Lecture 6

CSE250 - Circuits and Electronics

NODAL ANALYSIS

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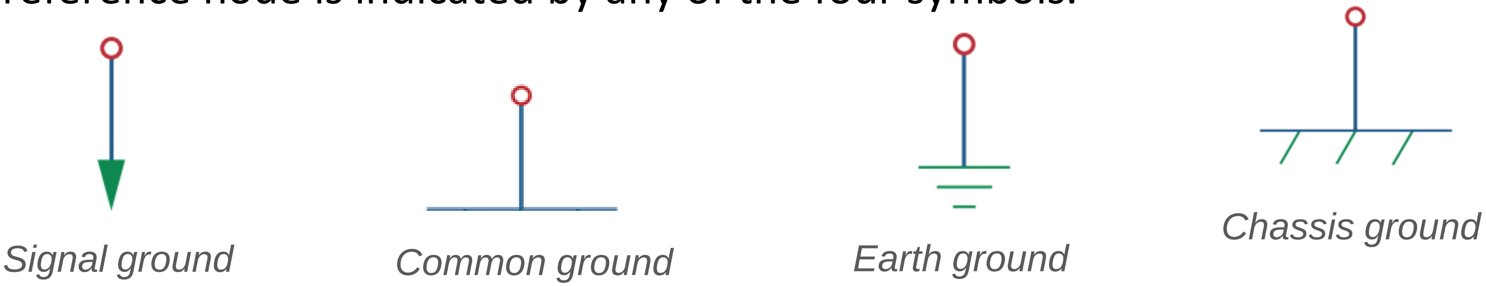
Ground

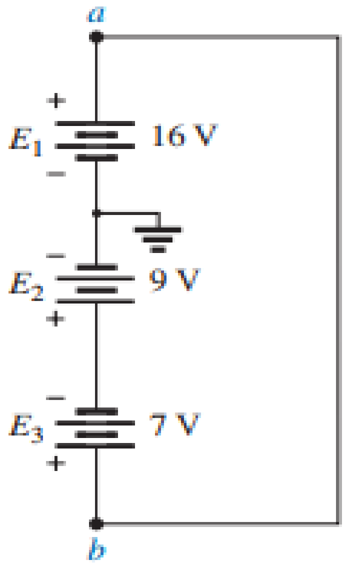
- Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes.
- It is called *ground* since it is assumed to have zero potential.
- In general, the placement of the ground connection will not affect the magnitude or polarity of the voltage across an element, but it may have a significant impact on the voltage from any point in the network to ground.
- A reference node is indicated by any of the four symbols.



Problem 1

For the series network shown below, determine,

i) The voltage V_a.
ii) The voltage V_b.
iii) The voltage V_{ab}



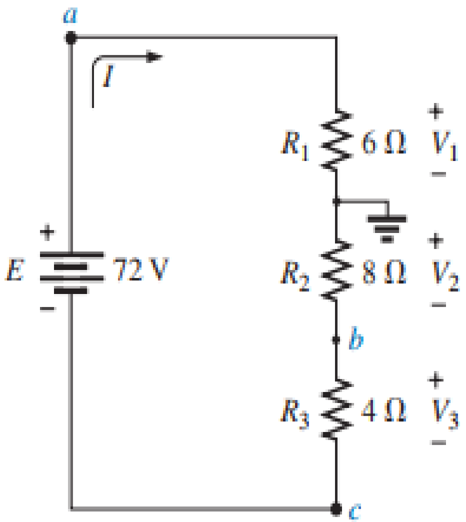
Hint: A node voltage is the potential difference between the given node and the reference node (ground in this case).

Ans: (i) $\boldsymbol{V_a = 16\,V}$
(ii) $\boldsymbol{V_b = 16\,V}$
(iii) $\boldsymbol{V_{ab} = 0\,V}$

Problem 2

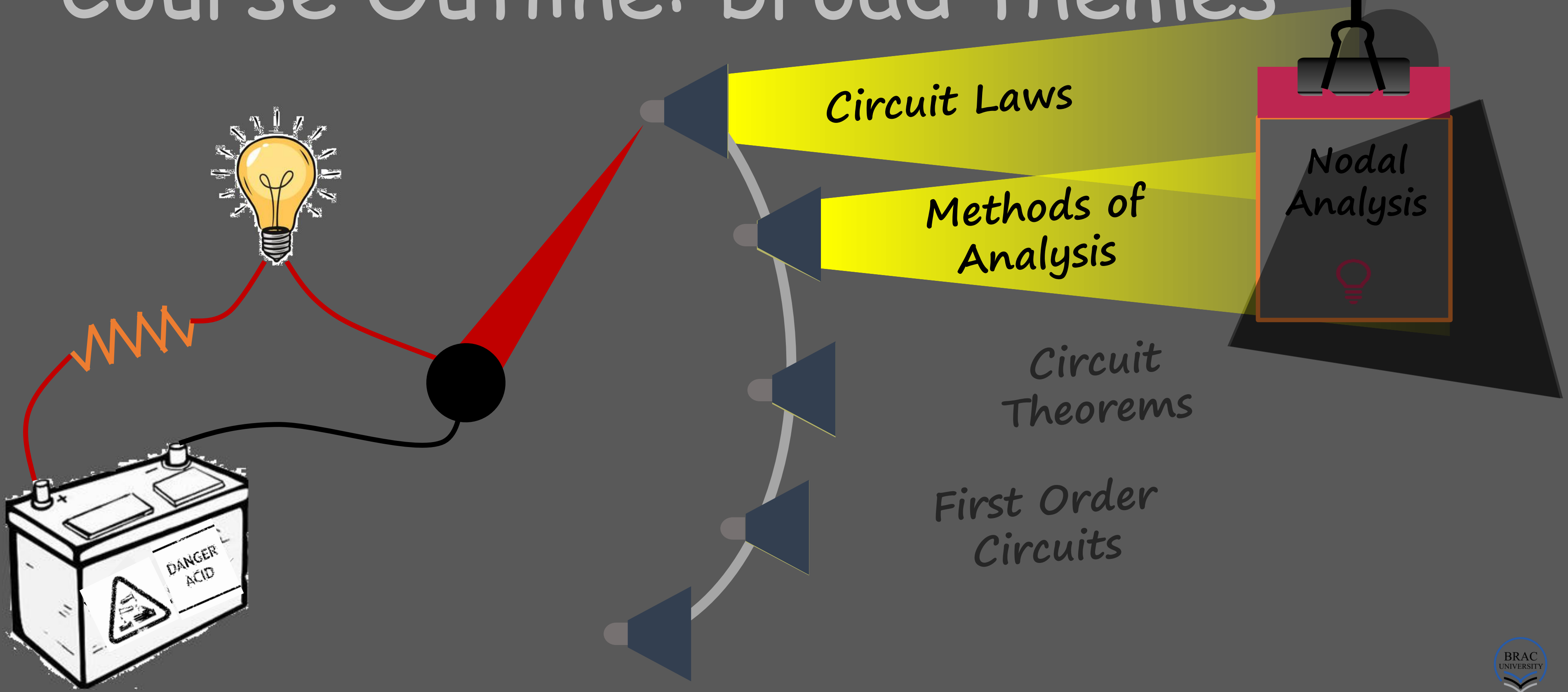
For the series network shown below, determine,

i) The voltage V_a.
ii) The voltages V_b and V_c
iii) The voltage V_{ab}



Ans: $(i)\ \boldsymbol{V_a = 24\ V}$
$(ii)\ \boldsymbol{V_b = -32\ V;\ V_c = -48\ V}$
$(iii)\ \boldsymbol{V_{ab} = 56\ V}$

Course Outline: broad themes



Nodal Analysis

- *Nodal analysis* provides a general procedure for analyzing circuits using node voltages as the circuit variables. Nodal analysis applies KCL to find unknown voltages in a given circuit.
- A *node voltage* is the potential difference between the given node and some other node that has been chosen as a reference node.
- *Remember that applying KCL to n-1 nodes produces n-1 variables and n-1 equations. As you will see, it is not necessary to apply KCL to every node in a circuit. So, being a little discreet can significantly reduce the number of variables. See an example.*
- *But first, we need to look at four cases.*

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

General Approach

Step 1: Identify all the nodes and place a ground

Recall that, 'Node' is a connection point of two or more branches. Make a node as the reference node. Appropriate placement of ground may provide advantage.

Step 2: Look for voltage sources directly connected to ground

If a voltage source is connected from a node to the ground, voltage of that node is equal to the value of the voltage source. Careful about the polarity of the node voltage.

Step 3: Selectively assign node variables

Assign node variables only to those remaining nodes where more than two branches are connected.

Step 4: Apply KCL

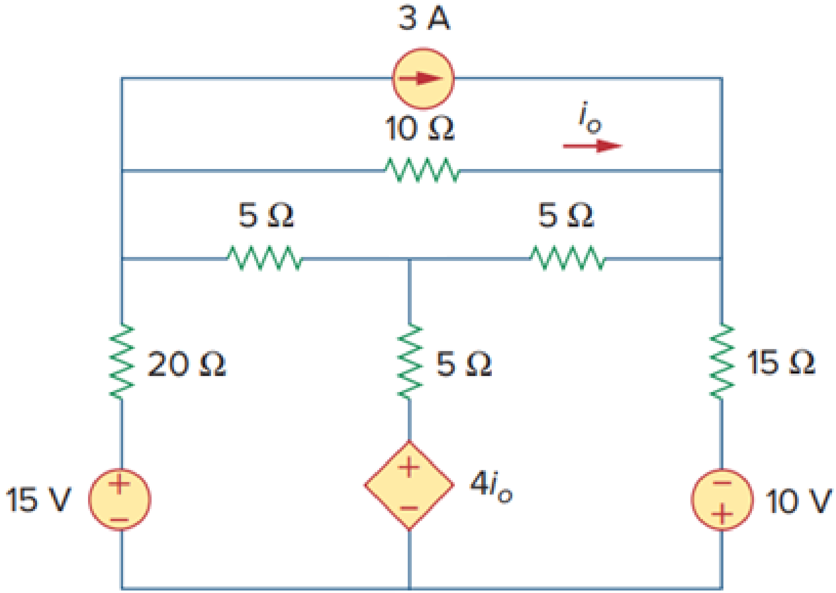
Apply KCL to the nodes where a variable is assigned. Follow the four cases. Look for Supernodes. No need to apply if a node voltage is known already.

Step 5: Solve the simultaneous equations

You should get a number of equations equal to the node variables in Step 4. Solve using a calculator.

Example 1

Use nodal analysis to determine the voltage across the $3\ A$ current source. What is the power of it? Is it absorbing or supplying?



Before solving the circuit using nodal analysis, remember that "*Current flows from a higher potential to a lower potential in a resistor*." This is true since resistor is a passive element, by the *passive sign convention*, current must always flow from a higher potential to a lower potential.

We can express this principle as,

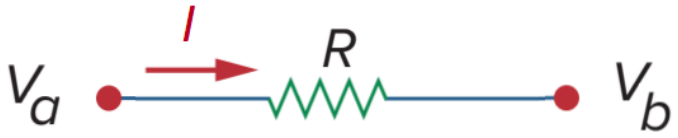
$$i = \frac{v_{higher} - v_{lower}}{R}$$

However, do we know which voltage is the higher one beforehand?

