Fall 2023

CSE250 - Circuits and Electronics

I-V Characteristics of Linear Circuits

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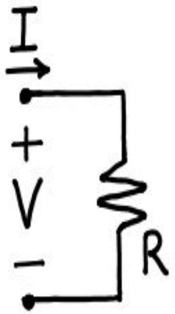
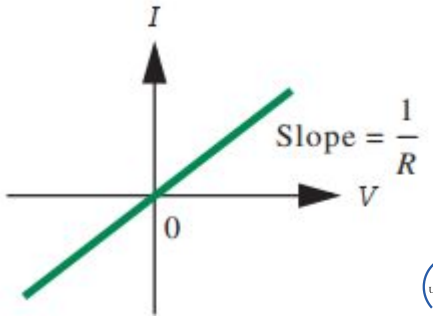
I-V Characteristics

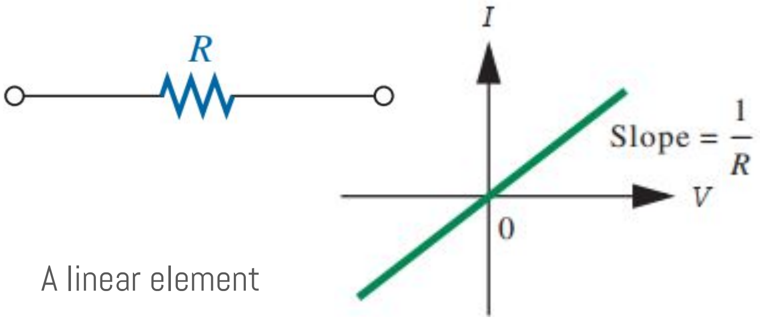
- The *current-voltage characteristics* or the $I - V$ *characteristics* is a relationship, typically expressed graphically, between the electrical current flowing through an element, circuit, device, or material and the corresponding voltage across it.
- From the viewpoint of circuit analysis, $I - V$ the most important characteristics of a two-terminal element, also called *Element Law*.
- So far, we have seen that the current voltage relationship for a resistor follows Ohm's Law, that is, $V = IR$. In an I vs. V plot it is a straight line with slope equal to $\frac{1}{R}$ that goes through the origin.
- It is important to note the direction of current to be plotted. Generally, the current plotted along the y-axis is the current drawn by the element, circuit, or device.



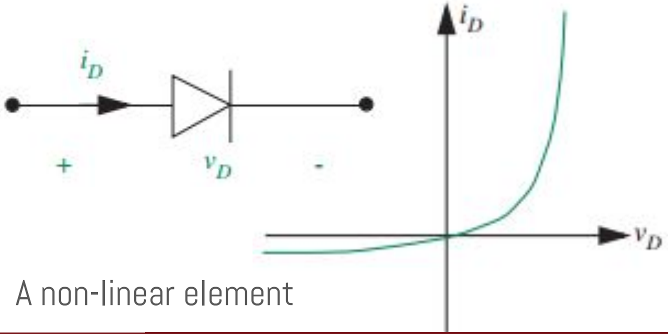


Linear and non-linear elements

- We can distinguish between the linear and non-linear circuit components from $I - V$ point of view.
- A two-terminal circuit element is said to be *linear* if it constitutes a linear (straight line) relationship between the current through and the voltage across it. Examples of linear circuit elements are ideal resistor, ideal voltage source and current source, open circuit, short circuit, capacitors, inductors etc. A circuit constructed with linear circuit elements is called a *linear circuit*.
- *Non-linear* devices, on the other hand, have a $I - V$ curve that is not straight line. Examples of non-linear elements include diodes, transistors, and nonlinear capacitors



