# 110

а) вариант 5

б) вариант 10

| хим эл-ты | $A_1$ | $A_2$ | $A_3$ | $A_4$ | хим состав сплава (%) | заданное кол-во сплава (т) |
|---|---|---|---|---|---|---|
| $B_1$ | 50 | 10 | 10 | 20 | 42 | 1200 |
| $B_2$ | 40 | 10 | 20 | 30 | 58 | |
| | содержание (%) хим эл-тов в шихт. материале | | | | | |
| стоимость 1т шихты (де) | 105 | 125 | 75 | 40 | | |

## Решение:

1) $A_1 \to x_1$, $A_2 \to x_2$, $A_3 \to x_3$, $A_4 \to x_4$

Составим целевую ф-цию:

$$105x_1 + 125x_2 + 75x_3 + 40x_4 \to \min$$

$$\begin{cases} 5x_1 + x_2 + x_3 + 2x_4 = 5040 \\ 4x_1 + x_2 + 2x_3 + 3x_4 = 6960 \end{cases}$$

$$-x_1 + x_3 + x_4 = 1920$$

$$x_3 = 1920 - x_4 + x_1 > 0$$

$6x_1 + x_2$

$x_2 = 3$

$105x_1$

$\cdot(1920 -$

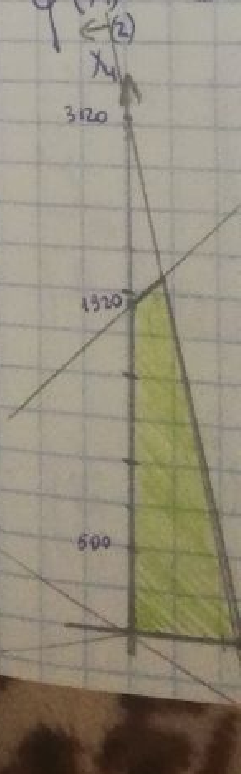
$105x_1$

$-75x_4 +$

$\varphi(x) = -570$

$\begin{cases} -x_4 + x_1 \\ -x_4 - 6x_1 \end{cases}$

$\varphi(X) = 57$



$$6x_1 + x_2 + x_4 = 3120$$

$$x_2 = 3120 - x_4 - 6x_1 > 0$$

$$105x_1 + 125(3120 - x_4 - 6x_1) + 75\cdot$$

$$\cdot(1920 - x_4 + x_1) + 40x_4 \to \min$$

$$105x_1 + 390000 - 125x_4 - 750x_1 + 144000 -$$

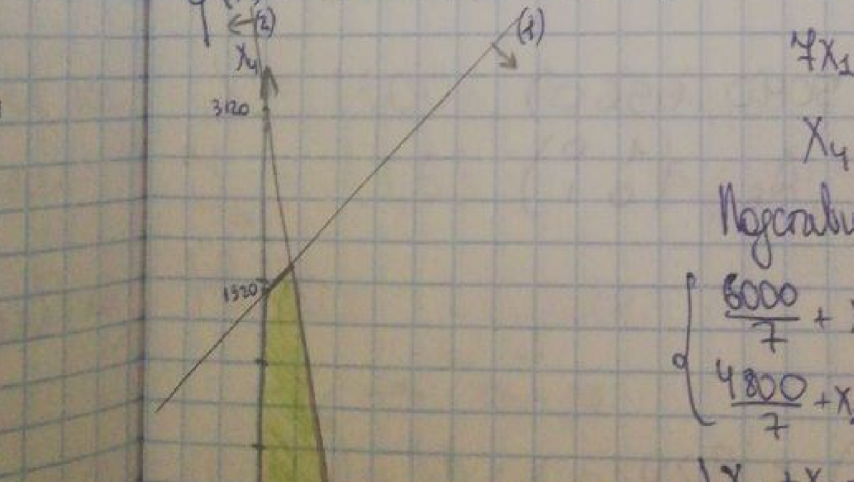
1200

$$-75x_4 + 75x_1 + 40x_4 \to \min$$

$$\varphi(x) = -570x_1 - 160x_4 + 534000 \to \min$$

$$\begin{cases} -x_4 + x_1 \geq -1920 \\ -x_4 - 6x_1 \geq -3120 \end{cases} \qquad \begin{cases} x_4 - x_1 \leq 1920 \quad (1) \\ x_4 + 6x_1 \leq 3120 \quad (2) \end{cases} \qquad \begin{matrix} x_1 \geq 0 \\ x_4 \geq 0 \end{matrix}$$

$$\varphi(x) = 570x_1 + 160x_4 \to \max$$




$$7x_1 = 1200 \Rightarrow x_1 = \frac{1200}{7}$$

$$x_4 = \frac{13440}{7} + \frac{1200}{7} = \frac{14640}{7}$$

Подставим $x_1, x_4$ в исход систему:

$$\begin{cases} \frac{6000}{7} + x_2 + x_3 + \frac{29280}{7} = \frac{35280}{7} \\ \frac{4800}{7} + x_2 + 2x_3 + \frac{43920}{7} = \frac{48720}{7} \end{cases}$$

$$\begin{cases} x_2 + x_3 = 0 \\ x_2 + 2x_3 = 0 \end{cases} \Rightarrow x_2 = x_3 = 0$$

$$X^0 = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}\right)$$

$6X_1 + X_2 + X_4 = 3120$

$X_2 = 3120 - X_4 - 6X_1 > 0$

$105X_1 + 125(3120 - X_4 - 6X_1) + 75 \cdot$

$\cdot (1920 - X_4 + X_1) + 40X_4 \to \min$

здайные кол-во сплава (т)

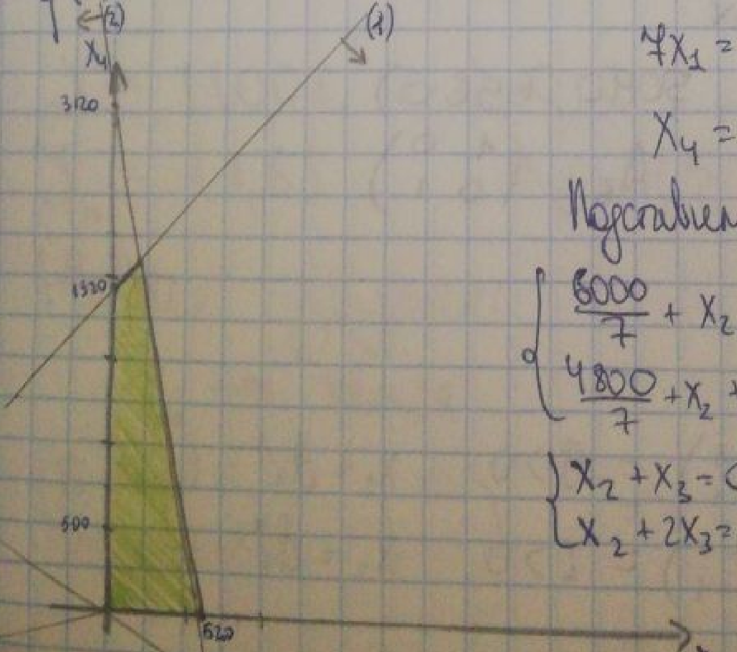
$105X_1 + 390000 - 125X_4 - 750X_1 + 144000 -$

