Department of Computer Science and Engineering (CSE)
BRAC University

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CSE250 - Circuits and Electronics

Thevenin's Theorem & Norton's Theorem

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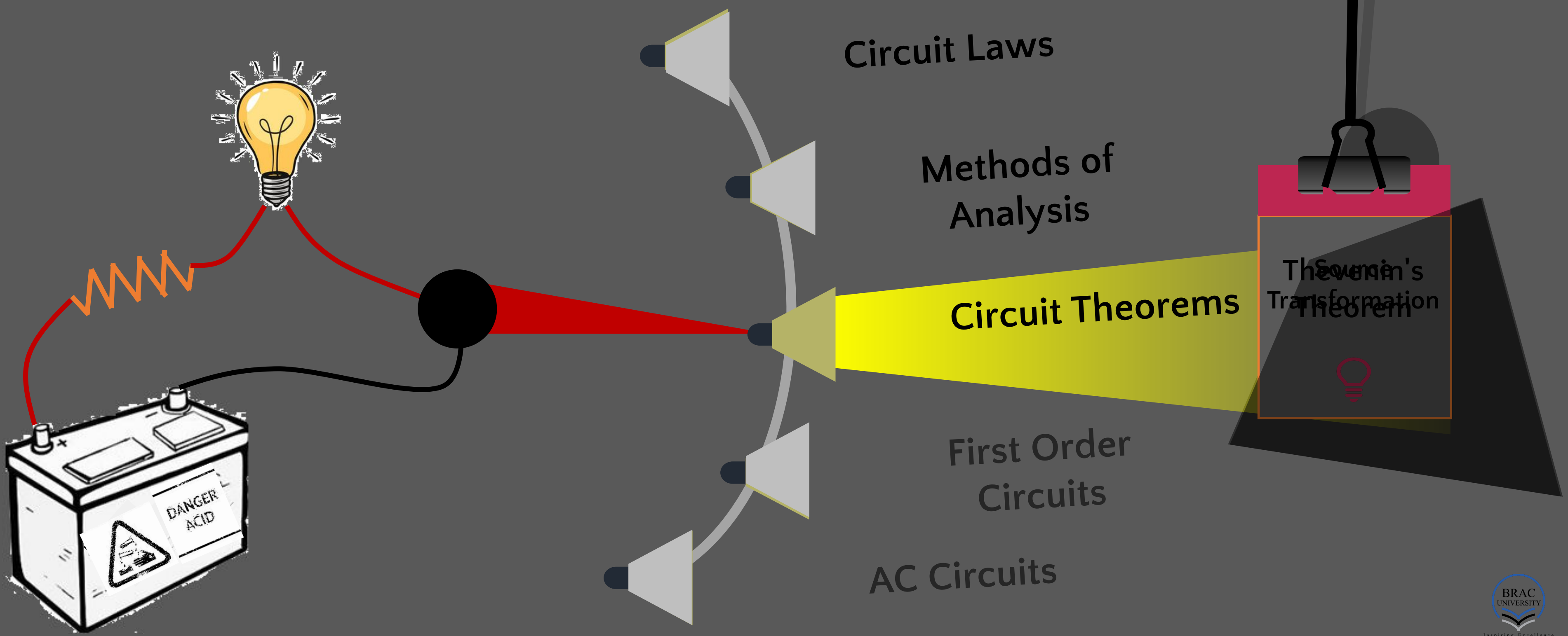
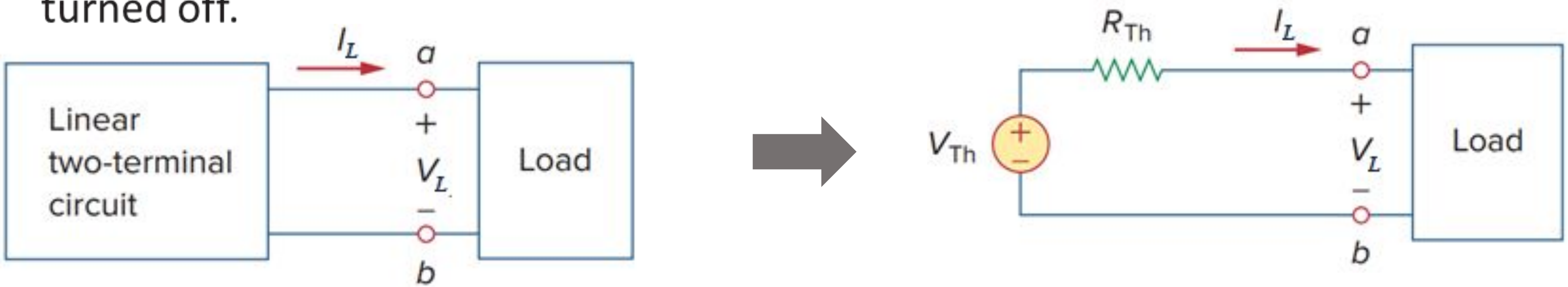
Course Outline: broad themes



Thevenin's Theorem

- *Thevenin's Theorem* states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



Original Circuit

The Thevenin equivalent circuit

- Two circuits are said to be equivalent if they have the same $I - V$ characteristics at their terminals.
- Let's find out what will make the two circuits equivalent!

I-V of the Thevenin Equivalent

- We recall that *an equivalent circuit is one whose $I-V$ characteristics are identical with the original circuit*.

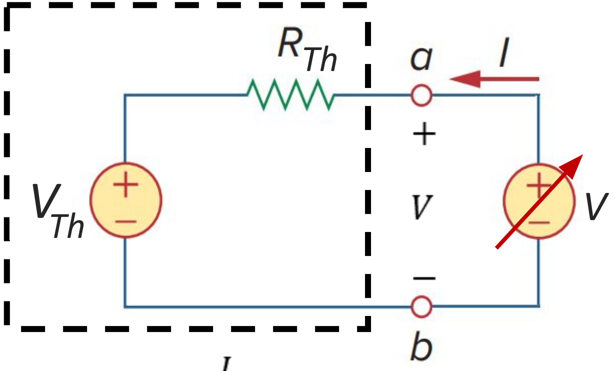
- Let's first find out the *$I-V$ characteristics of the reduced circuit* with respect to terminals $a-b$.

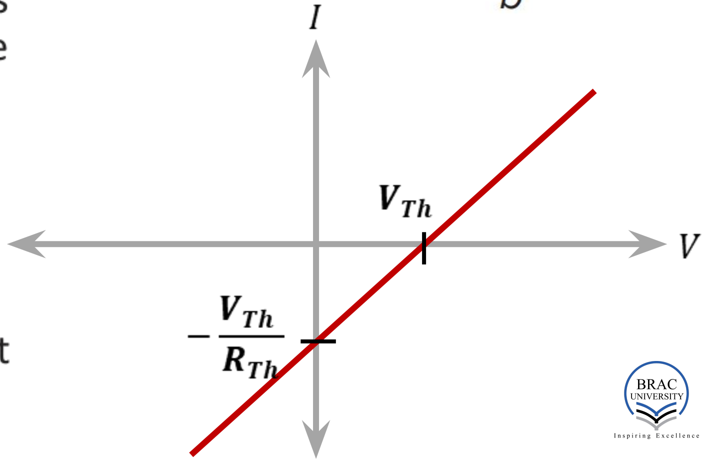
- The configuration is a voltage source (V_{Th}) in series with a resistor (R_{Th}). To determine the configuration's $I-V$ characteristics, if applying a voltage V gives rise to a current I, we can write using KVL,

$$V = V_{Th} + IR_{Th}$$

$$\Rightarrow I = \frac{1}{R_{Th}}V - \frac{V_{Th}}{R_{Th}}$$

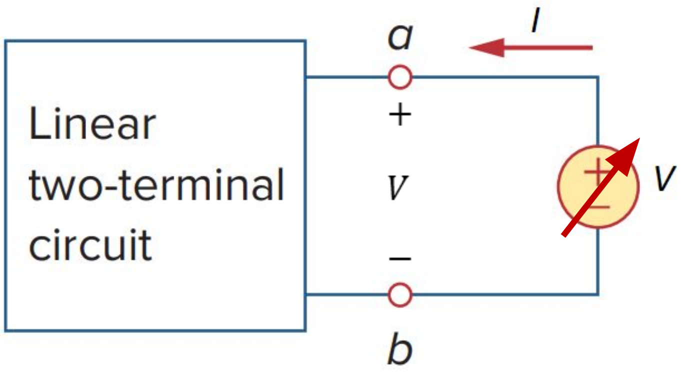
- The equation results in a linear I vs V plot that intersects the axes at V_{Th} and $-\frac{V_{Th}}{R_{Th}}$.





I-V of the Actual Circuit

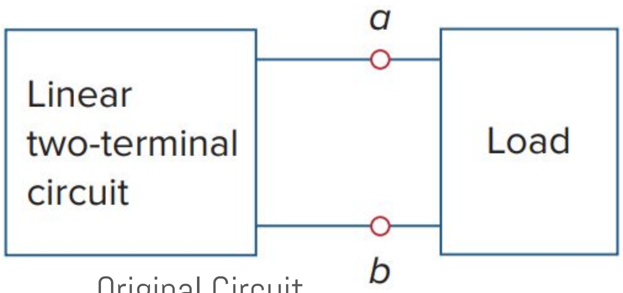
- Let's now find out the $I-V$ *characteristics of the original circuit* with respect to terminals $a-b$.
- The circuit is a combination of linear circuit elements. We cannot theoretically derive exactly the relation between I and V unless we know the actual circuitry. However, as the circuit is linear, the $I-V$ characteristic will be a straight line and the line can be drawn if minimum two points on the line are known.



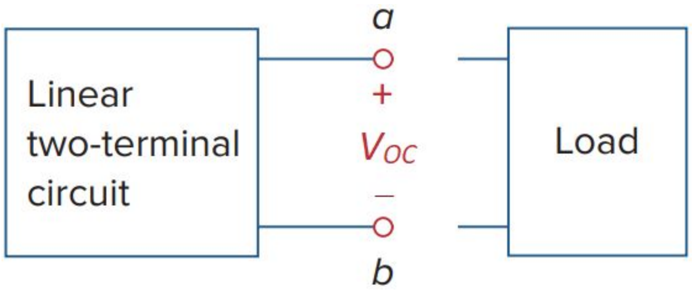
- The two points we can get are the intersecting points of x and y axis.
- To get the intersecting location on the voltage axis, current (I) at the terminals should be made equal to 0. That is, the terminals $a-b$ must be open circuited.
- Similarly, for the intersecting location on current axis, $V_{ab} = V = 0$. That is, the terminals $a-b$ must be shorted.

Open Circuit Voltage & Short Circuit Current

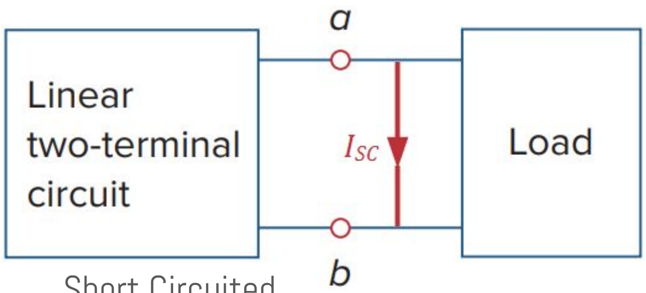
- Let's denote V_{oc} be the voltage at the open terminals upon disconnecting the load and I_{sc} be the current through the shorted terminals upon short circuiting the load.



Original Circuit



Open Circuited at the terminals

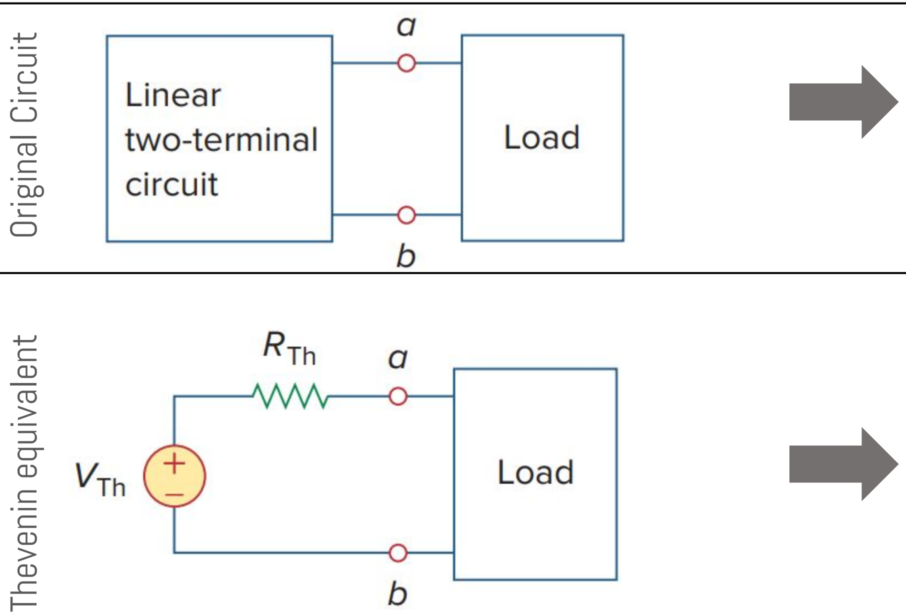


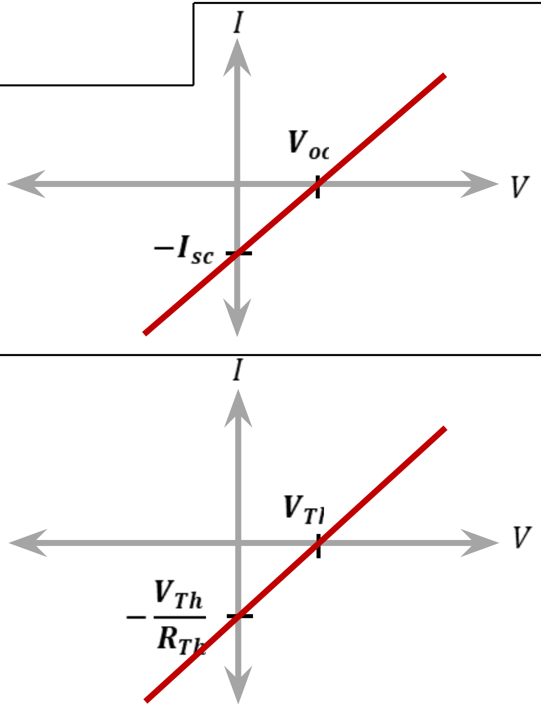
Short Circuited

So, the $I - V$ characteristic should be the straight line passing through the points $(V_{oc}, 0)$ and $(0, -I_{sc})$. The reason for the negative sign is that I_{sc} is opposite to the current (I) plotted along the y-axis.

Condition for Equivalency

- The original circuit and the reduced Thevenin equivalent circuit will be equivalent to each other if the $I - V$ characteristics of the two are identical. They will indeed be identical if the intersecting points $V_{oc} = V_{Th}$ and $-I_{sc} = -\frac{V_{Th}}{R_{Th}}$.





How to determine R_{Th}?

- We have seen in the previous slides that, Thevenin's conversion is valid if

i. $V_{oc} = V_{Th}$ *and*

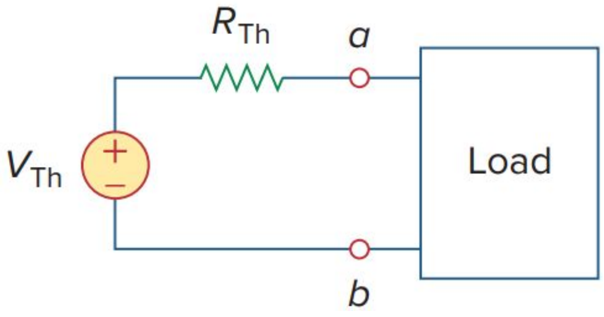
ii. $-I_{sc} = -\frac{V_{Th}}{R_{Th}}$ or $I_{sc} = \frac{V_{Th}}{R_{Th}}$

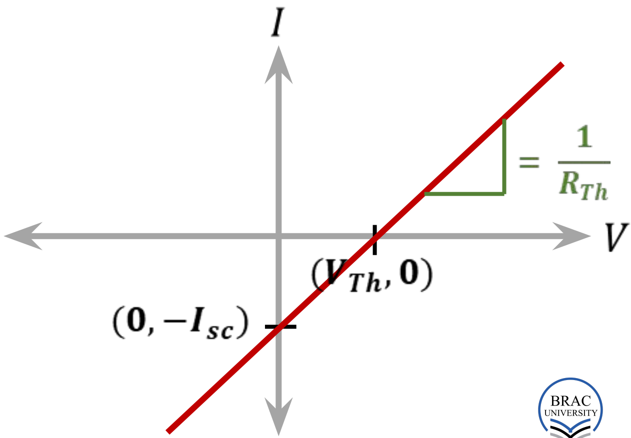
- For the linear $I - V$ characteristic, R_{Th} is the inverse of the slope of the straight line passing through the points $(V_{Th}, 0)$ and $(0, -I_{sc})$. That is,

$$Slope = \frac{\Delta I}{\Delta V} = \frac{1}{R_{Th}} = \frac{0-(-I_{sc})}{V_{Th}-0}$$

$$\Rightarrow \quad R_{Th} = \frac{V_{Th}}{I_{sc}}$$

- Thus, R_{Th} may be found from this ohmic relation between V_{Th} and I_{sc}.





Special Case: undefined R_{Th}

■ **Special Case ($V_{Th} = 0$):** If V_{Th} is zero, $I_{sc} = \frac{V_{Th}}{R_{Th}}$ is likewise zero, and the circuit becomes resistive with respect to the terminals where Thevenin conversion is taking place. In this situation, the $I - V$ characteristic line, as shown, passes through the origin.

- This can happen in two scenarios:

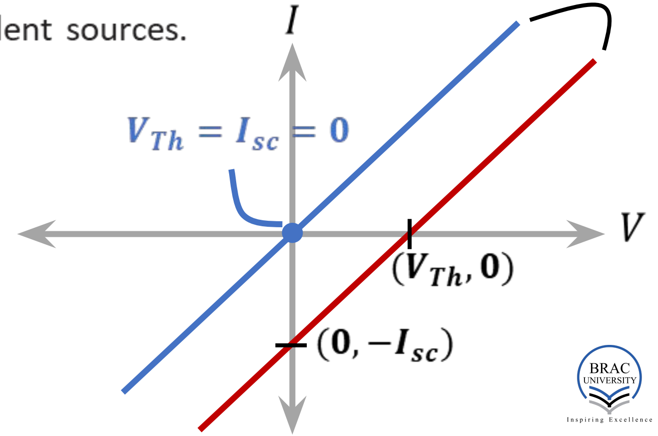
[See Example] i. if the network is erroneous in such a way that the load connected to the circuit gets no voltage and

[See Example] ii. if the portion of the network excluding load has no independent sources.

- This results in an undefined and indeterminant situation if we proceed to determine R_{Th}.

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{0}{0}$$

- Let's think of a different approach to tackle this situation to determine R_{Th}.

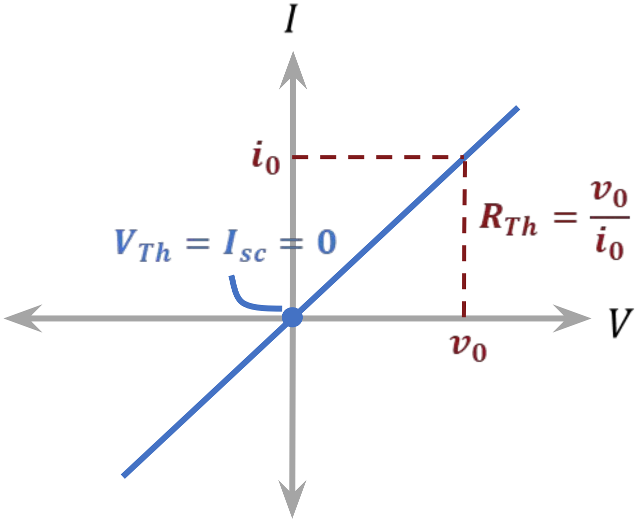


Universal Rule to determine R_{Th}

- As $V_{Th} = 0$ results in a resistive $I - V$ passing through the origin $(V_{Th},\ 0) = (0,\ -I_{sc}) = (0, 0)$, we may still find the value of R_{Th} by measuring the slope of the line with any other arbitrary point $(v_0,\ i_0)$ on the line.

- Interestingly, we can use this technique to determine R_{Th} whether or not V_{Th} is zero. This is what the term "*Universal Rule*" refers to.

- So, in general (whether or not V_{Th} is zero), the strategy is to forcefully make the $I - V$ characteristic line to go through the origin. Then calculating R_{Th} as $R_{Th} = \frac{v_0}{i_0}$.

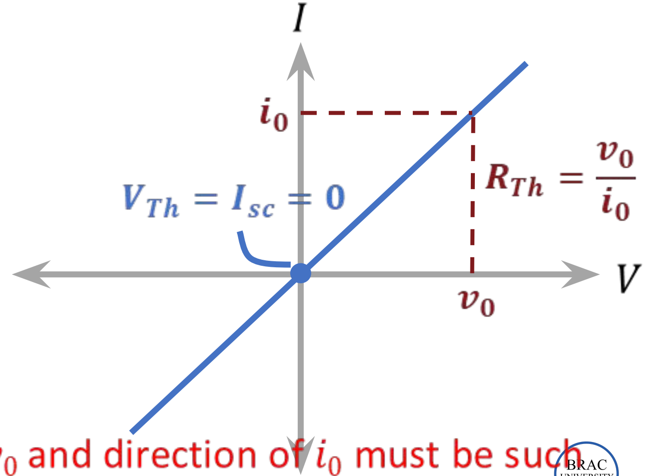


- This can be accomplished simply by turning off all the independent sources (or equivalently replacing them with their resistances). As a result, the circuit becomes resistive, and the characteristic line will pass through the origin.

Universal Rule w/wo dependent source

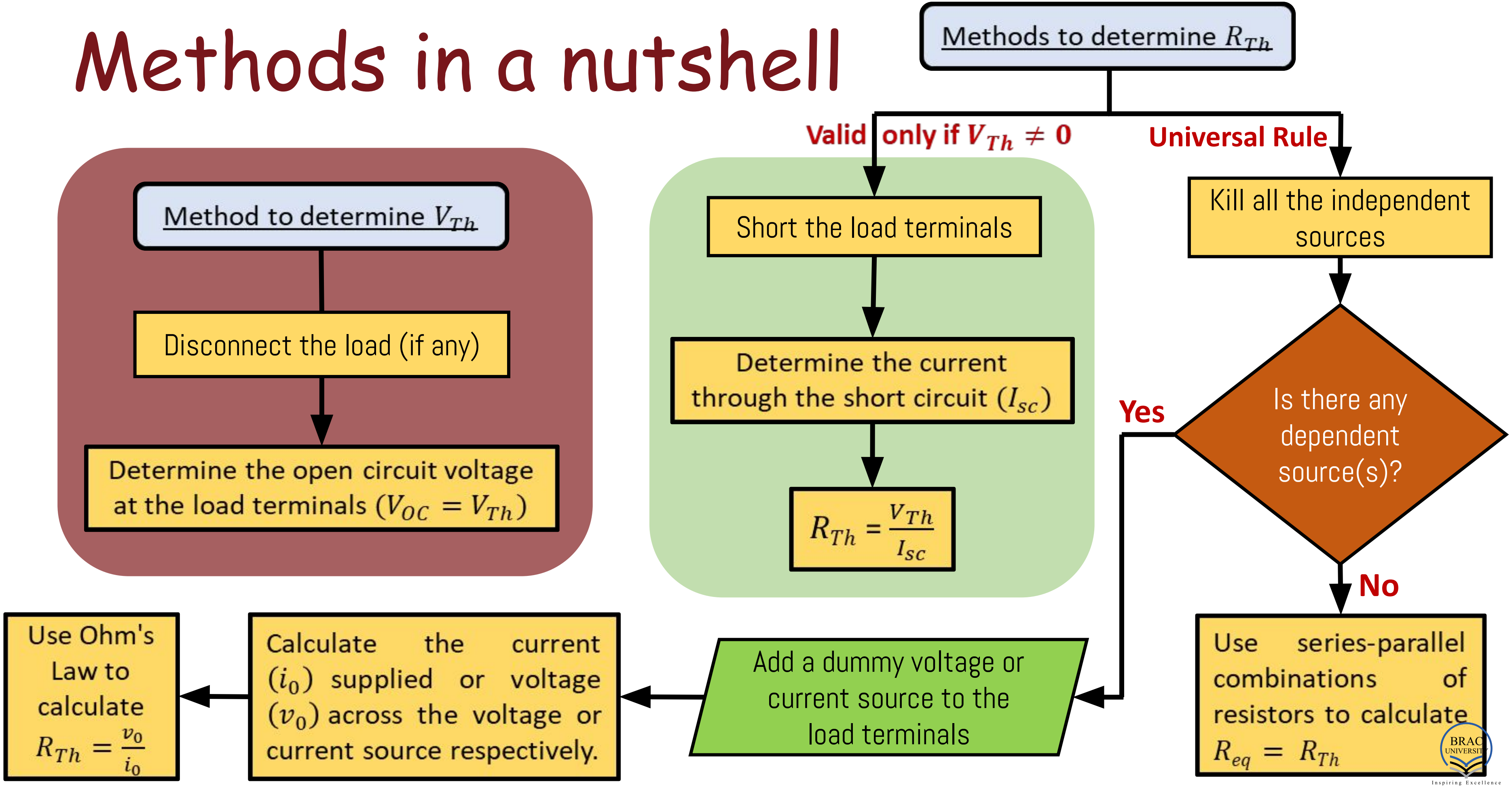
- **No dependent source:** If a network has no dependent sources, then after turning off all the independent sources, the circuit will be a combination of resistors only. This simplifies the procedure as that, to determine R_{Th}, it is not even required to apply a voltage v_0 (or current i_0) and determine the corresponding current i_0 (or voltage v_0). Instead, *use the series and/or parallel combinations of resistors to determine the equivalent resistance at the terminals, which is R_{Th}.*

- **Dependent source:** However, if the network has dependent sources, then to get any point (v_0, i_0) on the line, *apply a voltage source v_0 at load terminals and determine the resulting current i_0. Then $R_{Th} = v_0/i_0$.* Alternatively, *insert a current source i_0 at load terminals and find the terminal voltage v_0. Again $R_{Th} = v_0 / i_0$. We call the applied source as dummy or test source. In either approach, we may assume any value of v_0 or i_0.*



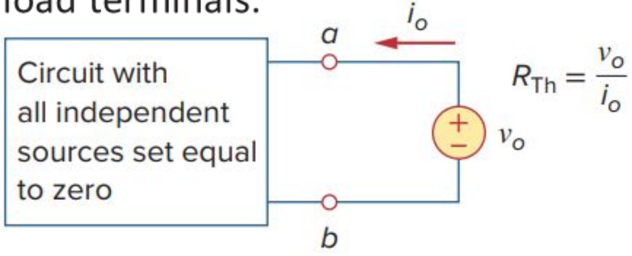
- Note that, for an applied dummy or test source, polarity of v_0 and direction of i_0 must be such that, the current i_0 leaves the $+ve$ terminal of v_0.

