Fall 2023

CSE250 – Circuits and Electronics

KVL AND KCL

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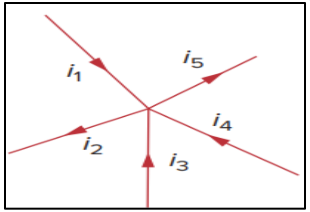
Kirchhoff's Current Law (KCL)

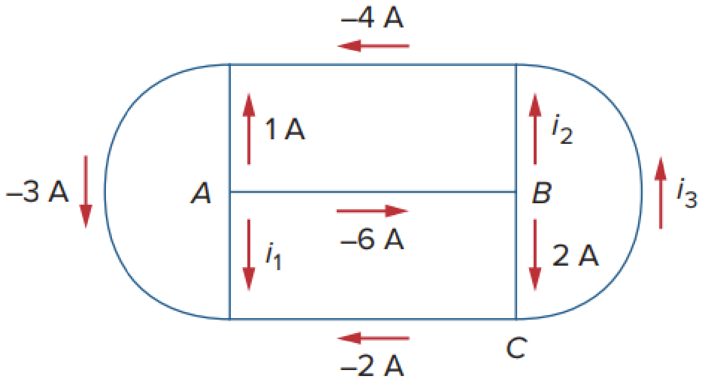
- Kirchhoff's current law (KCL) the algebraic sum of the currents entering a node is equal to the algebraic sum of the currents leaving the node.
- Mathematically, $\sum_{n=1}^{N} i_n = 0$, where N is the number of branches connected to the node and I_n is the nth current entering (or leaving) the node.
- Assume a set of currents $i_k(t), k = 1, 2, \dots,$ flow into a node. The algebraic sum of currents at the node is, $i_{total}(t) = i_1(t) + i_2(t) + i_3(t) + \dots \dots$
- Integrating both sides, $q_{total}(t) = q_1(t) + q_2(t) + q_3(t) + \dots \dots,\ [q_k(t) = \int i_k(t)\, dt]$
- The *law of conservation of electric charge* requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus, $q_{Total}(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.
- For the node shown beside, $i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

  $or, i_1 + i_3 + i_4 = i_2 + i_5$



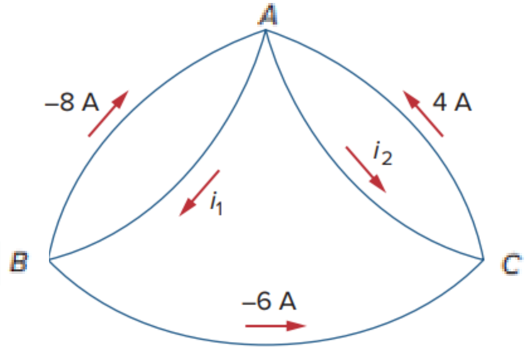
Example 1

(i) Find i_1, i_2, and i_3



Note that, in both the circuits A, B, and C are the same nodes. It is more appropriate to call them junctions in this case.

(ii) Find $i_1, and\ i_2$



KCL at junction A,

$$i_1 + 1 + (-6) = 0$$

$$\Rightarrow \quad i_1 = 5\ A$$

KCL at junction B,

$$i_2 + 2 = -6$$

$$\Rightarrow \quad i_2 = -8\ A$$

KCL at junction C,

$$2 = (-2) + i_3$$

$$\Rightarrow \quad i_3 = 4\ A$$

KCL at junction B,

$$i_1 = (-8) + (-6)$$

$$\Rightarrow \quad i_1 = -14\ A$$

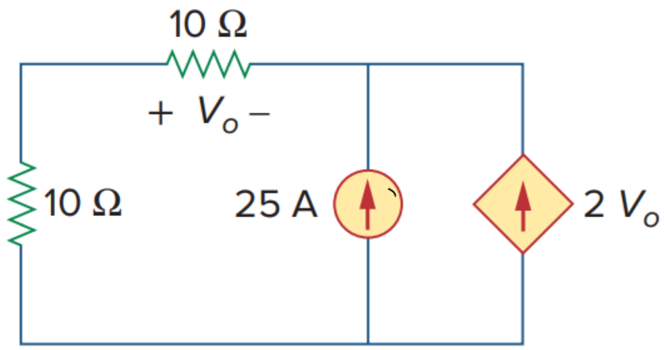
KCL at junction C,

$$i_2 + (-6) = 4$$

$$\Rightarrow \quad i_2 = 10\ A$$

Example 2

- Find V_0 and power absorbed/supplied by the dependent source with appropriate $\pm$ sign.



Current through the series resistances = $25 + 2V_0$

According to the Ohm's law,

$$V_0 = -10 \times (25 + 2V_0)$$
$$V_0 = -11.9\ V$$

The voltage across the dependent source is,

$$V_x = (10 + 10) \times (25 + 2V_0) = 24\ V$$

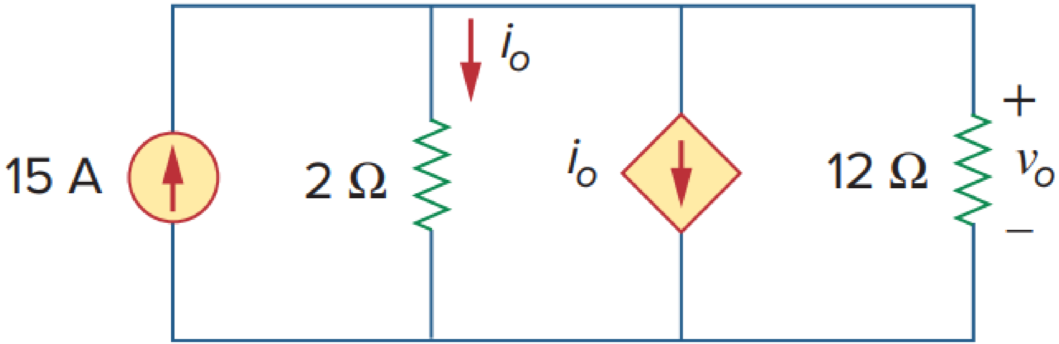
With the polarity of V_x and the direction of the current $(2V_0)$ given, according to the passive sign convention, the dependent source is supplying power. So,

$$p = -\,24 \times 2V_0 = 571.2\ W$$

The power is positive, hence, the dependent source is actually absorbing power. This is true as V_0 is negative, the current $2V_0$ is actually flowing in the opposite direction.

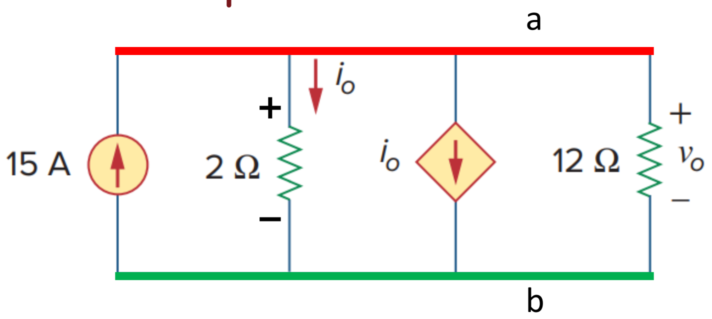
Problem 1

- Determine v_0 and i_0.



$\underline{\text{Ans}}$: $\boldsymbol{v_0 = 13.85\ V;\ i_0 = 6.92\ A.}$

Solution to problem 1



There are two nodes in the circuit (Red and Green).

From the direction of i_o, we can deduce,

$$v_o = 2i_o \quad \text{..........} \ (i)$$

Now applying KCL at node a,

$$15 = i_o + i_O + \frac{v_o}{12} \quad \text{..........} \ (ii)$$

(i) In (ii),

$$15 = i_o + i_O + \frac{2i_o}{12}$$

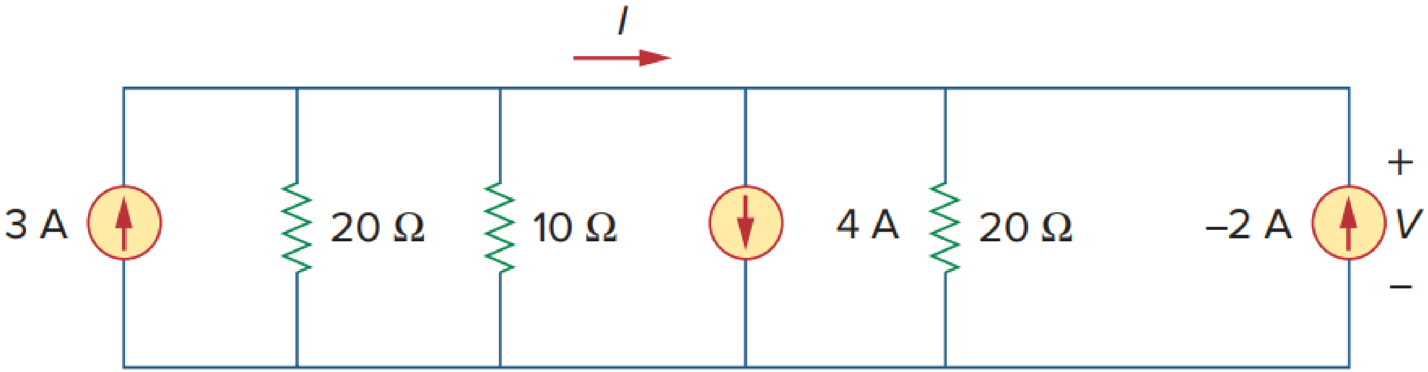
$$\gg 15 = \frac{(12i_o + 12i_o + 2i_o)}{12}$$