1200 $- 75X_4 + 75X_1 + 40X_4 \to \min$

$\varphi(x) = -570X_1 - 160X_4 + 534000 \to \min$

$$\begin{cases} -X_4 + X_1 \geq -1920 \\ -X_4 - 6X_1 \geq -3120 \end{cases} \qquad \begin{cases} X_4 - X_1 \leq 1920 \quad (1) \\ X_4 + 6X_1 \leq 3120 \quad (2) \end{cases} \qquad \begin{matrix} X_1 \geq 0 \\ X_4 \geq 0 \end{matrix}$$

$\varphi(X) = 570X_1 + 160X_4 \to \max$

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Подставим $X_1, X_4$ в исход систему:

$$\begin{cases} \frac{6000}{7} + X_2 + X_3 + \frac{29280}{7} = \frac{35280}{7} \\ \frac{4800}{7} + X_2 + 2X_3 + \frac{43920}{7} = \frac{48720}{7} \end{cases}$$

$$\begin{cases} X_2 + X_3 = 0 \\ X_2 + 2X_3 = 0 \end{cases} \Rightarrow X_2 = X_3 = 0$$

$X^0 = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}\right)$

$X_4$

Симплекс - метод:

$\varphi(x) = -105x_1 - 125x_2 - 75x_3 - 40x_4 \to max$

$$\begin{cases} 5x_1 + x_2 + x_3 + 2x_4 = 5040 \\ 4x_1 + x_2 + 2x_3 + 3x_4 = 6960 \end{cases} \qquad 0 \le x_i \le M \quad i = \overline{1,4}$$

$\tilde{x} = (0,0,0,0)$ $\qquad 0 \le x_5 \le 5040$

$\omega = \begin{pmatrix} 5040 \\ 6960 \end{pmatrix}$ $\qquad 0 \le x_6 \le 6960$

Переходим к задаче I фазы:

$$\begin{cases} 5x_1 + x_2 + x_3 + 2x_4 + x_5 = 5040 \\ 4x_1 + x_2 + 2x_3 + 3x_4 + x_6 = 6960 \end{cases}$$

$-x_5 - x_6 \to max$

$x^1 = (0, 0, 0, 0, 5040, 6960)$

$J_Б = \{5, 6\}$ $\qquad A_Б = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Итерация 1:

$u = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\Delta_1 = 0 - (5\ 4)\begin{pmatrix} -1 \\ -1 \end{pmatrix} = 9 > 0 \qquad x_1 = d_1^* \quad (-)$

$\Delta_2 = 0 - (1\ 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix} = 2 > 0 \qquad x_2 = d_2^* \quad (-)$

$\Delta_3 = 3 > 0 \quad x_3 = d_3^* \quad (-)$

$\Delta_4 = 5 > 0 \quad x_4 = d_4^* \quad (-)$

$j_0 = 1 \Rightarrow$

$l_1 = 1, \; l_2 = l_3 = l_4 = 0$

$$\begin{cases} 5 + l_5 = 0 \\ 4 + l_6 = 0 \end{cases} \Rightarrow \begin{matrix} l_5 = -5 \\ l_6 = -4 \end{matrix}$$

$\Theta_1 = \Theta_2 = \Theta_3 = \Theta_4 = \infty$

$\Theta_5 = \frac{0-5040}{-5} = 1008 \qquad \Theta_6 = \frac{0-6960}{-4} = 1740$

$\Theta^0 = \Theta_5 = 1008$

$\Rightarrow j_* = 5$

$X^2 = (1008, 0, 0, 0, 0, 2928)$

$J_Б = \{1, 6\} \qquad A_Б = \begin{pmatrix} 5 & 0 \\ 4 & 1 \end{pmatrix}$

Новая задача:

$-x_6 \to max$

$\begin{pmatrix} 5 & 0 \\ 4 & 1 \end{pmatrix}^T \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \Rightarrow u = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\Delta_2 = 1 > 0 \quad x_2 = d_2^* \; (-)$

$\Delta_3 = 2 > 0 \quad x_3 = d_3^* \; (-)$

$\Delta_4 = 3 > 0 \quad x_4 = d_4^* \; (-)$

$\Delta_5 = 0 \quad (+)$

$j_0 = 4 \Rightarrow$

$l_4 = 1, \quad l_2 = l_3 = l_5 = 0$

$\begin{cases} 5l_1 + 2 = 0 \\ 4l_1 + 3 + l_6 = 0 \end{cases} \quad \begin{cases} l_1 = -\frac{2}{5} \\ l_6 = -\frac{7}{5} \end{cases}$

$\Theta_2 = \Theta_3 = \Theta_5 = \infty = \Theta_4$

$\Theta_1 = \frac{0 - 1008}{-2/5} = 2520$

$\Theta_6 = \frac{0 - 2928}{-7/5} = \frac{14640}{7} \approx 2091$

$\Theta^0 = \Theta_6 = \frac{14640}{7}$

$\Rightarrow j_* = 6$

$X^3 = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}, 0, 0\right)$

$Y_5 = \{1, 4\} \qquad A_b = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$

$X^3$ – НБП, т.к занулились искусств. переменные => переходим ко 2 фазе.

$$\varphi(X) = -105X_1 - 125X_2 - 75X_3 - 40X_4 \to max$$

$$\begin{cases} 5X_1 + X_2 + X_3 + 2X_4 = 5040 \\ 4X_2 + X_2 + 2X_3 + 3X_4 = 6960 \end{cases} \quad 0 \le X_i \le M$$

$$X = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}\right)$$

$$J_Б = \{1, 4\}$$

$$\begin{pmatrix} 5 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -105 \\ -40 \end{pmatrix}$$

$$\begin{cases} 5u_1 + 4u_2 = -105 \\ 2u_1 + 3u_2 = -40 \end{cases} \qquad \begin{cases} -7u_2 = -10 \\ 2u_1 + 3u_2 = -40 \end{cases}$$

$$\begin{cases} u_2 = \frac{10}{7} \\ 2u_1 + \frac{30}{7} = -\frac{210}{7} \end{cases} \Rightarrow u = \begin{pmatrix} -120/7 \\ 10/7 \end{pmatrix}$$

$$\Delta_2 = -125 - (1\ 1)\begin{pmatrix} -120/7 \\ 10/7 \end{pmatrix} = -\frac{745}{7} < 0 \quad X_2 = d_{2*} \ (+)$$

$$\Delta_3 = -75 - (1, 2)\begin{pmatrix} -120/7 \\ 10/7 \end{pmatrix} = -\frac{385}{7} < 0 \quad X_3 = d_{3*} \ (+)$$

$$\Rightarrow X^0 = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}\right)$$

