

# Лабораторные работы №1

1. 27 (1. 3)

а) 3 б) 4

	A <sub>1</sub>	A <sub>2</sub>	Общ. приб.
Ф	6	8	504
Т	5	11	550
Ш	15	5	945

	A <sub>1</sub>	A <sub>2</sub>
Прибыль	40	120
нит. опр.	5	5
Верх. опр.	60	45

(1)

$$Y = 40X_1 + 120X_2 \rightarrow \max (\min)$$

$$\begin{cases} (60, 18), (130, 40, 5) \\ 6X_1 + 8X_2 \leq 504 \\ 5X_1 + 11X_2 \leq 550 \\ 15X_1 + 5X_2 \leq 945 \\ (63, 0), (123, 9) \end{cases}$$

$$5 \leq X_1 \leq 60$$

$$5 \leq X_2 \leq 45$$

Т. min - 1-я т. касания - Т. А

Т. MAX - послед-я т. касания -

C' = (40, 120) - Век-р. из цел. гр-ции

$$\operatorname{tg} d_{(C)} = 3$$

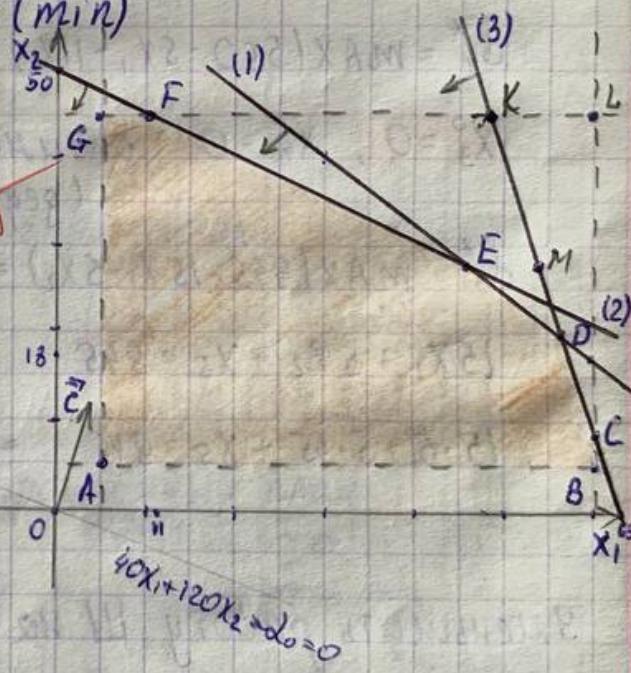
$$\operatorname{tg} d_{(1)} = \frac{4}{3}, \quad \operatorname{tg} d_{(2)} = \frac{11}{5}, \quad \operatorname{tg} d_{(3)} = \frac{1}{3}$$

$$\operatorname{tg} d_{(3)} < \operatorname{tg} d_{(1)} < \operatorname{tg} d_{(2)} < \operatorname{tg} d_{(C)} \Rightarrow T. \max - T. F$$

Найдём коорд. т. F: F(11, 45)  $\Rightarrow$  чтобы получить MAX прибыль надо произвести 11 единиц A<sub>1</sub>, 45 единиц A<sub>2</sub>

$$\varphi_{\max} = 440 + 5400 = 5840 \quad \varphi_{\min} = 800$$

T-использов. полностью, ФИЦI осталось неиспользов. ресурсы



(5)

$$② \quad \varphi(x) = 40x_1 + 120x_2 \rightarrow \max$$

$$\begin{cases} 6x_1 + 8x_2 + x_3 &= 504 \\ 5x_1 + 11x_2 + x_4 &= 550 \\ 15x_1 + 5x_2 + x_5 &= 945 \end{cases}$$

$$\begin{aligned} 5 \leq x_1 \leq 60 & \quad 0 \leq x_4 \leq 550 - 25 - 55 = \\ & = 470 \\ 5 \leq x_2 \leq 45 & \\ 0 \leq x_3 \leq 504 - 6 \cdot 5 - 8 \cdot 5 & = 434 \end{aligned}$$

$$0 \leq x_5 \leq 945 - 75 - 25 = 845$$

③ На рисунке

$$d_3^* = \max(504 - 6x_1 - 8x_2) = 504 - 6 \cdot 5 - 8 \cdot 5 = 434$$

$$d_4^* = \max(550 - 5x_1 - 11x_2) = 550 - 25 - 55 = 470$$

$x_1 = 0, x_2 = 0$  оптимальные свободные  
(дефицитные) ресурсы

$$d_5^* = \max(945 - 15x_1 - 5x_2) = 945 - 75 - 25 = 845$$

$$15x_1 + 5x_2 + x_5 = 945$$

$$15 \cdot 0 + 5 \cdot 45 + x_5 = 945 \Rightarrow x_5 = 555$$

$$x_5^* = 555 - \text{Или ф. оборуд.} \Rightarrow$$

Уменьшить работу III на 555 часов

$$d_3^* = 434$$

$$6x_1 + 8x_2 + x_3 = 504$$

$$6 \cdot 11 + 8 \cdot 45 + x_3 = 504 \Rightarrow x_3 = 78$$

$x_3^* = 78 \rightarrow \text{Презер. оборудование} \Rightarrow \text{уменьшил работу IV на 78 часов.}$

⑤

• A(5, 5)

$$X^A = (5, 5, 434, 470, 845) \quad A = \begin{pmatrix} 6 & 8 & 1 & 0 & 0 \\ 5 & 11 & 0 & 1 & 0 \\ 15 & 5 & 0 & 0 & 1 \end{pmatrix}$$

$J_H^1 = \{3, 4, 5\}$  вырожденный план

$$J_B^2 = \{1, 2, 3\}$$

• B(60, 5)

$$X^B = (60, 5, 104, 195, 20) - \text{невырожденный план}$$

$$J_H = \{1, 2\} \quad J_B = \{3, 4, 5\}$$

• C(60, 9)

$$X^C = (60, 9, 72, 151, 0) - \text{невырожденный план}$$

$$J_H = \{1, 5\} \quad J_B = \{2, 3, 4\}$$

• D(56, 21)