$$\gg 15 \times 12 = 26i_o$$

$$\gg \boldsymbol{i_o = 6.92\ \mathrm{A}}$$

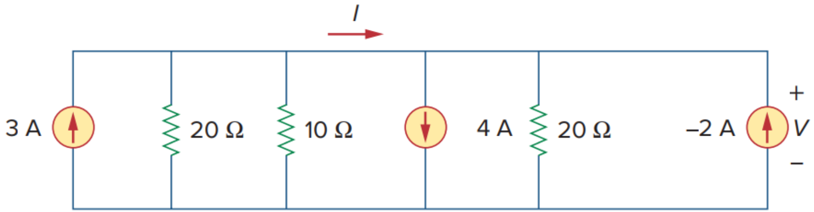
Problem 2

- Find the I and V shown in the following circuit.



Ans: $\boldsymbol{V = -15\,V;\ I = 5.25\,A.}$

Solution to problem 2



At first we have to find V. To find V, let us simplify the circuit. By the term 'simplify', we mean to do find the equivalent current source and equivalent resistance in the circuit.

We know how to find the equivalent resistance when the resistors are in parallel. If current sources are in parallel, they can be simply added algebraically to find the equivalent current source.

$$R_{eq} = \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{20}\right)^{-1}$$

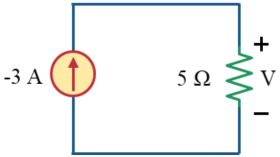
$$R_{eq} = 5\Omega$$

$$I_{eq} = 3 - 4 + (-2)$$

$$I_{eq} = -3A$$

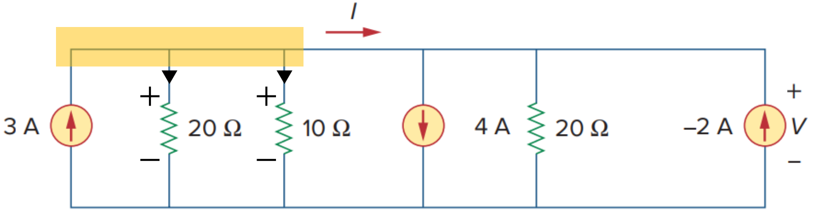
Assuming the upward current direction to be positive

The equivalent circuit becomes like this,



$$V = -3 \times 5 = \mathbf{-15\ V}$$

Going back to original circuit,



Applying KCL in the Yellow portion of the circuit,

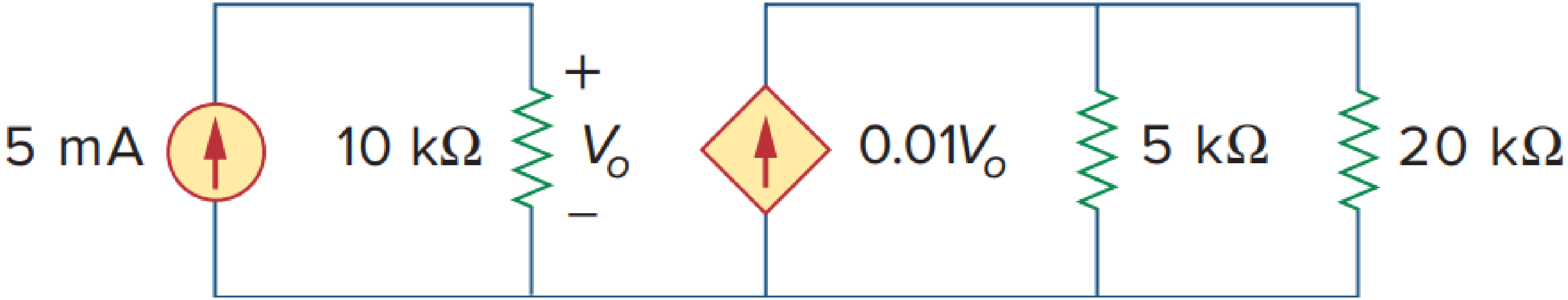
$$3 = \frac{V}{20} + \frac{V}{10} + I$$

$$\gg 3 = \frac{(-15)}{20} + \frac{(-15)}{10} + I$$

$$\gg \boldsymbol{I = 5.25\ A}$$

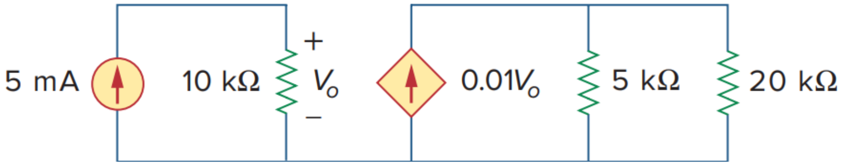
Problem 3

- For the network shown below, find the current, voltage, and power associated with the 20 $k\Omega$ resistor.



Ans: $\mathbf{0.1\ mA, 2\ V, 0.2\ mW}$

Solution to problem 3

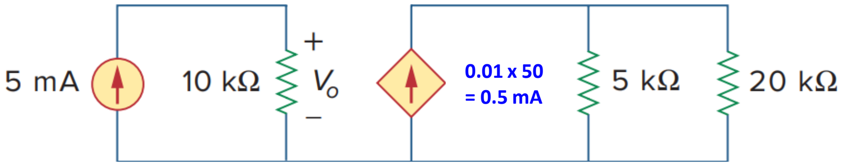


Let us find V_o first. See in the left portion of the circuit, the current 5 mA goes through the 10 kΩ.

$$V_o = (10 \times 10^3) \times (5 \times 10^{-3}) = 50V$$

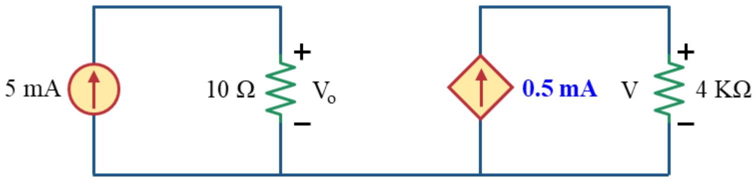
Here, the resistor is in KΩ and current is in mA. We have considered the unit during the calculation.

Now, we can put the value of V_o in the dependent current source in the right portion of the circuit,



Why 5 mA and not 5 A?. Because the resistors in the circuit is in KΩ, So the current in all the branches in bound to be in mA range.

To find the voltage across 20 KΩ, we need to find the equivalent resistance for 5 KΩ and 20 KΩ (as voltage across them are same). 5 || 20 = 4 KΩ



Voltage across 4 KΩ,

$$V = (0.5 \times 10^3) \times (4 \times 10^{-3}) = \mathbf{2V}$$

So, the voltage across 20 KΩ is 2 V

Going back to Circuit 2, The current across 20 KΩ is,

$$I = \frac{2V}{20\text{K}\Omega}$$

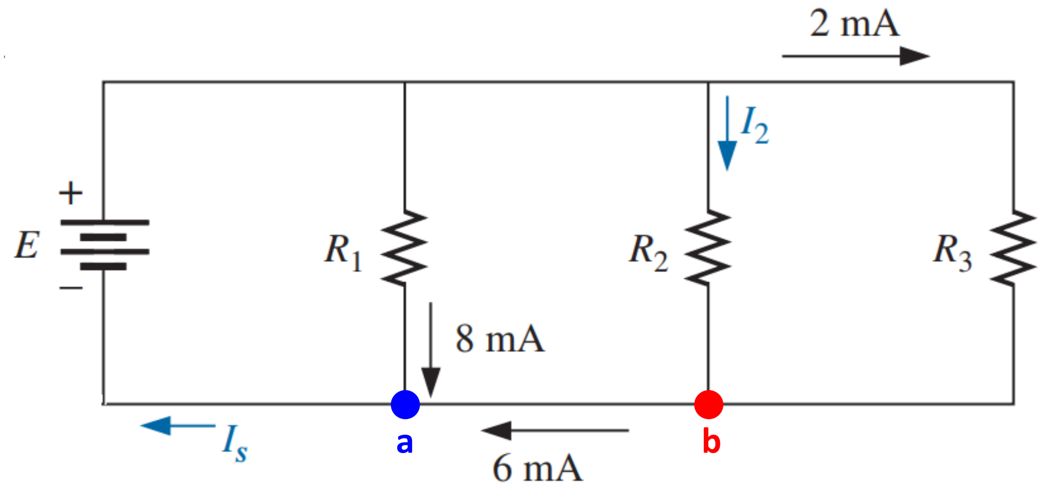
$$\gg I = \mathbf{0.1\ mA}$$

Power dissipated in 20 KΩ is,

$$P = VI = 2V \times 0.1mA = \mathbf{0.2\ mW}$$

Problem 4

- Using KCL, determine the unknown currents.



