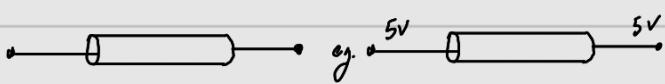


or Wattage

Power  $\rightarrow$  1 WattElectric Potential / Voltage  $\rightarrow$  1 Volt

$I \propto V$

$\Delta V = IR$

Just like force  $\xrightarrow{2W} \rightarrow 5V$   
 $(5-2)N = 3N \rightarrow$   
 Assuming passive element is power to the  
**# Passive sign convention**

• Current passes from +ve to -ve

Power  $\rightarrow +5W \rightarrow$  Consume  
 $\rightarrow -5W \rightarrow$  Supply

Conventional Current direction  
 is from High Voltage to Low voltage

Passive Component  
 (Consumes power)

$\Rightarrow$  Inductor Capacitor  
 Diode, LED

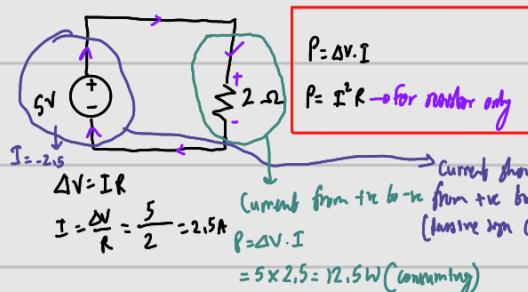
Active Components (Supplies power)

Resistor has no direction.

voltage can be +ve/-ve as charge can be +ve/-ve

Current +ve/-ve means opposite direction.

Power of Resistor is positive as it consumes power



$P = \Delta V \cdot I$

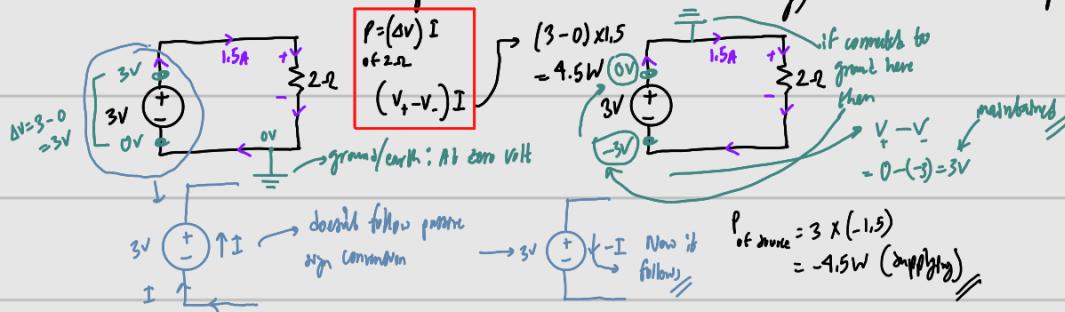
$P = I^2 R \rightarrow$  for Resistor only

~~$P=VI$~~   $P=VI$  works only for passive sign convention.

$$P = -\frac{dV}{dt} I, P = \frac{W}{t}, P = \frac{q \times V}{t}, P = VI, W = qV$$

Q) What do we mean when we want to find power?

= It is not about the power of the Whole circuit. Usually, we talk about the power of an appliance / Component.



$$P = (\Delta V) I$$

$$\text{or } 2 \cdot 2$$

$$(V_+ - V_-) I$$

$$(3-0) \times 1.5$$

$$= 4.5W$$

if connected to ground here

then  $V_+ - V_- = 0 - (-3) = 3V$

$$P_{\text{of source}} = 3 \times (-1.5)$$

$$= -4.5W \text{ (Supplying)}$$

[From L-14]

Q) What is the no. 2 voltage?  
 = no voltage, no current here  
 & no no. 2 voltage

Q) Voltage Source gtrm

$$I = \frac{3}{2+2} = 0.75$$

$$P_1 = (\Delta V) I$$

$$= (3-1.5) \times 0.75$$

$$= +1.125W$$

$$P_2 = (1.5-0) \times 0.75$$

$$= +1.125W$$

$P_3 = 3 \times (-0.75)$  eg Current Source gtrm

$$= -2.25W$$

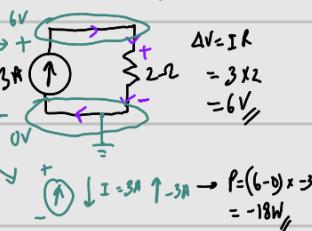
$$\text{OR } P_3 = -(P_1 + P_2)$$

$$= -2.25W$$

$$\Delta V = IR$$

$$= 3 \times 2$$

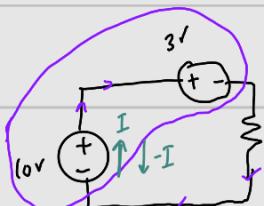
$$= 6V$$



$$I = 3A \uparrow -3A \rightarrow P = (6-0) \times 3 = -18W$$

e.g.  
 Source always work have -ve power  
 $\rightarrow$  less power  
 more power /  
 Consumes Power

# L-4

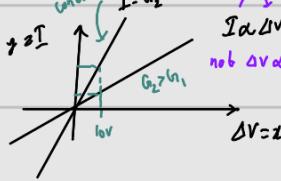


$$\# P_{10V} = (\Delta V)I \\ = 10 \times 1.4 \\ = 14W \text{ (supplying)} //$$

$$\# P_{5\Omega} = (\Delta V)I \\ = (V_+ - V_-)I \\ = (-7 - 0) \times 1.4 \\ = 9.8W \text{ (consuming)} //$$

$$P_{3V} = 19 - 9.8 \quad \text{here voltage source} \\ = +9.2W \quad \text{is considered power}$$

if Ohm's Story  $\rightarrow$  Object  $I$  means intensity  
Wheatstone

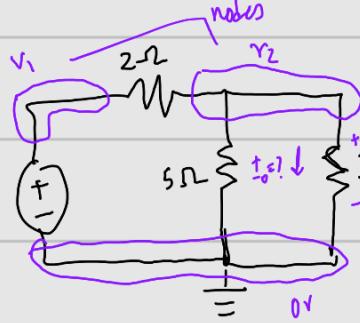


$G = \text{conductance}$   
how much current it allows

$$R = \frac{1}{G}$$

how much current it needs

$$\begin{aligned} & I \propto \Delta V \\ & I = G \Delta V \\ & I = \frac{1}{R} \Delta V \\ & I = \Delta V \end{aligned}$$

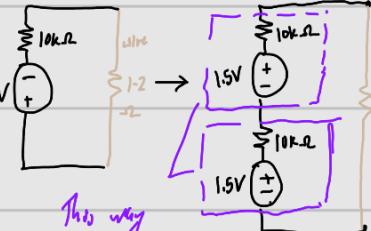


$$\# I = \frac{\Delta V}{R} = \frac{10}{3} \quad (\text{wrong})$$

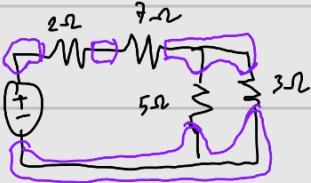
$$\# I = \frac{\Delta V}{R} = \frac{V_2 - 0}{3} = \frac{V_2}{3} //$$

# For Source,  $\Delta V = 10 = V_1 - 0 = 10V$

output Not zero



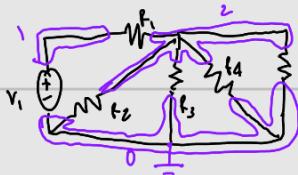
wire has very low resistance. They affect current flow insignificantly. They can be ignored.



⑧ How to Identify nodes?

= Put opposite branches on the terminal of every component. Then connect them without crossing a junction

## Parallel Connection



$R_4 \parallel R_5 ?$   
 $(2,0) = (2,0)$   
same nodes  
Yes //

$R_1 \parallel R_5 ?$   
 $(1,2) \neq (2,0)$   
different nodes  
No //

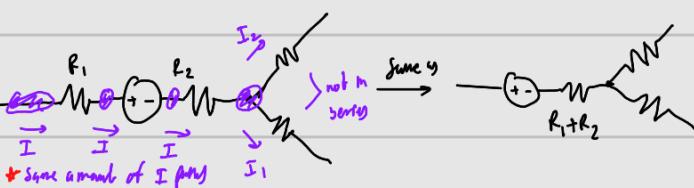
## L-5

### # Series / Parallel Connection Problems

- Swapping
- Equivalent Resistance

$$R_1 = \frac{1}{S_1} \parallel S_2 = \frac{1}{R_1 R_2} \parallel S_1 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

### # Series (Vanilla Water)



### # Voltage Source

$$(series) \quad 2V + 3V = 5V$$

$$\begin{aligned} 2V &= 3V \\ 2V &= -3V \\ 2V + (-3V) &= 1V \end{aligned}$$

### # Current Source

$$2A + 3A = 5A \quad \text{not possible} //$$

$$(parallel) \quad 2A \parallel 3A = 5A \quad (KCL)$$

$$\begin{aligned} 2A &= -3A \\ 2A + (-3A) &= -1A \end{aligned}$$

In reality, the voltage sources have internal resistance which is very small. Thus a lot of current passes. Thus affecting the source //

Not possible

## Kirchoff's Current Law (KCL)

- Applies at a Node / Supernode
- Algebraic Sum
- Currents
- $\sum I = 0$

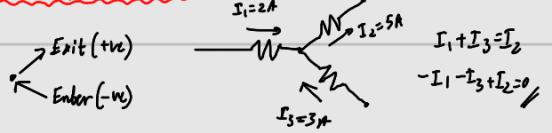
$$\sum I_{\text{enter}} = \sum I_{\text{exit}}$$

$$0 = -\sum I_{\text{enter}} + \sum I_{\text{exit}}$$

$$\sum I_{\text{exit}} + \sum (-I_{\text{enter}}) = 0$$

$$\sum I = 0$$

### # KCL Current Convention



KCL @ Node 1

$$-1.5 - 2 + I_4 = 0$$

$$\therefore I_4 = 2 + 1.5 = 3.5 \text{ A}$$

KCL Node 2

$$-2 + 2 + I_4 = 0$$

$$-3.5 + 2 + I_4 = 0$$

$$I_4 = 1.5 \text{ A}$$

$$-1.5 + I_4 = 0$$

$$I_4 = 1.5 \text{ A}$$

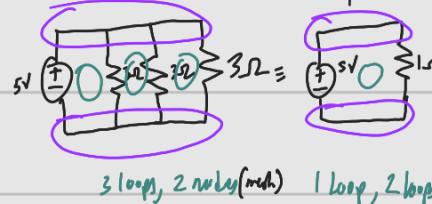
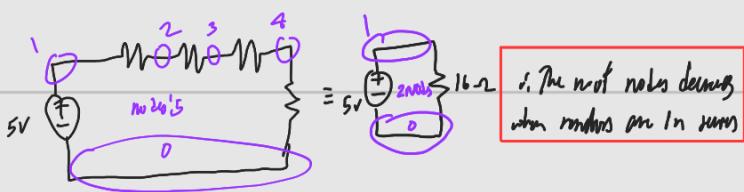
actually what happens:

$$\text{Node 2! } -I_4 + 2 + I_4 = 0$$

$$\text{Node 1! } -1.5 - 2 + I_4 = 0$$

$$-1.5 + I_4 = 0$$

$$I_4 = 1.5 \text{ A}$$



The no. of mesh necessary but nodes remain same in parallel connection of resistors

## KCL

- (1) node / supernode
- (2) Algebraic sum
- (3) Current
- (4) zero

$$\sum I = 0$$

## KVL

- (1) Mesh / Supernode
- (2) Algebraic sum
- (3) Voltage
- (4) zero

$$\sum V = 0$$

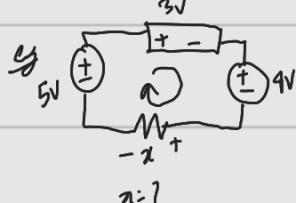
as loops in series



$$V, K\Omega, mA$$

(Algebraic sum)

$$\begin{aligned} & \# \text{ Mesh (Loop)} \quad \# -\Delta V_1 + \Delta V_2 - \Delta V_3 + \Delta V_4 = 0 \\ & \sum (\Delta V) = 0 \quad \Rightarrow -(10) + (5) - (2) + (7) = 0 \\ & \Rightarrow 0 = 0 \end{aligned}$$

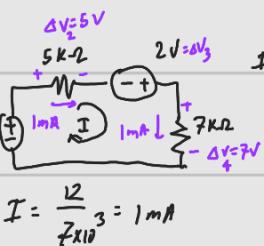


$$+3 + 4 + 7 - 5 = 0$$

$$x = -2 \text{ V}$$

$$x = ?$$

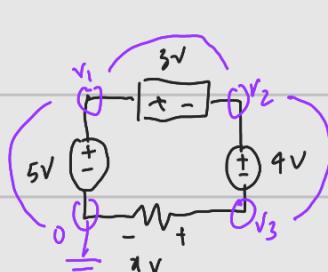
Q2



$$I = \frac{12}{7 \times 10} = 1 \text{ mA}$$

$$\begin{aligned} & \text{KCL: center(-ve), exit(+ve)} \\ & -x + (-3) + 2 - (5) + (-8) = 0 \\ & x = -14 \text{ A} \end{aligned}$$

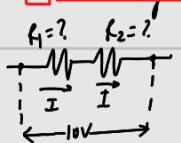
Voltage is of a node [1 node = 1 voltage]  
Voltage difference is b/w two nodes [1 Element (Terminal) = 1 current]  
Current is of an element



$$\begin{aligned} V_1 - 0 &= 5 \\ V_1 - V_2 &= 3 \\ V_2 - V_3 &= 4 \\ V_3 - 0 &= 1 \\ -4 - 3 + 5 - 1 &= 0 \end{aligned}$$

$$\begin{aligned} V_3 - V_2 &= -1 \\ V_2 - V_1 &= -3 \\ V_1 - 0 &= 5 \\ 0 - V_3 &= -1 \\ 0 = -4 - 3 + 5 - 1 \end{aligned}$$

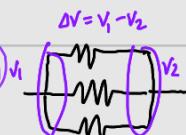
## Voltage Divider Rule (Only applicable for 2 resistors)



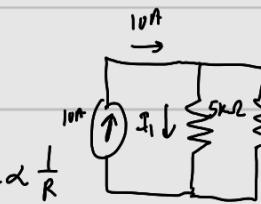
- Series: 1) I same  
2) V divide or R

$$\Delta V_1 = 10V \times \frac{R_1}{R_1+R_2}$$

$$\Delta V_2 = 10V \times \frac{R_2}{R_1+R_2}$$



- Parallel: 1) V same  
2) I divided or  $\frac{1}{R}$

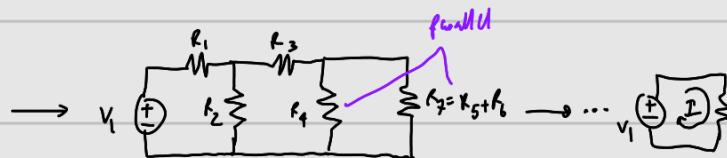
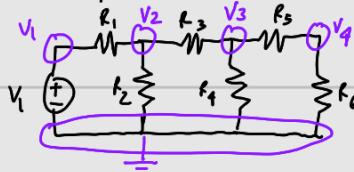


$$I_1 : I_2 = \frac{1}{R_1} : \frac{1}{R_2}$$

$$I_1 = 10 \times \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$I_2 = 10 \times \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

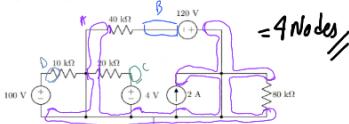
## # Lab Exp 5: Series-parallel



Brac University  
Semester Spring 2022  
Course No: CS2520  
Course Title: CIRCUITS AND ELECTRONICS  
Section: 07 & 08  
Faculty: SHS

Quiz 1  
Full Marks: 20  
Time: 20 minutes  
Date: February 15, 2022

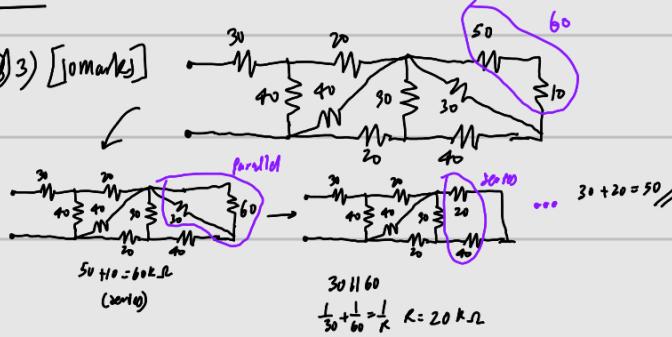
Question 1 [2 marks]



- How many nodes are there in this circuit (including the ground node)? (2 marks)