E(44, 30)

$$X^D = (56, 21, 0, 39, 0) - \text{невырожденный план}$$

$$J_H = \{3, 5\} \quad J_B = \{1, 2, 4\}$$

$$X^E = (44, 30, 0, 0, 135) - \text{невырожденный план}$$

• F(11, 45)

$$J_B = \{1, 2, 5\} \quad J_H = \{3, 4\}$$

$$X^F = (11, 45, 78, 0, 555) - \text{невырожденный план}$$

$$J_H = \{2, 4\} \quad J_B = \{1, 3, 5\}$$

• G(5, 45)

$$X^G = (5, 45, 114, 30, 645) - \text{невырожденный план}$$

$$J_H = \{1, 2\} \quad J_B = \{3, 4, 5\}$$

$$⑥ 40x_1 + 120x_2 \rightarrow \max$$

т. А:

$$A_B(Q_3, Q_4, Q_5) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C^I = (40, 120, 0, 0, 0)$$

$$Q_j^I U = C_j \Rightarrow U = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$X^A = (5, 5, 434, 470, 845)$$

$$\Delta_1 = C_1 - U^I Q_1 = 40 > 0 \quad x_1 = d_1^* (-)$$

$$\Delta_2 = C_2 - U^I Q_2 = 120 > 0 \quad x_2 = d_2^* (-)$$

план не оптимальный

т. F:

$$X^F = (11, 45, 78, 0, 555) \quad J_B = \{1, 3, 5\}$$

$$A_B = \begin{pmatrix} 6 & 1 & 0 \\ 5 & 0 & 0 \\ 15 & 0 & 1 \end{pmatrix}$$

$$(U, U_2, U_3) \begin{pmatrix} 6 & 1 & 0 \\ 5 & 0 & 0 \\ 15 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix} \Rightarrow U = (0, 8, 0)$$

$$\Delta_2 = 120 - 88 = 32 > 0 \quad x_2 = d_2^* (+)$$

$$\Delta_4 = 0 - 8 = -8 < 0 \quad x_4 = d_4^* (+)$$

план оптимальный

$$⑦ -40x_1 - 120x_2 \rightarrow \max$$

$$X^0 = (5, 5, 434, 470, 845)$$

$$II) A_B = (Q_1, Q_3, Q_4) = \begin{pmatrix} 6 & 1 & 0 \\ 5 & 0 & 1 \\ 15 & 0 & 0 \end{pmatrix} \quad J_B = \{1, 3, 4\}$$

$$\begin{pmatrix} 6 & 5 & 15 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} U = \begin{pmatrix} -40 \\ 0 \\ 0 \end{pmatrix} \Rightarrow U = \left(-\frac{8}{3}, 0, 0\right)$$

$$\Delta_2 = -120 + \frac{64}{3} = -\frac{296}{3} < 0 \quad x_2 = d_2^* (+)$$

$$\Delta_5 = 0 - 0 = 0$$

$\Rightarrow A = (Q_1, Q_3, Q_4)$  - оптимальные

$$2) A_B = \begin{pmatrix} 6 & 8 & 0 \\ 5 & 11 & 0 \\ 15 & 5 & 1 \end{pmatrix} \quad J_B = \{1, 2, 5\}$$

$$\begin{pmatrix} 6 & 5 & 15 \\ 8 & 11 & 5 \\ 0 & 0 & 1 \end{pmatrix} U = \begin{pmatrix} -40 \\ -120 \\ 0 \end{pmatrix} \Rightarrow U = \left( \frac{80}{13}, -\frac{200}{13}, 0 \right)$$

$$\Delta_3 = 0 - \frac{80}{13} = -\frac{80}{13} < 0 \quad x_3 = d_{3*} - (-)$$

$$\Delta_4 = 0 + \frac{200}{13} = \frac{200}{13} > 0 \quad x_4 = d_{4*} - (+)$$

$\Rightarrow A = (Q_1, Q_2, Q_5)$  - не оптимальный

$$⑧ \psi(x) = 40x_1 + 120x_2 \rightarrow \max$$

$$\bar{x} = (5, 5)$$

X

$$\begin{cases} 6x_1 + 8x_2 + x_3 = 504 \\ 5x_1 + 11x_2 + x_4 = 550 \\ 15x_1 + 5x_2 + x_5 = 945 \end{cases} \quad \begin{array}{l} 5 \leq x_1 \leq 60 \\ 5 \leq x_2 \leq 45 \\ 0 \leq x_3 \leq 434 \end{array} \quad \begin{array}{l} 0 \leq x_4 \leq 470 \\ 0 \leq x_5 \leq 845 \end{array}$$

$$w = b - A\bar{x} = \begin{pmatrix} 434 \\ 470 \\ 845 \end{pmatrix}$$

$$\bar{x} = (5, 5, 434, 470, 845) \quad J_B = \{3, 4, 5\}$$

1 шаг.

$$U = (0, 0, 0)$$

$$\Delta_1 = 40 \quad \Delta_2 = 120 \quad j_0 = 1$$

$$\ell = (1, 0, -6, -5, -15) \quad \Theta = (55, \infty, \frac{217}{3}, 94, 169) \quad j_* = 1$$

$$\tilde{J}_u = J_u \setminus j_0 = \{2\}$$

$$\bar{x} = (60, 5, 104, 195, 20)$$

2 шаг.

$$j_0 = 2, 2, 20$$

$$\ell = (0, 1, -8, -11, -5) \quad \Theta = (\infty, 40, 13, \frac{195}{11}, 4) \quad j_* = 5$$

$$J_6 = \{2, 3, 4\}$$

$$\bar{X} = (60, 9, 72, 151, 0)$$

3 uTep.

$$U = (0, 0, 24)$$

$$\Delta_1 = -320 \quad \Delta_5 = -24 \quad j_0 = 1$$

$$\ell = (-1, 3, -18, -28, 0) \quad \Theta = (55, 12, 4, \frac{151}{28}, \infty) \quad j_* = 3$$

$$J_6 = \{1, 2, 4\}$$

$$\bar{X} = (56, 21, 0, 39, 0)$$

4 uTep.

$$U = \left( \frac{160}{9}, 0, -\frac{40}{9} \right)$$

$$\Delta_3 = -\frac{160}{9} \quad \Delta_5 = \frac{40}{9} \quad j_0 = 5$$

$$\ell = \left( -\frac{4}{45}, \frac{1}{15}, 0, -\frac{13}{45}, 1 \right) \quad \Theta = \left( \frac{51 \cdot 45}{4}, 24 \cdot 15, \infty, 135, 845 \right)$$

$$J_6 = \{1, 2, 5\} \quad j_* = 4$$

$$\bar{X} = (44, 30, 0, 0, 135)$$

5 uTep.

