CSE230: Discrete Mathematics

SET - A Semester: **Spring 2024** Time: **30 minutes**
Examination: **Quiz 8** Full marks: **20**

Name: _____ ID: _____ Section: _____

*(There are **3** questions total. You must answer **all**.*
Feel free to use the back of the question paper, if needed.)

Q1. Write the count for the following terms:

(a) How many strings of 4 decimal digits do not end with 4?	
(b) How many strings of four lowercase letters have the same letter repeated 3 times?	
(c) How many strings of 6 length have lowercase letters in the first 2 positions and numbers from 1 to 9 in the last 3 positions, considering the third position can have either?	

[2+3+3=8 Marks]

Q2. Brac University Student bus has 10 rows of seats with 4 seats per row.

(a) How many <u>minimum</u> students must be on the bus to confirm that there is at least 1 row with 4 students?	
(b) How many <u>maximum</u> students must be on the bus to confirm that there exists 2 empty seats in a row?	

[2+2=4 Marks]

Q3. Assume the following scenario:
There are 36 students in your class. Through the semester there have been some friend circles forming in the class, and the most notable two friend circles are as follows:
- A group of 7 girls and 6 boys that includes Labonno,
- A group of 9 students that includes Arnob and Asif.

These 2 groups can be considered exclusive, so none of the students in Arnob's group are counted in Labonno's group and vice versa. The other students don't really mix with any of the groups.
The classroom seat arrangement allows for 4 rows of 9 seats for the 36 students.
Now answer the following questions:
- **(a)** Arnob's group wants to sit in the last row together, however, while Asif sits in the same group, he does not want to sit next to Arnob. How many ways can they sit in the row?
- **(b)** Labonno's group always sits taking the first 2 rows: one row with all the girls of the group and one row with all the boys of the group. How many ways can they sit?

[3+5=8 Marks]

End

Q1. A

How many strings of 4 decimal digits do not end with 4?

→ first 3 positions have 10 choices,
Last position have 9.

Count = $10^3 * 9 = \boxed{9000}$

Q1. B

How many strings of 4 lower case letters have the same letter repeated 3 times (exactly)?

→ First, choose 1 letter from 26 to be repeated. count = $C(26,1) = 26$

Next, choose 1 letter from remaining 25, that will not be repeated = $C(25,1) = 25$

finally, the strings can be ordered or arranged in $\frac{4!}{3!}$ ways. (since 3 letters are repeated and are indistinguishable.

∴ Total count = $26 * 25 * \frac{4!}{3!} = \boxed{2600}$

Q1. C How many strings of length 6 have lower case letters in the first 2 positions and numbers from 1 to 9 in the last 3 positions, considering the third position can have either?

$$\rightarrow \text{positions:} \quad \underset{26}{\frac{1}{}} \quad \underset{26}{\frac{2}{}} \quad \underset{35}{\frac{3}{}} \quad \underset{9}{\frac{4}{}} \quad \underset{9}{\frac{5}{}} \quad \underset{9}{\frac{6}{}}$$

choices : $26 \quad 26 \quad \underset{(26+9)}{35} \quad 9 \quad 9 \quad 9$

$\therefore$ Total choices $= 26 * 26 * 35 * 9 * 9 * 9$

$= 26^2 * 35 * 9^3$

$= \boxed{17248140}$

Q.2 . A How many minimum students ~~on~~ must be on the bus to confirm 1 row has 4 students?

$\rightarrow$ worst case $\rightarrow$ maximum students, no row has 4 students.

count : 30 (3 students on every row).

Now, $(30+1) = \boxed{31}$ students would confirm one row must have 4 students.

Q2.B How many maximum students must be on the bus to confirm ~~there~~ 2 empty seats in a row?

→ worst case → minimum students, no row has ~~2~~ 2 empty seats.

∴ count : 30 students (Every row has 1 empty seat)

∴ Now (30−1) students = $\boxed{29}$ students would confirm one row must have 2 empty seats.

Q3.A

→ Total possible arrangements = 9!

Arrangements where Arnob is next to Asif = 8! * 2! [Arnob and Asif as one block. And 2! ways to rearrange between themselves]

∴ Total arrangements where Arnob is Not next to Asif = 9! − 8! * 2!

= 362880 − 80640

= $\boxed{282240}$

Q3.B

→ Let, the counts for
girl's row = R_g = $P(9,7)$ = 181440

and boy's row = R_b = $P(9,6)$ = 60480

R_g and R_b can be ordered in $2!$ ways.
(Either girls on first row or boys on first row)

∴ Total count = $181440 * 60480 * 2!$

$= 21946982240$

$= P(9,6) * P(9,7) * 2!$