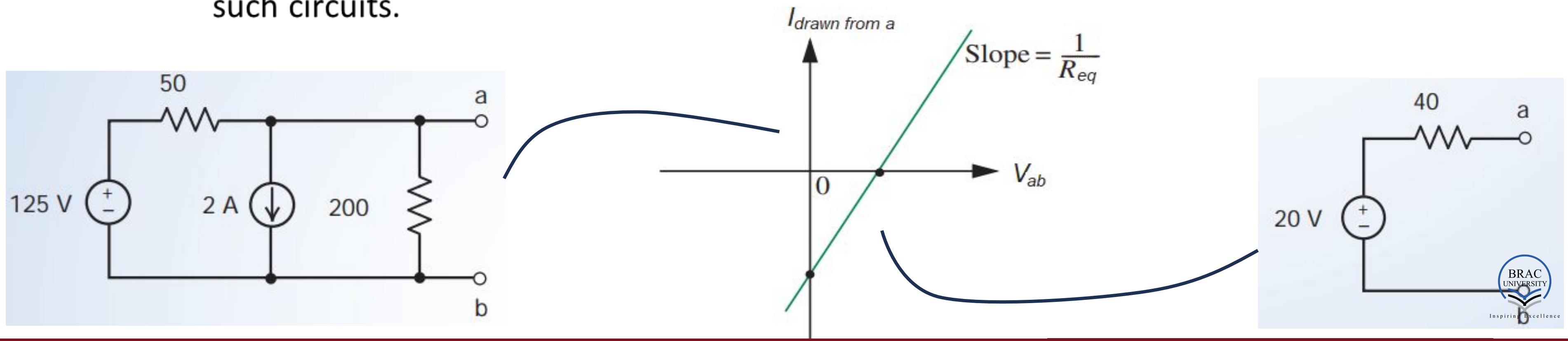
A linear element



A non-linear element

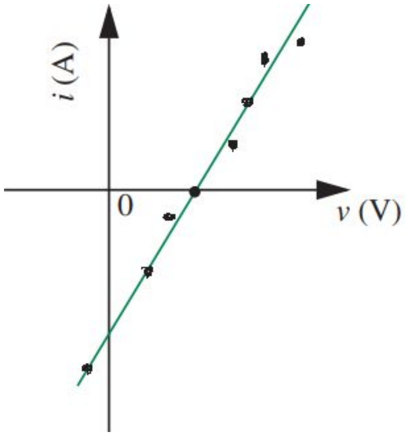
Circuit Equivalence

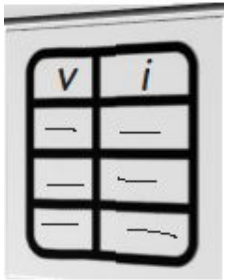
- Two circuits are said to be linear with respect to two particular terminals (or node) if they have identical $I - V$ relationships between those terminals.
- For example, a resistive series network can only be replaced with their equivalent resistance if the $I - V$ line remains identical after replacement. This requires the relation $R_{eq} = R_1 + R_2 + \cdots + R_N$ to be followed.
- Similarly, the following two linear circuits are equivalent as they have the identical $I - V$ relation between terminals $a - b$ as shown. Let's see how to derive $I - V$ plots for such circuits.

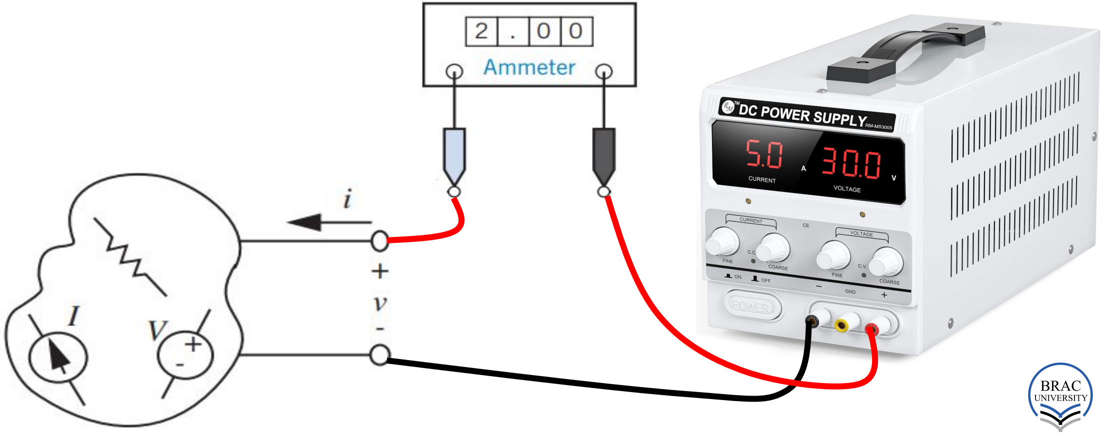


Deriving I-V: experimentally

- To determine the $I - V$ graph of a circuit experimentally, connect and vary a voltage source between the terminals where current and voltage are to be plotted.
- The varying voltages can be measured with a parallel voltmeter (or from the dc source's display), and the corresponding currents drawn by the circuitry can be measured with an ammeter in series. The more data points we collect, the more accurate the $I - V$, particularly for non-linear devices.







Deriving I-V: theoretically

Method 1

Assume V and I variables

Assume a voltage variable between the terminals to be considered (let's say $a-b$) and a current variable directed outward from the '+' of V. The direction ensures the current is drawn by the circuit.

↓

Derive an equation

Apply circuit laws or other solving methods to derive a relation between the variables I and V. I and V are the only variable of the equation.

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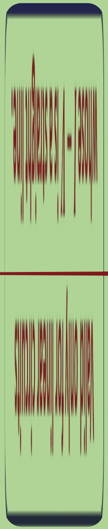
Plot the relation

In a I vs. V grid, plot the equation derived in step 2.



Method 2

Apply a known voltage

Apply a voltage source between the terminals with any arbitrary value.

↓

Solve for current

Solve the circuit and determine the current supplied by the voltage source.

↓

Repeat the previous steps

Again, apply another known voltage and solve for the same current.

↓

Connect the data points

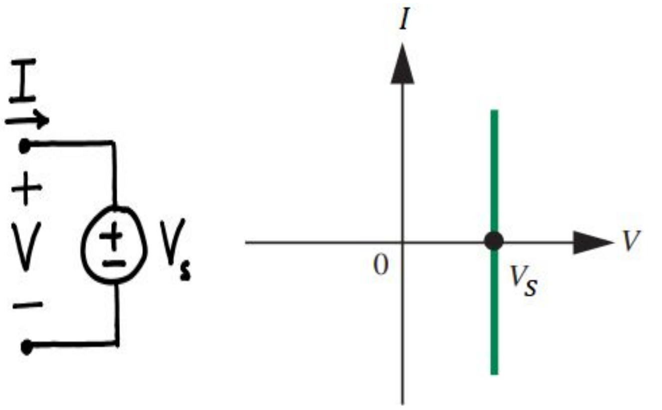
Place the two data points (v_1, i_1) and (v_2, i_2) in a grid and connect them with a line.