(2) Записать двойств. + решить двойств. методом исходную

Двойств. задача для исходной:

$$5040y_1 + 6960y_2 \to max$$

$$\begin{cases} 5y_1 + 4y_2 \geq 105 \\ y_1 + y_2 \geq 125 \\ y_1 + 2y_2 \geq 75 \\ 2y_1 + 3y_2 \geq 40 \end{cases}$$

$$-105x_1 - 125x_2 - 75x_3 - 40x_4 \to max$$

$$\begin{cases} 5x_1 + x_2 + x_3 + 2x_4 + x_5 = 5040 \\ 4x_1 + x_2 + 2x_3 + 3x_4 + x_6 = 6960 \end{cases}$$

$0 \leq x_i \leq M \quad i = \overline{1,4}$

$0 \leq x_5 \leq 0$

$0 \leq x_6 \leq 0$

$J_Б = \{5, 6\} \qquad A_б = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Итерация 1:

1) $u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2) $\delta_{u_1} = -105 < 0$

$\delta_{u_2} = -125 < 0$

$\delta_{u_3} = -75 < 0$

$\delta_{u_4} = -40 < 0$

3) $x_{u_1} = x_{u_2} = x_{u_3} = x_{u_4} = 0$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{u_5} \\ x_{u_6} \end{pmatrix} = \begin{pmatrix} 5040 \\ 6960 \end{pmatrix}$

$x_{u_5} = 5040 \quad (-)$

$x_{u_6} = 6960 \quad (-)$

4) $j_0 = 6$

5) $(0 \;\; 1) \begin{pmatrix} p_{u_1} \\ p_{u_2} \end{pmatrix} = -1$

$(1 \;\; 0) \begin{pmatrix} p_{u_1} \\ p_{u_2} \end{pmatrix} = 0 \quad \Rightarrow p = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

6) $p\delta_1 = -(0 \;\; -1) \begin{pmatrix} 5 \\ 4 \end{pmatrix} = 4$

$p\delta_2 = 1$

$p\delta_3 = 2$

$p_{\delta_4} = 3$

4) $\delta_1 = -\frac{\delta_{..}}{p_{\delta_1}} = \frac{105}{4}$

$\delta_2 = 125$

$\delta_3 = \frac{75}{2}$

$\delta_4 = \frac{40}{3}$

$\Rightarrow \delta' = \delta_4$

$y_Б = \{4, 5\}$ $\qquad A_Б = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$

Итерация 2

1) $(2\ 3)\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = -40$ $\Rightarrow u = \begin{pmatrix} 0 \\ -40/3 \end{pmatrix}$

$u_1 = 0$

2) $\delta_{u_1} = -105 - (5\ 4)\begin{pmatrix} 0 \\ -40/3 \end{pmatrix} = -105 - \frac{160}{3} = -\frac{475}{3} < 0$

$\delta_{u_2} = -125 - (1\ 1)\begin{pmatrix} 0 \\ -40/3 \end{pmatrix} = -\frac{335}{3} < 0$

$\delta_{u_3} = -75 - (1\ 2)\begin{pmatrix} 0 \\ -40/3 \end{pmatrix} = -\frac{225}{3} + \frac{80}{3} < 0$

3) $x_{u_1} = x_{u_2} = x_{u_3} = 0$

$\begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} x_{u_4} \\ x_{u_5} \end{pmatrix} = \begin{pmatrix} 5040 \\ 6960 \end{pmatrix}$

$\begin{cases} 2x_{44} + x_{45} = 5040 \\ 3x_{44} = 6960 \end{cases}$ $x_{45} = 400 \ (-)$

$x_{44} = 2320 \ (+)$

$x = (0, 0, 0, 2320, 400)$

$\sigma' = \sigma_4$

4) $j_0 = 5$

5) $(2 \ 3)\begin{pmatrix} p_{41} \\ p_{42} \end{pmatrix} = 0$ $3p_{42} = 2$

$(1 \ 0)\begin{pmatrix} p_{41} \\ p_{42} \end{pmatrix} = -1$ $p_{41} = -1$

$= \begin{pmatrix} 0 \\ -40/3 \end{pmatrix}$ $p_4 = \begin{pmatrix} -1 \\ 2/3 \end{pmatrix}$

$105 - \frac{160}{3} =$

6) $p_{\delta_1} = -(-1 \ \ 2/3)\begin{pmatrix} 5 \\ 4 \end{pmatrix} = 5 - \frac{8}{3} = \frac{7}{3}$

$p_{\delta_2} = -(-1 \ \ 2/3)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 - \frac{2}{3} = \frac{1}{3}$

$\frac{335}{3} < 0$

$p_{\delta_3} = -(-1 \ \ 2/3)\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 - \frac{4}{3} = -\frac{1}{3}$

$\frac{225}{3} + \frac{80}{3} < 0$

7) $\sigma_1 = -\frac{\delta_1}{p_{\delta_1}} = \frac{475}{7}$

$\sigma_2 = -\frac{\delta_2}{p_{\delta_2}} = 335$ $\Rightarrow \sigma' = \sigma_1$

$\sigma_3 = +\infty$

$J_б = \{1, 4\}$

<u>Итерация 3:</u>

1) $\begin{pmatrix} 5 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -105 \\ -40 \end{pmatrix}$

$\begin{cases} 5u_1 + 4u_2 = -105 \\ 2u_1 + 3u_2 = -40 \end{cases}$ $\begin{cases} 10u_1 + 8u_2 = -210 \\ 2u_1 + 3u_2 = -40 \end{cases}$

$\begin{cases} -7u_2 = -10 \\ u_1 = -\frac{155}{7} \end{cases}$

$u = \begin{pmatrix} -155/7 \\ 10/7 \end{pmatrix}$

2) $\delta_{u_2} = -125 - (1\ 1) \begin{pmatrix} -155/7 \\ 10/7 \end{pmatrix} = -125 + \frac{155}{7} - \frac{10}{7} = -\frac{740}{7} < 0$

$\delta_{u_3} = -75 - (1, 2) \begin{pmatrix} -155/7 \\ 10/7 \end{pmatrix} = -\frac{525}{7} + \frac{155}{7} - \frac{20}{7} =$

$= -\frac{400}{7} < 0$

3) $x_{u_2} = x_{u_3} = 0$

$\begin{cases} 5x_{u_1} + 2x_{u_4} = 5040 \\ 4x_{u_1} + 3x_{u_4} = 6960 \end{cases}$ $\begin{cases} 5x_{u_1} + 2x_{u_4} = 5040 \\ 8x_{u_1} + 6x_{u_4} = 13920 \end{cases}$

$u_1 + 8u_2 = -210$

$+ 3u_2 = -40$

$125 + \frac{155}{7} - \frac{10}{7} = -\frac{740}{7}$

$50$

$\frac{325}{7} + \frac{155}{7} - \frac{20}{7} =$

$_{u_4} = 5040$

$_{u_4} = 13920$

$$\begin{cases} 5x_{u_1} + 2x_{u_4} = 5040 \\ -7x_{u_1} = -1200 \end{cases}$$

$x_{u_1} = \frac{1200}{7}$ (+)