L-6

Q3 [10 marks]

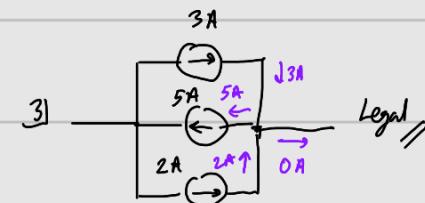


$$(i) P = \Delta V I = V_+ - V_- = V_o - V_i$$

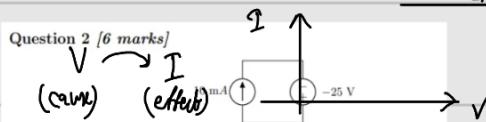
$$= -25 \times 10^{-3} W$$

$$(ii) P = \Delta V I = V_+ - V_- = V_o - V_i$$

$$= (-25) \times (-10 \times 10^{-3}) W$$



## I-V Characteristics



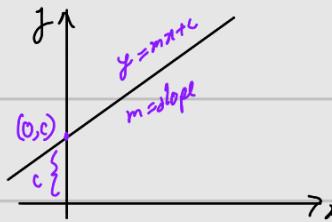
- What is the power of the current source (with appropriate ± sign)? (3 marks)  
• What is the power of the voltage source (with appropriate ± sign)? (3 marks)

We've already seen this in Ohm's Law.  $V = IR$

$$\text{Ohm's Law}$$

$$I = \frac{1}{R} V + 0$$

$$y = mx + c$$



## # Resistor $\rightarrow$ I-V Characteristics

$\hookrightarrow$  ① straight line

$$\textcircled{2} m = \text{slope} = \frac{1}{R}$$

$$\textcircled{3} c = 0 \Rightarrow \text{origin}$$

$$R = 20 \Omega$$

(Insulator)

$$m = \frac{1}{R} = \frac{1}{\lim_{R \rightarrow \infty} R} = 0$$

$I \uparrow$  Insulator

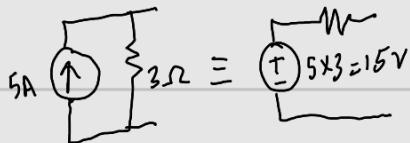
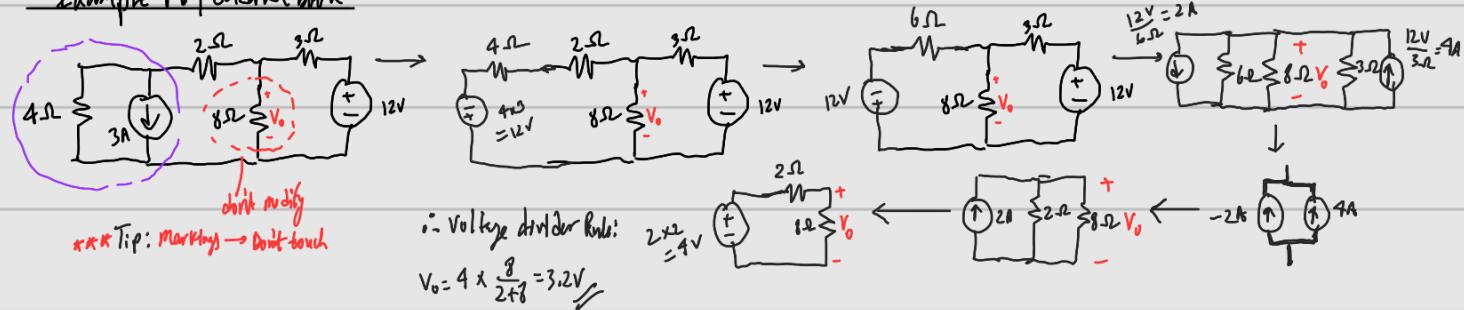
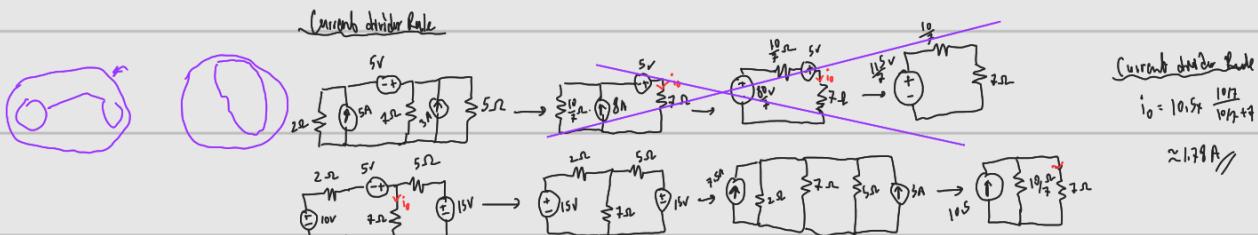
$m=0$   
for insulator  
also Open circuit

$I \uparrow$  Short Circuit

$$R = 0 \Omega$$

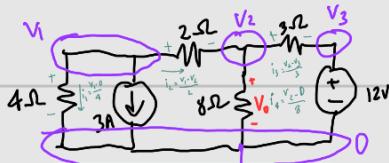
$$m = \frac{1}{R} = \frac{1}{\lim_{R \rightarrow 0} R} = \infty$$



Example 4.6 | Source transformationPractice Problem - 6Drawback of Source Transformation

work only when:

- Current source & resistor parallel
- Voltage source & resistor in series

Nodal Analysis (using KCL)Figure 4.19  
For Practice Prob. 4.6.

$$\text{Node 3: } V_3 \left( \frac{1}{3} \right) - \frac{V_2}{3} + \frac{V_1}{2} - 1 = 0$$

Voltage extra variable  
no need for RREF

Step 1: Node Identification

0 (ground)

$$\text{Step 2: Voltage source eqn: } 4V = V_3 - 0 = 12$$

Step 3: Nodal Currents (KCL)

$$\text{Nod 2: } V_2 \left( \frac{1}{2} + \frac{1}{3} \times \frac{1}{8} \right) - \frac{V_1}{2} - \frac{V_3}{3} - 0 = 0$$

$$\text{Step 4: Node 1} \quad i_1 + i_2 + i_3 = 0$$

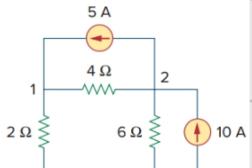
$$\Rightarrow \frac{V_1}{4} + \frac{V_1 - V_2}{2} + 3 = 0$$

$$\Rightarrow V_1 \left( \frac{1}{4} + \frac{1}{2} \right) - \frac{V_2}{2} + 3 = 0$$

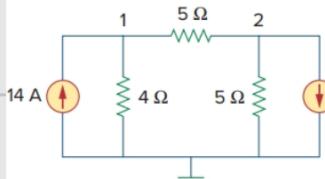
2 eqn, 2 variables

$$\text{Step 5: solve eqn} \quad (\text{no need for RREF})$$

(3.1.1)

Step 3) Identify nodStep 2) Voltage source eqn. Nod 1 to Nod 2

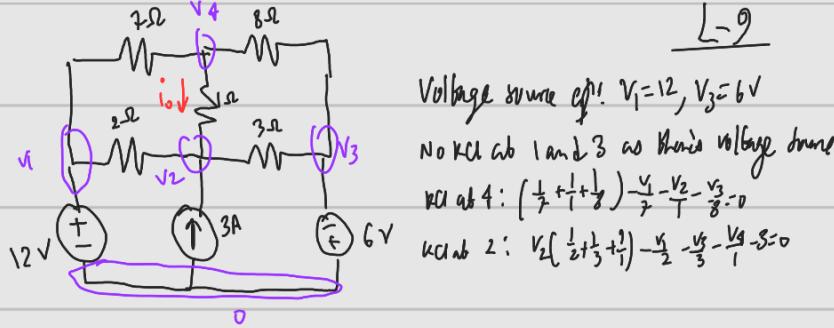
$$\text{Step 3) } \text{Nod 1: } V_1 \left( \frac{1}{2} + \frac{1}{4} \right) - \frac{V_2}{2} - 5 = 0 \quad \text{Nod 2: } V_2 \left( \frac{1}{4} + \frac{1}{6} \right) - \frac{V_1}{4} - \frac{10}{6} + 5 = 0$$



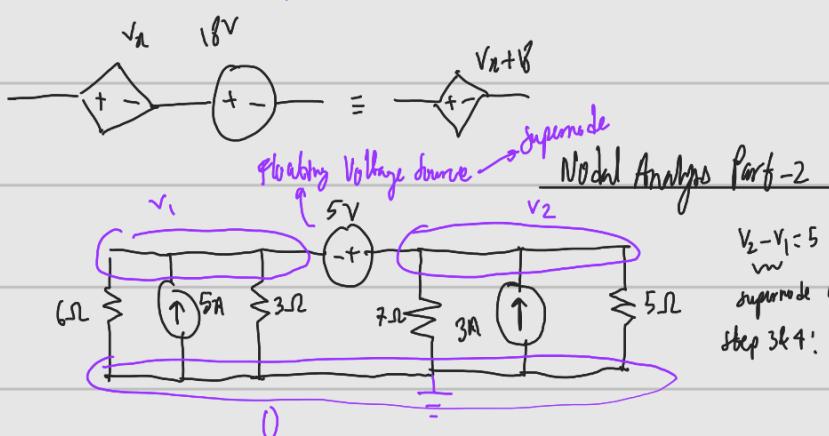
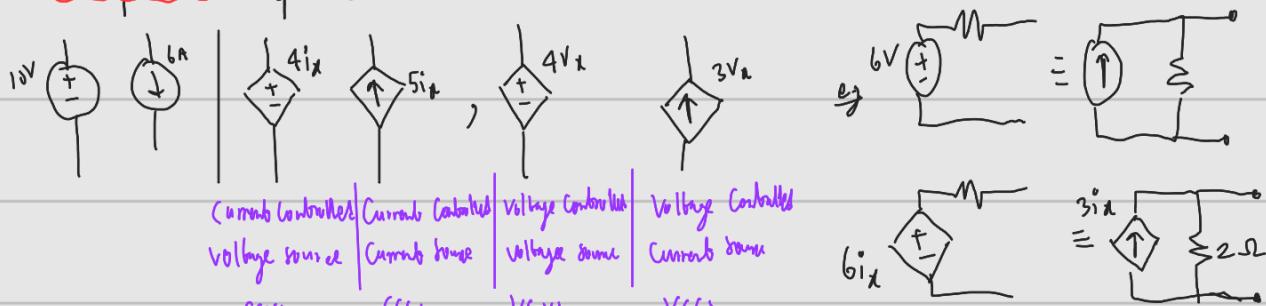
$$\text{Nod 1: } V_1 \left( \frac{1}{5} + \frac{1}{4} \right) - \frac{V_2}{5} - \frac{14}{4} - 19 = 0$$

$$V_2 \left( \frac{1}{4} + \frac{1}{5} \right) - \frac{V_1}{5} - \frac{7}{5} + 7 = 0$$

$$V_1 = 30V, V_2 = -2.5V$$

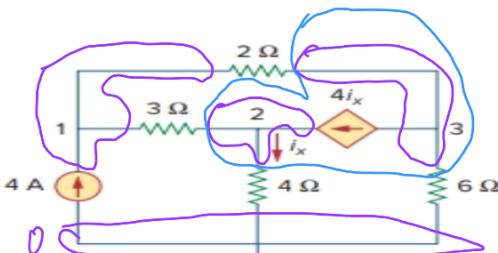


### # Independent Vs Dependent Source



$$\begin{aligned}
 V_2 - V_1 &= 5 \\
 \text{supernode abt } V_1, V_2 & \\
 \text{step 3 & 4: Supernode abt 1,2: } V_1 \left( \frac{1}{6} + \frac{1}{3} \right) - \frac{0}{6} - \frac{0}{3} &\rightarrow V_2 \left( \frac{1}{7} + \frac{1}{5} \right) - \frac{0}{7} - \frac{0}{5} - 5 - 3 = 0 \\
 \text{Node 1 + Node 2} &
 \end{aligned}$$

### Practice Problem 3.2



$$\begin{aligned}
 & \text{wrong. This is a current source not a voltage source!} \\
 & 4i_x = V_3 - V_2 \\
 & V_2 - 0 = i_{in} \times 4 \Rightarrow V_2 - 4i_x = V_3 - V_2 \therefore 2V_2 = V_3 \\
 & \text{KCL abt 1: } V_1 \left( \frac{1}{2} + \frac{1}{3} \right) - \frac{V_1}{2} - \frac{0}{3} - 4i_x \\
 & \text{KCL abt Supernode 2+3: } \left\{ V_2 \left( \frac{1}{3} + \frac{1}{4} \right) - \frac{V_1}{3} - \frac{0}{4} \right\} + \left\{ V_3 \left( \frac{1}{2} + \frac{1}{6} \right) - \frac{V_1}{2} - \frac{0}{6} \right\} = 0 \\
 & = V_2 \left( \frac{1}{3} + \frac{1}{4} \right) + V_1 \left( -\frac{1}{3} - \frac{1}{4} \right) + V_3 \left( \frac{1}{2} + \frac{1}{6} \right) = 0
 \end{aligned}$$

$$\begin{aligned}
 V_2 \left( \frac{1}{3} + \frac{1}{4} \right) - \frac{V_1}{3} - \frac{0}{4} - 4i_x &= 0 \\
 V_3 \left( \frac{1}{2} + \frac{1}{6} \right) - \frac{V_1}{2} - \frac{0}{6} + 4i_x &= 0 \\
 \therefore V_1 = 32, V_2 = -25.6, V_3 = 62.4 &
 \end{aligned}$$

No need for hyper node.  
 as there is no voltage source.  
 & it's dependent current source!

### Example 3.3

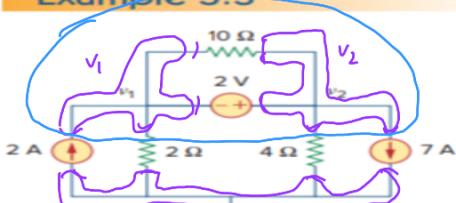
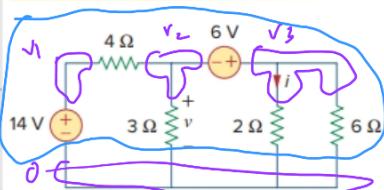


Figure 3.9  
For Example 3.3.

$$\begin{aligned}
 V_2 - V_1 &= 2 \\
 (1+2) \left\{ V_1 \left( \frac{1}{10} + \frac{1}{2} \right) - \frac{V_2}{10} - \frac{0}{2} - 2 \right\} + \left\{ V_2 \left( \frac{1}{10} + \frac{1}{4} \right) - \frac{V_1}{10} - \frac{0}{4} + 7 \right\} &= 0 \\
 V_1 \left( \frac{1}{10} + \frac{1}{2} - \frac{1}{10} \right) + V_2 \left( -\frac{1}{10} + \frac{1}{10} + \frac{1}{4} \right) + 5 &= 0 \\
 V_1 = -2.33V & \quad V_2 = 5.33V
 \end{aligned}$$

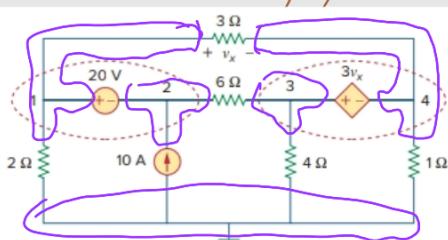
### Practice Problem 3.3



**Figure 3.11**  
For Practice Prob. 3.3.

(a) Find  $v$  and  $i$ .

$$\text{Ans! } -900 \text{ mV}, 2.9 \text{ A}$$



$$\underline{\text{L-10}}$$

### Mesh Analysis

#### Nodal Analysis

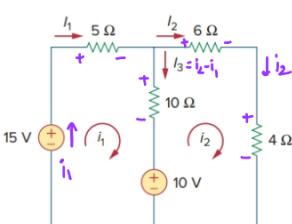
- ①  $V_1, V_2, V_3, \dots$  (Node voltage)
- ② KCL eq<sup>n</sup>
- ③ Step 2: Voltage source
- ④ Supernode [if there is/are floating voltage source]

#### Mesh Analysis

- ①  $i_1, i_2, i_3, \dots$  (mesh current)
- ② KVL eq<sup>n</sup>
- ③ Step 2: Current source
- ④ Supernode [if there is/are common current source]

Take Mesh (Loop) direction clockwise always

### Example 3.5



**Figure 3.18**  
For Example 3.5.

$$\text{Mesh-1: } -15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$\Rightarrow i_1(5+10) - 10i_2 - 15 + 10 = 0$$

$$\text{Mesh-2: } -10 - 10(i_1 - i_2) + 6i_2 + 4i_2 = 0$$

$$i_2(10+6+4) - 10i_1 - 10 = 0$$

$$i_1 = \square, i_2 = \square$$

Step wise!