$$U = \left( -\frac{80}{13}, \frac{200}{13}, 0 \right)$$

$$\Delta_3 = \frac{80}{13} \quad \Delta_4 = -\frac{200}{13} \quad j_0 = 3$$

$$\ell = \left( -\frac{11}{26}, \frac{5}{26}, 1, 0, \frac{70}{13} \right) \quad \Theta = \left( \frac{41 \cdot 26}{11}, 78, 434, \infty, \frac{169 \cdot 13}{14} \right)$$

$$J_6 = \{1, 3, 5\}$$

$$\bar{X} = (11, 45, 78, 0, 555) \quad j_* = 2$$

6 итер.

$$U = (10, 8, 0)$$

$$\Delta_2 = 32 \quad \Delta_4 = -8 \Rightarrow X^o = (11, 45, 78, 0, 555) -$$

- оптимальный план с  $A_B^o = \begin{pmatrix} 6 & 1 & 0 \\ 5 & 0 & 0 \\ 15 & 0 & 1 \end{pmatrix}$

$$(9) \quad \varphi(X) = 40X_1 + 120X_2 \rightarrow \text{MAX}$$

$$\bar{X} = (60, 45)$$

$$W = \begin{pmatrix} 504 - 60 \cdot 6 - 8 \cdot 45 \\ 550 - 60 \cdot 5 - 11 \cdot 45 \\ 945 - 60 \cdot 15 - 5 \cdot 45 \end{pmatrix} = \begin{pmatrix} -216 \\ -245 \\ -180 \end{pmatrix}$$

$$\varphi(X) = -X_3 - X_4 - X_5 \rightarrow \text{MAX}$$

$$\begin{cases} 6X_1 + 8X_2 - X_3 = 504 \\ 5X_1 + 11X_2 - X_4 = 550 \\ 15X_1 + 5X_2 - X_5 = 945 \end{cases} \quad \begin{array}{l} 0 \leq X_3 \leq 216 \\ 5 \leq X_4 \leq 60 \\ 5 \leq X_2 \leq 45 \\ 0 \leq X_5 \leq 180 \end{array}$$

$$\bar{X} = (60, 45, 216, 245, 180)$$

$$J_B = \{3, 4, 5\} \quad A_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$U = (1, 1, 1)$$

$$\Delta_1 = -26 \quad \Delta_2 = -24 \quad j_0 = 1$$

$$P = (-1, 0, -6, -5, -15) \quad \Theta = (55, \infty, 36, 49, 12)$$

$$J_B = \{1, 3, 4\}$$

$$\bar{X} = (48, 45, 144, 185, 0) \quad A_B = \begin{pmatrix} 6 & -1 & 0 \\ 5 & 0 & -1 \\ 15 & 0 & 0 \end{pmatrix}$$

$$-X_3 - X_4 \rightarrow \text{MAX}$$

$$\begin{cases} 6X_1 + 8X_2 - X_3 = 504 \\ 5X_1 + 11X_2 - X_4 = 550 \\ 15X_1 + 5X_2 + X_5 = 945 \end{cases} \quad \begin{array}{l} 0 \leq X_3 \leq 216 \\ 5 \leq X_4 \leq 60 \\ 5 \leq X_2 \leq 45 \\ 0 \leq X_5 \leq 845 \end{array}$$

$$\begin{pmatrix} 6 & 5 & 15 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} U = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \Rightarrow U = (1, 1, -\frac{11}{15})$$

$$\Delta_2 = -\frac{46}{3}, \quad \Delta_5 = \frac{11}{15}, \quad j_0 = 2$$

$$L = (\frac{1}{3}, -1, -6, -\frac{28}{3}, 0), \quad \Theta = (36, 40, 24, \frac{555}{28}, \infty)$$

$$J_B = \{1, 2, 3\}$$

$$\hat{x} = (54\frac{17}{28}, 25\frac{5}{28}, 25\frac{1}{14}, 0, 0) \quad A_B = \begin{pmatrix} 6 & 8 & -1 \\ 5 & 11 & 0 \\ 15 & 5 & 0 \end{pmatrix}$$

$-x_3 \rightarrow \text{MAX}$

$$\begin{cases} 6x_1 + 8x_2 - x_3 = 504 \\ 5x_1 + 11x_2 + x_4 = 550 \\ 15x_1 + 5x_2 + x_5 = 945 \end{cases} \quad \begin{array}{l} 5 \leq x_1 \leq 60 \\ 5 \leq x_2 \leq 45 \\ 0 \leq x_3 \leq 216 \end{array} \quad \begin{array}{l} 0 \leq x_4 \leq 470 \\ 0 \leq x_5 \leq 845 \end{array}$$

$$U = (1, -\frac{9}{14}, -\frac{13}{70})$$

$$\Delta_4 = \frac{9}{14}, \quad \Delta_5 = \frac{13}{70}, \quad j_0 = 5$$

$$L = (-\frac{11}{140}, \frac{1}{28}, -\frac{13}{70}, 0, 1), \quad \Theta = (\frac{6945}{11}, 471, 135, \infty, 845)$$

$$J_B = \{1, 2, 5\}$$

$$\hat{x} = (44, 30, 0, 0, 135) - T.E$$

Т.к. все искусств. нер-е = 0, то переходим ко 2-й  
стадии. Запишем зеркальную фигуру

$$\Psi = 40x_1 + 120x_2$$

$$\begin{cases} 6x_1 + 8x_2 + x_3 = 504 \\ 5x_1 + 11x_2 + x_4 = 550 \\ 15x_1 + 5x_2 + x_5 = 945 \end{cases} \quad \begin{array}{l} 5 \leq x_1 \leq 60 \\ 5 \leq x_2 \leq 45 \\ 0 \leq x_3 \leq 434 \end{array} \quad \begin{array}{l} 0 \leq x_4 \leq 470 \\ 0 \leq x_5 \leq 845 \end{array}$$

(10)

$$\begin{cases} 6x_1 + 8x_2 + x_3 = 11 \\ 5x_1 + 11x_2 + x_4 = 11 \\ 15x_1 + 5x_2 + x_5 = 11 \end{cases}$$

Огранич.

Двойств.