$x_{u_4} = \frac{14640}{7}$ (+)

$\Rightarrow x = \left(\frac{1200}{7}, 0, 0, \frac{14640}{7}\right) \Rightarrow$

$\underline{x = X^0}$

–1.20 (1.30 a) 2 варианта

18 варианта ① Используя I фазу, построить НБП и записать задачу II фазы

$\varphi = -2x_1 - x_2 + x_3$

$$\begin{cases} x_1 + 2x_2 + x_3 = 3 \\ x_1 + 3x_2 - 4x_3 = -2 \\ x_2 - 5x_3 = -5 \end{cases}$$

$-20 \le x_1 \le 2$
$0 \le x_2 \le 10$
$1 \le x_3 \le 3$

$-2x_1 - x_2 + x_3 \to max$

$\tilde{x} = (0, 0, 0)$

$\omega = b - A\tilde{x}$

$\omega_1 = 3 > 0$

$\omega_2 = -2 < 0 \quad \Rightarrow \omega = \begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix}$

$\omega_3 = -5 < 0$

$0 \le x_4 \le 3$
$0 \le x_5 \le 2$
$0 \le x_6 \le 5$

Решаем след. задачу:

$-x_4 - x_5 - x_6 \to max$

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 3 \\ x_1 + 3x_2 - 4x_3 - x_5 = -2 \\ x_2 - 5x_3 - x_6 = -5 \end{cases}$$

оить

фазы

2

10

3

3

2

5

$-10 \le x_1 \le 2$ $\quad 1 \le x_3 \le 3$

$0 \le x_2 \le 10$ $\quad x_{\_} \ge 0 \quad i = \overline{4,6}$

$J_Б = \{4, 5, 6\}$ $\quad A_Б = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$x^1 = (0, 0, 0, 3, 2, 5)$

<u>Итерация 1</u>

1) $u = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

2) $\Delta_1 = 0 - (1\ 1\ 0)\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 0 \quad (+)$

$\Delta_2 = 0 - (2\ 3\ 1)\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 2 - 3 - 1 = -2 < 0$

$x_2 = d_{2*} (+)$

$\Delta_3 = 0 - (1\ -4\ -5)\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 1 + 4 + 5 = 10 > 0 \quad x_3 = d_3^* \ (-)$

3) $j_0 = 3$

4) $l_3 = 1, \quad l_2 = l_1 = 0$

$\begin{cases} 1 + l_4 = 0 \\ -4 - l_5 = 0 \\ -5 - l_6 = 0 \end{cases} \Rightarrow \begin{matrix} l_4 = -1 \\ l_5 = -4 \\ l_6 = -5 \end{matrix}$

5) $l = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ -4 \\ -5 \end{pmatrix}$ $\quad \Theta_1 = \Theta_2 = \infty$

$\Theta_3 = 5 \quad \Theta_4 = 3 \quad \Theta_5 = \frac{1}{2}$

$\Theta_6 = 1$

$\Rightarrow \theta^0 = \theta_5 = \frac{1}{2}$

$\bar{x}_1 = 0, \ \bar{x}_2 = 0 \quad \bar{x}_3 = \frac{1}{2} \quad \bar{x}_4 = \frac{5}{2} \quad \bar{x}_5 = 0 \quad \bar{x}_6 = \frac{5}{2}$

$x^2 = (0, 0, \frac{1}{2}, \frac{5}{2}, 0, \frac{5}{2})$

$J_Б = \{3, 4, 6\}$

Новая задача:

$-x_4 - x_6 \to \max$

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 3 \\ x_1 + 3x_2 - 4x_3 + x_5 = -2 \\ x_2 - 5x_3 - x_6 = -5 \end{cases}$$

$-20 \le x_1 \le 2$
$0 \le x_2 \le 10$
$1 \le x_3 \le 3$
$0 \le x_4 \le 3$
$0 \le x_5 \le 0$
$0 \le x_6 \le 5$

$A_Б = \begin{pmatrix} 1 & 1 & 0 \\ -4 & 0 & 0 \\ -5 & 0 & -1 \end{pmatrix}$

1) $u = \begin{pmatrix} -1 \\ -3/2 \\ 1 \end{pmatrix}$

2) $\Delta_1 = 0 - (-1 \ \ -3/2 \ \ 1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 + \frac{3}{2} = \frac{5}{2} > 0$

$x_1 = d_1^* \ (-)$

$\Delta_2 = 0 - (-1 \quad -\frac{3}{2} \quad 1)\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 2 + \frac{9}{2} - 1 = \frac{11}{2} > 0$

$x_2 = d_2^* \ (-)$

$\frac{5}{2} \quad \bar{x}_5 = 0 \quad \bar{x}_6 = \frac{5}{2}$

3) $j_0 = 2$

4) $l_2 = 1, \quad l_1 = l_5 = 0$

$$\begin{cases} 2 + l_3 + l_4 = 0 \\ 3 - 4l_3 = 0 \\ 1 - 5l_3 - l_6 = 0 \end{cases} \qquad \begin{aligned} l_4 &= -\frac{11}{4} \\ l_3 &= 3/4 \\ l_6 &= 1 - \frac{15}{4} = -\frac{11}{4} \end{aligned}$$

$$l = \begin{pmatrix} 0 \\ 1 \\ 3/4 \\ -11/4 \\ 0 \\ -11/4 \end{pmatrix}$$

$= 3$

$= -2$

$= -5$

$-20 \leq x_1 \leq 2$
$0 \leq x_2 \leq 10$
$1 \leq x_3 \leq 3$
$0 \leq x_4 \leq 3$
$0 \leq x_5 \leq 0$
$0 \leq x_6 \leq 5$

5) $$\theta = \begin{pmatrix} \infty \\ 10 \\ 10/3 \\ 10/11 \\ \infty \\ 10/11 \end{pmatrix} \Rightarrow \theta^0 = \theta_6 = 10/11$$

$x^3 = (0, 10/11, 13/11, 0, 0, 0)$

$J_б = \{2, 3, 4\}$

$\frac{3}{2} = \frac{5}{2} > 0$

$x_1 = d_1^* \ (-)$

Новая задача:

$$-x_4 \to \max$$

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 3 \\ x_1 + 3x_2 - 4x_3 + x_5 = -2 \\ x_2 - 5x_3 + x_6 = -5 \end{cases}$$

$$\begin{pmatrix} -20 \le x_1 \le 2 \\ 0 \le x_2 \le 10 \\ 1 \le x_3 \le 3 \\ 0 \le x_4 \le 0 \\ 0 \le x_5 \le 0 \\ 0 \le x_6 \le 0 \end{pmatrix}$$

