BRAC University
Department of Computer Science and Engineering (CSE)

CSE230: Discrete Mathematics

SET - B Semester: **Spring 2024** Time: **20 minutes**
Examination: **Quiz 4** Full marks: **20**

Name: Solution ID: ______________ Section: _____

*(There are 2 questions total. You must answer **both**.*
Feel free to use the back of the question paper, if needed.)

Q1. You are given the following 2 sets:
A = $\{x \in \mathbf{Z} \mid x$ is even and $0 < x < 10\}$ and B = $\{x \in \mathbf{Z} \mid x$ is a perfect square and $0 < x < 20\}$
Now find the following sets:

(a) B × (A − B)	$\{(1,2), (1,6), (1,8), (4,2), (4,6), (4,8), (9,2), (9,6), (9,8), (16,2), (16,6), (16,8)\}$
(b) (A − B) × $P(\emptyset)$	$\{(2,\emptyset), (6,\emptyset), (8,\emptyset)\}$
(c) P(A − B)	$\{\emptyset, \{2\}, \{6\}, \{8\}, \{2,6\}, \{2,8\}, \{6,8\}, \{2,6,8\}\}$
(d) P(A) − P(B)	$\{\{2\}, \{6\}, \{8\}, \{2,4\}, \{2,6\}, \{2,8\}, \{4,6\}, \{4,8\}, \{6,8\}, \{2,4,6\}, \{2,4,8\}, \{2,6,8\}, \{4,6,8\}, \{2,4,6,8\}\}$

[2+2+2+2=8 Marks]

Q2. Consider the following function: $f : \mathbf{R}^+ \to \mathbf{R}, f(x) = 2 - x^4$.
(a) Identify the domain, codomain and range of the function f.
(b) Determine whether f is a one-to-one function.
(c) Determine whether f is a onto function.
(d) Determine whether f is a bijection.

[3+4+4+1=12 Marks]

End

2.a Domain = R^+ , Codomain = R
Range = $\{x \in R \mid x \leq 2\}$

2.b Assume $f(a)$, $f(b)$ are two range elements,
and $f(a) = f(b) \Rightarrow 2 - a^4 = 2 - b^4 \Rightarrow a^4 - b^4 = 0$
$\Rightarrow (a^2 - b^2)(a^2 + b^2) = 0$ $\cancel{\Rightarrow (a^2 - b^2) = 0}$ [~~since a, b > 0~~
if $a^2 + b^2 = 0$, then a, b both 0, $\boxed{a = b}$
if $a^2 - b^2 = 0$, then $(a+b)(a-b) = 0$. ~~Here~~

Again, if $a+b=0$, $a=b=0$, since $a,b \in R^+$
and if $a-b=0$, $a=b$.
$\therefore$ for any $f(a)=f(b)$, $a=b$. showed.
$\therefore f$ is one to one.

2.c Here, ~~gs~~ given codomain is R.
But, with a domain of R^+, the function f cannot output/generate all values of R, as x^4 will always be positive, or zero ~~making~~ meaning, $(2-x^4)$ can only be 2 at most.
$\therefore$ Range $= \{x \in R \mid x \leq 2\}$, Codomain $= R$
$\therefore f$ is not onto.

2.d f is one-to-one but not onto.
$\therefore f$ is not a bijection.