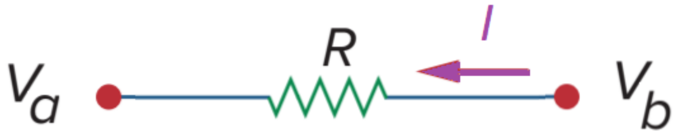
Case 1: R bet^n nodes

There are four scenarios that we may encounter while writing currents in terms of node voltages throughout the nodal analysis procedure. We will arbitrarily choose the direction of the current flowing through a wire.

- **Case 1** In case of only a resistor connected between two nodes of voltages V_a and V_b, the current, assumed to be flowing in a particular direction, can be written as,



$$\boxed{I = \frac{V_a - V_b}{R}}$$

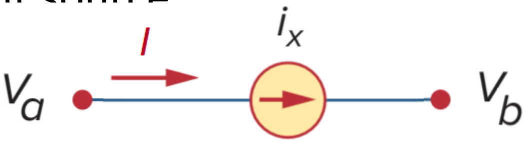


$$\boxed{I = \frac{V_b - V_a}{R}}$$

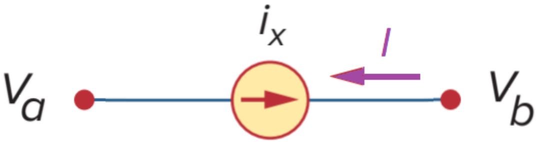
The actual direction of the current can be known after solving for the node voltages.

Case 2: current src betⁿ nodes

- **Case 2** In case of a current source connected between two nodes of voltages V_a and V_b, current flowing between the nodes will be equal to the current supplied by the current source.

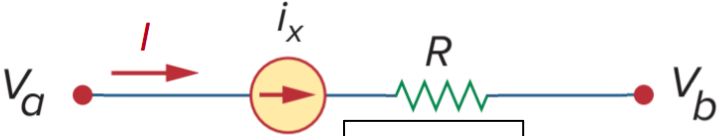


$\boxed{I = i_x}$

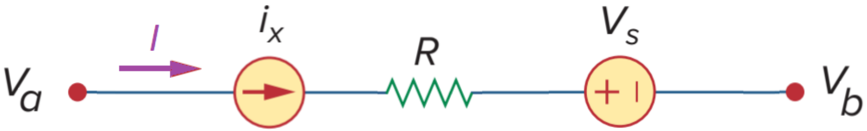


$\boxed{I = -i_x}$

If any other elements are connected in series with a current source, the current between the nodes will still be equal to the current supplied by the source.



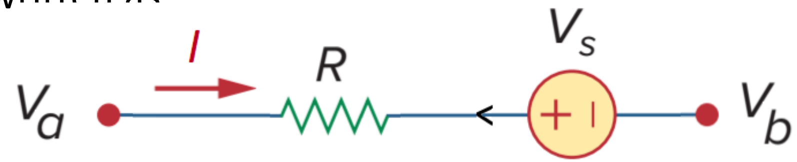
$\boxed{I = i_x}$



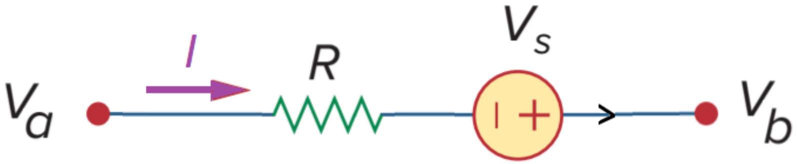
$\boxed{I = i_x}$

Case 3: R & V src in series bet^n nodes

- **Case 3** In case of a resistor and a voltage source in series connected between two nodes under consideration, the current, assumed to be flowing in a particular direction, can be written as



$$I = \frac{V_a - V_b - V_s}{R}$$

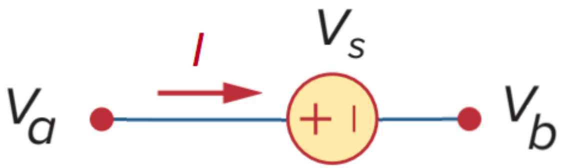


$$I = \frac{V_a - V_b + V_s}{R}$$

This is how we might perceive the scenario. We'll assume the current flows from V_a to V_b. Given that voltage sources tend to produce power, we add V_s with the term $(V_a - V_b)$ in the numerator if the current contributed by the source (indicated in black arrow) is in the same direction (from V_a to V_b), otherwise we deduct V_s.

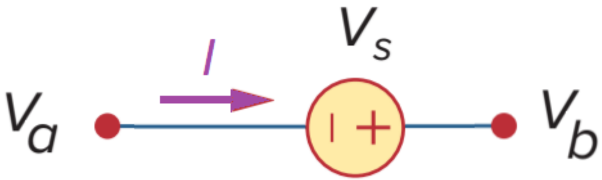
Case 4: voltage source bet^n nodes

- **Case 4** Because Ohm's Law cannot be applied in the absence of a resistor, in the case of a voltage source linked between two nodes, we don't know the current of a voltage source in advance. This is a unique case in which the condition is known as a *Supernode*. This is handled differently, as demonstrated by an example later. We may still write KVL equation as,



$$\boxed{I = ?}$$

$$\boxed{V_a - V_b = V_s}$$



$$\boxed{I = ?}$$

$$\boxed{V_a - V_b = -V_s}$$

Example 1 - 1/8

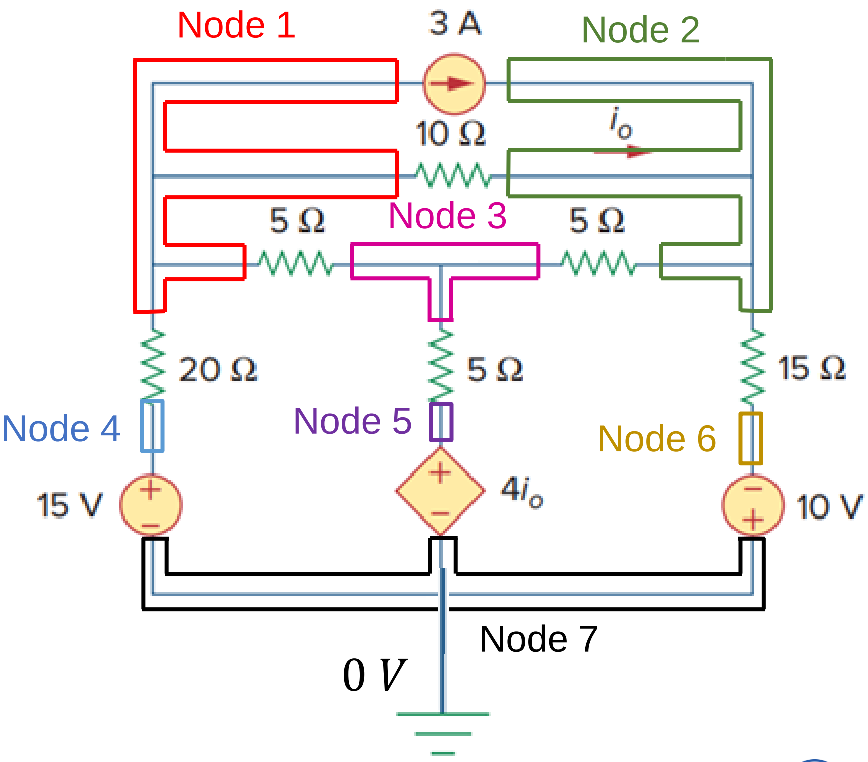
👉 First identify all the nodes in this circuit. Recall that, A *node* is the point of connection between two or more branches. A node is an equipotential portion of a circuit.

There are 7 nodes as identified in the circuit.

👉 Make one of the nodes as the reference node. It is most convenient (not mandatory) to choose the node that has the maximum number of circuit elements connected to it.

Let's assign the node 7 as the reference node.

👉 Place a ground to the reference node.



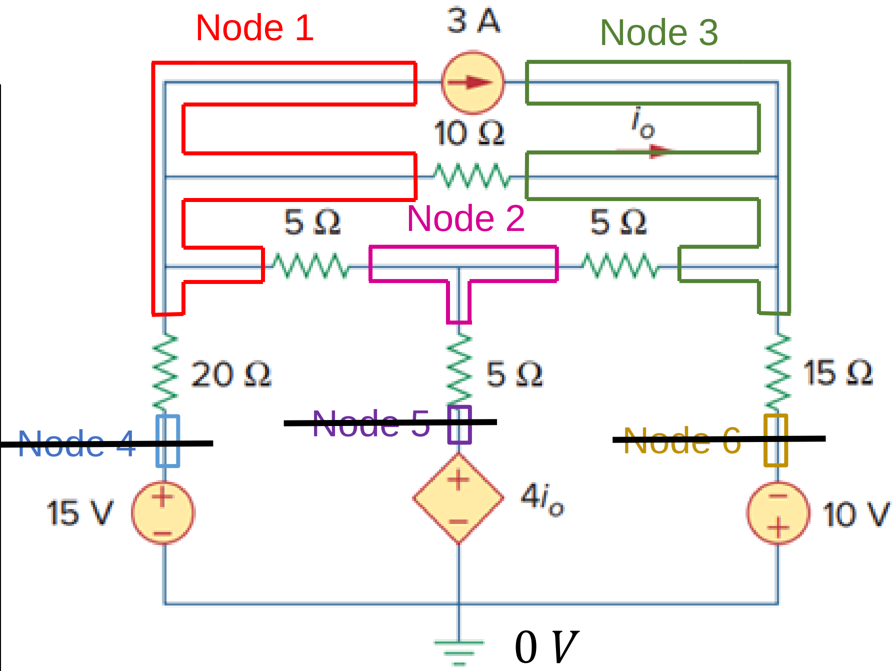
Example 1 - 2/8

👉 The 2nd step is to assign node variables to the remaining nodes.

There are 6 nodes apart from the ground.

We don't need to apply KCL separately to all the remaining nodes.

👉 One thumb rule is that, assign node variable (apply KCL) to the nodes where at least three or more branches are connected, if the node voltage is not already known.



This enables us to put the nodes 4, 5, and 6 out of consideration. Assign variables V_1, V_2, and V_3 to the nodes 1, 2, and 3 respectively.

