Going Deep

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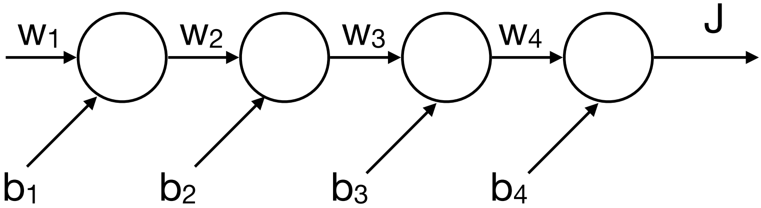
$F(x) = \sum_{i=1}^{N} v_i \phi(w_i^T x + b_i)$ as an approximate realization of the function f ; that is,

$|F(x) - f(x)| < \epsilon$
for all $x \in I_m$.

Then Why Go Deep?

- There are functions you can compute with a deep neural network that shallow networks require exponentially more hidden units to compute.
- The following function is more efficient to implement using a deep neural network: $y = x_1 \oplus x_2 \oplus x_3 \oplus \cdots \oplus x_n$

Issues with Going Deep



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- This issue affects many-layered networks (feed-forward), as well as recurrent networks.

Residual Connections

- One of the most effective ways to resolve the vanishing gradient problem is with residual neural networks (ResNets)[1].

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- ResNets are artificial neural networks that use *skip connections* to jump over layers.
- The vanishing gradient problem is mitigated in ResNets by reusing activations from a previous layer.

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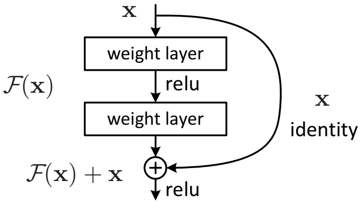

Figure 2. Residual learning: a building block.[2]

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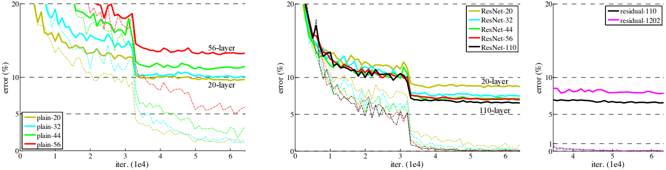

Figure 6. Training on **CIFAR-10**. Dashed lines denote training error, and bold lines denote testing error. **Left**: plain networks. The error of plain-110 is higher than 60% and not displayed. **Middle**: ResNets. **Right**: ResNets with 110 and 1202 layers. [3]

[3] K. He, X. Zhang, S. Ren and J. Sun, "Deep Residual Learning for Image Recognition," CVPR, Las Vegas, NV, 2016, pp. 770-778.

Regularization in Deep Networks

- $L1$ and $L2$ norm
- Dropout

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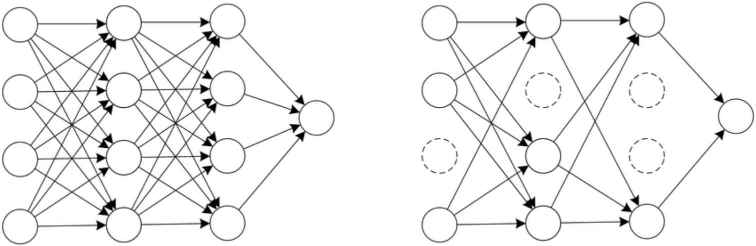
- Dropout is a form of regularization
- The key idea in dropout is to randomly drop neurons, including all of the connections, from the neural network during training.

Dropout



(a) Standard Neural Network (b) Network after Dropout [4]

[4] Image from: https://www.researchgate.net/figure/Dropout-neural-network-model-a-is-a-standard-neural-network-b-is-the-s fig3_309206911

Why Does Dropout Work?

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- By breaking co-adaptation, each unit will ultimately find more general features