Let us define the points a and b for convenience

Applying KCL at point a,

$$I_s = 8 + 6$$

$$\gg I_s = \mathbf{14\ mA}$$

Applying KCL at point b,

$$I_2 + 2 = 6$$

$$\gg I_2 = \mathbf{4\ mA}$$

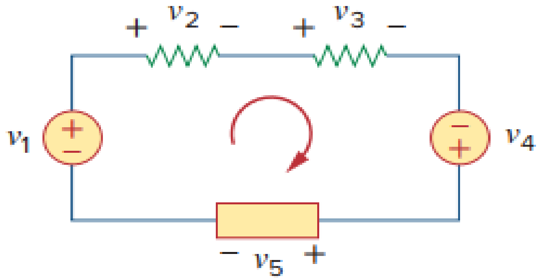
Kirchhoff's Voltage Law (KVL)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a path (or loop) is zero. KVL can be applied both in loop or path consisting of open circuits.
- Mathematically, $\sum_{m=1}^{M} v_m = 0$, where M is the number of voltages (or branches) in the loop and v_m is the m^{th} voltage.
- To illustrate KVL, consider the circuit shown. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- If we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $+v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

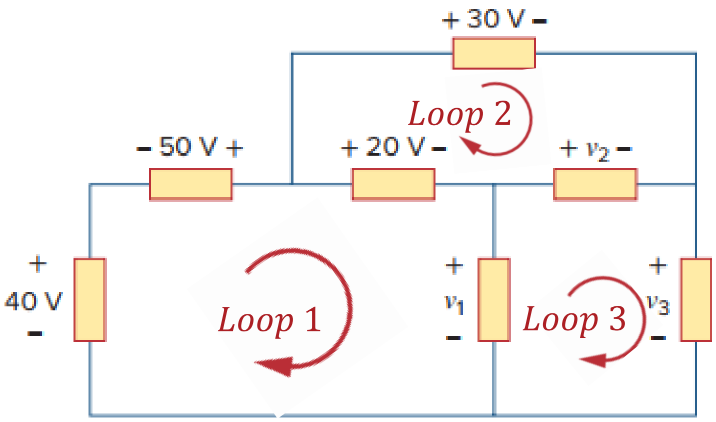
$$or, v_2 + v_3 + v_5 = v_1 + v_4$$

Sum of voltage drops = Sum of voltage rises



Example 3

- Determine v_1, v_2, v_3 using KVL



KVL at loop 1,

$$-40 - 50 + 20 + v_1 = 0$$
$$v_1 = 70\ V$$

KVL at loop 2,

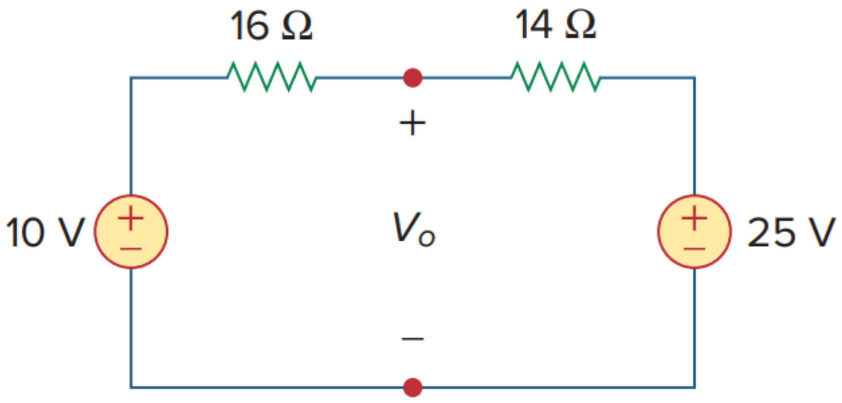
$$-20 + 30 - v_2 = 0$$
$$v_2 = 10\ V$$

KVL at loop 3,

$$-v_1 + v_2 + v_3 = 0$$
$$-70 + 10 + v_3 = 0$$
$$v_3 = 60\ V$$

Example 4

- Determine V_0 using KVL.



Let's assume that the current through the series circuit is i.

Applying KVL around the loop,

$$-10 + 16i + 14i + 25 = 0$$

$$i = -0.5\ A$$

V_0 can be found either by applying KVL through the loop consisting of V_0, 14 Ω, and 25 V or applying KVL through the loop consisting of V_0, 16 Ω, and 10 V. That is,

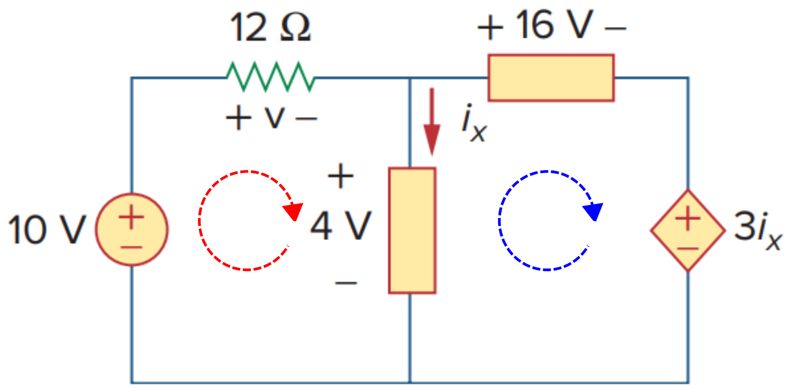
$-V_0 + 14i + 25 = 0$, or $V_0 = 18\ V$

Or,

$-10 + 16i + V_0 = 0$, or $V_0 = 18\ V$

Problem 5

- Find v and i_x in the following circuit.



Applying KVL in left mesh,

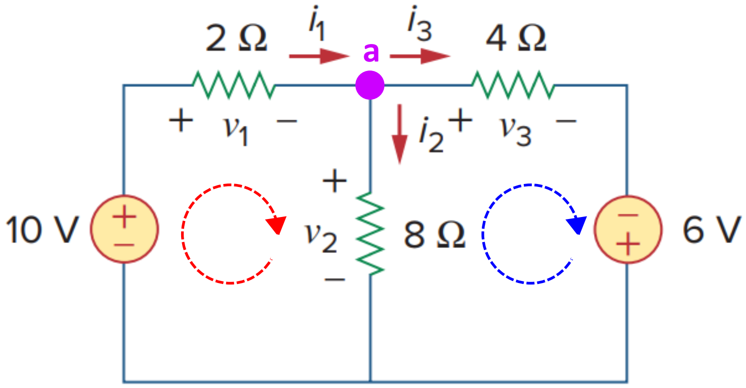
$$-10 + V + 4 = 0$$
$$\gg V = \mathbf{6\ V}$$

Applying KVL in Right mesh,

$$-4 + 16 + 3i_x = 0$$
$$\gg i_x = \mathbf{-4\ A}$$

Problem 6

- Find the voltages and currents shown in the following circuit.



We can solve this problem 2 ways. Either we can find all the currents and then using the currents we will find the voltage, or We can find all the voltages and then using the voltages we will find the currents.

We will use the first procedure

Applying KVL in left mesh,

$$-10 + 2i_1 + 8i_2 = 0$$

$$\gg 2i_1 + 8i_2 = 10 \text{ .... (i)}$$

Applying KVL in right mesh,

$$-8i_2 + 4i_3 - 6 = 0$$

$$\gg 8i_2 - 4i_3 = -6 \text{ .... (ii)}$$

Applying KCL at node a,

$$i_1 = i_2 + i_3$$

$$\gg i_1 - i_2 - i_3 = 0 \text{ .... (ii)}$$

Solving (i), (ii) and (iii),

$$i_1 = \mathbf{3\ A}$$

$$i_2 = \mathbf{0.5\ A}$$

$$i_1 = \mathbf{2.5\ A}$$

Now, using Ohm's law, we can find the unknown voltages.

$$v_1 = 2 \times i_1$$

$$\gg v_1 = 2 \times 3$$

$$\gg v_1 = \mathbf{6\ V}$$

$$v_2 = 8 \times i_2$$

$$\gg v_2 = 8 \times 0.5$$

$$\gg v_2 = \mathbf{4\ V}$$

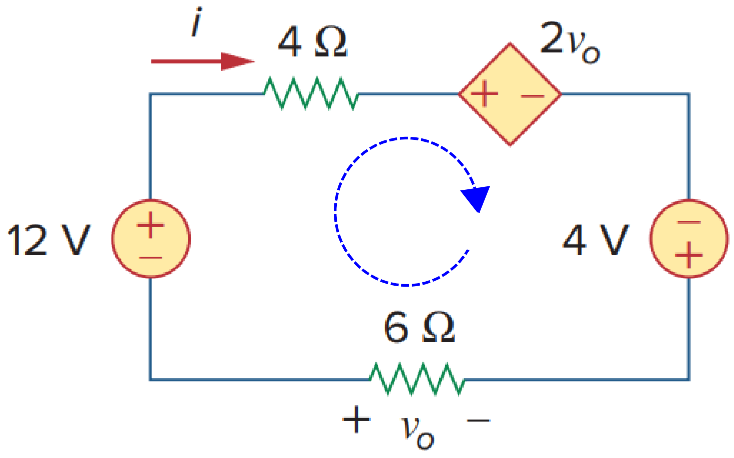
$$v_3 = 4 \times i_3$$

$$\gg v_3 = 4 \times 2.5$$

$$\gg v_1 = \mathbf{10\ V}$$

Problem 7

- Find v_0 and i in the circuit



From the circuit,

$$v_o = -6i \quad .... \text{(i)}$$

The minus sign is because of the predefined voltage polarity of the 6 Ω the circuit