Methods in a nutshell



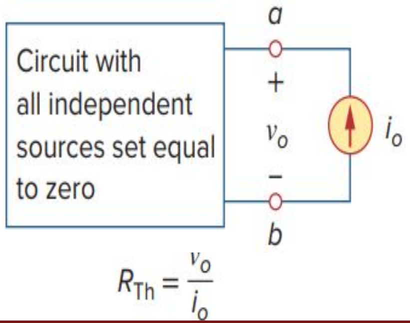
Procedure to find parameters

- *Finding* $V_{oc} = V_{Th}$: Disconnect the load and use nodal/mesh or other circuit solving techniques to find the open circuit voltage at the load terminals.
- *Finding* I_{sc}: Disconnect the load, short the terminals, use nodal/mesh or other circuit solving techniques to find the short circuit current at the load terminals.
- *Finding* R_{Th}



Case 1: If $V_{Th} \neq 0$, Use $R_{Th} = \frac{V_{Th}}{I_{sc}}$

Case 2: If $V_{Th} = 0$, turn off all the independent sources.

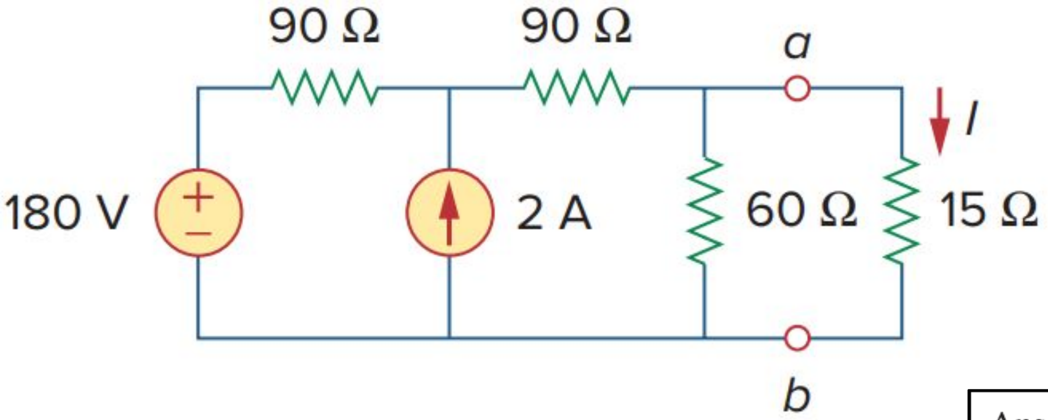


If the network has no dependent sources, R_{Th} is the input resistance of the network looking between the load terminals.

If the network has dependent sources, apply a voltage source v_0 at load terminals determine the resulting current i_0. Then $R_{Th} = v_0 / i_0$. Alternatively, we may insert a current source i_0 at load terminals and find the terminal voltage v_0. Again $R_{Th} = v_0 / i_0$. In either approach we may assume any value of v_0 and i_0.

Example 1

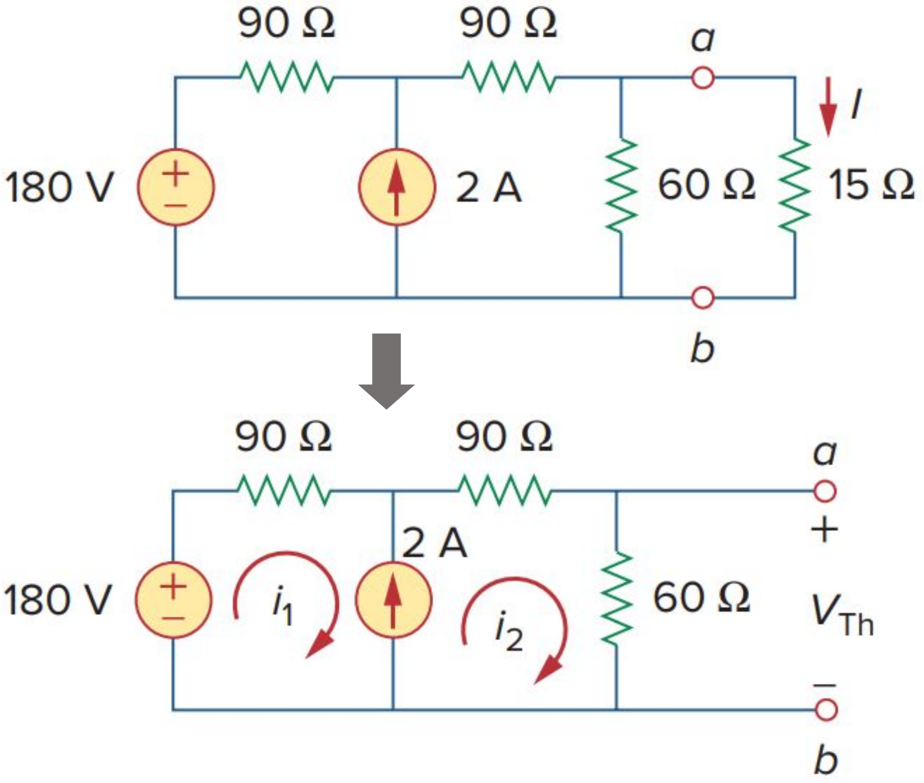
- Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit. Then find I.



Ans: $V_{Th} = 90\ V$; $R_{Th} = 45\ \Omega$; $i_x = 1.5\ A$

* *See solution in the next slide if necessary*

Example 1: finding V_{Th}



Step 1: Disconnecting the load and finding the open circuit voltage.

Let's use mesh analysis to find the V_{Th}.

KVL at mesh 1 and mesh 2 (forming supermesh),

$$-180 + 90i_1 + 90i_2 + 60i_2 = 0$$

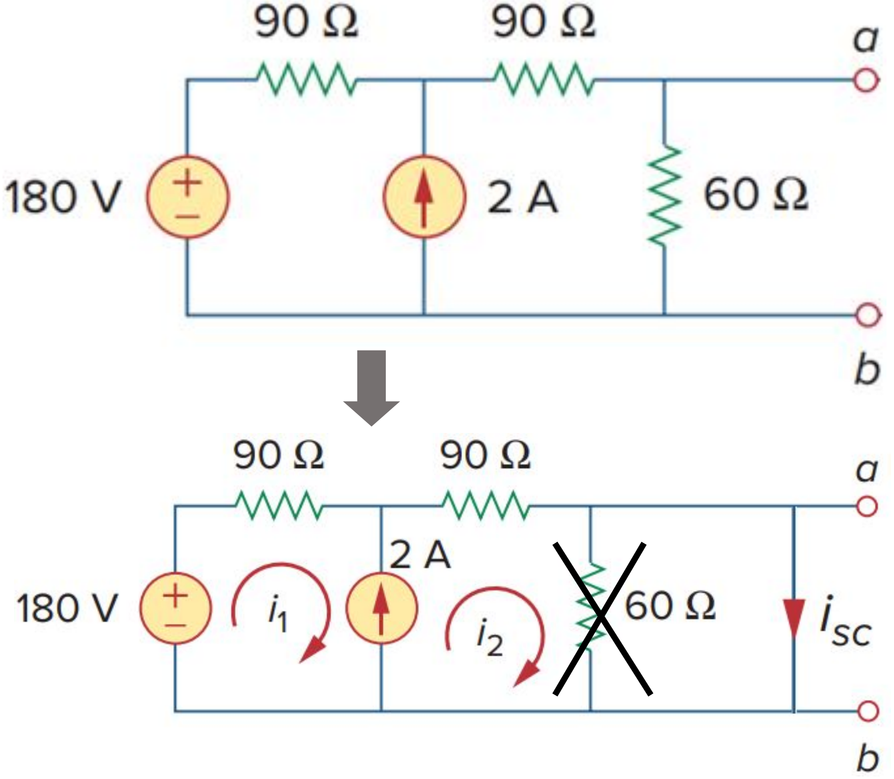
$$\Rightarrow \; 90i_1 + 150i_2 = 180 ------(i)$$

KCL at the supermesh, $i_1 - i_2 = -2 \; ----(ii)$

Solving … …, $i_1 = -0.5\,A;\qquad i_2 = 1.5\,A$

So, $V_{Th} = 60i_2 = 60 \times 1.5 = 90\,V$

Example 1: finding R_{Th}



<u>Step 2</u>: As $V_{Th} \neq 0$, with the load disconnected, we find the short circuit current I_{sc}. The terminals a-b are shorted.

Let's use mesh analysis to find the $I_{sc.}$ Note that the 60 Ω resistance is shorted out.

KVL at mesh 1 and mesh 2 (forming supermesh),

$$-180 + 90i_1 + 90i_2 = 0$$

$$\Rightarrow \; i_1 + i_2 = 2 ----- (i)$$

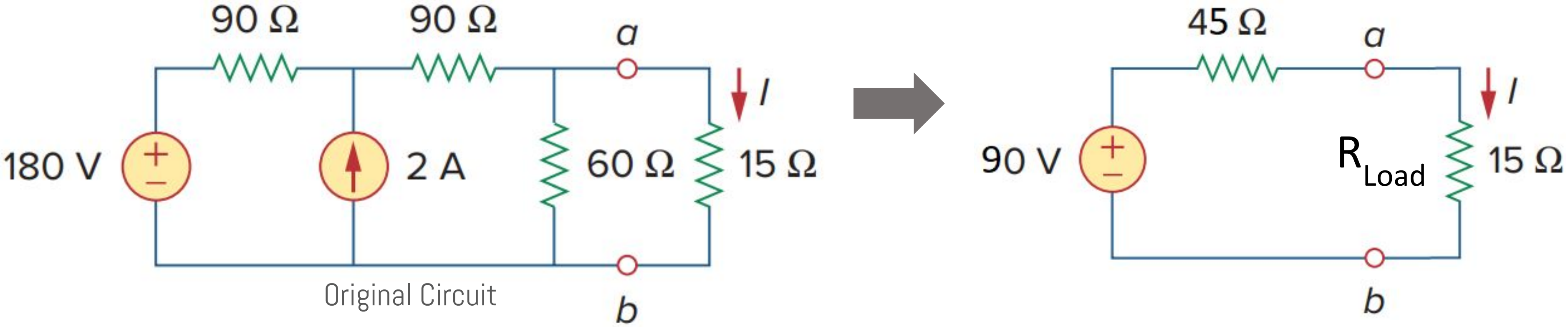
KCL at the supermesh,

$$i_1 - i_2 = -2 \; ---- (ii)$$

Solving … …, $i_1 = 0\ A;\quad i_2 = 2\ A$

So, $I_{sc} = i_2 = 2\ A$

Example 1: Thevenin equivalent



Original Circuit

Step 3: With V_{Th} and I_{sc} known, we can find R_{Th} as follows,

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{90}{2} = 45\ \Omega$$

So, the Thevenin equivalent circuit looks like the one shown above.

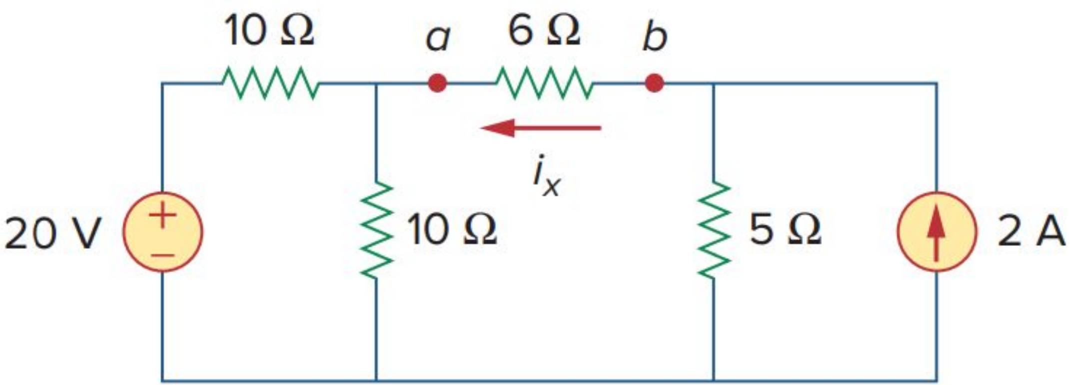
The load current I can be found as follows,

$$I = \frac{90}{45 + 15} = 1.5\ A$$

[Calculate I from the original circuit and verify the Thevenin's theorem]

Example 2

- Find the Thevenin equivalent looking into terminals $a - b$ of the circuit and solve for i_x.



Ans: $V_{Th} = 0$ V; $R_{Th} = 10\ \Omega$; $i_x = 0$ A

* *See solution in the next slide if necessary*

Example 2: finding V_{Th}

Step 1: Disconnecting the load and finding the open circuit voltage.

No current flows through the open circuit. So, the voltage across the 10 Ω resistance can be found by voltage division, that is,

$$V_{10\Omega} = \frac{10}{10+10} \times 20 = 10\ V.$$

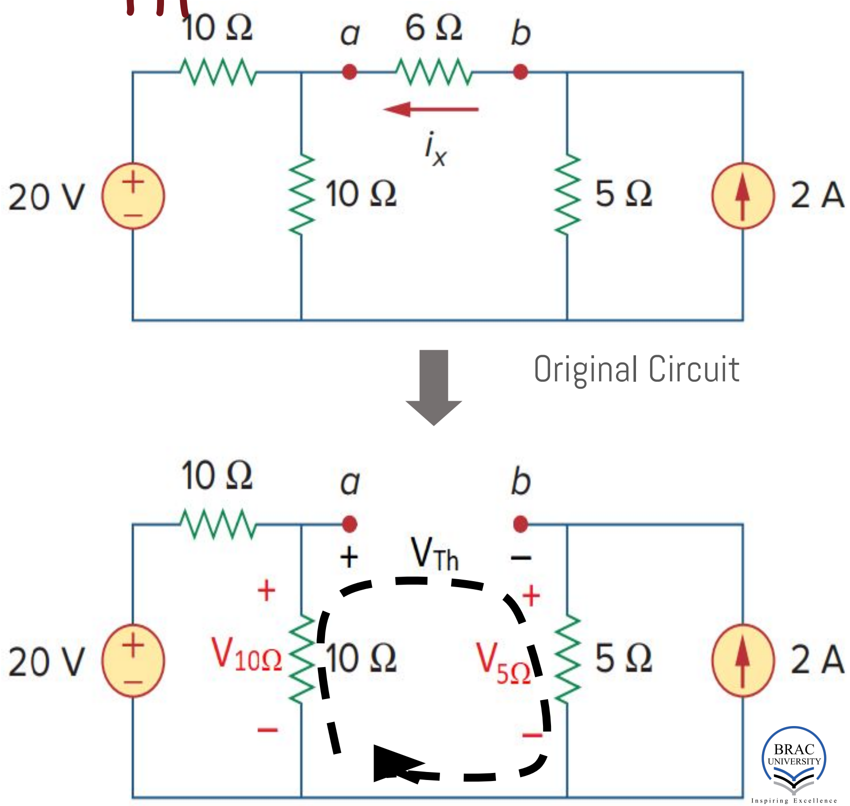
The current 2 A flows through the 5 Ω resistor.

$$V_{5\Omega} = 5 \times 2 = 10\ V$$

The voltages are indicated in the figure. V_{Th} can be found by applying KVL along the black dashed line shown. That is,

$$-10 + V_{Th} + 10 = 0$$

$$\Rightarrow \quad V_{Th} = 0\ V$$



Example 2: finding R_{Th}

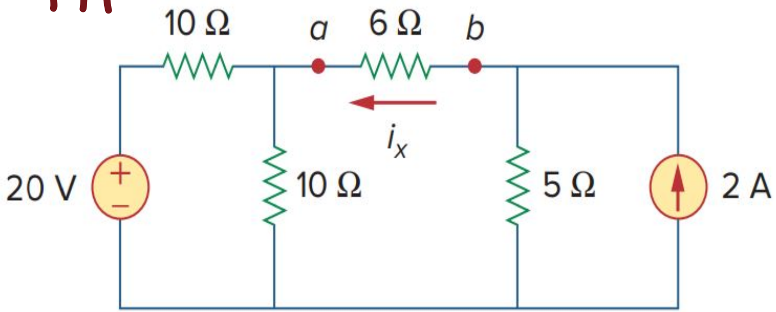
Step 2: As $V_{Th} = 0$, I_{sc} will also be zero, hence, $R_{Th} = \frac{V_{Th}}{I_{sc}}$ will result in a $\frac{0}{0}$ situation. In this case we find R_{Th} by killing all the independent sources [*Replace voltage sources by short circuits and current sources by open circuits*].

Now check if there are dependent sources in the reduced circuit. As there is none, we simply use series-parallel combination to find the equivalent resistance seen from the load terminal. That is,

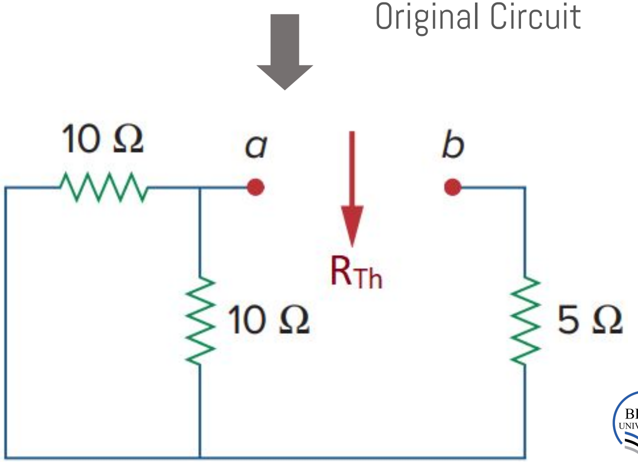
$$R_{Th} = (10 \,||\, 10) + 5$$

$$\Rightarrow \quad R_{Th} = 10\ \Omega$$

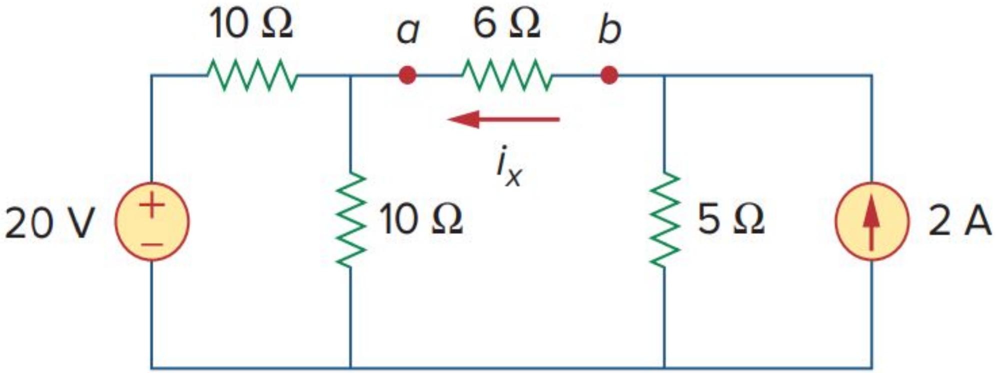
[Keep in mind that, this method of determining R_{Th} by killing independent sources always works regardless of whether V_{Th} is equal to zero or not.]



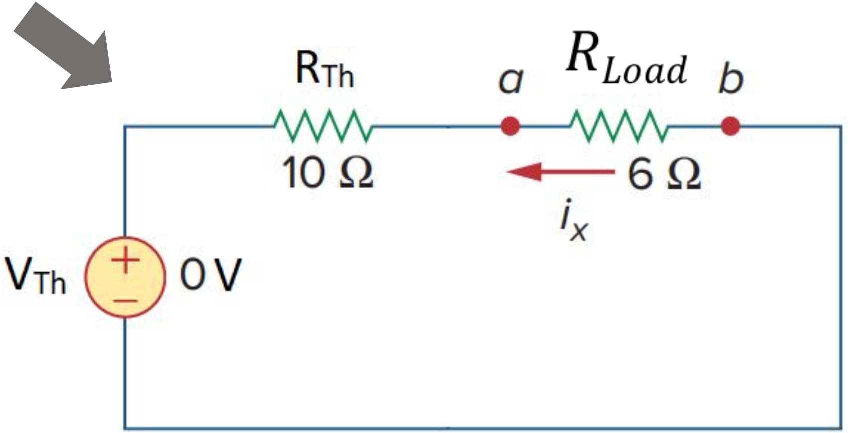
Original Circuit



Example 2: Thevenin equivalent



Original Circuit

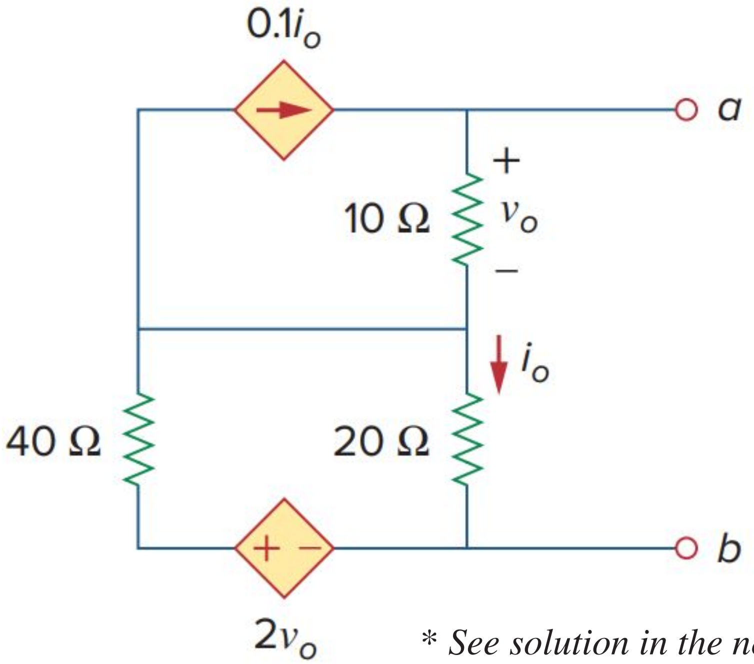


The Thevenin equivalent circuit is shown in the figure above. The $0\,V$ Thevenin voltage can be represented by a voltage source with $0\,V$ value or by a short circuit (not shown).

Thus, the load current i_x is equal to zero.

Example 3

- Obtain the Thevenin equivalent circuit at terminals $a - b$.