$\psi(\lambda)$

+ 434  $\omega_3$

$$\begin{cases} 6y_1 + 8y_2 + y_3 = 11 \\ 8y_1 + 11y_2 + y_4 = 11 \\ y_1 - 7y_2 - 5y_3 - 2y_4 = 11 \end{cases}$$

$U^0$ :

$\omega_1$ :

$\Omega_1$ :

$\lambda^0$ :

$\psi_1$

$\psi_1$

$$⑩ \quad \psi(x) = 40x_1 + 120x_2 \rightarrow \max$$

$$\begin{cases} 6x_1 + 8x_2 + x_3 = 504 \\ 5x_1 + 11x_2 + x_4 = 550 \\ 15x_1 + 5x_2 + x_5 = 845 \end{cases} \quad \begin{array}{l} 5 \leq x_1 \leq 60 \\ 5 \leq x_2 \leq 45 \\ 0 \leq x_3 \leq 434 \end{array} \quad \begin{array}{l} 0 \leq x_4 \leq 470 \\ 0 \leq x_5 \leq 845 \end{array}$$

Оптимальный план первой задачи:  $x^* = (11, 45, 78, 0, 558)$

$$J_B = \{1, 3, 5\}$$

Двойственность к ней:

$$\begin{aligned} \psi(\lambda) = & 504y_1 + 550y_2 + 845y_3 - 5v_1 - 5v_2 + 60w_1 + 45w_2 + \\ & + 434w_3 + 470w_4 + 845w_5 \rightarrow \min \end{aligned}$$

$$\begin{cases} 6y_1 + 5y_2 + 15y_3 - v_1 + w_1 = 40 \\ 8y_1 + 11y_2 + 5y_3 - v_2 + w_2 = 120 \\ y_1 - v_3 + w_3 = 0 \\ y_2 - v_4 + w_4 = 0 \\ y_3 - v_5 + w_5 = 0 \end{cases} \quad y_i, v_i, w_i \geq 0$$

$$U^* = (0, 8, 0) \Rightarrow y_1 = 0, \quad y_2 = 8, \quad y_3 = 0$$

$$w_1 = 0, \quad w_2 = 32, \quad w_3 = 0, \quad w_4 = 0, \quad w_5 = 0$$

$$v_1 = 0, \quad v_2 = 0, \quad v_3 = 0, \quad v_4 = 8, \quad v_5 = 0$$

$$\lambda^* = (U^*, V^*, W^*)$$

$$\begin{aligned} \psi(X^*) = 5840 \\ \psi(\lambda^*) = 5840 \end{aligned} \quad \Rightarrow \quad \psi(X^*) = \psi(\lambda^*)$$

$$(11) \quad \psi(x) = 40x_1 + 120x_2 \rightarrow \max \quad \begin{matrix} 40x_1 + 120x_2 = 1150 \\ 5x_1 + 11x_2 = 550 \\ 15x_1 + 5x_2 = 945 \end{matrix}$$

$$\begin{cases} 6x_1 + 8x_2 + x_3 = 504 \\ 5x_1 + 11x_2 + x_4 = 550 \\ 15x_1 + 5x_2 + x_5 = 945 \end{cases} \quad \begin{matrix} 5 \leq x_1 \leq 60 \\ 5 \leq x_2 \leq 45 \\ 0 \leq x_3 \leq 434 \end{matrix} \quad \begin{matrix} 0 \leq x_4 \leq 470 \\ 0 \leq x_5 \leq 845 \end{matrix}$$

$$J_B = \{3, 4, 5\}$$

1 step

$$U = (0, 0, 0)$$

$$\delta_1 = 40 \quad \delta_2 = 120 \quad j_* = 4$$

$$x = \begin{pmatrix} 60 \\ 45 \\ -216 \\ -245 \\ -180 \end{pmatrix} \quad P = \begin{pmatrix} -5 \\ -11 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{\delta}_1 = 8 \quad \bar{\delta}_2 = \frac{120}{11} \quad j_0 = 1$$

$$J_B = \{1, 3, 5\}$$

2 step

$$U = (0, 8, 0)$$

$$\delta_2 = 32 \quad \delta_4 = -8$$

$$x = \begin{pmatrix} 11 \\ 45 \\ 78 \\ 0 \\ 555 \end{pmatrix} \quad -T.F$$

$$x = (11, 45, 78, 0, 555) = X^0$$

(12)

$$y^0 = (0, 8, 0)$$

$$w^0 = (0, 32, 0, 0, 0)$$

$$v^0 = (0, 0, 0, 8, 0)$$

CORE i5  
8th Gen

Увеличение на 1 единицу ресурса  $\Phi$  не приведёт к увеличению прибыли

Увеличение на 1 единицу ресурса  $T$  приведёт к увеличению прибыли на 8 единиц.

Увеличение на 1 единицу ресурса  $W$  не приведёт к увеличению прибыли.

$\Rightarrow$  Если надо увеличить прибыль, надо выбирать 2-й ресурс.

$y_2^* \neq 0 \Rightarrow$  дефицитный ресурс

Рассмотрим  $y_1^*$  и  $y_3^*$

Обратимся к оптимальным планам!

$x_3^* = 78, x_5^* = 555 \Rightarrow$  ресурсы не дефицитные

1. 10

a) 3      b) 1

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	%	
B <sub>1</sub>	50	20	50	10	62,5	400
B <sub>2</sub>	40	10	10	10	37,5	

A <sub>1</sub>	100
A <sub>2</sub>	120
A <sub>3</sub>	70
A <sub>4</sub>	30

$$\textcircled{1} \quad 100x_1 + 120x_2 + 70x_3 + 30x_4 \rightarrow \min$$

$$\psi = -100x_1 - 120x_2 - 70x_3 - 30x_4 \rightarrow \max$$

$$\begin{cases} 5x_1 + 2x_2 + 5x_3 + x_4 = 2500 \\ 4x_1 + x_2 + x_3 + x_4 = 1500 \end{cases}$$

$$\begin{cases} x_1 + x_2 + 4x_3 = 1000 \\ 4x_1 + x_2 + \frac{1000 - x_1 - x_2}{4} + x_4 = 1500 \end{cases} \quad \begin{cases} x_3 = \frac{1000 - x_1 - x_2}{4} \geq 0 \\ x_4 = \frac{5000 - 15x_1 - 3x_2}{4} \geq 0 \end{cases}$$

$$\psi = 30x_1 - 80x_2 - 55000 \rightarrow \max$$

$$\begin{cases} (500, 500), (1000, 0) \\ x_1 + x_2 \leq 1000 \end{cases}$$

$$\begin{cases} (333\frac{1}{3}, 0), (200, 666\frac{2}{3}) \\ 15x_1 + 3x_2 \leq 5000 \end{cases}$$

