_i x_i - y_i)^2$$

$$\frac{\partial \mathcal{L}}{\partial W} = \frac{2}{N}(X^t X W - X^t Y) = 0$$

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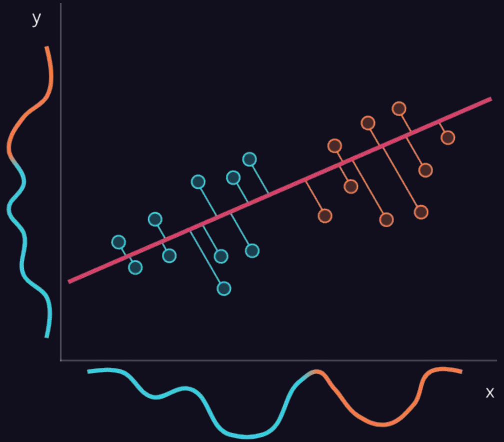

LINEAR DISCRIMINANT ANALYSIS

$$Y = f(W, X) = XW^t + \epsilon$$

$$Objective(W) = \frac{|\tilde{\mu_1} - \tilde{\mu_2}|}{\tilde{s_1^2} + \tilde{s_2^2}}$$

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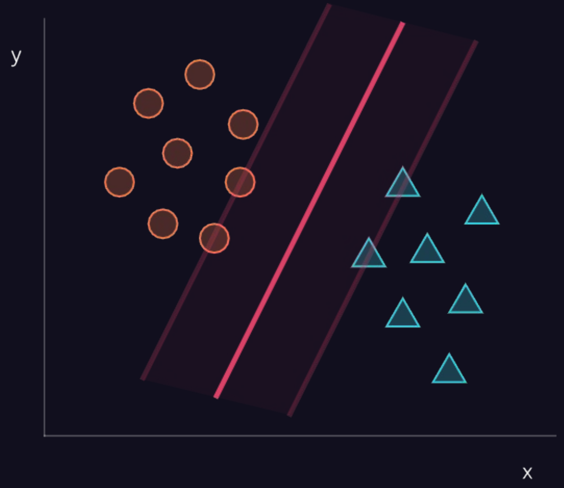


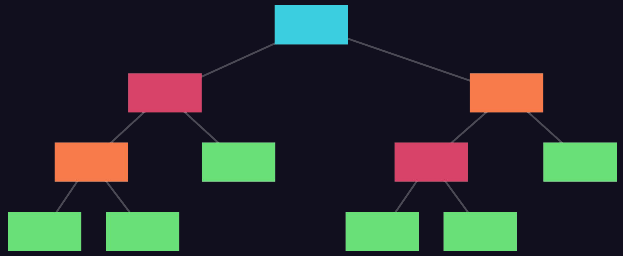
SUPPORT VECTOR MACHINE

$$Y_i(WX + b) - 1 \geq 0$$

$$margin = (X_+ - X_-)\frac{W}{\|W\|}$$

$$J(W) = \frac{1}{2}\|W\|^2 + C(\frac{1}{N}\sum_i max(0, 1 - y_i(wx_i + b)))$$

DECISION TREE



Steps

1) Compute all **Gini Impurity** Scores
2) **Lowest** Impurity Score **Becomes a Leaf**
3) Else **if Improvement** pick the Separation with the **Lowest Score**

$$Gini(K) = \sum_i P_{i,K}(1 - P_{i,K}) = 1 - \sum_i P_{i,K}^2$$

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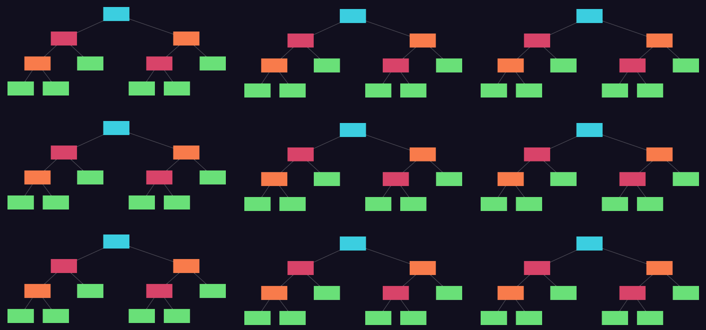
RANDOM FOREST

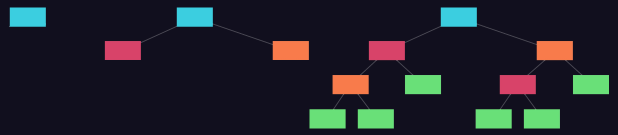


Intuition

Forest of **Dense Decision Trees**
Like **Cross Validation**
Ensemble Methods give Better Results
Use Voting, Max or Average

|05 SUPERVISED LEARNING

GRADIENT BOOSTING



Steps

1) Start from a Single Leaf of the **Average Prediction**
2) Build a **New Tree** to Predict the **Residual Error** from the Previous Tree and **Weight** its Contribution by a **Learning Rate**
3) Do **Until Convergence** Condition