Recurrent Neural Networks

Kate Farrahi

ECS Southampton

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Batch Normalisation

- Batch normalization (BN) is a technique for improving the performance of ANNs
- It improves the stability of ANNs by adjusting and scaling the activations
- BN makes your ANN more robust to the choice of hyperparameters (larger range of parameters that will work well)
- It was introduced by Sergey Ioffe and Christian Szegedy in 2015 [1]

[1] https://arxiv.org/pdf/1502.03167.pdf

Batch Normalisation

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

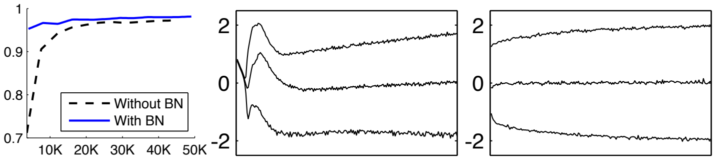
$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m} (x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Algorithm 1: Batch Normalizing Transform, applied to activation x over a mini-batch.

Batch Normalisation



(a) (b) Without BN (c) With BN

Figure 1: (a) *The test accuracy of the MNIST network trained with and without Batch Normalization, vs. the number of training steps. Batch Normalization helps the network train faster and achieve higher accuracy.* (b, c) *The evolution of input distributions to a typical sigmoid, over the course of training, shown as* $\{15, 50, 85\}$*th percentiles. Batch Normalization makes the distribution more stable and reduces the internal covariate shift.*

Batch Normalisation

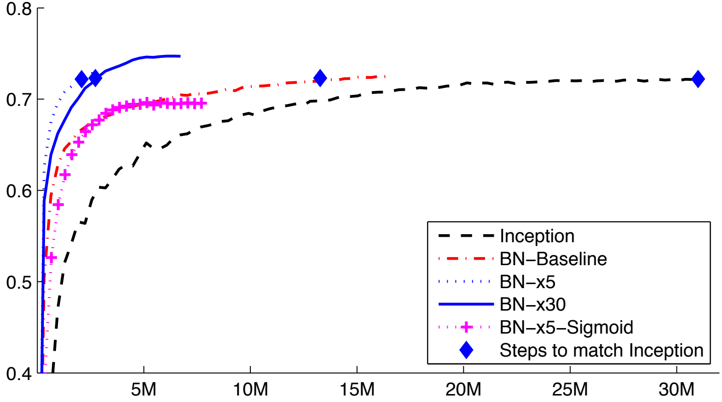


Figure 2: *Single crop validation accuracy of Inception and its batch-normalized variants, vs. the number of training steps.*

Recurrent Neural Networks - Motivation

x:	Kate	Farrahi	and	Jonathon	Hare	teach	deep	learning
y:	1	1	0	1	1	0	0	0

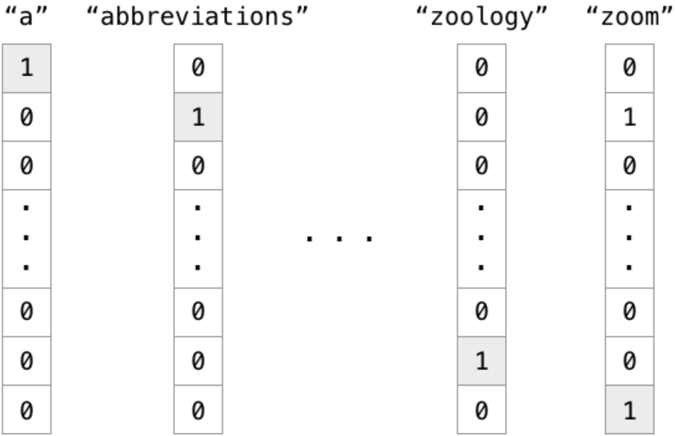
Recurrent Neural Networks - Motivation

x:	$x^{<1>}$	$x^{<2>}$	...	$x^{<t>}$	...	$x^{<T_x>}$
x:	Kate	Farrahi	...	Hare	...	learning
y:	$y^{<1>}$	$y^{<2>}$	...	$y^{<t>}$	...	$y^{<T_y>}$
y:	1	1	...	1	...	0

In this example, $T_x = T_y = 8$ but T_x and T_y can be different.

