BRAC University

Department of Computer Science and Engineering (CSE)

SET - A

CSE230: Discrete Mathematics

Semester: **Spring 2024**
Examination: **Midterm**

Time: **75 minutes**
Full marks: **30+5**

Name: ______________________ ID: ______________________ Section: _____

*(There are 4 questions total in 2 pages. Questions Q1 through Q3 are mandatory. Question Q4 is an **optional bonus question**. Answer all the sub-parts of a question together. Numbers on the right denote the marks assigned to that question.)*

Q1. **(a)** Determine whether the following four statements are consistent with each other:

CO1

(i) If Shourav gets the promotion, he will get the Eid bonus.
(ii) Shourav's salary will increase, if he gets the promotion.
(iii) If Shourav gets the Eid bonus, then his salary will not increase.
(iv) Shourav gets the promotion.

(b) Show that $(r \rightarrow p) \vee (p \rightarrow q) \equiv r \rightarrow (\neg q \vee (p \rightarrow q)) \equiv \mathbf{T}$.
(Show that both propositions are Tautology. You may use any method preferred.)

[5+5=10 Marks]

Q2. **(a)** Let A = $\{x \in \mathbf{N} \mid 2^{x-1} < 40\}$ and B = $\{x \in \mathbf{N} \mid 10 < 3^{x+1} < 1000\}$.
Now, showing the necessary calculations, find $P(A - B) \times P(B - A)$.

CO2

(b) Consider the following functions:

$$f : \mathbf{R} \rightarrow \mathbf{R}, f(x) = \frac{1}{x^5 - 1};$$

$$g : \mathbf{R} \rightarrow \mathbf{R}, g(x) = \frac{1}{2}(x - 3).$$

Determine whether the function $g \circ f(x)$ is a surjection.

[5+5=10 Marks]

Q3. **(a)** Use mathematical induction to show that:

CO3

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \ . \ . \ . \ + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

when n is a positive integer greater than 1.

(b) Prove by contraposition that if a, b and c are integers and $a^3 + 3b^2 + 3c$ is even, then at least one of a, b or c is even.

[5+5=10 Marks]

Q4. [Bonus Question: Answer only one]

Determine the truth value of each of the following statements if the domain for all variables consists of all integers:

(a) $\exists x \exists y(x^2 + y^2 = 6)$

(b) $\exists y \forall x(x + y = x - y)$

(c) $\exists x \exists y((x^2 > y) \land (x < y))$

(d) $\forall x \forall y \exists z(x + y = z)$

(e) $\forall x \exists y(x + y = 1)$

or,

On the island of Knights and Knaves, everyone is either a Knave or a Knight. Knights always tell the truth and Knaves always lie. There, encountering a group of 3 islanders: Alice, Bob and Carol, you asked them to make some statements about one another. Their statements were as follows.

Alice says:	Exactly two of us are knights.
Bob says:	Alice and Carol are opposite types.
Carol says:	Both Alice and Bob are lying.

Consider the statements as independent, i.e., each is either a true statement or a false statement. Your task is to categorize each islander as either a knight or a knave.

[Bonus: 5 Marks]

End

Set A

Q1 (a)

Translating each statement into propositions:

Let, $p :=$ Shourav gets the promotion.

$q :=$ Shourav will get the Eid bonus.

$r :=$ Shourav's salary will increase.

Then,

(i) $\Rightarrow$ $p \rightarrow q$

(ii) $\Rightarrow$ $p \rightarrow r$

(iii) $\Rightarrow$ $q \rightarrow \sim r$

(iv) $\Rightarrow$ p

Assuming (iv) is T, $p \equiv T$, we can have the following cases for different truth values of q and r:

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$q \rightarrow \sim r$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	T

There are no cases with all (i), (ii), (iii), (iv) are True ~~to~~ simultaneously. Therefore, the 4 statements are $\boxed{\text{Not consistent}}$

Q1. (b) Here, $(r \to p) \vee (p \to q)$

$\equiv \sim r \vee p \vee \sim p \vee q$

$\equiv \sim p \vee (p \vee \sim p) \vee q$

$\equiv \sim r \vee T \vee q$

$\equiv T$

Also, $r \to (\sim q \vee (p \to q))$

$\equiv r \to (\sim q \vee \sim p \vee q)$

$\equiv r \to (\sim p \vee (\sim q \vee q))$

$\equiv r \to (\sim p \vee T)$

$\equiv r \to T$

$\equiv T$

$\therefore (r \to p) \vee (p \to q) \equiv r \to (\sim q \vee (p \to q)) \equiv T$