$$T. \min - T. D(0, 1000)$$

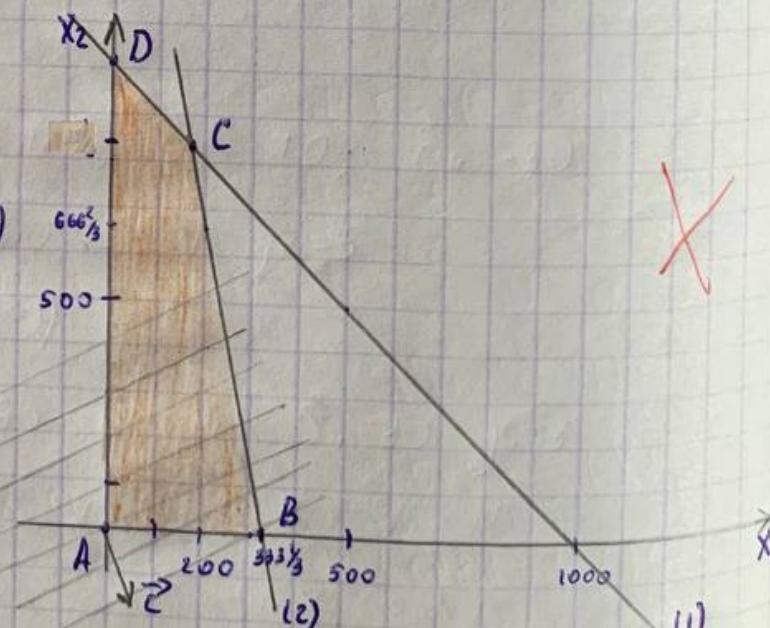
$$T. \max - T. B\left(\frac{1000}{3}, 0\right)$$

$$x^D = (0, 1000, 0, 500)$$

$$x^B = \left(\frac{1000}{3}, 0, \frac{500}{3}, 0\right)$$

$$\bar{x} = (0, 0, 0, 0)$$

$$w = \begin{pmatrix} 2500 \\ 1500 \end{pmatrix}$$



$$\begin{cases} 5x_1 + 2x_2 \leq 2500 \\ 4x_1 + x_2 \leq 1500 \end{cases}$$

$$\bar{x} = (1, -x)$$

1. uTeP

U

$\Delta =$

$\ell =$

$J_B$

$\bar{x} :$

2. uTeP

U

$\Delta$

$\ell =$

$J_B$

$\bar{x}$

T. w

page

L

$$\begin{cases} 5x_1 + 2x_2 + 5x_3 + x_4 + x_5 = 2500 \\ 4x_1 + x_2 + x_3 + x_4 + x_6 = 1500 \end{cases} \quad \begin{array}{l} 0 \leq x_i \leq M, i=1,4 \\ 0 \leq x_5 \leq 2500 \\ 0 \leq x_6 \leq 1500 \end{array}$$

$$\bar{x} = (0, 0, 0, 0, 2500, 1500) \quad J_B = \{5, 6\}$$

$$-x_5 - x_6 \rightarrow \max$$

1 итер.

$$U = (-1, -1)$$

$$\Delta = (9, 3, 6, 2, 0, 0) \quad j_0 = 1$$

$$\ell = (1, 0, 0, 0, -5, -4) \quad Q = (M, \infty, \infty, \infty, 500, 375)$$

$$J_B = \{1, 5\} \quad j_x = 6$$

$$\bar{x} = (375, 0, 0, 0, 625, 0)$$

2 итер.

$$U = (-1, 5/4)$$

$$\Delta = (0, 3/4, 15/4, -1/4, 0, -5/4) \quad j_0 = 3$$

$$\ell = (-1/4, 0, 1, 0, -15/4, 0) \quad Q = (1500, \infty, M, \infty, \frac{500}{3}, \infty)$$

$$J_B = \{1, 3\}$$

$$\bar{x} = (\frac{1000}{3}, 0, \frac{500}{3}, 0, 0, 0)$$

т.к. зменились исходные. не р-e  $\Rightarrow$  перейти на 2-й шаг

$$\psi(x) = -100x_1 - 120x_2 - 70x_3 - 30x_4 \rightarrow \max$$

$$\begin{cases} 5x_1 + 2x_2 + 5x_3 + x_4 = 2500 \\ 4x_1 + x_2 + x_3 + x_4 = 1500 \end{cases} \quad 0 \leq x_i \leq M \quad i=1,4$$

$$\bar{x} = \left( \frac{1000}{3}, 0, \frac{500}{3}, 0 \right) \quad J_B = \{1, 3\}$$

$$U = (-12, -10)$$

$$\Delta = (0, -8, 0, -8) \Rightarrow X^0 = \left( \frac{1000}{3}, 0, \frac{500}{3}, 0 \right)$$

②

$$-100x_1 - 120x_2 - 70x_3 - 30x_4 \rightarrow \text{MAX}$$

$$\begin{cases} 5x_1 + 2x_2 + 5x_3 + x_4 + x_5 = 2500 \\ 4x_1 + x_2 + x_3 + x_4 + x_6 = 1500 \end{cases} \quad 0 \leq x_i \leq M \quad i=1,4$$

$$J_B = \{5, 6\}, A_B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

1. Iter.

$$U = (0, 0)$$

$$\delta_1 = -100, \delta_2 = -120, \delta_3 = -70, \delta_4 = -30$$

$$x = (0, 0, 0, 0, 2500, 1500) \quad j_* = 5$$

$$P = (5, 2, 5, 1, -1, 0)$$

$$\bar{\sigma}_1 = 20, \bar{\sigma}_2 = 60, \bar{\sigma}_3 = 14, \bar{\sigma}_4 = 30 \quad j_a = 3$$

$$J_B = \{3, 6\}$$

2. Iter.

$$U = (-14, 0)$$

$$\delta_1 = -30, \delta_2 = -92, \delta_3 = -16, \delta_5 = 14$$

$$\mathbf{x} = (0, 0, 500, 0, 0, 1000) \quad j_* = 6$$

$$\mathbf{p} = (3, \frac{3}{5}, \frac{1}{5}, \frac{1}{5}, -\frac{1}{5}, -1)$$

$$G_1 = 10 \quad G_2 = \frac{400}{3} \quad G_4 = 20 \quad G_5 = 70 \quad j_0 = 1$$

3 итер.