One Hot Encoding

How can we represent individual words?



[2]

2 https://ayearofai.com

Why Not a Standard Feed Forward Network?

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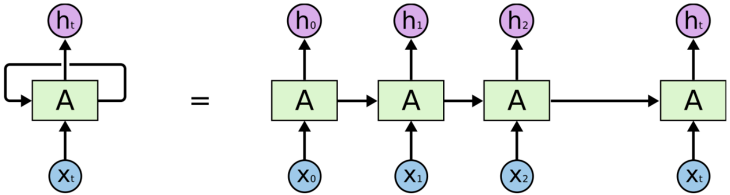
- The inputs and outputs may have varying lengths
- The features wouldn't be shared across different positions in the network

Recurrent Neural Networks

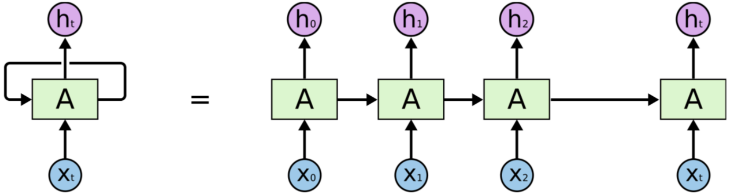


[3]

- RNNs are a family of ANNs for processing sequential data

[3] Image taken from `https://towardsdatascience.com`

Recurrent Neural Networks



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- RNNs have directed cycles in their computational graphs

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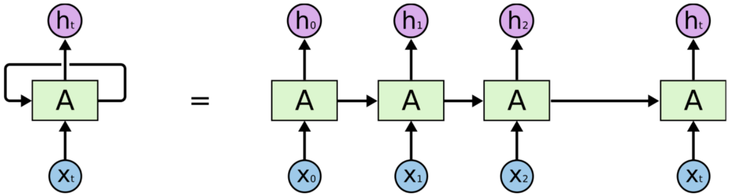
Recurrent Neural Networks



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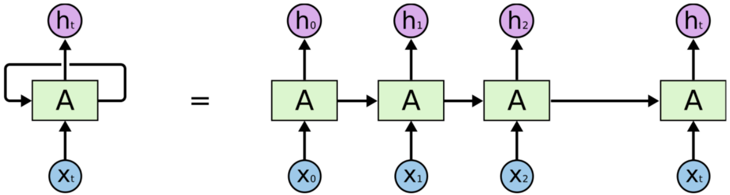
Recurrent Neural Networks



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- RNNs are a family of ANNs for processing sequential data
- RNNs have directed cycles in their computational graphs
- They can have complicated dynamics, difficult to train
- They are more biologically realistic

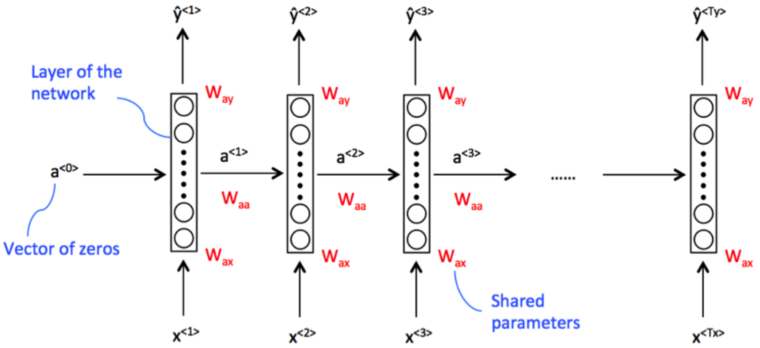
[3]Image taken from `https://towardsdatascience.com`

Recurrent Neural Networks

RNNs combine two properties which make them very powerful.