Q2.(a)

Given,

$A = \{x \in N \mid 2^{x-1} < 40\}$ and $B = \{x \in N \mid 10 < 3^{x+1} < 1000\}$

For A, we find for different elements of N,

$2^{1-1} = 2^0 = 1 < 40 \quad \therefore 1 \in A$

$2^{2-1} = 2^1 = 2 < 40 \quad \therefore 2 \in A$

$2^{3-1} = 2^2 = 4 < 40 \quad \therefore 3 \in A$

$2^{4-1} = 2^3 = 8 < 40 \quad \therefore 4 \in A$

$2^{5-1} = 2^4 = 16 < 40 \quad \therefore 5 \in A$

$2^{6-1} = 2^5 = 32 < 40 \quad \therefore 6 \in A$

$2^{7-1} = 2^6 = 64 \not< 40 \quad \therefore 7 \notin A$

~~Th~~ For subsequent values of N, the condition remains false.

$\therefore A = \{1, 2, 3, 4, 5, 6\}$

For B, we find for different elements of N,

$3^{1+1} = 3^2 = 9 \not> 10, 9 < 1000 \quad \therefore 1 \notin B$

$3^{2+1} = 3^3 = 27 > 10, 27 < 1000 \quad \therefore 2 \in B$

$3^{3+1} = 3^4 = 81 > 10, 81 < 1000 \quad \therefore 3 \in B$

$3^{4+1} = 3^5 = 243 > 10, 243 < 1000 \quad \therefore 4 \in B$

$3^{5+1} = 3^6 = 729 > 10, 729 < 1000 \quad \therefore 5 \in B$

$3^{6+1} = 3^7 = 2187 > 10, 2187 \not< 1000 \quad \therefore 6 \notin B$

For subsequent values of N, the condition remains false.

$\therefore B = \{2, 3, 4, 5\}$

$\therefore A - B = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\} = \{1, 6\}$

$\therefore B - A = \{2, 3, 4, 5\} - \{1, 2, 3, 4, 5, 6\} = \{\} = \phi$

$\therefore P(A-B) = \{\phi, \{1\}, \{6\}, \{1, 6\}\}$

$\therefore P(B-A) = \{\phi\}$

$\therefore P(A-B) \times P(B-A) = \{\phi, \{1\}, \{6\}, \{1, 6\}\} \times \{\phi\}$

$$= \{(\phi, \phi), \cancel{\phi}(\{1\}, \phi), (\{6\}, \phi), (\{1, 6\}, \phi)\}$$

Q2.(b) $f(x) = \frac{1}{x^5-1}$

$g(x) = \frac{1}{2}(x-3) = \frac{x-3}{2}$

$\therefore\ g\circ f(x) = g\left(\frac{1}{x^5-1}\right) = \frac{\frac{1}{x^5-1} - 3}{2}$

$= \frac{\frac{1-3x^5+3}{x^5-1}}{2} = \frac{-3x^5+4}{x^5-1} \times \frac{1}{2}$

$= -\frac{3x^5-4}{2x^5+2} = -\frac{2x^5+2+x^5-6}{2x^5+2}$

$= -\left(1 + \frac{x^5-6}{2x^5+2}\right)$

$= -1 - \frac{x^5-6}{2x^5+2}$

Now, let, $y = g\circ f(x) = -1 - \frac{x^5-6}{2x^5+2}$

$\Rightarrow y+1 = \frac{-x^5+6}{2x^5+2} \Rightarrow (2x^5+2)(y+1) = -x^5+6$

$\Rightarrow 2x^5y + 2x^5 + 2y + 2 = -x^5 + 6$

$\Rightarrow 2x^5y + 2x^5 + x^5 = -2y \cancel{-2} + 4$

$\Rightarrow 2x^5y + 3x^5 = -2y \cancel{-2} + 4 \Rightarrow x^5 = -\frac{2y \cancel{+2} - 4}{2y+3}$

$\Rightarrow x^5(2y+3) = -(2y-4)$

$\Rightarrow x = \sqrt[5]{-\frac{2y \cancel{+2} - 4}{2y+3}}$

as $x \in R$, $2y+3 \neq 0$

$\Rightarrow 2y \neq -3$

$\Rightarrow y \neq -\frac{3}{2}$

$\therefore$ Range of $gof(x) = R - \{-3/2\}$

But codomain of $gof(x)$ = Codomain of $g(x) = R$.