Step 1: Mesh identification

Step 2: Current source eq<sup>n</sup>

Step 3/4: KVL at mesh

$$\text{Mesh-1: } i_1(5+10) - 10(i_2) - 15 + 10 = 0$$

$$\text{Mesh-2: } i_2(6+4+10) - 10i_1 - 10 = 0$$

Step 5: solve

$$\text{Mesh-1: } i_1(6+10+4) - 6 \times 0 - 10 \times (-i_2) - 9 \times 0 - 10 = 0 \Rightarrow i_1(20) - 10i_2 = 90$$

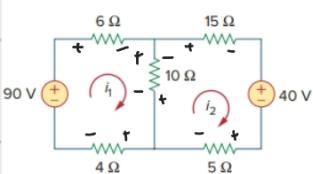
$$\text{Mesh-2: } i_2(15+5+10) - 15 \times 0 - 5 \times 0 - 10 \times (\cancel{i_1}) + 10 = 0 \Rightarrow i_2(30) - 10i_1 = -40$$

$$i_1 = 6.2 \text{ A}, i_2 = -3.4 \text{ A}$$

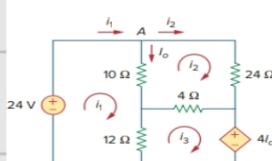
$$i_1 = 4.6 \text{ A}, i_2 = 0.2 \text{ A}$$

$$i_0 = i_3$$

### Practice Problem 3.5



#### Example 3.6



Step 1: Mesh Identification

Step 2: Current source eq<sup>n</sup>  $i_3 = 4i_0$

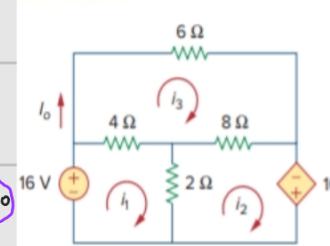
Step 3/4: Mesh 1:  $i_1(10+i_2) - 10 \times i_2 - 12 \times i_3 - 24 = 0$

Mesh 2:  $i_2(24+4+i_0) - 24 \times i_0 - 4 \times i_3 - 10 \times i_1 = 0$

Mesh 3:  $i_3(4+i_1) - 4 \times i_2 - 12 \times i_1 + 4i_0 = 0$

3eq<sup>n</sup>, 3 variables

### Practice Problem 3.6



**Figure 3.21**  
For Practice Prob. 3.6.

$$i_1(4+i_2) - 4(i_1) - 2(i_2) - 16 = 0$$

$$i_2(2+i_3) - 2(i_1) - 8(i_3) - 10i_0 = 0$$

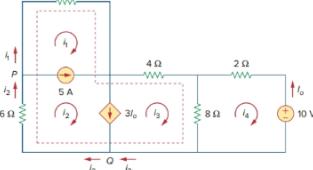
$$i_3(6+4+i_2) - 4(i_1) - 8(i_2) = 0$$

$$i_1 = -2.57, i_2 = -7.77$$

$$i_3 = -4.4 = i_0$$

for the circuit in Fig. 3.24, find  $i_1$  to  $i_4$  using mesh analysis.

Example  
3.7



Step 2:  $i_2 - i_1 = 5$

$$i_2 - i_3 = 3i_0$$

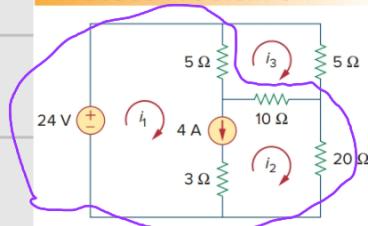
$$i_0 = -i_4$$

Step 3 & 4:

$$i_4(2+8) - 2(5) - 8(i_3) + 10 = 0$$

$$\text{Super node 1,2,3: } \{i_1(2) - 0\} + \{i_2(6)\} + \{i_3(4+8) - 4 \times 2 - 8 \times i_4\} = 0$$

### Practice Problem 3.7



$$\text{Ans! } i_1 = 12.379, i_2 = 5.69 \text{ mA}, i_3 = 3.284 \text{ A}$$

(Wrong ans in book)

$$i_1 - i_2 = 9 \Rightarrow i_1 = 4 + i_2$$

$$\text{mesh 3: } i_3(5+10+3) - 5i_1 - 5 \times 0 - 10i_2 = 0$$

$$\Rightarrow i_3(20) - 5(4+i_2) - 10i_2 = 0 \Rightarrow -15i_2 + i_3(20) = 20 \quad (1)$$

$$\text{Super node 1,2: } \{i_1(5+3) - 5i_3 - 3i_2 - 24\} + \{i_2(10+20+3) - 10i_3 - 20(0) - 3(i_1)\} = 0$$

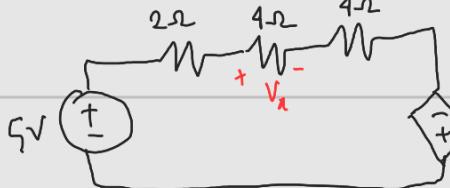
$$i_1(5) + i_2(30) + i_3(-15) = 24$$

$$5(4+i_2) + 30i_2 - 15i_3 = 24 \Rightarrow 35i_2 - 15i_3 = 24 \quad (2)$$

$$i_1 = 9.8, i_2 = 0.6, i_3 = 1.6 \quad //$$

26/11/2024

Tuesday



Mid Review

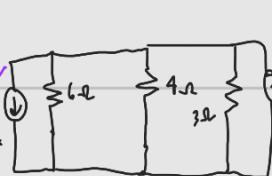


$$\# V_R = 4i_1 \quad \# i_1(4+6) - (5+6V_x) = 0$$

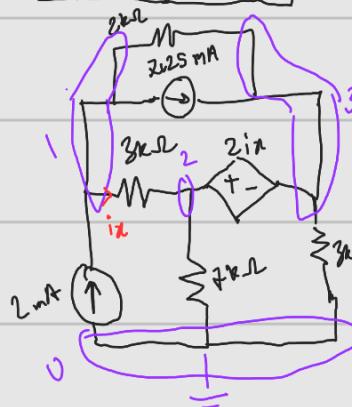
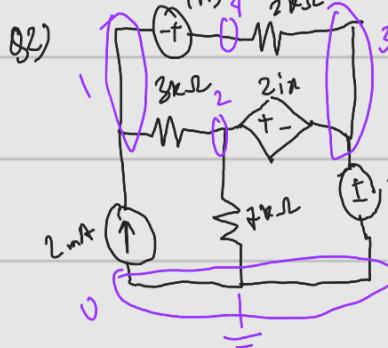
$$\Rightarrow i_1 = \frac{V_R}{4} \Rightarrow \frac{V_R}{4}(10) - (5+6V_x) = 0$$

$$\Rightarrow -3.5V_x - 5 = 0$$

$$\therefore V_x = \frac{5}{3.5} = -\frac{10}{7} \approx -1.43 \text{ V}$$



$$\begin{aligned} & (\text{current divider rule}) \\ & i_1 = 1 \times \frac{2}{2+4} = \frac{1}{3} \approx 0.33 \text{ A} \\ & \text{OR } (1-i_1)x = i_1 \times 2 \Rightarrow i_1 = \frac{1}{3} \\ & 1 - i_1 = 2i_1 \\ & 1 = 3i_1 \end{aligned}$$



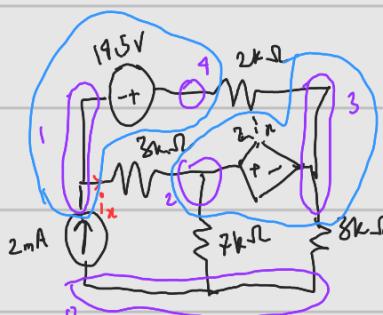
$$\# i_1 = V_1 - \frac{V_2}{3}$$

$$(\text{mA})$$

$$\# V_2 - V_3 = 2i_2$$

$$\# V_1 \left( \frac{1}{3} + \frac{1}{2} \right) - \frac{V_2}{3} - \frac{V_3}{2} + 7.25 - 2 = 0$$

$$\# \left\{ V_2 \left( \frac{1}{3} + \frac{1}{2} \right) - \frac{V_1}{3} - \frac{0}{2} \right\} + \left\{ V_3 \left( \frac{1}{2} + \frac{1}{3} \right) - \frac{V_1}{2} - \frac{0}{3} - 7.25 \right\} = 0$$

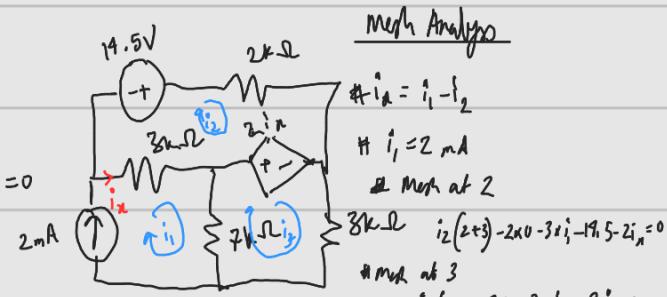


Nodal Analysis

$$\# i_2 = \frac{V_1 - V_2}{3}$$

$$\# 2i_1 = V_2 - V_3$$

$$\# \text{Super node at 2+3}$$



Mesh Analysis

$$\# i_1 = i_1 - i_2$$

$$\# i_1 = 2 \text{ mA}$$

# Mesh at 2

$$\# 8k\Omega \quad i_2(2+3) - 2 \times 0 - 3k(i_1) - 19.5 - 2i_1 = 0$$

# Mesh at 3

$$i_3(3+2) - 3k0 - 7 \times \frac{1}{2} + 2i_2 = 0$$

Note: No node for hyper mesh as there is no common current source



$$\# i_3 - i_1 = 2 \text{ mA}$$

$$\# i_2(2+3) - 2 \times 0 - 3k(i_1) - 14.5 - 2i_1 = 0$$

# Super node at 1+3!

$$\# \{i_1(2+3) - 2 \times 0 - 3k \frac{i_2}{2}\} + \{i_3(3) - 3k0 + 2i_2\} = 0$$

### Practice Problem 3.7

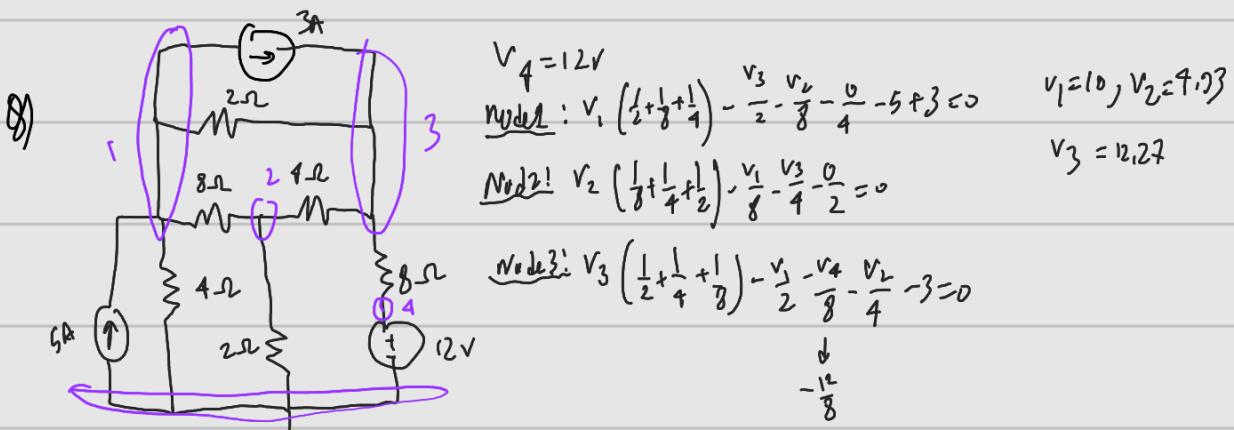
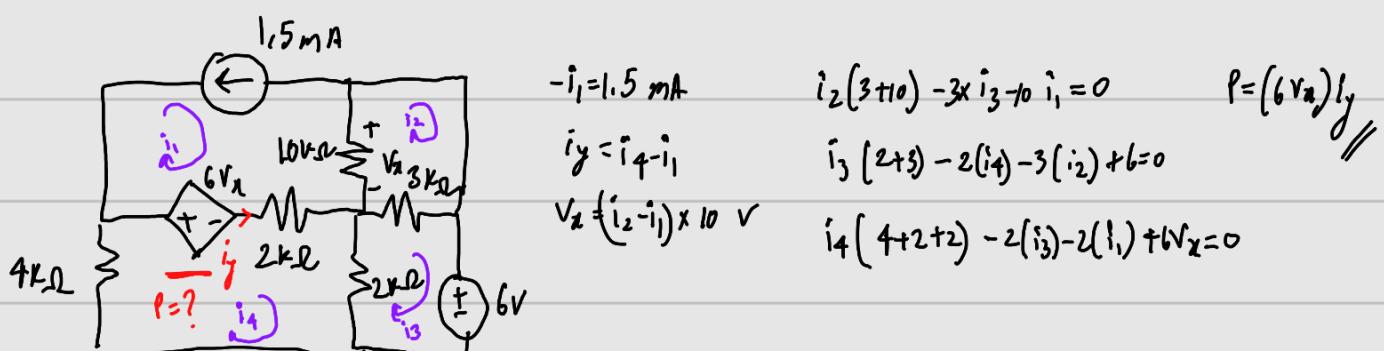
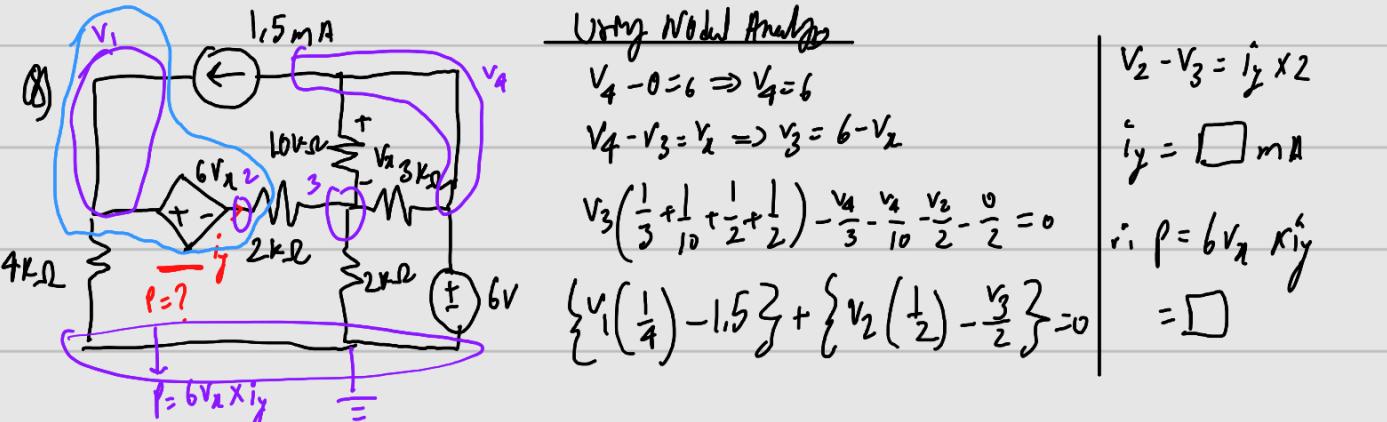


$$\# i_1 - i_2 = 9$$

$$\# i_3(5+5+10) - 5 \times 0.5 \times i_1 - 10i_2 = 0$$

$$\# \{i_1(5+3) - 5k i_3 - 3k i_2 - 2i_1\} = 0$$

$$\# \{i_2(3+10+2) - 3i_1 - 10i_2\} = 0$$



L-1

### Practical Problems

Problem 1) (i)  $1e^- \rightarrow -1.602 \times 10^{-19} \text{ C}$

$$\therefore 1 \text{ C} \rightarrow \frac{1}{1.602 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

(ii) (a)  $1e^- \rightarrow -1.6 \times 10^{-19} \text{ C}$

(iii)  $V = \frac{W}{q} = \frac{1.2}{2 \times 10^{-19}} = 60,000 \text{ V} = 60 \text{ kV}$

$1.24 \times 10^{-18} \rightarrow (1.24 \times 10^{-18} \times -1.6 \times 10^{-19}) = -0.198648 \text{ C} \approx -198.65 \text{ mC}$

(b)  $2.46 \times 10^{-19} \rightarrow 2.46 \times 10^{-19} \times (-1.6 \times 10^{-19}) = -3.9402 \text{ C} \approx -3.94 \text{ C}$