Ans: $V_{Th} = 0$ V; $R_{Th} = 31.73\ \Omega$

* *See solution in the next slide if necessary*

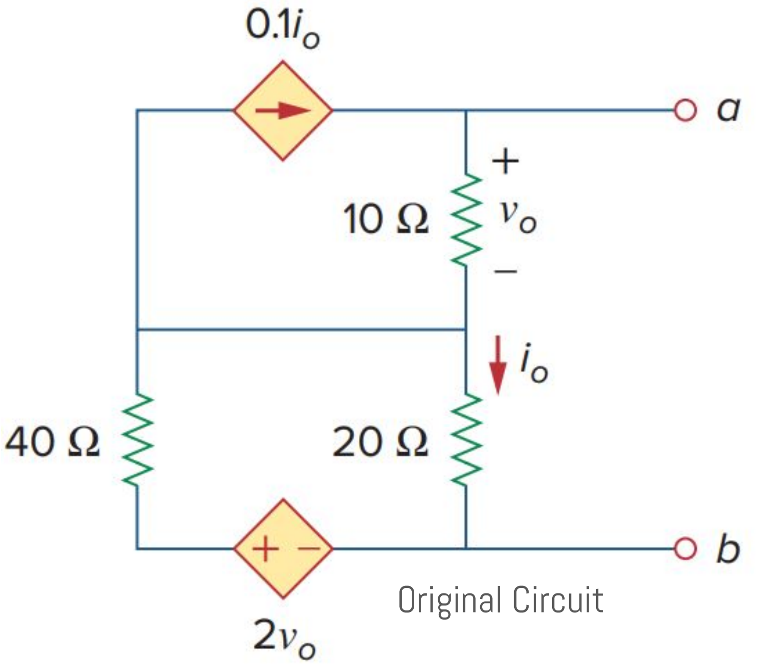
Example 2: finding V_{Th}

<u>Step 1</u>: Disconnecting the load and finding the open circuit voltage.

There are two dependent sources but no independent sources in this circuit. This means that all currents and voltages, including those on which dependent sources rely, will be zero. That is, $i_0 = 0, v_0 = 0$. As a result, there will be no contributions from the dependent sources. So, we can write,

$$V_{Th} = V_{ab} = 0\,V.$$

[Circuit analysis can be used to confirm that $V_{Th} = 0$*]*



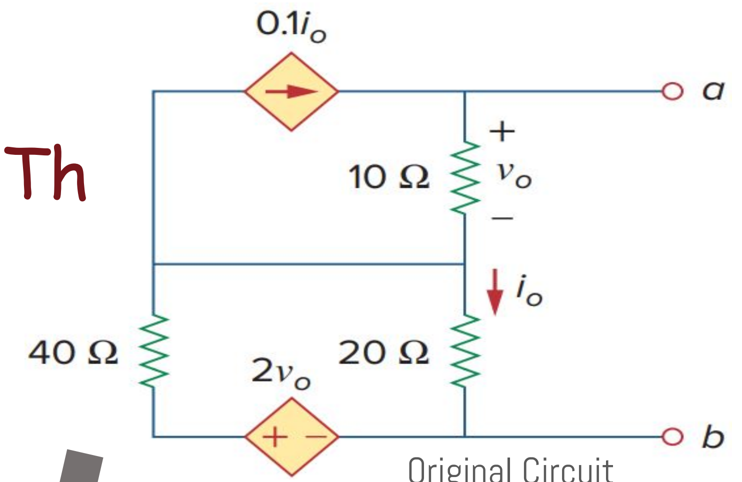
Original Circuit

Example 3: finding R_{Th}

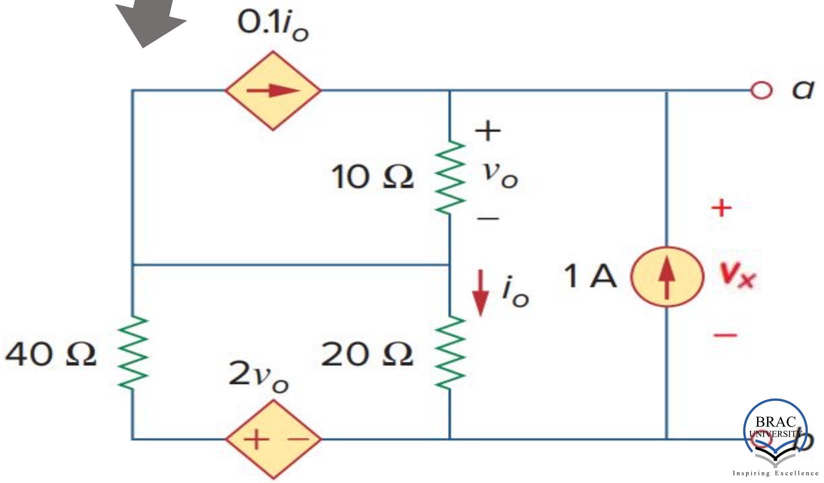
Step 2: As $V_{Th} = 0$, I_{sc} will also be zero, hence, $R_{Th} = \frac{V_{Th}}{I_{sc}}$ will result in a $\frac{0}{0}$ situation. In this case we find R_{Th} by killing all the independent sources.

There are no independent sources in this circuit.

There are two dependent sources. So, we must add a dummy voltage/current source between terminals *a-b*. Let's add a current source of 1 A between the terminals *a-b*. We have to find the voltage v_x across the current source as shown in the circuit diagram.



Original Circuit



Example 3: finding R_{Th} (contd … 2)

Let's apply mesh analysis to solve for v_0.

It can be seen from loop 3 that,

$$i_3 = -1\ A$$

Also from loop 1,

$$i_1 = 0.1 i_0 = 0.1(i_2 - i_3) \quad [i_0 = i_2 - i_3]$$

$$\Rightarrow \quad i_1 - 0.1 i_2 + 0.1 i_3 = 0$$

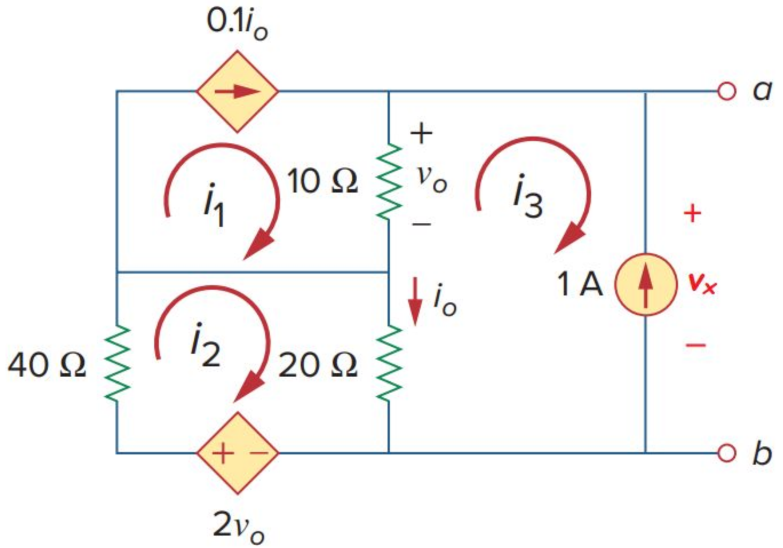
$$\Rightarrow \quad i_1 - 0.1 i_2 = 0.1 \text{ ---}(i) \quad [i_3 = -1\ A]$$

KVL at loop 2,

$$40 i_2 + 20(i_2 - i_3) - 2 v_0 = 0$$

$$\Rightarrow \quad 40 i_2 + 20(i_2 - i_3) - 2 \times 10(i_1 - i_3) = 0$$

$$\Rightarrow \quad 20 i_1 - 60 i_2 = 0 \text{ -----}(ii)$$



Solving (i) and (ii) yields,

$$i_1 = 0.103\ A,$$

$$i_2 = 0.034\ A.$$

Let's find v_0 now!

Example 3: finding R_{Th} (contd … 3)

Now,

$$v_0 = 10 \times (i_1 - i_3)$$

$$\Rightarrow \quad v_0 = 10 \times \{0.103 - (-1)\} = 11.03\ V$$

$$i_0 = i_2 - i_3$$

$$\Rightarrow \quad i_0 = 0.034 - (-1) = 1.034\ A$$

The voltage across the 20 Ω is $= 20i_0 = 20.68\ V$.

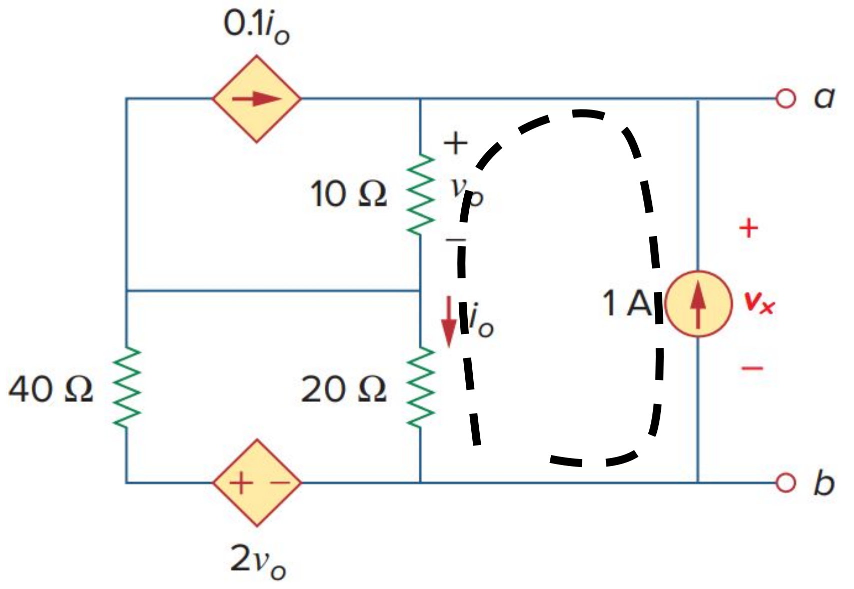
Applying KVL along the black dotted line,

$$-v_x + v_0 + 20.68 = 0$$

$$\Rightarrow \quad v_x = 31.71\ V \quad [v_0 = 11.03\ V]$$

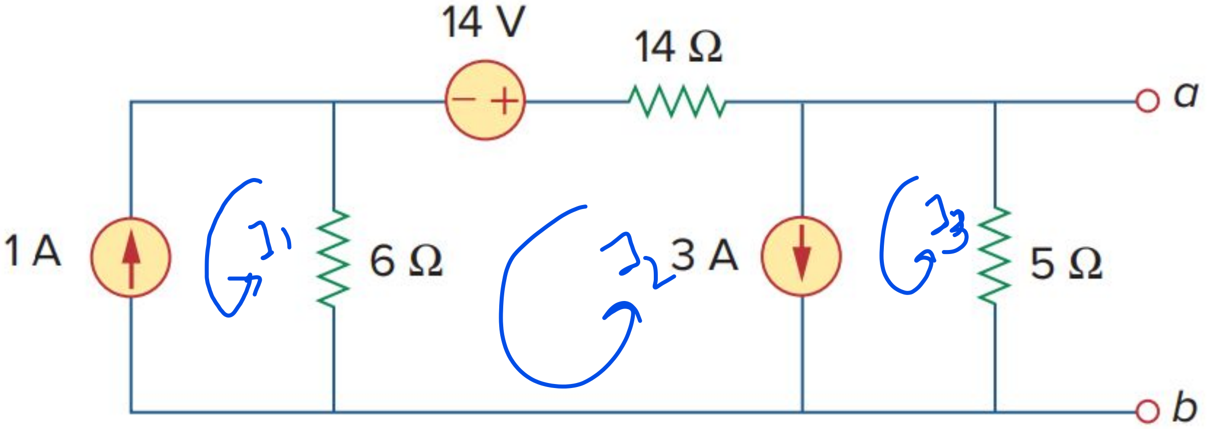
So,

$$R_{Th} = \frac{v_x}{1} = 31.71\ \Omega$$



Problem 1

- Find the Thevenin equivalent at terminals $a - b$.



Ans: $V_{Th} = -8$ V; $R_{Th} = 4\ \Omega$

Step 1: finding V_{th}

$I_1 = -1A$

$I_3 - I_2 = 3A$

Loop 3, 2:

$20I_2 + 5I_3 - 6I_1 + 14 = 0$

$-6I_1 + 20I_2 + 5I_3 = -14$

$-6\times(-1) + 20I_2 + 5I_3 = -14$

$20I_2 + 5I_3 = -20$

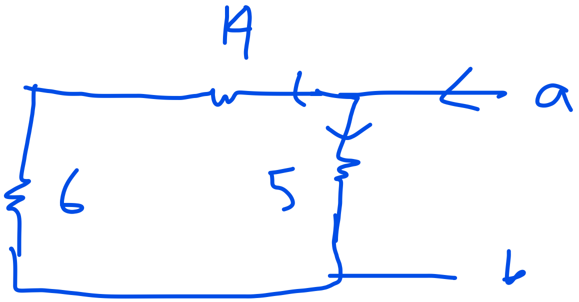
$I_2 = -7/5$

$I_3 = -8/5$

So, $V_{th} = -8/5 \times 5$

$= -8V$

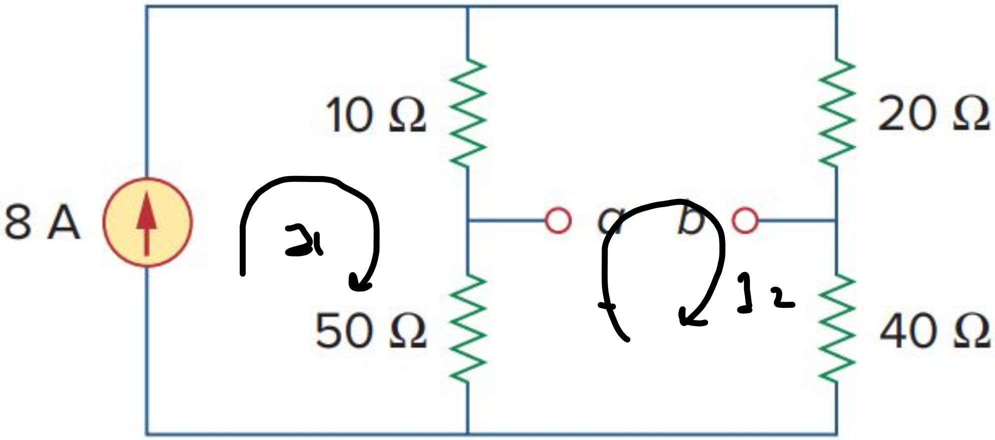
Step 2: for R_{th}:



$$R_{th} = (20 \parallel 5) = 4\,\Omega$$

Problem 2

- Find the Thevenin equivalent at terminals $a - b$.



Ans: $V_{Th} = 40\text{ V}$; $R_{Th} = 22.5\ \Omega$

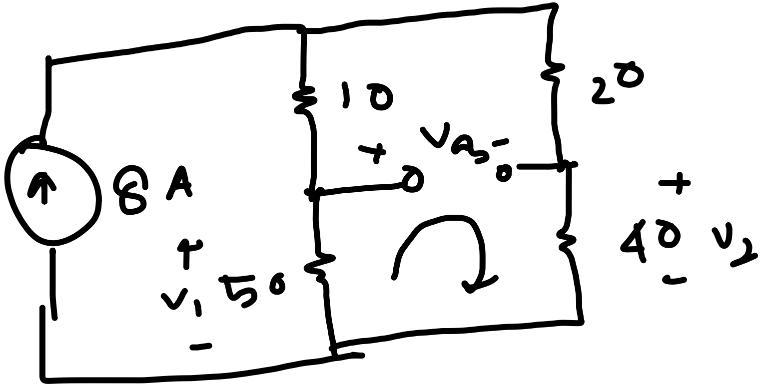
Step 1: finding V_{th}:

$I_1 = 8A$

$(10+50+20+40)I_2 - 60I_1 = 0$

$I_2 = 4\ A$

So,



$-V_1 + V_{ab} + V_2 = 0$

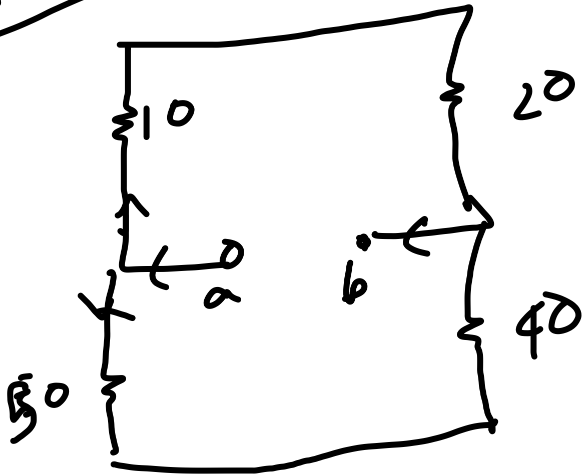
$-50(I_2 - I_1) + V_{ab}$
$+ 40I_2 = 0$

$V_{ab} = 50(8-4) - 40\times4$

$= 40V$

Step 2:

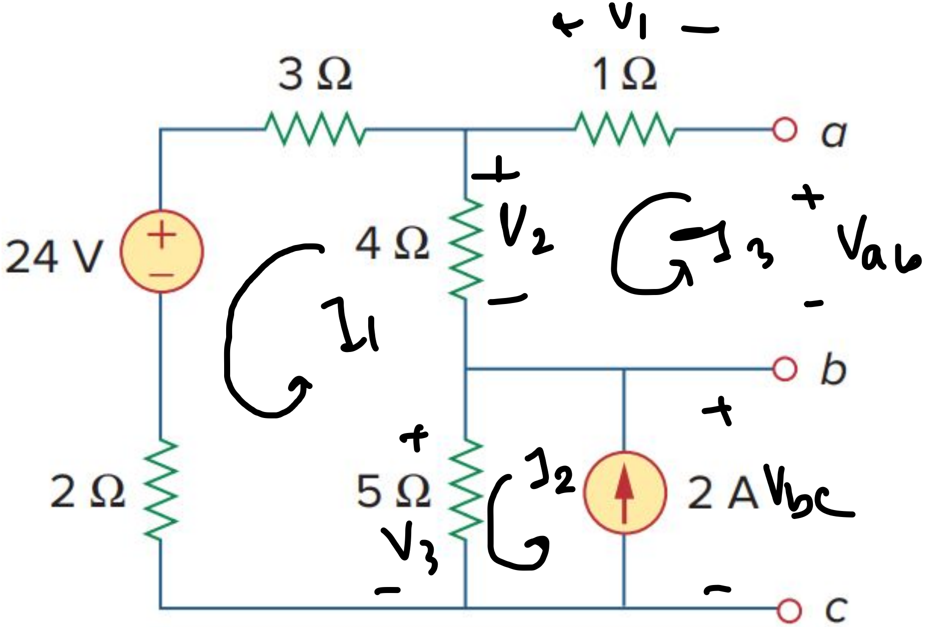
Rth



$R_{th} = (30 \| 90) = 22.5\,\Omega$

Problem 3

- Find the Thevenin equivalent as seen from terminals (i) $a - b$ and (ii) $b - c$.



Ans: (i) $V_{Th} = 4\ V;\ R_{Th} = 3.857\ \Omega;$
(ii) $V_{Th} = 15\ V;\ R_{Th} = 3.214\ \Omega;$

Step 1: V_{th}

$I_2 = 2A$

$(4+5+3+2) I_1 - 5I_2 + 24 = 0$

$14I_1 = -24 + 5I_2$

$I_1 = -1A$

So, $-V_1 + V_2 = V_{ab}$

$V_{ab} = 0 + 4(I_3 - I_1)$

$V_{ab} = 4 \times 1$

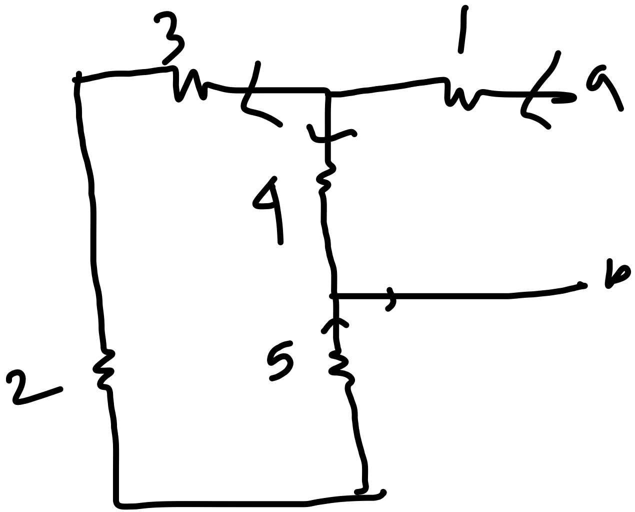
$= 4V$

$V_{bc} = V_3$

$= 5(I_2 - I_1)$

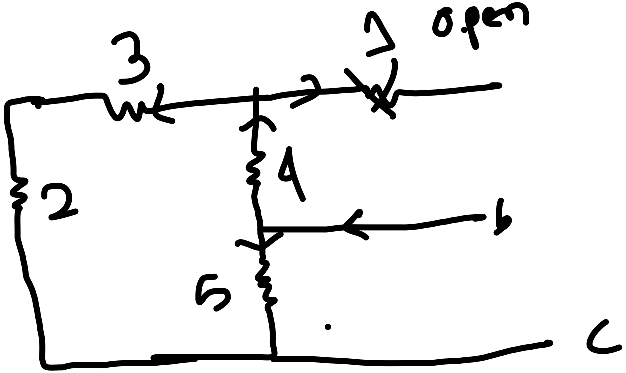
$= 5 \times 3$

$= 15V$



$R_1 = 4 \parallel (2+3+5) = \frac{20}{7}$

$R_{th} = 1 + R_1 = 3.857$

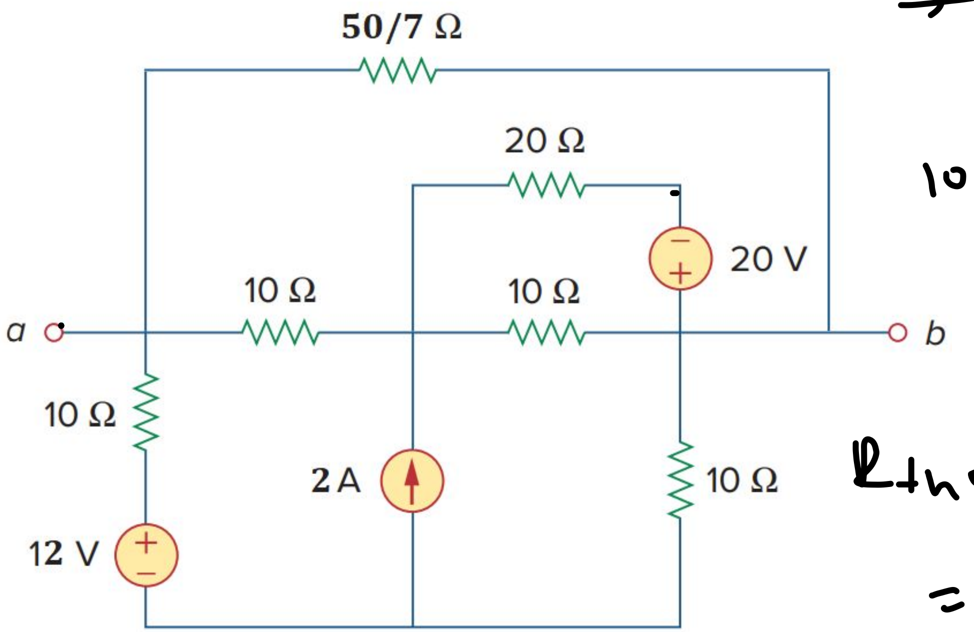


So,

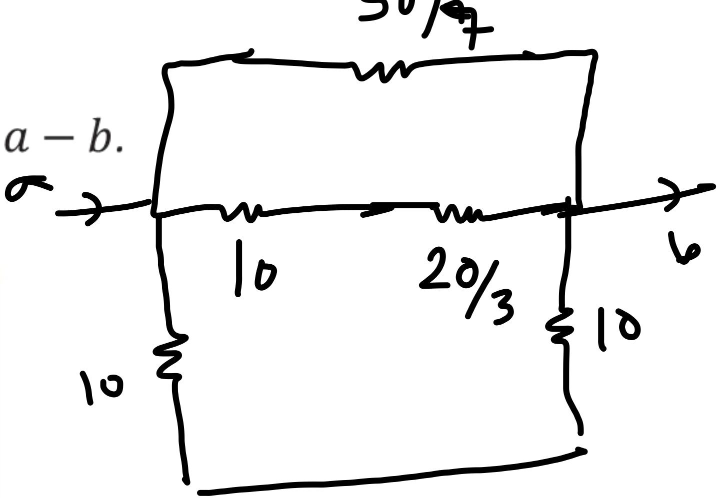
$R_{th(b,c)} = 5 \parallel (4+5) = 3.214A$

Problem 4

- Find the Thevenin equivalent at terminals $a - b$.