Applying KVL in left mesh,

$$-12 + 4i + 2v_o - 4 + 6i = 0 \quad .... \text{(ii)}$$

(i) In (ii),

$$-12 + 4i + 2(-6i) - 4 + 6i = 0$$

$$\gg -12 + 4i - 12i - 4 + 6i = 0$$

$$\gg \boldsymbol{i = -8\ A} \quad .... \text{(iii)}$$

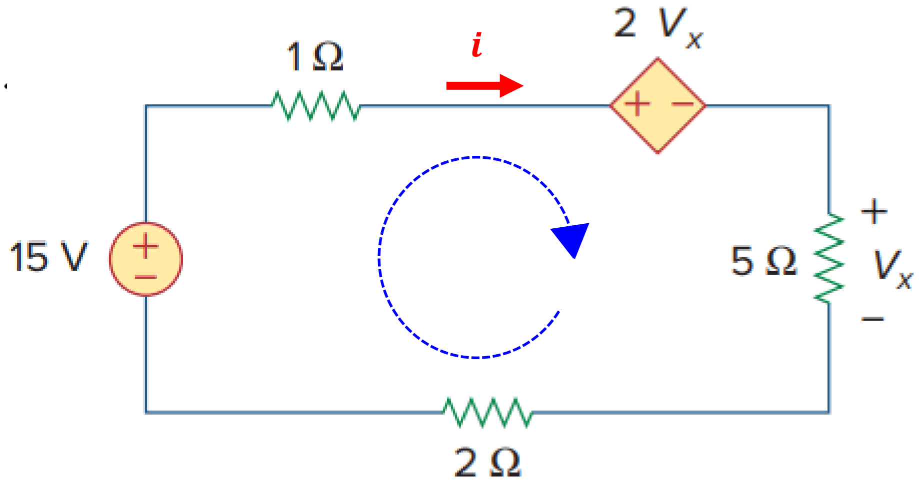
(iii) In (i),

$$v_o = -6 \times -8$$

$$\boldsymbol{v_o = 48\ V}$$

Problem 8

- Find V_x



Before solving the circuit, let current i flows through the circuit.

From the circuit,

$$V_x = 5i \quad \text{.......... (i)}$$

Applying KVL in the mesh,

$$-15 + 1i + 2V_x + 5i + 2i = 0 \quad \text{.......... (ii)}$$

(i) In (ii),

$$-15 + 1i + 2 \times 5i + 5i + 2i = 0$$

$$\gg i = 0.833\ A \quad \text{.......... (iii)}$$

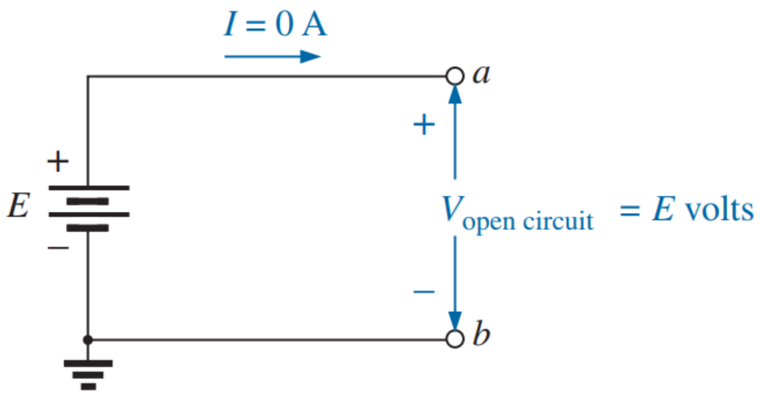
(i) In (ii),

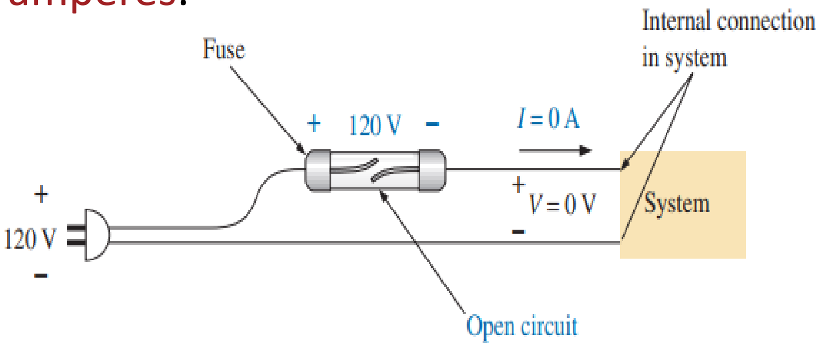
$$V_x = 5 \times 0.833$$

$$\gg V_x = \mathbf{4.167\ V}$$

Open circuit

- An *open circuit* is two isolated terminals not connected by an element of any kind. It is the limiting case of a resistor where the resistance approaches infinite.
- Any element with $R \to \infty$ is an open circuit. $i = 0 = \lim_{R \to \infty} \frac{v}{R}$
- Indicating that, an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

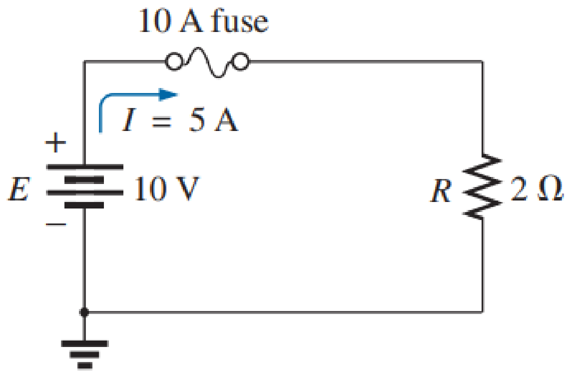


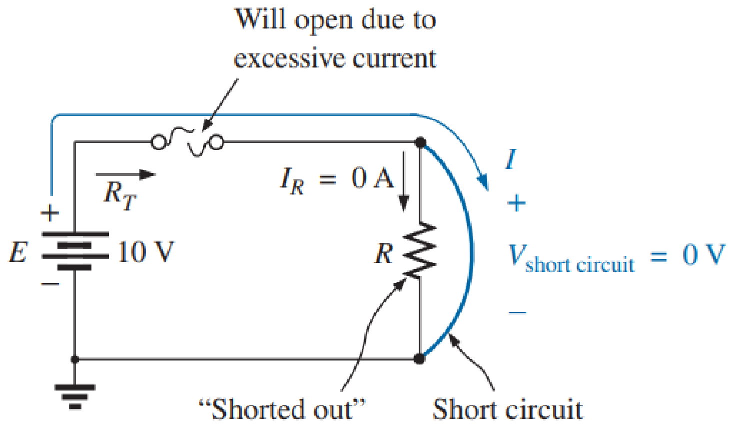


In the event of an excessive current flow, a fuse opens to protect appliances.

Short circuit

- A *short circuit* is a very low resistance, direct connection between two terminals of a network. It is the limiting case of a resistor where the resistance approaches zero.
- Any element with $R = 0$ is a short circuit. $v = 0 = \lim_{R \to 0} iR$
- Indicating that, a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.



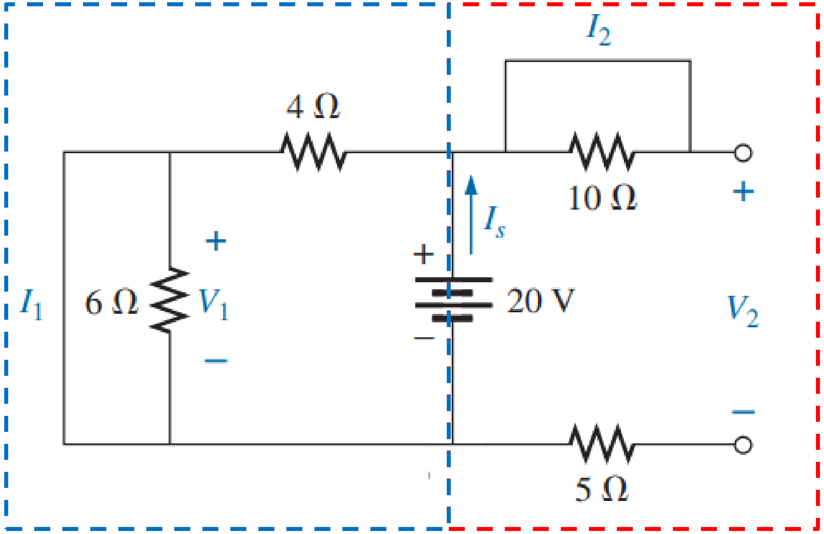


Problem 9

a. Determine the short circuit currents I_1 and I_2.

b. The voltages V_1 and V_2.

c. The source current I_s



For calculating I_1, let us concentrate on the left portion of the circuit (blue marked).