Problem 2) (i)  $W = q(V_B - V_A)$

$$-10 = -2(V_B - 22)$$

Work done by the charge  $\rightarrow V_B = 27 \text{ V}$

$$\therefore W = -4 \times (-5) = 20 \text{ J}$$

### Problem 3

$$V_B - V_A = \frac{W}{q} \rightarrow \therefore W = 0.3 \text{ J}$$

### Problem 4)

### Problem 5)

$$P_1 = VI = 15 \times (-10) = -150 \text{ W} \text{ (Supplying)}$$

$$P_{5V} = 5 \times (-7) = -35 \text{ W} \text{ (Supplying)}$$

$$P_{2V} = 2 \times (-2) = -4 \text{ W} \text{ (Supplying)}$$

$$P_2 = 15 \times 4 = 60 \text{ W}$$

$$P_3 = 9 \times (-6) = 54 \text{ W} \text{ (Absorbing)}$$

$$P_4 = -6 \times (-6) = 36 \text{ W}$$

$$P_{3V} = 3 \times (-3) = -9 \text{ W} \text{ (Supplying)}$$

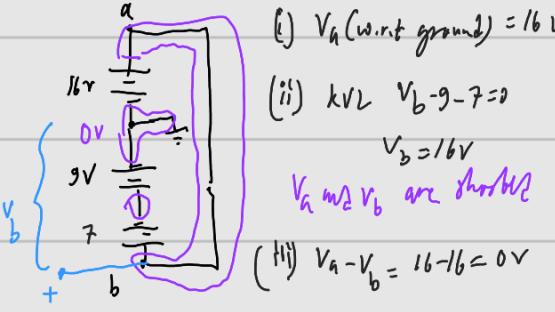
$$P_{4V} = -8 \times (-5) = 40 \text{ W} \text{ (Absorbing)}$$

$$P_{1V} = 1 \times (-4) = -4 \text{ W} \text{ (Supplying)}$$

$$P_{6V} = (-6) \times (-2) = 12 \text{ W} \text{ (Absorbing)}$$

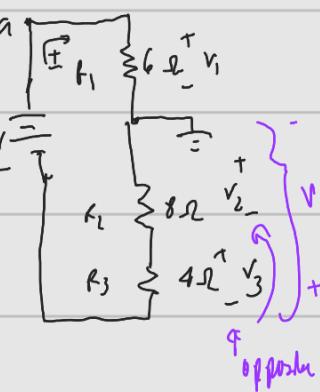


### Problem-1



### L-6

#### Problem-2



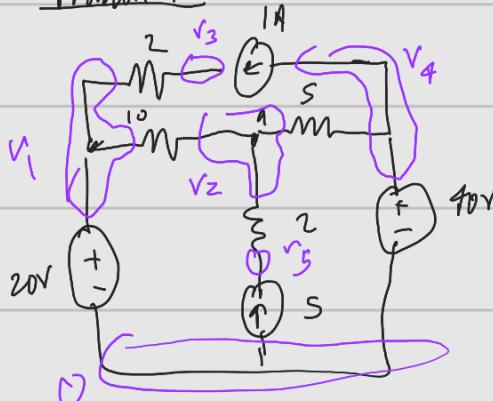
$$(i) V_a = 72 \times \frac{6}{6+8+4} = 24V$$

$$(ii) V_b = -V_2 = -72 \times \frac{8}{8+6+4} = -32V$$

$$V_c = -(V_2 + V_3) - (32 + \frac{72 \times 4}{8+9+4}) = -48V$$

$$V_{ab} = V_c - V_b = 24 - (-32) = 56V$$

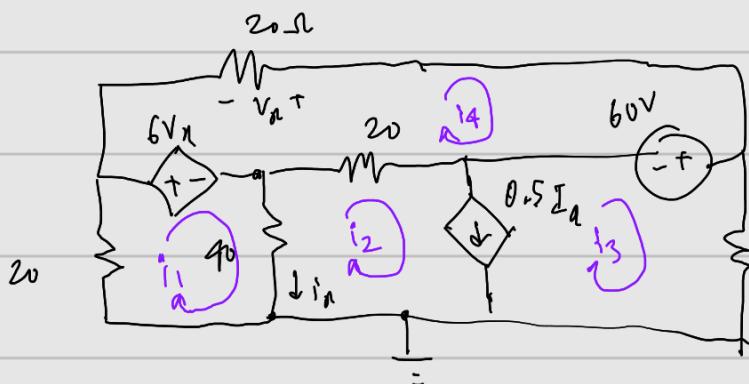
### Problem-4



$$V_1 - 0 = 20 \quad V_3 \left( \frac{1}{2} \right) - \frac{V_1}{2} - 1 = 0$$

$$V_4 - 0 = 40$$

$$V_2 \left( \frac{1}{5} + \frac{1}{10} + \frac{1}{2} \right) - \frac{V_4}{5} - \frac{V_1}{10} - \frac{V_5}{2} = 0$$



$$\# i_2 - i_3 = 0.5 I_1$$

$$\# i_1 - i_2 = i_3$$

$$10 \# V_1 = -i_4 \times 20$$

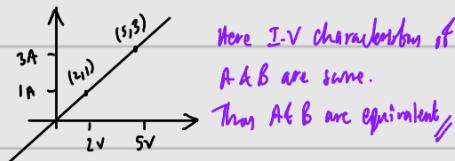
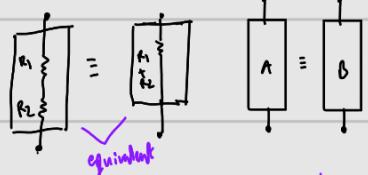
$$\text{mesh ab } i_1: i_1 (20 + 40) - 20(0) - 40(i_2) + 6V_x = 0$$

$$\text{mesh ab } i_4: i_4 (20 + 20) - 20(0) - 20(i_2) + 60 - 6V_x = 0$$

$$\text{super mesh ab } i_2 \& i_3: \left\{ i_2 (40 + 20) - 40(i_1) - 20(i_4) \right\} + \left\{ i_3 (10) - 10(0) - 60 \right\} = 0$$

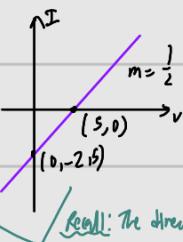
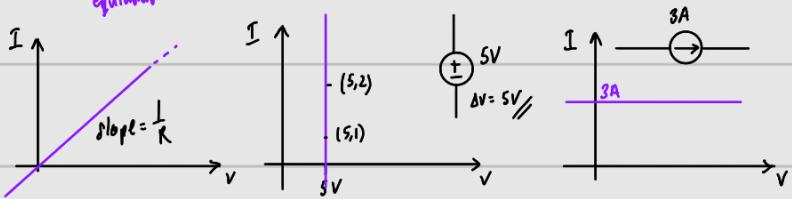
## Equivalent Circuit

L-11

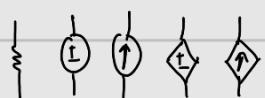


Here I-V characteristics if A & B are same.  
Thus A & B are equivalent.

In this course, we'll deal with  
Linear Devices only.  
They give straight line I-V Characteristics



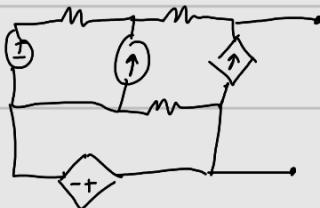
Recall: The direction is opposite  
of positive sign convention.



Dependent sources will also  
give a straight line.  
Not necessarily  
a positive slope  
every time



Logic gates are made  
using non-linear circuits



## Thevenin's Theorem

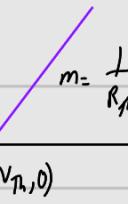
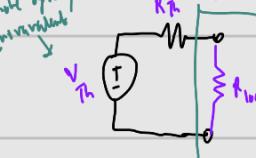
According to Thevenin any complicated (linear) circuit can be reduced and be equivalent to a voltage source and a resistor in series

[Practically]  
Safe Test

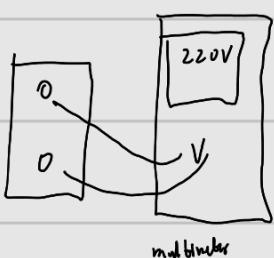


lights up for the moment

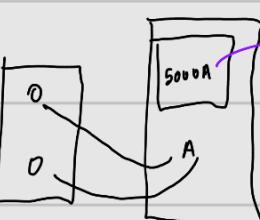
whole system (working)  
equivalent



(Result: Load shedding)  
There is no open circuit current,  
no short circuit voltage



Voltmeter works in open circuit mode

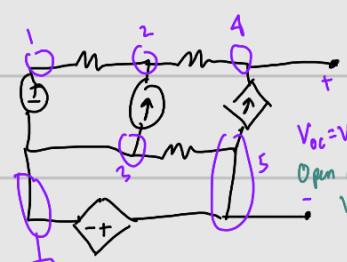


Ammeter works in short circuit mode

This is not the resistance of the hours  
but of the whole system (ex. whole BD)

$$\therefore R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{220V}{5000A} = 0.099\Omega$$

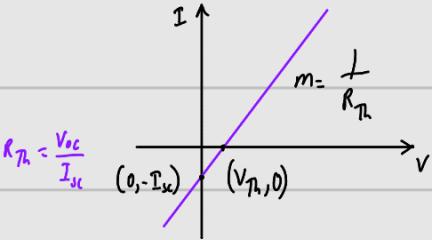
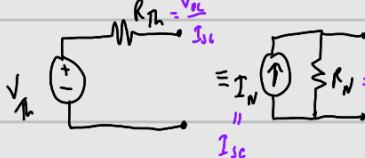
very small



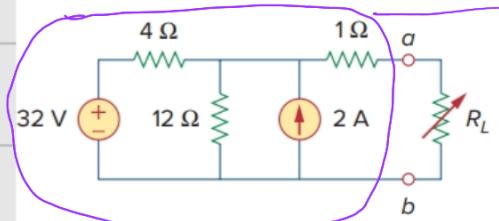
$$V_{Th} = V_{oc}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

## Norton's Theorem



## Example 4.8



Part 1:  $V_{oc}/V_{Th}$

This way we don't need to do mesh analysis every time for new values of  $R_L$ .

$V_2 = V_1$

$V_2 = \frac{1}{4+12+1} - \frac{32}{4} = \frac{V_3}{1} - 8 = V_3$

$\therefore V_{Th} = V_{oc} = V_3 = 50V$

Part 2:  $I_{sc}$

$i_1 = i_2 = 2$

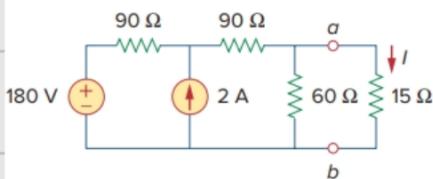
$i_2 + i_3 + i_4 = 2 \Rightarrow i_3 + i_4 = 0$

$i_3 = i_4 = 2.5A$

$i_1 = i_2 + i_3 + i_4 = 2 + 2.5 = 4.5A$

$i_1 = \frac{V_{oc}}{R_{Th}} = \frac{50}{4.5} = 11.1A$

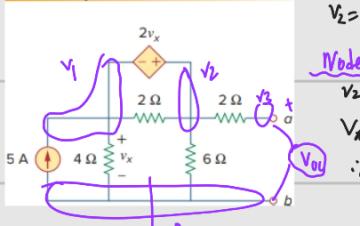
## Practice Problem 4.8



$$V_{Th} = 90V, R_{Th} = 45\Omega$$

$$I = 1.5A$$

## Example 4.9



Part 1:  $V_{oc}$

$$V_2 = V_3$$

Node 1, 2:

$$V_1 \left( \frac{1}{2} + \frac{1}{2} \right) - \frac{6}{2} - \frac{V_3}{2} + V_2 \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) - \frac{V_1}{2} - \frac{V_3}{2} = 0$$

$$V_A = V_1 \quad (\text{Node 1, 2})$$

$$\therefore V_2 - V_1 = 2V_1$$

$$3V_1 = V_2$$

Solving these eqns

$$V_1 = 6.67V$$

$$V_2 = 20V$$

$$V_{oc} = V_3 - 0 = 20V = V_{Th}$$

L-12

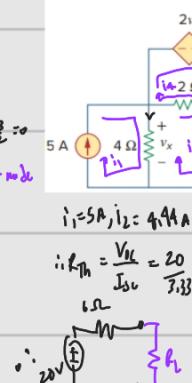
Separate 1/2

$$V_1 \left( \frac{1}{2} + \frac{1}{2} \right) - \frac{6}{2} - \frac{V_3}{2} + V_2 \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) - \frac{V_1}{2} - \frac{V_3}{2} = 0$$

No need As common in super node

$$\text{Node 3} \quad V_3 \left( \frac{1}{2} \right) - \frac{V_1}{2} = 0$$

## Example 4.9



$$I_{sc} = i_3$$

Step 1: 4 nodes. No supernode.

Step 2: mesh 1:  $i_1 = 3$

Step 3/4:

$$Mesh 2: i_2 + (4+2)i_1 - 4i_1 - 2i_2 - 6i_3 = 0$$

$$Mesh 3: i_3 + (6+2) - 6(i_2) = 0$$

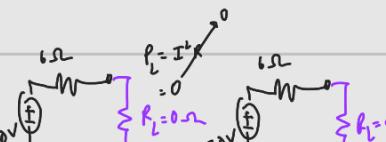
$$Mesh 4: i_4 + (2) - 2(i_2) - 2\sqrt{6}i_2 = 0$$

$$V_{oc} = (i_1 - i_2) \times 9$$

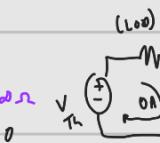
## # Maximum Power Transfer Theorem



happening when  $R_L = R_{Th}$



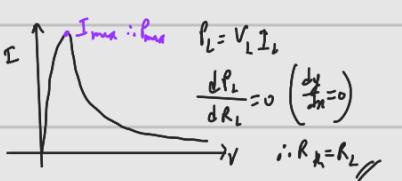
So more currents is somewhere  
In both  $R=0$  &  $R=\infty$   
This is when  $R_L = R_{Th}$



$$P_L = V_L I_L$$

$$V_L = V_{Th} \times \frac{R_L}{R_{Th} + R_L}$$

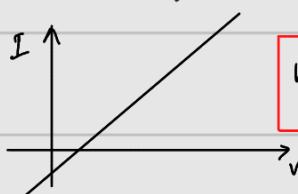
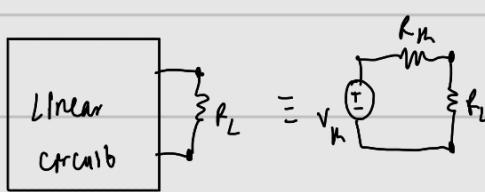
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$



## Thevenin's Theorem (+Norton's Theorem)

We are able to do equivalent circuit of complicated circuit because they are linear and have same characteristic eqn