Все искусств переменные занулились $\Rightarrow$ переходим к решению буферной задачи

$$-2x_1 - x_2 + x_3 \to \max$$

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 3 \\ x_1 + 3x_2 - 4x_3 = -2 \\ x_2 - 5x_3 = -5 \end{cases}$$

$$-20 \le x_1 \le 2$$
$$0 \le x_2 \le 10$$
$$1 \le x_3 \le 3$$
$$0 \le x_4 \le 0$$

мние
дим
дачи

$\leq 2$
$_2 \leq 10$
$_3 \leq 3$
$_4 \leq 0$

# 1.20 (1.30 б)

1.20 – 18 вариант

1.30 (б) – 4 вариант

$\varphi = -2x_1 - x_2 + x_3 + 2x_4 \to ?$

$$\begin{cases} -2x_1 - 4x_2 - 3x_3 + 15x_4 = 7 \\ 20x_1 + 30x_2 + x_3 + x_4 \geq 30 \\ 7x_1 + 10x_2 + 2x_3 + 7x_4 = 10 \end{cases}$$

$0 \leq x_1 \leq 5$
$0 \leq x_2 \leq 3$
$1 \leq x_3 \leq 5$
$2 \leq x_4 \leq 10$

(1) Решить прямым СМ и двойственным

$-2x_1 - x_2 + x_3 + 2x_4 \to max$

$$\begin{cases} -2x_1 - 4x_2 - 3x_3 + 15x_4 = 7 \\ -20x_1 - 30x_2 - x_3 - x_4 + x_5 = -30 \\ 7x_1 + 10x_2 + 2x_3 + 7x_4 = 10 \end{cases}$$

(в (2) для ноб задачи построена двойственная)

$0 \leq x_1 \leq 5$ $\quad$ $1 \leq x_3 \leq 5$

$0 \leq x_2 \leq 3$ $\quad$ $2 \leq x_4 \leq 10$

$d_5^* = max(-30 + 20x_1 + 30x_2 + x_3 + x_4) =$

$= -30 + 100 + 90 + 5 + 10 = 175$

$0 \leq x_5 \leq 175$

$\tilde{x} = (0, 0, 1, 2, 0)$

$\omega_1 = 7 - 27 = -20$

$\omega_2 = -30 + 3 = -27$

$\omega_3 = 10 - 16 = -6$

Решаем задачу I фазы:

$-x_6 - x_7 - x_8 \to max$

$$\begin{cases} -2x_1 - 4x_2 - 3x_3 + 15x_4 - x_6 = 7 \\ -20x_1 - 30x_2 - x_3 - x_4 + x_5 - x_7 = -30 \\ 7x_1 + 10x_2 + 2x_3 + 7x_4 - x_8 = 10 \end{cases}$$

$0 \le x_1 \le 5$ $\quad 1 \le x_3 \le 5$ $\quad 0 \le x_5 \le 175$ $\quad 0 \le x_7 \le 27$

$0 \le x_2 \le 3$ $\quad 2 \le x_4 \le 10$ $\quad 0 \le x_6 \le 20$ $\quad 0 \le x_8 \le 6$

$x' = (0, 0, 1, 2, 0, 20, 27, 6)$

$J_Б = \{6, 7, 8\}$

Итерация 1:

1) $u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

2) $\Delta_1 = 0 - (1\ 1\ 1)\begin{pmatrix} -2 \\ -20 \\ 7 \end{pmatrix} = 2 + 20 - 7 = 15 > 0 \quad x_1 = l_1^* \ (-)$

3) $j_0 = 1$

4) $l_1 = 1, \ l_2 = l_3 = l_4 = l_5 = 0$

$$\begin{cases} -2 - l_6 = 0 \\ -20 - l_7 = 0 \\ 7 - l_8 = 0 \end{cases}$$

$l_6 = -2$

$l_7 = -20$

$l_8 = 7$

5) $\theta_1 = 5$

$\theta_2 = \theta_3 = \theta_4 = \theta_5 = \infty$

$\theta_6 = \frac{0-20}{-2} = 10$ $\theta_7 = \frac{0-27}{-20} = \frac{27}{20}$

$\theta_8 = 0$

$\Rightarrow \theta^0 = \theta_8 = 0$

6) $x^2 = (0, 0, 1, 2, 0, 20, 27, 6)$

$y_5 = \{1, 6, 7\}$

<u>Итерация 2:</u>

1) $\begin{pmatrix} -2 & -20 & 7 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \Rightarrow u = \begin{pmatrix} 1 \\ 1 \\ 22/7 \end{pmatrix}$

2) $\Delta_2 = 0 - (1 \ 1 \ 22/7) \begin{pmatrix} -4 \\ -30 \\ 10 \end{pmatrix} = 4 + 30 - \frac{227}{7} =$

$= \frac{28 + 210 - 227}{7} = \frac{8}{7} > 0$ $x_2 = d_2^*$ $(-)$

3) $j_0 = 2$

---

$= 7$

$-x_7$ $= -30$

$-x_8 = 10$

$0 \le x_7 \le 27$

$0 \le x_8 \le 6$

$7 = 15 > 0$ $x_1 = d_1^*$ $(-)$

4) $l_2 = 1, \quad l_3 = l_4 = l_5 = l_8 = 0$

$$\begin{cases} -2l_1 - l_6 = 4 \\ -20l_1 - l_7 = 30 \\ 7l_1 = -10 \end{cases} \qquad \begin{cases} l_6 = -4 - \frac{20}{7} \\ l_7 = -30 - \frac{200}{7} \\ l_1 = -\frac{10}{7} \end{cases}$$

$l_1 = -\frac{10}{7}$

$l_6 = -8/7$

$l_7 = -10/7$

5) $\Theta_3 = \Theta_4 = \Theta_5 = \Theta_8 = \infty$

$\Theta_2 = 3$

$\Theta_1 = 0$

$\Theta_6 = \frac{0-20}{8/7} = \frac{140}{8}$

$\Theta_7 = \frac{27 \cdot 7}{10}$

$\Theta^0 = \Theta_1 = 0$

6) $X^3 = (0, 0, 1, 2, 0, 20, 27, 6)$

$J_6 = \{2, 6, 7\} \qquad A_6 = \begin{pmatrix} -4 & -1 & 0 \\ -30 & 0 & -1 \\ 10 & 0 & 0 \end{pmatrix}$

Итерация 3:

$$\begin{pmatrix} -4 & -30 & 10 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \Rightarrow u = \begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix}$$

$$\Delta_1 = 0 - (-2\ {-20}\ 7)\begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix} = 2 + 20 - 23{,}8 = -1{,}8 < 0 \quad x_1 = d_{1*}\ (+)$$

$$\Delta_3 = 0 - (3\ {-1}\ 2)\begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix} = \frac{15 + 5 - 34}{5} = -2{,}8 < 0 \quad x_3 = d_{3*}\ (+)$$

$$\Delta_4 = 0 - (15\ {-1}\ 7)\begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix} = -15 + 1 - 23{,}8 = -37{,}8 < 0 \quad x_4 = d_{4*}\ (+)$$

$$\Delta_5 = 0 - (0\ 1\ 0)\begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix} = -1 < 0 \quad x_5 = d_{5*}\ (+)$$

$$\Delta_8 = -1 - (0\ 0\ {-1})\begin{pmatrix} 1 \\ 1 \\ 34/10 \end{pmatrix} = \frac{34}{10} - 1 = \frac{24}{10} > 0 \quad x_8 = d_8^*\ (+)$$

$\Rightarrow$ план $x^3$ является оптимальным для задачи. Однако $x_4^* \neq 0 \Rightarrow$ задача не имеет решения из-за пустоты мн-ва планов.

