

In the GG02 protocol, Alice prepares an ensemble of coherent states  $|\alpha = q + ip\rangle$  with probabilities  $p_A(\alpha)$  distributed according to Gaussian law,

$$p_A(\alpha) = \frac{1}{2\pi V_A} \exp \left[ -\frac{|\alpha|^2}{2V_A} \right] \quad (1)$$

i.e.

$$\rho_A = \int d^2\alpha p_A(\alpha) |\alpha\rangle\langle\alpha| \quad (2)$$

! As the phase-rotated quadrature operator is defined as (see preprint):

$$\hat{x}_\varphi = \hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}, \quad (3)$$

i.e.

$$\hat{q} = \hat{a} + \hat{a}^\dagger, \quad \hat{p} = -i [\hat{a} - \hat{a}^\dagger], \quad (4)$$

the first two moments of each component of  $\alpha$  read

$$\langle \hat{q} \rangle = \text{Tr} \hat{q} \rho_A = \int d^2\alpha p_A(\alpha) \underbrace{\langle \alpha | \hat{q} | \alpha \rangle}_{2 \text{Re } \alpha = 2q} = 0, \quad (5a)$$

$$\langle \hat{q}^2 \rangle = \text{Tr} \hat{q}^2 \rho_A = \int d^2\alpha p_A(\alpha) \underbrace{\langle \alpha | \hat{q}^2 | \alpha \rangle}_{4q^2+1} = 4 \underbrace{\int d^2\alpha p_A(\alpha) q^2}_{V_A} + 1 = 4V_A + 1, \quad (5b)$$

$$\langle \hat{p} \rangle = 0, \quad (5c)$$

$$\langle \hat{p}^2 \rangle = 4V_A + 1. \quad (5d)$$

After transmission through a Gaussian channel, which attenuates the coherent amplitude by a factor of  $\sqrt{T}$ , where  $T$  is the channel transmission, the state transforms as  $|\alpha\rangle \mapsto |\sqrt{T}\alpha\rangle \equiv |\tilde{\alpha} = \tilde{q} + i\tilde{p}\rangle$ . The ensemble reads

$$\tilde{\rho}_A = \frac{1}{T} \int d^2\tilde{\alpha} p_A(\tilde{\alpha}) |\tilde{\alpha}\rangle\langle\tilde{\alpha}|. \quad (6)$$

Then, Bob performs a measurement described by POVM  $\{\hat{\Pi}_x\}$ , where the index  $x$  parametrizes the measurement outcomes (quadrature values  $q$  and  $p$  in homodyne detection). The conditional probability that Bob obtains measurement outcome  $x$  given that Alice sent the specific coherent state  $|\tilde{\alpha}\rangle$  is given by the Born rule:

$$p_B(x|\tilde{\alpha}) = p_A(\tilde{\alpha}) \text{Tr} \left[ |\tilde{\alpha}\rangle\langle\tilde{\alpha}| \hat{\Pi}_x \right] = p_A(\tilde{\alpha}) Q_x(\tilde{\alpha}), \quad (7)$$

where  $Q_x(\tilde{\alpha})$  is the  $Q$ -function of POVM used.

# 1 Homodyne

If homodyne detection is used

$$Q_x(\tilde{\alpha}) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left[-\frac{(x - \overbrace{\langle \hat{x}_\varphi \rangle}^{2\tilde{q}})^2}{2\sigma_x}\right], \quad (8)$$

$$p_B(x = q|\tilde{\alpha}) = p_A(\tilde{\alpha})Q_x(\tilde{\alpha}) \sim \exp\left[-\frac{(x - 2\tilde{q})^2}{2\sigma_x} - \frac{\tilde{q}^2}{2TV_A}\right]. \quad (9)$$

