Fall 2023

CSE250 - Circuits and Electronics

SUPERPOSITION PRINCIPLE

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Linearity property

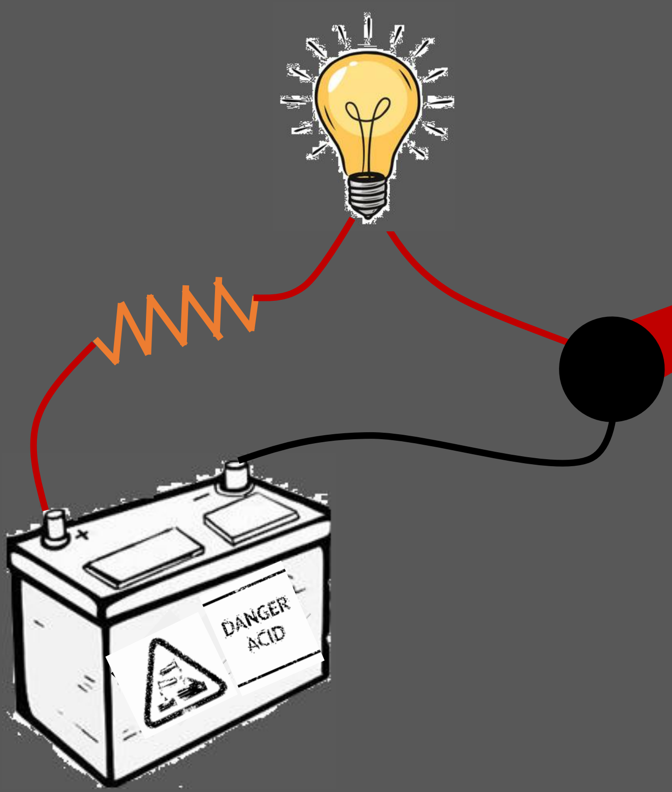
- *Linearity* is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the *homogeneity* (scaling) property and the *additivity* property.

- Homogeneity property
  For a resistor, for example, Ohm's law relates the input i to the output v as $v = iR$. If the current is increased by a constant k, then the voltage increases correspondingly by k; that is, $kv = (ki)R$.

- Additivity property
  If applying v_1 & v_2 separately to a resistor gives rise to currents i_1 & i_2 respectively, then applying $(v_1 + v_2)$ should give rise to the current $(i_1 + i_2)$.

- A *linearity circuit* is one whose output is linearly related (or directly proportional) to its input.

Course Outline: broad themes



- Circuit Laws
- Methods of Analysis
- Circuit Theorems
  - Superposition Principle
- First Order Circuits
- AC Circuits

Superposition Principle

- The *superposition principle* states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source.

$$P_{Total}^2 \neq P_1^2 + P_2^2 + \ldots + P_N^2$$

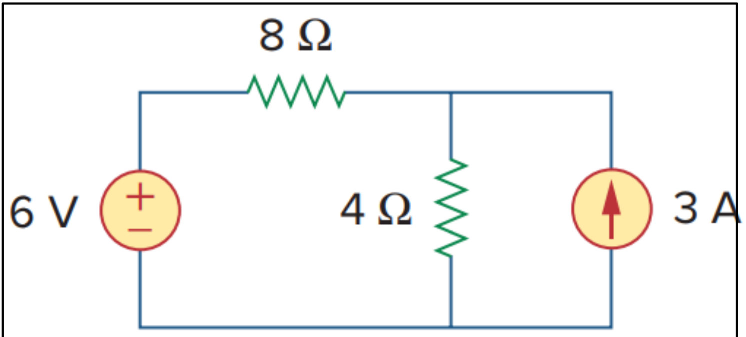
- If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Steps to Apply Superposition Principle:

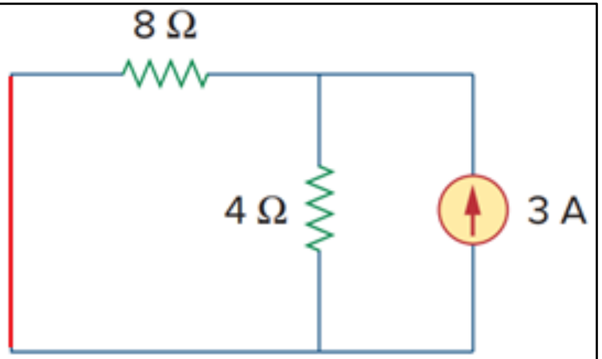
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Equivalence with inactive I/V sources

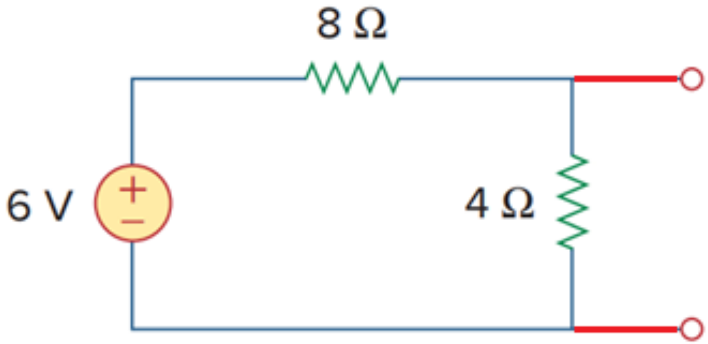
- In superposition principle, we consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by $0\ V$ (or a short circuit), and every current source by $0\ A$ (or an open circuit).
- Dependent sources are left intact because they are controlled by circuit variables.