# Двойств симплекс-метод

$$-2x_1 - x_2 + x_3 + 2x_4 \to \max$$

$$\begin{cases} -2x_1 - 4x_2 - 3x_3 + 15x_4 + x_6 = 7 \\ -20x_1 - 30x_2 - x_3 - x_4 + x_5 = -30 \\ 7x_1 + 10x_2 + 2x_3 + 7x_4 + x_7 = 10 \end{cases}$$

$0 \le x_1 \le 5 \quad 1 \le x_3 \le 5 \quad 0 \le x_5 \le 175 \quad 0 \le x_7 \le 0$

$0 \le x_2 \le 3 \quad 2 \le x_4 \le 10 \quad 0 \le x_6 \le 0$

$J_Б = \{5, 6, 7\}$

Итерация 1

1) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

2) $\delta_{u_1} = -2 < 0 \qquad \delta_{u_3} = 1 > 0$

$\delta_{u_2} = -1 < 0 \qquad \delta_{u_4} = 2 > 0$

3) $\varkappa_{u_1} = 0 \quad \varkappa_{u_2} = 0 \quad \varkappa_{u_3} = 5 \quad \varkappa_{u_4} = 10$

$\varkappa_{u_5} = -30 + 5 + 10 = -15 \ (-)$

$\varkappa_{u_6} = 7 + 15 - 150 = -128 \ (-)$

$\varkappa_{u_7} = 10 - 10 - 70 = -70 \ (-)$

4) $j_* = 6$

$P = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

5) $p_{\delta_1} = -(1\ 0\ 0)\begin{pmatrix}-2\\-20\\7\end{pmatrix} = 2$

$p_{\delta_2} = 4$

$p_{\delta_3} = 3$

$p_{\delta_4} = -15$

6) $\sigma_1 = -\frac{\delta_{u_1}}{p_{\delta_1}} = -\frac{-2}{2} = 1$ $\Rightarrow \sigma^1 = \sigma_4$

$\sigma_2 = \frac{1}{4}$, $\sigma_3 = +\infty$, $\sigma_4 = \frac{2}{15}$

$J_б = \{4, 5, 7\}$ $A_б = \begin{pmatrix}15 & 0 & 0\\-1 & 1 & 0\\7 & 0 & 1\end{pmatrix}$

Итерация 2:

1) $(15\ -1\ 7)\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix} = 2$ $\Rightarrow u = \begin{pmatrix}2/15\\0\\0\end{pmatrix}$

$u_2 = 0$

$u_3 = 0$

2) $\delta_{u_1} = -2 - (-2\ -20\ 7)\begin{pmatrix}2/15\\0\\0\end{pmatrix} = -\frac{30}{15} + \frac{4}{15} = -\frac{26}{15} < 0$

$\delta_{u_2} = -1 - (-4\ -30\ 10)\begin{pmatrix}2/15\\0\\0\end{pmatrix} = -\frac{15}{15} + \frac{8}{15} = -\frac{7}{15} < 0$

$\delta_{u_3} = 1 - (-3\ -1\ 2)\begin{pmatrix}2/15\\0\\0\end{pmatrix} = \frac{15}{15} + \frac{6}{15} = \frac{21}{15} > 0$

3) $æ_{u_1} = 0$, $æ_{u_2} = 0$, $æ_{u_3} = 5$

$$\begin{cases} 15x_{u_4} = 22 \\ -x_{u_4} + x_{u_5} = -25 \\ 7x_{u_4} + x_{u_7} = 0 \end{cases}$$

$x_{u_4} = \frac{22}{15}$ (-)

$x_{u_7} = -\frac{154}{15}$ (-)

$x_{u_5} = -25 + \frac{22}{15} = -\frac{353}{15}$ (-)

4) $j^* = 7$

$(0\ 0\ 1)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 1 \Rightarrow p_{u_3} = 1$

$(0\ 1\ 0)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0 \Rightarrow p_{u_2} = 0$

$(15\ -1\ 7)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0 \quad p_{u_1} = -\frac{7}{15}$

5) $p_{\delta_1} = -(-7/15\ 0\ 1)\begin{pmatrix} -2 \\ 20 \\ 7 \end{pmatrix} = -\frac{14}{15} - 7 = -\frac{119}{15}$

$p_{\delta_2} = -(-7/15\ 0\ 1)\begin{pmatrix} -4 \\ -30 \\ 10 \end{pmatrix} = -\frac{28}{15} - 10 = -\frac{178}{15}$

$p_{\delta_3} = -(-7/15\ 0\ 1)\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} = -\frac{21}{15} - 2 = -\frac{51}{15}$

6) $\tilde{\sigma}_1 = +\infty$

$\tilde{\sigma}_2 = +\infty \Rightarrow \tilde{\sigma}^1 = \tilde{\sigma}_3$

$\tilde{\sigma}_3 = -\frac{21}{15} \cdot \frac{-15}{51} = \frac{21}{51}$

$Y_Б = \{3, 4, 5\}$

Итерация 3:

1) $\begin{cases} -3u_1 - u_2 + 2u_3 = 1 \\ 15u_1 - u_2 + 7u_3 = 2 \\ u_2 = 0 \end{cases}$ $\begin{cases} -3u_1 + 2u_3 = 1 \\ 15u_1 + 7u_3 = 2 \end{cases}$

$u_3 = \frac{7}{17} \Rightarrow -3u_1 = 1 - \frac{14}{17} = \frac{3}{17} \Rightarrow u_1 = -\frac{1}{17}$

$u = \begin{pmatrix} -1/17 \\ 0 \\ 7/17 \end{pmatrix}$

2) $\delta_{u_1} = -2 - (-2, -20, 7)\begin{pmatrix} -1/17 \\ 0 \\ 7/17 \end{pmatrix} = -\frac{34}{17} - \frac{2}{17} - \frac{49}{17} =$

$= -5 < 0$

$\delta_{u_2} = -1 - (-4, -30, 10)\begin{pmatrix} -1/17 \\ 0 \\ 7/17 \end{pmatrix} = -\frac{17}{17} - \frac{4}{17} - \frac{7}{17} =$

$= -\frac{91}{17} < 0$

3) $x_{u_1} = 0 \quad x_{u_2} = 0$

$\begin{cases} -3x_{u_3} + 15x_{u_4} = 7 \\ -x_{u_3} - x_{u_4} + x_{u_5} = -30 \\ 2x_{u_3} + 7x_{u_4} = 10 \end{cases}$