1. Distributed hidden state that allows them to store a lot of information about the past efficiently. This is because several different units can be active at once, allowing them to remember several things at once.
2. Non-linear dynamics that allows them to update their hidden state in complicated ways. They can however have complicated dynamics, making them difficult to train

Backpropagation Through Time (BPTT)



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[4] Image taken from Andrew Ng

BPTT - Forward Pass

$$a^{<t>} = g(w_{aa}a^{<t-1>} + w_{ax}x^{<t>} + b_a) \quad (1)$$

BPTT - Forward Pass

$$a^{<t>} = g(w_{aa}a^{<t-1>} + w_{ax}x^{<t>} + b_a) \quad (1)$$

$$\hat{y}^{<t>} = g(w_{ya}a^{<t>} + b_y) \quad (2)$$

BPTT - Forward Pass

$$
\begin{aligned}
a^{<t>} &= g(w_{aa}a^{<t-1>} + w_{ax}x^{<t>} + b_a) &(1)\\
\hat{y}^{<t>} &= g(w_{ya}a^{<t>} + b_y) &(2)\\
\mathcal{L}^{<t>} &= -y^{<t>}log(\hat{y}^{<t>}) - (1 - y^{<t>})log(1 - \hat{y}^{<t>}) &(3)
\end{aligned}
$$

BPTT - Forward Pass

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\mathcal{L}^{<t>} &= -y^{<t>}log(\hat{y}^{<t>}) - (1 - y^{<t>})log(1 - \hat{y}^{<t>}) && (3)\\
\mathcal{L} &= \sum_{t=1}^{T_y} \mathcal{L}^{<t>} && (4)
\end{aligned}$$

BPTT - Backwards Pass

$$\frac{\partial \mathcal{L}^{<3>}}{\partial w_{ya}} = \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial w_{ya}} \tag{5}$$

BPTT - Backwards Pass

$$
\begin{aligned}
\frac{\partial \mathcal{L}^{<3>}}{\partial w_{ya}} &= \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial w_{ya}} && (5) \\
\frac{\partial \mathcal{L}^{<3>}}{\partial w_{aa}} &= \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial a^{<3>}} \frac{\partial \hat{a}^{<3>}}{\partial w_{aa}} && (6) \\
& && (7)
\end{aligned}
$$

BPTT - Backwards Pass

$$\frac{\partial \mathcal{L}^{<3>}}{\partial w_{ya}} = \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial w_{ya}} \tag{5}$$

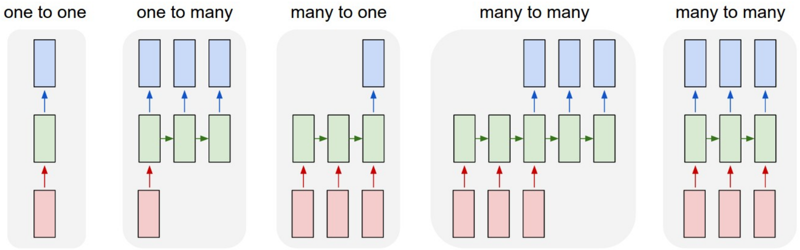
$$\frac{\partial \mathcal{L}^{<3>}}{\partial w_{aa}} = \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial a^{<3>}} \frac{\partial \hat{a}^{<3>}}{\partial w_{aa}} \tag{6}$$

$$\tag{7}$$

$$\text{Recall } a^{<3>} = g(w_{aa}a^{<2>} + w_{ax}x^{<3>} + b_a)$$

$$\frac{\partial \mathcal{L}^{<3>}}{\partial w_{aa}} = \frac{\partial \mathcal{L}^{<3>}}{\partial \hat{y}^{<3>}} \frac{\partial \hat{y}^{<3>}}{\partial a^{<3>}} \frac{\partial a^{<3>}}{\partial a^{<2>}} \frac{\partial a^{<2>}}{\partial a^{<1>}} \frac{\partial a^{<1>}}{\partial w_{aa}} \tag{8}$$

Recurrent Neural Networks



[5]

[5] http://karpathy.github.io/2015/05/21/rnn-effectiveness/