Example 1 – 3/8

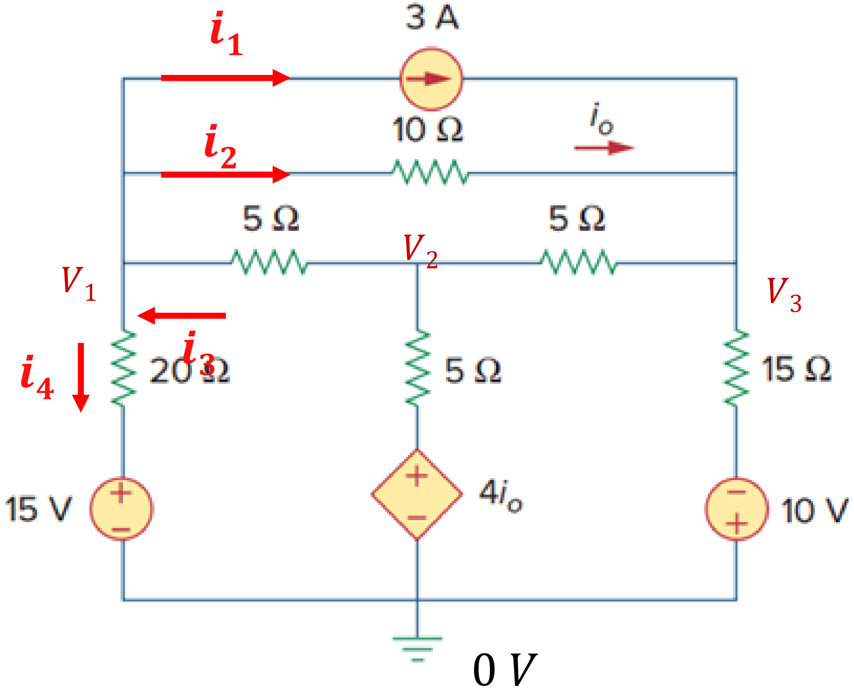
👉 The 3rd step is to apply KCL separately to each of the nodes in consideration.

<u>Applying KCL to the node 1</u>

Let's add currents to all the wires (4 wires) connected to node 1. The direction of the currents are taken arbitrarily.

According to the KCL,

$$\underbrace{i_1 + i_2 + i_4}_{\text{Sum of currents entering the node}} = \underbrace{i_3}_{\text{Sum of currents leaving the node}}$$



Example 1 - 4/8

$$i_1 + i_2 + i_4 = i_3$$

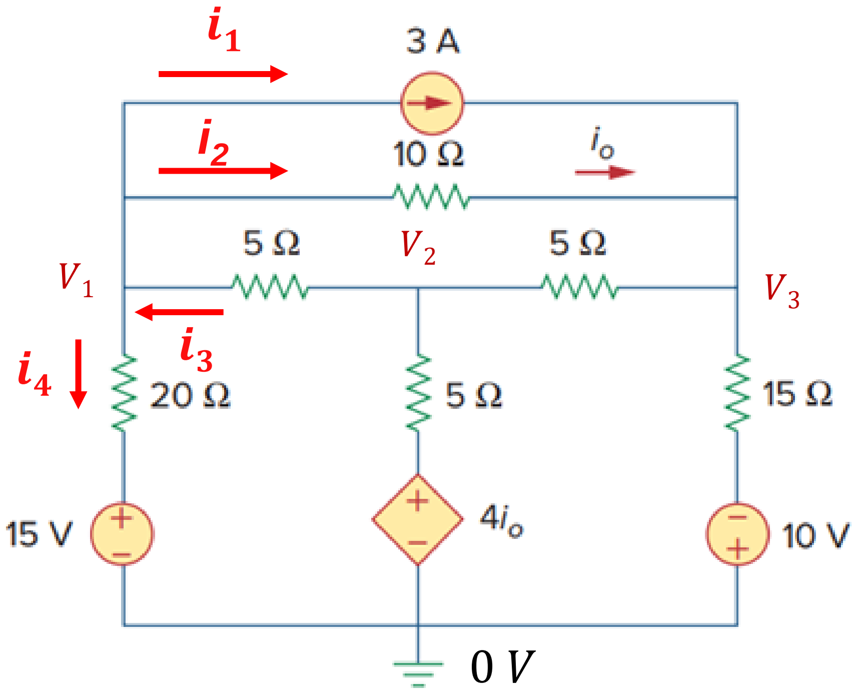
Now express the unknown currents in terms of node voltages and resistances using Ohm's law and recall the cases.

$$\underbrace{3}_{i_1} + \underbrace{\frac{V_1 - V_3}{10}}_{i_2} + \underbrace{\frac{V_1 - 0 - 15}{20}}_{i_4} - \underbrace{\frac{V_2 - V_1}{5}}_{i_3} = 0$$

case 2 case 1 case 3 case 2

Simplifying the equation yields,

$$7V_1 - 4V_2 - 2V_3 = -45 \quad \text{-------------} \quad (i)$$



Example 1 - 5/8

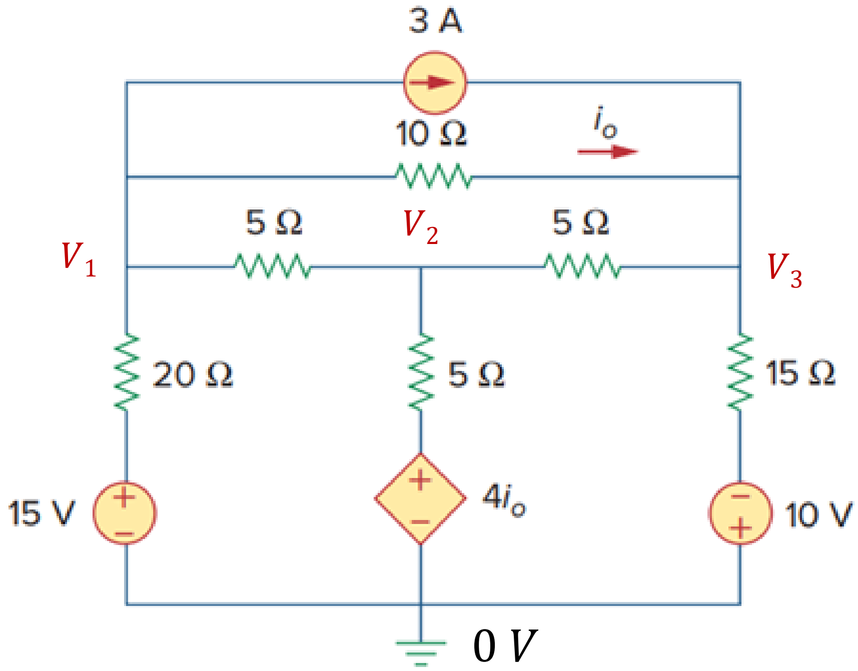
In a similar way, apply KCL to node 2

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 0 - 4i_0}{5} + \frac{V_2 - V_3}{5} = 0$$

where, all the currents are assumed to be leaving the node 2 (arbitrary assumption)

Due to the gain $(4i_0)$ of the dependent source, the parameter i_0 is present in the equation. We need to replace i_0 in terms of the node voltages. i_0 can be written as,

$$i_0 = \frac{V_1 - V_3}{10} \; [see\ the\ direction\ of\ i_0\ in\ the\ circuit\ diagram]$$



Example 1 - 6/8

Replace i_0 in the equation for node 2 by $\frac{V_1-V_3}{10}$

$$\frac{V_2-V_1}{5}+\frac{V_2-4\left(\frac{V_1-V_3}{10}\right)-0}{5}+\frac{V_2-V_3}{5}=0$$

Simplifying the equation yields,

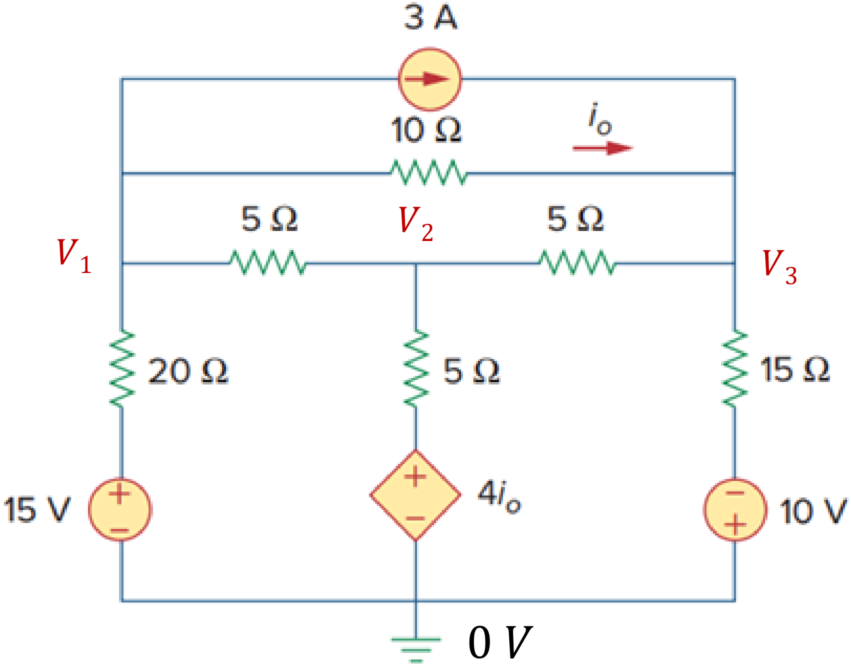
$$7V_1-15V_2+3V_3=0 \text{ ---------- } (ii)$$

Next, apply KCL to node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3-V_1}{10}+\frac{V_3-0+10}{15}=3$$

Or,

$$3V_1+6V_2-11V_3=-70 \text{ --------- } (iii)$$



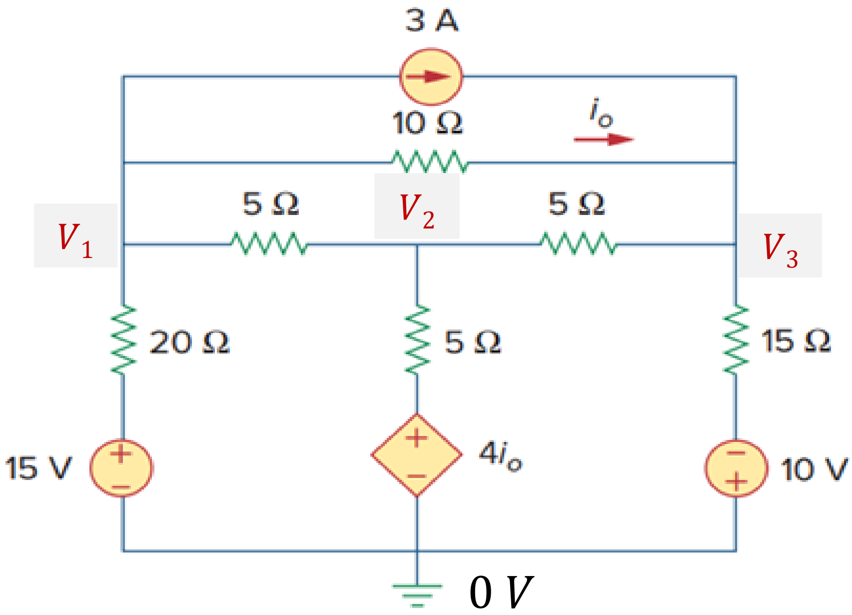
Example 1 - 7/8

We have derived the three node equations consisting of three variables.

$$7V_1 - 4V_2 - 2V_3 = -45$$
$$7V_1 - 15V_2 + 3V_3 = 0$$
$$3V_1 + 6V_2 - 11V_3 = -70$$

Solving the three simultaneous equations yields,

$$V_1 = -7.19\ V$$
$$V_2 = -2.78\ V$$
$$V_3 = 2.89\ V$$



Example 1 - 8/8

Determining the voltage and power of the 3 A source.