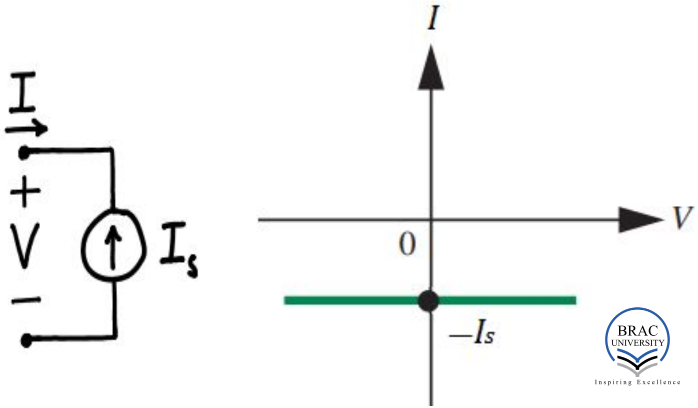
I-V of Voltage and Current Source

- An ideal *independent voltage source* always holds a constant potential difference between its terminals irrespective of the current drawn from it.
- So, the constituent relation for an independent voltage source supplying a voltage of V_s is,

$$V = V_s$$

- This is a straight line that is parallel to the I-axis and intersects the V-axis at V_S.
- Similarly, an ideal *independent current source* always supplies a constant current to the wire it is connected irrespective of the voltage across it.
- The constituent relation is then $I = -I_S$, which is a straight line parallel to V-axis.

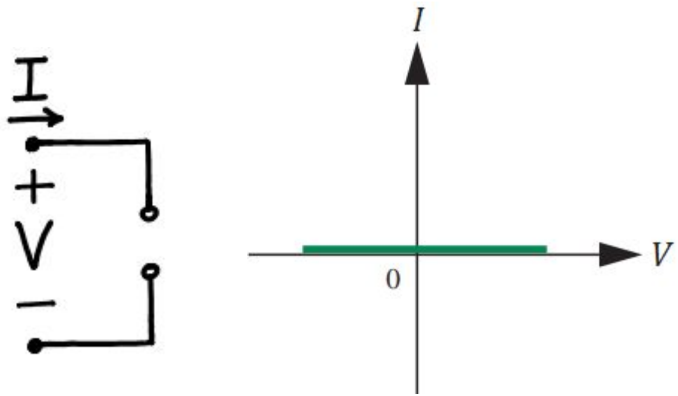




I-V of Open and Short Circuits

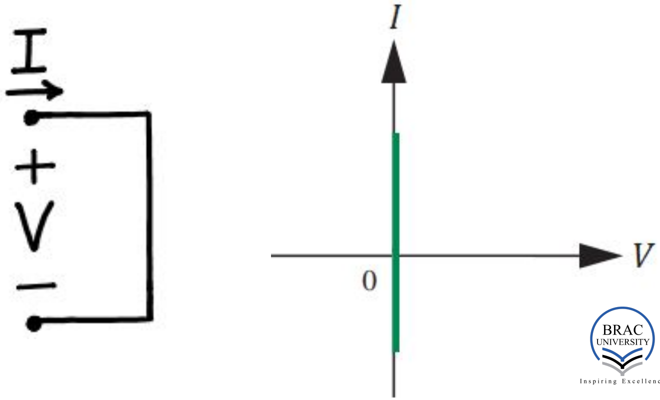
- Recall that, an *ideal open circuit* is the limiting case of a resistor where the resistance approaches infinite.
- As infinite resistance means zero current according to the Ohm's law, the constituent relation for an open circuit is,

$$I = 0$$



- An *ideal short circuit (or a wire)* is the limiting case of a resistor where the resistance approaches zero.
- As zero resistance means there can be no voltage difference according to the Ohm's law, the constituent relation for a short circuit is,

$$V = 0$$



Example 1

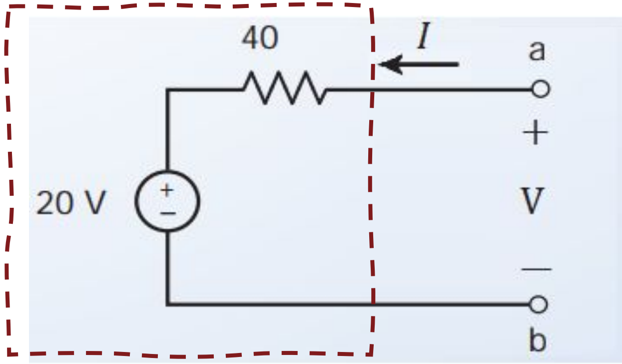
- Derive and plot the $I - V$ relationships of the following configurations: a $20\ V$ voltage source in series with a $40\ \Omega$ resistor.

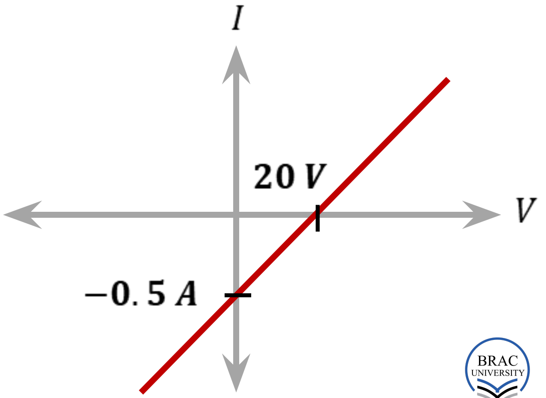
- Let's say we have a $20\ V$ voltage source in series with a $40\ \Omega$ resistor between terminals $a - b$ as shown.
- Applying KVL to the loop yields,