In real life  $R=0\Omega$  and  $R=-\infty\Omega$  is not possible



$$V_{Th} = V_{oc \text{ open circuit}}, R_{Th} = \frac{V_{oc}}{I_{sc}}$$

Maximum Power Transfer Condition  
 $R_{Th} = R_L$

#  $R_{Th}$  can be negative



$$P = I^2 R = \frac{V^2}{R}$$

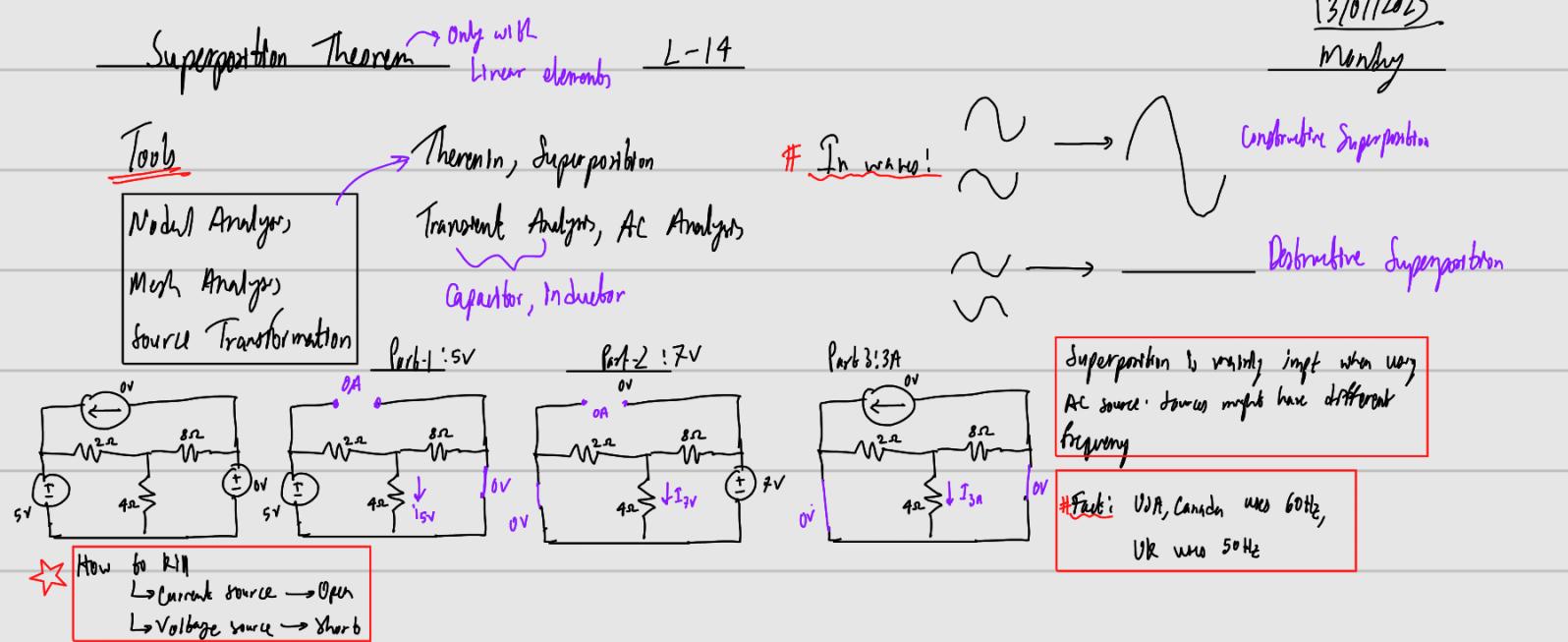
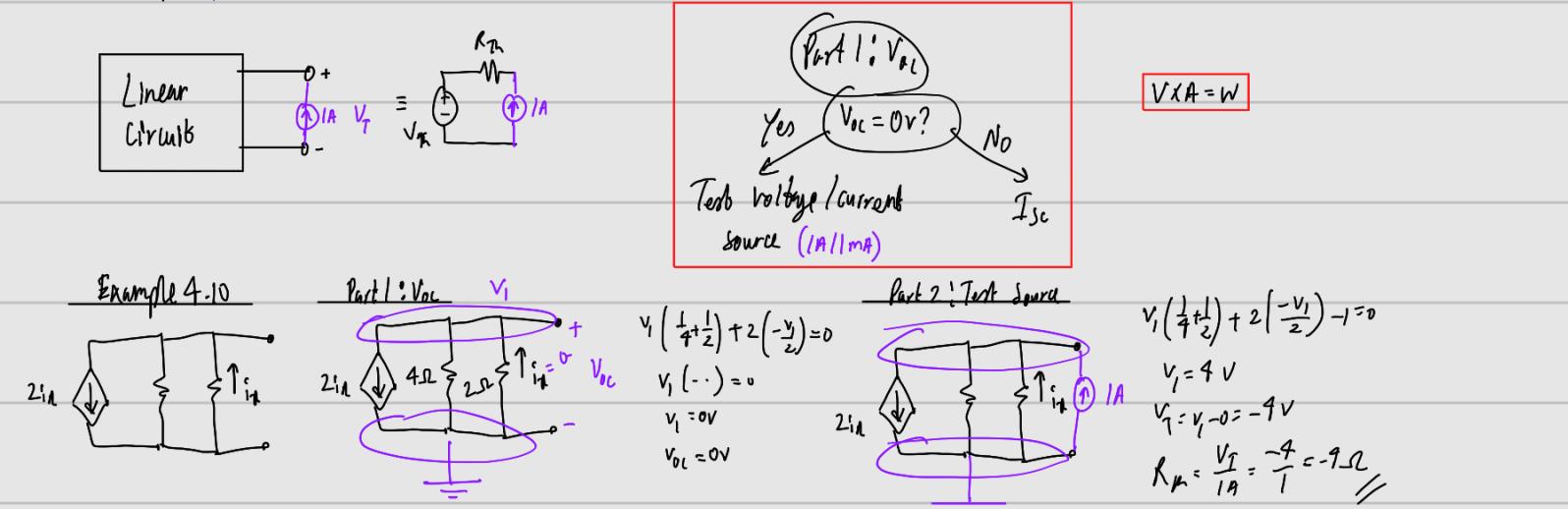
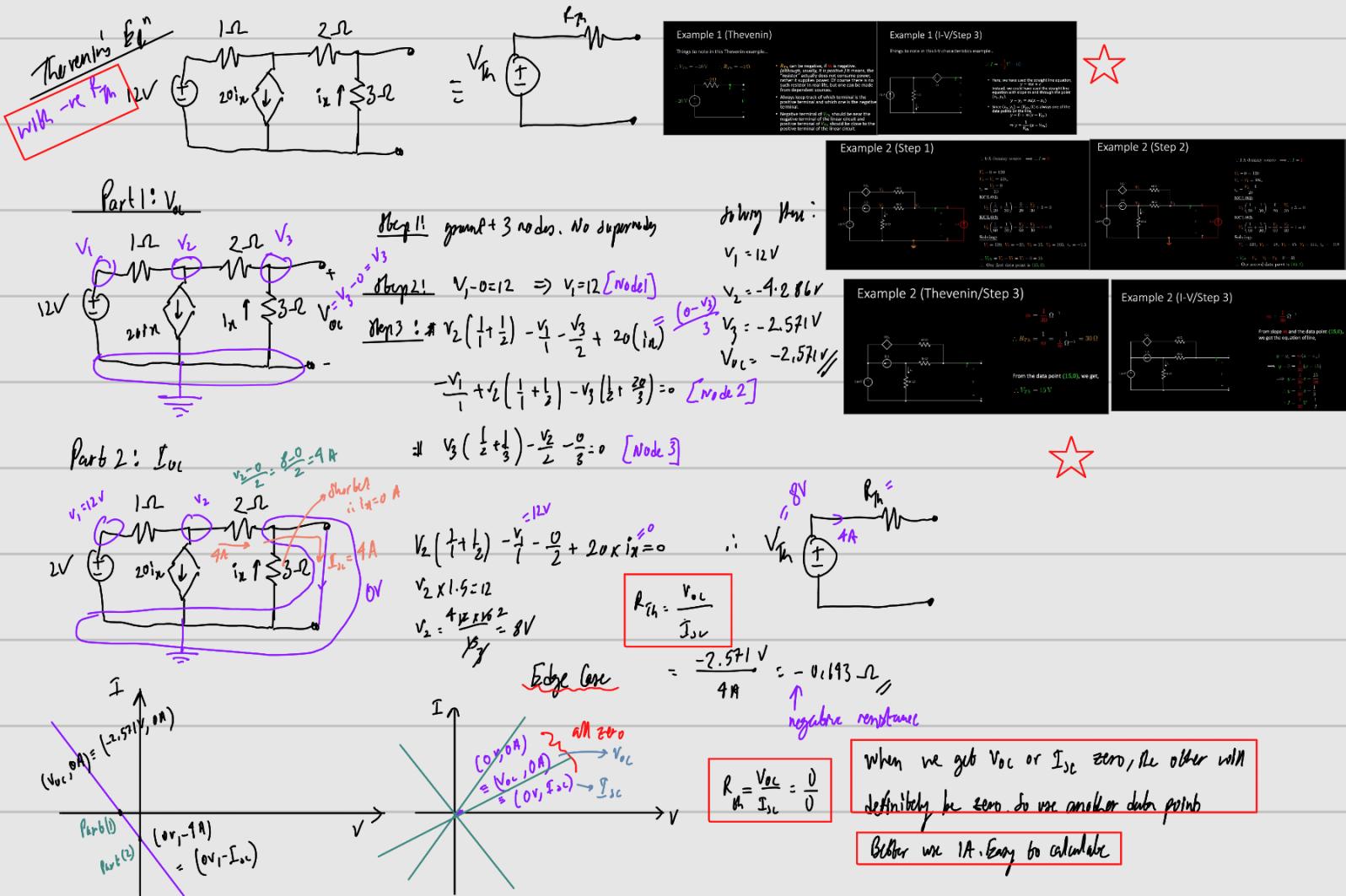
$$= (V_{oc})(V_{oc}) = \frac{(V_{oc})}{(V_{oc})}$$

$$= (V_{oc})$$

$$(Power \text{~consuming})$$

$$= (-V_{oc})$$

$$(Power \text{~dissipating})$$



## Chapter 4.3: Superposition Theorem. Example 4.3

### Example 4.3



Part 1: 6V

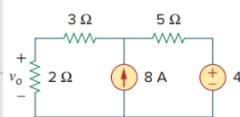
$$V_{6V} = 4i_1 \quad i_1(8+4) - 6 = 0 \quad i_1 = 0.5A$$

$$i_{6V} = -i_1 = -0.5A$$

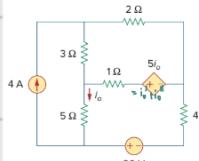
$$V_{6V} = 4i_1 = (4 \times 0.5)V = 2V$$

Do not kill the dependent sources

### Practice Problem 4.3



### Example 4.4



### Example 4.4



$$i_1 = 4A \text{ (mesh 1)}$$

$$i_2 = (3+2+i) - i_3 - 3i_1 - 5i_6 = 0 \quad (\text{mesh 2})$$

$$i_4 = i_1 - i_3 \quad -5(i_1 - i_2)$$

$$i_5 = (5 + 1 + 4) - 5i_1 - i_2 + 5i_6 = 0 \quad (\text{mesh 3})$$

$$i_1 = 4A$$

$$i_2 = 4.706A$$

$$i_3 = 0.941A$$

$$i_4 = 4 - 0.941A$$

$$\sim 3.059A$$

### Example 4.4



Using superposition Theorem,

$$V = V_{6V} + V_{3A} = 2V + 8V = 10V$$

$$I = I_{6V} + I_{3A} = -0.5 + 1 = 0.5A$$



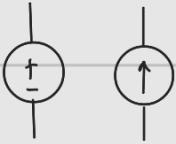
Q) What is the node voltage?  
= no voltage, no current thru  
no node voltage



As no independent source  
so dependent source  
behaves like a resistor

### Summary

- Kill only independent source



- How to kill i:



# Superposition Theorem can be applied to find

↳ Voltage ✓  $P \neq P_{1A} + P_{20V}$

↳ Current ✓  $I^2 \neq I_{1A}^2 + I_{20V}^2$

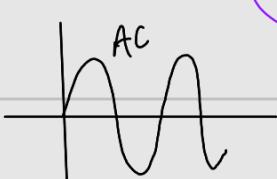
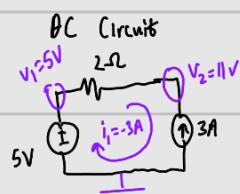
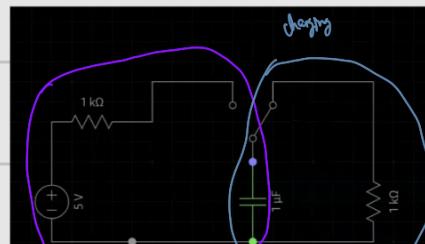
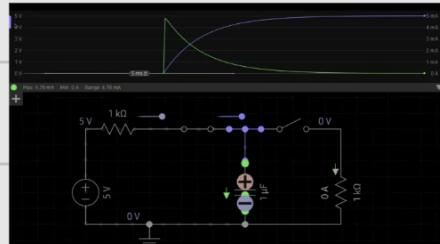
↳ Power X  $(I_{1A} + I_{20V})^2 \neq I_{1A}^2 + I_{20V}^2$

20/12/2029

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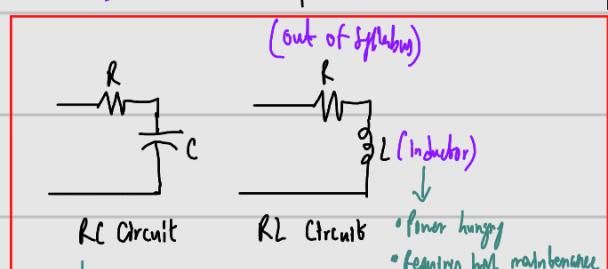
Online class

### RC Circuit Intuition



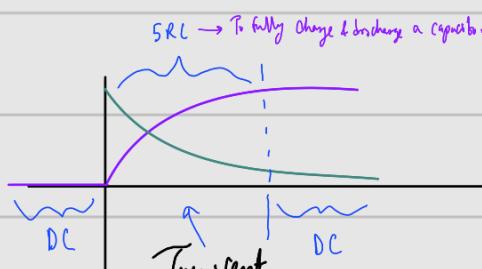
A lot of transient circuit is used  
inside RAM

Indeed every bit is a transient circuit:  
resistor & capacitor circuit



↳ Speed at which CPU can work, is determined by the values of R and C.

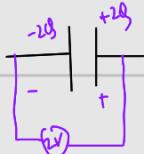
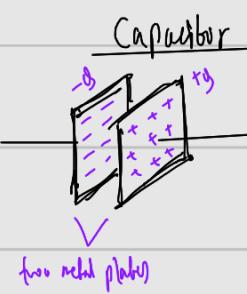
\* 5GHz → One of the fastest in today's time



$$5RL = 5 \times 1k\Omega \times 1\mu F$$

$$= 15ms$$

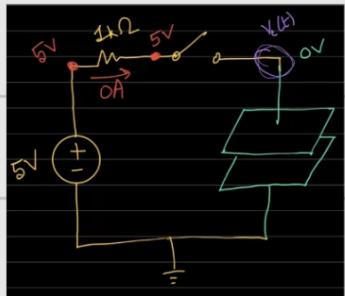
To fully charge & discharge 1μF



Capacitor wants to be an open circuit but breakers force to open. This time is transient behaviour

$$Q = CV$$

↳ Capacitor finally becomes open  
↳ Capacitor's voltage always changes gradually

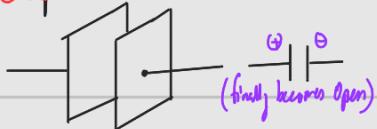


↳ DC Analysis

↳ Transient Analysis

↳ AC Analysis

### # Capacitor



Transient → Temporary

L-15

14/01/2025  
Tuesday



$V = RI$   
resistor resists current

$$\begin{aligned} Q &= CV \\ \Rightarrow \frac{dQ}{dt} &= C \frac{dV}{dt} \\ I &= C \frac{dV}{dt} \end{aligned}$$

$$\frac{E_{rt}}{R+i} = E_0 e^{-\frac{Rt}{L}}$$

C [For AF]

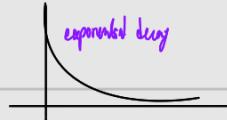
capacitor resists the change in voltage

### Inductor

(finally becomes short circuit)  
SI Unit: H (Henry)

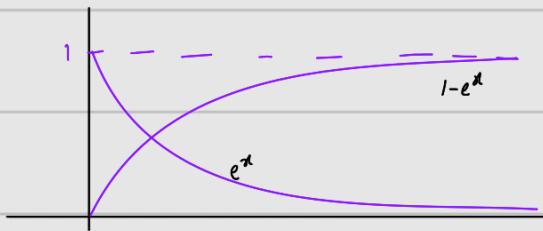
\* \* Whenever current through Inductor is changed, a voltage difference is created

$$\begin{aligned} \text{Here } V &\propto \frac{dI}{dt} \\ V &= L \frac{dI}{dt} \end{aligned}$$



Inductance resists change in current

$$T = \frac{L}{R} \quad I_s = \frac{1H}{1Ω}$$

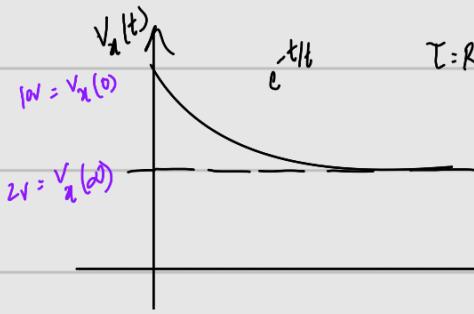
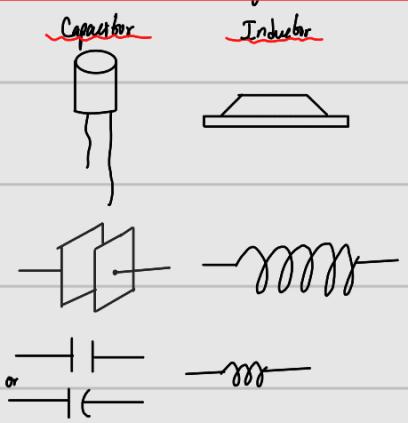


$$V(t) = (V_f - V_i)(1 - e^{-\frac{t}{T}}) + V_i$$

$$T = RC$$

$$\therefore V(t) = (V_f - V_i)(1 - e^{-\frac{t}{RC}}) + V_i$$

$$I(t) = (I_f - I_i)(1 - e^{-\frac{t}{RC}}) + I_i$$



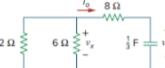
$T = RC$  # Chapter 7: First-Order Circuits

$RC/RL$



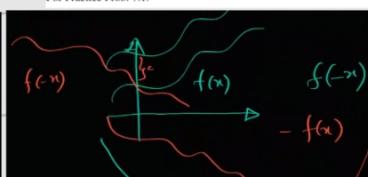
### Practice Problem 7.1

Refer to the circuit in Fig. 7.7. Let  $v_C(0) = 60 \text{ V}$ . Determine  $v_C$ ,  $v_A$ , and  $i_A$  for  $t \geq 0$ .



