



DE VINCI  
INNOVATION  
CENTER

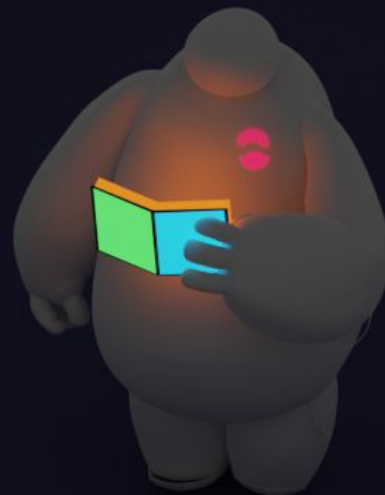
# MACHINE LEARNING

Learning from Data

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**LÉONARD  
DE VINCI**  
PARIS-LA DÉFENSE



ÉCOLE  
D'INGÉNIEURS  
PARIS-LA DÉFENSE



INSTITUT DE  
L'INTERNET  
ET DU MULTIMÉDIA  
PARIS-LA DÉFENSE



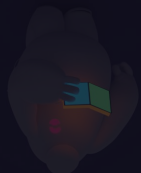
ÉCOLE DE  
MANAGEMENT  
PARIS-LA DÉFENSE



## Pattern Recognition

Study **Correlations** Between **Data Domains**

**Tune a Parametric Model** with **Data**



# |00 INTRODUCTION



## SUPERVISED

Learn parameters from  
**Labeled Data**

Regression

Classification

## UNSUPERVISED

Learn parameters from  
**Unlabeled Data**

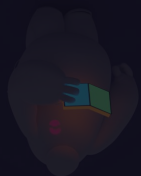
Clustering

Latent Space

## REINFORCEMENT

**Autonomous Agent** Learning  
from **Experience**

Q-Learning



# 01|

## LINEAR ALGEBRA

The Last Space Bender

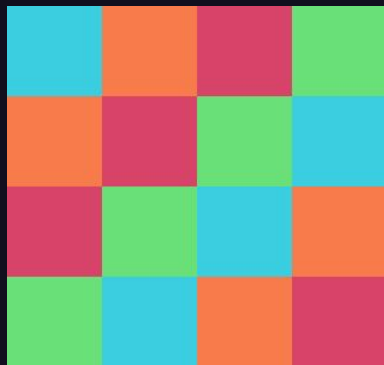


# |01 LINEAR ALGEBRA

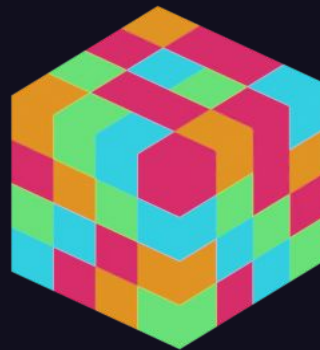
**VECTOR**



**MATRIX**



**TENSOR**



# |01 LINEAR ALGEBRA

VECTOR

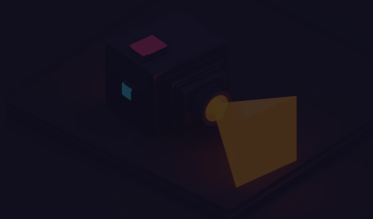


$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

LENGTH

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

# |01 LINEAR ALGEBRA



## VECTOR

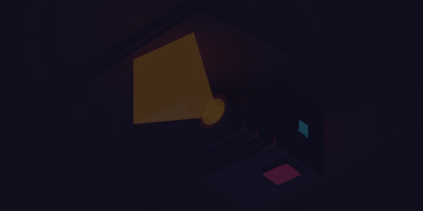


$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

## ADDITION / SUBTRACTION

$$\vec{v} + \vec{u} = \begin{pmatrix} v_x + u_x \\ v_y + u_y \end{pmatrix}$$



# |01 LINEAR ALGEBRA

## VECTOR



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

## SCALAR (DOT) PRODUCT

$$\vec{v} \cdot \vec{u} = v_x u_x + v_y u_y = |v| |u| \cos \theta$$



# |01 LINEAR ALGEBRA

## VECTOR



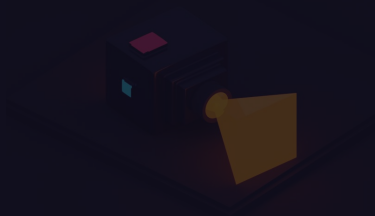
$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