We can see that the 6 Ω shorted (no current flows through it). So,

$$I_1 = \frac{20}{4} = \mathbf{5\ A}$$

$$V_1 = \mathbf{0\ V}$$

For calculating I_2, let us concentrate on the right portion of the circuit (red marked)

We can see that the portion is open circuit. So no current flows through it.

$$I_2 = \mathbf{0\ A}$$

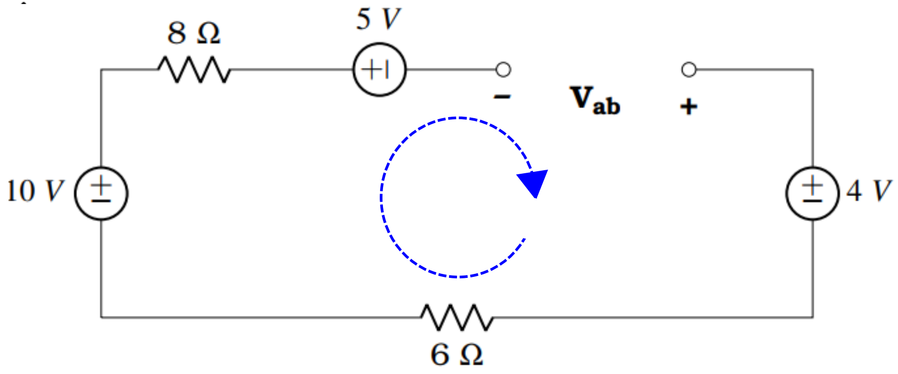
$$V_2 = \mathbf{20\ V}$$

As $I_1 = 0$,

$$I_s = I_2 = \mathbf{5\ A}$$

Problem 10

- Determine the voltage V_{ab} as indicated.



Applying KVL in the mesh,

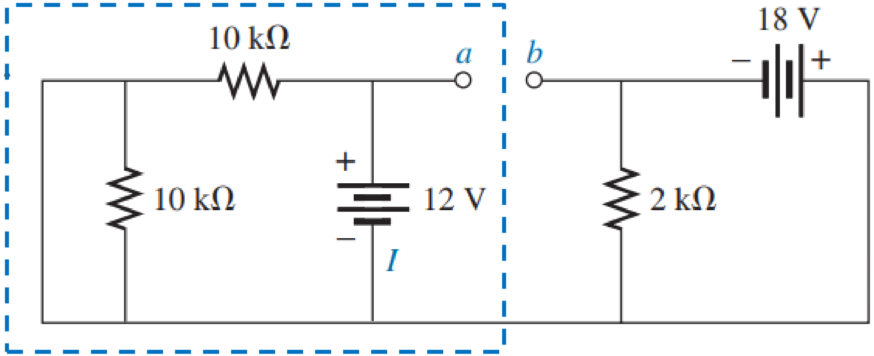
$$-10 + 0 + 5 - V_{ab} + 4 + 0 = 0$$

$$\gg V_{ab} = \mathbf{-1V}$$

Why the voltage across 8 Ω and 6 Ω is zero? Because the circuit is open. Current is zero. So, voltage across the 8 Ω and 6 Ω zero.

Problem 11

- Determine the voltage between terminals a and b and the current I for the network shown below.



The 12 V battery is connected to the node a

&

The 18 V battery is connected to the node b

$$V_a = 12\ V$$

$$V_b = -18\ V$$

$$V_{ab} = V_a - V_b$$

$$\gg V_{ab} = 12 - (-18)$$

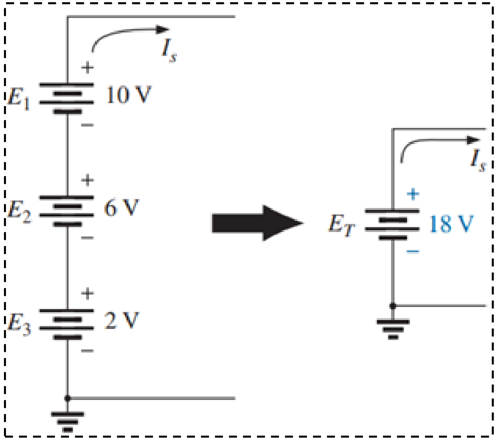
$$\gg V_{ab} = \mathbf{30\ V}$$

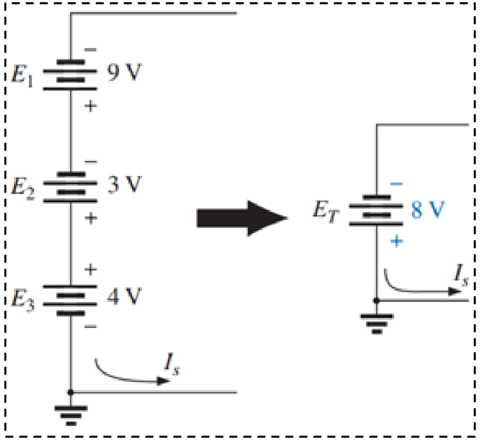
To find the current I, let us concentrate on the blue portion of the circuit. We can see that the 10 KΩ is shorted, so no current flows through it. Hence,

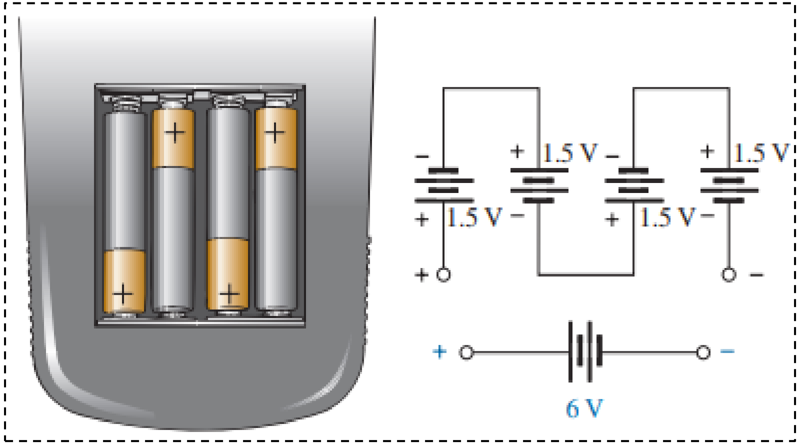
$$I = \frac{12V}{10\text{K}\Omega} = \mathbf{1.2\ mA}$$

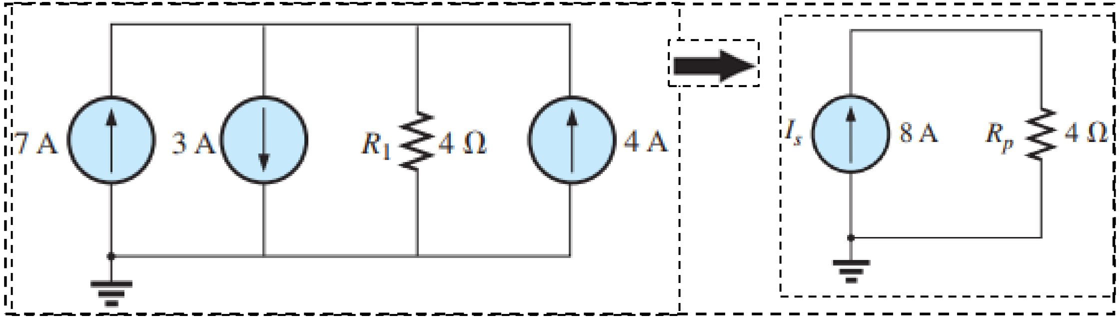
Ans: $\boldsymbol{V_{ab} = 30\ V; I = 1.2\ mA.}$

Series and Parallel sources





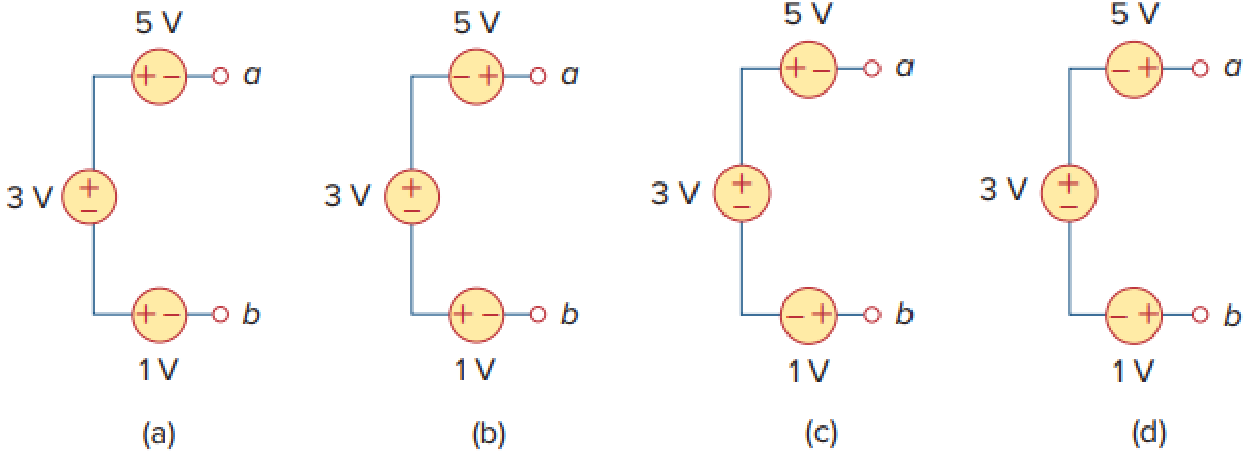




It is not practical to connect voltage sources of unequal ratings in parallel and current sources of unequal currents in series due to the direct violation of KVL and KCL respectively.