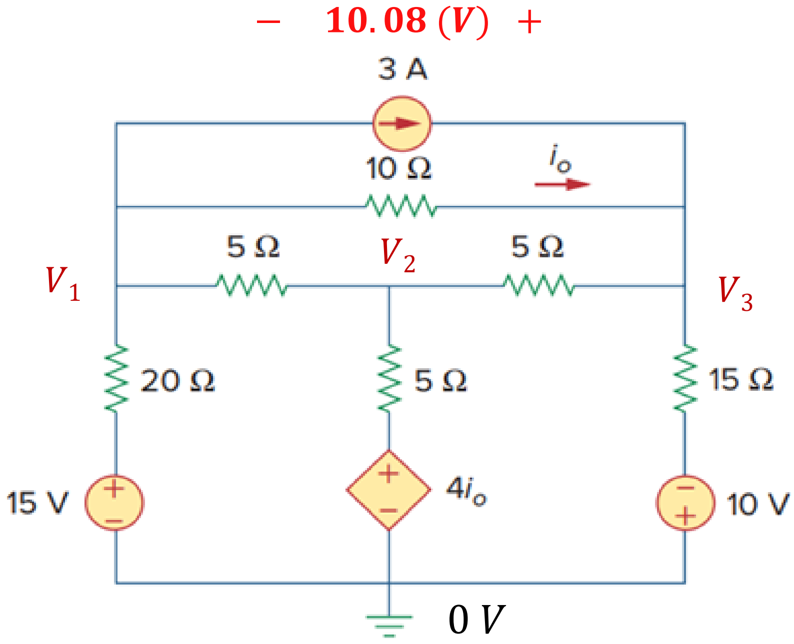
The voltage across the voltage 3 A source is either $V_1 - V_3$ or $V_3 - V_1$. With $V_3 > V_1$, we calculate the voltage as a positive quantity to be,

$$V_3 - V_1 = 2.89 - (-7.19) = 10.08\ V$$

The polarity of the voltage is such that V_3 is at a higher potential than V_1, as shown in the figure.

According to the passive sign convention, the power supplied by the 3A source is thus,

$$p = -10.08 \times 3 = -30.24\ (Watt)$$



Format Approach

- Nodal analysis using *Format approach* allows to write nodal equations rapidly and in a form that is convenient for the use of determinants.
- The first node equation from *Example 1* can be written in this form,

$$3+\frac{V_1-V_3}{10}+\frac{V_1-0-15}{20}-\frac{V_2-V_1}{5}=0\ (from\ example\ 1)$$

$$\Downarrow$$

$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{15}{20}-\frac{V_2}{5}-\frac{V_3}{10}+3=0$$

- Note that, each node voltage variable is multiplied by the sum of the conductances (reciprocal of R) attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the same side of the equals sign with a positive sign if they leaves the node and with a negative sign if they draw enter to the node. So, to summarize the procedure …

Format Approach: procedure

- **Steps**

1. Choose a reference node and assign a subscripted voltage label to all the $(N - 1)$ remaining nodes of the network.
2. The number of equations required for a complete solution is equal to the number of subscripted voltages $(N - 1)$. Column 1 of each equation is formed by summing the conductances (reciprocal of R) tied to the node of interest and multiplying the result by that subscripted nodal voltage.
3. We must now consider the mutual terms, which, as noted in the preceding slide, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This is demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage, tied to that conductance.
4. A current source is assigned a positive sign if it draws current from a node and negative sign if it supplies current from the node.
5. Solve the resulting simultaneous equations for the desired voltages.

Example 2 - 1/5

👉 Identify all the nodes and label them (with ground being the 0th node).

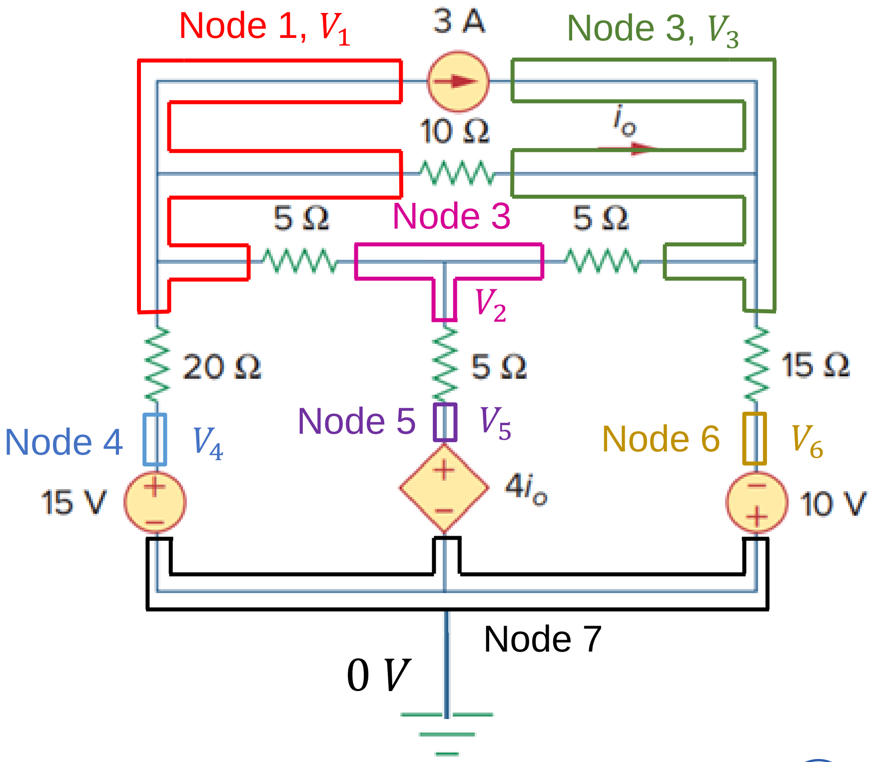
👉 Write the component equations for all the voltage sources (voltage difference = labeled variable).

$$V_4 = 15\ V -----(i)$$

$$V_5 = 4i_0 = 4 \times \frac{V_1 - V_3}{10}$$

$$\Rightarrow \quad 4V_1 - 4V_3 - 10V_5 = 0 -----(ii)$$

$$V_6 = -10\ V -----(iii)$$



Example 2 - 2/5

👉 Node equation formation.

***Node* 1, V_1** : There are 3 resistors (20 Ω, 5 Ω, 10 Ω) connected to V_1. We write,

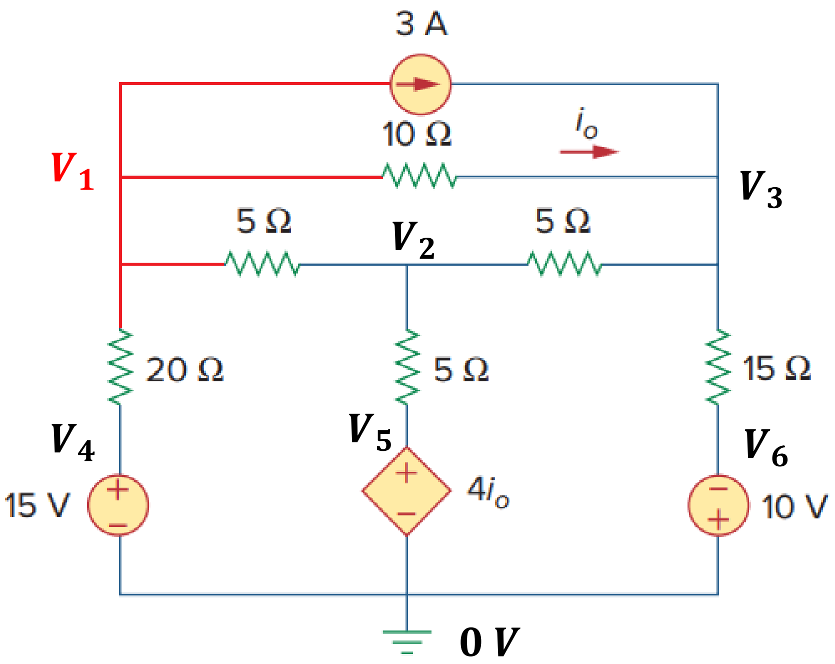
$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)\ldots=0$$

The other end of the 20 Ω, 5 Ω, and 10 Ω resistors are connected to the nodes V_4, V_2, and V_3 respectively. So, we subtract,

$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{V_4}{20}-\frac{V_2}{5}-\frac{V_3}{10}\ldots=0$$

Finally, we subtract any currents entering to that node (or add if leaving),

$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{V_4}{20}-\frac{V_2}{5}-\frac{V_3}{10}+3=0$$



Example 2 - 3/5

Substituting $15\ V$ for V_4 from equation (i),

$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{15}{20}-\frac{V_2}{5}-\frac{V_3}{10}+3=0$$

$$\Rightarrow\quad V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{V_3}{10}-\frac{V_2}{5}=-\frac{9}{4}$$

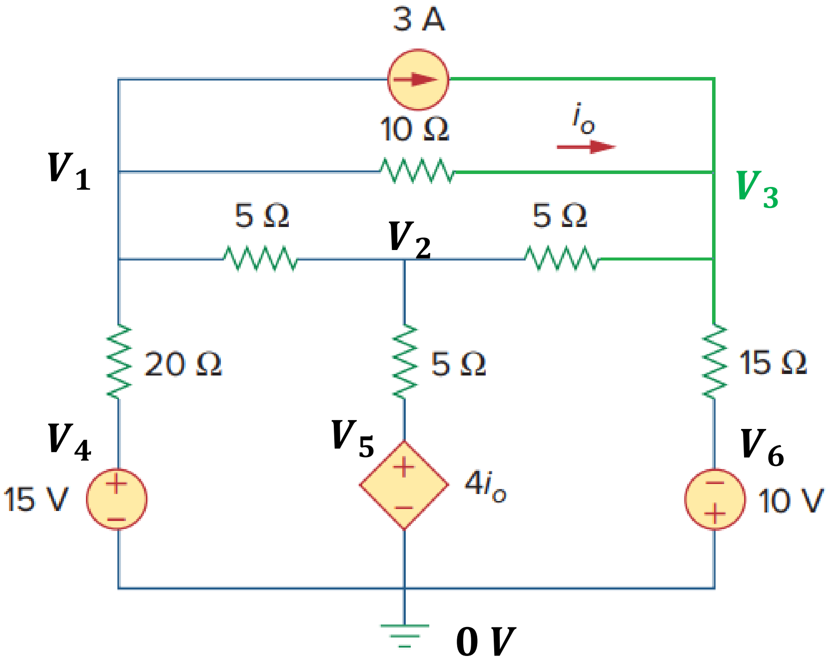
$$------(iv)$$

Node 3, V_3: Similarly, $10\ \Omega, 5\ \Omega$, and $15\ \Omega$ resistors are connected between V_3 and V_1, V_3 and V_2, and V_3 and V_6 respectively. Also, the $3\ A$ current is entering to V_3.

$$V_3\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{15}\right)-\frac{V_1}{10}-\frac{V_2}{5}-\frac{V_6}{15}-3=0$$

Substituting $-10\ V$ for V_6 from equation (iii),

$$V_3\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{15}\right)-\frac{V_1}{10}-\frac{V_2}{5}=\frac{7}{3}-----(v)$$



Example 2 - 4/5

Node 2, V_2 : Similarly, three $5\,\Omega$ resistors are connected between V_2 and V_1, V_2 and V_3, and V_2 and V_5 respectively. So,

$$V_2\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\right)-\frac{V_1}{5}-\frac{V_3}{5}-\frac{V_5}{5}=0$$