$x_{u_3} = \frac{101}{51}$ (+)

$x_{u_4} = \frac{44}{51}$ (–)

$x_{u_5} = -\frac{1385}{51}$ (–)

4) $j_x = 5$

5) $(0,1,0)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 1 \qquad p_{u_2} = 1$

$(-3,-1,2)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0 \qquad \begin{cases} -3p_{u_1} + 2p_{u_3} = 1 \\ 15p_{u_1} + 7p_{u_3} = 1 \end{cases}$

$(15,-1,7)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0$

$\begin{cases} -3p_{u_1} = \frac{5}{17} \\ 17p_{u_3} = 6 \end{cases} \qquad p_{u_1} = -\frac{5}{51} \quad p_{u_3} = \frac{6}{17} \qquad \Rightarrow p = \begin{pmatrix} -5/51 \\ 1 \\ 6/17 \end{pmatrix}$

6) $p_{\delta_1} = -\left(-\frac{5}{51}, 1, \frac{6}{17}\right)\begin{pmatrix} -2 \\ -20 \\ 7 \end{pmatrix} = -\frac{10}{51} + 20 - \frac{42}{17} =$

$= -\frac{136}{51} + \frac{1020}{51} = \frac{884}{51} > 0$

$p_{\delta_2} = -\left(-\frac{5}{51}, 1, \frac{6}{17}\right)\begin{pmatrix} -4 \\ -30 \\ 10 \end{pmatrix} = -\frac{20}{51} + \frac{1530}{51} - \frac{180}{51} =$

$= \frac{1330}{51} > 0$

7) $\sigma_1 = \frac{255}{884}$ $\qquad \Rightarrow \sigma' = \sigma_2$

$\sigma_2 = \frac{273}{1330}$

$y_5 = \{2; 3, 4\}$

$= 1$

$= 1$

$p = \begin{pmatrix} -5/51 \\ 1 \\ 6/17 \end{pmatrix}$

$- \frac{42}{17} =$

$\frac{1530}{51} - \frac{180}{51} =$

## Итерация 4:

1) $\begin{cases} -4u_1 - 30u_2 + 10u_3 = -1 \\ -3u_1 - u_2 + 2u_3 = 1 \\ 15u_1 - u_2 + 7u_3 = 2 \end{cases} \Rightarrow u = \begin{pmatrix} 3/38 \\ 39/190 \\ 46/95 \end{pmatrix}$

2) $\delta_{u_1} = -2 - (-2, -20, 7)\begin{pmatrix} -3/38 \\ 39/190 \\ 46/95 \end{pmatrix} = -\frac{380}{190} - \frac{30}{190} +$

$+ \frac{780}{190} - \frac{644}{190} = -\frac{274}{190} < 0$

$\delta_{u_5} = -(0, 1, 0)\begin{pmatrix} -3/38 \\ 39/190 \\ 46/95 \end{pmatrix} = -\frac{39}{190} < 0$

3) $x_{u_1} = 0, \quad x_{u_5} = 0$

$\begin{cases} -4x_{u_2} - 3x_{u_3} + 15x_{u_4} = 7 \\ -30x_{u_2} - x_{u_3} - x_{u_4} = -30 \\ 10x_{u_2} + 2x_{u_3} + 7x_{u_4} = 10 \end{cases}$

$x_{u_2} = \frac{277}{266}$ (+)

$x_{u_3} = -\frac{220}{133}$ (−)

$x_{u_4} = \frac{55}{133}$ (−)

4) $j_* = 4$

5) $(15, -1, 7)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = -1$

$(-4, -30, 10)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0$

$(-3, -1, 2)\begin{pmatrix} p_{u_1} \\ p_{u_2} \\ p_{u_3} \end{pmatrix} = 0$

$$\begin{cases} 15p_{u_1} - p_{u_2} + 7p_{u_3} = 1 \\ -4p_{u_1} - 30p_{u_2} + 10p_{u_3} = 0 \\ -3p_{u_1} - p_{u_2} + 2p_{u_3} = 0 \end{cases} \Rightarrow p = \begin{pmatrix} 5/133 \\ 11/665 \\ 43/665 \end{pmatrix}$$

6) $p_{\delta_1} = -\left(25/665,\ 11/665,\ 43/665\right)\begin{pmatrix} -2 \\ -20 \\ 7 \end{pmatrix} = \frac{50}{665} +$

$+ \frac{220}{665} - \frac{301}{665} = -\frac{31}{665} < 0$

$p_{\delta_5} = -\frac{11}{665} < 0$

7) $\sigma_1 = +\infty$

$\sigma_5 = +\infty$ $\Rightarrow \sigma' = +\infty$

$\Rightarrow$ двойственная целевая ф-ция неогр. убывает и $\psi(\lambda(\sigma)) \to -\infty$

$\Rightarrow$ мн-во планов пусто

② Записать двойств. задачу

$$\psi(\lambda) = 7y_1 - 30y_2 + 10y_3 - \vartheta_3 - 2\vartheta_4 + 5\omega_1 + 3\omega_2 + 5\omega_3 + 10\omega_4 + 175\omega_5 \to \min$$

$$\begin{cases} -2y_1 - 20y_2 + 7y_3 - \vartheta_1 + \omega_1 = -2 \\ -4y_1 - 30y_2 + 10y_3 - \vartheta_2 + \omega_2 = -1 \\ -3y_1 - y_2 + 2y_3 - \vartheta_3 + \omega_3 = 1 \\ 15y_1 + y_2 + 7y_3 - \vartheta_4 + \omega_4 = 2 \\ y_2 - \vartheta_5 + \omega_5 = 0 \end{cases}$$