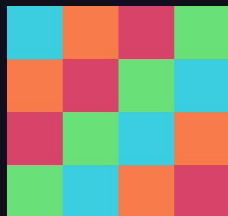
## CROSS PRODUCT

$$\vec{v} \times \vec{u} = \begin{pmatrix} v_y u_z - v_z u_y \\ v_z u_x - v_x u_z \\ v_x u_y - v_y u_x \end{pmatrix}$$

# |01 LINEAR ALGEBRA



## MATRIX

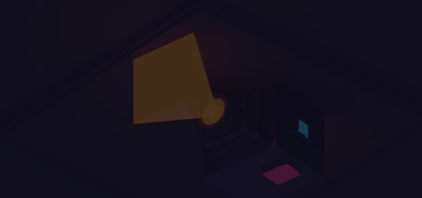


## ADDITION / SUBTRACTION

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

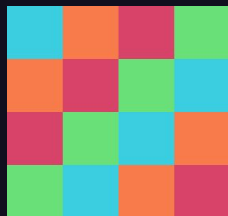
$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A + B = \begin{pmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{pmatrix}$$



# |01 LINEAR ALGEBRA

## MATRIX



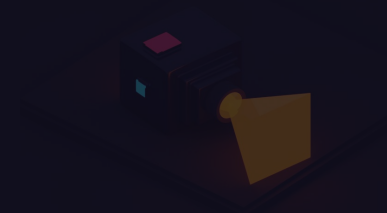
## HADAMARD PRODUCT

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

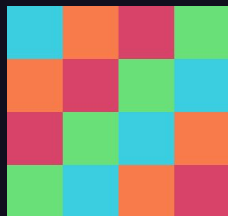
$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$A \odot B = \begin{pmatrix} a_{1,1}b_{1,1} & a_{1,2}b_{1,2} \\ a_{2,1}b_{2,1} & a_{2,2}b_{2,2} \end{pmatrix}$$

# |01 LINEAR ALGEBRA



## MATRIX

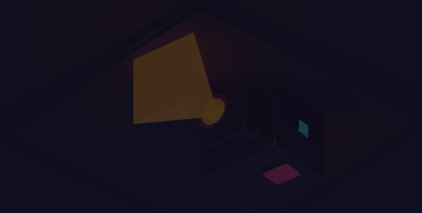


$$A = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

## MATRIX MULTIPLICATION

$$A \cdot B = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$



# |01 LINEAR ALGEBRA

**MATRIX**

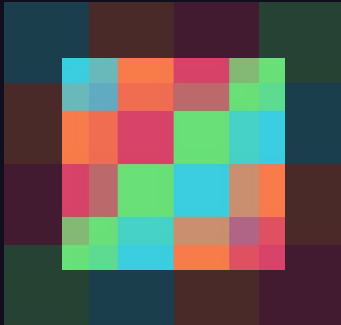


**EYE**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# |01 LINEAR ALGEBRA

## MATRIX



## SCALE

$$\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# |01 LINEAR ALGEBRA

## MATRIX



## ROTATION

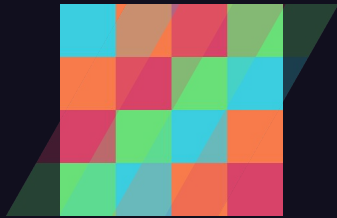
$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# |01 LINEAR ALGEBRA

**MATRIX**



**SHEAR**

$$\begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



# |01 LINEAR ALGEBRA

## MATRIX

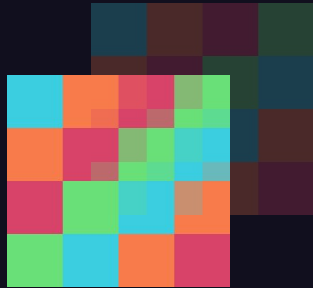


## REFLECTION

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# |01 LINEAR ALGEBRA

## MATRIX



## TRANSLATION

$$\begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

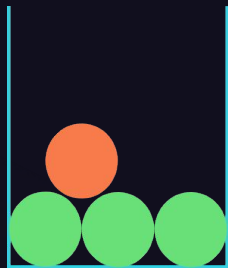
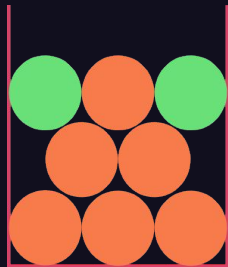
# |02