Step 2: R_{th}



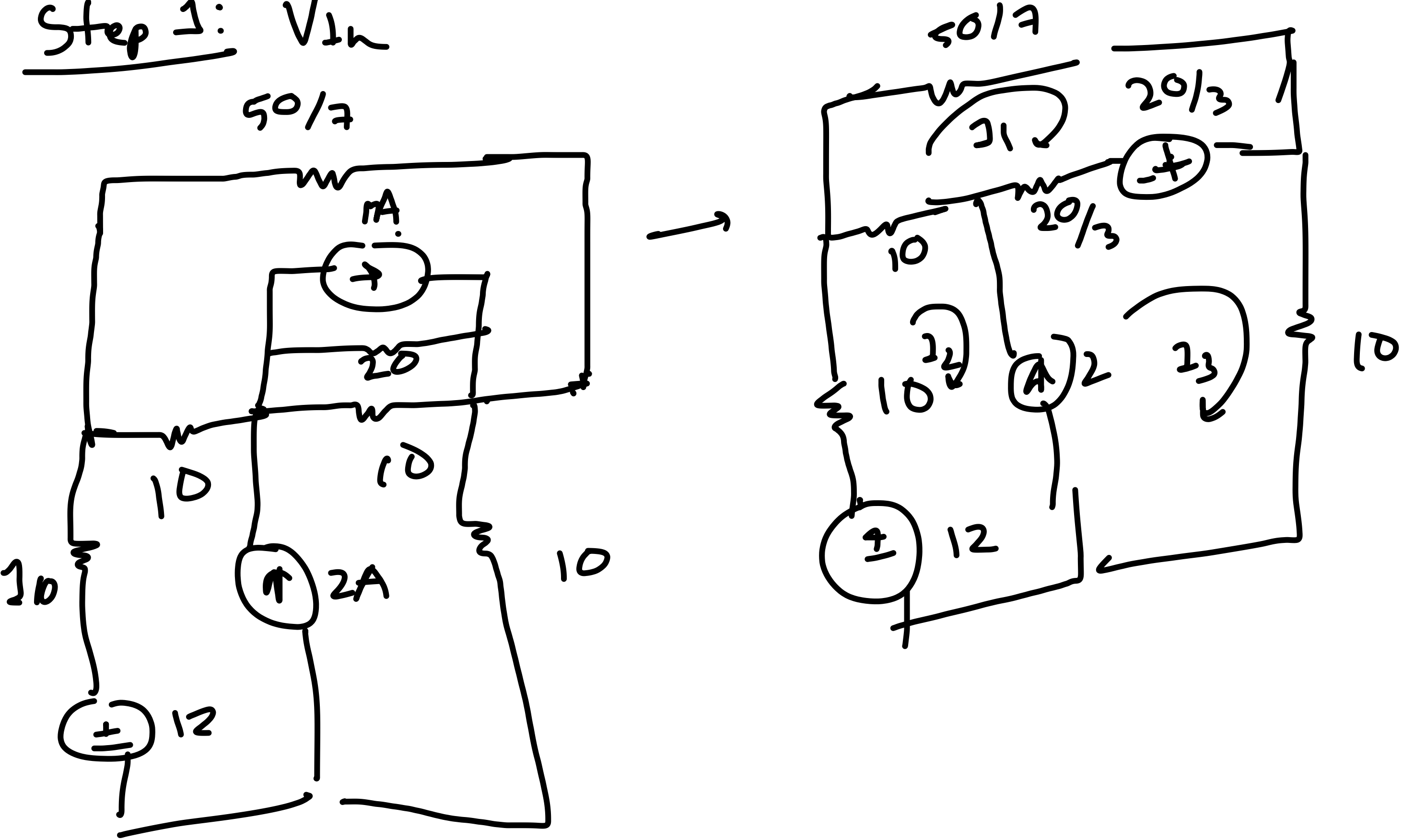
$R_1 = 10 \| 20 = 20/3$

$R_{th} = 50/7 \| (10 + \frac{20}{3}) \| (10+10)$

$= 4\,\Omega$

$\boxed{\text{Ans: } V_{Th} = 0\ V;\ R_{Th} = 4\ \Omega}$

Step 1: V_{th}



$$I_3 - I_2 = 2 \quad \text{—— } 1$$

$$\left(\frac{50}{7} + \frac{20}{3} + 10\right) I_1 - 10 I_2 - \frac{20}{3} I_3 = \frac{20}{3} \quad \text{—— } 11$$

Loop 2,3:

$$-\left(10 + \frac{20}{3}\right) I_1 + 20 I_2 + \left(10 + \frac{20}{3}\right) I_3 = \frac{20}{3} + 12$$

$I_1 = 0$, $I_2 = -2/5$, $I_3 = 8/5$

So, $V_{th} = V_{ab}$

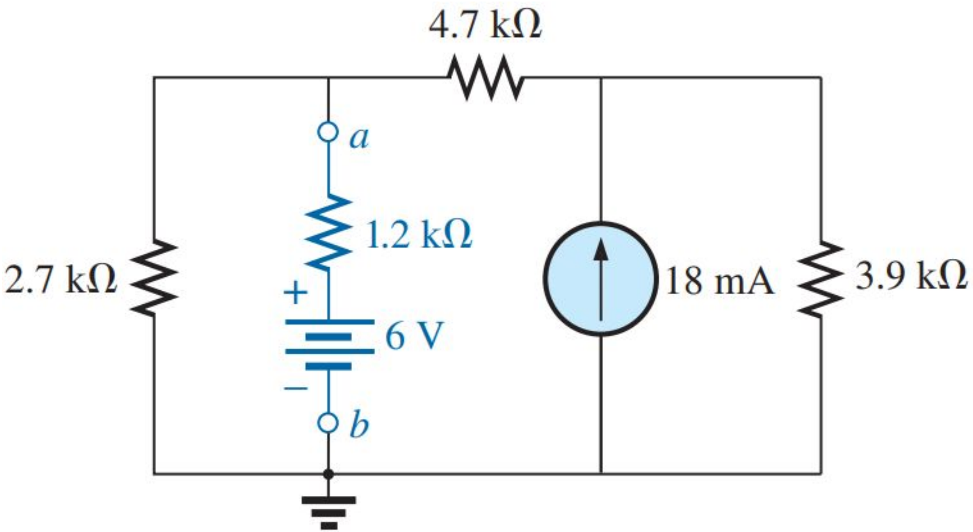
$$= \frac{50}{7} \times 0$$

$$= 0\,V$$

Problem 5

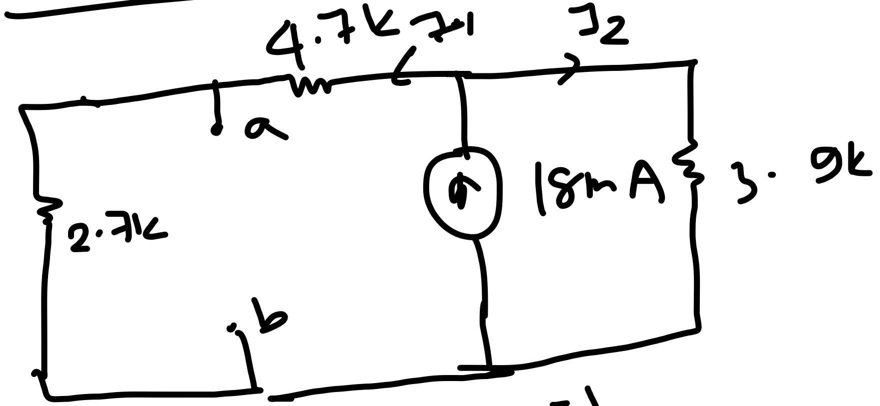
i. Find the Thevenin equivalent circuit for the portions of the network below external to points a and b.

ii. Redraw the network with the Thevenin circuit in place and find the current through the resistor.



Ans: $(i)\ V_{Th} = 16.77\ V;\ R_{Th} = 2.054\ k\Omega;$
$(ii)\ \pm 3.31\ A$

Step 1:



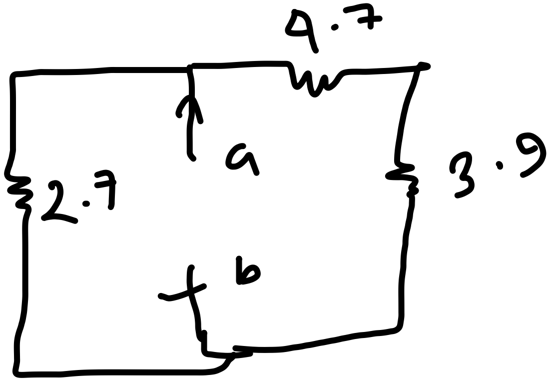
$$I_1 = \frac{(4.7+2.7)^{-1} \times 18}{(4.7+2.7)^{-1} + 3.9^{-1}} = 6.212\,mA$$

So,

$V_{th} = 2.7 \times I_1$

$= 16.77V$

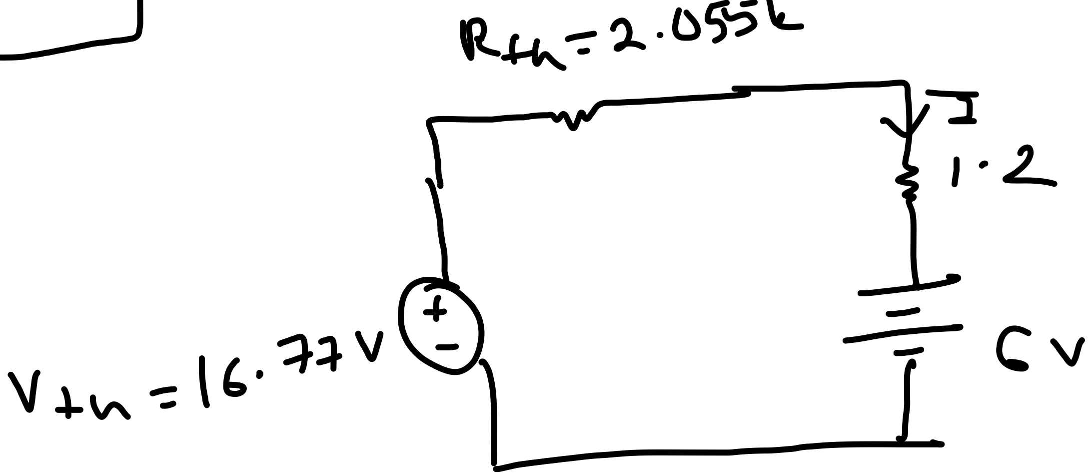
R_{th}:



$R_{th} = 2.7 \,||\, (4.7 + 3.9)$

$= 2.055k$

Now,

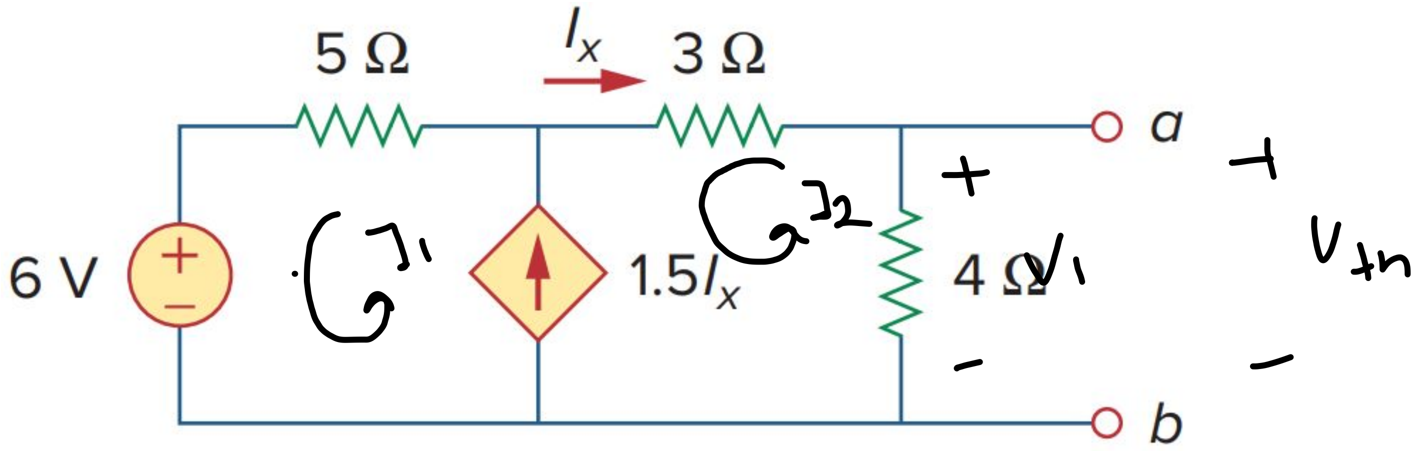


$$I = \frac{16.77 - 6}{1.2 + 2.055}$$

$$= 3.31 \, mA$$

Problem 6

- Find the Thevenin equivalent circuit of the circuit to the left of the terminals.



Ans: $V_{Th} = 5.333$ V; $R_{Th} = 0.44\ \Omega$

V_{th}:

$I_x = -I_2$

$I_1 - I_2 = 1.5 I_x$

$I_1 - I_2 = -1.5 I_2$

$I_1 + 0.5 I_2 = 0$

Loop 1,2:

$5 I_1 + 7 I_2 = -6$

$I_1 = 2/3$

$I_2 = -4/3$

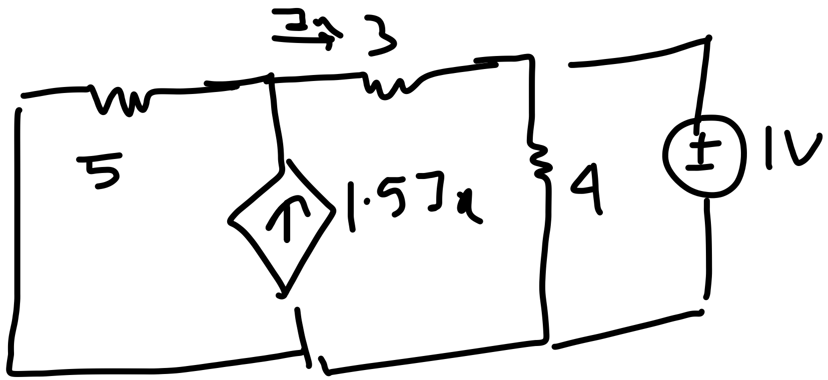
So, $V_1 = -4 I_2$

$= -4 \times \left(-\frac{4}{3}\right)$

$= -\frac{16}{3}$

So, $V_{th} = V_1$

$= \frac{16}{3}$ V



$I_1 = -I_x$

$12I_1 - 4I_2 + 7.5I_x = 0$

$4.5I_1 - 4I_2 = 0$ — 1

$4I_2 - 4I_1 = 1$

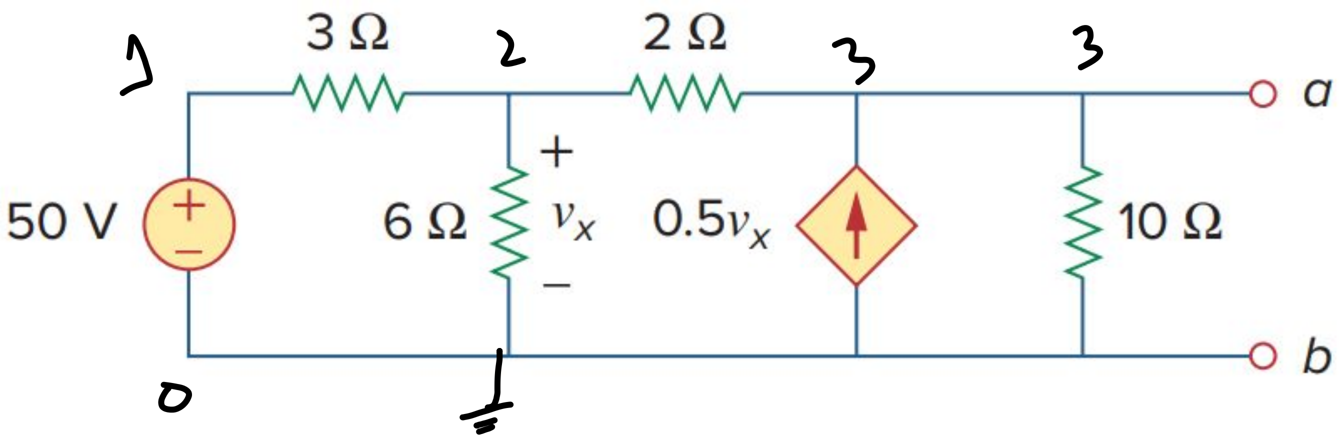
$I_1 = 2A$

$I_2 = 9/4$

$$R_{th} = \frac{1}{9/4} = 0.44\ \Omega$$

Problem 7

- Obtain the Thevenin equivalent circuit at terminals $a - b$.



Ans: $V_{Th} = 166.67\ V$; $R_{Th} = 10\ \Omega$

V_{th}:

$V_1 = 50V$, $V_x = V_2$

$$V_2\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{2}\right) - \frac{V_1}{3} - \frac{V_3}{2} = 0$$

$$V_2 \times 1 - \frac{V_3}{2} = 50/3$$

$$V_3\left(\frac{1}{10}+\frac{1}{2}\right) - \frac{V_2}{2} - 0.5V_x = 0$$

$$V_3\left(\frac{1}{10}+\frac{1}{2}\right) - \frac{V_2}{2} - 0.5 \times V_2 = 0$$

$$V_3 \times \frac{3}{5} - V_2 = 0$$

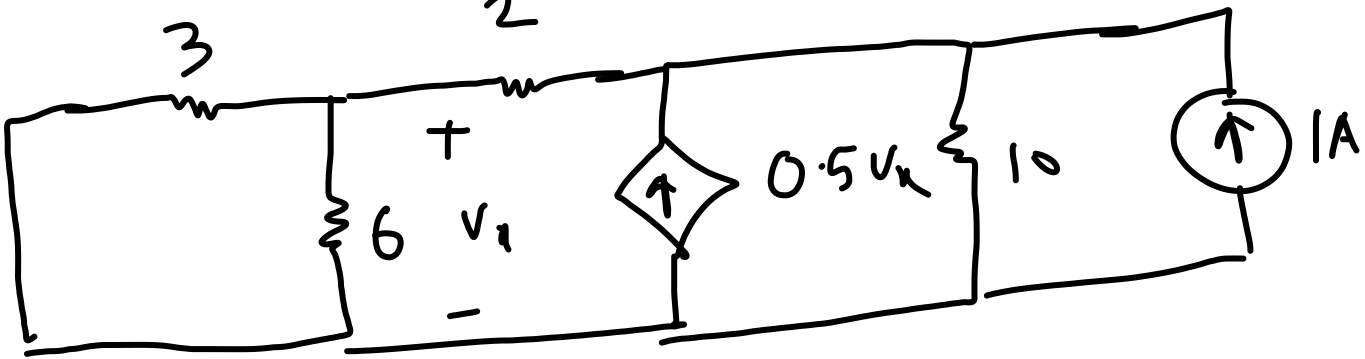
$$V_2 = 100V$$

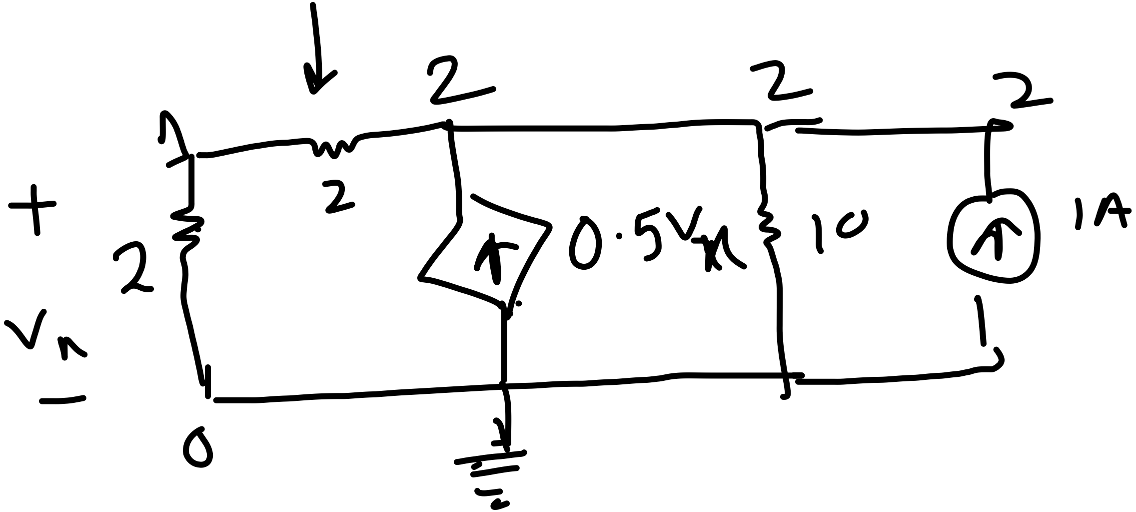
$$V_3 = 500/3$$

$$V_{th} = V_{ab}$$

$$= V_3$$

$$= 166.67V$$





$$V_x = V_1$$

$$V_1\left(\frac{1}{2}+\frac{1}{2}\right)-\frac{V_2}{2}=0$$

$$V_2\left(\frac{1}{2}+\frac{1}{10}\right)-\frac{V_1}{2}-1-0.5V_x=0$$

$$V_2\left(\frac{1}{2}+\frac{1}{10}\right)-\frac{V_1}{2}-\frac{V_1}{2}=1$$

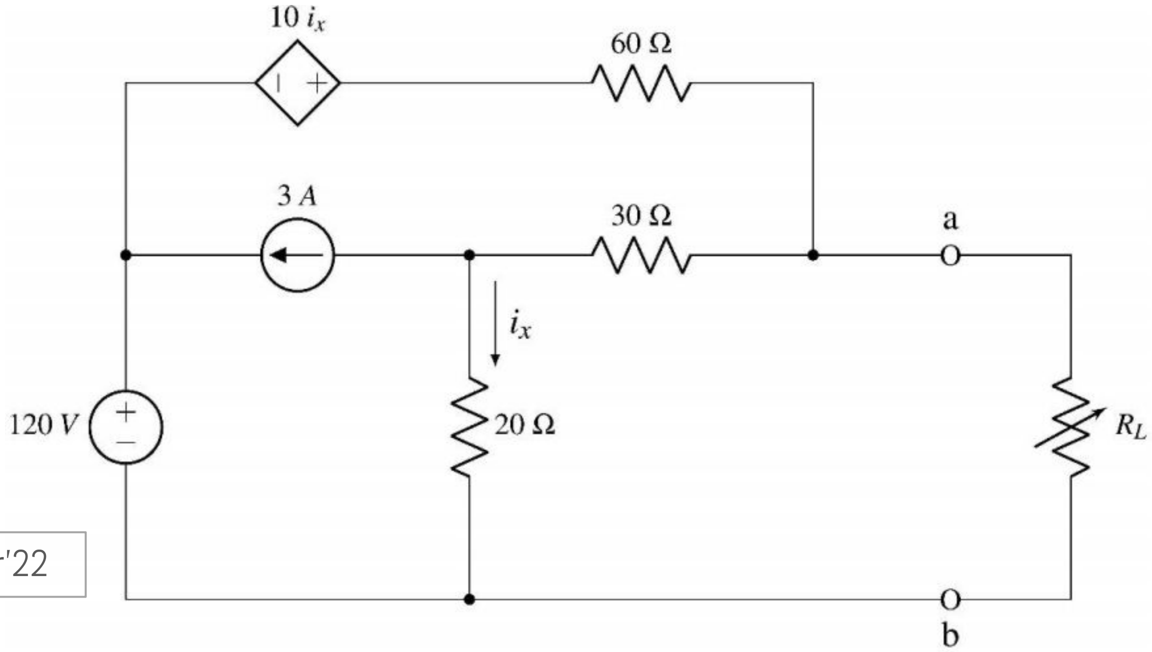
$V_1 = 5V$

$V_2 = 10V$

So, $R_{th}=\frac{V_2}{1}=\frac{10}{1}=10\Omega$

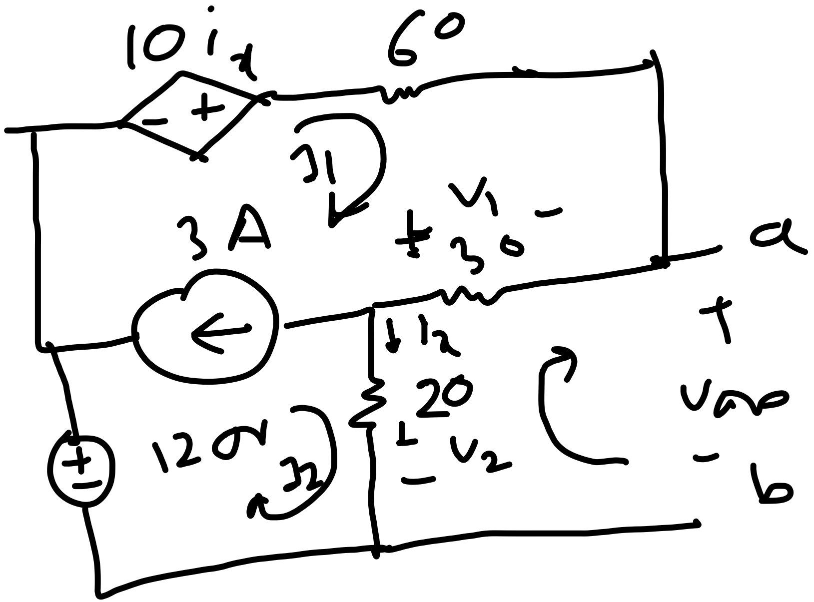
Problem 8

- Obtain the Thevenin equivalent circuit at terminals $a - b$.