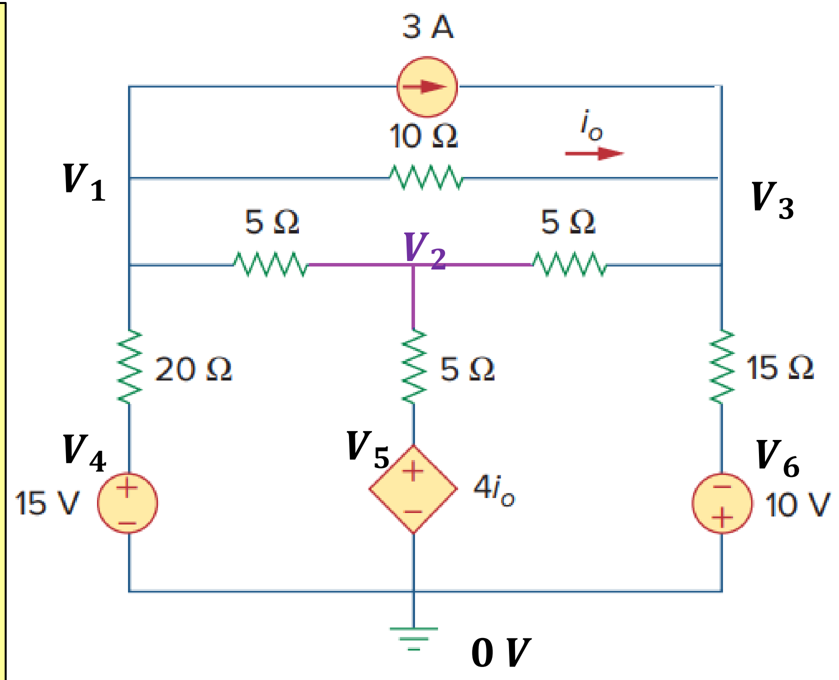
From equation (ii),

$$V_5=\frac{4V_1-4V_3}{10}$$

Substituting for V_5 from equation (ii),

$$V_2\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\right)-\frac{V_1}{5}-\frac{V_3}{5}-\frac{4V_1-4V_3}{10\times 5}=0$$

$$V_2\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\right)-V_1\left(\frac{1}{5}+\frac{2}{25}\right)-V_3\left(\frac{1}{5}-\frac{2}{25}\right)=0 \quad ---------(vi)$$



Example 2 - 5/5

We got three equations with three variables.

$$V_1\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right)-\frac{V_3}{10}-\frac{V_2}{5}=-\frac{9}{4}$$

$$V_3\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{15}\right)-\frac{V_1}{10}-\frac{V_2}{5}=\frac{7}{3}$$

$$V_2\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\right)-V_1\left(\frac{1}{5}+\frac{2}{25}\right)-V_3\left(\frac{1}{5}-\frac{2}{25}\right)=0$$

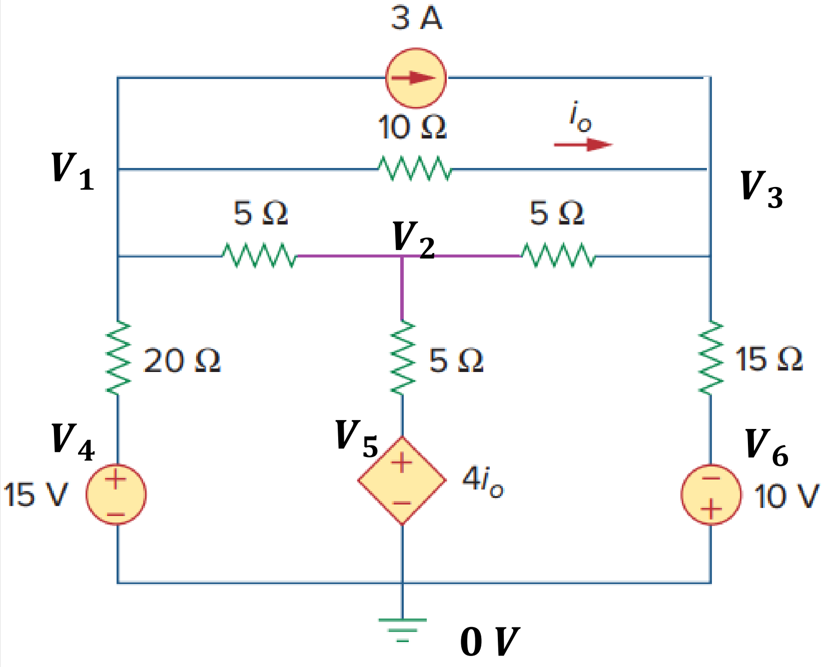
Solving the three equations we get,

$$V_1=-7.19\ V$$

$$V_2=-2.78\ V$$

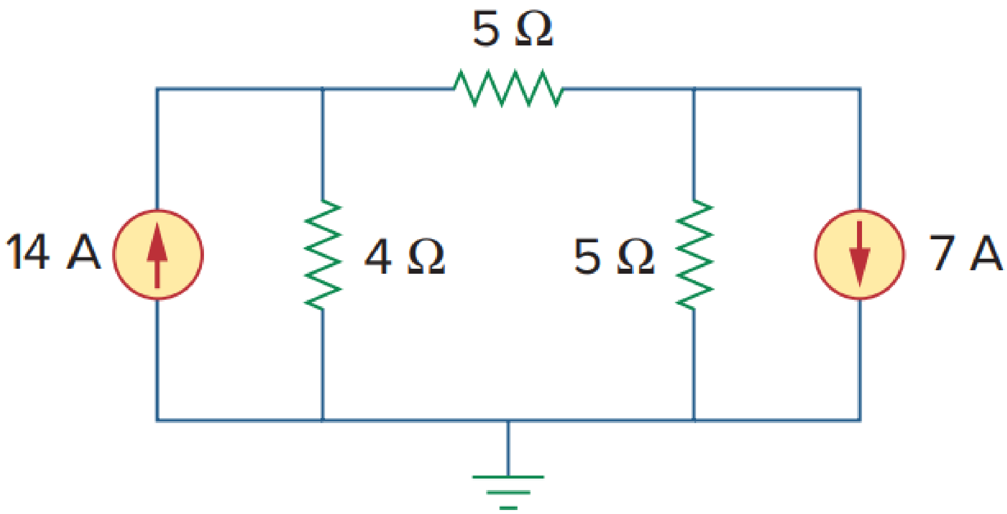
$$V_3=2.89\ V$$

$$i_0=\frac{V_1-V_3}{10}=-1.008\ A$$



Problem 3

- Find all the node voltages.



Ans: $\mathbf{0\ V;\ 30\ V;\ -2.5\ V}$

Solution to Problem 3

Applying KCL at node 1,

$$\frac{V_1 - V_2}{5} + \frac{V_1}{4} - 14 = 0$$

$$\Rightarrow \frac{V_1}{5} + \frac{V_1}{4} - \frac{V_2}{5} - 14 = 0$$

$$\Rightarrow \frac{9}{20} V_1 - \frac{1}{5} V_2 = 14 \quad \text{.......... (i)}$$

Applying KCL at node 2,

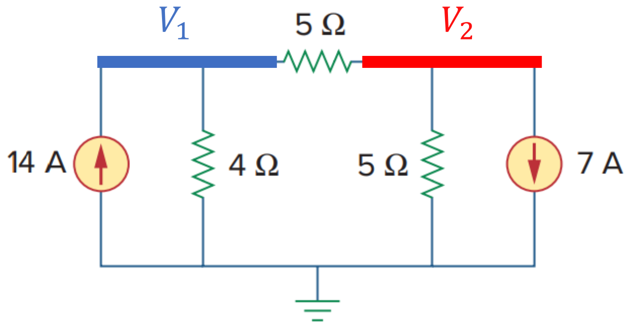
$$\frac{V_2 - V_1}{5} + \frac{V_2}{5} + 7 = 0$$

$$\Rightarrow \frac{V_2}{5} + \frac{V_2}{5} - \frac{V_1}{5} + 7 = 0$$

$$\Rightarrow -\frac{1}{5} V_1 + \frac{2}{5} V_2 = -7 \quad \text{.......... (ii)}$$

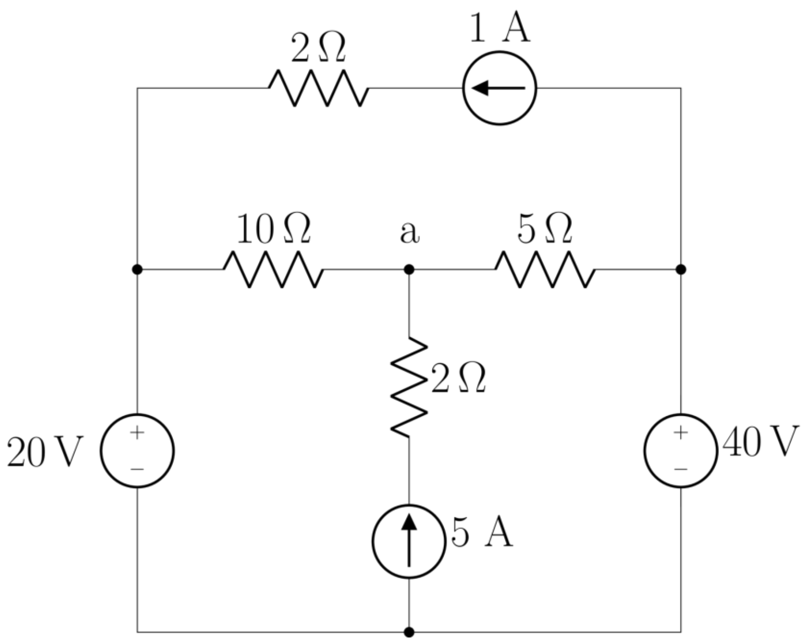
Solving (i) and (ii),

$$\boxed{\begin{aligned} V_1 &= 30\,V \\ V_2 &= -2.5\,V \end{aligned}}$$



Problem 4

- Find the voltage of node a using nodal analysis.



Note that, the problem does not have a specific answer as the node voltage depends on the placement of ground.

- If the ground is placed on node a, then $V_a = 0\ V$.
- If the ground is placed on the bottom-most node, then $V_a = 50\ V$.

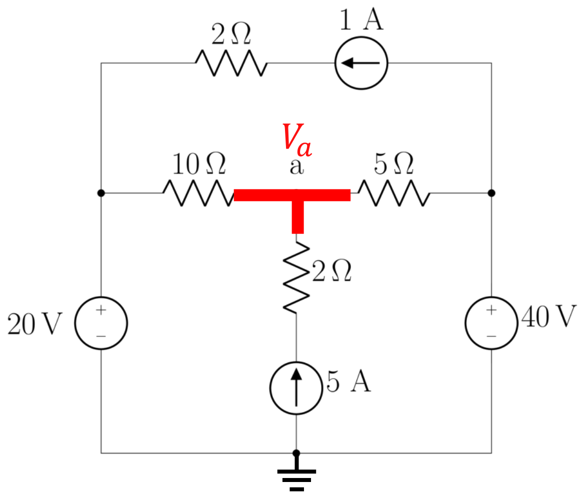
So, node voltages depend on the position of the ground, however, elemental voltages do not. Wherever the ground is placed, voltage across the elements and their currents will be the same.

