104 „Задача о рюкзаке"

$C = 48$

i	1	2	3	4	5
c_i	15	10	12	18	20
p_i	3	5	8	4	6

Мат. модель:

$$f(x) = 3x_1 + 5x_2 + 8x_3 + 4x_4 + 6x_5 \to max$$

$$15x_1 + 10x_2 + 12x_3 + 18x_4 + 20x_5 \leq 48$$

$$x_i = 0 \vee 1 \qquad i = \overline{1,5}$$

p_i/c_i	1/5	1/2	4/3	2/9	3/10
посл. загрузки	1	4	5	2	3

$$\varphi(x) = \min(3x_1 + 5x_2 + 8x_3 + 4x_4 + 6x_5)$$

$$15x_1 + 10x_2 + 12x_3 + 18x_4 + 20x_5 \geq 48$$

$$0 \leq x_i \leq 1 \qquad i = \overline{1,5}$$

$x_1 = 1$

$x_4 = 1$

$x_5 = \frac{3}{4}$

$x_2 = 0$

$x_3 = 0$

$g(x) = 3 + 4 + 4{,}5 = 11{,}5$

т.к x_5 дробная, разбиваем на 2 мн-ва:

$X_1 = \{x \in X : x_1 = 0\}$ $\quad X = X_1 \cup X_2$

$X_2 = \{x \in X : x_1 = 1\}$

1) $g(X_1) = \min(5x_2 + 8x_3 + 4x_4 + 6x_5)$

$10x_2 + 12x_3 + 18x_4 + 20x_5 \geq 48 \quad 0 \leq x_i \leq 1 \; i \overline{2,5}$

2) $g(X_2) = \min(3 + 5x_2 + 8x_3 + 4x_4 + 6x_5)$

$10x_2 + 12x_3 + 18x_4 + 20x_5 \geq 33 \quad 0 \leq x_i \leq 1 \; i \overline{2,5}$

Решим 1): $c_4 < c \quad x_4 = 1$; $c_5 < c - c_4 \quad x_5 = 1$

$x_2 = 1$, $x_3 = 0$

$x = (1, 0, 1, 1)$

Решим 2): $x = (0, 0, 1, 3/4)$

$g(X_1) = 15$

$g(X_2) = 11,5$

$X_2 = X_3 \cup X_4$

$X_3 = \{x \in X_2 : X_2 = 0\}$

$X_4 = \{x \in X_2 : X_2 = 1\}$

3) $g(X_3) = \min(3 + 8X_3 + 4X_4 + 6X_5) = 11,5$

$12X_3 + 18X_4 + 20X_5 \geq 33$

4) $g(X_4) = \min(8 + 8X_3 + 4X_4 + 6X_5) = 13,5$

$12X_3 + 18X_4 + 20X_5 \geq 23$

$0 \leq X_i \leq 1 \quad i = \overline{3,5}$

Решение 3): $(0, 1, 3/4)$

Решение 4): $(0, 1, 1/4)$

$X_3 = X_5 \cup X_6$

$X_5 = \{x \in X_3 : X_3 = 0\}$

$X_6 = \{x \in X_3 : X_3 = 1\}$

5) $\xi(X_5) = \min(3 + 4X_4 + 6X_5) = 11,5$

$18X_4 + 20X_5 \geq 33$

6) $\xi(X_6) = \min(11 + 4X_4 + 6X_5) = 16,8$

$18X_4 + 20X_5 \geq 21$

Решение 5): $X = (1, 3/4)$

Решение 6): $X = (1, 3/20)$

$X_5 = X_7 \cup X_8$

$X_7 = \{x \in X_5 : X_4 = 0\}$

$X_8 = \{x \in X_5 : X_4 = 1\}$

7) $\xi(X_7) = \min(3 + 6X_5) = \infty$

$20X_5 \geq 33$

8) $\xi(X_8) = \min(7 + 6X_5) = 11,5$

$20X_5 \geq 15$

Решение 7): $X_5 \geq 1 \Rightarrow X_7 = \emptyset \Rightarrow \xi(X_7) = \infty$