Rewriting exponential's power in Eq.(9) in quadratic form results in

$$\begin{aligned} -\frac{1}{2} \left[ \frac{(x - 2\tilde{q})^2}{\sigma_x} + \frac{\tilde{q}^2}{TV_A} \right] &= -\frac{1}{2} \left[ \frac{x^2 - 4x\tilde{q} + 4\tilde{q}^2}{\sigma_x} + \frac{\tilde{q}^2}{TV_A} \right] = \\ &= -\frac{1}{2} \left[ \frac{x^2}{\sigma_x} - x\tilde{q}\frac{4}{\sigma_x} + \tilde{q}^2 \left( \frac{4}{\sigma_x} + \frac{1}{TV_A} \right) \right] = \\ &= -\frac{1}{2} \begin{pmatrix} \tilde{q} & x \end{pmatrix} \begin{pmatrix} \frac{4}{\sigma_x} + \frac{1}{TV_A} & -\frac{2}{\sigma_x} \\ -\frac{2}{\sigma_x} & \frac{1}{\sigma_x} \end{pmatrix} \begin{pmatrix} \tilde{q} \\ x \end{pmatrix} = \\ &= / \det^{-1} = \sigma_x TV_A / = -\frac{1}{2} \begin{pmatrix} \tilde{q} & x \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & 2TV_A \\ 2TV_A & 4TV_A + \sigma_x \end{pmatrix}^{-1}}_{\Sigma^H} \begin{pmatrix} \tilde{q} \\ x \end{pmatrix}, \end{aligned} \quad (10)$$

Mutual information between Alice and Bob can be calculated as follows [1]

$$I_{AB}^H = \frac{1}{2} \log \frac{\Sigma_{11}^H \Sigma_{22}^H}{\det \Sigma^H} = \frac{1}{2} \log \frac{4TV_A + \sigma_x}{\sigma_x}. \quad (11)$$

**This result is fine**, as in Ref. [2] modulation variance is defined per quadrature component (see Eq. (8) in Ref.), corresponding to variable change  $V_A \mapsto \frac{V_A}{4}$  (see Eq. (5), resulting in formula (50) from Ref.

# 2 Double homodyne

If double homodyne detection is used, i.e.

$$Q_x(\tilde{\alpha}) = \frac{1}{2\pi\sqrt{\sigma_G^{(1)}\sigma_G^{(2)}}} \exp\left[-\frac{(x_1 - \tilde{q})^2}{\sigma_1} - \frac{(x_2 - \tilde{p})^2}{\sigma_2}\right], \quad (12)$$

$$p_A(\tilde{\alpha})Q_x(\tilde{\alpha}) \sim \exp\left[-\frac{(x_1 - \tilde{q})^2}{\sigma_1} - \frac{\tilde{q}^2}{2TV_A} - \frac{(x_2 - \tilde{p})^2}{\sigma_2} - \frac{\tilde{p}^2}{2TV_A}\right], \quad (13)$$

analogously,

$$\begin{aligned} -\frac{(x_1 - \tilde{q})^2}{\sigma_1} - \frac{\tilde{q}^2}{2TV_A} &= -\frac{1}{2} \begin{pmatrix} \tilde{q} & x_1 \end{pmatrix} \begin{pmatrix} \frac{2}{\sigma_1} + \frac{1}{TV_A} & -\frac{2}{\sigma_1} \\ -\frac{2}{\sigma_1} & \frac{2}{\sigma_1} \end{pmatrix} \begin{pmatrix} \tilde{q} \\ x_1 \end{pmatrix} = \\ &= \left/ \det^{-1} = \frac{\sigma_1 TV_A}{2} \right/ = -\frac{1}{2} \begin{pmatrix} \tilde{q} & x_1 \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & TV_A \\ TV_A & TV_A + \frac{\sigma_1}{2} \end{pmatrix}^{-1}}_{\Sigma^{\text{DH}(1)}} \begin{pmatrix} \tilde{q} \\ x_1 \end{pmatrix}, \end{aligned} \quad (14)$$

$$-\frac{(x_2 - \tilde{p})^2}{\sigma_1} - \frac{\tilde{q}^2}{TV_A} = -\frac{1}{2} \begin{pmatrix} \tilde{p} & x_2 \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & TV_A \\ TV_A & TV_A + \frac{\sigma_2}{2} \end{pmatrix}^{-1}}_{\Sigma^{\text{DH}(2)}} \begin{pmatrix} \tilde{p} \\ x_2 \end{pmatrix}, \quad (15)$$

$$I_{\text{AB}}^{\text{DH}} = \frac{1}{2} \sum_{i=1,2} \log \frac{\Sigma_{11}^{\text{DH}(i)} \Sigma_{22}^{\text{DH}(i)}}{\det \Sigma^{\text{DH}(i)}} = \frac{1}{2} \sum_i \log \frac{2TV_A + \sigma_i}{\sigma_i}. \quad (16)$$

This result is fine by the same reasoning.