## PROBABILITIES

Roll the Dice



## |02 PROBABILITIES



### RANDOM VARIABLES

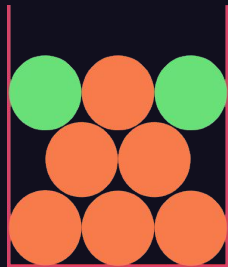
$$B = \begin{cases} r & \text{if Box is red} \\ b & \text{if Box is blue} \end{cases}$$

$$F = \begin{cases} a & \text{if Fruit is an apple} \\ o & \text{if Fruit is an orange} \end{cases}$$

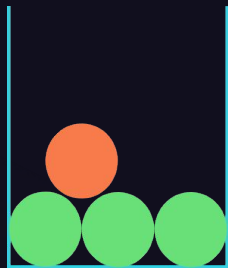
## |02 PROBABILITIES



### PROBABILITIES



$$p(B = r) = \frac{\# \text{ red boxes picked}}{\# \text{ total boxes picked}} \in [0; 1]$$



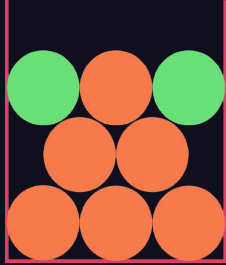
$$p(B = r) + p(B = b) = 1$$



# |02 PROBABILITIES



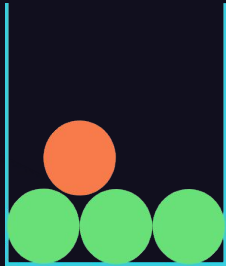
## JOINT & CONDITIONAL



### Sum Rule

Joint probabilities  $p(B = r, F = a)$

Marginal probability  $p(B = r) = p(B = r, F = a) + p(B = r, F = o)$



### Product Rule

Conditional probabilities  $p(B = r|F = o)$

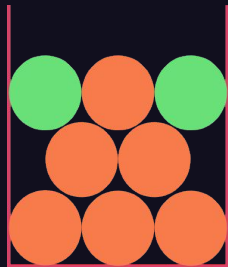
$p(B = r, F = o) = p(F = o|B = r) p(B = r)$



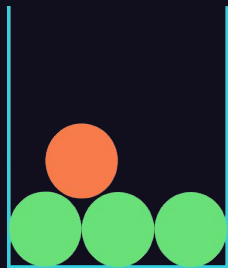
## |02 PROBABILITIES



### BAYES THEOREM



$$p(B = r | F = o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o)}$$



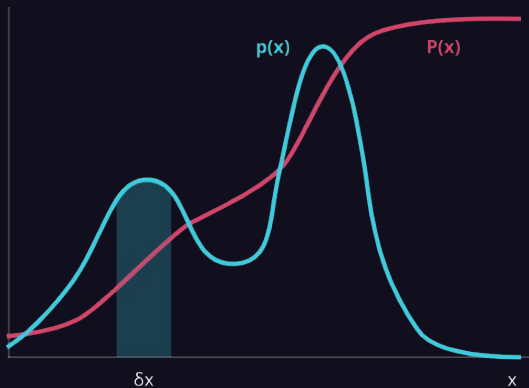
$$p(B = r | F = o) = \frac{p(F=o|B=r)p(B=r)}{p(F=o|B=r)p(B=r) + p(F=o|B=b)p(B=b)}$$



## |02 PROBABILITIES



### DENSITY



$$p(x \in (a, b)) = \int_a^b p(x) dx$$

$$\int_{-\infty}^{+\infty} p(x) dx = 1$$

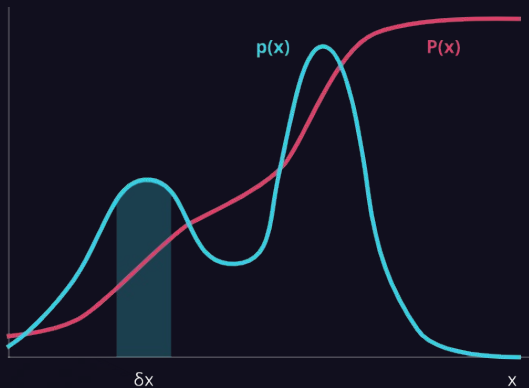
$$p(x) \geq 0$$







## EXPECTATION & COVARIANCE



$$\mathbb{E}[f] = \int p(x) f(x) dx$$

$$\text{var}[f] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$

$$\text{var}[f] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

$$\text{cov}[x, y] = \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y]$$





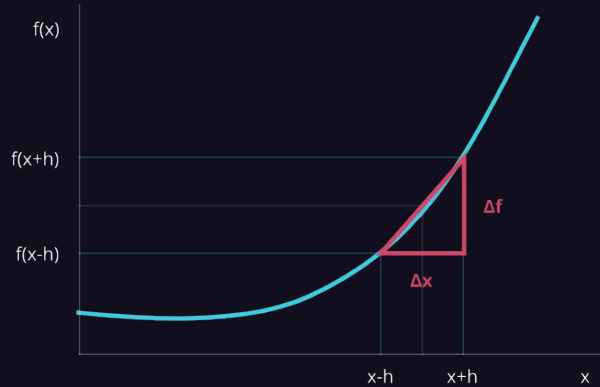
# 03|

## OPTIMIZATION

One Method to Rule them All



## DERIVATIVES



$$f'(x) = \lim_{h \rightarrow 0} \frac{\Delta f}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