Решение 8): $X = \left(\frac{3}{4}\right)$

$X_7 = X_9 \cup X_{10}$

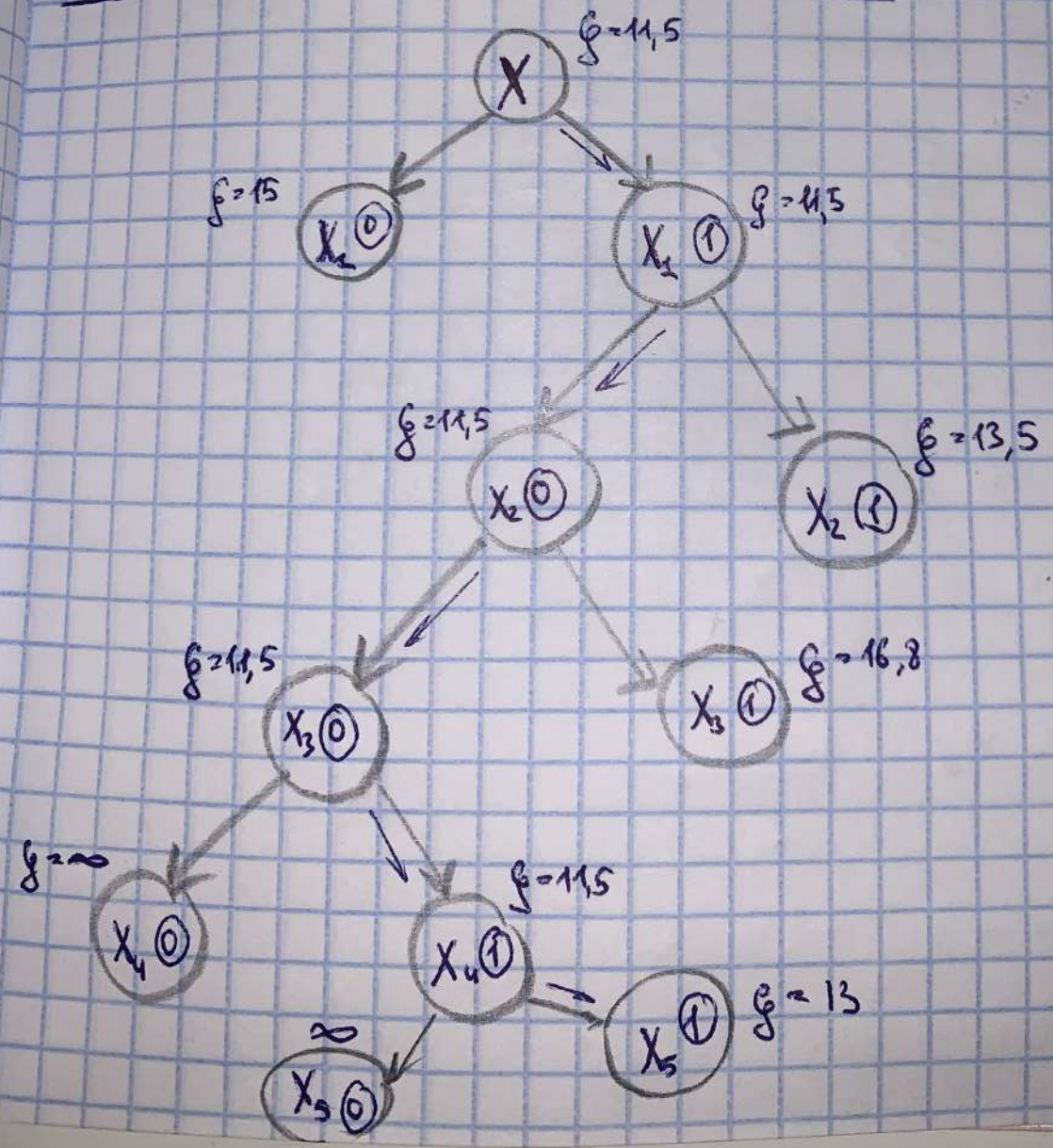
$g(X_9) = \infty$

$g(X_{10}) = 13$

$20 \geq 15$

$X^* = (1, 0, 0, 1, 1)$

$\min f(x) = 13$



№ 14.1

$n = 4 \quad C = 25$

x	0	5	10	15	20	25
f_1	0	18	36	54	72	90
f_2	0	20	40	60	80	89
f_3	0	22	45	66	80	92
f_4	0	15	30	45	60	75

$$B_k(y) = \max\left(B_{k-1}(y-z) + f_k(z)\right)$$

y	0	5	10	15	20	25
$B_1(y)$	0	18	36	54	72	90
$B_2(y)$	0	20	40	60	80	98
$x_2^\circ(y)$		5	10	15	20	20
$B_3(y)$	0	22	45	66	86	106
$x_3^\circ(y)$		5	10	15	15	15
$B_4(y)$	0	22	45	80	86	106
$x_4^\circ(y)$		5	10	50	0	0

