Make a forward pass before the backward pass

Backpropagation: Understanding the implications of the chain rule

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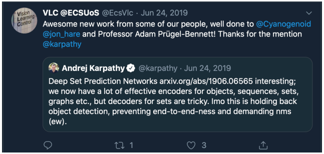
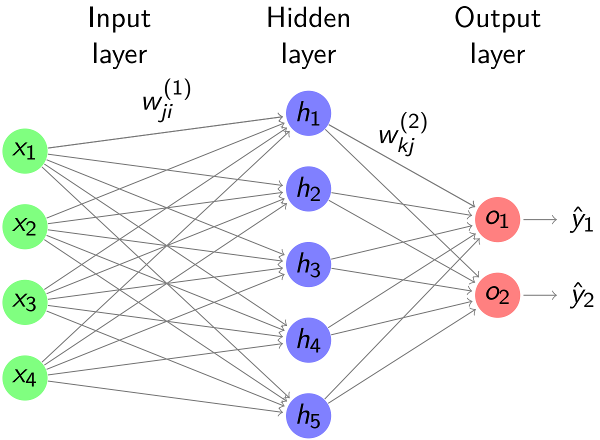
A lot of the ideas in this lecture come from Andrej Karpathy's blog post on backprop (`https://medium.com/@karpathy/yes-you-should-understand-backprop-e2f06eab496b`) and his CS231n Lecture Notes (`http://cs231n.github.io/optimization-2/`)



Topics

- A quick look at an MLP again
- The chain rule (again)
- A closer look at basic stochastic gradient descent algorithms

The unbiased Multilayer Perceptron (again)...



Without loss of generality, we can write the above as:

$$\hat{\boldsymbol{y}} = g(f(\boldsymbol{x}; \boldsymbol{W}^{(1)}); \boldsymbol{W}^{(2)}) = g(\boldsymbol{W}^{(2)} f(\boldsymbol{W}^{(1)} \boldsymbol{x}))$$

where f and g are activation functions.

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- (But we're not that crazy!)

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so $\nabla_{[x,y,z]} f = [z, z, q]$

A computational graph