$$-V + 40I + 20 = 0$$

$$\Rightarrow\ I = \frac{1}{40}V - 0.5$$

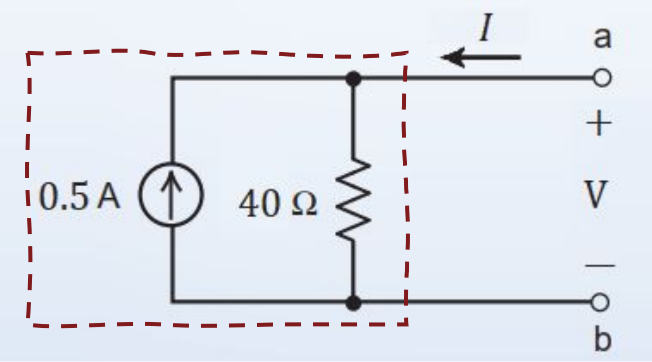
- This is straight line that intersects the current and voltage axes at $(20\ V,\ 0)$ and $(0,\ \ -0.5\ A)$ respectively.
- *It is important to notice here that, I is the current resulting from the application of a bias V. One must not interpret the $a - b$ terminals as open circuit with 0 current in this case. Think V as a applied voltage source connected between a and b.*





Example 2

- Derive and plot the $I - V$ relationships of the following configurations: a $0.5\ A$ current source in parallel with a $40\ \Omega$ resistor.

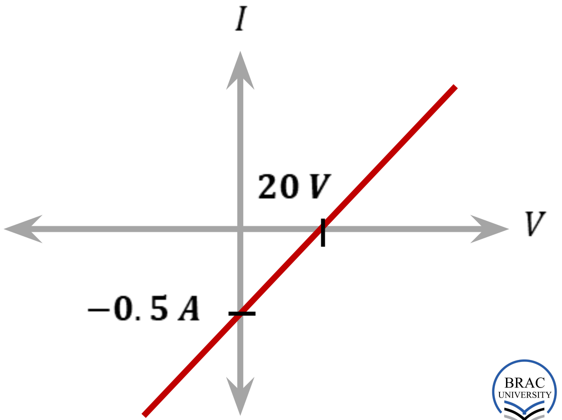


- Let's say we have a $0.5\ A$ current source in parallel with a $40\ \Omega$ resistor between terminals $a - b$ as shown.
- Applying KCL to the node a yields,

$$0.5 + I - \frac{V}{40} = 0$$

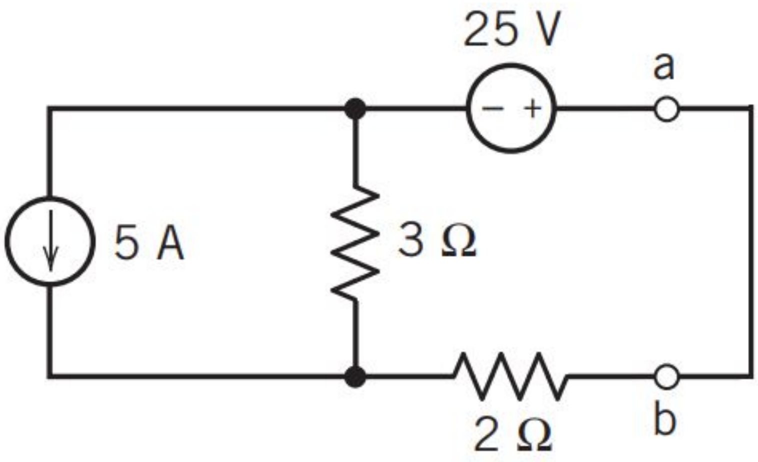
$$\Rightarrow \quad I = \frac{1}{40}V - 0.5$$

- This is straight line that intersects the current and voltage axes at $(20\ V,\ \ 0)$ and $(0,\ \ -0.5\ A)$ respectively.
- *Notice that the $I - V$ curve is identical to that derived in [Example 1](). Thus, the two circuits are equivalent to each other.*



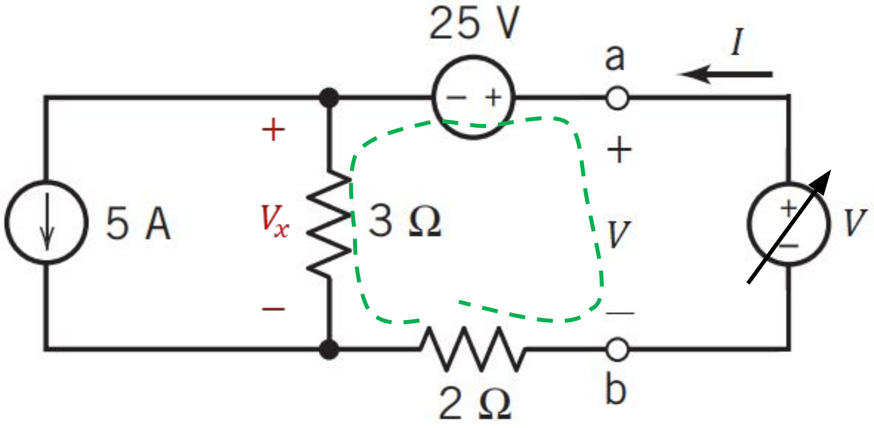
Example 3

- Derive and plot the $I - V$ relationship of the left portion of $a - b$ in the following circuit



- The first step is to consider only the left portion of the terminal $a - b$ disconnect anything connected to the right (a short circuit in this case).
- Then we have to apply a voltage (taken as a variable V) between terminals $a - b$ and determine the current supplied by V (denoted as variable I).