### First order Derivative

Direction of the Slope

### Second order Derivative

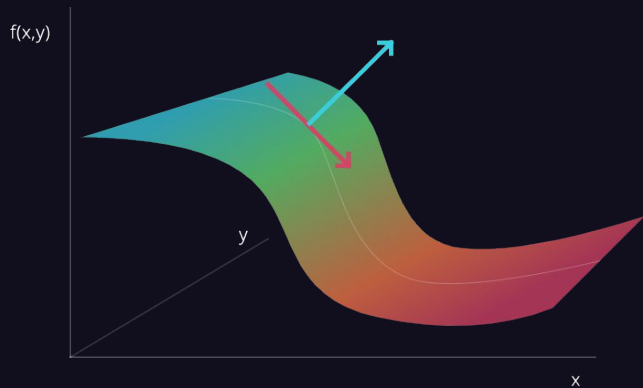
Rate of Changes in the Slope



# |03 OPTIMIZATION



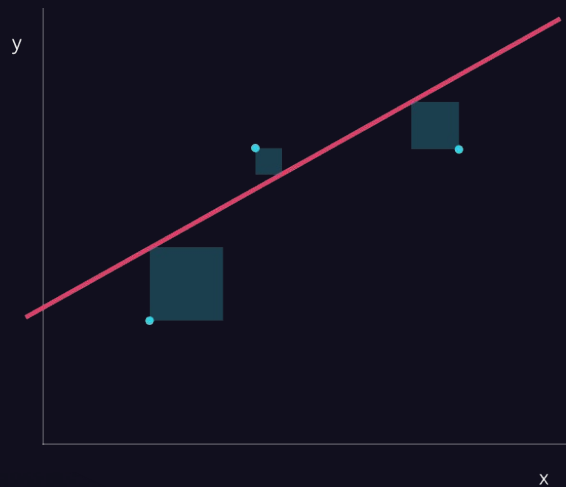
## GRADIENTS



$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$



# |03 OPTIMIZATION



## LEAST SQUARES

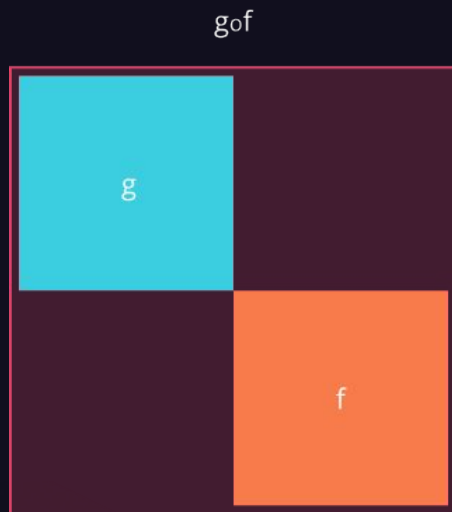
### Objective Function

Minimize Squared Distances  
from Expected Value

$$\hat{y} = ax + b$$

$$\sum_i (y_i - \hat{y}_i)^2 = 0$$





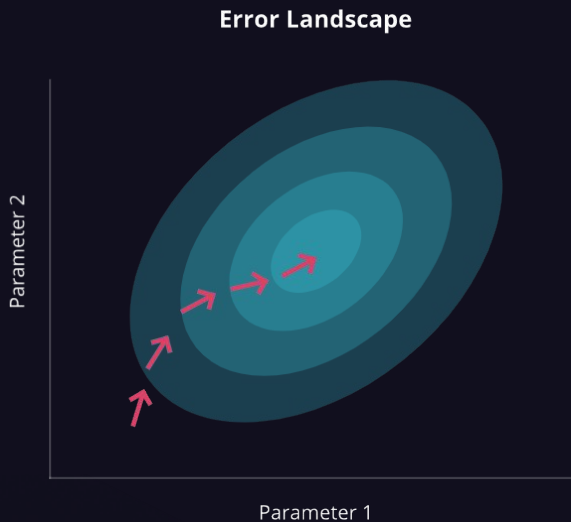
## CHAIN RULE

$$h = g \circ f$$

$$h'(x) = g'(f(x))f'(x)$$



# |03 OPTIMIZATION



## GRADIENT DESCENT

### Steps

- 1) **Forward Propagate** Through the Chain
- 2) Compute Output **Error**
- 3) **Backpropagate** Error Through the Chain
- 4) **Update** Weights w/ Learning Rate
- 5) **Repeat** Until Convergence Threshold