$$U = (-12, -10)$$

$$J_5 = \{1, 3\}$$

$$\delta_2 = -86 \quad \delta_4 = -8 \quad \delta_5 = 12 \quad \delta_6 = 0$$

$$\mathbf{x} = (\frac{1000}{3}, 0, \frac{500}{3}, 0, 0, 0) \Rightarrow \mathbf{x} = X^0$$

Двойств. задача о лин. уксп. залоги:  $100x_1 + 120x_2 + 20x_3 +$

$$2500y_1 + 1500y_2 \rightarrow \max \quad + 30x_4 \rightarrow \min$$

$$\begin{cases} 5y_1 + 4y_2 \geq 100 \\ 2y_1 + y_2 \geq 120 \\ 5y_1 + y_2 \geq 20 \\ y_1 + y_2 \geq 30 \end{cases}$$



$$(0, 0) = X$$

$$\begin{pmatrix} F \\ F \\ F \end{pmatrix} = G$$

$$F = \{0, 500, 800, 20\}$$

$$\begin{cases} F_1 = 0 + 120x_1 + 20x_3 - 2 \\ F_2 = 0 + 30x_4 - 10 \end{cases}$$

$$(0, F_1, F_2, 0, 0) = X$$

1. 17

а) 11      б) 3

$$\textcircled{1} \quad \psi = 4x_1 + 3x_2$$

$$\begin{cases} -2x_1 + 3x_2 \leq 7 \\ -2x_1 + 5x_2 \leq 17 \\ x_1 - 25x_2 \leq 0 \end{cases} \quad x_1, x_2 \geq 0$$

т. min - т. A(0,0)

$\psi(x)$  - не ограничен  $\Rightarrow$

нет решения ид max

$$\textcircled{2} \quad \begin{cases} -2x_1 + 3x_2 + x_3 = 7 \\ -2x_1 + 5x_2 + x_4 = 17 \\ x_1 - 25x_2 + x_5 = 0 \end{cases}$$

$$\begin{aligned} & x_1, x_2 \geq 0 \\ & 0 \leq x_3 \leq 2M+7 \\ & 0 \leq x_4 \leq 2M+17 \\ & 0 \leq x_5 \leq 25M \end{aligned}$$

$$\textcircled{3} \quad 4x_1 + 3x_2 \rightarrow \max$$

$$\begin{cases} -2x_1 + 3x_2 \leq 7 \\ -2x_1 + 5x_2 \leq 17 \\ x_1 - 25x_2 \leq 0 \end{cases} \quad x_1, x_2 \geq 0$$

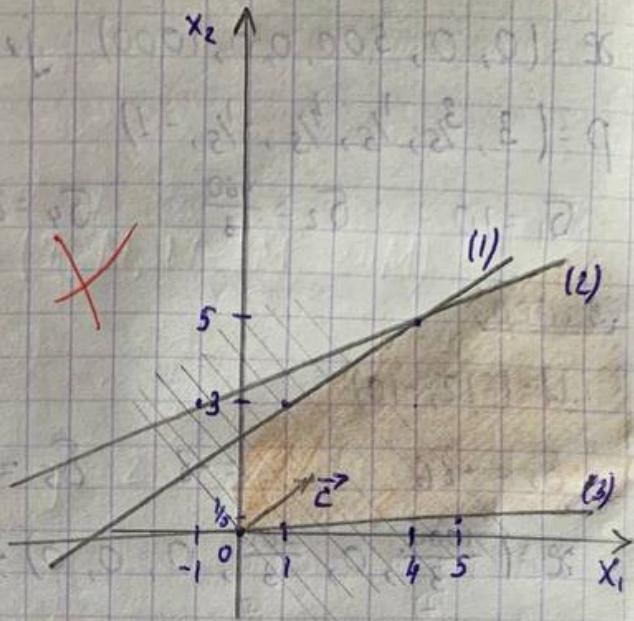
$$\bar{x} = (0, 0)$$

$$w = \begin{pmatrix} 7 \\ 17 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2x_1 + 3x_2 + x_3 = 7 \\ -2x_1 + 5x_2 + x_4 = 17 \\ x_1 - 25x_2 + x_5 = 0 \end{cases} \quad \begin{aligned} & 0 \leq x_1, x_2 \leq M \\ & 0 \leq x_3 \leq 2M+7 \\ & 0 \leq x_4 \leq 2M+17 \\ & 0 \leq x_5 \leq 25M \end{aligned}$$

$$\bar{x} = (0, 0, 7, 17, 0)$$

$$J_6 = \{3, 4, 5\}$$



1) итер,

$$U = 10$$

$$\Delta = (4, 3)$$

$$\rho = (1)$$

$$\bar{x} = (0, 0)$$

2) итер,

$$U = 10$$

$$\Delta = (1)$$

$$\rho = (1)$$

$$\Rightarrow \text{загор}$$

левой гр.

$$\textcircled{4} \quad \psi = 4x_1 + 3x_2$$

$$\begin{cases} -2x_1 + 3x_2 \leq 7 \\ -2x_1 + 5x_2 \leq 17 \\ x_1 - 25x_2 \leq 0 \end{cases}$$

$$\psi(\lambda)$$

$$\begin{cases} -2y_1 + 3y_2 \leq 7 \\ -2y_1 + 5y_2 \leq 17 \\ y_1 - 25y_2 \leq 0 \end{cases}$$

$$\textcircled{5} \quad 4x_1 + 3x_2 \rightarrow \max$$

$$\begin{cases} -2x_1 + 3x_2 \leq 7 \\ -2x_1 + 5x_2 \leq 17 \\ x_1 - 25x_2 \leq 0 \end{cases}$$

1 итер.

$$U = (0, 0, 0)$$

$$\Delta = (4, 3, 0, 0, 0) \quad j_0 = 1$$

$$l = (1, 0, 2, 2, -1)$$

$$\Omega = (M, \infty, M, M, 0), j_* = 5$$

$$\bar{x} = (0, 0, 7, 17, 0)$$

$$J_B = \{1, 3, 4\}$$

2 итер.

$$U = (0, 0, 4)$$

$$\Delta = (0, 103, 0, 0, -4) \quad j_0 = 2$$

$$l = (25, 1, 47, 25, 0) \quad \Omega = (\frac{M}{25}, M, \frac{2M}{47}, \frac{2M}{25}, \infty)$$

$\Rightarrow$  задача не имеет решений из-за неограниченности уравнений в левой части на множестве планов.