$\therefore$ gof is not a surjection/ onto.

Q3. (a)

Here, Proposition is

$$P(n) = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

where n is a positive integer, greater than 1.

Basis: ~~since~~

$$P(2) \equiv \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots + \frac{1}{(3 \cdot (2)-2)(3(2)+1)} = \frac{2}{3(2)+1}$$

for the ~~seriei~~ series on the left,

$$\text{last term} = \frac{1}{(6-2)(6+1)} = \frac{1}{4 \cdot 7}$$

$$\therefore L.S = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} = \frac{7+1}{4 \cdot 7} = \frac{8}{4 \cdot 7} = \frac{2}{7}$$

$$R.S = \frac{2}{6+1} = \frac{2}{7}$$

$\therefore P(2) \equiv T$.

Induction Step :

Here, $P(k) \equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{(3k+1)}$

$$P(k+1) \equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+3-2)(3k+3+1)} = \frac{k+1}{(3k+3+1)}$$

$$\equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{(3k+4)}$$

Assuming $P(k) \equiv T$, we find from $P(k+1)$,

$$L.S = \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)} = \frac{3k^2+3k+k+1}{(3k+1)(3k+4)} = \frac{3k(k+1)+(k+1)}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)(3k+1)}{(3k+1)(3k+4)} = \frac{k+1}{3k+4} = R.S$$

$\therefore P(k) \rightarrow P(k+1)$

$\therefore$ Proved by Induction.

Q.3 (b)

Here, Hypothesis; $p :=$ "$a^3 + 3b^2 + 3c$ is even"

Conclusion, $q :=$ "At least one of a, b, c is even"

Assuming q is false, $\sim q$ is True.

a, b, c are even,

$\sim q$ = None of a, b, c are all odd numbers.

= a, b, c are all odd numbers.

a, b, c can be written as,

Then, $a = 2i + 1$; $b = 2j + 1$, $c = 2k + 1$

where, i, j and k are integers.

Now

$$a^3 + 3b^2 + 3c$$

$$= (2i+1)^3 + 3(2j+1)^2 + 3(2k+1)$$

$$= 8i^3 + 3 \cdot 4i^2 + 3 \cdot 2i + 1 + 3(4j^2 + 4j + 1) + 6k + 3$$

$$= 8i^3 + 12i^2 + 6i + 1 + 12j^2 + 12j + 3 + 6k + 3$$

$$= 8i^3 + 12i^2 + 6i + 12j^2 + 12j + 6k + 1 + 3 + 3$$

$$= 8i^3 + 12i^2 + 6i + 12j^2 + 12j + 6k + 6 + 1$$

$$= 2(4i^3 + 12i^2 + 3i + 6j^2 + 6j + 3k + 3) + 1$$

= odd Number.

$\therefore$ p is False.

$\therefore$ $\sim q \rightarrow \sim p$.

$\therefore \forall_a \forall_b \forall_c \, (p(a,b,c) \rightarrow q(a,b,c)) \equiv T$.

$\therefore$ Proved by contraposition.

Q4. Part 1

(a) False. (Not possible)

(b) True, (when $y=0$)

(c) True, $(2^2 > 3,\ 2 < 3)$

(d) True, (Any two integer sum)

(e) True. (for $y = 1-x$)

Q4. Part 2

Alice, A $\longrightarrow$ Exactly 2 true

Bob, B $\longrightarrow$ $A \not\equiv C$

Carol, C $\longrightarrow$ $A = B = F$

If A is T, B or C is true, not both.

if B is T & C is F,

then, $A \not\equiv C$ holds, and "$A = B = F$"

being false is justified.

$\therefore A, B \equiv T,\ C \equiv F$.

$\therefore$ Alice and Bob are Knights and Carol is a Knave.

BRAC University

Department of Computer Science and Engineering (CSE)

SET - B

CSE230: Discrete Mathematics

Semester: **Spring 2024** Time: **75 minutes**

Examination: **Midterm** Full marks: **30+5**

Name: ______________________ ID: ______________________ Section: _____

(There are 4 questions total. Questions Q1 through Q3 are mandatory. Question Q4 is an ***optional bonus question****.*
Answer all the sub-parts of a question together.)