Summer'22

Ans: $V_{Th} = 15\ V$; $R_{Th} = 30\ \Omega$



Sol $I_1 = 3/2$

$I_2 = -\frac{3}{2}$

$V_1 = -30 \times I_1$

$V_2 = 20 \times I_2$

So, $I_1 - I_2 = 3$, $I_x = I_2$

$1,2 \Rightarrow \quad -10 i_x + 60 I_1 + 30 I_1 + 20 I_2 = 120$

$-10 I_2 + 90 I_1 + 20 I_2 = 120$

so1

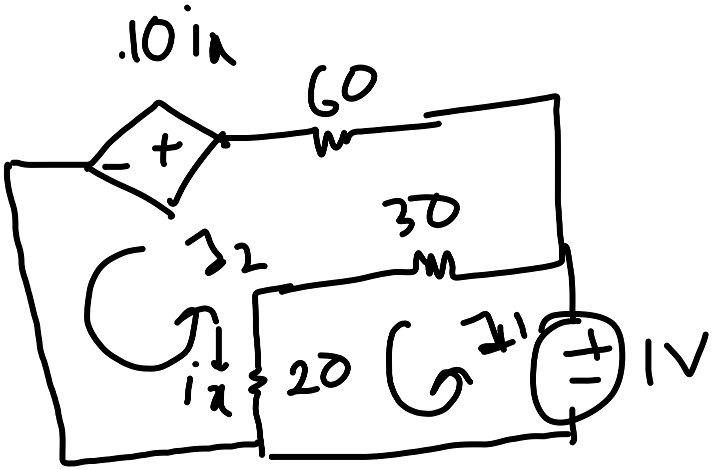
$-V_2 + V_1 + V_{ab} = 0$

$V_{th} = V_{ab} = V_2 - V_1 = 20 \times I_2 + 30 \times I_1$

$= 20 \times (-\frac{3}{2}) + 30 \times (\frac{3}{2}) = 15V$

$I_x = I_1 - I_2$

Rth!



$50I_1 - 50I_2 = 1$

$110 I_2 - 50I_1 = -10I_1$

$110I_2 - 50I_1 + 10I_1 - 10I_2 = 0$

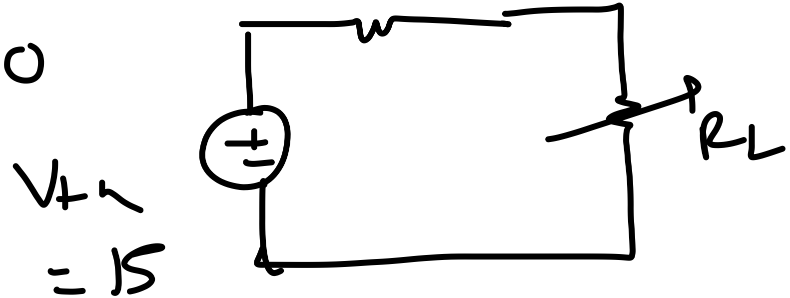
$I_1 = 1/30 \; A$

$I_2 = \frac{1}{75} A$

So, $R_{th} = 1/I_1 = 30 \; \Omega$

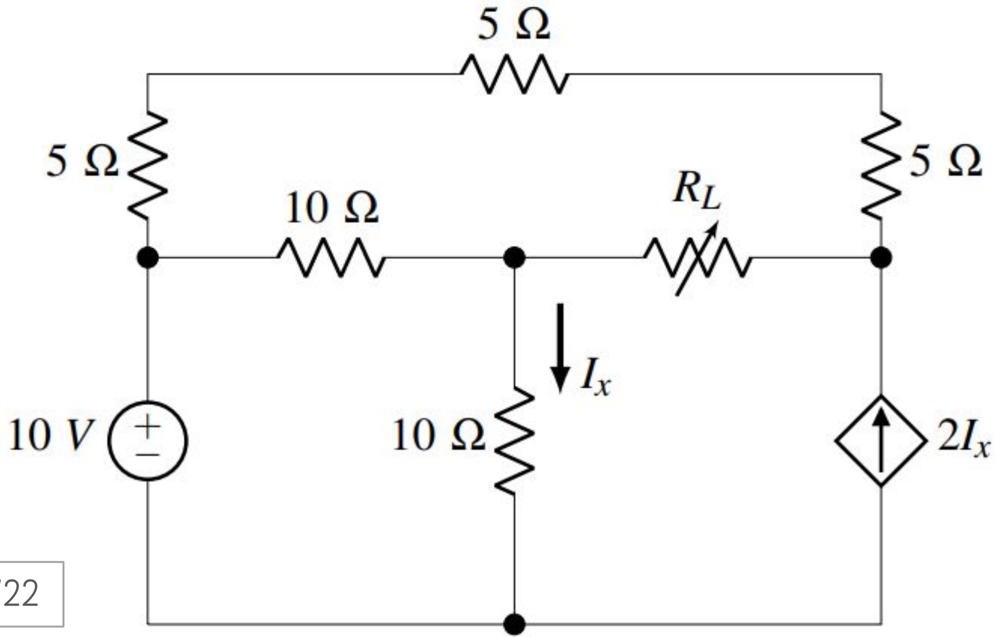
thevenin

$R_{th} = 30$



Problem 9

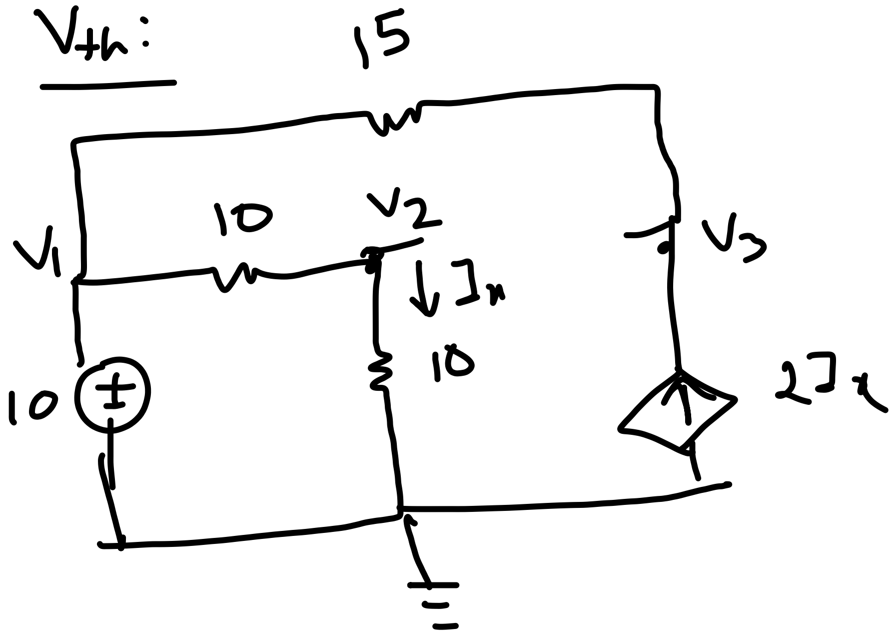
- Obtain the Thevenin equivalent circuit at the load (R_L) terminals.



Fall'22

Ans: $V_{Th} = \pm 20\ V$; $R_{Th} = 5\ \Omega$

$\underline{V_{th}:}$



$$V_{th} = \pm(V_3 - V_2) = \pm 20V$$

$V_1 = 10V$

$$V_2\left(\frac{1}{10} + \frac{1}{10}\right) - \frac{V_1}{10} = 0$$

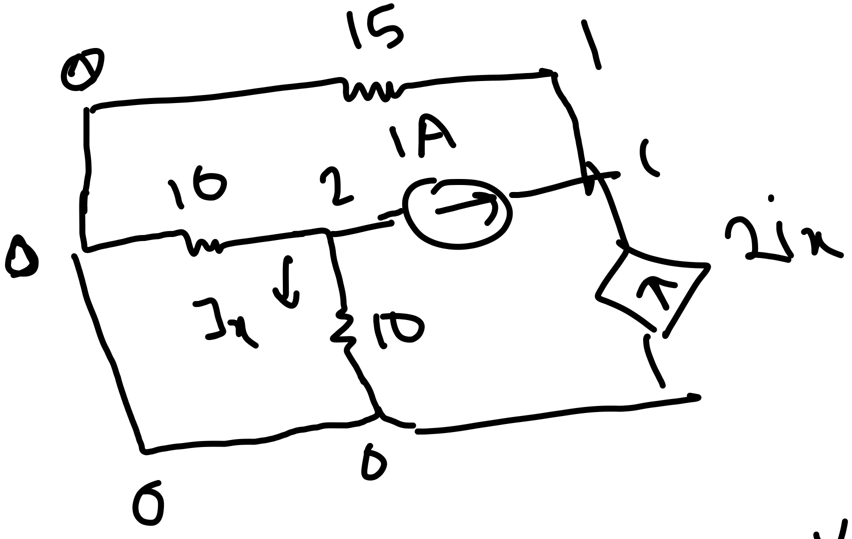
$$V_2 \cdot \frac{1}{5} = 1 \qquad \therefore V_2 = 5V$$

$$V_3\left(\frac{1}{15}\right) - \frac{V_1}{15} = 2I_x$$

$$\frac{V_3}{15} - \frac{10}{15} = 2\frac{V_2}{10}$$

$$V_3 = 25V$$

Exr:



$$V_2\left(\frac{1}{10}+\frac{1}{10}\right) = -1$$

$$V_2 = -5V$$

$$V_1\left(\frac{1}{15}\right) - 1 - 2I_x = 0$$

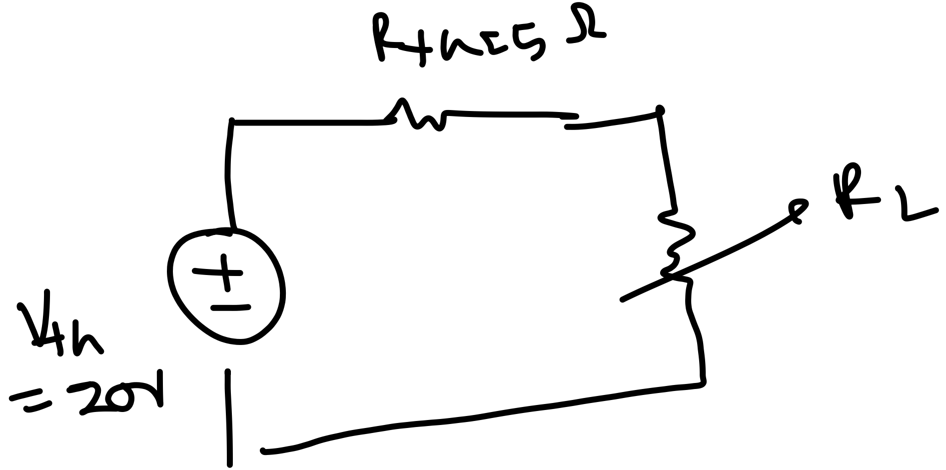
$$\frac{V_1}{15} = 1 + 2\times\frac{V_2}{10}$$

$$= 0$$

$$\therefore V_1 = 0V$$

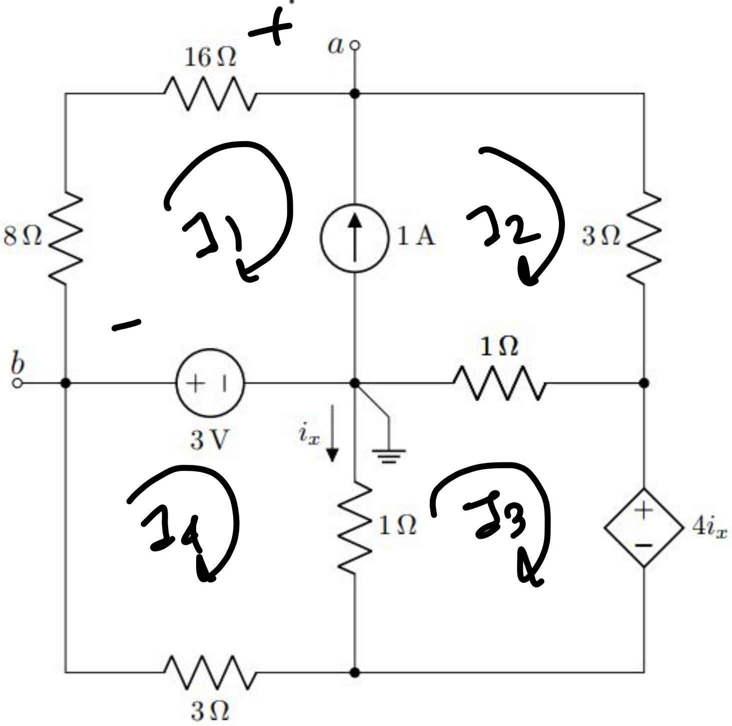
Sol

$$R_{th} = \frac{V_1 - V_2}{1} = \frac{\cancel{V_1} + 5}{1} = 5\,\Omega$$



Problem 10

- Obtain the Thevenin equivalent circuit with respect to terminals $a - b$.



$I_x = I_4 - I_3$

Ans: $V_{Th} = 3\ V$; $R_{Th} = 4\ \Omega$

Spring'23

V_{th}:

$I_2 - I_1 = 1A$ — (i)

Loop 4:

$4I_4 - I_3 = -3$ — (ii)

$2I_3 - I_4 - I_2 + 4I_x = 0$

$2I_3 - I_4 - I_2 + 4(I_4 - I_3) = 0$

$-2I_3 + 3I_4 - I_2 = 0$ — (iii)

Loop 1,2

$24I_1 + (3+1)I_2 - I_3 = 3$ — (iv)

$I_1 = -\frac{1}{8}$

$I_2 = 7/8$

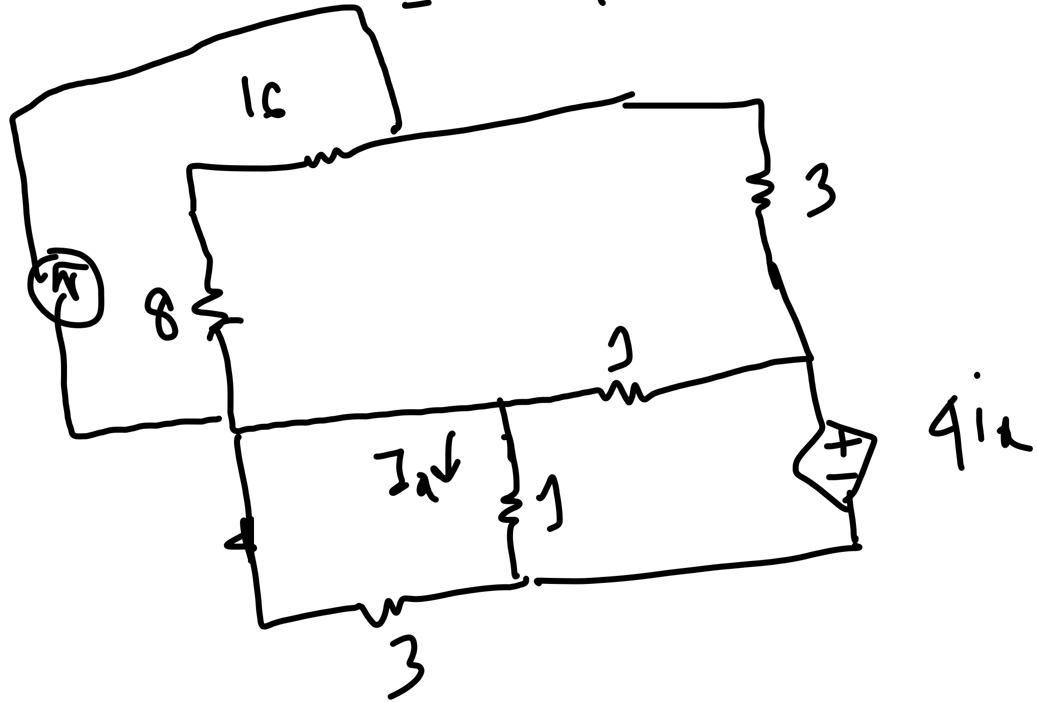
$I_3 = -5/2$

$I_4 = -\frac{11}{8}$

$V_{th} = V_{ab} = -24 I_1$

$= -24 \times \left(-\frac{1}{8}\right) = 3V$

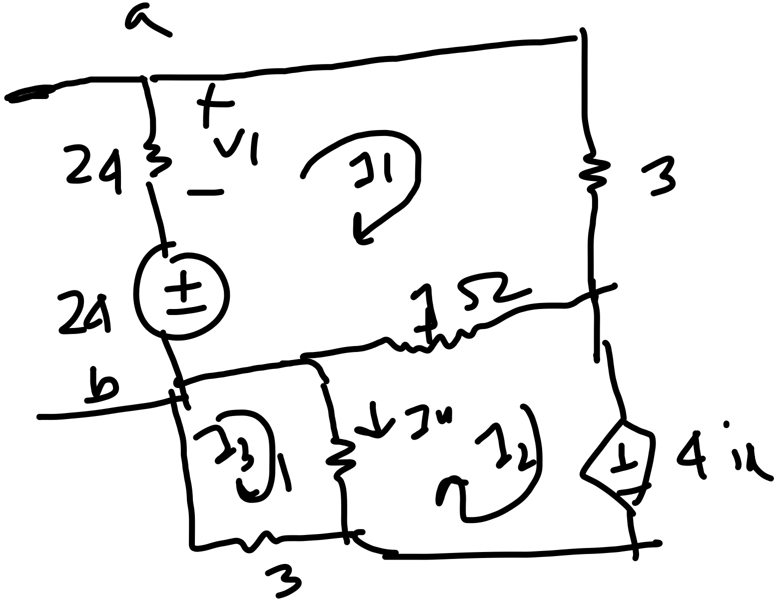
R_{th}:



So, $R_{th} = \frac{V_{ab}}{1}$

$= \frac{4}{1}$

$= 4\,\Omega$



$$V_{ab} = 24 + V_1$$
$$= 24 + 24(-I_1)$$
$$= 24 +$$
$$= 4^V$$

I_3

$$4I_3 - I_2 = 0$$

I_2: $2I_2 - I_3 - I_1 + 4I_x = 0$

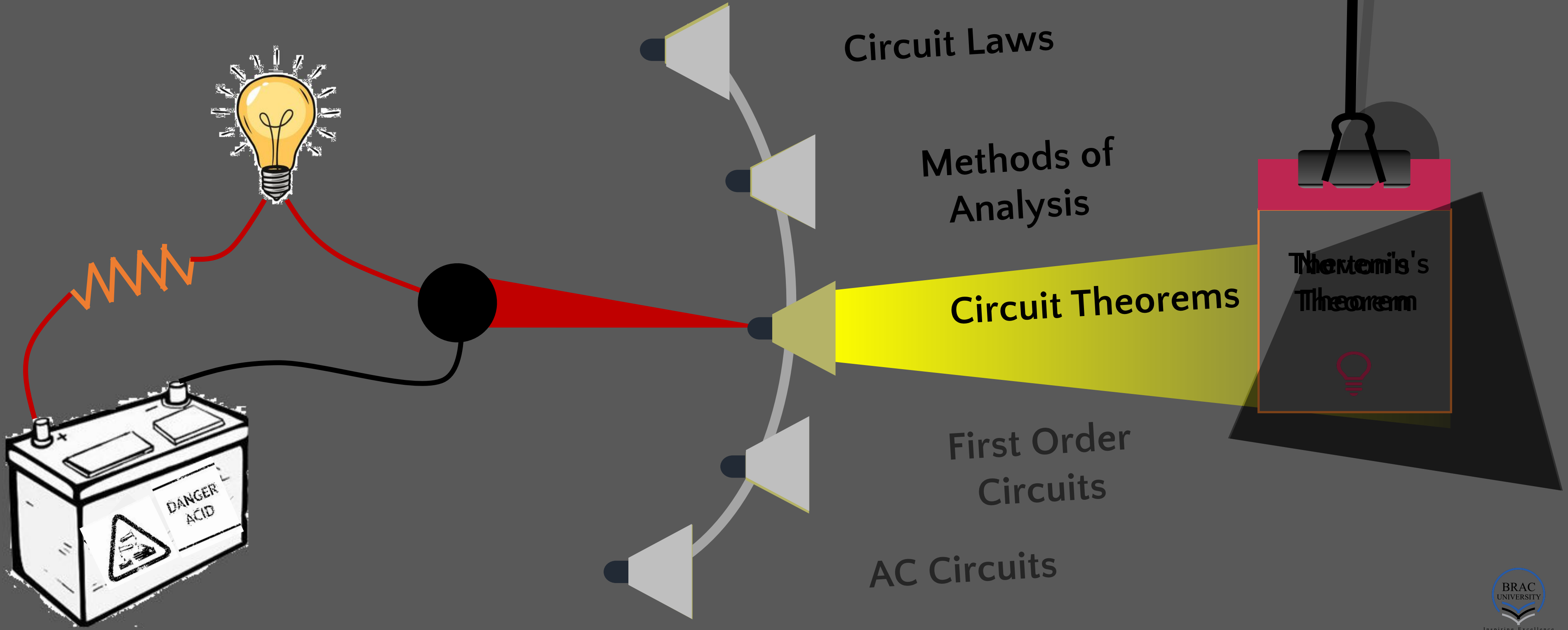
$$2I_2 - I_3 - I_1 + 4(I_3 - I_2) = 0$$

I_1: $28 I_1 - I_2 = 29$

$$I_1 = 5/6 \;,\; I_2 = -2/3$$

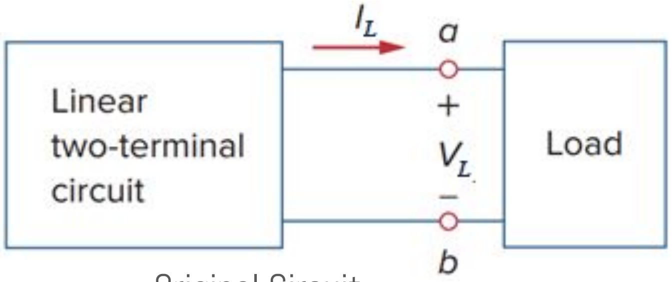
$$I_3 = -1/6$$

Course Outline: broad themes

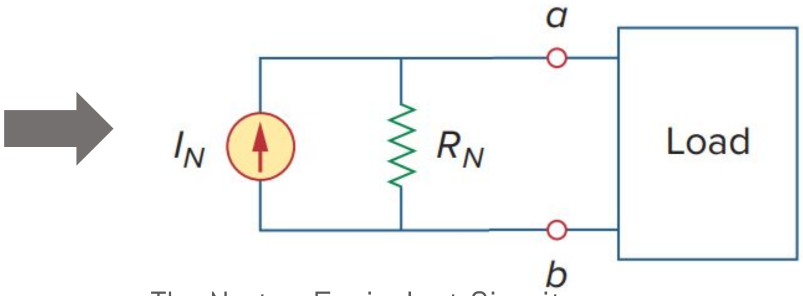


Norton’s Theorem

- *Norton’s theorem* states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and I_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



Original Circuit



The Norton Equivalent Circuit

- Two circuits are said to be equivalent if they have the same $I - V$ characteristics at their terminals.
- Let's find out what will make the two circuits equivalent!