Original circuit



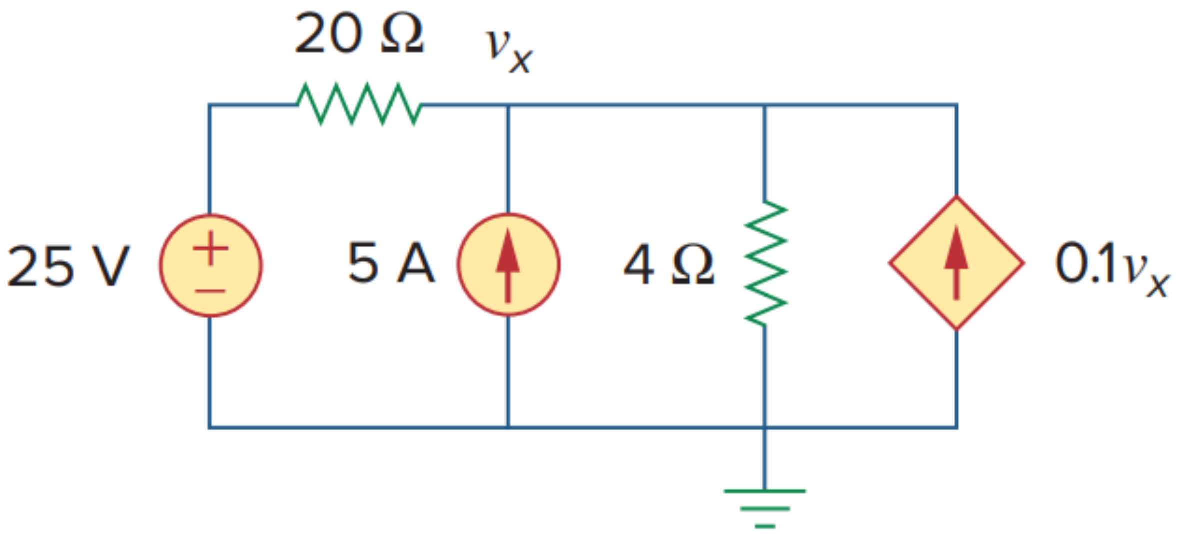
The voltage source has been replaced with a short circuit



The current source has been replaced with an open circuit

Example 1

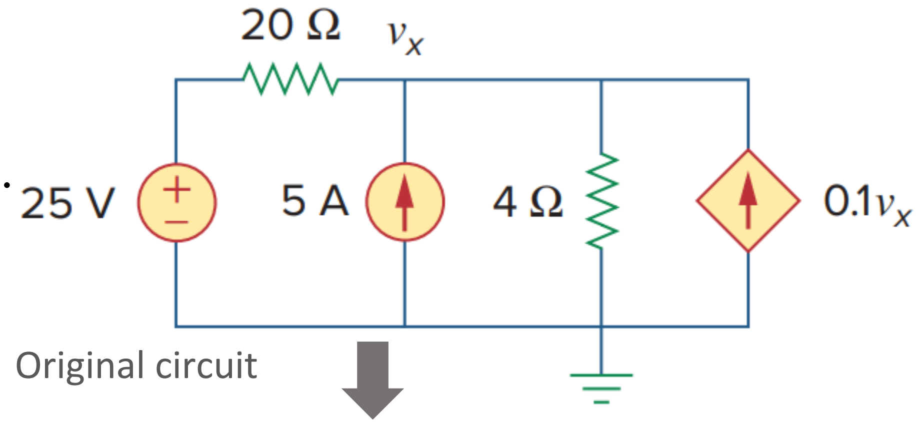
- Use Superposition Principle to find v_x.



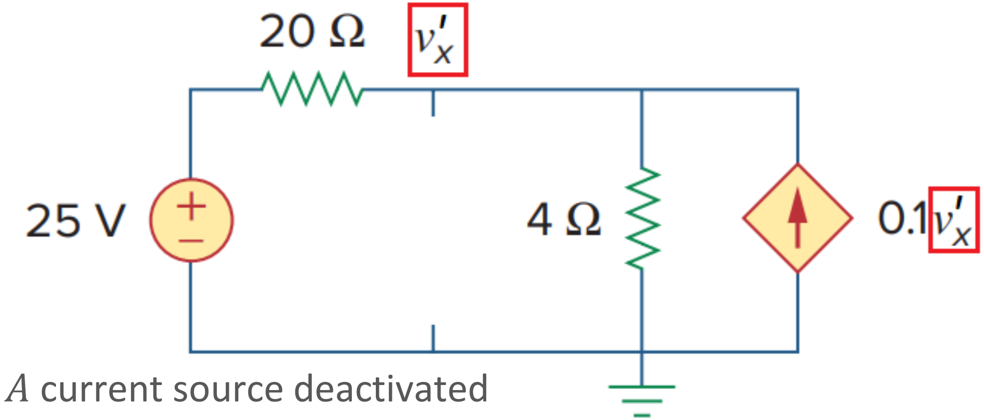
There are two independent and one dependent sources. The principle requires us to determine the individual contributions of the two independent sources to the node voltage v_x. If v_x' and v_x'' are the contributions from the $25\ V$ voltage source and $5\ A$ current source respectively, then

$$v_x = v_x' + v_x''.$$

Example 1: 25 V source is active



Original circuit



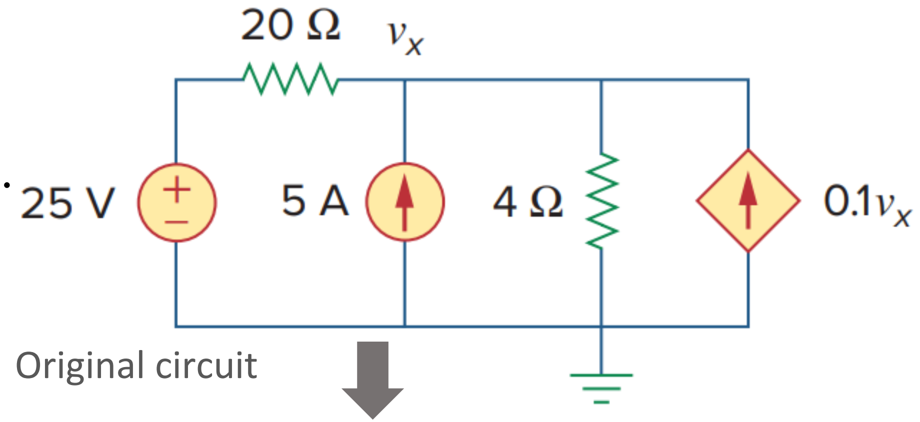
5 A current source deactivated

- The 5 A current source has been replaced by an open circuit. The notation v_x is replaced by v_x'.
- Different circuit solving techniques (nodal analysis or mesh analysis or source transformation or voltage division) can be applied to solve for v_x'. Nodal analysis may be the easiest one.
- KCL at the node v_x',