### 3 Channel noise

Channel noise is modeled as independent Gaussian noise of variance  $\xi$ , modifying Eq. (6) as

$$\tilde{\rho}_A = \frac{1}{T} \int d^2\alpha' d^2\tilde{\alpha} \exp \left[ -\frac{|\tilde{\alpha} - \alpha'|^2}{2\xi} \right] p_A(\tilde{\alpha}) |\tilde{\alpha}\rangle \langle \tilde{\alpha}| \quad (17)$$

$$p_B(x|\alpha) \sim \exp \left[ -\frac{(x - 2\tilde{q})^2}{2(\sigma_x + \xi)} - \frac{\tilde{q}^2}{2TV_A} \right], \quad (18)$$

$$\begin{aligned} &-\frac{1}{2} \left[ \frac{(x - 2\tilde{q})^2}{\sigma_x + \xi} + \frac{\tilde{q}^2}{TV_A} \right] = \\ &= -\frac{1}{2} \left[ \frac{x^2}{\sigma_x + \xi} - x\tilde{q} \frac{4}{\sigma_x + \xi} + \tilde{q}^2 \left( \frac{4}{\sigma_x + \xi} + \frac{1}{TV_A} \right) \right] = \\ &\quad -\frac{1}{2} \begin{pmatrix} \tilde{q} & x \end{pmatrix} \begin{pmatrix} \frac{4}{\sigma_x + \xi} + \frac{1}{TV_A} & \frac{2}{\sigma_x + \xi} \\ \frac{2}{\sigma_x + \xi} & \frac{1}{\sigma_x + \xi} \end{pmatrix} \begin{pmatrix} \tilde{q} \\ x \end{pmatrix} = \\ &= \left/ \det^{-1} = TV_A(\sigma_x + \xi) \right/ = -\frac{1}{2} \begin{pmatrix} \tilde{q} & x \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & 2TV_A \\ 2TV_A & 4TV_A + \sigma_x + \xi \end{pmatrix}^{-1}}_{\Sigma^{\text{H}}} \begin{pmatrix} \tilde{q} \\ x \end{pmatrix}, \end{aligned} \quad (19)$$

resulting in

$$I_{AB}^H = \frac{1}{2} \log \frac{\Sigma_{11}^H \Sigma_{22}^H}{\det \Sigma^H} = \frac{1}{2} \log \frac{4TV_A + \sigma_x + \xi}{\sigma_x + \xi} \quad (20)$$

and for double homodyne, we have

$$p_A(\tilde{\alpha})Q_x(\tilde{\alpha}) \sim \exp \left[ -\frac{(x_1 - \tilde{q})^2}{2\left(\frac{\sigma_1}{2} + \xi\right)} - \frac{\tilde{q}^2}{2TV_A} - \frac{(x_2 - \tilde{p})^2}{2\left(\frac{\sigma_2}{2} + \xi\right)} - \frac{\tilde{p}^2}{2TV_A} \right], \quad (21)$$

$$\begin{aligned} -\frac{(x_1 - \tilde{q})^2}{2\left(\frac{\sigma_1}{2} + \xi\right)} - \frac{\tilde{q}^2}{2TV_A} &= -\frac{1}{2} \begin{pmatrix} \tilde{q} & x_1 \end{pmatrix} \begin{pmatrix} \frac{2}{\sigma_1+2\xi} + \frac{1}{TV_A} & -\frac{2}{\sigma_1+2\xi} \\ -\frac{2}{\sigma_1+2\xi} & \frac{2}{\sigma_1+2\xi} \end{pmatrix} \begin{pmatrix} \tilde{q} \\ x_1 \end{pmatrix} = \\ &= \left/ \det^{-1} = \frac{TV_A(\sigma_1 + 2\xi)}{2} \right/ = -\frac{1}{2} \begin{pmatrix} \tilde{q} & x_1 \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & TV_A \\ TV_A & TV_A + \frac{\sigma_1}{2} + \xi \end{pmatrix}^{-1}}_{\Sigma^{\text{DH}(1)}} \begin{pmatrix} \tilde{q} \\ x_1 \end{pmatrix}, \end{aligned} \quad (22)$$

$$-\frac{(x_2 - \tilde{p})^2}{\sigma_1} - \frac{\tilde{p}^2}{TV_A} = -\frac{1}{2} \begin{pmatrix} \tilde{p} & x_2 \end{pmatrix} \underbrace{\begin{pmatrix} TV_A & TV_A \\ TV_A & TV_A + \frac{\sigma_2}{2} + \xi \end{pmatrix}^{-1}}_{\Sigma^{\text{DH}(2)}} \begin{pmatrix} \tilde{p} \\ x_2 \end{pmatrix}, \quad (23)$$

$$I_{AB}^{\text{DH}} = \frac{1}{2} \sum_i \log \frac{2TV_A + \sigma_i + 2\xi}{\sigma_i + 2\xi}. \quad (24)$$

## 4 Equivalency between EB and PM

The goal is to show that the prepared ensemble sent through channel is the same as Eq. (2) after some variable change.

