BRAC University
Department of Computer Science and Engineering (CSE)

CSE230: Discrete Mathematics

SET - A

Semester: **Spring 2024**
Examination: **Quiz 4**

Time: **20 minutes**
Full marks: **20**

Name: Solution ID: _____ Section: _____

*(There are 2 questions total. You must answer **both**.*
Feel free to use the back of the question paper, if needed.)

Q1. You are given the following 2 sets:
$A = \{x \in \mathbf{Z} \mid x \text{ is even and } 0 < x < 10\}$ and $B = \{x \in \mathbf{Z} \mid x \text{ is a perfect square and } 0 < x < 20\}$
Now find the following sets:

(a) $A \times (B - A)$	$\{(2,1), (2,9), (2,16), (4,1), (4,9), (4,16), (6,1), (6,9), (6,16), (8,1), (8,9), (8,16)\}$
(b) $(B - A) \times P(\emptyset)$	$\{(1,\emptyset), (9,\emptyset), (16,\emptyset)\}$
(c) $P(B - A)$	$\{\emptyset, \{1\}, \{9\}, \{16\}, \{1,9\}, \{1,16\}, \{9,16\}, \{1,9,16\}\}$
(d) $P(B) - P(A)$	$\{\{1\}, \{9\}, \{16\}, \{1,4\}, \{1,9\}, \{1,16\}, \{4,9\}, \{4,16\}, \{9,16\}, \{1,4,9\}, \{1,9,16\}, \{4,9,16\}, \{1,4,16\}, \{1,4,9,16\}\}$

[2+2+2+2=8 Marks]

Q2. Consider the following function: $f: \mathbf{R}^+ \to \mathbf{R}, f(x) = 3 - x^3$.
(a) Identify the domain, codomain and range of the function f.
(b) Determine whether f is a one-to-one function.
(c) Determine whether f is a onto function.
(d) Determine whether f is a bijection.

[3+4+4+1=12 Marks]

End

2.a) Domain = R^+, codomain = R, Range = $\{x \in R \mid x \leq 3\}$

2.b) Assume $f(a) = f(b) \Rightarrow 3 - a^3 = 3 - b^3 \Rightarrow a^3 - b^3 = 0$
$\Rightarrow (a-b)(a^2+ab+b^2) = 0$. Here, since a, b are both greater than or ~~both~~ equal to zero, (Domain R^+)
if $a^2+ab+b^2 = 0$, then $a = b = 0$.
if $a - b = 0$, then $a = b$.
therefore, for all $f(a) = f(b)$, ~~b~~ it must be $a = b$.
∴ f in one to one.

2.c) Domain $= \mathbb{R}^+$, Codomain $= \mathbb{R}$.

However, Range $= \mathbb{R} \{x \in \mathbb{R} \mid x \leq 3\}$.

Because, for $x \in \mathbb{R}^+$, $x^3 \geq 0$

$\Rightarrow 3 - x^3 \leq 3 - 0 \Rightarrow 3 - x^3 \leq 3$.

$\therefore$ f is not onto.

2.d) since f is one-to-one but not onto, it is not a bijection.