Applying KCL at node a,

$$\frac{V_a - 20}{10} + \frac{V_a - 40}{5} - 5 = 0$$

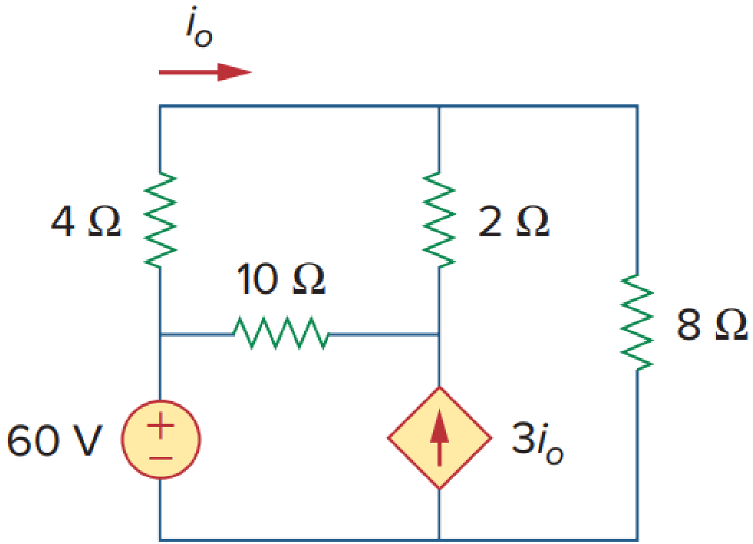
$$\Rightarrow \frac{V_a}{10} - \frac{20}{10} + \frac{V_a}{5} - \frac{40}{5} - 5 = 0$$

$$\Rightarrow \boldsymbol{V_a = 50\,V}$$



Problem 5

- Find i_0 using nodal analysis. Determine the current supplied by the $60\ V$ source.



Ans: $\boldsymbol{i_0 = 1.73\ A; 1.262\ A}$

From the figure,

$$i_o = \frac{60 - V_1}{4}$$

Applying KCL at node 1,

$$\frac{V_1 - 60}{4} + \frac{V_1 - V_2}{2} + \frac{V_1}{8} = 0$$

$$\Rightarrow \frac{V_1}{4} + \frac{V_1}{2} + \frac{V_1}{8} - \frac{60}{4} - \frac{V_2}{2} = 0$$

$$\Rightarrow \frac{7}{8}V_1 - \frac{1}{2}V_2 = 15 \text{ ……….. (i)}$$

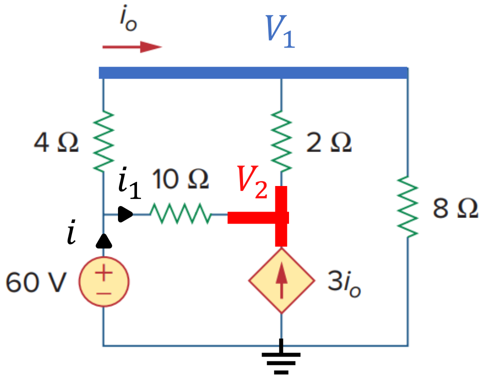
Applying KCL at node 2,

$$\frac{V_2 - 60}{10} + \frac{V_2 - V_1}{2} - 3i_o = 0$$

$$\Rightarrow \frac{V_2 - 60}{10} + \frac{V_2 - V_1}{2} - 3\frac{60 - V_1}{4} = 0$$

$$\Rightarrow \frac{V_2}{10} + \frac{V_2}{2} - \frac{60}{10} - \frac{V_1}{2} - 3 \times \frac{60}{4} + 3\frac{V_1}{4} = 0$$

$$\Rightarrow \frac{1}{4}V_1 + \frac{3}{5}V_2 = 51 \text{ ……….. (ii)}$$



Solving (i) and (ii),

$$\boxed{\begin{aligned} V_1 &= 53.07\ V \\ V_2 &= 62.89\ V \end{aligned}}$$

Now,

$$i_o = \frac{60 - V_1}{4} = \frac{60 - 53.07}{4} = 1.73\ A$$

$$i_1 = \frac{60 - V_2}{10} = \frac{60 - 62.89}{10} = -0.289\ A$$

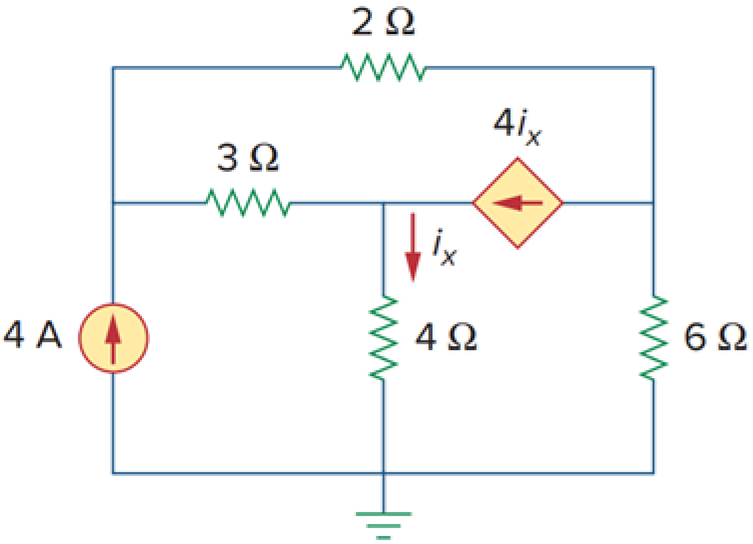
$$i = i_0 + i_1$$

$$i = 1.73 - 0.289$$

$$\Rightarrow \boldsymbol{i = 1.441\ A}$$

Problem 6

- Find the node voltages.



Ans: $\mathbf{0\ V};\ \mathbf{32\ V};\ -\mathbf{25.6\ V};\ \mathbf{62.4\ V};$

Solution to Problem 6

From the figure,

$$i_x = \frac{V_2}{4}$$

Applying KCL at node 1,

$$\frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3} - 4 = 0$$

$$\Rightarrow \frac{V_1}{2} + \frac{V_1}{3} - \frac{V_2}{3} - \frac{V_3}{2} - 4 = 0$$

$$\Rightarrow \frac{5}{6}V_1 - \frac{1}{3}V_2 - \frac{V_3}{2} = 4 \qquad \ldots\ldots\ldots \text{(i)}$$

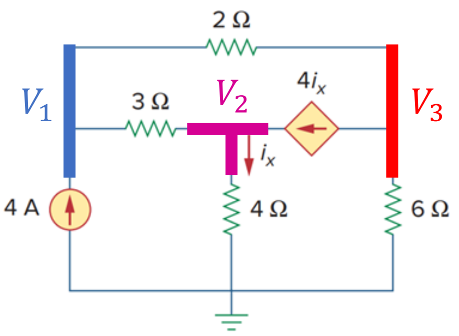
Applying KCL at node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{4} - 4i_x = 0$$

$$\Rightarrow \frac{V_2 - V_1}{3} + \frac{V_2}{4} - 4\frac{V_2}{4} = 0$$

$$\Rightarrow -\frac{V_1}{3} + \frac{V_2}{3} + \frac{V_2}{4} - V_2 = 0$$

$$\Rightarrow -\frac{1}{3}V_1 - \frac{5}{12}V_2 = 0 \qquad \ldots\ldots\ldots\ldots \text{(ii)}$$



Applying KCL at node 3,

$$\frac{V_3 - V_1}{2} + \frac{V_3}{6} + 4i_x = 0$$

$$\Rightarrow \frac{V_3 - V_1}{2} + \frac{V_3}{6} + 4\frac{V_2}{4} = 0$$

$$\Rightarrow -\frac{V_1}{2} + V_2 + \frac{V_3}{2} + \frac{V_3}{6} = 0$$

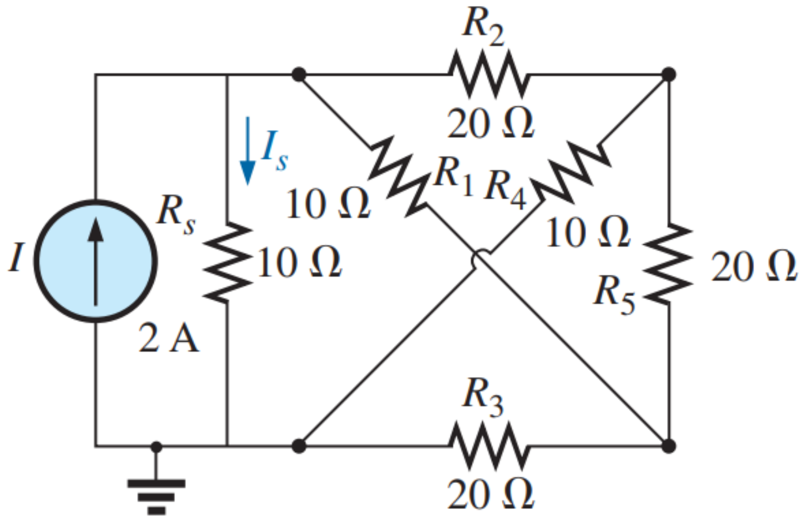
$$\Rightarrow -\frac{1}{2}V_1 + V_2 + \frac{2}{3}V_3 = 0 \qquad \ldots\ldots\ldots\ldots \text{(iii)}$$

Solving (i), (ii) and (iii),

$$\boxed{\begin{aligned} V_1 &= 32\ V \\ V_2 &= -25.6\ V \\ V_3 &= 62.4\ V \end{aligned}}$$

Problem 7

- Determine the current through the source resistor R_s using nodal analysis.