To calculate Holevo information, first, consider two mode squeezed vacuum state (TMSVS), in ket notation written as [3]

$$|\Psi\rangle_{AB} = \sqrt{1 - \lambda^2} \sum_{n=0}^{\infty} (-\lambda)^n |n, n\rangle_{AB}, \quad (25)$$

where  $\lambda = \tanh r$ ,  $r$  is the squeezing parameter. Let Alice hold the state  $\varsigma_{AB} = |\Psi\rangle_{AB} \langle\Psi|_{AB}$ . In the entanglement-based (EB) protocol, Alice measures one mode using a double measurement, which corresponds to a POVM of coherent state projectors  $\left\{ \hat{\Pi}_\beta = \frac{|\beta\rangle\langle\beta|}{\pi} \right\}$ , and sends the second mode to Bob. The probability that Alice observes double homodyne outcome  $\beta$  is given by the Born rule:

$$p_A^{\text{EB}}(\beta) = \text{Tr } \hat{\Pi}_\beta \varsigma_A, \quad (26)$$

where  $\varsigma_A = \text{Tr}_B \varsigma_{AB}$  is the reduced density matrix of Alice's mode. Straightforward calculations

$$\text{Tr}_B \varsigma_{AB} = (1 - \lambda^2) \sum_{n,m} (-\lambda)^{n+m} |n\rangle_A \langle m|_A \underbrace{\text{Tr}_B |n\rangle_B \langle m|_B}_{\delta_{nm}} = (1 - \lambda^2) \sum_n \lambda^{2n} |n\rangle_A \langle n|_A \quad (27)$$

lead to the expression of the reduced matrix as

$$\varsigma_A = (1 - \lambda^2) \sum_n \lambda^{2n} |n\rangle \langle n|. \quad (28)$$

Substituting Eq. (28) into into Eq. (26) yields

$$\begin{aligned} \langle \beta | \varsigma_A | \beta \rangle &= (1 - \lambda)^2 \sum_n \lambda^{2n} \underbrace{|\langle \beta | n \rangle|^2}_{e^{-|\beta|^2} \frac{|\beta|^{2n}}{n!}} = (1 - \lambda)^2 e^{-|\beta|^2} \underbrace{\sum_n \frac{(\lambda |\beta|)^{2n}}{n!}}_{e^{(\lambda |\beta|)^2}} = \\ &= (1 - \lambda)^2 \exp [(\lambda^2 - 1) |\beta|^2], \end{aligned} \quad (29)$$

$$p_A^{\text{EB}}(\beta) = \text{Tr} \hat{\Pi}_\beta \varsigma_A = \frac{(1 - \lambda)^2}{\pi} \exp [(\lambda^2 - 1) |\beta|^2]. \quad (30)$$

After Alice obtains outcome  $\beta$ , second mode before channel transmission reads

$$\begin{aligned} \varsigma_B^\beta &= \frac{\text{Tr}_A [\hat{\Pi}_\beta \otimes \mathbb{I}] \varsigma_{AB}}{p_A^{\text{EB}}(\beta)} = \exp [-(\lambda^2 - 1) |\beta|^2] \sum_{nm} (-\lambda)^{n+m} \underbrace{\langle \beta | n \rangle}_{e^{-\frac{|\beta|^2}{2}} \frac{\beta^* n}{\sqrt{n!}}} \underbrace{\langle m | \beta \rangle}_{e^{-\frac{|\beta|^2}{2}} \frac{\beta^m}{\sqrt{m!}}} |n\rangle_B \langle m|_B = \\ &= \underbrace{\sum_n e^{-\frac{(\lambda |\beta|)^2}{2}} \frac{(-\lambda \beta^*)^n}{\sqrt{n!}} |n\rangle_B}_{|-\lambda \beta^*\rangle} \underbrace{\sum_m e^{-\frac{(\lambda |\beta|)^2}{2}} \frac{(-\lambda \beta)^m}{\sqrt{m!}} \langle m|_B}_{\langle -\lambda \beta^* |} = \\ &= |-\lambda \beta^*\rangle \langle -\lambda \beta^*| \end{aligned} \quad (31)$$