Q1. **(a)** Determine whether the following four statements are consistent with each other:

CO1

(i) If Shovon gets the job, he will get the big office.
(ii) If Shovon gets the big office, then he will not get a car.
(iii) Shovon will get a car, if he gets the job.
(iv) Shovon gets the job.

(b) Show that $(p \rightarrow q) \vee (q \rightarrow r) \equiv p \rightarrow (\neg r \vee (q \rightarrow r)) \equiv \mathbf{T}$.
(Show that both propositions are Tautology. You may use any method preferred.)

[5+5=10 Marks]

Q2. **(a)** Let $A = \{x \in \mathbf{N} : 7 < 2^{x+1} < 70\}$ and $B = \{x \in \mathbf{N} : 3^{x-1} < 300\}$.
Now, showing the necessary calculations, find $P(A - B) \times P(B - A)$.

CO2

(b) Consider the following functions:

$$f: \mathbf{R} \rightarrow \mathbf{R}, f(x) = \frac{1}{3}(x - 2);$$

$$g: \mathbf{R} \rightarrow \mathbf{R}, g(x) = \frac{1}{x^5 - 3};$$

Determine whether the function $f \circ g(x)$ is a surjection.

[5+5=10 Marks]

Q3. **(a)** Use mathematical induction to show that:

CO3

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \ \ldots \ + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

when n is a positive integer greater than 1.

(b) Prove by contraposition that if x, y and z are integers and $x + y^2 + z^3$ is even, then at least one of x, y or z is even.

[5+5=10 Marks]

Q4. [Bonus Question: Answer only one]
Determine the truth value of each of these statements if the domain for all variables consists of all integers:

(a) $\exists x \exists y (x^2 + y^2 = 5)$
(b) $\exists y \forall x (x + y \neq x - y)$
(c) $\exists x \exists y (x + y = 4 \wedge x - y = 1)$
(d) $\forall x \forall y \exists z (z = (y - x)/2)$
(e) $\forall y \exists x (x + y = -1)$

or,
On the island of Knights and Knaves, everyone is either a Knave or a Knight. Knights always tell the truth and Knaves always lie. There, encountering a group of 3 islanders – Alice, Bob and Carol, you asked them to make some statements about one another. Their statements were as follows.

Alice says:	At most one of us is a knave.
Bob says:	Alice and Carol are the same type.
Carol says:	Alice is correct but Bob is not.

Consider the statements as independent, i.e., each is either a true statement or a false statement. Your task is to categorize each islander as either a knight or a knave.

[Bonus: 5 Marks]

End

Set B

Q1. (a)

Translating the statements into propositions;

Let, p :: Sharon gets the job.

q :: Sharon will get the big office

r :: Sharon will get a car.

then,

(i) $\Rightarrow$ $p \rightarrow q$

(ii) $\Rightarrow$ $q \rightarrow \sim p$

(iii) $\Rightarrow$ $p \rightarrow r$

(iv) $\Rightarrow$ p

Assuming (iv) is T, $p \equiv T$. we can have the following cases for different truth values of q and r :

p	q	r	$\sim r$	$p \rightarrow q$	$q \rightarrow \sim r$	$p \rightarrow r$
T	T	T	F	T	F	T
T	T	F	T	T	T	F
T	F	T	F	F	T	T
T	F	F	T	F	T	F

There are no cases with all (i), (ii), (iii) and (iv) are True.

$\therefore$ The statements are not consistent.

Q1.(b)

Here, $(p \to q) \vee (q \to r) = \sim p \vee q \vee \sim q \vee r$

$= \sim p \vee (q \vee \sim q) \vee r = \sim p \vee T \vee r$

$= T$

Also, $p \to (\sim r \vee (q \to r)) = p \to (\sim r \vee \sim q \vee r)$

$= p \to (\sim q \vee \sim r \vee r) = p \to (\sim q \vee T)$

$= p \to T = T$

Q2.(a) Given $A = \{x \in N \mid 7 < 2^{x+1} < 70\}$ and $B = \{x \in N \mid 3^{x-1} < 300\}$

for A, we find.