④  $\psi = 4x_1 + 3x_2$

$$\begin{cases} -2x_1 + 3x_2 \leq 7 \\ -2x_1 + 5x_2 \leq 17 \\ x_1 - 25x_2 \leq 0 \end{cases} \quad x_1, x_2 \geq 0$$

$$\psi(\lambda) = 7y_1 + 17y_2 \rightarrow \min$$

$$\begin{cases} -2y_1 - 2y_2 + y_3 \geq 4 \\ 3y_1 + 5y_2 - 25y_3 \geq 3 \end{cases} \quad y_i \geq 0$$

⑤  $4x_1 + 3x_2 \rightarrow \max$

$$\begin{cases} -2x_1 + 3x_2 + x_3 = 7 \\ -2x_1 + 5x_2 + x_4 = 17 \\ x_1 - 25x_2 + x_5 = 0 \end{cases} \quad x_1, x_2, x_3, x_4 \geq 0$$

$$J_B = \{3, 4, 5\}$$

1utep.

$$U = (0, 0, 0)$$

$$\delta_1 = 4$$

$$\delta_2 = 3$$

$$\mathcal{D}_H = (M, M)$$

$$\mathbf{x}_5 = (7-M, 17-3M, 24M)$$

$$\ell = (2, -5, 0, 1, 0)$$

$$j^* = 4$$

$$G_1 = \infty$$

$$\sigma_2 = \frac{3}{5} \quad j_0 = 2$$

$$J_5 = \{2, 3, 5\}$$

Zutep.

$$U = \left(0, \frac{3}{5}, 0\right)$$

$$\delta_1 = 26\%$$

$$\delta_4 = -\frac{3}{5}$$

$$\mathcal{X}_M = (M, \mathcal{O})$$

$$\mathbf{x}_6 = \left( \frac{17+2M}{5}, \frac{4M-16}{5}, 9M+85 \right)$$

Условие оптимальности выполнено. Однако имеем неогр. цел-й ф-ции  $\Rightarrow$  задача не имеет решения.

2018.8.18

卷之三

$$EY + SYCF \cdot YS^2$$

1 524 833 27

$5 \times 24 = 120$

1.20 (1.30 S)

1.20 - 18B. 1.30(S) - 6B.

$$\varphi = -2x_1 - x_2 + x_3 + 2x_4$$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 7 \\ -3x_1 - 2x_2 - x_3 + x_4 = -10 \\ -x_1 + x_2 - 2x_3 + 2x_4 \geq -2 \end{cases} \quad \begin{array}{ll} 0 \leq x_1 \leq 5 & 1 \leq x_3 \leq 5 \\ 0 \leq x_2 \leq 3 & 2 \leq x_4 \leq 10 \end{array}$$

①  $-2x_1 - x_2 + x_3 + 2x_4 \rightarrow \text{MAX}$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 7 \\ -3x_1 - 2x_2 - x_3 + x_4 = -10 \\ -x_1 + x_2 - 2x_3 + 2x_4 + x_5 = 2 \end{cases} \quad \begin{array}{ll} 0 \leq x_1 \leq 5 & 1 \leq x_3 \leq 5 \\ 0 \leq x_2 \leq 3 & 2 \leq x_4 \leq 10 \\ 0 \leq x_5 \leq 23 \end{array}$$

$$\bar{x} = (0, 0, 1, 2, 0)$$

$$W = \begin{pmatrix} 6 \\ -11 \\ 4 \end{pmatrix}$$

Peważeli zogorzy I φQzbł:

$$-x_6 - x_7 - x_8 \rightarrow \text{MAX}$$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 + x_6 = 7 \\ -3x_1 - 2x_2 - x_3 + x_4 - x_7 = -10 \\ -x_1 + x_2 - 2x_3 - 2x_4 + x_5 + x_8 = 2 \end{cases} \quad \begin{array}{ll} 0 \leq x_1 \leq 5 & 2 \leq x_4 \leq 10 \\ 0 \leq x_2 \leq 3 & 0 \leq x_5 \leq 23 \\ 1 \leq x_3 \leq 5 & 0 \leq x_6 \leq 6 \end{array}$$

$$\bar{x} = (0, 0, 1, 2, 0, 6, 11, 4) \quad J_B = \{6, 7, 8\} \quad \begin{array}{l} 0 \leq x_7 \leq 11 \\ 0 \leq x_8 \leq 4 \end{array}$$

1 użer.

$$U = (-1, 1, -1)$$

$$\Delta = (6, 4, 2, -2, 1, 0, 0, 0) \quad j_0 = 1$$

$$L = (1, 0, 0, 0, 0, -2, -3, -1) \quad \Theta = (5, \infty, \infty, \infty, \infty, 3, \frac{11}{3}, 4)$$

$$\bar{x} = (3, 0, 1, 2, 0, 0, 2, 1) \quad J_B = \{1, 7, 8\} \quad j^* = 6$$

2 итер.

$$U = (2, 1, -1)$$

$$\Delta = (0, -5, 5, -5, 1, -3, 0, 0) \quad j_0 = 3$$

$$L = (0, 0, 1, 0, 0, 0, -1, -2) \quad \Theta = (\infty, \infty, 4, \infty, \infty, \infty, 2, \frac{1}{2})$$

$$\bar{X} = (3, 0, \frac{3}{2}, 2, 0, 0, \frac{3}{2}, 0) \quad j_* = 8$$

$$J_B = \{1, 3, 7\}$$

2 итер.3 итер.

$$U = (1, 1, 1)$$

$$\Delta = (0, 0, 0, 0, -1, -2, 0, -2)$$

$X^0 = (3, 0, \frac{3}{2}, 2, 0, 0, \frac{3}{2}, 0)$ . Однако  $x_7 \neq 0 \Rightarrow$  задача не имеет решения из-за ненулевой кон-бд именов.  $\times$

ДБ-й симпл. метод

$$-2x_1 - x_2 + x_3 + 2x_4 \rightarrow \max$$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 & +x_6 = 7 \\ -3x_1 - 2x_2 - x_3 + x_4 & +x_7 = -10 \\ x_1 - x_2 + 2x_3 - 2x_4 + x_5 & = 2 \end{cases} \quad \begin{array}{l} 0 \leq x_1 \leq 5 \\ 1 \leq x_3 \leq 5 \\ 0 \leq x_2 \leq 3 \\ 2 \leq x_4 \leq 10 \\ 0 \leq x_5 \leq 23 \end{array}$$

1 итер.

$$U = (0, 0, 0)$$

$$\delta_1 = -2 \quad \delta_2 = -1 \quad \delta_3 = 1 \quad \delta_4 = 2$$

$$x_H = (0, 0, 5, 20) \quad x_B = (2, -15, 12) \quad j_* = 6$$

$$P = (-2, -3, 1, -1, 1, 0, 0)$$

$$\delta_1 = \infty \quad \delta_2 = \infty \quad \delta_3 = \infty \quad \delta_4 = 2 \quad j_0 = 4$$

3 итер.