Problem 12

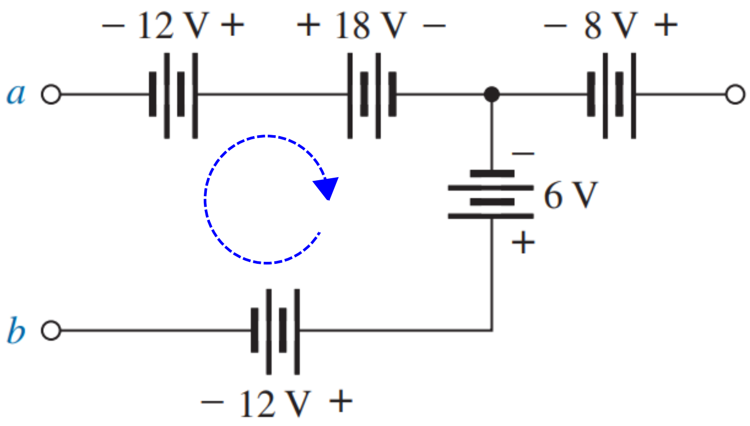
- For each of the circuits shown below, calculate V_{ab}



Ans: $(a)\ \boldsymbol{V_{ab} = -1\ V};\ (b)\ \boldsymbol{V_{ab} = 9\ V};\ (c)\ \boldsymbol{V_{ab} = -3\ V};\ (d)\ \boldsymbol{V_{ab} = 7\ V};$

Problem 13

- For each of the circuits shown below, calculate V_{ab}

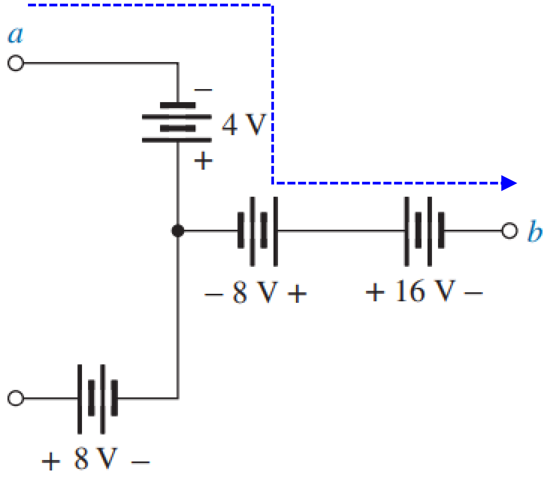


Applying KVL in the loop,

$$-V_{ab} - 12 + 18 - 6 + 12 = 0$$

$$V_{ab} = -12 + 18 - 6 + 12$$

$$\boldsymbol{V_{ab} = 12\ V}$$



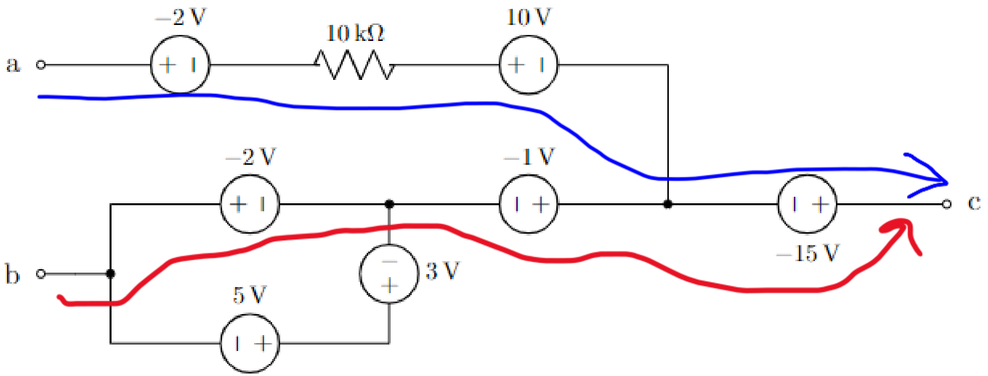
Applying KVL in the path,

$$-V_{ab} - 4 - 8 + 16 = 0$$

$$\boldsymbol{V_{ab} = 4\ V}$$

Problem 14

- For the circuit shown below, calculate V_{ac} and V_{bc}



For V_{ac}
Applying KVL in the blue path,

$$-V_{ac} + (-2) + 10 - (-15) = 0$$

$$\boldsymbol{V_{ac} = 23\ V}$$

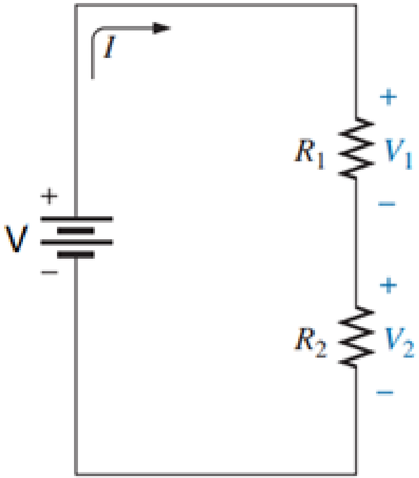
For V_{bc}
Applying KVL in the Red path,

$$-V_{bc} + (-2) - (-1) - (-15) = 0$$

$$\boldsymbol{V_{bc} = 14\ V}$$

Voltage Division Rule

- The voltage division rule permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.



- The current through the series circuit can be found using Ohm's law as,

$$I = \frac{V}{R_1 + R_2}$$

- Applying Ohm's law to each of the resistors,

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

$$\Rightarrow \quad V_1 = \frac{V}{R_1+R_2} R_1 \quad \text{and} \quad V_2 = \frac{V}{R_1+R_2} R_2$$

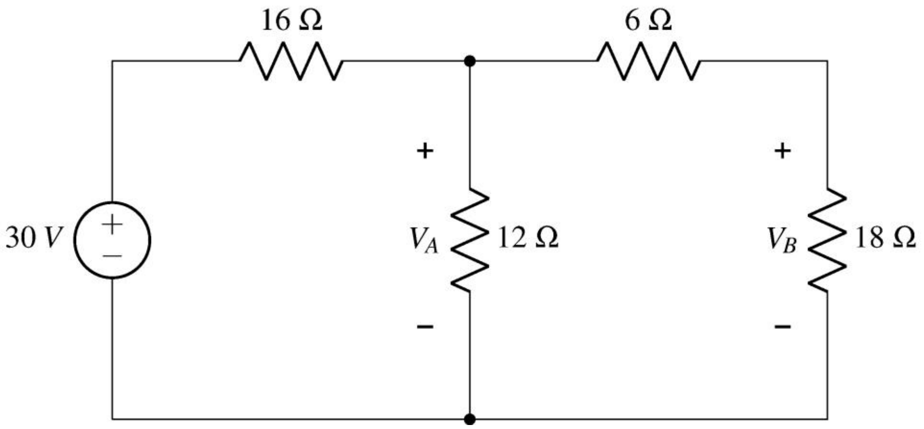
$$\Rightarrow \quad V_1 = \frac{R_1}{R_1+R_2} \times V \quad \text{and} \quad V_2 = \frac{R_2}{R_1+R_2} \times V$$

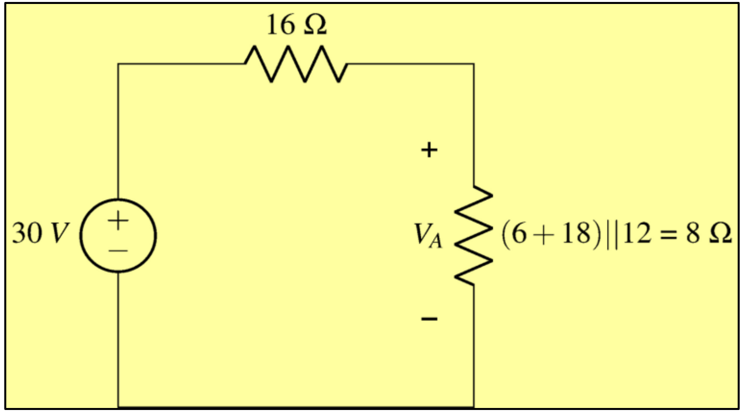
- In general, for any number of resistors connected in series to a supply voltage, the voltage across any particular resistor R_x is,

$$\boxed{V_x = \frac{R_x}{R_1 + R_2 + R_3 + \ \ldots \ldots + R_N} \times V}$$

Example 5

- Using the voltage divider rule, find the voltages V_A and V_B. Don't calculate currents.