$B_4(25) = 106$

$X_4^\circ(25) = 0$

$X_3^\circ(25) = 15$

$X_2^\circ(10) = 10$

$X_1^\circ(0) = 0$

а) $n = 3$

$X_3^\circ(25) = 15$

$X_2^\circ(10) = 10$

$X_1^\circ(0) = 0$

б) $C = 20$

$X_4^\circ(20) = 0$

$X_3^\circ(20) = 15$

$X_2^\circ(5) = 5$

$X_1^\circ(0) = 0$

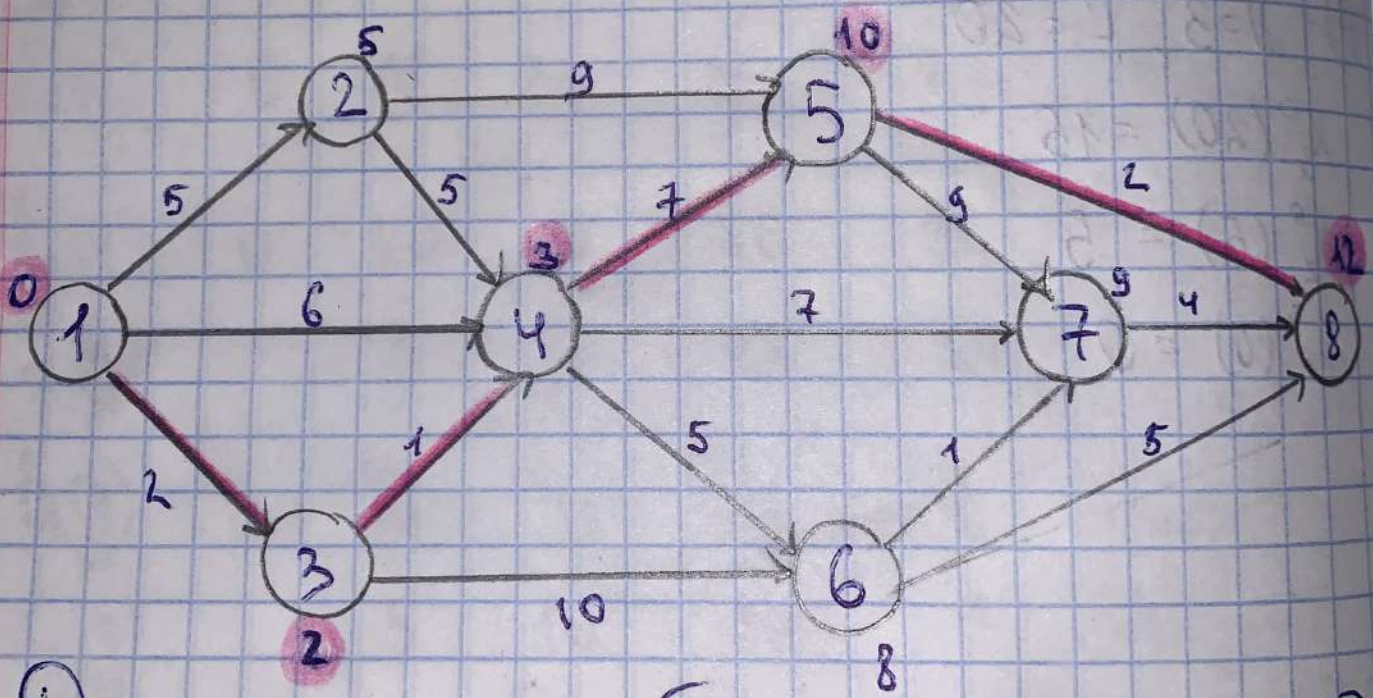
в) $n = 3$, $C = 20$

$X_3^\circ(20) = 15$

$X_2^\circ(5) = 5$

$X_1^\circ(0) = 0$

№ 14.2



Длина кратчайшего пути = 12