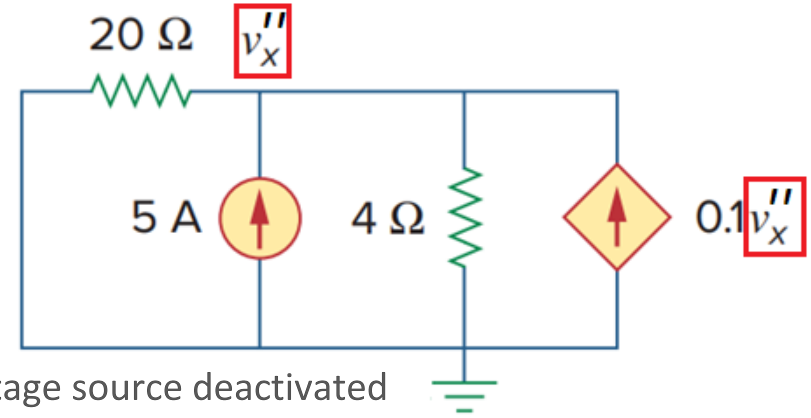
$$\frac{v_x' - 25}{20} + \frac{v_x'}{4} = 0.1v_x'$$

Simplification yields, $v_x' = 6.25\ V$

Example 1: 5 A source is active



Original circuit



25 V voltage source deactivated

- The 25 V voltage source has been replaced by a short circuit. The notation v_x is replaced by v_x''.
- KCL at the node v_x'',

$$\frac{v_x''}{20} + \frac{v_x''}{4} = 5 + 0.1v_x''$$

Simplification yields, $v_x'' = 25\ V$

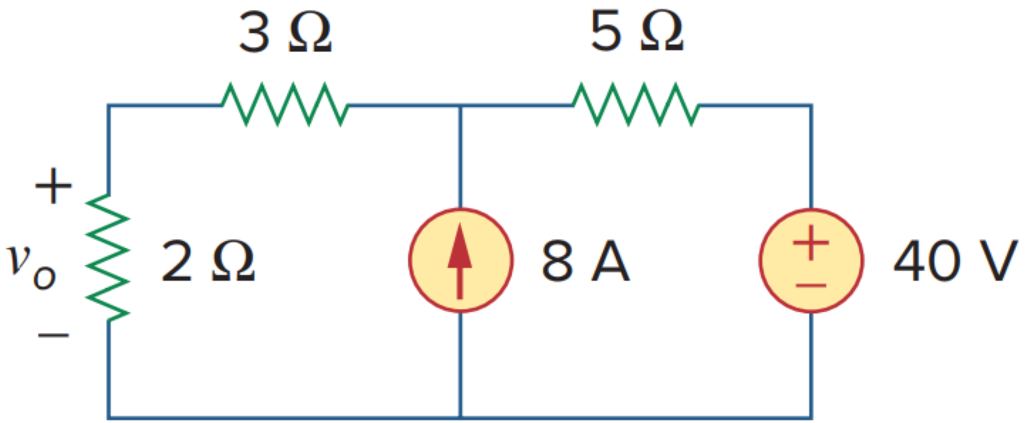
So, according to the Superposition Principle,

$$v_x = v_x' + v_x''$$

$$\Rightarrow \quad v_x = 6.25 + 25 = 31.25\ V$$

Problem 1

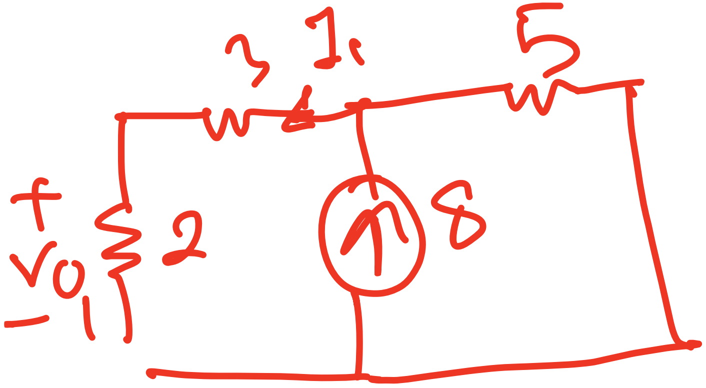
- Using the Superposition Theorem, find v_o.



Ans: $\boldsymbol{v_0 = 16\ V}$

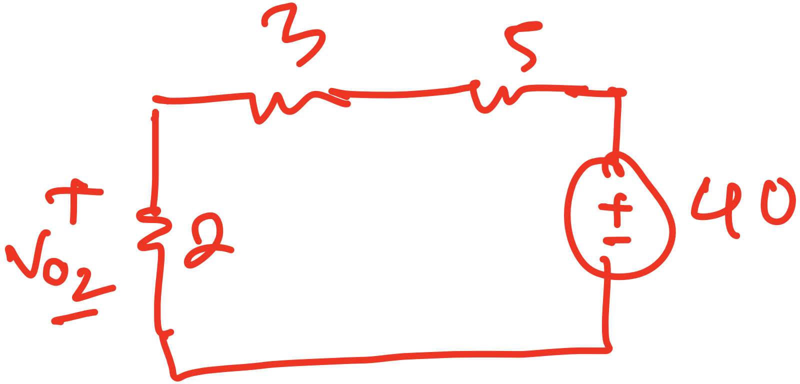
When 8A is active:

$$I_1 = \frac{5^{-1}}{5^{-1}+5^{-1}} \times 8 = 4$$



$$V_{o_1} = 4 \times 2 = 8$$

When 40V is active:

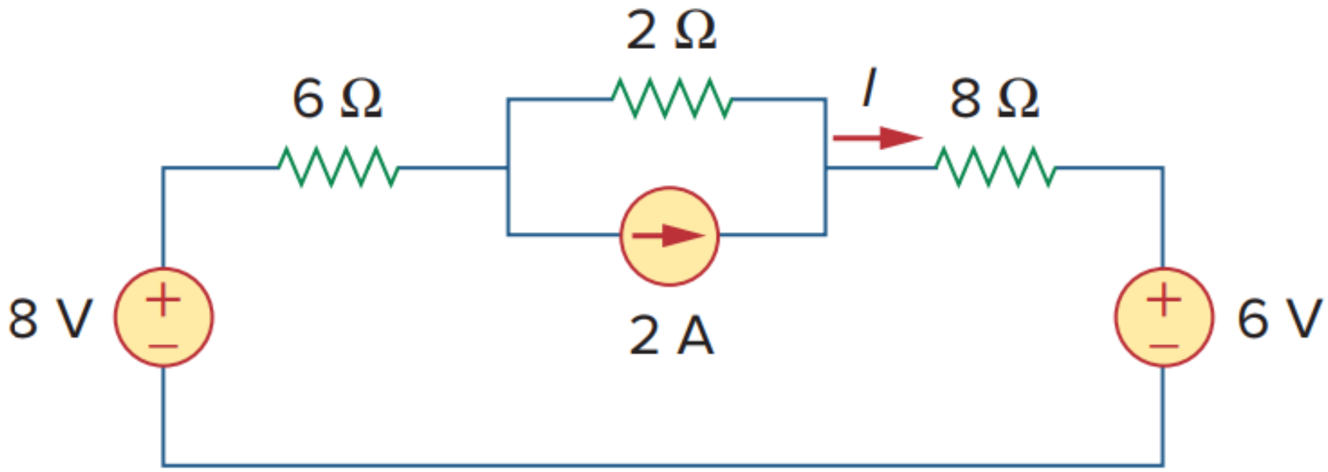


$$V_{o_2} = \frac{2}{2+3+5} \times 40 = \frac{2}{10} \times 40 = 8V$$

$$\therefore V_o = V_{o_1} + V_{o_2} = 16V$$

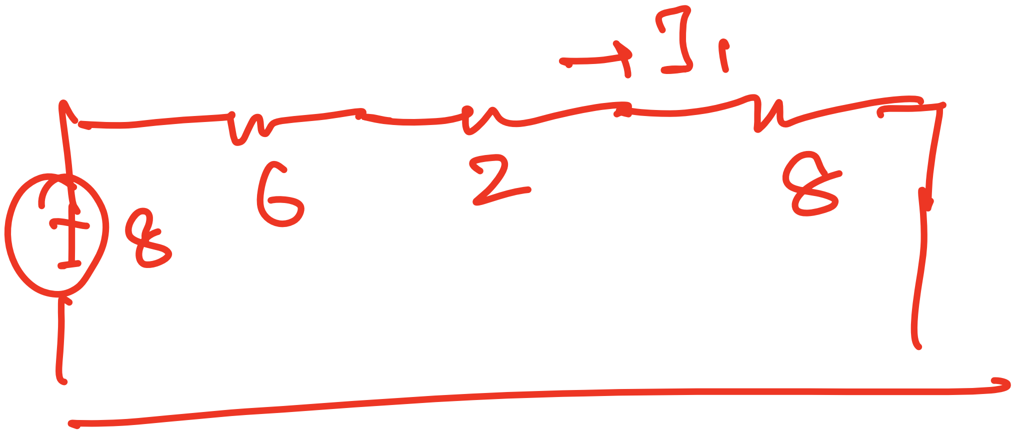
Problem 2

- Find I in the circuit using the Superposition Principle.



Ans: $i_0 = 0.375\ A$

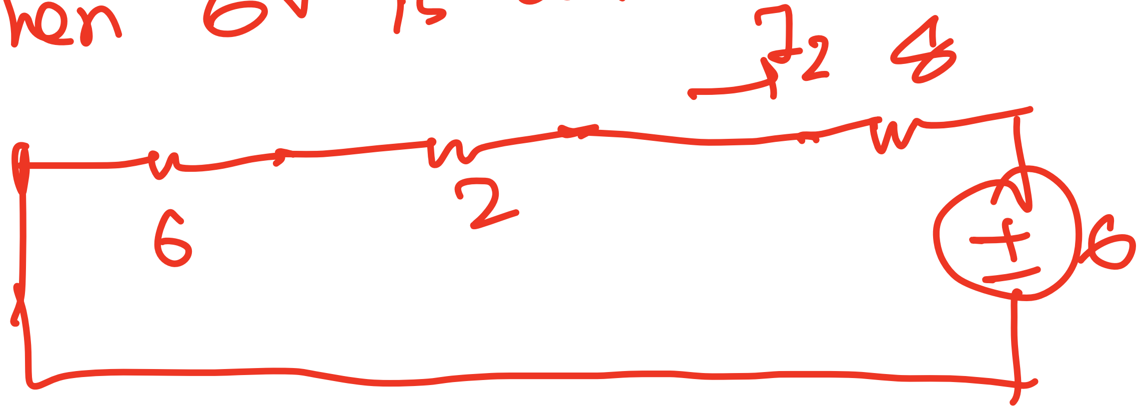
When 8V is active:



$$\therefore I_1 = \frac{8}{6+2+8}$$

$$= \frac{1}{2} A$$

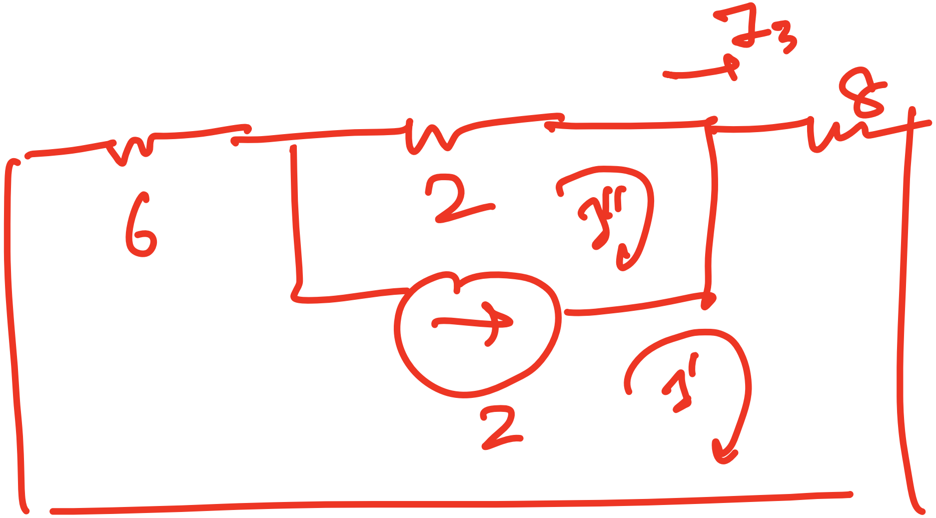
When 6V is active:



$$I_2 = \frac{-6}{8+8} A$$

$$= -0.375 A$$

When 2A is active:



$\therefore I_0 = I_1 + I_2 + I_3$

$= 0.375A$

$I' - I'' = 2$

$(6+8)I' + 2I'' = 0$

$14I' + 2\cdot(I' - 2) = 0$

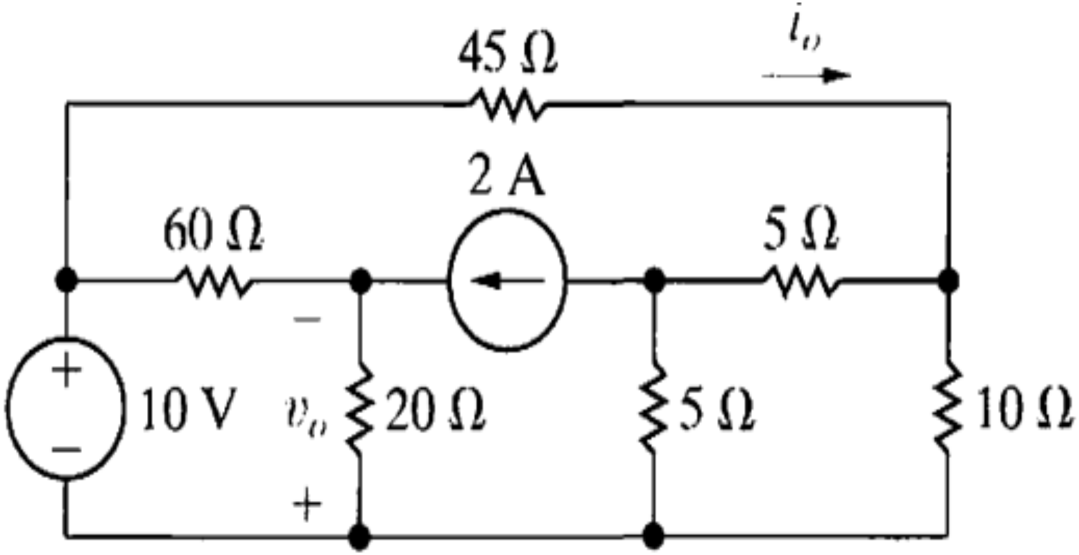
$14I' + 2I' - 4 = 0$

$I' = \frac{4}{16}$

$\therefore I_3 = \frac{4}{16}$

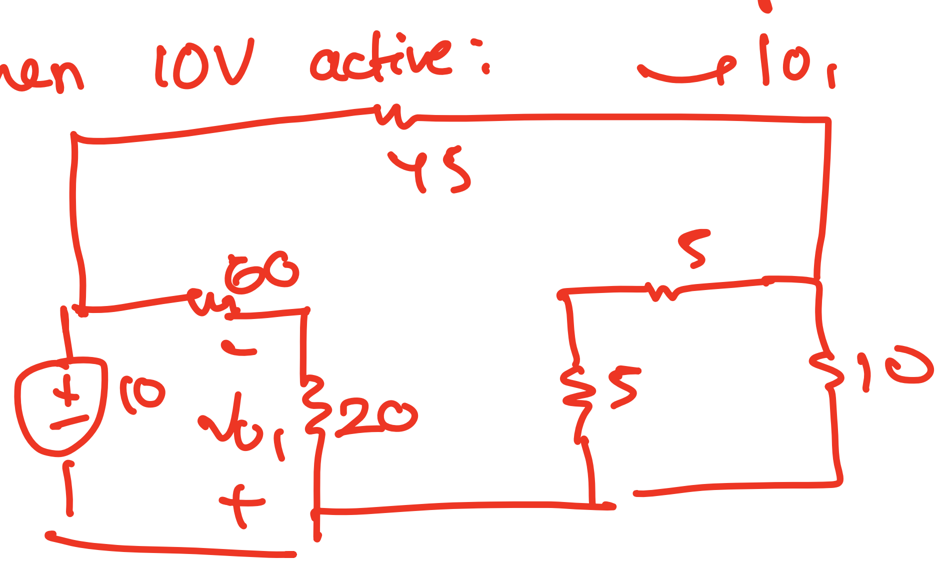
Problem 3

- Use Superposition Principle to solve for i_0 and v_0.



Ans: $\boldsymbol{i_0 = 0.2 + 0.1 = 0.3\ A;\ v_0 = -2.5 - 30\ V = -32.5\ V}$

when 10V active:

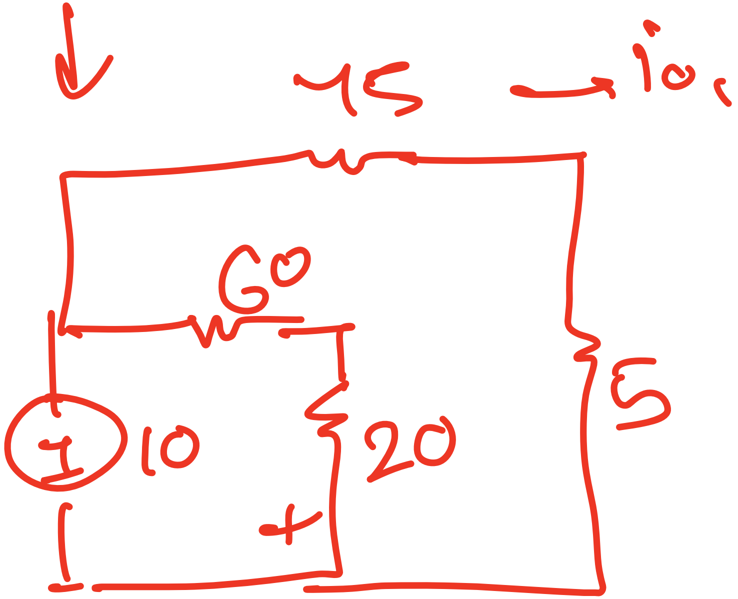


$$\therefore V_{o_1} = -\frac{20}{60+20} \times 10$$

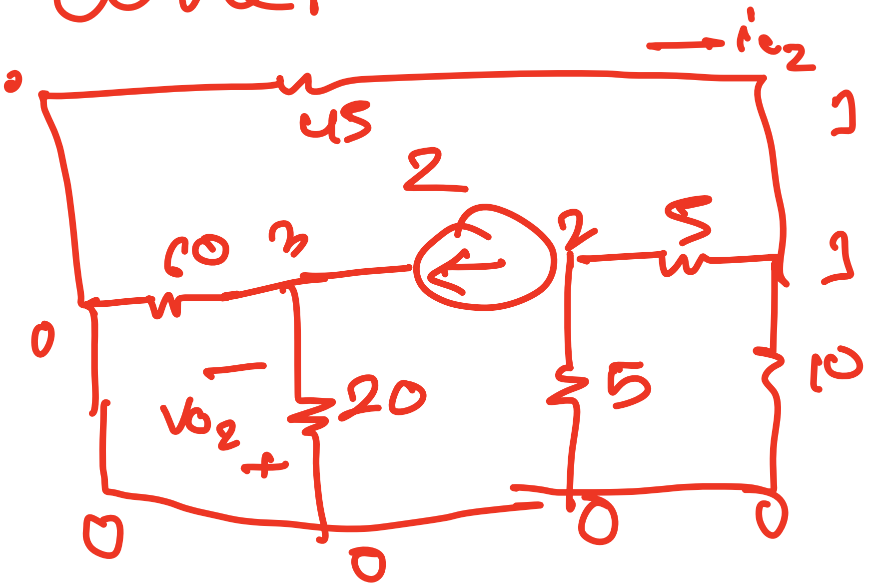
$$= -2.5V$$

$$i_{o_1} = \frac{10}{50}$$

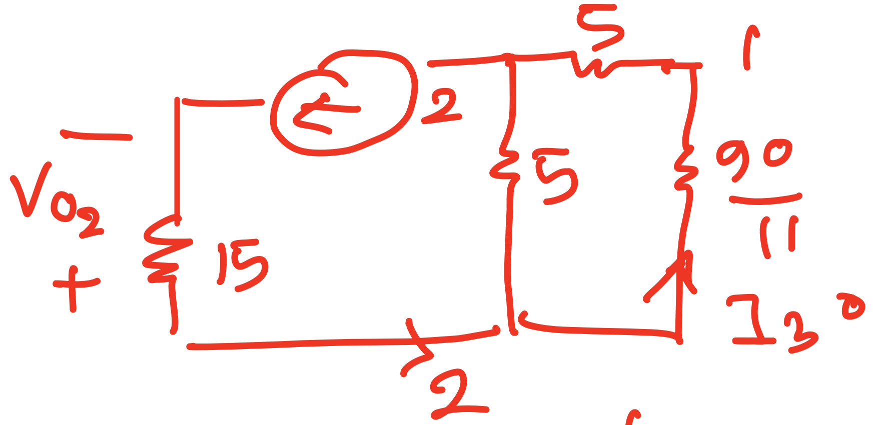
$$= 0.2A$$



When 2A active:



20 || 60 and 45 || 10



$$I_3 = \frac{(5+90/11)^{-1}}{(5+90/11)^{-1}+5^{-1}} \times 2$$

So, $V_{o_2} = -2 \times 15 = -30$

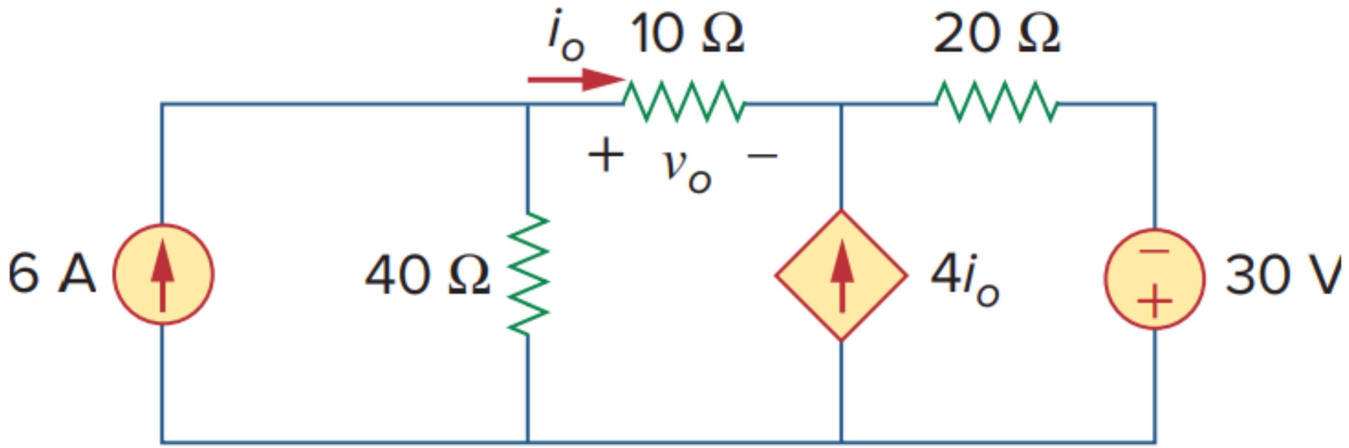
$$i_{o_2} = \frac{I_3 \times 90/11}{45} = 0.1A$$