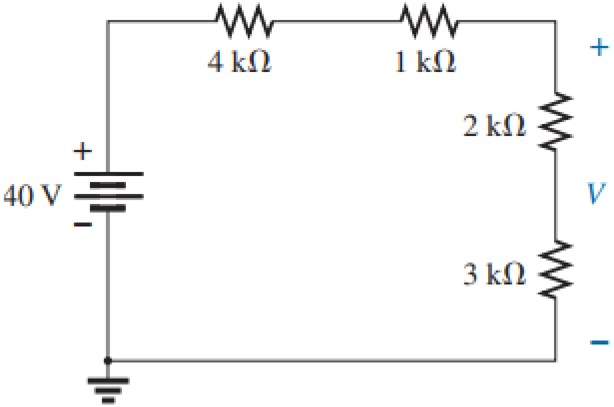


$$V_A = \frac{8}{8+16} \times 30 = 10\ V$$

$$V_B = \frac{18}{18+6} \times V_A = 7.5\ V$$

Problem 15

- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



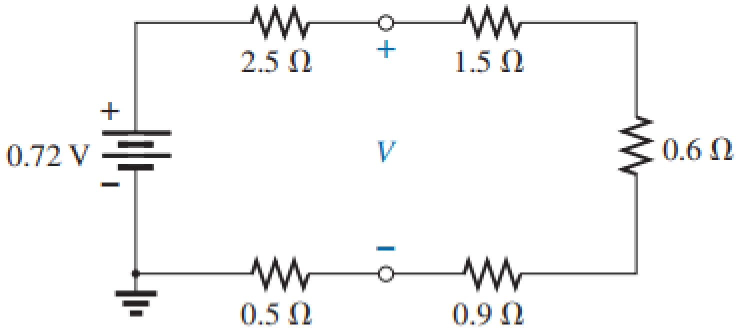
$$V = 40 \times \frac{2+3}{4+1+2+3}$$

$$\boldsymbol{V = 20V}$$

Ans: $\boldsymbol{V}$ = $\boldsymbol{20\ V}$

Problem 16

- Using the voltage divider rule, find the indicated voltage. Don't calculate current.



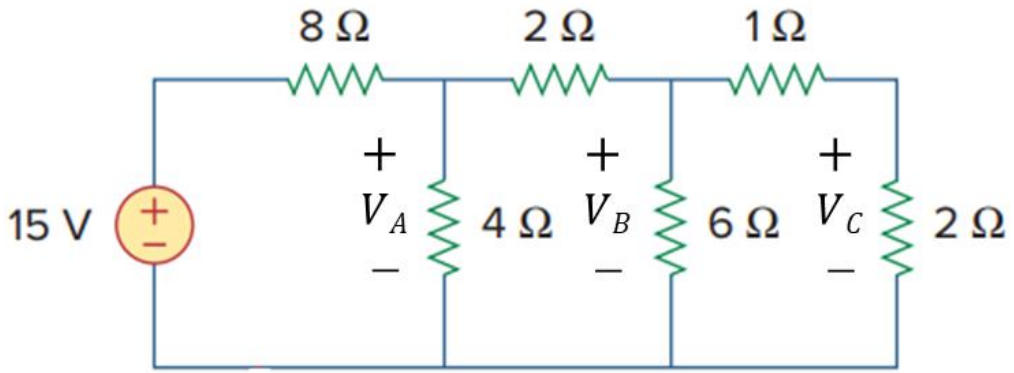
$$V = 0.72 \times \frac{1.5 + 0.6 + 0.9}{2.5 + 1.5 + 0.6 + 0.9 + 0.5}$$

$$\boldsymbol{V = 0.36\ V}$$

Ans: $\boldsymbol{V = 0.36\ V}$

Problem 17

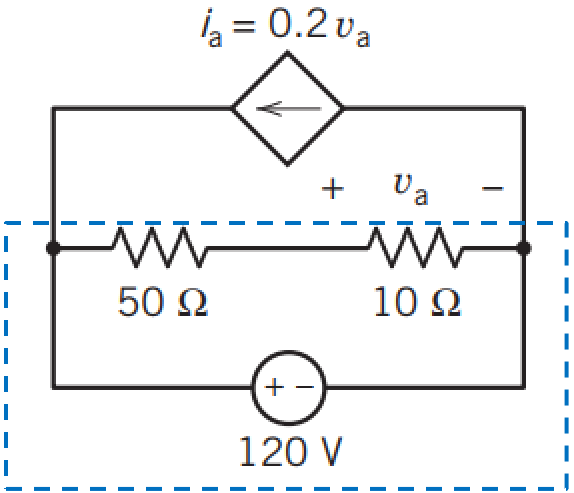
- Using the voltage divider rule, find the voltages V_A, V_B, and V_C. Don't calculate currents.



Ans: $\boldsymbol{V_A = 3\,V;\ V_B = 1.5\,V; V_C = 1\,V}$

Problem 18

- Determine the power of the dependent source. Don't use Ohm's Law.



Applying Voltage divider rule in the blue portion of circuit,

$$V_a = 120 \times \frac{10}{50+10}$$

$$\boldsymbol{v_a = 20\ V}$$

$$i_a = 0.2 \times v_a$$

$$i_a = 0.2 \times 20$$

$$\boldsymbol{i_a = 4\ A}$$

Power of the dependent source,

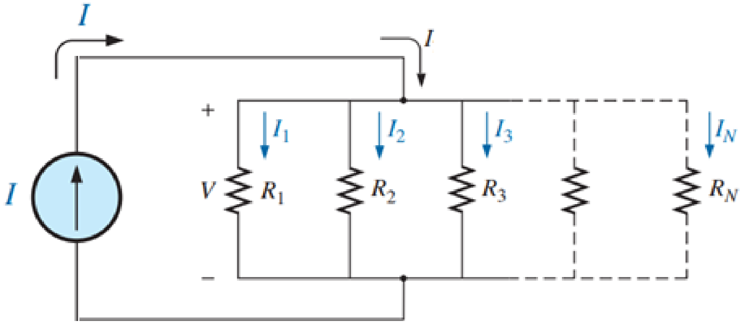
$$P = -(V \times i_a)$$

$$P = -(120 \times 4)$$

$$\boldsymbol{P = -480\ W}$$

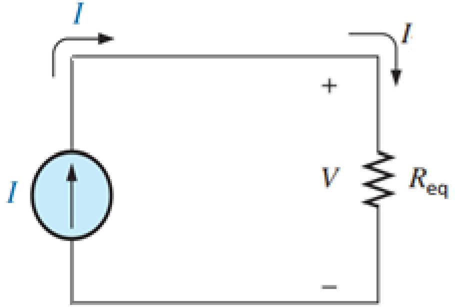
P = - VI if the current enter the negative polarity of the element

Current Division Rule



- The current division rule permits the determination of the currents through resistors connected in parallel without first having to determine the voltage across them.
- Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \ \dots\dots = I_NR_N$$



- Substituting V with $V = IR_{eq}$,

$$IR_{eq} = I_1R_1 = I_2R_2 = I_3R_3 = \ \dots\dots = I_NR_N$$

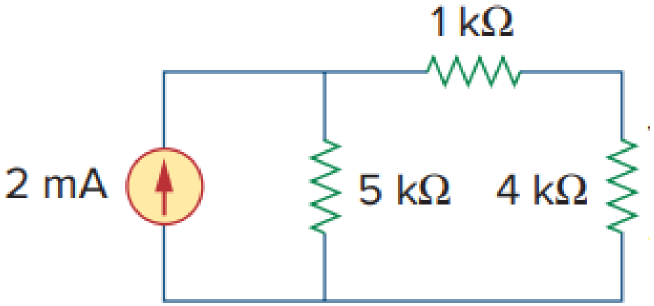
$$\Rightarrow \quad I_1 = \frac{R_{eq}}{R_1} \times I, \qquad I_2 = \frac{R_{eq}}{R_2} \times I, \qquad I_3 = \frac{R_{eq}}{R_3} \times I$$

- In general, for any number of resistors connected in parallel to a supply current, the current through any particular resistor R_x is,

$$\boxed{I_x = \frac{R_{eq}}{R_x} \times I, \text{or}, I_x = \frac{(R_x)^{-1}}{(R_1)^{-1} + (R_2)^{-1} + \dots + (R_N)^{-1}} \times I}$$

Example 6

- Calculate the current through the $5\ k\Omega$ resistor using current division rule. Do not use Ohm's Law.



