

# Optimisation

Kate Farrahi

ECS Southampton

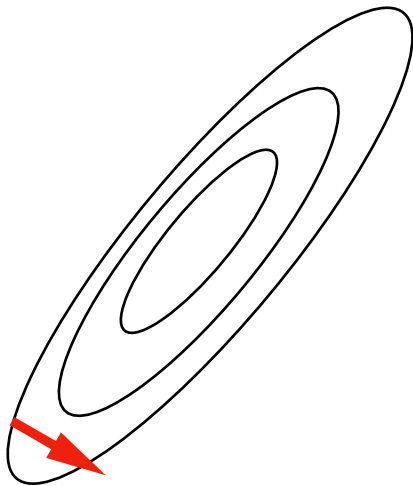
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<sup>1</sup>Some of the material in this lecture is based on Andrew Ng's lectures on Optimisation

## Why learning can be slow



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- ▶ If the ellipse is very elongated, the direction of steepest descent is almost perpendicular to the direction towards the minimum
- ▶ The gradient vector will have a large component along the short axis of the ellipse and a small component along the long axis of the ellipse.
- ▶ This is the opposite of what we want to optimise efficiently

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$$v_{100} = 0.1\theta_{100} + 0.9 * 0.1 * \theta_{99} + 0.1 * (0.9)^2 * \theta_{98} + 0.1(0.9)^3\theta_{97} + \dots$$



# Momentum

- ▶ The momentum method allows to accumulate velocity in directions of low curvature that persist across multiple iterations
- ▶ This leads to accelerated progress in low curvature directions compared to gradient descent

# Gradient Descent (GD) with Momentum

Learning with momentum is given by

On iteration  $t$ :

Compute  $dW_t$  on the current mini-batch

$$V_t = \beta V_{t-1} + (1 - \beta) dW_t \quad (1)$$

$$w_t = w_{t-1} - \eta V_t \quad (2)$$

Note that  $dW_t$  represents the gradient of the cost function (as computed in standard GD).  $\eta$  is the learning rate and  $\beta = 0.9$  is a good choice for the exponentially weighted average parameter.

# RMSProp

Learning with RMSProp is given by

On iteration  $t$ :

Compute  $dW$  on current mini-batch

$$S_{dW_t} = \beta S_{dW_{t-1}} + (1 - \beta) dW_t^2 \quad (3)$$

$$w_t = w_{t-1} - \eta \frac{dW_t}{\sqrt{S_{dW_t}}} \quad (4)$$

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- ▶ ...
- ▶  $t = 10$ :  $\frac{v_{10}}{1-(0.9)^{10}} = 1.535 * v_{10}$
- ▶ ...
- ▶  $t = 20$ :  $\frac{v_{20}}{1-(0.9)^{20}} = 1.138 * v_{20}$

# Adam

Initialize parameters:  $V_{dW} = 0, S_{dW} = 0$

On iteration  $t$ :

Compute  $dW_t$  on current mini-batch

$$V_{dW} = \beta_1 V_{dW} + (1 - \beta_1) dW, \quad V_{dW}^{corr} = \frac{V_{dW}}{(1 - \beta_1^t)} \quad (5)$$

$$S_{dW} = \beta_2 S_{dW} + (1 - \beta_2) dW^2, \quad S_{dW}^{corr} = \frac{S_{dW}}{(1 - \beta_2^t)} \quad (6)$$

$$w := w - \eta \frac{V_{dW}^{corr}}{\sqrt{(S_{dW}^{corr} + \epsilon)}} \quad (7)$$