and the ensemble reads

$$\varsigma_B = \int d^2 \beta p_A^{\text{EB}}(\beta) |-\lambda \beta^*\rangle \langle -\lambda \beta^*|, \quad (32)$$

exactly the same as Eq. (2) after substituting

$$\alpha = -\lambda \beta^* \implies |\beta|^2 = \frac{|\alpha|^2}{\lambda^2}, \quad (33)$$

$$\frac{1 - \lambda^2}{\lambda^2} = \frac{1}{2V_A} \implies 2V_A = \frac{\lambda^2}{1 - \lambda^2} \quad (34)$$

It follows that covariance matrix of TMSVS can be used to calculate Holevo information in PM scheme. It is defined as follows

$$\Sigma^{\text{TMSVS}} = \begin{pmatrix} V\mathbb{I} & \sqrt{V^2 - 1}\sigma_Z \\ \sqrt{V^2 - 1}\sigma_Z & V\mathbb{I} \end{pmatrix}, \quad (35)$$

where  $V = \cosh 2r$ . Relating  $V$  to  $\lambda = \tanh r$ :

$$V = \frac{1 + \lambda^2}{1 - \lambda^2} = 1 + 2 \underbrace{\frac{\lambda^2}{1 - \lambda^2}}_{\text{Eq. (34)}} = 1 + 4V_A. \quad (36)$$

After transmitting through noisy channel,

$$\Sigma_{ABC} = [\mathbb{I} \oplus \text{BS}] [\Sigma^{\text{TMSVS}} \oplus N\mathbb{I}] [\mathbb{I} \oplus \text{BS}]^T, \quad (37)$$

$$\text{BS} = \begin{pmatrix} \sqrt{T}\mathbb{I} & \sqrt{1 - T}\mathbb{I} \\ -\sqrt{1 - T}\mathbb{I} & \sqrt{T}\mathbb{I} \end{pmatrix}, \quad N = 1 + \frac{\xi}{1 - T}, \quad (38)$$

(see Ref. [2] app. C3) covariance matrix reads

$$\Sigma_{AB}^{\text{EB}} = \begin{pmatrix} V\mathbb{I} & \sqrt{T(V^2 - 1)}\sigma_Z \\ \sqrt{T(V^2 - 1)}\sigma_Z & [T(V - 1) + 1 + \xi]\mathbb{I} \end{pmatrix} \equiv \begin{pmatrix} a\mathbb{I} & c\sigma_Z \\ c\sigma_Z & V_B\mathbb{I} \end{pmatrix}. \quad (39)$$

## 5 Partial Gaussian measurement

See Ref. [4] (5.4.5).

Consider covariance matrix

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \quad (40)$$

and the goal is to calculate  $\sigma_{A|B}$  describing system  $A$  after projection measurement of system  $B$ .

$$\sigma_{A|B} = A - C(B + \sigma_m)^{-1}C^T, \quad (41)$$

Writing Eq. (5.140)[4] for one mode case for the coherent state ( $\sigma_c = \mathbb{I}_2$ ,  $\bar{\mathbf{r}}^T = (2 \text{Re } \alpha, 2 \text{Im } \alpha)^T$ ) and  $\bar{\mathbf{r}}_m^T = (q, p)^T$

$$p \sim \exp \left[ -(\bar{\mathbf{r}}_m - \bar{\mathbf{r}})^T (\sigma + \sigma_m)^{-1} (\bar{\mathbf{r}}_m - \bar{\mathbf{r}}) \right] \quad (42)$$

and using Eq. (8), as well as ideal covariance matrix of homodyne detection,  $\lim_{z \rightarrow 0} \text{diag}(z^2, z^{-2})$ , we see that

$$\lim_{z \rightarrow 0} \begin{pmatrix} 1 + z^2 & 0 \\ 0 & 1 + \frac{1}{z^2} \end{pmatrix}^{-1} = \lim_{z \rightarrow 0} \begin{pmatrix} \frac{1}{1+z^2} & 0 \\ 0 & \frac{1}{1+\frac{1}{z^2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (43)$$

so we may write

$$(B + \sigma_m)^{-1} \mapsto \Pi_{qp} B \Pi_{qp} + \sigma_N \Pi_{qp}. \quad (44)$$