Answer:  $60e^{-0.25t} \text{ V}$ ,  $20e^{-0.25t} \text{ V}$ ,  $-5e^{-0.25t} \text{ A}$ .

Figure 7.7  
For Practice Prob. 7.1.

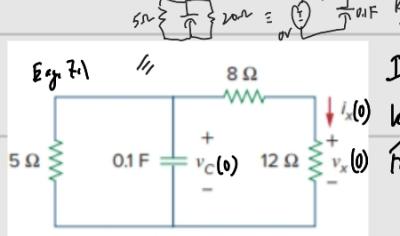


$$\begin{aligned} v_C(t) &= v_C(0) + [v_C(0) - v_x(0)] e^{-t/RC} \\ i_A(t) &= i_A(0) + [i_A(0) - i_x(0)] e^{-t/RC} \\ v_x(t) &= v_x(0) + [v_x(0) - v_A(\infty)] e^{t/RC} \end{aligned}$$

### Initial Circuit

### Final Circuit

after infinite time  
Complete Eqn



In this circuit,  $5\Omega$  let  $V_C(0) = 15 \text{ V}$   
Find  $V_C$ ,  $V_A$ ,  $i_A$

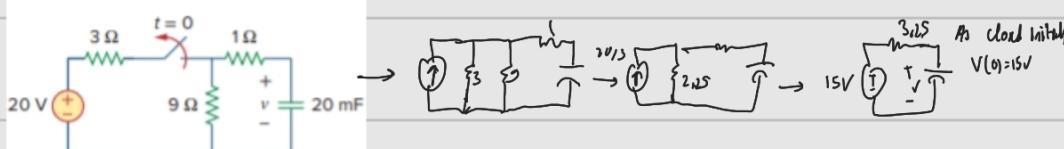
$$\begin{aligned} \# V_C(0) &= 15 \text{ V} \\ i_A(0) &= \frac{15 \text{ V}}{8 + 12} = 0.75 \text{ A}, \\ V_A &= 12 \times i_A(0) = 12 \times \frac{3}{4} = 9 \text{ V}, \end{aligned}$$



$$\begin{aligned} v_A(t) &= v_A(0) + [v_A(0) - v_A(\infty)] e^{-t/RC} \\ &= 2e^{-2.5t} \text{ V} \end{aligned}$$

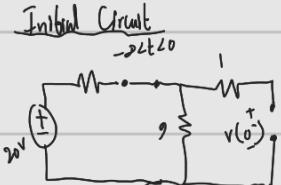
$$\begin{aligned} i_A(t) &= [i_A(0) - i_A(\infty)] e^{-t/RC} \\ &= [0.75 - 0] e^{-t/0.4} \\ &= 0.75e^{-2.5t} \text{ A} \end{aligned}$$

### Example 7.2



replace capacitor/inductor with prop. element

Initial Circuit  $\rightarrow x(0)$   $\rightarrow$  actually  $x(0)$   
Intermediate situation  $\rightarrow T = R_P C / L$   
Final Circuit  $\rightarrow x(\infty)$   
 $x(t) = [x(0) - x(\infty)] e^{-t/T} + x(\infty)$



### Intermediate Circuit

### Final Circuit

$$\begin{aligned} \# T &= R_P C = 10 \times 20 \times 10^{-3} \text{ s} \\ &= 0.2 \text{ s}, \\ v(t) &= 0 + [15 - 0] e^{-t/0.2} \\ &= 15e^{-5t} \text{ V} \end{aligned}$$

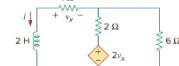
Capacitor stores energy as electrical energy  $\rightarrow$  Electrical energy

Inductor stores energy magnetically.  $\rightarrow$  Magnetic energy

Find  $i$  and  $v_A$  in the circuit of Fig. 7.15. Let  $i(0) = 7 \text{ A}$ .

Answer:  $7e^{-2t} \text{ A}$ ,  $-7e^{-2t} \text{ V}$ ,  $t > 0$ .

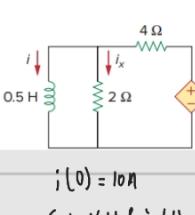
### Practice Problem 7.3



$$\begin{aligned} \text{Energy stored initially} &= \frac{1}{2} (V(0))^2 \\ (\text{Capacitor}) &= \frac{1}{2} C V^2 \\ &= \frac{1}{2} \times 20 \text{ mF} \times (15 \text{ V})^2 \\ &= 2.25 \text{ J} \end{aligned}$$

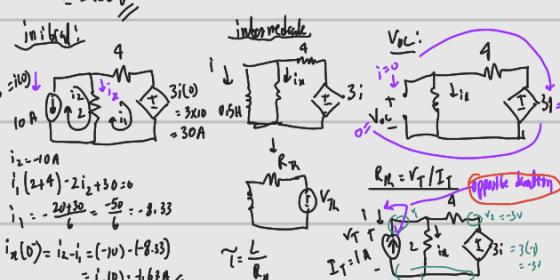
$$\begin{aligned} \text{Energy stored} &= \frac{1}{2} L I^2 \\ (\text{Inductor}) &= \frac{1}{2} L I^2 \end{aligned}$$

### Example 7.3



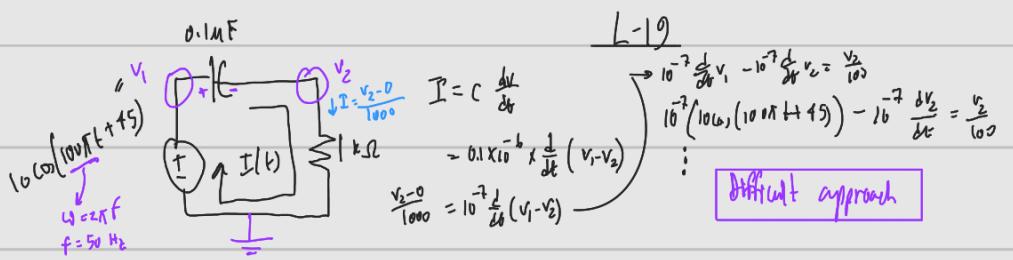
$$i(0) = 10 \text{ A}$$

Find  $i(t)$  &  $i_A(t)$



$$\begin{aligned} \# i(t) &= [i(0) - i(\infty)] e^{-t/RC} + i(\infty) \\ &= [7 - 0] e^{-t/0.5} + 0 \\ &= 7e^{-2t} \text{ A} \\ i_A(t) &= i_A(0) + [i_A(0) - i_A(\infty)] e^{-t/RC} \\ &= 10 + [10 - 0] e^{-t/0.5} \\ &= 10 + 10e^{-2t} \text{ A} \end{aligned}$$





Quarantine  
antibiotic  
higher dimension

# Impedance,  $z(\omega)$ : In AC analysis capacitor, resistor, inductor produces impedance. They work like complex resistor.

			$\rightarrow$ All 3 branch the same
$Z = R$	$Z = \frac{1}{j\omega C} = -\frac{j}{\omega C}$	$Z = j\omega L = \omega L < 90^\circ$	
$= R < 0^\circ$	$= \frac{1}{\omega C} < -90^\circ$		

- \* shift  $\rightarrow$  2: CMPLX
- \* Diff + mode
- \* 3: Deg
- \* go down
- \* 3: Complex
- \* Complex result?
- 1: abs 2:  $\angle$

0.1MF  $\rightarrow z_1 = \frac{1}{\omega C} < 90^\circ = 1 < -90^\circ$

$100\pi \times 0.1 \times 10^{-6} = \frac{10^5}{\pi} < -90^\circ \Omega$

$z_2 = 100 < 0^\circ \Omega$

phasor form

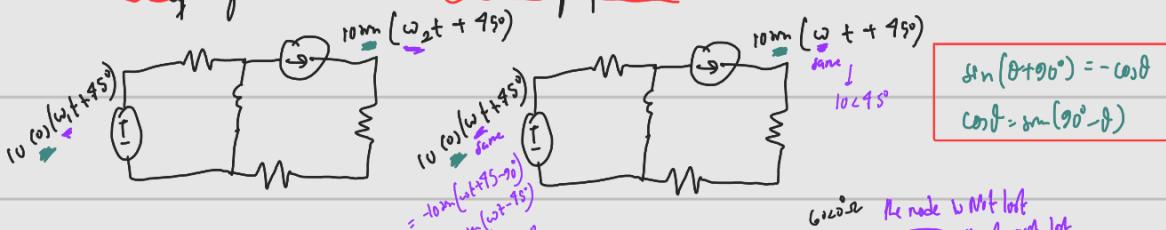
$$I = \frac{V}{Z_1 + Z_2} = \frac{10 \angle 90^\circ}{\frac{10^5}{\pi} \angle -90^\circ + 100 \angle 0^\circ} A$$

$$= 3.14 \times 10^{-4} < 133.2^\circ A$$

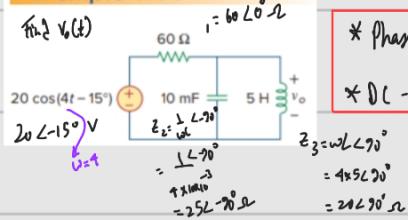
$$= 0.314 \cos(100t + 133.2^\circ) \text{ mA (Complex form)}$$

$$I(t) = 0.314 \cos(100t + 133.2^\circ) \text{ mA}$$

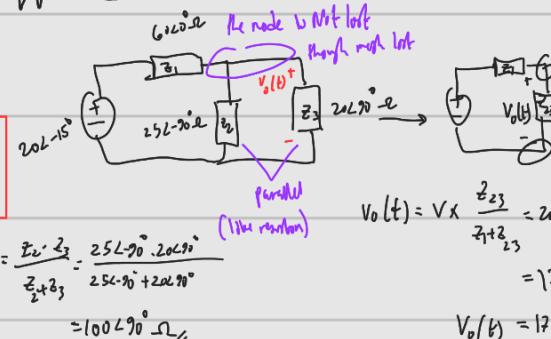
# When frequency is different we've to do superposition



### Example 9.11



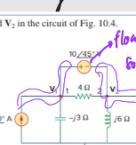
\* Phasor (Impedance) circuit  
\* DC-like Calculation



### AC Analysis

#### Chapter 10: Sinusoidal Steady State Analysis

L-20

Example 10.2 Compute  $V_1$  and  $V_2$  in the circuit of Fig. 10.4.

$$V_1 - V_2 = 10 L 45^\circ \text{ Supercede } 1 \text{ A}$$

$$V_1 \left( \frac{1}{4} + \frac{1}{3 L - 90^\circ} \right) - \frac{V_2}{4} - \frac{1}{3 L - 90^\circ} - 3 L 0^\circ + V_2 \left( \frac{1}{6} + \frac{1}{4 L 90^\circ} + \frac{1}{12} \right) - \frac{V_1}{12} = 0 \dots V_2 = 31.7 L - 86.7^\circ$$

$$V_1 = 10 L 45^\circ + 26.2 L - 67.7^\circ$$

$$r_1 \angle \theta_1 \times r_2 \angle \theta_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

OR

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$V_1 \cdot 0.33 L 70^\circ + V_2 \cdot 0.186 L - 63.1^\circ = 3 L 0^\circ \quad V_1 - V_2 = 10 L 45^\circ$$

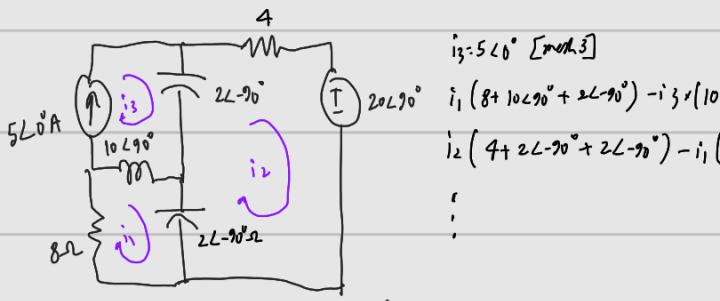
$$V_1 = \frac{\begin{vmatrix} 10 L 45^\circ & -1 \\ 3 L 0^\circ & 0.186 L \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 0.33 L 70^\circ & 0.186 L - 63.1^\circ \end{vmatrix}} = \frac{10 L 45^\circ \times 0.186 L - 63.1^\circ - 3 L 0^\circ \times 1 - 1}{1 \times 0.186 L - 63.1^\circ - (-1) \times 0.33 L 70^\circ} \approx 26.14 L - 70^\circ$$

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$J = \frac{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

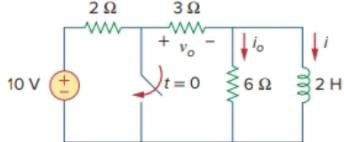


$$i_3 = 5L0^\circ \text{ [initial]}$$

$$i_1(8 + 10L90^\circ + 2L-90^\circ) - i_3(10L90^\circ) - i_2(2L-90^\circ) = 0$$

$$i_2(4 + 2L-90^\circ + 2L-90^\circ) - i_1(2L-90^\circ) - i_3(2L-90^\circ) + 20L90^\circ = 0$$

### Example 7.5



$$i(t) = [i(0^+) - i(\infty)] e^{-\frac{t}{\tau}} + i(\infty)$$

$i(0^-) = i(0^+)$  → same for Inductor

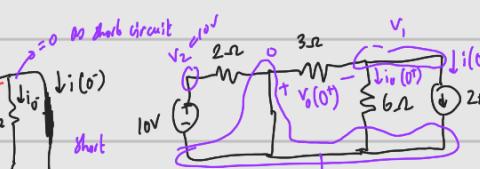
$t=0^- (t \neq 0)$



$$i(0^-) = 0A$$

$$i(0^+) = \frac{10}{2+3} = 2A$$

$$v_o(0^-) = 10 \times \frac{3}{2+3} = 6V$$



$\# T = 1s$

$$i(t) = [i(0^+) - i(\infty)] e^{-\frac{t}{T}} + i(\infty)$$

$$= [-0.67 - 0] e^{-\frac{t}{1}} + 2A$$

$$= -0.67 e^{-t} A_{\parallel}$$

$$v_1(0^+) = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad i_o(0^+) = 0A$$

$$v_1 = \frac{-2}{1/6 + 1/3} = -4$$

$$v_o(0^+) = 0 - v_1 = 4V, i_o(0^+) = \frac{v_1}{1.1} = -0.67A$$

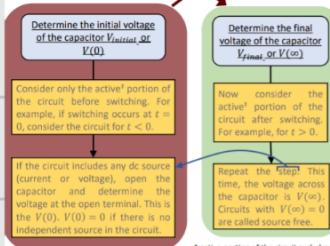
$$\begin{cases} i_o(0^-) = 0A & (t < 0) \\ i_o(0^+) = -0.67A & (t = 0) \\ i_o(\infty) = 0A \end{cases}$$

$$i_o(t) = \begin{cases} 0A & t < 0 \\ -0.67e^{-t} & t \geq 0 \end{cases}$$

[SUS] YT

### Transient Analysis Problem Solving

#### Procedure



Consider only the active portion of the circuit before switching. For example, if switching occurs at  $t = 0$ , consider the circuit for  $t < 0$ . If the circuit includes any dc source (not open), open the capacitor and determine the voltage at the open terminal. This is the  $V(0)$ .  $V(0) = 0$  if there is no independent source in the circuit.

The active portion of the circuit excludes everything that has no influence on the capacitor.

Repeat the step! This time, consider the active portion after switching. For example, for  $t > 0$ .

Now consider the active portion of the circuit after switching. For example, if switching occurs at  $t = 0$ , consider the circuit for  $t > 0$ .

Again, only consider the active portion after switching. For example, for  $t > 0$ .

Determine the Thevenin resistance ( $R_{TH}$ ) seen from the capacitor terminals.

$\tau = R_{TH} C$

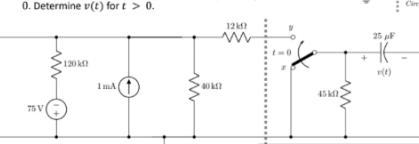
Determine any other voltages or currents in the circuit using  $v(t)$  and the circuit laws.

Plug in  $V(0)$ ,  $V(\infty)$ , and  $\tau$  into the equation for  $v(t)$ .