Example 3: solution



- The arrow symbol with a voltage source means it is a voltage to be varied, variable in our case.
- To solve the circuit using KVL, KCL, and Ohm's law, let the voltage across the $3\ \Omega$ resistor be V_x.

- Applying KCL at the positive node of V_x,

$$5 + \frac{V_x}{3} - I = 0$$

$$\Rightarrow \ V_x = 3I - 15$$

- Now, for the 25 V source, we can write using KVL to the loop consisting of $3\ \Omega$, $25\ V$, $2\ \Omega$, and V as shown by the dashed arrow,

$$-V_x - 25 + V - 2I = 0$$

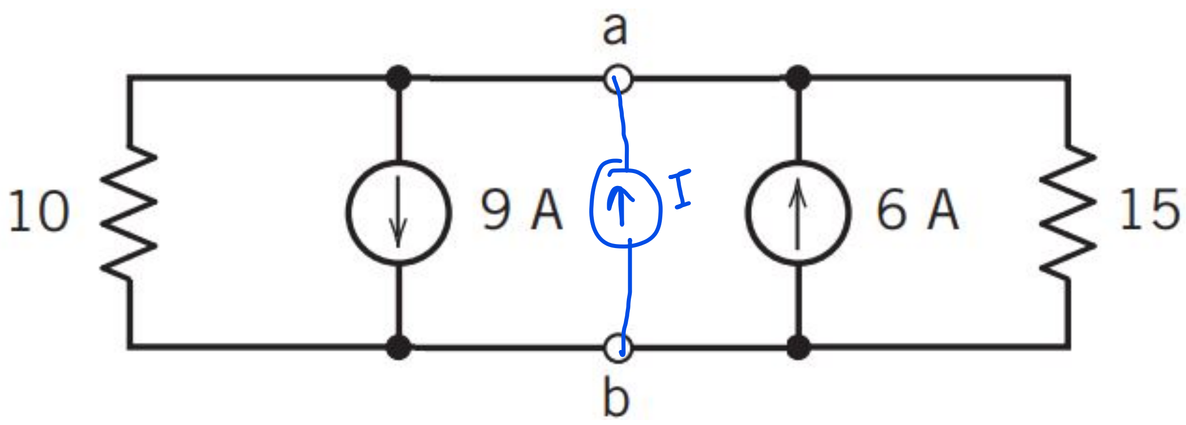
$$\Rightarrow \ -(3I - 15) - 25 + V - 2I = 0$$

$$\Rightarrow \ I = \frac{1}{5}V - 2$$

- This constitutes a straight line of slope ${}^{1}/_{5}\ (\Omega^{-1})$ that intersects the axes at $(10\ V,\ 0)$ and $(0,\ -2\ A)$.

Problem 1

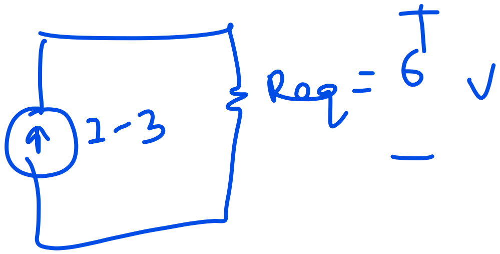
- Derive the $I - V$ characteristics of the following circuit with respect to the terminals $a - b$.



$R_{eq} = 10 || 15$

$I_{eq} = I + 6 - 9$
$= I - 3$

$\boxed{\text{Ans: } \boldsymbol{I = \frac{1}{6}V + 3}}$



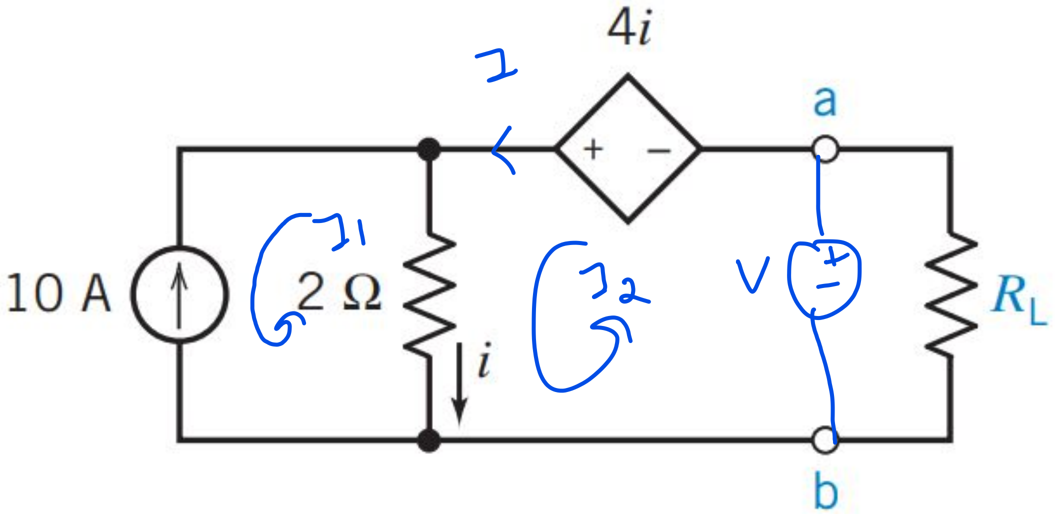
$$\therefore \; V = (I-3)\times 6$$

$$\frac{V}{6} = I-3$$

$$\therefore \; I = \frac{V}{6} + 3$$

Problem 2

- Derive the $I - V$ characteristics of the following circuit with respect to the terminals $a - b$.



