

Challenge 4

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Sabendo e_1, e_2, c_1, c_2, N e :

$$C_1 \equiv M^{e_1} \pmod{N}$$

$$C_2 \equiv M^{e_2} \pmod{N}$$

Como $\gcd(e_1, e_2) = 1$ podemos aplicar o extended Euclid's algorithm $e_1 s_1 + e_2 s_2 = 1$

Visto que:

$$C_1^{s_1} * C_2^{s_2} \Leftrightarrow (M^{e_1})^{s_1} * (M^{e_2})^{s_2} \Leftrightarrow$$

$$M^{e_1 s_1 + e_2 s_2} \Leftrightarrow M^1 \Leftrightarrow M$$

Logo:

$$M \equiv C_1^{s_1} * C_2^{s_2} \pmod{N}$$

Para calcular s_1 e s_2 :

$$s_1 = e_1^{-1} \pmod{e_2}$$

$$s_2 = (\gcd(e_1, e_2) - e_1 s_1) / e_2$$

Como o s_2 é negativo, é necessário calcular o inverso de C_2 :

$$\text{inv_}C_2 = C_2^{-1} \pmod{N}$$

Para obter a mensagem final M :

$$M = C_1^{s_1} \pmod{N} * (\text{inv_}C_2^{-s_2} \pmod{N})$$

Por fim foi necessário converter a mensagem para hexadecimal e depois para ascii.

$M = \text{"Let's shaft Mallory! He's a lazy and ugly SOAB."}$

Source Code

```
import sys

def gcdExtended(a, b):
    if a == 0 :
        return b, 0, 1
    gcd, x1, y1 = gcdExtended(b%a, a)
    x = y1 - (b//a) * x1
    y = x1

    return gcd, x, y

def find_a_b(e1,e2):
    a = modinv(e1,e2)
    b = (1-e1*a)/e2
    return a,b

def euclidean_gcd(x,y):
    while(y):
        x, y = y, x %y
    return x

def modinv(a, m):
    g, x, y = gcdExtended(a, m)
    if g != 1:
        return -1
    else:
        return x % m

def findResult(c1,a,b,N,inv):
    return (pow(c1,a,N) * pow(inv,-int(b),N)) % N

def main():
    sys.setrecursionlimit(15000)
    # Parsing data.txt file
    datafile = open("data.txt","r")
    lineList = datafile.readlines()
    for line in lineList:
```

```

    if("N=" in line):
        N = int(line.replace("N=", ""))
    elif("e1=" in line):
        e1 = int(line.replace("e1=", ""))
    elif("e2=" in line):
        e2 = int(line.replace("e2=", ""))
    elif("C1=" in line):
        c1 = int(line.replace("C1=", ""))
    elif("C2=" in line):
        c2 = int(line.replace("C2=", ""))

#Start

a,b = find_a_b(e1,e2)

inv = modinv(c2,N)

result = findResult(c1,a,b,N,inv)

plain_text = str(result)

hex_array = hex(int(plain_text)).replace("0x", "")

text_inverted = bytes.fromhex(hex_array).decode('UTF-8')
# Final Result
print(''.join(reversed(text_inverted)))

if __name__ == "__main__":
    main()

```