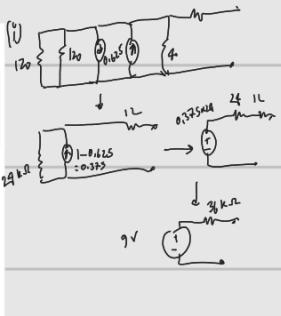
Spring 2023

#### Problem 14

- Reduce the left portion with respect to the dashed grey line of Circuit 1 so that it takes the form of Circuit 2 as shown. Write down the values of  $V$  and  $R$ .
- Now, analyse the Transient Behaviour of the circuit assuming that the switch moves from position  $x$  to position  $y$  at  $t = 0$ . Determine  $v(t)$  for  $t > 0$ .



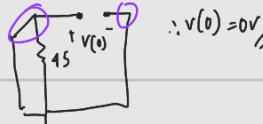
$$\text{Ans: } V_1 = 9V; R_1 = 36k\Omega; v(t) = 5(1 - e^{-2t})V$$



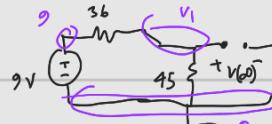
1) Initial Circuit  $\xrightarrow{\text{capacitor open}} V_C(0)$

2) Final Circuit  $\xrightarrow{\text{capacitor open}} V(\infty)$

$$KCL@1: V_1 \left( \frac{1}{36} + \frac{1}{45} \right) - \frac{9}{36} = 0 \quad 3) \text{ Time Constant } \tau'$$



\* Open the circuit and find the voltage across it



$$V_1 = 5V$$

$$\therefore V_\infty = 5V$$

$$\tau' = R_{\parallel} C \rightarrow \text{of final circuit}$$

$$R_{\parallel} = 20k\Omega \times 25\mu F = 0.5s$$

$$4) V(t) = \begin{cases} V(0); t \leq 0 \\ V(\infty) + [V(0) - V(\infty)] e^{-\frac{t}{\tau'}}; t \geq 0 \end{cases}$$

$$V(t) = \begin{cases} 0V; t \leq 0 \\ 5 + [0 - 5] e^{-\frac{t}{0.5}} V; t \geq 0 \end{cases}$$

$$\text{when } b = 5 \gamma$$

$$-\frac{5}{0.5} = -10$$

$$= 0.0067$$

$$V(0) = 5 - 5e^{-2(0)} = 5V$$

$$V(1) = 5 - 5e^{-2 \cdot 1} = 4.323V$$

$$V(2.5) = 4.966V \approx 99.3\%$$

(almost 5)

$$5) I(t) = C \frac{dV}{dt}$$

$$I(t) = C \frac{d}{dt}(0) = 0A$$

$$I(t) = C \frac{d}{dt}(5 - e^{-2t})$$

$$= C \left( \frac{d}{dt}(5) - \frac{d}{dt}(e^{-2t}) \right)$$

$$= -C \times 5(-2)e^{-2t}$$

$$= 10 \times 2.5 \times 10^{-5} t e^{-2t} A$$

$$= 0.125 \times 10^{-3} t e^{-2t} A = 0.125 t e^{-2t} mA$$

$$V(t) \uparrow$$

$$V(\infty) = 5V$$

(fully charged)

$$t \leq 0: V(t) = 0V$$

$$DC$$

$$5\gamma = 2.5s$$

$$P(t) = V(t) i(t)$$

$$t \geq 0$$

$$I(t) = 0.25mA$$

$$I(0) = 0.25mA$$

$$I(t) = 0.25 e^{-2t} mA$$

$$I(0) = 0.25mA$$

$$I(t) = 0.25 e^{-2t} mA$$

$$I(0) = 0.25mA$$

$$I(t) = 0.25 e^{-2t} mA$$

$$I(0) = 0.25mA$$

Initial and final current of capacitor is always zero except for transients period.

At rest of the time it acts as an open circuit when it stabilizes.

# Final Review (fall 2024)

- { 1) Transient
- { 2) Superposition/ Source Transformation
- { 3) I-V
- { 4) AC Sinusoidal

→ No dependent source  
 →  $R_L$  by killing all sources (Superposition)  
 and find equivalent resistance  
 → Source Transformation

$$\text{Atn}(\omega t + \phi)$$

$$\downarrow$$

$$A > 0 \quad \omega > 0 \quad -180^\circ \leq \phi \leq 180^\circ$$

$$\sin \theta = \cos(90^\circ - \phi) = \cos(\phi - 90^\circ)$$

$$\sin(\theta + 90^\circ) = \cos \phi, \quad \frac{\sin \theta}{\sin(\theta + 90^\circ)} = \frac{\cos \phi}{\cos(\phi + 90^\circ)}$$

$$\sin(-\phi) = -\sin \phi$$

$$\cos(-\phi) = \cos \phi,$$

Always keep  $\phi$  between  $-180^\circ$  and  $180^\circ$   
 (Add/Subtract  $360^\circ$  when needed to adjust)

$$\sin(180 + \phi) = -\sin \phi$$

$$\cos(180 + \phi) = -\cos \phi$$

## Problem 3

- I.  $V_s = 45 \cos(5\pi t + 36^\circ) V$
- II.  $I_s = 15 \cos(25t + 25^\circ) A$
- III.  $I_s = -20 \cos(314t - 30^\circ) A$
- IV.  $V_s = -4 \sin(628t + 55^\circ) V$

Ans: I.  $45 \cos(5\pi t + 36^\circ) V$   
 II.  $15 \cos(25t + 25^\circ) A$   
 III.  $15 A; 25^\circ$  or  $115^\circ; 25 \pi \text{ rad/s}; 80 \text{ ms}; 12.5 \text{ Hz}; -6.34 A$   
 IV.  $4 V; -125^\circ; 628 \text{ rad/s}; 10 \text{ ms}; 100 \text{ Hz}; -3.26 V$

(i)  $V_s = 45 \cos(5\pi t + 36^\circ) V$

$A = 45V, \omega = 45, \phi = 36^\circ$

OR

 $v_s = 45 \cos(5\pi t + 36 + 90^\circ)$ 
 $= 45 \sin(5\pi t + 126^\circ)$

(ii)  $I_s = 15 \cos(25t + 25^\circ) A$

$A = 15A, \omega = 25\pi \text{ rad/s}, \phi = 25^\circ$

OR

 $i_s = 15 \cos(25t + 25 + 90^\circ)$ 
 $= 15 \sin(25t + 125^\circ)$

(iii)  $I_s = -20 \cos(314t - 30^\circ) A$

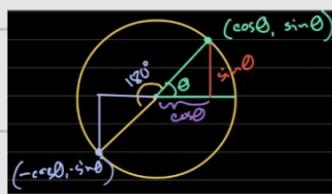
$A = 20A, \omega = 314 \text{ rad/s}, \phi = -30^\circ$

OR

 $i_s = 20 \cos(314t - 30 + 90^\circ)$ 
 $= 20 \sin(314t + 60^\circ)$

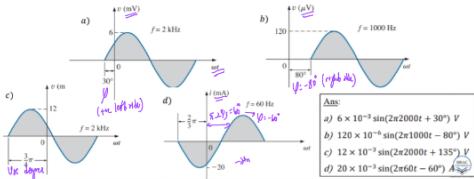
(iv)  $V_s = -4 \sin(628t + 55^\circ) V$

$A = 4V, \omega = 628 \text{ rad/s}, \phi = -125^\circ$



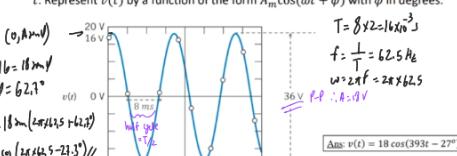
## Problem 4 $\phi = \omega t$

- Write analytical expressions for the waveforms with the initial phase in degrees.



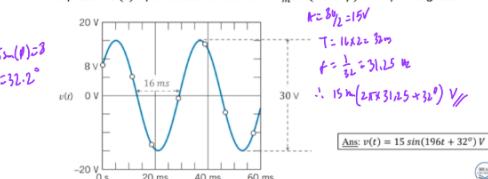
## Problem 5

- The following figure shows a sinusoidal voltage  $v(t)$ , plotted as a function of time  $t$ . Represent  $v(t)$  by a function of the form  $A_m \cos(\omega t + \phi)$  with  $\phi$  in degrees.



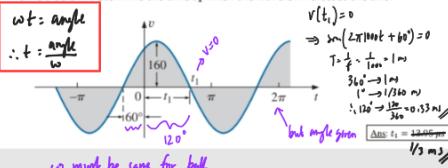
## Problem 6

- The following figure shows a sinusoidal voltage  $v(t)$ , plotted as a function of time  $t$ . Represent  $v(t)$  by a function of the form  $A_m \sin(\omega t + \phi)$  with  $\phi$  in degrees.



## Problem 7

- A sinusoidal voltage  $v(t) = 160 \sin(2\pi 1000t + 60^\circ)$  is plotted as a function of time  $t$  below. Determine the time  $t_1$  when the waveform crosses the axis.



we must be same for both

## Leading and Lagging Sinusoids

- The equation for a sinusoid in more general form,  
 $v(t) = V_m \sin(\omega t + \phi)$
- where  $(\omega t + \phi)$  is the argument and  $\phi$  is the initial phase. Both argument and phase can be in radians or degrees
- Let's examine two sinusoids  $v_1(t) = V_m \sin(\omega t)$  and  $v_2(t) = V_m \sin(\omega t + \phi)$
- The starting point of  $v_2$  occurs first in time. Or  $v_2$  passes the zero-crossing line first if compared between two same phase points of  $v_1$  and  $v_2$ .
- Therefore, we say that  $v_2$  leads  $v_1$  by  $\phi$  or that  $v_1$  lags  $v_2$  by  $\phi$ .
- If  $\phi \neq 0$ ,  $v_1$  and  $v_2$  are out of phase.
- If  $\phi = 0$ ,  $v_1$  and  $v_2$  are in phase.

## Problem 8

- Consider the sinusoidal voltage  $v(t) = 80 \cos(1000t - 30^\circ) V$ .

- a) What is the first time after  $t = 0$  that  $v(t) = 80 V$ ?
- b) If the sinusoidal voltage is shifted  $1/2$  ms to the left along the time axis, what will be the expression for  $v(t)$ ?
- c) What is the minimum number of microseconds that the function must be shifted to the right/left? If the expression for  $v(t)$  is  $80 \sin(1000t) V$ ?

Ans:  
 (a)  $t = 166.67 \mu s$   
 (b)  $-80 \sin(1000t) V$   
 (c)  $t = 333.33 \mu s$

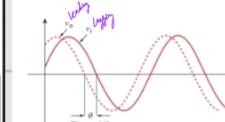
## Problem 9

- At  $t = -2$  ms, a sinusoidal voltage  $v(t)$  is known to be zero and going positive. The voltage is next zero at  $t = 8$  ms. It is also known that the voltage is  $80.9 V$  at  $t = 0$ . What is the expression for  $v(t)$ ?

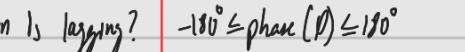
$\text{Ans: } v(t) = 20 \cos(400\pi t - 45^\circ) V$

## Problem 10

- Determine for each of the plots, which one is leading/lagging.



Ans: (a)  $v_1$  leading; (b)  $v_2$  leading



## Problem 11

- Determine for each of the plots, which one ( $v$  or  $t$ ) is leading/lagging and by how much.

8) Which one is leading and which one is lagging?

$$V_s = 45 \cos(5\pi t + 36^\circ) = 45 \sin(5\pi t + 126^\circ)$$

$$I_s = 80 \sin(5\pi t - 20^\circ)$$

$$\# V_s \text{ leads } I_s \text{ by } (126 - (-20)) = 146^\circ //$$

$$\# I_s \text{ lags } V_s \text{ by } 146^\circ //$$

$$\# I_s \text{ leads } V_s \text{ by } (-20 - 126) = -146^\circ$$

correct but not the convention

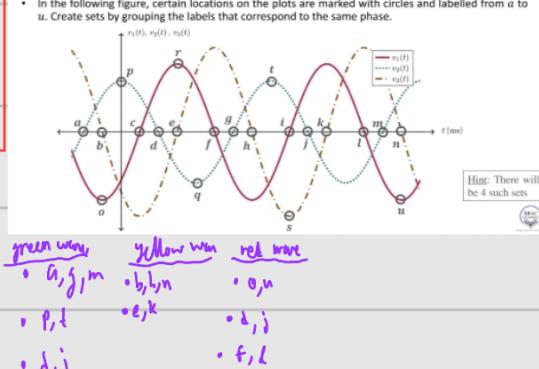
$$-180^\circ \leq \text{phase } (\phi) \leq 180^\circ$$

$$\text{phase difference } (\phi_A - \phi_B) \leq 180^\circ$$

$$A \text{ leads } B \text{ by } (\phi_A - \phi_B)$$

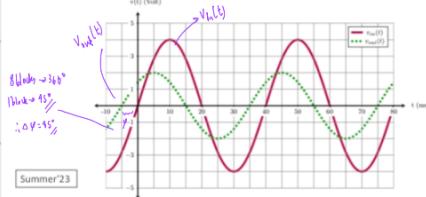
$$> 0, \leq 180$$

## Problem 12



## Problem 12

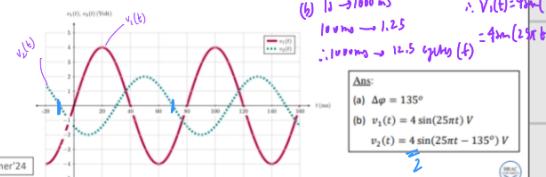
- Two ac voltages  $v_{in}(t)$  and  $v_{out}(t)$  from an ac circuit are plotted below. Which one is leading and by how much in degrees?



## Problem 13

Two ac voltages  $v_1(t)$  and  $v_2(t)$  from an ac circuit are plotted below.

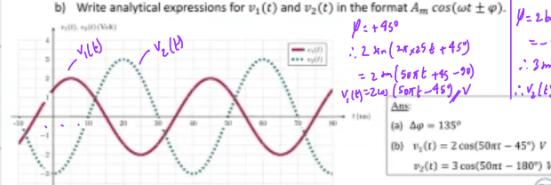
- Which one is leading and by how much in degrees?  $\rightarrow v_1(t)$  leads  $v_2(t)$  by  $135^\circ$
- Write analytical expressions for  $v_1(t)$  and  $v_2(t)$ .



## Problem 17

Two ac voltages  $v_1(t)$  and  $v_2(t)$  from an ac circuit are plotted below.

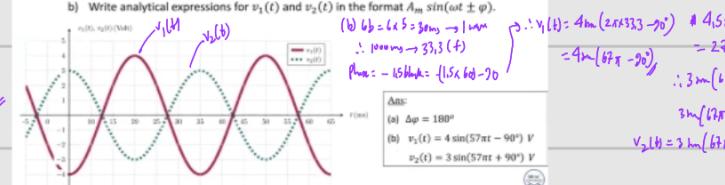
- Which one is leading and by how much in degrees?  $\rightarrow v_1(t)$  leads  $v_2(t)$  by  $135^\circ$
- Write analytical expressions for  $v_1(t)$  and  $v_2(t)$  in the format  $A_m \cos(\omega t \pm \phi)$ .



## Problem 18

Two ac voltages  $v_1(t)$  and  $v_2(t)$  from an ac circuit are plotted below.

- Which one is leading and by how much in degrees?
- Write analytical expressions for  $v_1(t)$  and  $v_2(t)$  in the format  $A_m \sin(\omega t \pm \phi)$ .

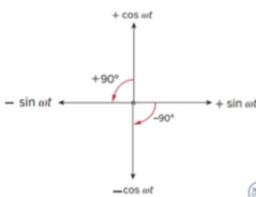


## Sine-Cosine Conversion

- A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is expedient to express both as either sine or cosine with positive amplitudes.

- The following trigonometric identities can be used to convert from sine to cosine or vice versa.