Or simply using derived in Ref. relation

$$\sigma_m = X^* S S^T X^T + Y^* \quad (45)$$

with  $X = \mathbb{I}$  and  $Y^* = \sigma_N \mathbb{I}$ . For double homodyne, we have ( $S S^T = \mathbb{I}$ )

$$\sigma_m = \mathbb{I} + \begin{pmatrix} \sigma_N^{(1)} & 0 \\ 0 & \sigma_N^{(2)} \end{pmatrix} = \mathbb{I} + \begin{pmatrix} \frac{\sigma_1}{2} - \frac{1}{2} & 0 \\ 0 & \frac{\sigma_2}{2} - \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1+1}{2} & 0 \\ 0 & \frac{\sigma_2+1}{2} \end{pmatrix}. \quad (46)$$

Note that, using generalization of POVM as superposition of projectors onto squeezed coherent states,  $S S^T$  in Eq. (45) becomes  $\text{diag}(e^{-2r}, e^{2r})$ , and  $2\sigma_N^{(i)} = \sigma_i - \frac{1+e^{\pm 2r}}{2}$ .

## 6 Holevo information

Eve's entropy,  $S_E = S_{AB}$ , is calculated from the symplectic eigenvalues of  $\Sigma_{AB}^{\text{EB}}$  (39) as [3]

$$\begin{aligned} \nu_{1,2} &= \frac{1}{2} (z \pm [V_B - a]), \\ z &= \sqrt{(a + V_B)^2 - 4c^2}. \end{aligned} \quad (47)$$

Bob's measurement partial measurement of a single quadrature, described by the matrix  $\Pi_{q,p}$ , where  $\Pi_q = \text{diag}(1, 0)$  and  $\Pi_p = \text{diag}(0, 1)$  are measurements of  $\hat{q}$  and  $\hat{p}$  quadratures, respectively, transforms Alice's state covariance matrix as [2]

$$\begin{aligned} \Sigma_{A|B}^H &= a\mathbb{I} - c^2 \sigma_Z [\Pi_{q,p} (V_B + \sigma_N) \mathbb{I} \Pi_{q,p}]^{-1} \sigma_Z^T \\ &= a\mathbb{I} - \frac{c^2}{V_B + \sigma_N} \Pi_{q,p}, \end{aligned} \quad (48)$$

with symplectic eigenvalue  $\nu_3^H$ :

$$\nu_3^H = \sqrt{a \left( a - \frac{c^2}{V_B + \sigma_N} \right)} \quad (49)$$

which allows us to calculate the conditional entropy  $S_{E|B} = S_{A|B}$ .

If double homodyne detection is used by Bob's, Alice's mode is transformed as follows

$$\begin{aligned}\Sigma_{A|B}^{\text{DH}} &= a\mathbb{I} - c^2\sigma_Z[V_B\mathbb{I} + \frac{1}{2}\text{diag}(\sigma_1 + 1, \sigma_2 + 1)]^{-1}\sigma_Z^T \\ &= \begin{pmatrix} a - \frac{c^2}{V_B + \frac{\sigma_1 + 1}{2}} & 0 \\ 0 & a - \frac{c^2}{V_B + \frac{\sigma_2 + 1}{2}} \end{pmatrix}\end{aligned}\quad (50)$$

resulting in symplectic eigenvalue  $\nu_3^{\text{DH}}$  in the form

$$\nu_3^{\text{DH}} = a\sqrt{\frac{(V_B + \frac{\sigma_1 + 1}{2} - \frac{c^2}{a})(V_B + \frac{\sigma_2 + 1}{2} - \frac{c^2}{a})}{(V_B + \frac{\sigma_1 + 1}{2})(V_B + \frac{\sigma_2 + 1}{2})}}\quad (51)$$

The Holevo information is calculated as

$$\begin{aligned}\chi_{EB} &\equiv S_E - S_{E|B} = S_{AB} - S_{A|B} \\ &= \sum_{i=1,2} g(\nu_i) - g(\nu_3),\end{aligned}\quad (52)$$

where

$$g(\nu) = \frac{\nu + 1}{2} \log \frac{\nu + 1}{2} - \frac{\nu - 1}{2} \log \frac{\nu - 1}{2},\quad (53)$$

with appropriate substitution of  $\nu_i$  depending on the measurement scheme, and the asymptotic secret fraction is then calculated as

$$R = \beta I_{AB} - \chi_{EB},\quad (54)$$

where  $\beta$  is the reconciliation efficiency.

## References

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