и

 $\Rightarrow$ 

(2)

+100

$$\begin{cases} 2 \\ 3 \\ - \end{cases}$$

$$J_B = \{4, 5, 7\}$$

2ніер.

$$U = (2, 0, 0)$$

$$\delta_1 = -6 \quad \delta_2 = -7 \quad \delta_3 = 3$$

$$x_H = (0, 0, 5) \quad x_B = (12, -5, -8) \quad j_* = 7$$

$$P = (5, 5, 0, -1, 1, 0)$$

$$\bar{\delta}_1 = \frac{6}{5} \quad \bar{\delta}_2 = \frac{7}{5} \quad \bar{\delta}_3 = \infty \quad j_0 = 1 \quad J_B = \{1, 4, 5\}$$

3 ніер.

$$U = (\frac{4}{5}, \frac{6}{5}, 0)$$

$$\delta_2 = -1 \quad \delta_3 = 3$$

$$x_2 = 0 \quad x_3 = 5 \quad x_1 = 12 \quad x_4 = -5 \quad x_5 = -8 \quad j_* = 5$$

$$P = (1, 0, 0, 1, 1)$$

$$\bar{\delta}_2 = \infty \quad \bar{\delta}_3 = \infty$$

$\Rightarrow$  3-желдік жүйе көрсетілгенде үзбіліктері и қылғалады

$\rightarrow -\infty \Rightarrow$  минимумы пусто

(2)

$$\psi(\lambda) = 7y_1 - 10y_2 + 2y_3 - v_3 - 2v_4 + 5w_1 + 3w_2 + 5w_3 + \\ + 10w_4 + 2s w_5 \rightarrow \min$$

$$\begin{cases} 2y_1 - 3y_2 + y_3 - v_1 + w_1 = -2 \\ 3y_1 - 2y_2 - y_3 - v_2 + w_2 = -1 \\ -y_1 - y_2 + 2y_3 - v_3 + w_3 = 1 \\ y_1 + y_2 - 2y_3 - v_4 + w_4 = 2 \\ y_3 - v_5 + w_5 = 0 \end{cases} \quad y_i, v_i, w_i \geq 0$$

1. 20 (1. 30α)

$$1.20 - 18B \quad 1.30\alpha - 6B.$$

①  $\psi = -2x_1 - x_2 + x_3 + 2x_4$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1 \\ -3x_1 + x_2 - x_3 - 2x_4 = -2 \\ 10x_1 + 4x_2 + 6x_4 = 6 \end{cases} \quad \begin{array}{l} -7 \leq x_1 \leq 11 \\ -4 \leq x_2 \leq 5 \\ -19 \leq x_3 \leq 11 \end{array}$$

$$\bar{x} = (-7, -4, -7, 11)$$

$$\omega = \begin{pmatrix} 9 \\ -4 \\ 26 \end{pmatrix}$$

$$-x_5 - x_6 - x_7 \rightarrow \text{MAX}$$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 + x_5 = 1 \\ -3x_1 + x_2 - x_3 - 2x_4 - x_6 = -2 \\ 10x_1 + 4x_2 + 6x_4 + x_7 = 6 \end{cases} \quad \begin{array}{l} 0 \leq x_5 \leq 9 \\ 0 \leq x_6 \leq 4 \\ 0 \leq x_7 \leq 26 \end{array}$$

$$\bar{x} = (-7, -4, -7, 11, 9, 4, 26) \quad J_B = \{5, 6, 7\}$$

1 итер.

$$U = (-1, 1, -1)$$

$$\Delta = (15, 6, 0, 9, 0, 0, 0) \quad j_0 = 2$$

$$\ell = (0, 1, 0, 0, -3, 1, -4) \quad \Theta = (\infty, 9, \infty, \infty, 3, 0, \frac{13}{2})$$

$$\bar{x} = (-7, -4, -7, 11, 9, 4, 26)$$

$$J_B = \{2, 5, 7\}$$

2 итер.

$$U = (-1, 7, -1)$$

$$\Delta = (33, 0, 6, 21, 0, 6, 0) \quad j_0 = 3$$

$$L = (0, 1, 1, 0, -2, 0, -4) \quad Q = (\infty, 9, 21, \infty, \frac{9}{2}, \infty, \frac{13}{2})$$

$$\bar{x} = (-7, \frac{1}{2}, -\frac{5}{2}, 11, 0, 4, 8) \quad J_B = \{2, 3, 7\}$$

3 итер.

$$U = (2, -2, -1)$$

$$\Delta = (0, 0, 0, 0, -3, -3, 0) \quad j_0 = 6$$

$$L = (0, \frac{1}{2}, \frac{3}{2}, 0, 0, -1, -2) \quad Q = (\infty, 9, \frac{23}{3}, \infty, \infty, 4, 4)$$

$$\bar{x} = (-7, \frac{5}{2}, \frac{7}{2}, 11, 0, 0, 0) \quad J_B = \{2, 3, 6\}$$

4 итер.

$$U = (-1, 1, \frac{1}{2})$$

$$\Delta = (0, 0, 0, 0, 0, 0, -\frac{1}{2}) \Rightarrow X^0 = (-7, \frac{5}{2}, \frac{3}{2}, 11, 0, 0, 0)$$

Все искусств. пер-е запнулись  $\Rightarrow$  переход к решению буферной задачи

+

$$2x_1 - x_2 + x_3 + 2x_4 \rightarrow \text{MAX}$$

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1 & -7 \leq x_1 \leq 11 & -19 \leq x_3 \leq 11 \\ -3x_1 + x_2 - x_3 - 2x_4 + x_6 = -2 & -4 \leq x_2 \leq 5 & 0 \leq x_6 \leq 0 \\ 10x_1 + 4x_2 + 6x_4 = 6 & -7 \leq x_3 \leq 14 \end{cases}$$

$$\bar{x} = (-7, \frac{5}{2}, \frac{3}{2}, 11, 0) \quad J_B = \{2, 3, 6\}$$