$\boxed{\text{Ans: } \boldsymbol{I = -\frac{1}{2}V - 10}}$

$I_2 = I$

$I_1 = -10$

$i = I_2 - I_1$

$= I + 10$

So,

$-V - 4i + 2I_2 - 2I_1 = 0$

$-V - 4(I+10) + 2\times I + 20 = 0$

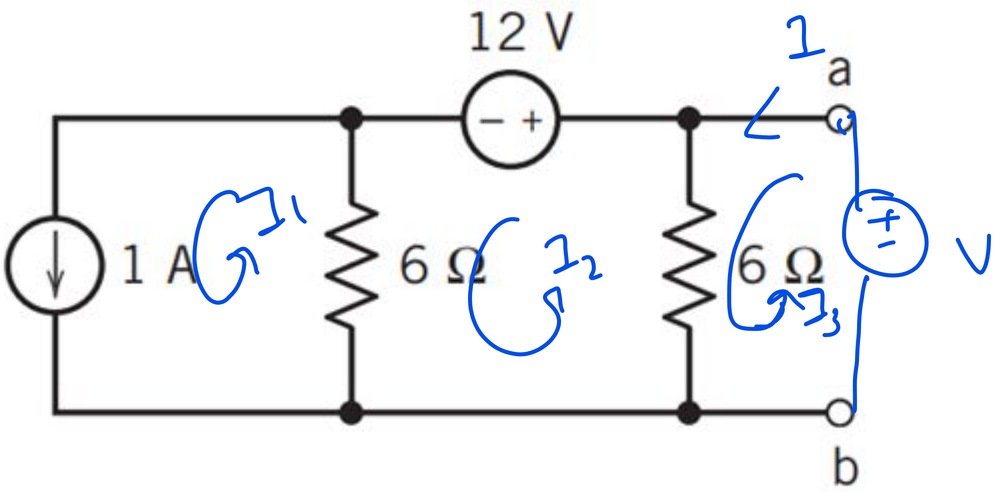
$-4I - 40 + 2I = V - 20$

$-2I = V + 20$

$I = -\frac{V}{2} - 10$

Problem 3

- From the following circuit, derive the current–voltage characteristics equation between the terminals $a - b$.



Ans: $I = \frac{1}{3}V - 1$

$I_1 = 1A$, $I_3 = I$

$12 I_2 - 6 I_1 - 6 I_3 = -12$

$-6 + 12 I_2 - 6 I = -12$

$12 I_2 = 6I - 6$

Loop 3:

$6 I_3 - 6 I_2 = V$

$6 I - 6 I_2 = V$

$12 I - 12 I_2 = 2V$

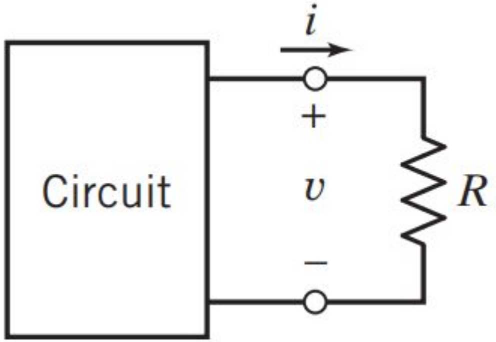
$12 I - 6I + 6 = 2V$

$6I = 2V - 6$

$I = \frac{V}{3} - 1$

Problem 4

- A resistor, R, was connected to a circuit box as shown below. The current i was measured. The resistance was changed, and the current was measured again. The results are shown in the table.
  i. Plot the relationship between i and v.
  ii. Draw a circuit diagram with minimum number of circuit elements that can give rise to the same $i - v$ curve derived in i.



R	i
2 kΩ	4 mA
4 kΩ	3 mA

Ans: $\boldsymbol{i = -\frac{1}{4}v + 6}$

$V = IR$

$V_1 = I_1 R_1 = 8$

$V_2 = I_2 R_2 = 12$

So, $\frac{1}{R_s} = \frac{4-3}{8-12} = -\frac{1}{4}$

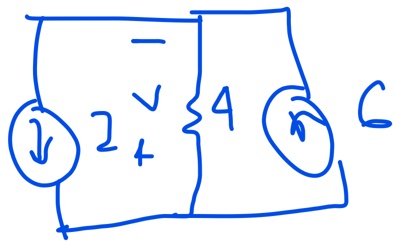
So, slope $= \frac{-1}{4}$

$I = -\frac{V}{4} + I_s$

$4 = \frac{-8}{4} + I_s$

$I_s = 6$

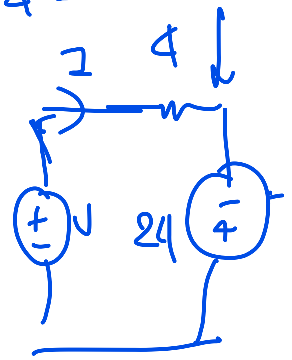
$I = -\frac{V}{4} + 6$



(11)