Ans: $i_s = 1.18\ A$

Solution to Problem 7

Applying KCL at node 1,

$$\frac{V_1}{10}+\frac{V_1-V_2}{20}+\frac{V_1-V_3}{10}-2=0$$

$$\Rightarrow \frac{V_1}{10}+\frac{V_1}{20}+\frac{V_1}{10}-\frac{V_2}{20}-\frac{V_3}{10}-2=0$$

$$\Rightarrow \frac{1}{4}V_1-\frac{1}{20}V_2-\frac{1}{10}V_3=2 \quad \text{........ (i)}$$

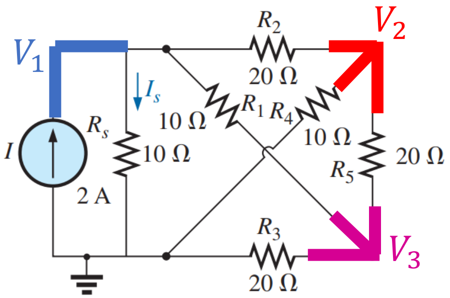
Applying KCL at node 2,

$$\frac{V_2}{10}+\frac{V_2-V_1}{20}+\frac{V_2-V_3}{20}=0$$

$$\Rightarrow -\frac{1}{20}V_1+\frac{V_2}{10}+\frac{V_2}{20}+\frac{V_2}{20}-\frac{V_3}{20}=0$$

$$\Rightarrow -\frac{1}{20}V_1+\frac{1}{5}V_2-\frac{1}{20}V_3=0 \quad \text{........ (ii)}$$

Applying KCL at node 3,



$$\frac{V_3}{20}+\frac{V_3-V_1}{10}+\frac{V_3-V_2}{20}=0$$

$$\Rightarrow -\frac{V_1}{10}-\frac{V_2}{20}+\frac{V_3}{20}+\frac{V_3}{10}+\frac{V_3}{20}=0$$

$$\Rightarrow -\frac{1}{10}V_1-\frac{1}{20}V_2+\frac{1}{5}V_3=0 \quad \text{.......... (iii)}$$

Solving (i), (ii) and (iii),

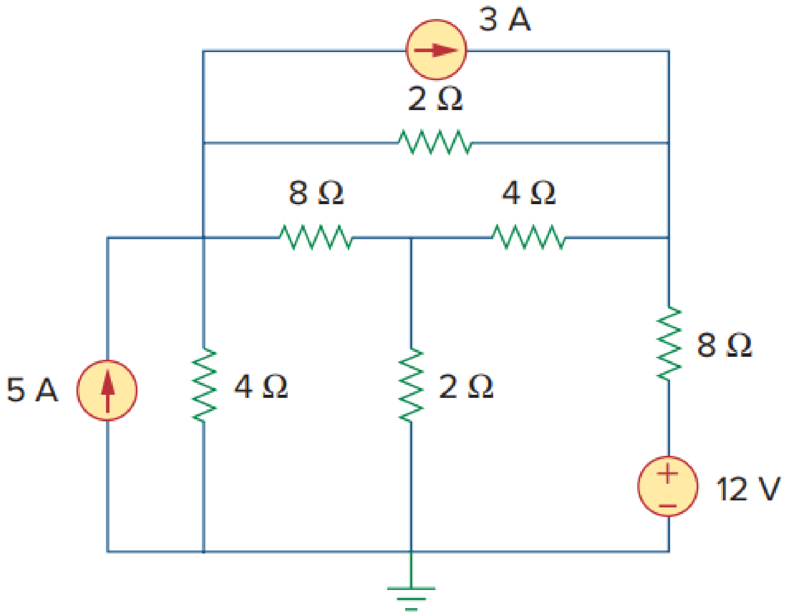
$$\boxed{\begin{aligned} V_1 &= 11.76\,V \\ V_2 &= 4.70\,V \\ V_3 &= 7.06\,V \end{aligned}}$$

Current through R_s resistor,

$$I_s=\frac{V_1}{10}=\frac{11.76}{10}=\mathbf{1.176\,A}$$

Problem 8

Use nodal analysis to determine the voltage across the $3\ A$ current source. What is the power of it? Is it absorbing or supplying?



Ans: Node voltages = $\mathbf{0\ V;\ 10\ V;\ 4.933\ V;\ 12.267\ V}$

$$\boldsymbol{v_{3A} = \pm 2.267\ V}$$

$$\boldsymbol{P_{3A} = -6.801\ W,}\ \textbf{Supplying}$$

Prepared By [PDS] Purbayan Das

Applying KCL at node 1,

$$\frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{2} + \frac{V_1}{4} + 3 - 5 = 0$$

$$\Rightarrow \frac{V_1}{2} + \frac{V_1}{8} + \frac{V_1}{4} - \frac{V_2}{8} - \frac{V_3}{2} - 2 = 0$$

$$\Rightarrow \frac{7}{4}V_1 - \frac{1}{8}V_2 - \frac{1}{2}V_3 = 2 \qquad \text{........ (i)}$$

Applying KCL at node 2,

$$\frac{V_2 - V_1}{8} + \frac{V_2 - V_3}{4} + \frac{V_2}{2} = 0$$

$$\Rightarrow -\frac{V_1}{8} + \frac{V_2}{8} + \frac{V_2}{4} + \frac{V_2}{2} - \frac{V_3}{4} = 0$$

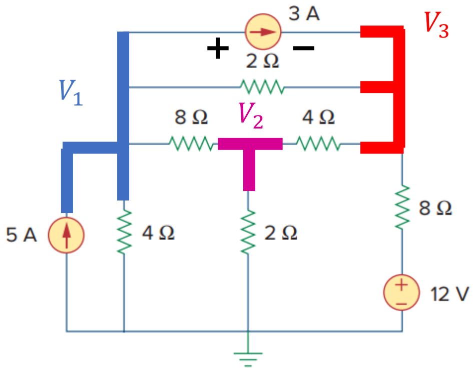
$$\Rightarrow -\frac{1}{8}V_1 + \frac{7}{4}V_2 - \frac{1}{4}V_3 = 0 \qquad \text{........ (ii)}$$

Applying KCL at node 3,

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{4} + \frac{V_3 - 12}{8} - 3 = 0$$

$$\Rightarrow -\frac{V_1}{2} - \frac{V_2}{4} + \frac{V_3}{2} + \frac{V_3}{4} + \frac{V_3}{8} - \frac{12}{8} - 3 = 0$$

$$\Rightarrow -\frac{1}{2}V_1 - \frac{1}{4}V_2 + \frac{7}{5}V_3 = \frac{9}{2} \qquad \text{.......... (iii)}$$



Solving (i), (ii) and (iii),

$$\boxed{\begin{aligned} V_1 &= 10\ V \\ V_2 &= 4.93\ V \\ V_3 &= 12.26\ V \end{aligned}}$$

Voltage across 3A current source

$$v_{3A} = V_1 - V_3$$

$$\Rightarrow v_{3A} = 10 - 12.26 = \mathbf{-2.267\ V}$$

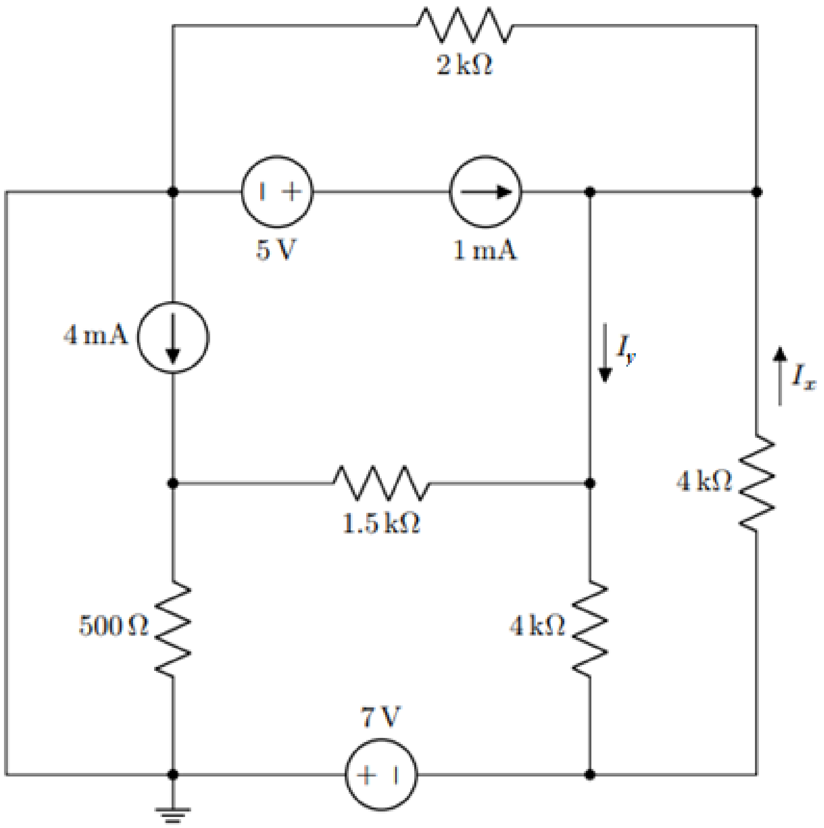
Power of 3A current source

$$P_{3A} = VI = v_{3A} \times 3$$

$$\Rightarrow P_{3A} = -2.267 \times 3 = \mathbf{-6.801\ W}$$

Problem 9

- Use nodal analysis to analyze the circuit. Find I_x.
- Determine the current I_y.



Ans: $I_x = -1.5\ mA; I_y = 0\ mA$

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Solution to Problem 9

Applying KCL at node 1,

$$\frac{V_1 - V_2}{1.5} + \frac{V_1}{0.5} - 4 = 0$$

$$\Rightarrow \frac{V_1}{1.5} + \frac{V_1}{0.5} - \frac{V_2}{1.5} - 4 = 0$$

$$\Rightarrow \frac{8}{3}V_1 - \frac{2}{3}V_2 = 4 \quad \text{........ (i)}$$

Applying KCL at node 2,

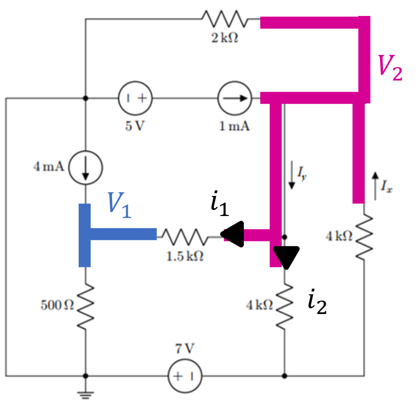
$$\frac{V_2 - V_1}{1.5} + \frac{V_2 - (-7)}{4} + \frac{V_2 - (-7)}{4} + \frac{V_2}{2} - 1 = 0$$

$$\Rightarrow -\frac{V_1}{1.5} + \frac{V_2}{1.5} + \frac{V_2}{4} + \frac{V_2}{4} + \frac{V_2}{2} + \frac{7}{4} + \frac{7}{4} - 1 = 0$$

$$\Rightarrow -\frac{2}{3}V_1 + \frac{5}{3}V_2 - \frac{1}{4}V_3 = -\frac{5}{2} \quad \text{........ (ii)}$$

Solving (i) and (ii)

$$\boxed{\begin{aligned} V_1 &= 1.25\ V \\ V_2 &= -1\ V \end{aligned}}$$



Value of I_x

$$I_x = \frac{-7 - V_2}{4} = \frac{-7 - (-1)}{4} = \mathbf{-1.5\ mA}$$

To find I_y, let,

$$I_y = i_1 + i_2$$

Now,

$$i_1 = \frac{V_2 - V_1}{1.5} = \frac{-1 - 1.25}{1.5} = -1.5\ mA$$

$$i_2 = \frac{V_2 - (-7)}{4} = \frac{-1 + 7}{4} = 1.5\ mA$$

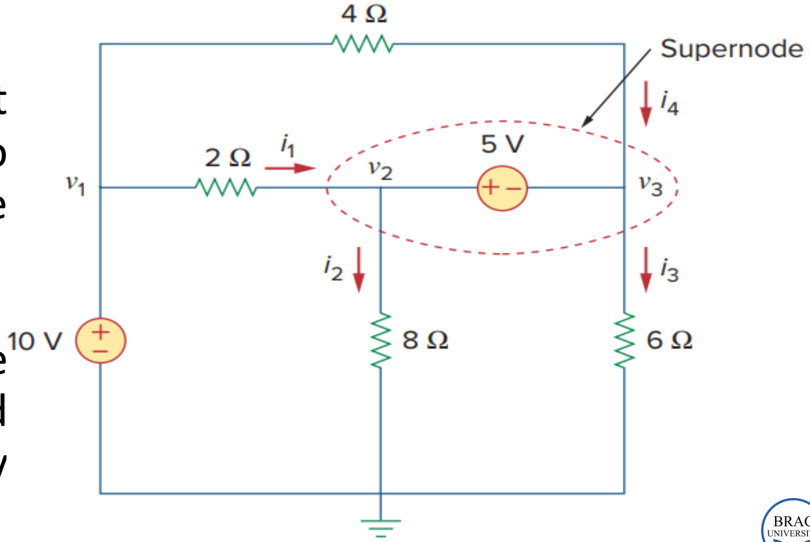
$$I_y = -1.5 + 1.5 = \mathbf{0\ mA}$$

Nodal with Voltage Src bet^n Nodes: (Case 4)

- **Scenario 1** If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. For example. $v_1 = 10\ V$.