I-V of the Norton Equivalent

- We can derive the *$I - V$ characteristics of the Norton equivalent* in a similar way as we did in for Thevenin.
- The configuration is a current source (I_N) in series with a resistor (R_N). To determine the configuration's $I - V$ characteristics, if applying a voltage V gives rise to a current i_x through the resistor, we can write using KCL,

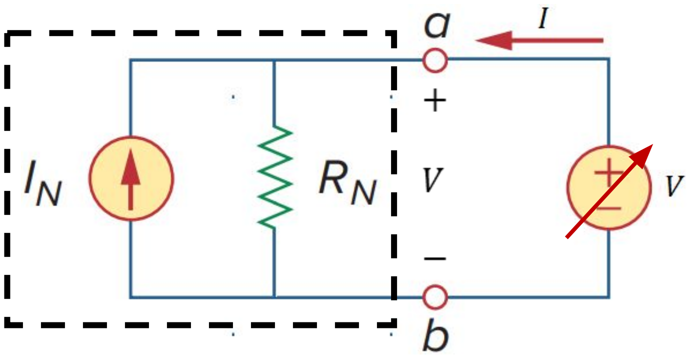
$$i_x = I_N + I$$

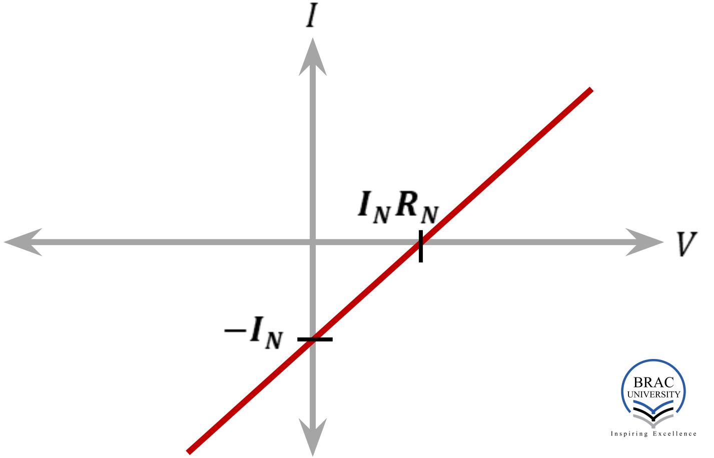
- So, voltage across the resistor can be written as,

$$V = i_x R_N = (I_N + I)R_N$$

$$\Rightarrow I = \frac{1}{R_N}V - I_N$$

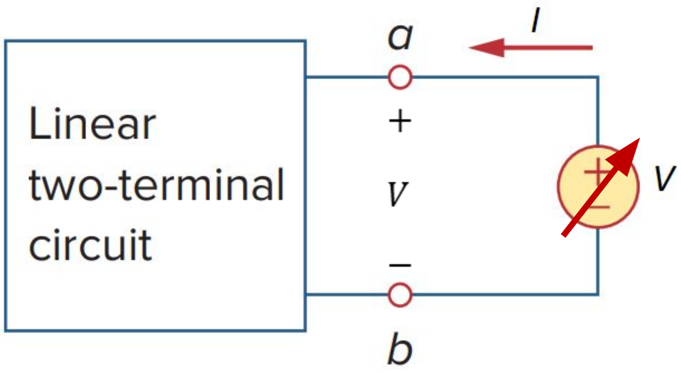
- The equation results in a linear I vs V plot that intersects the axes at $I_N R_N$ and $-I_N$





I-V of the Actual Circuit

- The procedure to derive the *$I - V$ characteristics of the original circuit* is exactly the same as done in the Thevenin part. This is described here again.



- To theoretically derive exactly the relation between I and V it is required to know the actual circuitry. As the circuit is linear, the $I - V$ characteristic will be a straight line and the line can be drawn if minimum two points on the line are known.

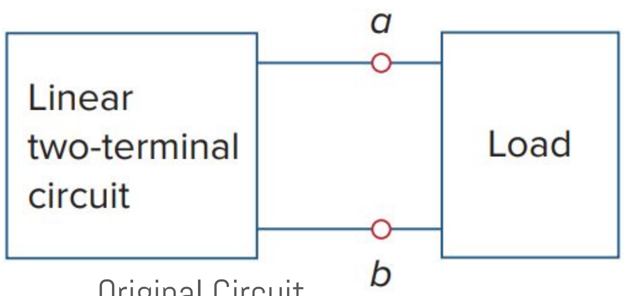
- The two points we can get are the intersecting points of x and y axis.

- To get the intersecting location on the voltage axis, current (I) at the terminals should be made equal to 0. That is, the terminals $a - b$ must be open circuited.

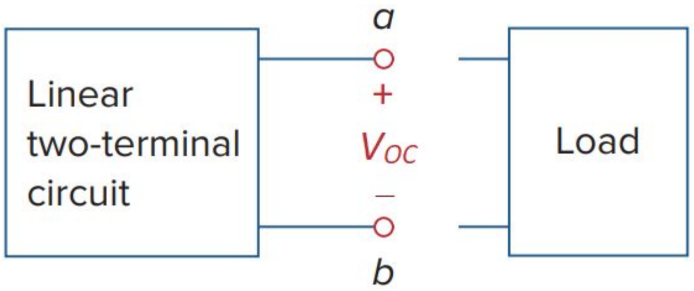
- Similarly, for the intersecting location on current axis, $V_{ab} = V = 0$. That is, the terminals $a - b$ must be shorted.

Open Circuit Voltage & Short Circuit Current

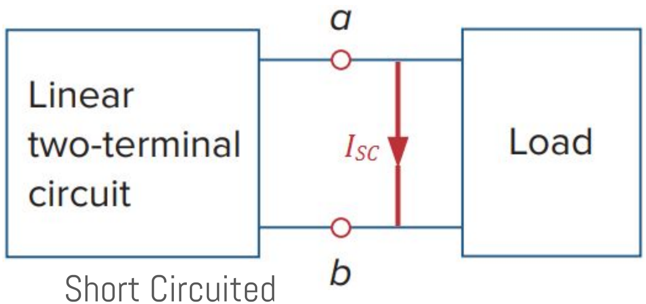
- Let's denote V_{oc} be the voltage at the open terminals upon disconnecting the load and I_{sc} be the current through the shorted terminals upon short circuiting the load.



Original Circuit



Open Circuited at the terminals

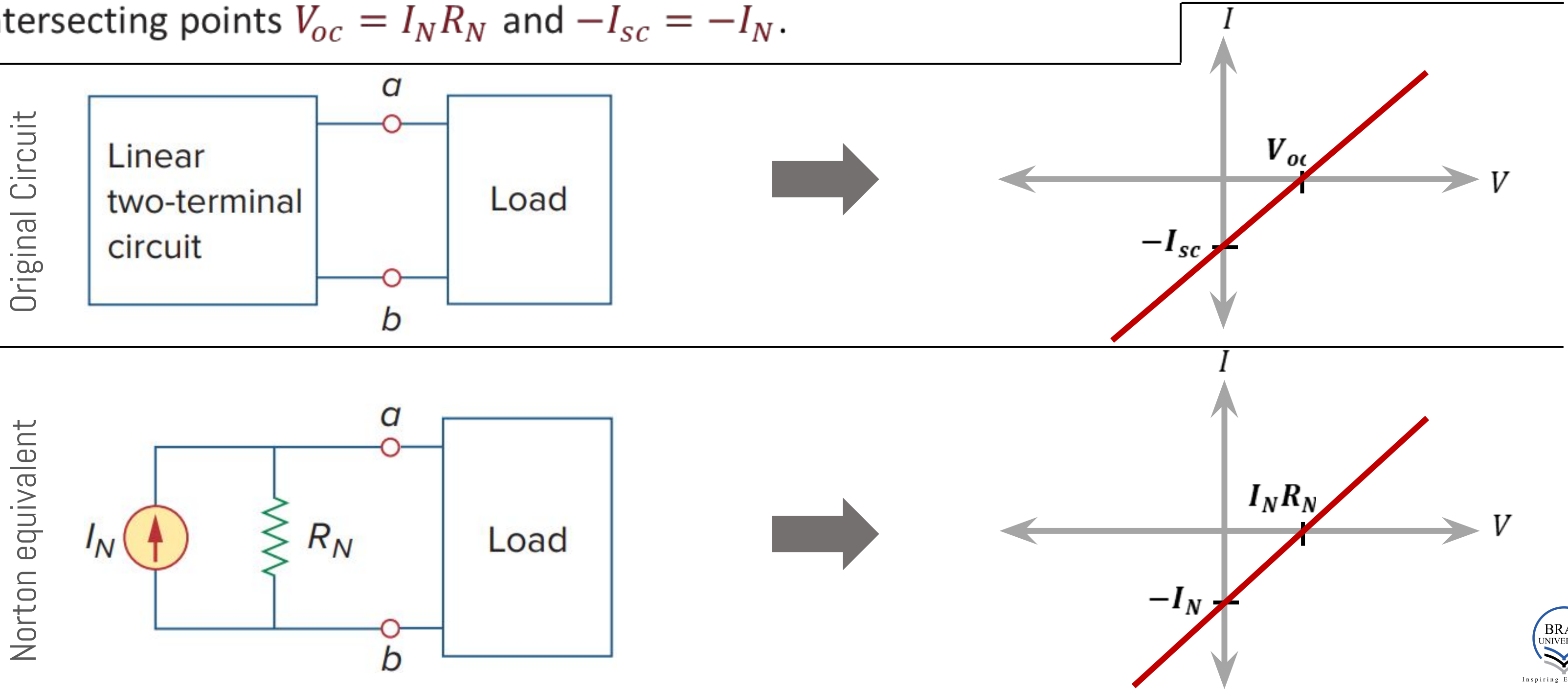


Short Circuited

So, the $I-V$ characteristic should be the straight line passing through the points $(V_{oc}, 0)$ and $(0, -I_{sc})$. The reason for the negative sign is that I_{sc} is opposite to the current (I) plotted along the y-axis.

Circuit Equivalency

- The original circuit and the reduced Norton equivalent circuit will be equivalent to each other if the $I - V$ characteristics of the two are identical. They will indeed be identical if the intersecting points $V_{oc} = I_N R_N$ and $-I_{sc} = -I_N$.

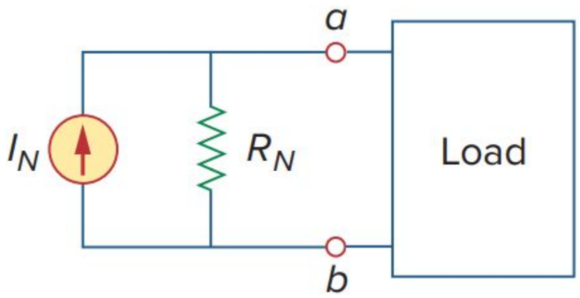


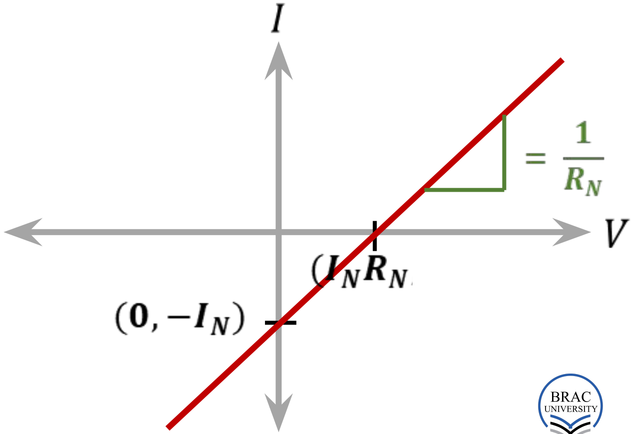
How to determine R_N?

- Refer to the previous slides, Norton's conversion is valid if
  i. $V_{oc} = I_N R_N$ *or* $I_N = \frac{V_{oc}}{R_N}$
  ii. $-I_N = -I_{sc}$ *or* $\frac{V_{oc}}{R_N} = I_{sc}$
- For the linear $I - V$ characteristic, R_N is the inverse of the slope of the straight line passing through the points $(I_N R_N,\ 0)$ and $(0,\ -I_N)$. That is,

$$Slope = \frac{\Delta I}{\Delta V} = \frac{0 - (-I_N)}{I_N R_N - 0} = \frac{1}{R_N}$$

- Thus, R_N may be found from the open circuit voltage V_{oc} and the Norton current I_N.
- The undefined scenario that occurs when determining R_{Th} when V_{Th} is zero (see here) also occurs when determining R_N when $I_{sc} = 0$. In that situation, the Universal Rule used to derive R_{Th} applies exactly to R_N.



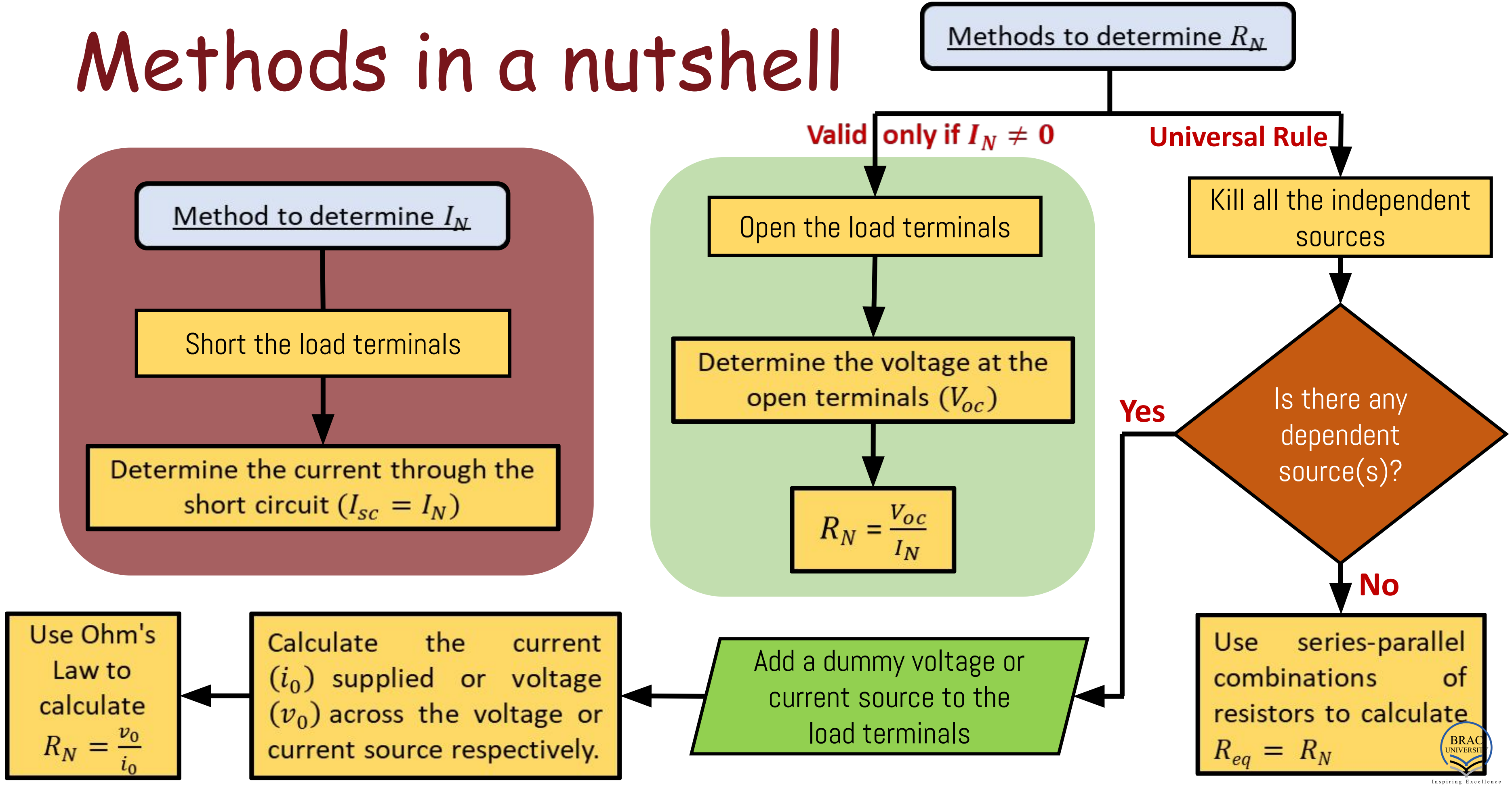


Thevenin ↔ Norton

- As you may have already noticed, Norton equivalent of a circuit can be derived from the Thevenin equivalent (or vice versa) of the same circuit by performing a source transformation.
- The requirement is that the two must have the same $I - V$ characteristics.
- From the conditions for which source transformation is valid (shown in slide 7 of Source Transformation) or by comparing the $I - V$ characteristics of the two, it can be seen that the conversion is valid if and only if,

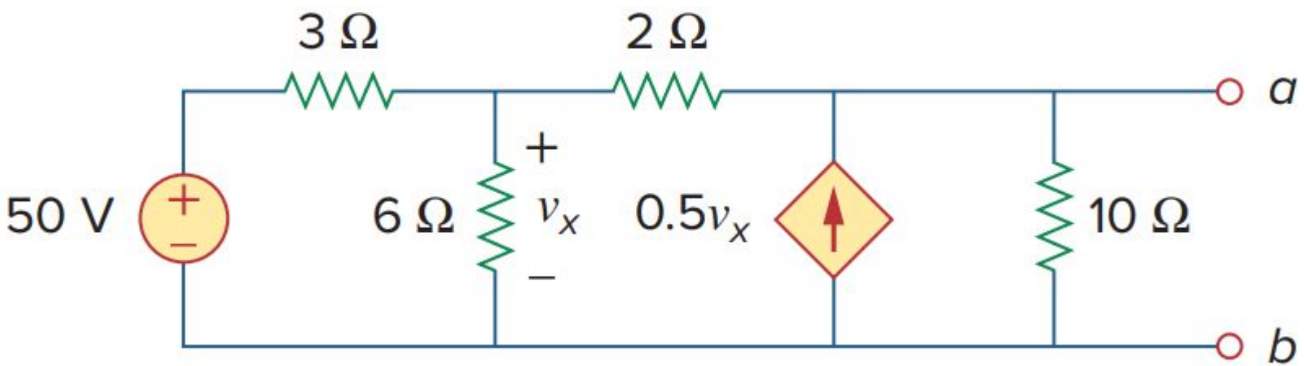
$$R_N = R_{Th} \quad \& \quad I_N R_N = V_{Th}$$

Methods in a nutshell



Example 4

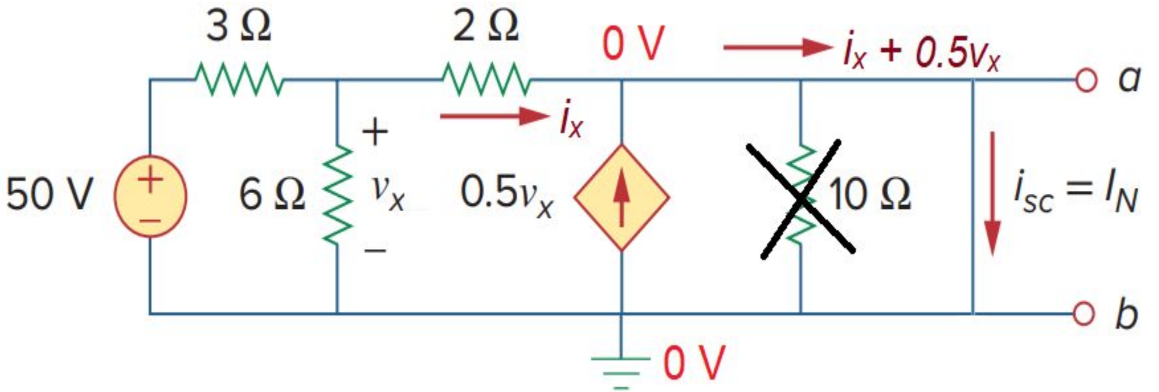
- Obtain the Norton equivalent circuit at terminals $a - b$.



Ans: $\boldsymbol{R_N = 10\ \Omega;\ I_N = 16.667\ A}$

* *See solution in the next slide if necessary*

Example 4: finding I_N



The 1st step is to disconnect the load and short the terminals.

Upon short circuiting the terminals a-b, the 10 Ω is shorted out. The whole circuit to the left of the dependent source is shorted with respect to it. As a result, the $0.5v_x$ current supplied by the dependent source will only flow through the short circuit.

Let's use nodal analysis to solve for the current i_x going towards the short circuit though the 2 Ω resistor.

KCL at v_x

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x}{2} = 0$$

$$\Rightarrow \quad v_x = 16.667\ V$$

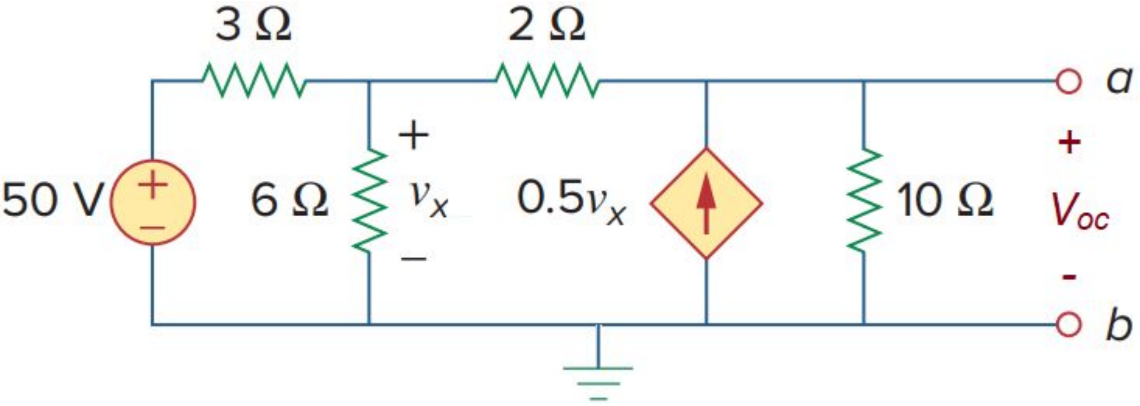
Now,

$$i_x = \frac{v_x - 0}{2} = 8.334\ A$$

So,

$$I_N = i_x + 0.5v_x = 16.667\ A$$

Example 4: finding R_N



R_N can be found by (i) determining V_{oc} and then using $R_N = \frac{V_{oc}}{I_N}$ (as $I_N \neq 0$) or (ii) first turning off all the independent sources and determining the R_{eq} at the terminals.

Let's employ the first method here.

Nodal analysis:

KCL at v_x,

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x - V_{oc}}{2} = 0$$

$$\Rightarrow \quad 6v_x - 3V_{oc} = 100 \ ---- (i)$$

KCL at V_{oc},

$$\frac{V_{oc} - v_x}{2} + \frac{V_{oc}}{10} = 0.5v_x$$

$$\Rightarrow \quad 10v_x - \ 6V_{oc} = 0 \ ---- (ii)$$

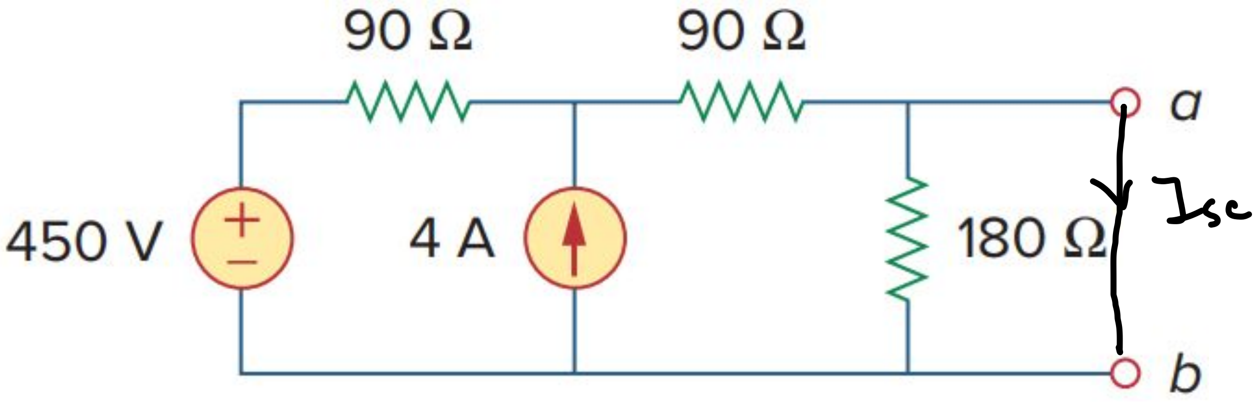
Solving (i) and (ii),

$$V_{oc} = 166.667\ V$$

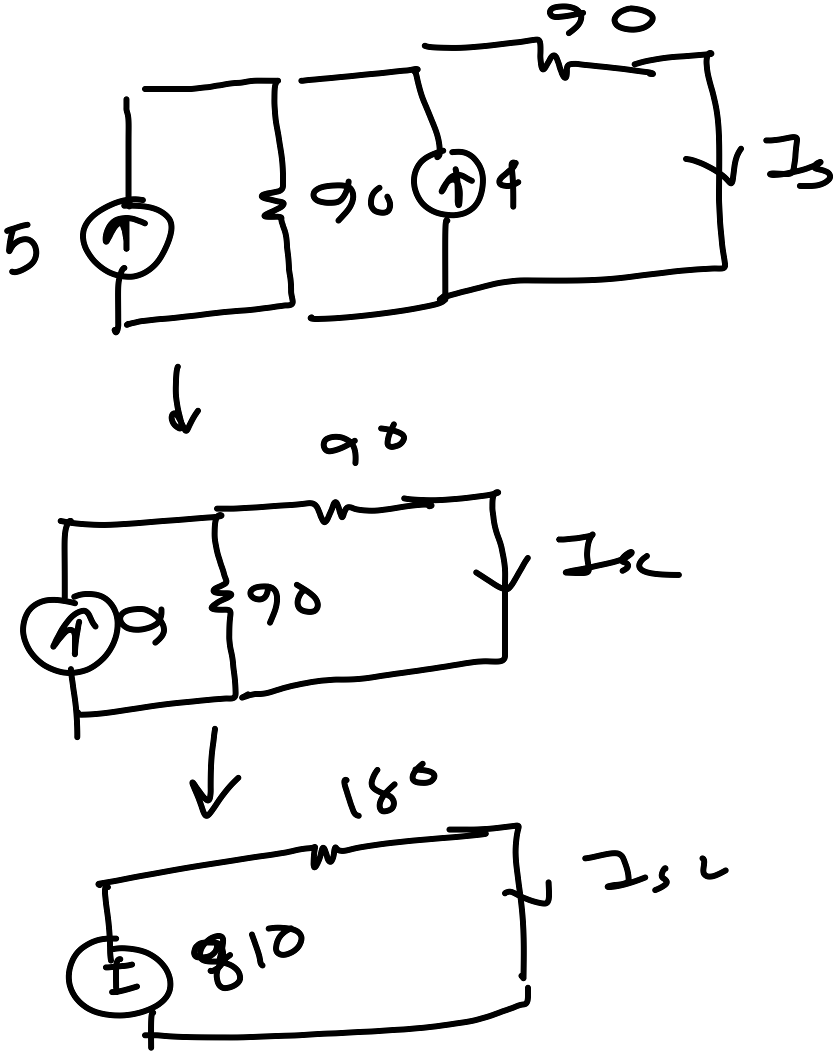
$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{166.667}{16.667} = 10\ \Omega$$

Problem 11

- Find the Norton equivalent circuit for the circuit at terminals $a - b$.