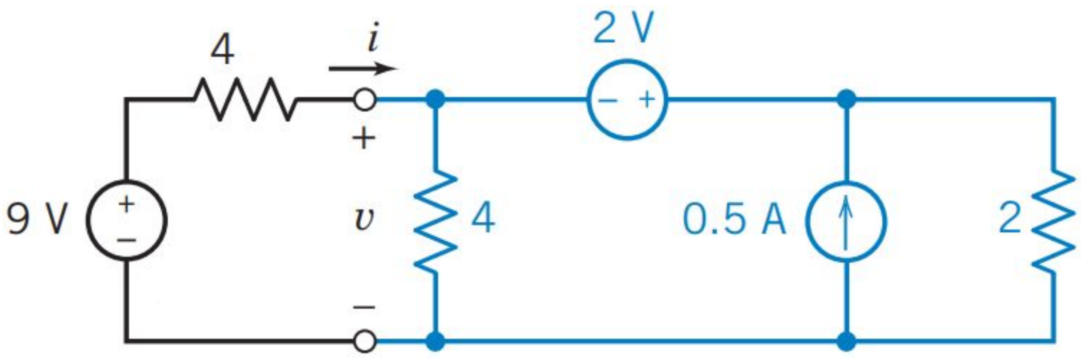
$4I = -V + 24$

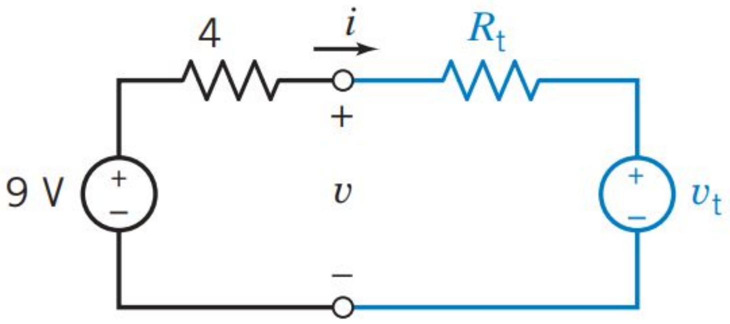
$4I + V - 24 = 0$



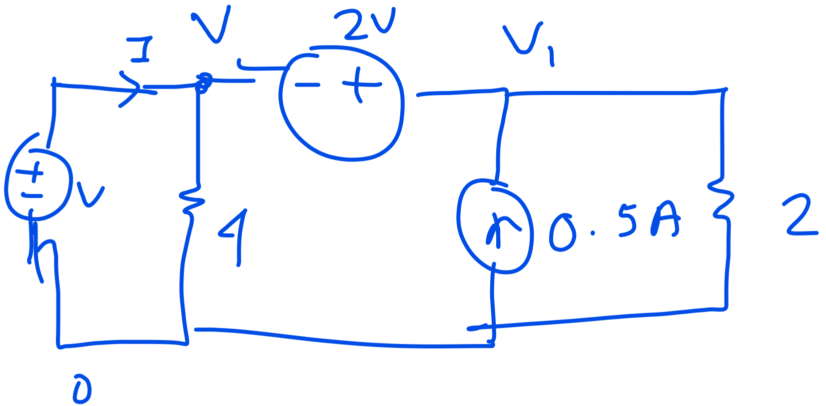
Problem 5

- Determine the values of R_t and v_t, if the following two circuits are equivalent to each other.





$\boxed{\underline{\text{Ans}}\text{: } v_t = -{}^{2}/_{3}\, V,\ R_t = {}^{4}/_{3}\ \Omega}$



Super node V, V_1

$$\frac{V}{4} - 0.5 + \frac{V_1}{2} - I = 0$$

$$I = -\frac{V}{4} + 0.5 - \frac{2+V}{2}$$

$$= -\frac{V}{4} + 0.5 - 1 - \frac{V}{2}$$

$$V_1 - V = 2 \quad \text{——} \quad (1)$$

$$I = -\frac{3V}{4} - 0.5$$

So, $\frac{1}{R_t} = 3/4$

$\therefore R_t = 4/3$

for, V_t, $I = 0$

So, $0.5 = -\frac{3V_t}{4}$

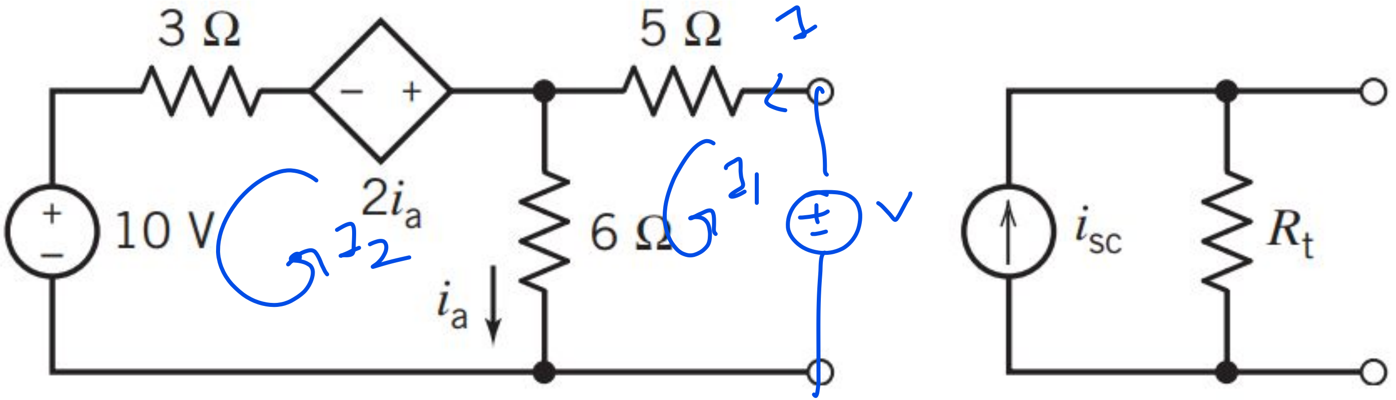
$\therefore V_t = -2/3$

Problem 6

for, i_{sc} put $v=0$ at ①

So, $I_{sc} = 1.13$

- Determine the values of R_t and i_{sc}, if the following two circuits are equivalent to each other.