So, $V_o = V_{o_1} + V_{o_2} = -30 - 2.5$

$i_o = i_{o_1} + i_{o_2} = 0.1 + 0.2$

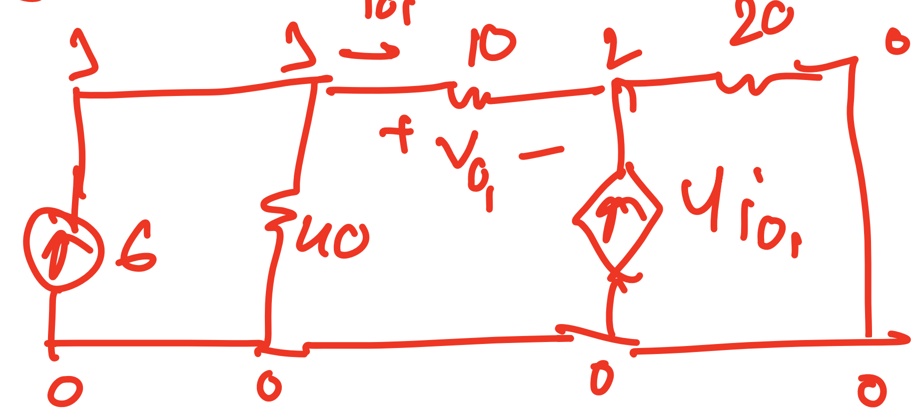
Problem 4

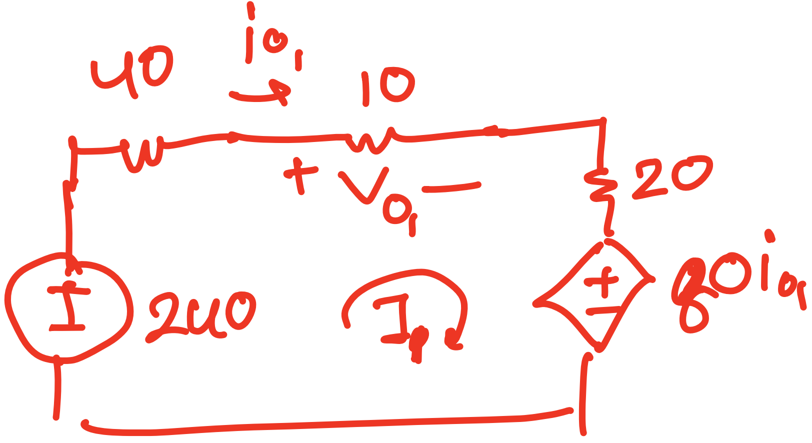
- Use the Superposition Principle to find i_o and v_o.



Ans: $\boldsymbol{i_0 = 1.8\,A;\ v_0 = 18\,V}$

6A active:





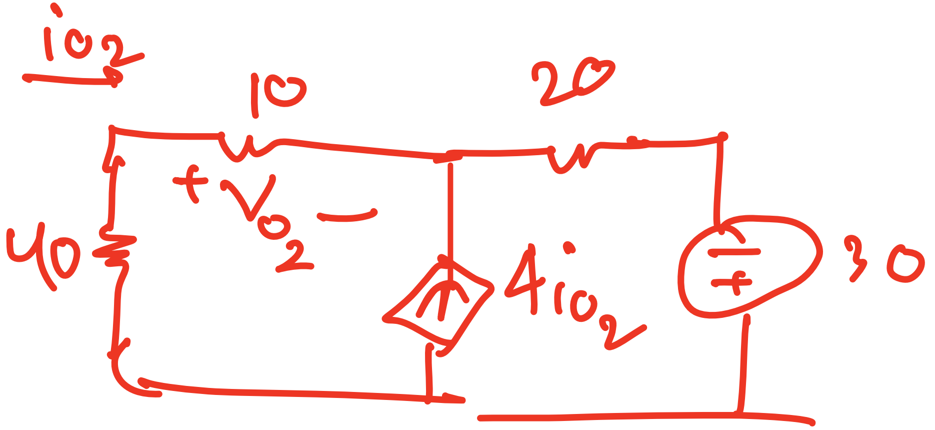
$I_1 = I_{o_1}$

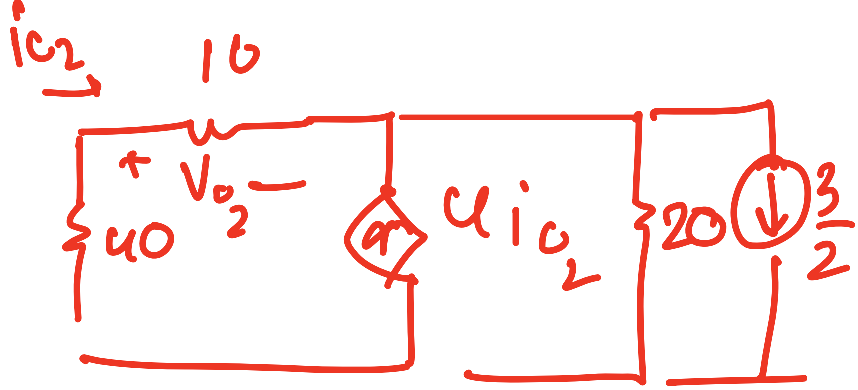
$$-240 + (40+10+20)I_{o_1} + 80I_{o_1} = 0$$

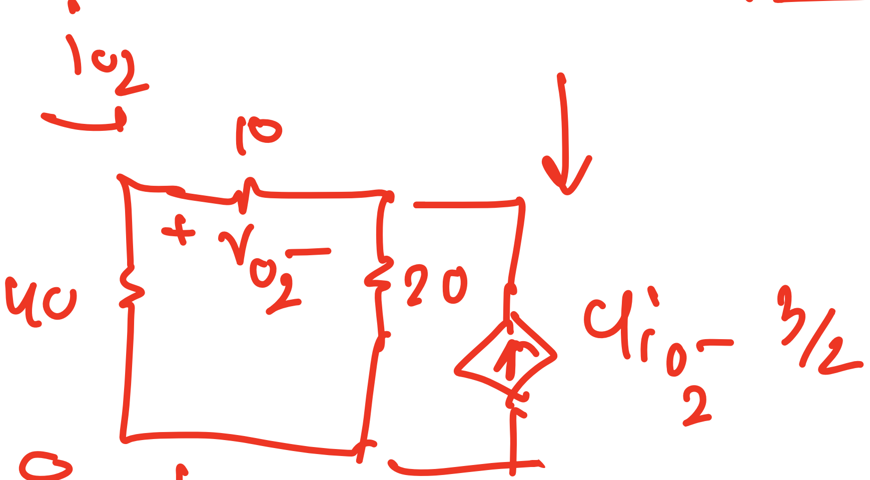
$$\therefore I_{o_1} = 1.6A$$

$$V_{o_1} = 10 \times 1.6 = 16V$$

when 30V active:







$$i_{o_2} = -\frac{50^{-1}}{50^{-1}+20^{-1}} \times \left(4i_{o_2} - 3/2\right)$$

$$\therefore i_{o_2} + \frac{50^{-1}}{50^{-1}+20^{-1}} \times \left(4i_{o_2} - 3/2\right) = 0$$