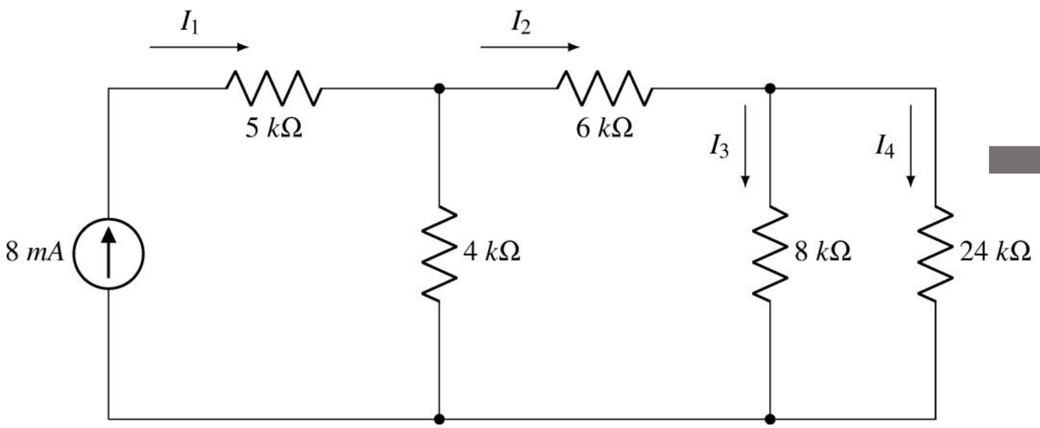
<u>Solution</u>

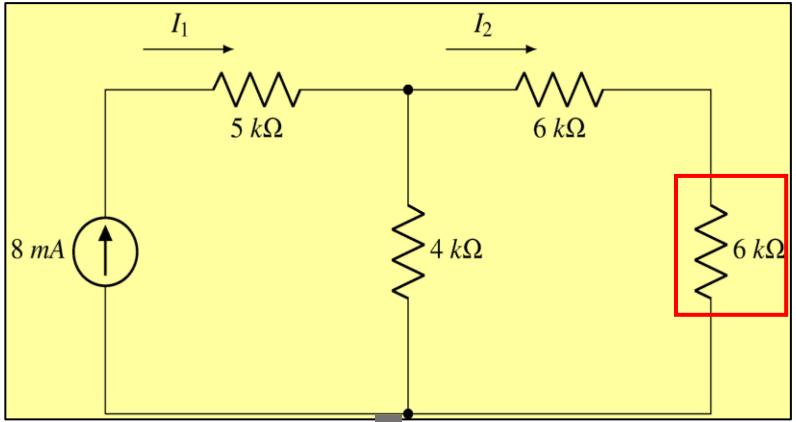
Current through the $5\ k\Omega$ resistor is,

$$\frac{5^{-1}}{(1+4)^{-1}+5^{-1}} \times 2\ mA$$
$$= 1\ mA$$

Example 7

- Calculate the currents I_1 to I_4 using current division rule. Don't calculate voltage.





$$I_1 = 8\ mA$$

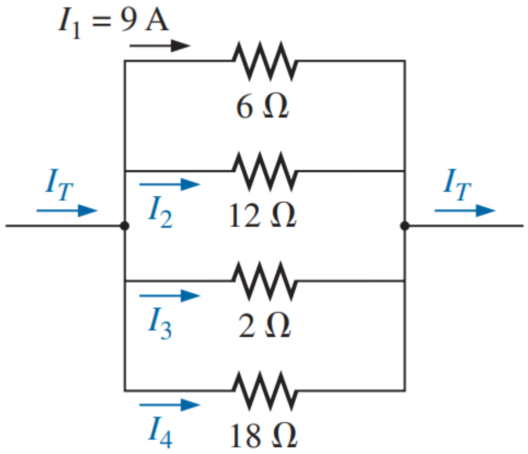
$$I_2 = \frac{12^{-1}}{4^{-1} + 12^{-1}} \times 8\ mA = 2\ mA$$

$$I_3 = \frac{8^{-1}}{24^{-1} + 8^{-1}} \times I_2 = 1.5\ mA$$

$$I_4 = I_2 - I_3 = 0.5\ mA$$

Problem 19

- Based solely on the resistor values, determine all the currents. Do not use Ohm's law.



$$R_{eq} = \left(\frac{1}{6} + \frac{1}{12} + \frac{1}{2} + \frac{1}{18}\right)^{-1} = \mathbf{1.24\ \Omega}$$

From the circuit,

$$I_1 = I_T \times \frac{R_{eq}}{6}$$

$$\gg 9 = I_T \times \frac{1.24}{6}$$

$$\gg \boldsymbol{I_T = 43.5\ A}$$

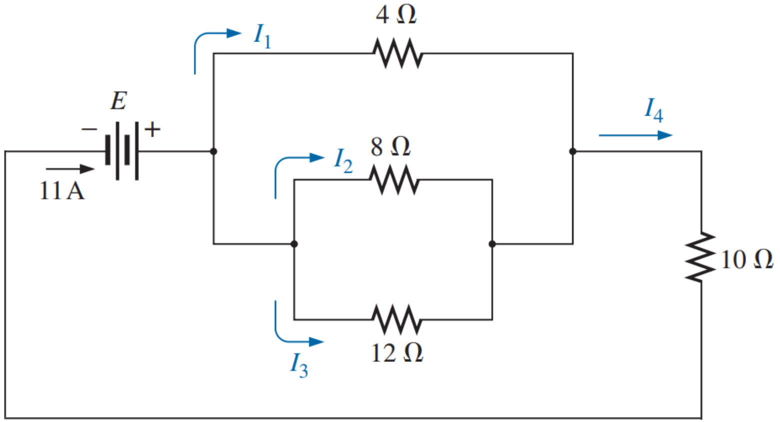
$$I_2 = I_T \times \frac{R_{eq}}{12} = 43.5 \times \frac{1.24}{12} = \mathbf{4.5\ A}$$

$$I_3 = I_T \times \frac{R_{eq}}{2} = 43.5 \times \frac{1.24}{2} = \mathbf{27\ A}$$

$$I_3 = I_T \times \frac{R_{eq}}{18} = 43.5 \times \frac{1.24}{18} = \mathbf{43\ A}$$

Problem 20

- Determine the unknown currents. Do not use Ohm's law.



From the circuit,

$$I_4 = 11\ A$$

The equivalent resistance for 4 Ω, 8 Ω and 12 Ω is,

$$R_{eq} = \left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right)^{-1} = 2.18\ \Omega$$

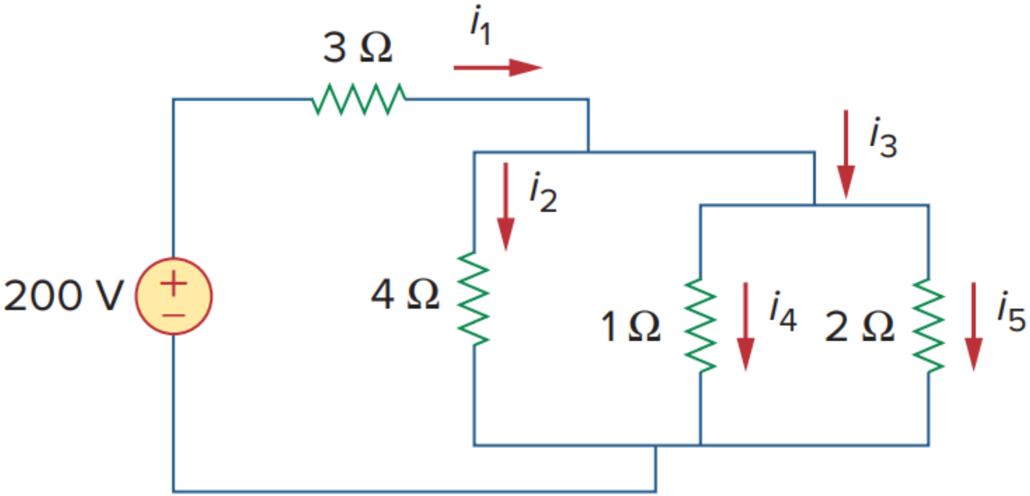
$$I_1 = I_4 \times \frac{R_{eq}}{4} = 11 \times \frac{2.18}{12} = \mathbf{6\ A}$$

$$I_2 = I_4 \times \frac{R_{eq}}{8} = 11 \times \frac{2.18}{2} = \mathbf{3\ A}$$

$$I_3 = I_4 \times \frac{R_{eq}}{12} = 11 \times \frac{2.18}{12} = \mathbf{2\ A}$$

Problem 21

- Determine the currents i_1 to i_5 using current division rule.



Ans: $\boldsymbol{i_1 = 56\,A;\ i_2 = 8\,A;\ i_3 = 48\,A;\ i_4 = 32\,A;\ i_5 = 16\,A.}$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here