Ans: $\boldsymbol{i_{sc} = 1.13\ A,\ R_t = 7.57\ \Omega}$

$I_1 = I$

$I_1 - I_2 = i_a$

Loop 2:

$6I_2 - 6I_1 + 2i_a + 3I_2 = -10$

$6I_2 - 6I_1 + 2I_1 - 2I_2 + 3I_2 = -10$

$7I_2 = -10 + 4I_1$

$\therefore I_2 = \frac{-10 + 4I}{7}$

Loop 1:

$11I_1 - 6I_2 = V$

$\frac{11}{6}I - I_2 = \frac{V}{6}$

$\frac{11}{6}I + \frac{10}{7} - \frac{4I}{7} = \frac{V}{6}$

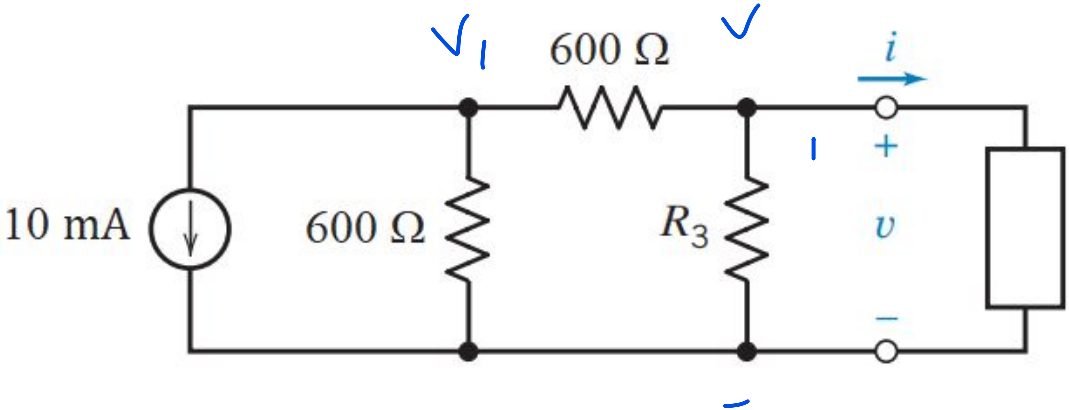
$\frac{53}{42}I = \frac{V}{6} - \frac{10}{7}$

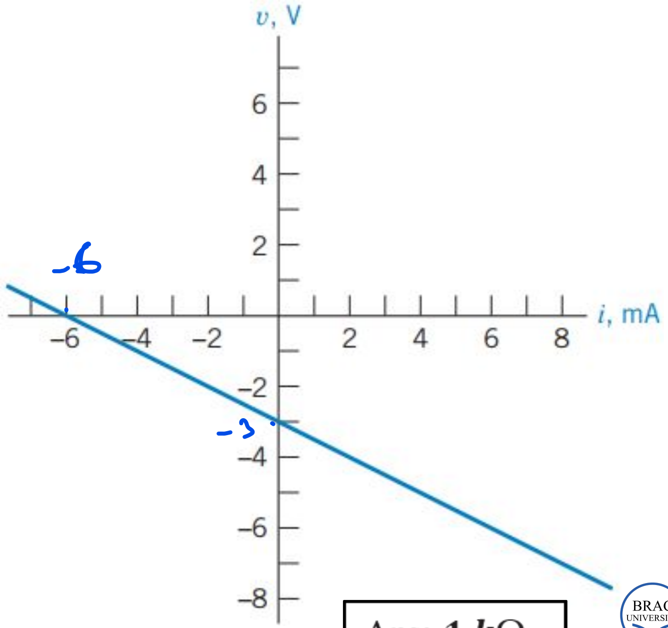
$\therefore I = \frac{42}{53} \times \frac{V}{6} - \frac{10 \times 42}{53 \times 7}$ — (1)

So, $\frac{1}{R_t} = \frac{42}{53} \times \frac{1}{6}$ $\therefore R_t = 7.57$

Problem 7

- If the voltage v vs. current i has the following relationship expressed graphically, determine the value of R_3.





Ans: $1\ k\Omega$

$$\frac{V}{-3} + \frac{I}{-6} = I$$

$$2V + I = -6$$

$$I = -2V - 6$$

$$V_1\left(\frac{1}{0.6} + \frac{1}{0.6}\right) - \frac{V}{0.6} = -10$$

$$V_1 \times \frac{10}{3} - \frac{V \times 10}{6} = -10$$

$$\frac{V_1}{3} - \frac{V}{6} = -1$$

$$2V_1 - V = -6$$

$$2V_1 = -6 + V$$

$$V\left(\frac{1}{0.6} + \frac{1}{R_3}\right) - \frac{V_1}{0.6} + i = 0$$

$$V\left(\frac{1}{0.6} + \frac{1}{R_3}\right) - \frac{1}{0.6}\left(-3 + \frac{V}{2}\right) + i = 0$$

$$i = -V\left(\frac{1}{0.6} + \frac{1}{R_3}\right) + \frac{1}{0.6}\left(-3 + \frac{V}{2}\right)$$

$$i = -V\left(\frac{1}{0.6} + \frac{1}{R_3} - \frac{1}{1.2}\right) - \frac{3}{0.6}$$

$$\frac{1}{0.6} + \frac{1}{R_3} - \frac{1}{1.2} = 2$$

So, $R_3 = 0.85k$

Thank you for your attention