$\vartheta \geq 0, \ \omega \geq 0$

## 1.17

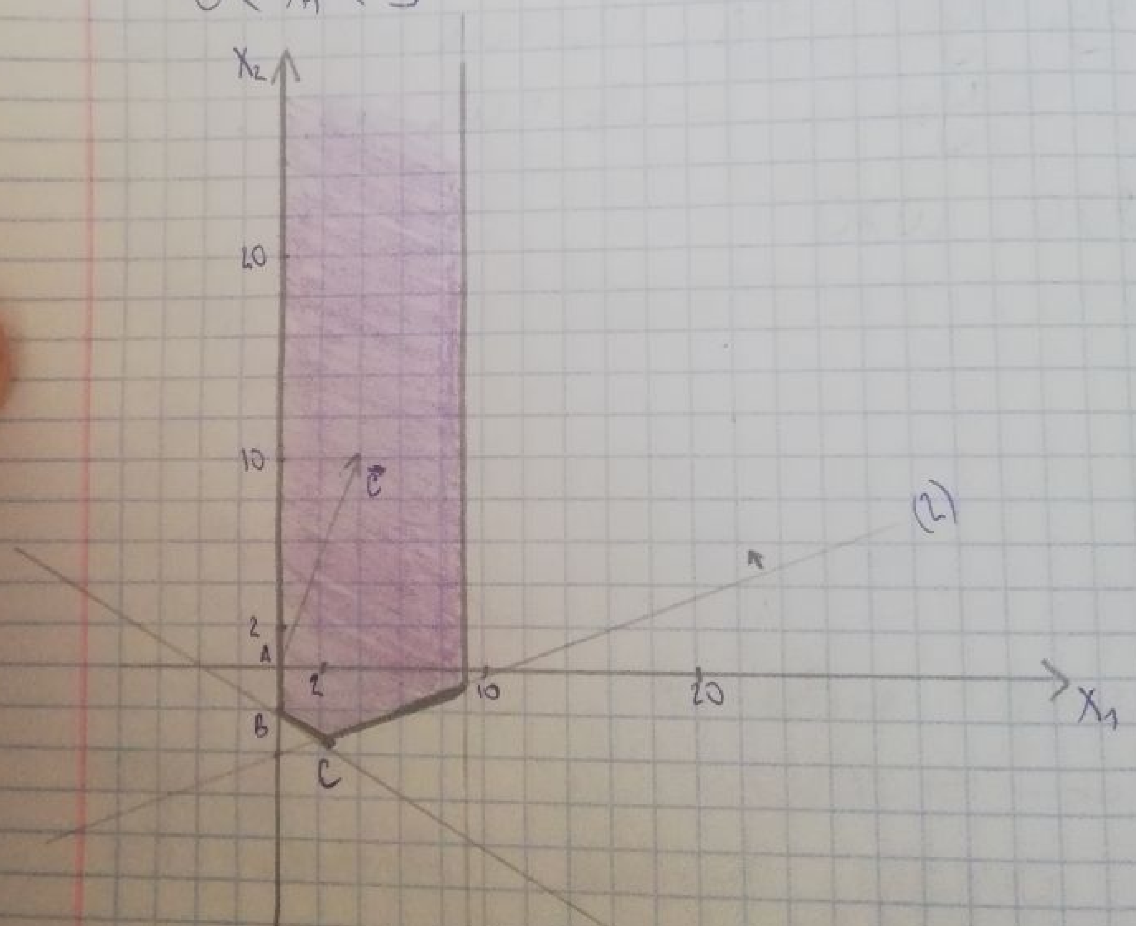
а) - 12в

б) - 9в

1 $\varphi = x_1 + 2x_2$

$$\begin{cases} x_1 + 2x_2 \geq -4 & (1) \\ 2x_1 - 5x_2 \leq 20 & (2) \end{cases}$$

$0 \leq x_1 \leq 9$



Ф-ция $\varphi(x)$ не ограничена => нет решения на максимум

$A(0,0)$ $\qquad tg\,\alpha_{(c)} = 2$

$B(0,-2)$ $\qquad tg\,\alpha_{(1)} = 2$

$C\left(\frac{20}{9}, -\frac{28}{9}\right)$ $\qquad tg\,\alpha_{(2)} = -2,5$

=> любая точка отрезка BC – решение задачи на min

② Записать канонич. форму для задачи на максимум

$$x_1 + 2x_2 \to max$$

$$\begin{cases} -x_1 - 2x_2 + x_3 = 4 \\ 2x_1 - 5x_2 + x_4 = 20 \end{cases}$$

$0 \leq x_1 \leq 9 \qquad 0 \leq x_3 \leq 13 + 2M$

$-M \leq x_2 \leq M \qquad 0 \leq x_4 \leq 20 + 5M$

$\to x_1$

$$d_3^* = max(4 + x_1 + 2x_2) = 13 + 2M$$

$$d_4^* = max(20 - 2x_1 + 5x_2) = 20 + 5M$$

③ $x_1 + 2x_2 \to max$

$$\begin{cases} -x_1 - 2x_2 \leq 4 \\ 2x_1 - 5x_2 \leq 20 \end{cases}$$

$0 \le x_1 \le 9 \qquad -M \le x_2 \le M$

$\tilde{x}_1 = 0$

$\tilde{x}_2 = 0$

$\omega_1 = 4 \ge 0$

$\omega_2 = 20 > 0$

$x = (0, 0, 4, 20)$

$J_Б = \{3, 4\} \qquad A_Б = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{cases} -x_1 - 2x_2 + x_3 = 4 \\ 2x_1 - 5x_2 + x_4 = 20 \end{cases}$$

$0 \le x_1 \le 9 \qquad 0 \le x_3 \le 13 + 2M$

$-M \le x_2 \le M \qquad 0 \le x_4 \le 20 + 5M$

<u>Итерация 1:</u>

1) $u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2) $\Delta_1 = 1 > 0 \qquad x_1 = d_1^* \ (-)$

$\Delta_2 = 2 > 0 \qquad x_2 = d_2^* \ (-)$

3) $j_0 = 2$

4) $l_1 = 0 \qquad l_2 = 1$

$l_3 = 2$ $\quad l_4 = 5$

5) $\Theta_1 = \infty$ $\quad \Theta_3 = \frac{13+2M-4}{2} = \frac{9+2M}{2}$

$\Theta_2 = \frac{M}{2}$ $\quad \Theta_4 = \frac{20+5M-20}{5} = M$

$\Theta = (\infty, \frac{M}{2}, \frac{9+2M}{2}; M)$

$\Rightarrow$ задача не имеет решение из-за неограниченности целевой ф-ции на мн-ве планов

(4) $x_1 + 2x_2 \to max$

$$\begin{cases} -x_1 - 2x_2 \le 4 \\ 2x_1 - 5x_2 \le 20 \end{cases}$$

$0 \le x_1 \le 9$

Двойственная задача:

$\psi(\lambda) = 4y_1 + 20y_2 + 9\omega_1 \to min$

$$\begin{cases} -y_1 + 2y_2 - \cancel{\theta_1} + \omega_1 \ge 1 \\ -2y_1 - 5y_2 \cancel{- \theta_1 + \omega_2} \ge 2 \end{cases}$$

$y_{1,2} \ge 0$, $\cancel{\theta \ge 0}$, $\omega_1 \ge 0$

В случае неогр. прямых ограничений, мн-во двойств. планов пусто

5 Решить на максимум двойств. методом

$x_1 + 2x_2 \to max$

$$\begin{cases} -x_1 - 2x_2 + x_3 = 4 \\ 2x_1 - 5x_2 + x_4 = 20 \end{cases}$$

$0 \le x_1 \le 9 \qquad x_3 \ge 0$

$-M \le x_2 \le M \qquad x_4 \ge 0$

$Y_б = \{3, 4\}$

1) $u = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2) $\delta_{u_1} = 1 > 0$

$\delta_{u_2} = 2 > 0$

3) $\varkappa_{u_1} = 9$

$\varkappa_{u_2} = M$

$\varkappa_{u_3} = 13 + 2M \quad (+)$

$\varkappa_{u_4} = 2 + 5M \quad (+)$

Условие оптимальности выполнено. Однако, имеем неогр. целевой ф-ции ⇒ задача не имеет решени