BRAC University
Department of Computer Science and Engineering (CSE)

CSE230: Discrete Mathematics

SET - B Semester: Spring 2024 Examination: Quiz 3 | Time: 20 minutes Full marks: 20

Name: ______________________ ID: ____________________ Section: _____

(There are 2 questions total. You must answer both. Feel free to use the back of the question paper, if needed.)

Q1. For $x \in \mathbb{Z}$, proof that if $x^2 - 6x + 5$ is even, then x^2 is odd. (Use any proof technique)

[10 Marks]

Q2. Prove that if x, y and z are integers and $x + y + z$ is odd, then at least one of x, y and z is odd.

[10 Marks]

End

Q1. $P(x) = x^2 - 6x + 5$ is even., $Q(x) := x^2$ is odd.
where domain is $\mathbb{Z}$.
Assuming $P(x) \equiv T$, $x^2 - 6x + 5$ is even.
$\Rightarrow x^2 - 6x + 6 - 1 = \text{even}$. $\Rightarrow x^2 - 6x + 6 = \text{odd}$
since, even + 1 = odd.
$\Rightarrow$ Now, $x^2 - 2(3x + 3) = \text{odd} \Rightarrow x^2 - \text{even} = \text{odd}$
$\Rightarrow x^2 = \text{odd} + \text{even}$. $\Rightarrow x^2 = \text{odd}$.
$\therefore Q(x) \equiv T$. (proved)

Q2. $P(x, y, z) :=$ '$x + y + z$ is odd', $Q(x, y, z) :=$ x is odd or y is odd or z is odd.
where domain is $\mathbb{Z}$.
Assuming $Q(x, y, z) \equiv F$, x is even and y is even and z is even.
$\therefore x + y + z = 2a + 2b + 2c$, where a, b, c are integer.
$\Rightarrow x + y + z = 2(a + b + c) = \text{even}$.
$\therefore P(x, y, z) \equiv F$.
$\therefore \sim Q \to \sim P$
Proved, contraposition.