$2^{1+1} = 2^2 = 4 \not> 7, 4 < 70 \quad \therefore 1 \notin A$

$2^{2+1} = 2^3 = 8 > 7, 8 < 70 \quad \therefore 2 \in A$

$2^{3+1} = 2^4 = 16 > 7, 16 < 70 \quad \therefore 3 \in A$

$2^{4+1} = 2^5 = 32 > 7, 32 < 70 \quad \therefore 4 \in A$

$2^{5+1} = 2^6 = 64 > 7, 64 < 70 \quad \therefore 5 \in A$

$2^{6+1} = 2^7 = 128 > 7, 128 \not< 70 \quad \therefore 6 \notin A$

The condition remains false for subsequent values of N.

$\therefore A = \{2, 3, 4, 5\}$

For B, we find,

$3^{1-1} = 3^0 = 1 < 300$, $\therefore 1 \in B$

$3^{2-1} = 3^1 = 3 < 300$, $\therefore 2 \in B$

$3^{3-1} = 3^2 = 9 < 300$, $\therefore 3 \in B$

$3^{4-1} = 3^3 = 27 < 300$, $\therefore 4 \in B$

$3^{5-1} = 3^4 = 81 < 300$, $\therefore 5 \in B$

$3^{6-1} = 3^5 = 243 < 300$, $\therefore 6 \in B$

$3^{7-1} = 3^6 = 729 \nless 300$. $\therefore 7 \notin B$

The ~~cod~~ condition remains false for subsequent values of N.

$\therefore \cancel{A}\, B = \{1, 2, 3, 4, 5, 6\}$

$\therefore A - B = \{2, 3, 4, 5\} - \{1, 2, 3, 4, 5, 6\}$

$= \emptyset$

$\therefore B - A = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\} = \{1, 6\}$

$\therefore P(A-B) = \{\emptyset\}$

$\therefore P(B-A) = \{\emptyset, \{1\} \{6\}, \{1 \cdot 6\}\}$

$\therefore P(A-B) \times P(B-A) = \{\emptyset\} \times \{\emptyset, \{1\}, \{6\}, \{1, 6\}\}$

$= \{(\emptyset, \emptyset), (\emptyset, \{1\}), (\emptyset, \{6\}), (\emptyset, \{1, 6\})\}$

Q2.(b) Given, $f(x) = \frac{1}{3}(x-2) = \frac{x-2}{3}$

$g(x) = \frac{1}{x^5-3}$

$\therefore f \circ g(x) = f\left(\frac{1}{x^5-3}\right) = \frac{\frac{1}{x^5-3} - 2}{3}$

$= \frac{1 - 2x^5 + 6}{(x^5-3)3} = \frac{-2x^5+7}{3x^5-9} = \frac{7-2x^5}{3x^5-9}$

Let, $y = f \circ g(x) = \frac{7-2x^5}{3x^5-9} \Rightarrow 3x^5y - 9y = 7 - 2x^5$

$\Rightarrow 3x^5y + 2x^5 = 7 + 9y \Rightarrow x^5(3y+2) = 7 + 9y$

$\Rightarrow x^5 = \frac{7+9y}{3y+2} \Rightarrow x = \sqrt[5]{\frac{7+9y}{3y+2}}$

Here, Domain of $f \circ g(x) =$ ~~R~~ Domain of $g(x) = R$

$\therefore x \in R$, $\therefore 3y + 2 \neq 0 \Rightarrow 3y \neq -2 \Rightarrow y \neq -\frac{2}{3}$

$\therefore$ Range of $f \circ g(x) = R - \{-2/3\}$

But Codomain of $f \circ g(x) =$ Codomain of ~~R~~ $f(x) = R$.

$\therefore$ ~~B~~ The function $f \circ g(x)$ is not a surjection.

Q3.(a).

Here, Proposition is as follows:

$$P(n) := \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

where, n is positive integer, greater than 1.

Basis: $P(2) \equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3(2)-2)(3(2)+1)} = \frac{2}{3(2)+1}$

Here, L.S is a series, with last term

$$= \frac{1}{(3(2)-2)(3(2)+1)} = \frac{1}{(6-2)(6+1)} = \frac{1}{4\cdot 7}$$

$$\therefore L.S = \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} = \frac{7+1}{1\cdot 4\cdot 7} = \frac{8}{1\cdot 4\cdot 7} = \frac{2}{7}$$

$$R.S = \frac{2}{3(2)+1} = \frac{2}{6+1} = \frac{2}{7} = L.S$$

$\therefore P(2) \equiv T$.