Ans: $I_N = 4.5$ A; $R_N = 90\ \Omega$

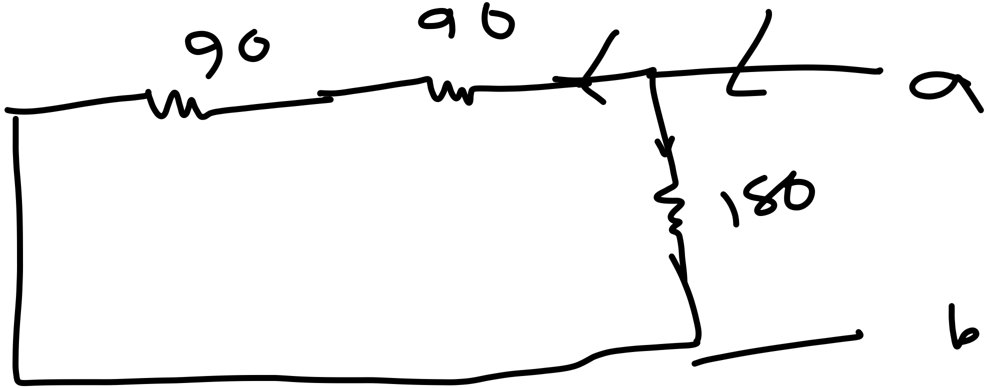


So,

$$I_{sc} = \frac{810}{180}$$

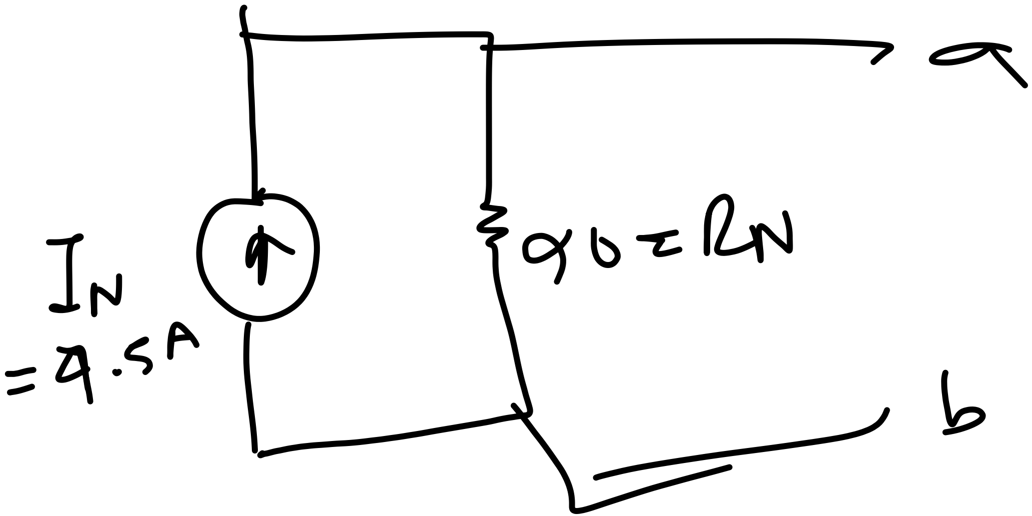
$$I_N = 4.5A$$

R_N



$R_{ab} = R_N = 180 \parallel 180$

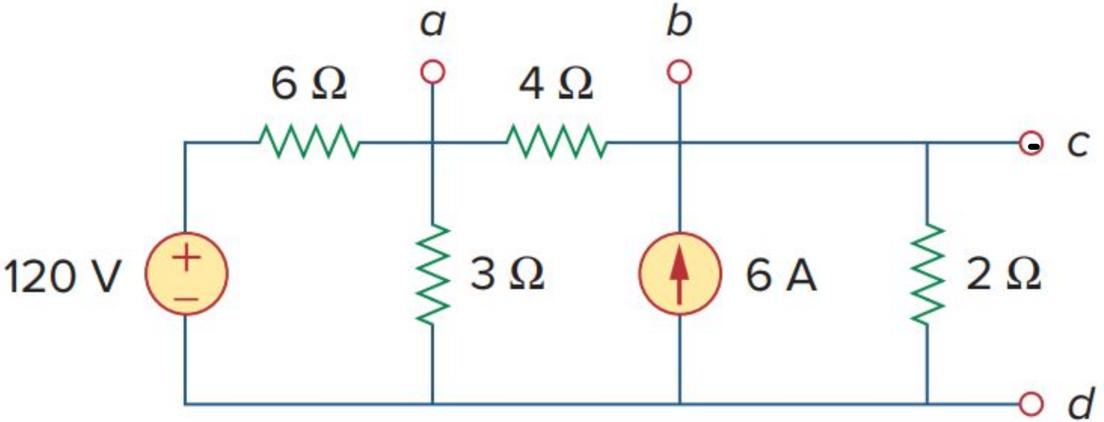
$= 90\,\Omega$



I_N
$= 4.5A$
$90 = R_N$
a
b

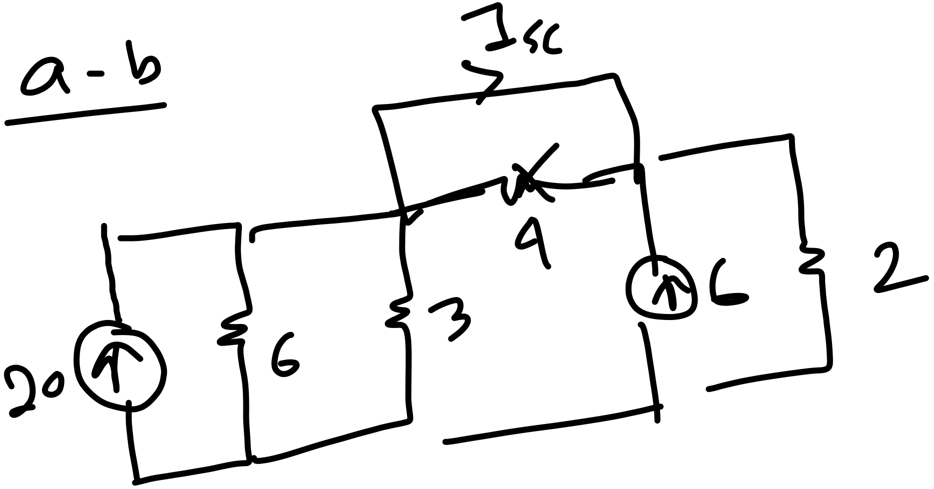
Problem 12

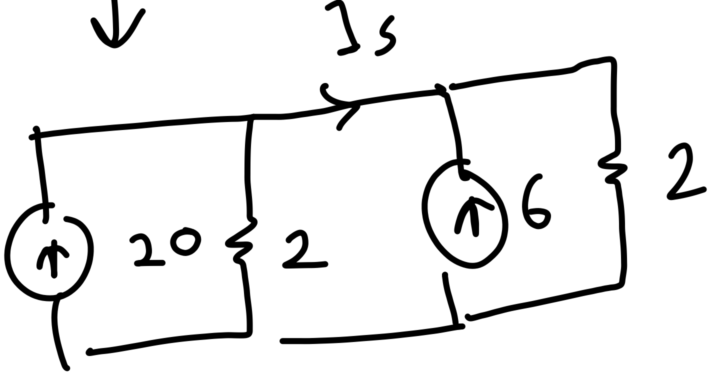
- Find the Norton equivalent circuit for the circuit at terminals $(i)\ a - b\ and$ $(ii)\ c - d$.



Ans: (i) $I_N = 7$ A; $R_N = 2\ \Omega$
(ii) $I_N = 12.667$ A; $R_N = 1.5\ \Omega$

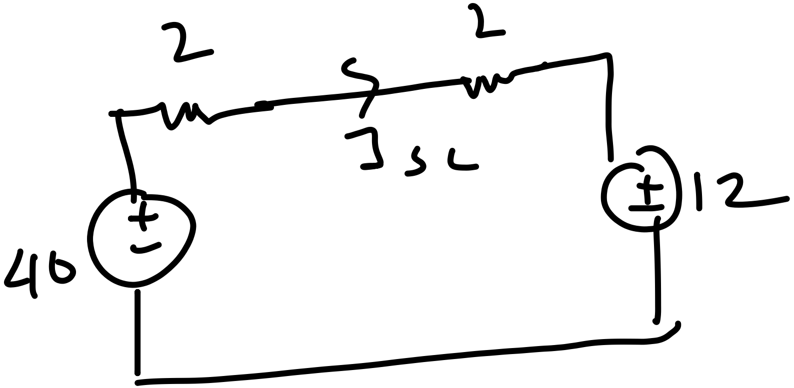
a - b



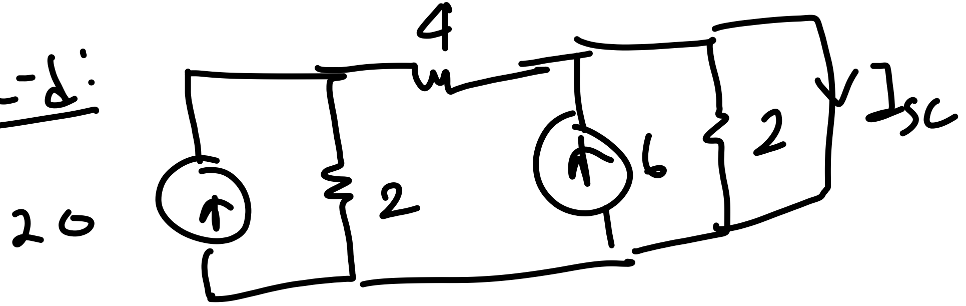


So, $I_{sc} = \dfrac{40-12}{4}$

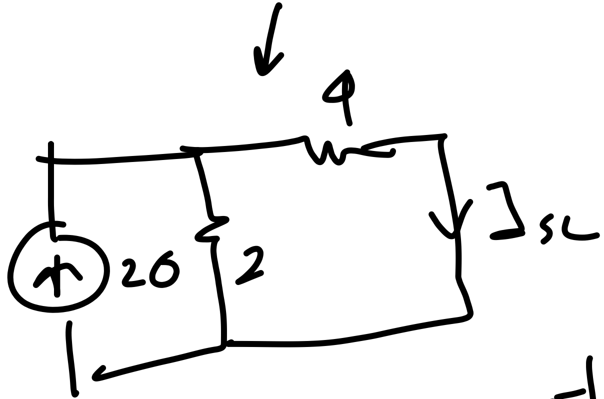
$I_{N_{ab}} = 7A$



$I_{N(c-d)}$:



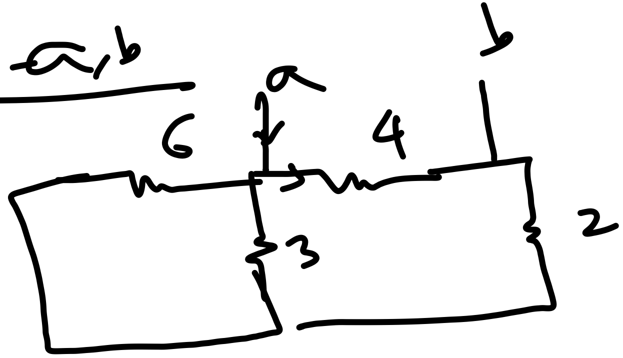
$2\Omega, 6A \rightarrow$ exclude (S.C)

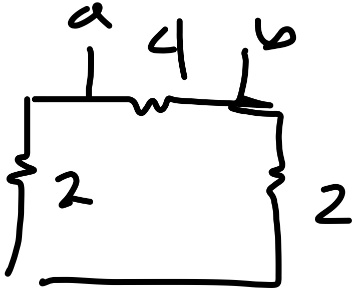


$$I_{sc} = \frac{4^{-1}}{2^{-1}+4^{-1}} \times 20$$

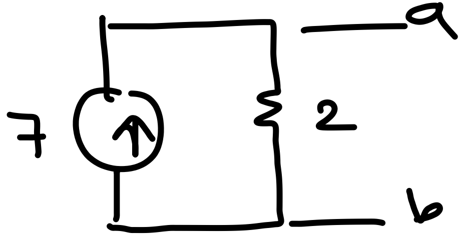
$$= 20/3\ A$$

R_N a,b

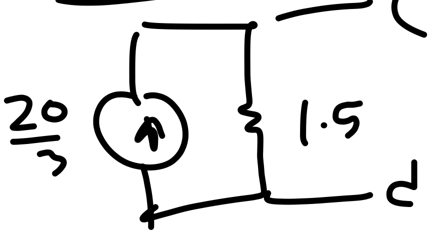




a,b



c,d

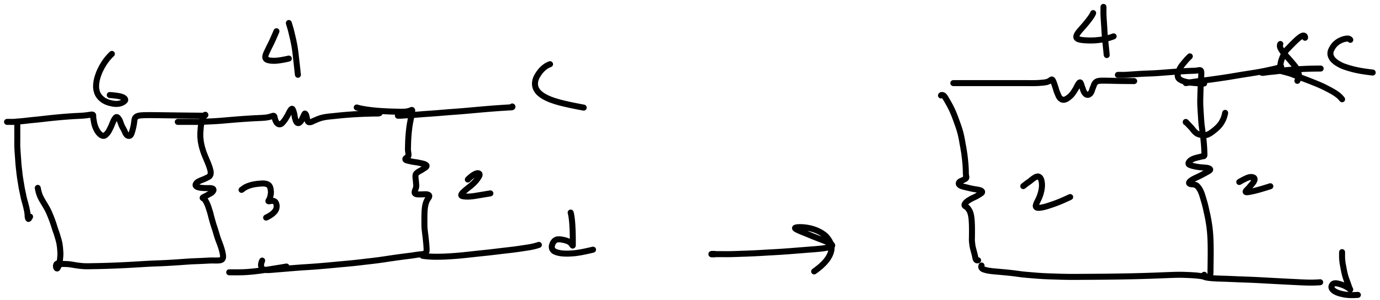


So, $R_{N\,a,b} = 4 \| 4$

$= 2\Omega$

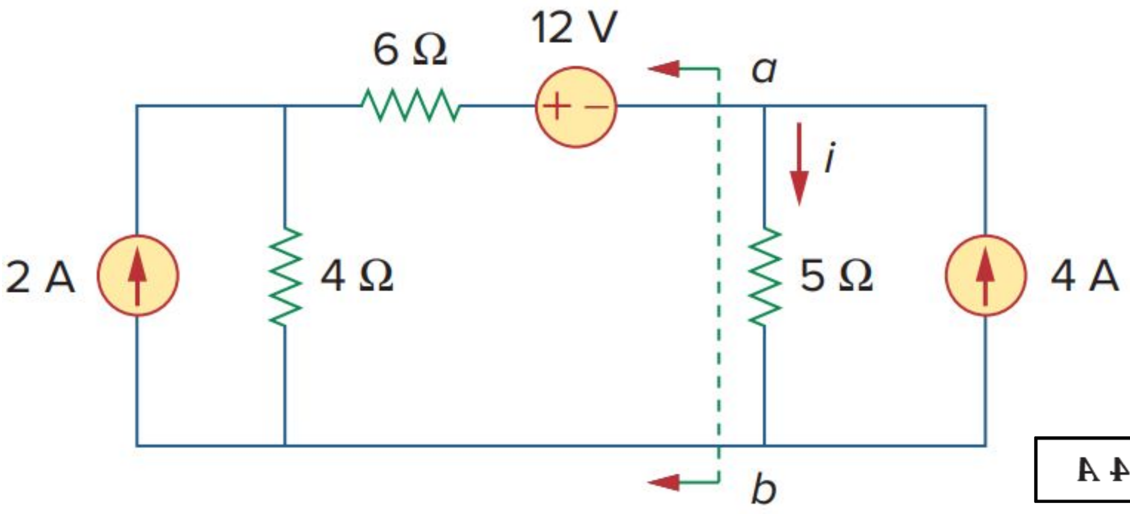
$R_{N\,c,d} = 6 \| 2 = 1.5\Omega$

R_N c,d



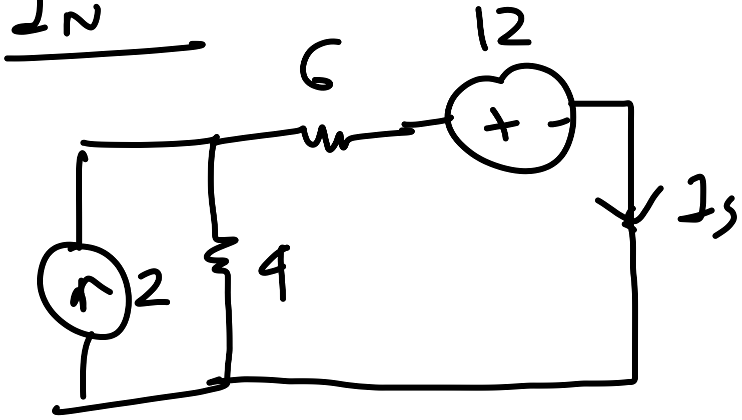
Problem 13

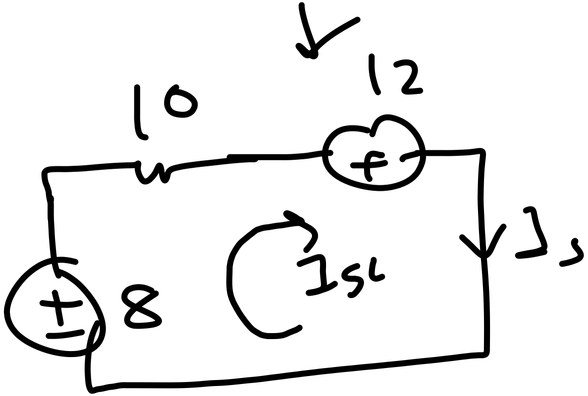
- Obtain the Thevenin/Norton equivalent of the circuit to the left of terminals $a - b$. Use the result to find current i.



Ans: $I_N = -0.4$ A; $R_N = 10\ \Omega$; $i = 2.4$ A

I_N

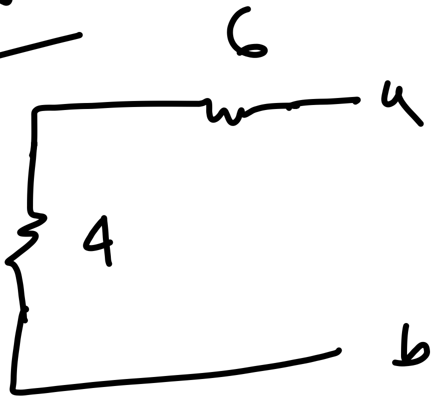




So, $I_{sc} = \frac{8-12}{10}$

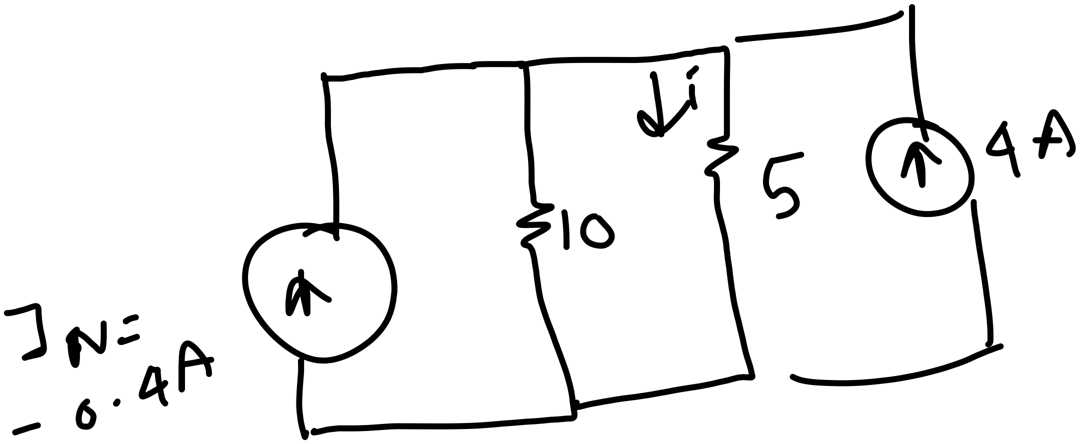
$= -0.4A$

R_N



$R_{ab} = 10\Omega$

$R_N = 10\Omega$

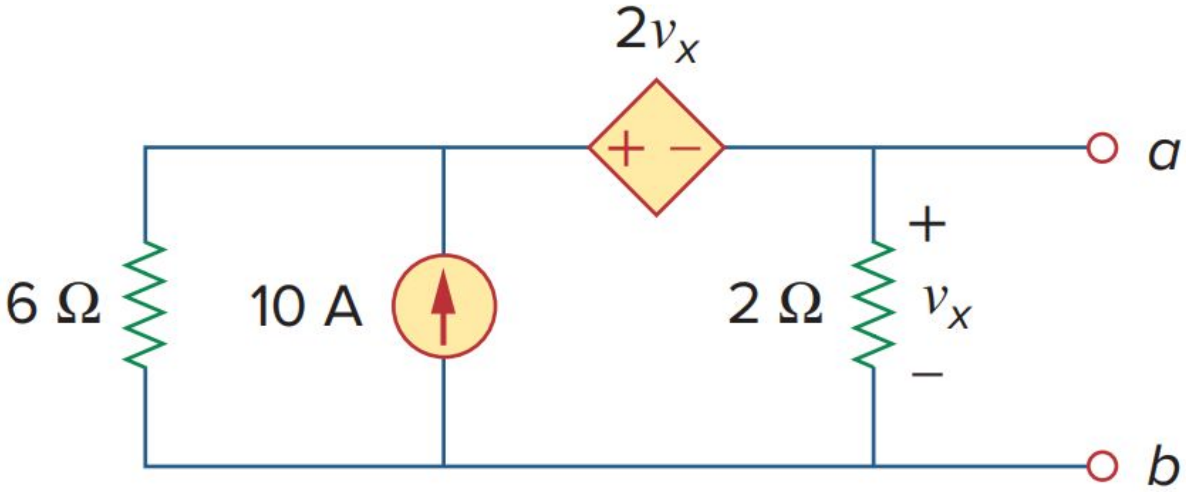


Sol $\quad i = \dfrac{5^{-1}}{10^{-1} + 5^{-1}} (4 - 0.4)$

$= 2.4A$

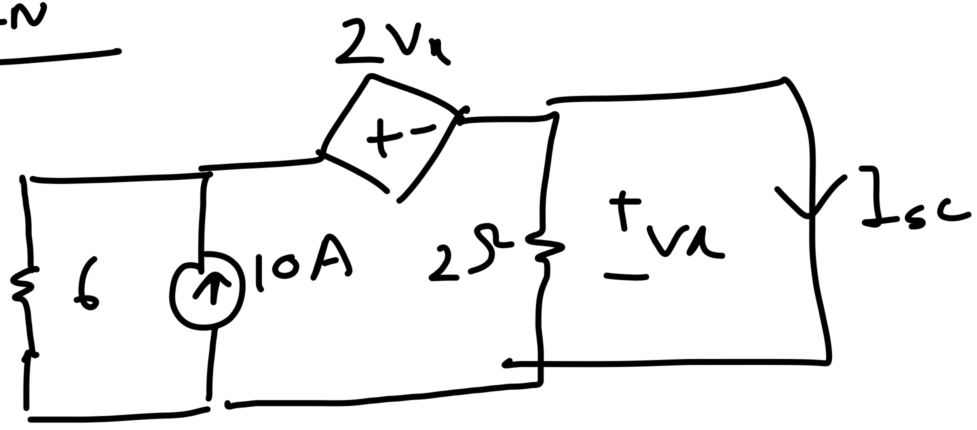
Problem 14

- Find the Norton equivalent circuit for the circuit at terminals $a - b$.