- $\sin(\omega t \pm 180^\circ) = -\sin\omega t$
- $\cos(\omega t \pm 180^\circ) = -\cos\omega t$
- $\sin(\omega t \pm 90^\circ) = \pm \cos\omega t$
- $\cos(\omega t \pm 90^\circ) = \mp \sin\omega t$



## Problem 14

- For the following pairs of sinusoids, determine which one leads and by how much ( $0^\circ \leq 180^\circ$ ).

$$I. \quad v(t) = 10 \cos(4\pi t - 60^\circ) \& i(t) = 4 \sin(4t + 50^\circ) \quad \rightarrow v(t) \text{ leads } i(t) \text{ by } 20^\circ$$

$$II. \quad v_1(t) = 4 \cos(377t + 10^\circ) \& v_2(t) = -20 \cos 377t = 20 \sin(377t + 90^\circ) \quad \rightarrow v_1(t) \text{ leads } v_2(t) \text{ by } 80^\circ$$

$$III. \quad v_1(t) = 45 \sin(\omega t + 30^\circ) V \& v_2(t) = 50 \cos(\omega t - 30^\circ) V \quad \rightarrow v_1(t) \text{ leads } v_2(t) \text{ by } 60^\circ$$

$$IV. \quad i_1(t) = -4 \sin(377t + 55^\circ) \& i_2(t) = 5 \cos(377t - 65^\circ) \quad \rightarrow i_2(t) \text{ leads } i_1(t) \text{ by } 120^\circ$$

$$V. \quad x(t) = (13 \cos 2t + 5 \sin 2t) \& y(t) = 15 \cos(2t - 11.8^\circ) \quad \rightarrow y(t) \text{ leads } x(t) \text{ by } 9.24^\circ$$

Hint: convert both the sinusoids into sine or cosine form  $\rightarrow$  convert them into phasors  $\rightarrow$  add them in frequency domain  $\rightarrow$  convert them back in the time domain and compare

$$\sin(\omega t + \phi) = \cos(\omega t + \phi - 90^\circ)$$

$$\cos(\omega t + \phi) = \sin(\omega t + \phi + 90^\circ)$$

$$\sin(\omega t + \phi) = -\sin(\omega t - \phi)$$

$$\cos(\omega t + \phi) = -\cos(\omega t - \phi)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 90^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 90^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 180^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 180^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 180^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 180^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 270^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 270^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 270^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 270^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 360^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 360^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 360^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 360^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 450^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 450^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 450^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 450^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 540^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 540^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 540^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 540^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 630^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 630^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 630^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 630^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 720^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 720^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 720^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 720^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 810^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 810^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 810^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 810^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 900^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 900^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 900^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 900^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 990^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 990^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 990^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 990^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1080^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1080^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1080^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1080^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1170^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1170^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1170^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1170^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1260^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1260^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1260^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1260^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1350^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1350^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1350^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1350^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1440^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1440^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1440^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1440^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1530^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1530^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1530^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1530^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1620^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1620^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1620^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1620^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1710^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1710^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1710^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1710^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1800^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1800^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1800^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1800^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1890^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1890^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1890^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1890^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 1980^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 1980^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 1980^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 1980^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2070^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2070^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2070^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2070^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2160^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2160^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2160^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2160^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2250^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2250^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2250^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2250^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2340^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2340^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2340^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2340^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2430^\circ)$$

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$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2430^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2520^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2520^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2520^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2520^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2610^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2610^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2610^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2610^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2700^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2700^\circ)$$

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$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2700^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2790^\circ)$$

$$\cos(\omega t + \phi) = \mp \cos(\omega t \pm 2790^\circ)$$

$$\sin(\omega t + \phi) = \pm \cos(\omega t \pm 2790^\circ)$$

$$\cos(\omega t + \phi) = \mp \sin(\omega t \pm 2790^\circ)$$

$$\sin(\omega t + \phi) = \pm \sin(\omega t \pm 2880^\circ)$$

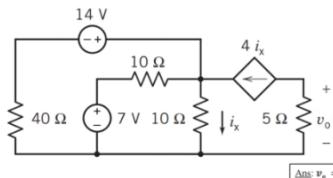
$$\cos(\omega t + \phi) = \mp \cos$$

## Superposition Theorem

\* Do not kill dependent source

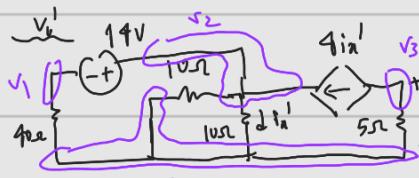
### Problem 7

- Use Superposition Principle to solve for  $v_o$ .



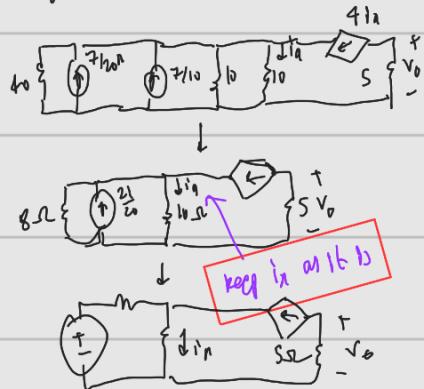
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$$\# \text{By superposition, } v_o = v_o^1 + v_o^{\parallel}, i_a = i_a^1 + i_a^{\parallel}$$



$$\begin{aligned} & v_2 - v_1 = 14 \text{ V} \\ & i_a^1 = v_2 / 10 \\ & v_o^1 = v_3 = 0 \\ & v_1 = v_2 - 14 \\ & \# v_3 \left( \frac{1}{5} \right) - 0 + 4i_a^1 = 0 \\ & v_3 \left( \frac{1}{5} \right) + 4 \left( \frac{v_2}{10} \right) = 0 \\ & v_3 \left( \frac{1}{5} \right) + 4 \left( \frac{v_2}{10} \right) - 4i_a^1 = 0 \\ & \# (v_1 + v_2) \left( \frac{1}{10} + \frac{1}{10} \right) - 4i_a^1 = 0 \\ & (v_2 - 14) \left( \frac{1}{10} \right) + v_2 \left( \frac{1}{10} + \frac{1}{10} \right) - 4 \left( \frac{v_2}{10} \right) = 0 \\ & v_2 \left( \frac{1}{40} + \frac{1}{10} + \frac{1}{10} - \frac{4}{10} \right) = \frac{14}{40} \\ & v_2 = 2 \text{ V} \end{aligned}$$

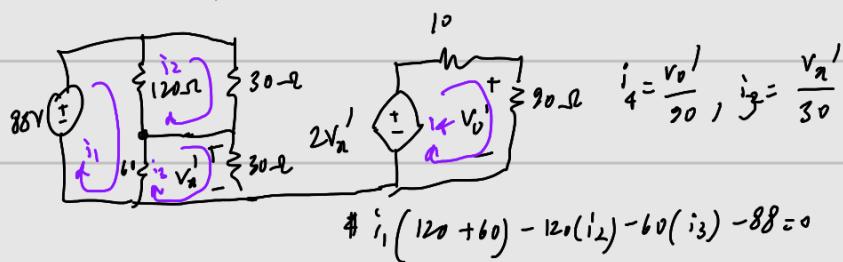
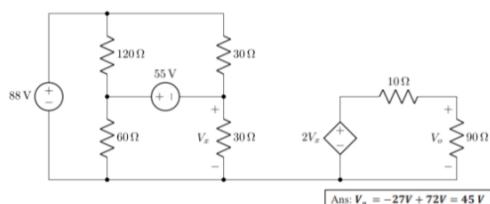
### Using Source Transform



$$\begin{aligned} & v_o^{\parallel} = v_2 \\ & v_2 = 2 \\ & v_2 = v_3 \\ & i_a^{\parallel} = \frac{v_2}{10} \\ & v_1 \left( \frac{1}{40} + \frac{1}{10} + \frac{1}{10} \right) - \frac{v_2}{10} - 4i_a^{\parallel} = 0 \\ & v_1 \left( \frac{1}{40} + \frac{1}{10} + \frac{1}{10} \right) - \frac{2}{10} - 4 \left( \frac{2}{10} \right) = 0 \\ & \therefore v_1 \left( -\frac{4}{40} \right) = \frac{2}{10} \quad \therefore v_1 = -4 \text{ V} \end{aligned}$$

### Problem 8

- Use Superposition Principle to solve for  $v_o$ .



$$\# i_1 (120 + 60) - 120(i_2) - 60(i_3) - 88 = 0$$

$$i_1 = 2, i_2 = \frac{8}{3}, i_3 = \frac{4}{3}$$

$$\# i_3 (60 + 30) - 60(i_1) = 0$$

$$V_o^1 = i_3 \times 90$$

$$\# i_4 (10 + 90) - 2V_a^1 = 0$$

$$= 0.8 \times 90$$

$$i_4 (10 + 90) - 2(30 i_3) = 0$$

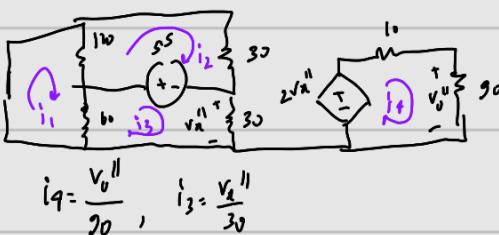
$$= 72 \text{ V}$$

$$i_4 (100) - 2 \left( 30 \times \frac{4}{3} \right) = 0$$

$$i_4 (100) - 80 = 0$$

$$i_4 = \frac{80}{100} = 0.8$$

$$\therefore V_o = V_o^1 + V_o^{\parallel} = -27 + 72 = 45 \text{ V}$$



$$i_1 (120 + 60) - 120(i_2) - 60(i_3) = 0$$

$$i_2 (120 + 30) - 120(i_1) - 55 = 0$$

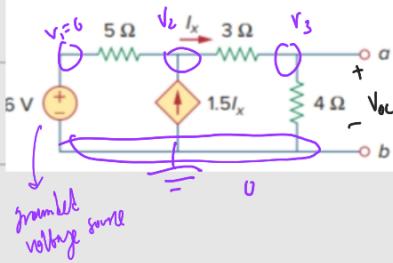
$$i_3 (60 + 30) - 60(i_1) + 55 = 0$$

$$i_4 (10 + 90) - 2V_a^{\parallel} = 0$$

$$i_4 (100) - 2(i_3 + 30) = 0$$

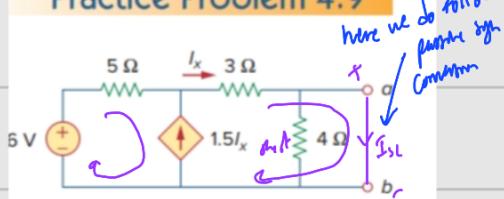
$$\begin{aligned} & i_1 = \frac{1}{6}, i_2 = \frac{1}{2}, i_3 = \frac{1}{2} \\ & i_4 (100) - 2 \left( -\frac{1}{2} \times 30 \right) = 0 \\ & i_4 (100) + 30 = 0 \\ & i_4 = -0.3 \\ & \therefore V_o^{\parallel} = -0.3 \times 90 \\ & = -27 \text{ V} \end{aligned}$$

## Practice Problem 4.9



$$\begin{aligned}
 & V_{01} = V_3 - 0 = V_3, \quad l_2 = \frac{V_2 - V_3}{3} \\
 & \text{and } V_3 \left( \frac{1}{3} + \frac{1}{4} \right) - \frac{V_2}{3} - \frac{V_3}{4} = 0 \quad \therefore V_{01} = 5.33 \text{ V} \\
 & \text{And } V_2 \left( \frac{1}{5} + \frac{1}{3} \right) - \frac{6}{5} - \frac{V_3}{3} - 1.5l_2 = 0 \\
 & \quad V_3 \left( \frac{1}{3} + \frac{1}{4} \right) + V_2 \left( -\frac{1}{3} \right) = 0 \quad (0, V_{in}) = (0, 5.33) \\
 & V_2 \left( \frac{1}{3} + \frac{1}{3} \right) - \frac{6}{5} - \frac{V_3}{3} - 1.5 \left( \frac{V_2}{3} - \frac{V_3}{3} \right) = 0 \quad V_2 = \frac{28}{3}, \quad V_3 = \frac{16}{3} \\
 & V_2 \left( \frac{1}{5} + \frac{1}{3} - \frac{1.5}{3} \right) + V_3 \left( -\frac{1}{3} + \frac{1.5}{3} \right) = \frac{6}{5} \quad = 7.33 \quad = 5.33
 \end{aligned}$$

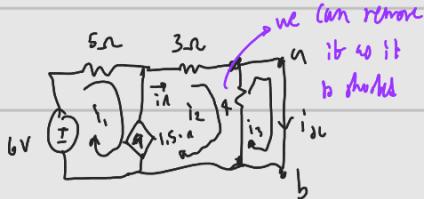
### Practice Problem 4.9



We don't follow people w/ communication:

↳ During source transformation

$\hookrightarrow$  Test current source



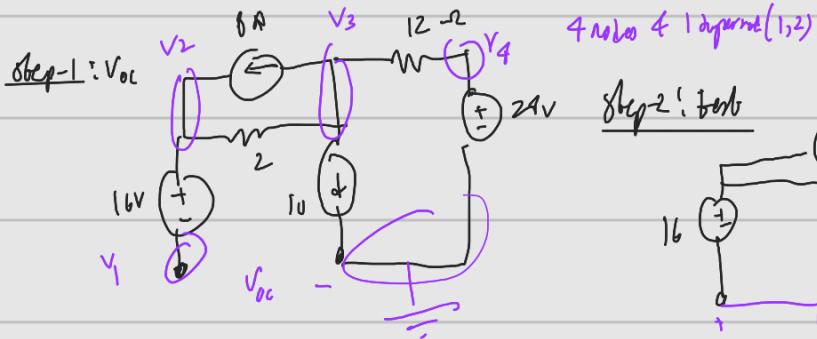
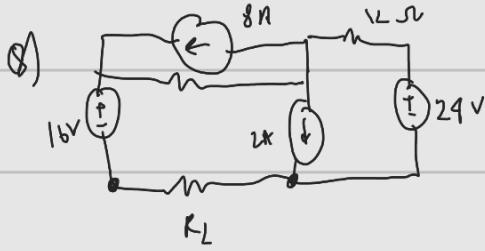
3 mesh, 1 supermesh (1,2)

$$\begin{aligned} \# i_2 - i_1 &= 1.5 i_2, \quad \# i_3 = i_3, \\ &\Rightarrow 1.5 i_2 \longrightarrow 0.5 i_2 + i_1 = 0 \\ \# \{i_1(5) - 6\} &\Rightarrow \{i_2(3+4) - 4i_3\} = 0 \end{aligned}$$

$$\therefore R_{th} = \frac{5.33}{12} = 0.44 \Omega$$

$$\# i_3(4) - 4(i_2) = 0$$

$i_1 = -6A_1, i_2 \in I_2, i_3 \in I_2$



$$4 \quad V_{BL} = V_1$$

$$A \quad |b = v_2 - v_1$$

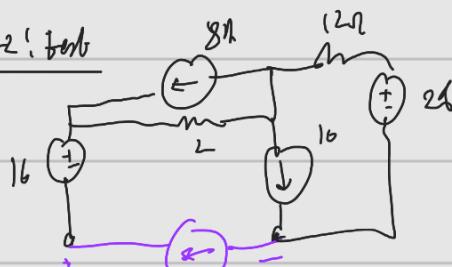
$$v_1(0) + v_2\left(\frac{1}{2}\right) - \frac{v_3}{2} - 8 = 0$$

$$A \cdot v_3 \left( \frac{1}{12} + \frac{1}{2} \right) - \frac{v_4^{24}}{12} - \frac{v_2}{2} + 10 = 0$$

$$\#V_4 = 24$$

$$\therefore V_1 = 6V, V_2 = 16V, V_3 = 0$$

$\delta \cdot V_{AC} = 0V \rightarrow$  hence put best current source



$$|k| = v_3 - v_1$$

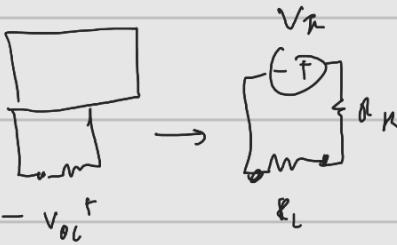
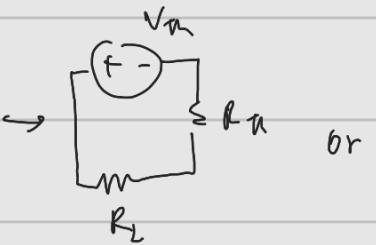
$$v_1(0) + v_2 \left(\frac{1}{2}\right) - \frac{v_3}{2} - 8 = 0$$

$$\sqrt{3} \left( \frac{1}{12} + \frac{1}{2} \right) - \frac{\sqrt{4}}{11} - \frac{\sqrt{2}}{2} + 10 = 0$$

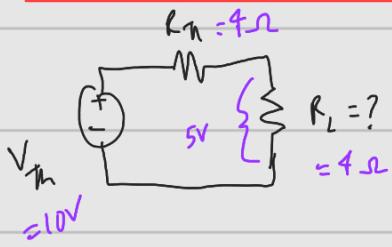
$$V_4=24$$

$$\therefore V_1 : V_f = 14V$$

$$\therefore R_{Th} = \frac{14}{1} = 14\Omega$$



## Maximum Power Transfer Theorem



It is when  $R_L = R_{Th}$

$$P_{L_{max}} = ? \quad P_{circuit} = ?$$

$$\text{Ex } P_{max} = \frac{V_L^2}{R_L} = \frac{5^2}{4} = 6.25W$$

$$(Load) \quad P_{max} = \frac{(V_{Th}^2)^2}{R_L} = \frac{V_{Th}^2}{4R_{Th}}$$