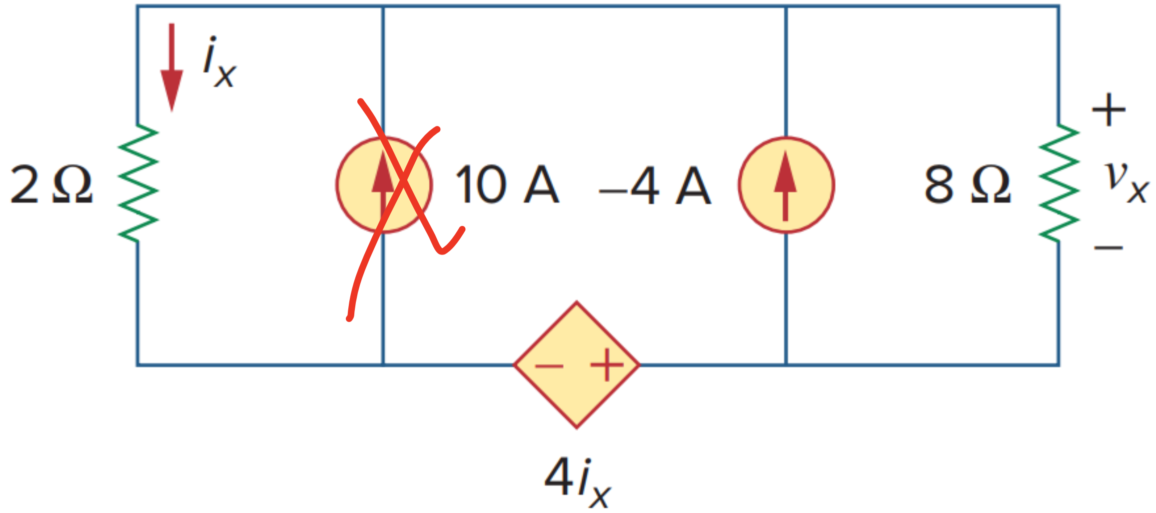
$\therefore i_{o_2} = 0.2A$ So, $V_{o_2} = 2V$

So, $i_o = 1.6 + 0.2 = 1.8$

$V_o = 16 + 2 = 18$

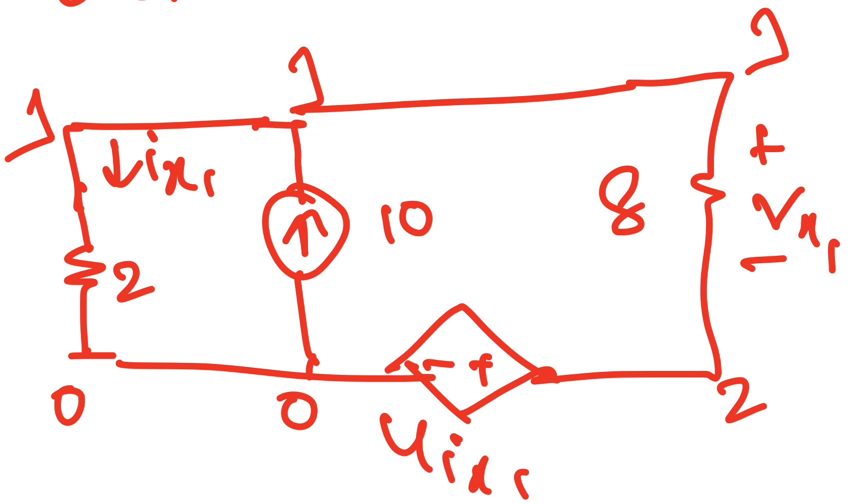
Problem 5

- Use Superposition Principle to solve for v_x.



Ans: $v_x = -16\ V$

10A active:



$\rightarrow$ $i_{x_1} = v_1/2$

$v_2 = 4i_{x_1}$

$v_2 = 4 \cdot \frac{v_1}{2} \Rightarrow 2v_1 - v_2 = 0$ — ©

At node 1:

$$v_1\left(\frac{1}{2}+\frac{1}{8}\right) - \frac{v_2}{8} = 10$$

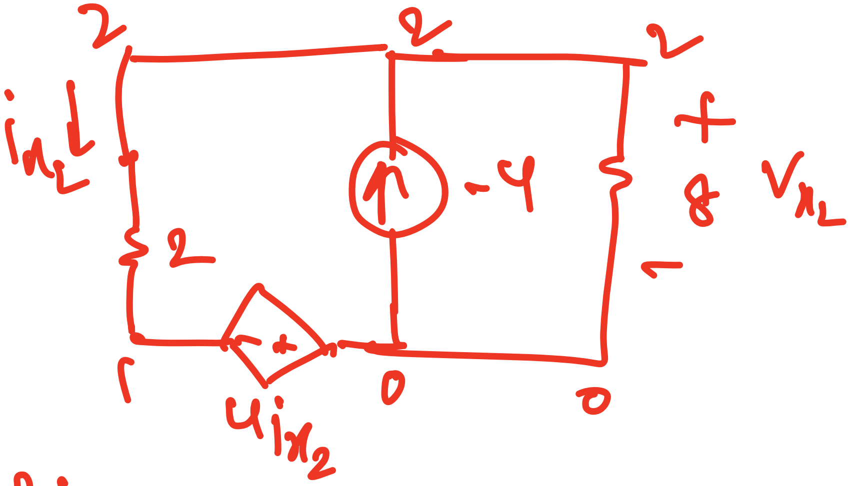
$$\frac{5}{8}v_1 - v_2/8 = 10$$

$v_1 = 80/3$

$v_2 = 160/3$

$v_{x_1} = -80/3$

$-4A$



node 2:

$$V_2\left(\frac{1}{2}+\frac{1}{8}\right)-\frac{V_1}{2}=-4$$

$$V_{x_2}=V_2=\frac{32}{3}$$

So, $V_{x}=\frac{32}{3}-\frac{80}{3}=-16$ ✓

$$i_{x_2}=\frac{V_2-V_1}{2}$$

$$V_0-V_1=4i_{x_2}$$

$$-V_1=4\cdot\frac{V_2-V_1}{2}$$

$$-V_1=2V_2-2V_1$$

$$V_1-2V_2=0 \quad (1)$$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the text book: here

Thank you for your attention

Course Outline: broad themes