Induction Step:

$$P(k) \equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$$

$$P(k+1) \equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+3-2)(3k+3+1)}$$

$$= \frac{k+1}{3k+3+1}$$

$$\equiv \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k+1}{3k+4}$$

Assuming $P(K) \equiv T$, we find from $P(K+1)$

$$L.S = \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \cdots \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{(3k+1)} + \frac{1}{(3k+1)(3k+4)} = \frac{3k^2+4k+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+3k+k+1}{(3k+1)(3k+4)} = \frac{3k(k+1)+(k+1)}{(3k+1)(3k+4)} = \frac{(k+1)(3k+1)}{(3k+1)(3k+4)}$$

$$= \frac{k+1}{3k+4} = R.S.$$

$\therefore P(K) \rightarrow P(K+1)$

$\therefore$ Proved by Induction.

Q3. (b)

Here,

Hypothesis, $P(x,y,z) :=$ " $x + y^2 + z^3$ is even"

Conclusion, $q(x,y,z) :=$ "At least one of x, y, z is even"

where the domain for x, y and z are all integers.

Now, Assuming q is false, None of x, y, z is even.

$\therefore x, y, z$ are all odd. $\therefore x, y, z$ can be re-written as follows.

$x = 2a+1$, $y = 2b+1$, $z = 2c+1$

where, a, b, c are all integers.

Now, $x + y^2 + z^3 = 2a+1 + (2b+1)^2 + (2c+1)^3$

$= 2a+1 + 4b^2+4b+1 + 8c^3+12c^2+6c+1$

$= 2a + 4b^2 + 4b + 8c^3 + 12c^2 + 6c + 2 + 1$

$= 2(a + 2b^2 + 2b + 4c^3 + 6c^2 + 3c + 1) + 1 =$ odd Number.

$\therefore p(x,y,z)$ is false.

$\therefore \sim q(x,y,z) \rightarrow \sim p(x,y,z)$

$\therefore \forall_x \forall_y \forall_z (p(x,y,z) \rightarrow q(x,y,z))$

$\therefore$ Proved by contraposition.

Q.4. part 1

(a) True. $(1^2 + 2^2 = 1 + 4 = 5)$

(b) True. (For any int, add 1, sub 1)

(c) false. $(2x = 5 \Rightarrow x = 2.5 \notin \mathbb{Z})$

(d) false. (~~No~~ $\mathbb{Z}$ for odd x, even y)

(e) True. $(x = -1 - y)$

Q4. Part 2

~~Here~~

~~Alice, $A :=$ At most 1 T~~

~~Bob, $B :=$ $A = C$~~

~~Carol, $C :=$ $A = T$, $B = F$ / $(A \wedge \sim B)$~~

~~If $A = T$, B, C must be false.~~

~~$\therefore C$ being false proves $A = C$ is false.~~

~~But $(A = T, B = F)$ cannot be true.~~

~~$\therefore A \neq T$~~

If $B = T$,

$A = C$, $A \neq T$ [showed before]

$\therefore A = C = F$

C being False, $A = T$, $B = F$ cannot happen.

This holds true for $B = T$. But,

A = At most 1 true also cannot happen.

This is a contradiction as $B = T$

for this case, $B = T$, $A, C = F$.

If $C = T$,

$A = T$, $B = F$.

For $A = T$, There must be 1 true at most which is not possible, as both C and A are True for this case.

$\therefore$ None of A, B, C are possible to be T.

$\therefore$ A, B, C all are F.

$\therefore$ Alice, Bob, Charlie are all Knaves.

Q4. Part 2

Here,

Alice, A:= At most 1 F

Bob, B:= A = C

Carol, C:= A=T, B = F

If A=T, there are At most 1 F among B, C

Assuming B = T, A = C ⇒ C = T

Then, Because of C, B = F → is a contradict.

∴ B ≠ T, B = F, A ≠ C ⇒ C ≠ T, C = F

But, B and C both can not be False,

as there should be At Most 1 F.

∴ A ≠ T, $\boxed{A = F}$

If B = T, A = C ⇒ C = F

which means A = T and B = F is not true. As we already shown, A = F,

This case is satisfiable.

∴ $\boxed{B = T}$, $\boxed{C = F}$

Alice, Carol = Knave, Bob = Knight.