# |03 OPTIMIZATION



## DATASET SPLIT

### Training Set

Samples used to Fit/Train the Model

### Validation Set

Samples used to provide an Unbiased Evaluation of the Model  
Becomes Biased during Training

### Testing Set

Samples used to provide an Unbiased Evaluation of the Model after Training



## |03 OPTIMIZATION



### CROSS-VALIDATION

Iteration 1	Valid	Train	Train	Train
Iteration 2	Train	Valid	Train	Train
Iteration 3	Train	Train	Valid	Train
Iteration 4	Train	Train	Train	Valid
	Fold 1	Fold 2	Fold 3	Fold 4

#### Method

- 1) Split Dataset into **k-Folds**
- 2) **Train on k-1 Folds** and Validate w/ Last
- 3) **Repeat** k-times
- 4) Use **Ensemble** Method for **Inference** or **Retrain** on all Dataset



# |04

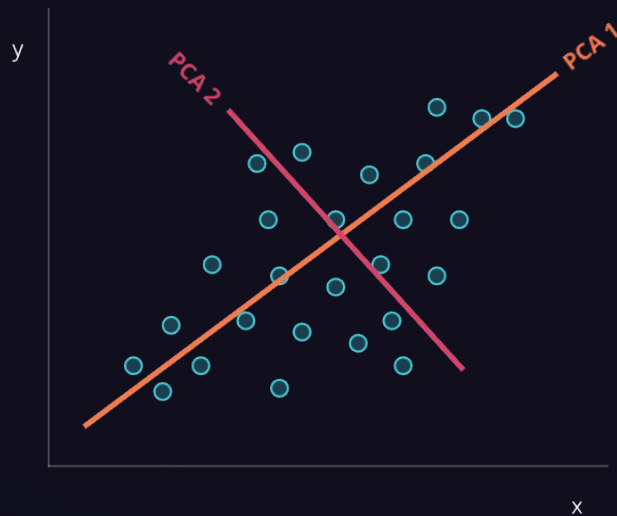
## **DIMENSIONALITY REDUCTION**

Small Worlds are Filled with  
Things to See

Paul Safranek



# |04 DIMENSIONALITY REDUCTION

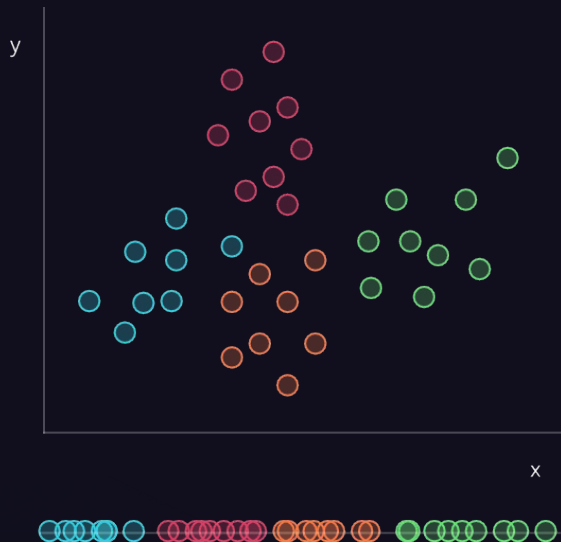


## PCA

$$C = \begin{pmatrix} Var(x) & Cov(x, y) & Cov(x, z) \\ Cov(y, x) & Var(y) & Cov(y, z) \\ Cov(z, x) & Cov(z, y) & Var(z) \end{pmatrix}$$

$v_1$	$\lambda_1$	↑ Big  Small
$v_2$	$\lambda_2$	
$v_3$	$\lambda_3$	

# |04 DIMENSIONALITY REDUCTION



## t-SNE

### Steps

- 1) Use **Normal Distribution** to Estimate **Similarity** b/ Data Points
- 2) Create another Distribution (**t-distribution**) Capturing the **Same Similarity** Between Data Points using **Gradient Descent** on **KL-divergence**