Ans: $\boldsymbol{I_N = 10\ A}$; $\boldsymbol{R_N = 1\ \Omega}$

I_N



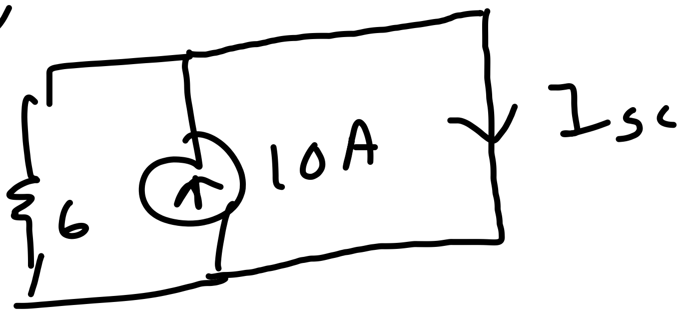
$I_{sc} = 10A$

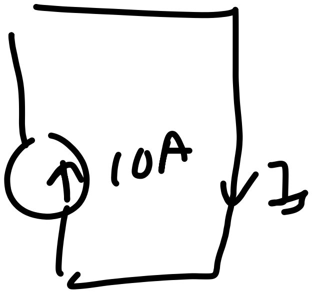
$I_N = 10A$

$2\Omega \rightarrow S.C$

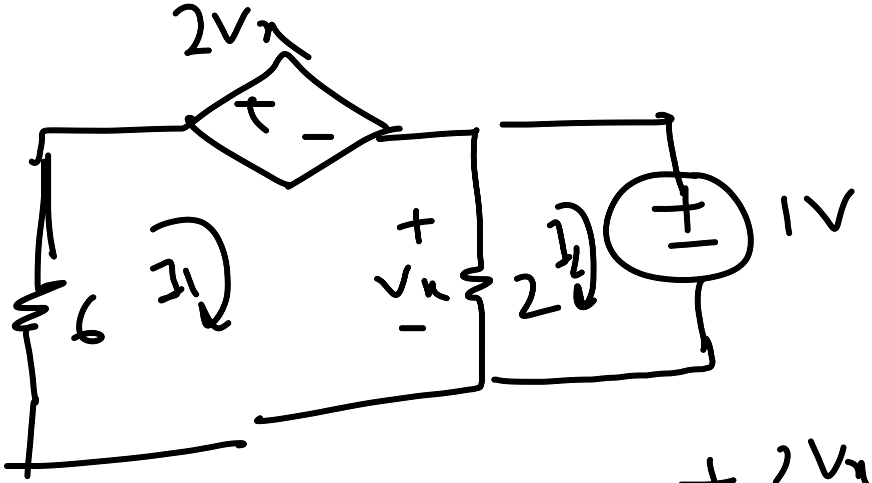
So $V_x = 0$

$2V_x = 0$





RN



$$V_2 = 2(I_1 - I_2)$$

L-1

$$8I_1 - 2I_2 + 2V_x = 0$$

$$8I_1 - 2I_2 + 4I_1 - 4I_2 = 0$$

$$I_1 = -\frac{1}{2}$$

$$I_2 = -1$$

L-2

$$2I_2 - 2I_1 = -1$$

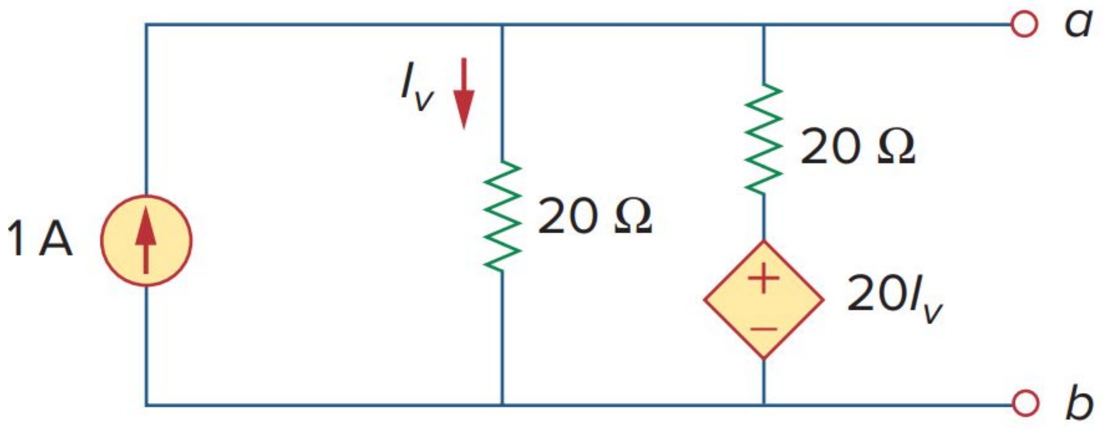
Sol

$$R_{TH} = -\frac{1}{I_2}$$

$$= -\frac{1}{-1} = 1\,\Omega$$

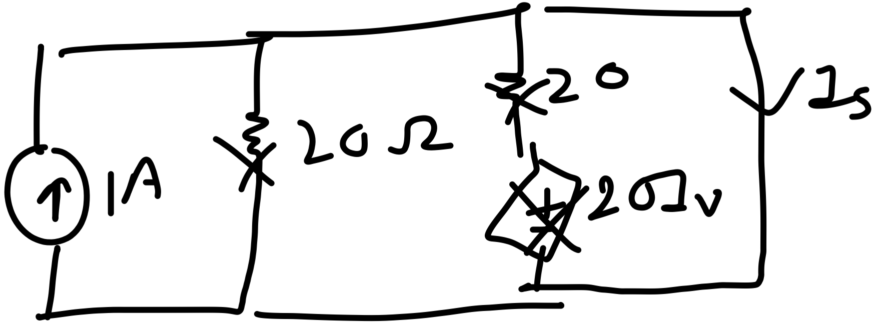
Problem 15

- Obtain the Norton equivalent circuit with respect to terminals a and b.



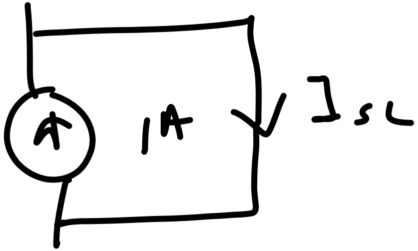
Ans: $\boldsymbol{I_N = 1\ A;\ R_N = 20\ \Omega}$

$\underline{I_N}$



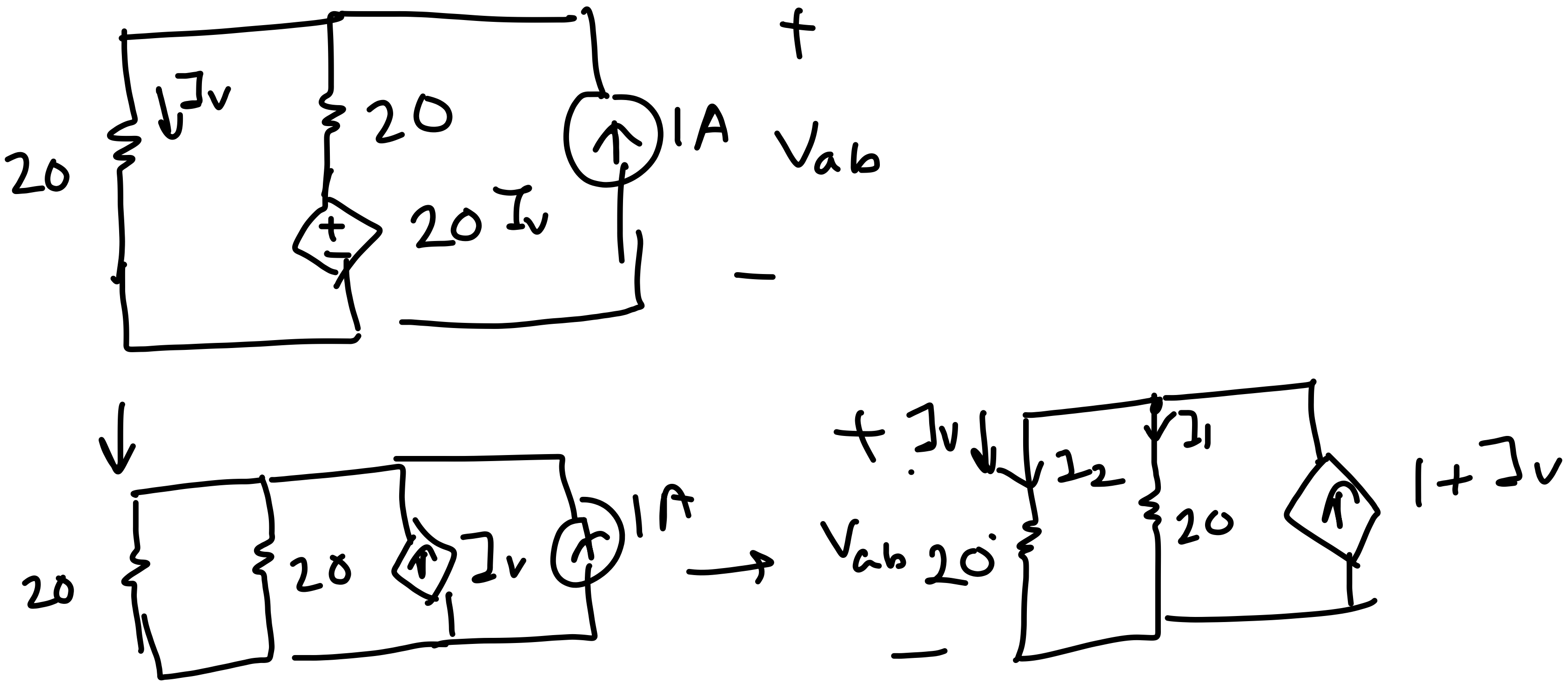
X → short

↓



$\therefore I_{sc} = 1A$

R_N



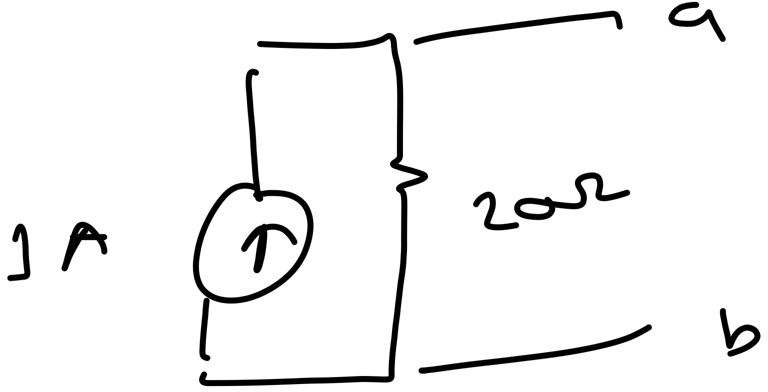
$1 + I_V = I_1 + I_2$

$1 + I_V = I_1 + I_V$

$I_1 = 1A$

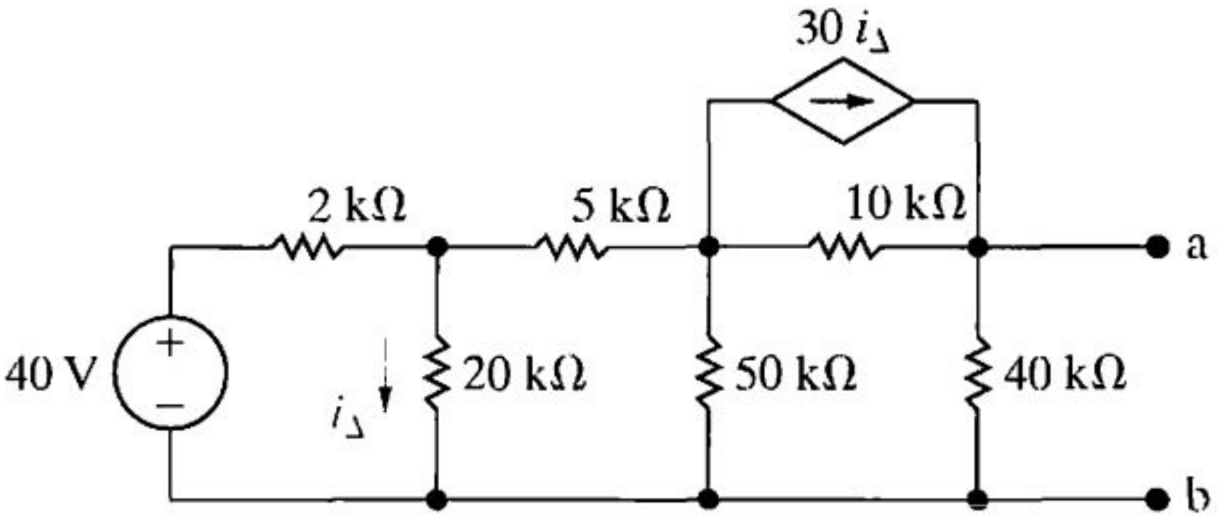
$V_{ab} = 20V$

So, $R_N = \frac{20}{1} = 20\Omega$

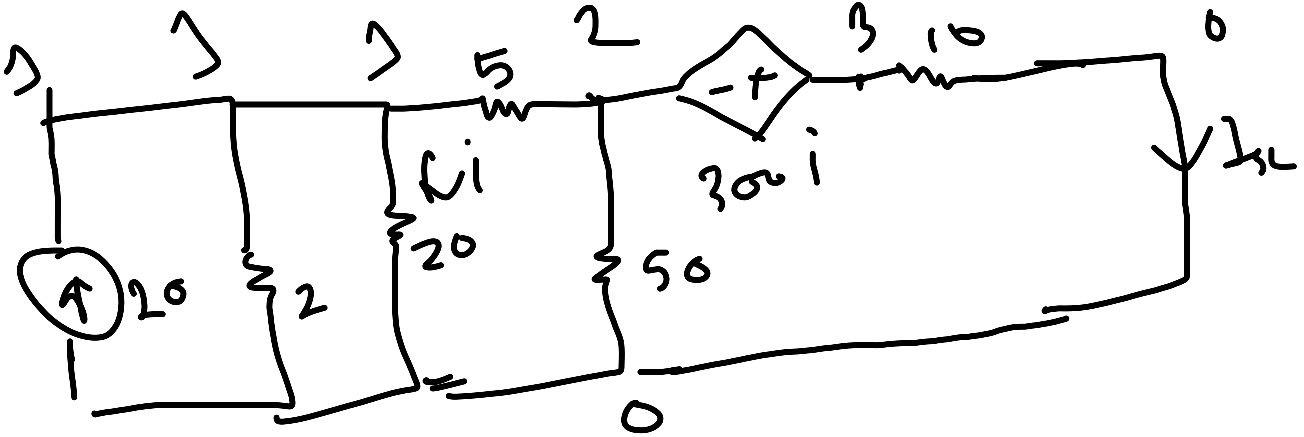


Problem 16

- Obtain the Norton equivalent circuit with respect to terminals a and b.



Ans: $I_N = 14\ A;\ R_N = 20\ k\Omega$



At V_1 $\quad V_1\left(\frac{1}{2}+\frac{1}{20}+\frac{1}{5}\right)-\frac{V_2}{5}=20$ — (I)

At V_2, V_3 $\quad V_3-V_2=300i$ $\quad \therefore -15V_1-V_2+V_3=0$ — (II)

$V_3-V_2=300\frac{V_1}{20}$

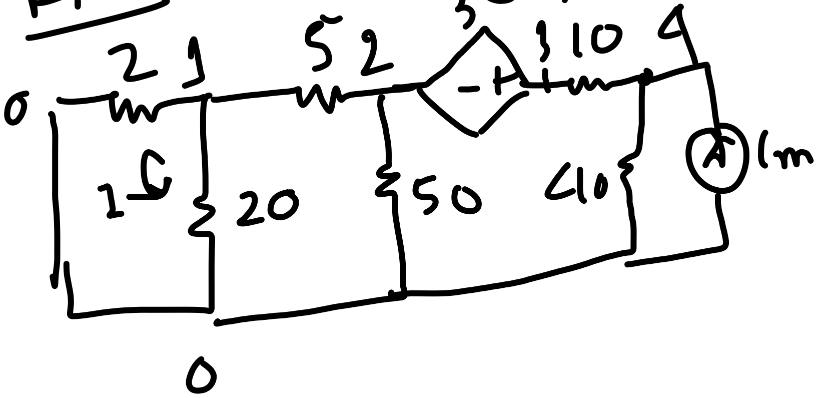
At 2,3

$$V_2\left(\frac{1}{5}+\frac{1}{50}\right)-\frac{V_1}{5}+\frac{V_3}{10}=0$$

$$V_1=\frac{64}{5},\ V_2=-52,\ V_3=140$$

$$\therefore I_{sc}=\frac{V_3-V_0}{10}=14A$$

R_N



$$V_1\left(\frac{1}{2}+\frac{1}{20}+\frac{1}{5}\right)-\frac{V_2}{5}=0$$

$$V_3-V_2=300i$$

$$V_3-V_2=300\times\frac{V_1}{20}$$

$$-15V_1+V_3-V_2=0$$

$$V_4\left(\frac{1}{40}+\frac{1}{10}\right)-\frac{V_3}{10}=1$$

So, $R_N=\frac{V_4}{1}$

$=20k\Omega$

2,3

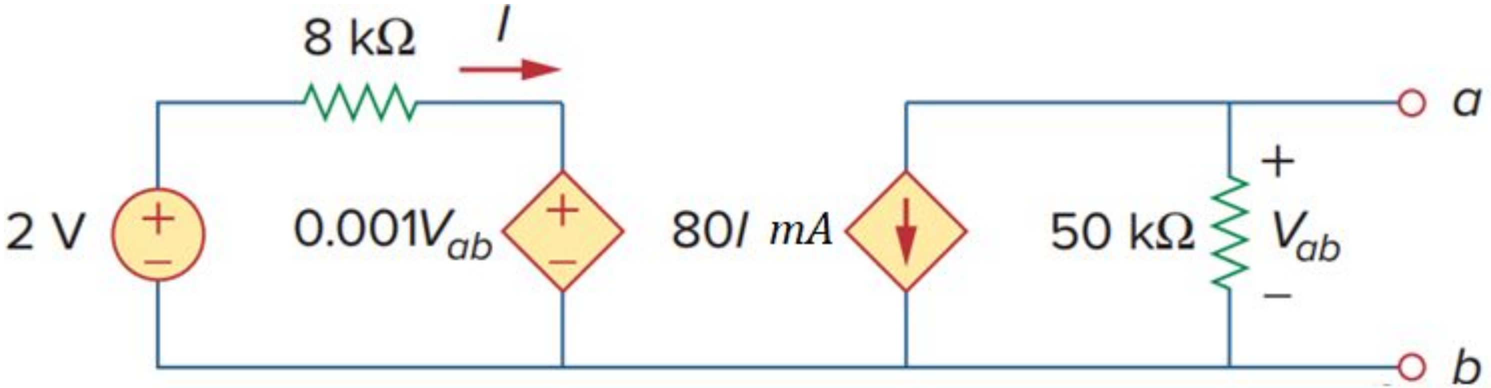
$$v_2\left(\frac{1}{50}+\frac{1}{5}\right)-\frac{V_1}{5}+\frac{V_3}{10}-\frac{V_4}{10}=0$$

$V_1 = 4.5$, $V_2=3$, $V_3=15$

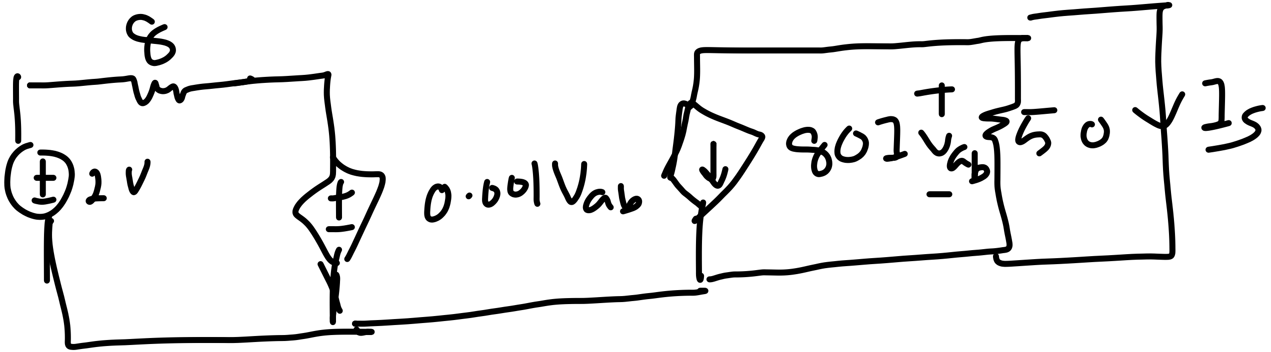
$V_4=20$

Problem 17

- Obtain the Thevenin/Norton equivalent of the circuit to the left of terminals $a - b$.

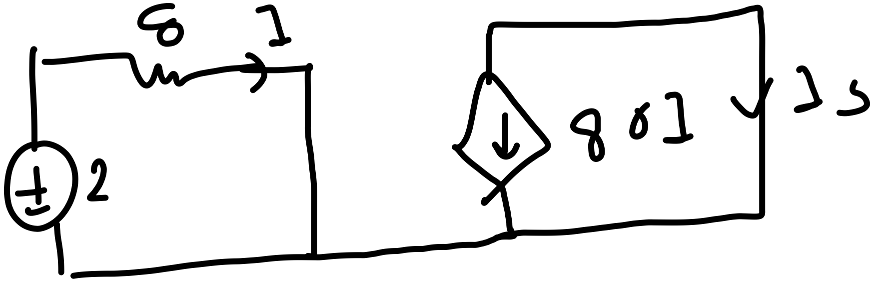


Ans: $V_{Th} = -2000\ V;\ I_N = -20\ mA;\ R_{Th} = R_N = 100\ k\Omega$



$$V_{ab} = 0 \quad \therefore \; 0.001 V_{ab} = 0$$

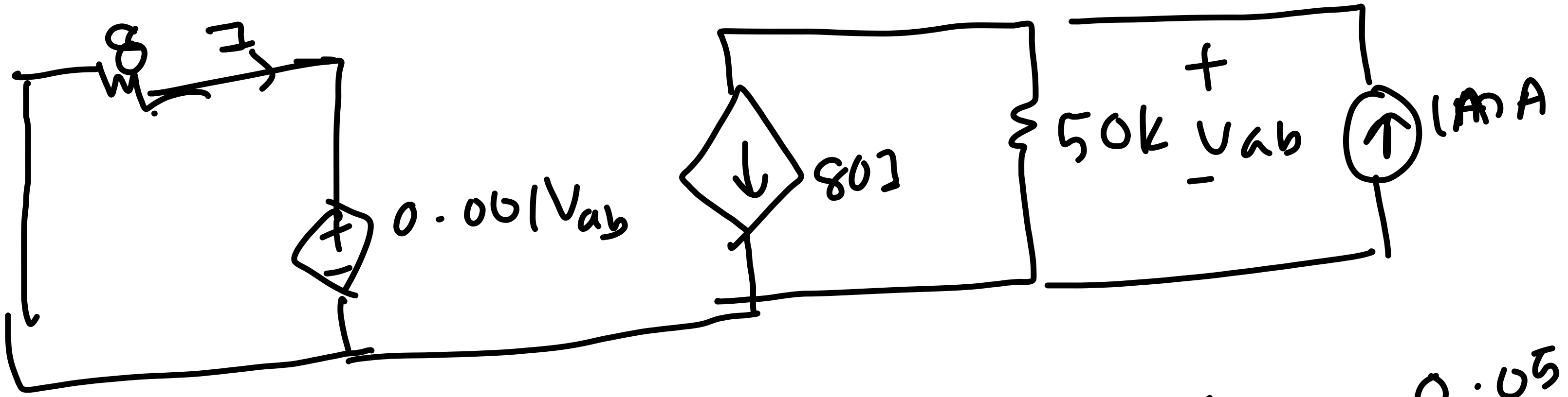
$$I_{sc} = -80I$$



$$= -80 \times \frac{2}{8}$$

$$I_N = -20\,\text{mA}$$

R_N



$$V_{ab} = (1 - 80I) \times 50$$

$$8I = -0.001 V_{ab}$$

$$8I = -0.001 \times 50 \, (1 - 80I)$$

$$8I = -$$

$$8I - 4I = -0.05$$

$$I = -0.0125$$

$\therefore V_{ab} = 100$

So, $R_N = \dfrac{100}{1} = 100k$

$V_{th} = I_{sc} \times R_N = -20 \times 100 = -2000V$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Thank you for your attention