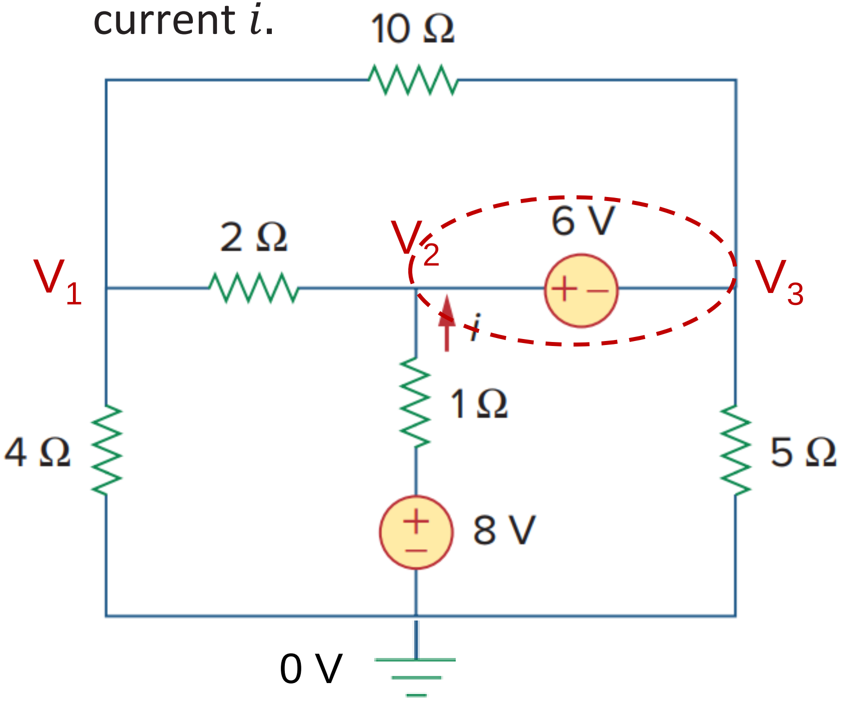
- **Scenario 2** If a voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode.

A *supernode* is formed when a voltage source (dependent or independent) is connected between two nonreference nodes and any elements connected in parallel with it.



Example 3: General Approach - 1/8

Use nodal analysis to determine the current i.



<u>Step 1</u>: Select a node as the reference node and place a ground to that node.

<u>Step 2</u>: Assign node variables to the remaining nodes.

Check for supernodes. Check if a voltage source (dependent or independent) is connected between two nonreference nodes under consideration. There can be multiple supernodes in a circuit.

In this circuit, the 6 V voltage source forms a supernode between nodes 2 and 3.

Example 3 - 2/8

Use nodal analysis to determine the current i.

We need to handle such conditions differently because there is no way to know the current through a voltage source in advance.

Consider the supernode as a "Whole" node and apply KCL to the node ignoring the source forming supernode and anything in parallel with it. There are 4 wires connected to the supernode, therefore, the KCL equation for the supernode should contain 4 terms.

V_1 V_2 V_3

Applying KCL to the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 8 - 0}{1} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{10} = 0$$

After simplification,

$$6V_1 - 15V_2 - 3V_3 = -80 ---- (i)$$

We need to handle such conditions differently because there is no way to know the current through a voltage source in advance.

Consider the supernode as a "Whole" node and apply KCL to the node ignoring the source forming supernode and anything in parallel with it. There are 4 wires connected to the supernode, therefore, the KCL equation for the supernode should contain 4 terms.

Applying KCL to the supernode,

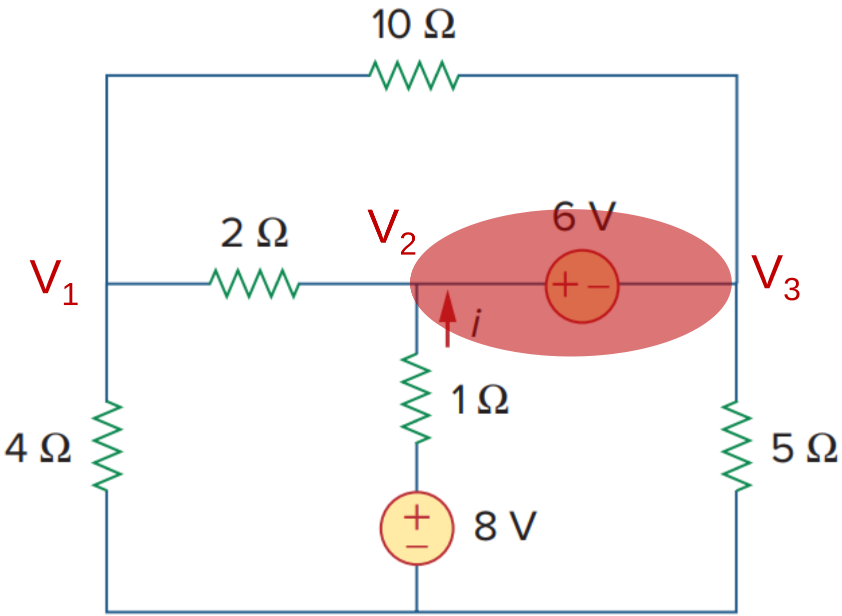
$$\frac{V_2 - V_1}{2} + \frac{V_2 - 8 - 0}{1} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{10} = 0$$

After simplification,

$$6V_1 - 15V_2 - 3V_3 = -80 ---- (i)$$

Example 3 - 3/8

Use nodal analysis to determine the current i.



The next step is to apply KCL to the other remaining nonreference nodes except for the nodes forming the Supernode.

Applying KCL to the node 1,

$$\frac{V_1 - 0}{4} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10} = 0$$

After simplification,

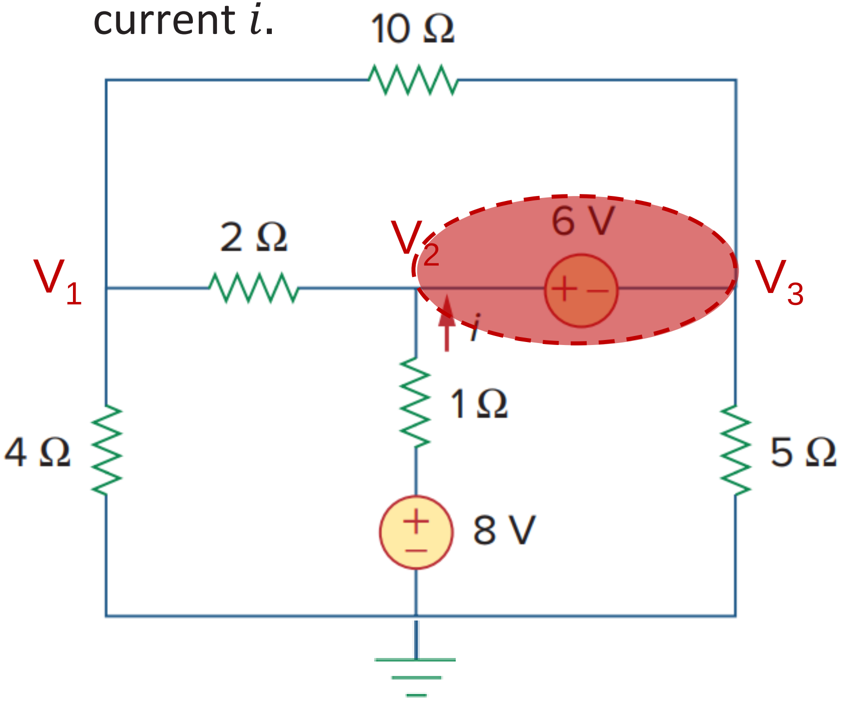
$$17V_1 - 10V_2 - 2V_3 = 0 ----(ii)$$

We have 2 equations, 3 variables, and no remaining nodes for KCL.

The 3rd equation required, can be found by applying KVL to the Supernode.

Example 3 - 4/8

Use nodal analysis to determine the current i.



Applying KVL to the supernode,

$$V_2 - V_3 = 6 - - - - (iii)$$

We got the three equations,

$$6V_1 - 15V_2 - 3V_3 = -80$$
$$17V_1 - 10V_2 - 2V_3 = 0$$
$$V_2 - V_3 = 6$$

Solving … …

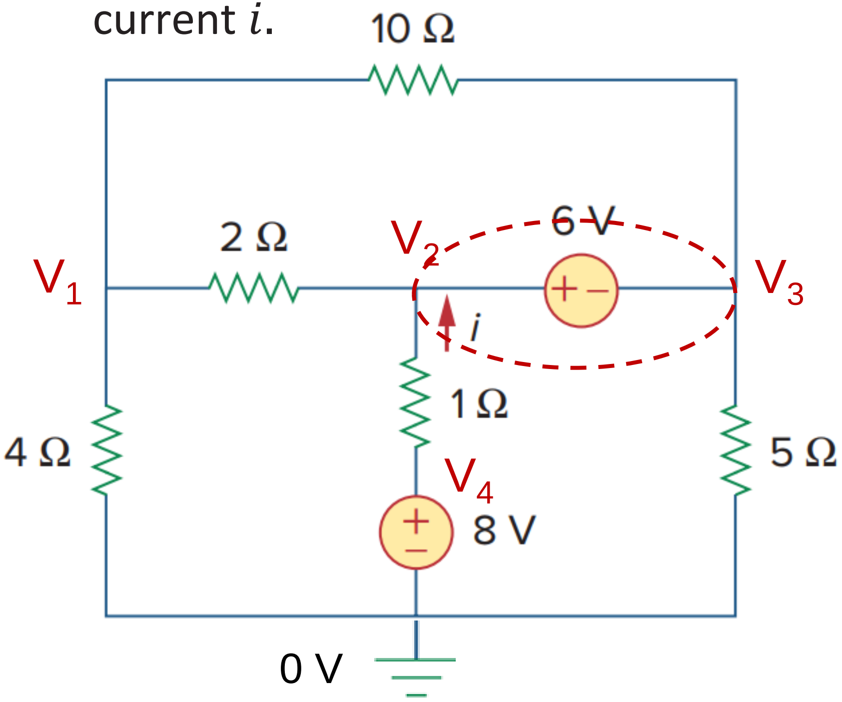
$V_1 = 4.1\ V;$ $\quad V_2 = 6.8\ V;$ $\quad V_3 = 0.8\ V;$

The current i can be written as,

$$i = \frac{0 - (-8) - V_2}{1} = 1.2\ A$$

Example 3: Format Approach - 5/8

Use nodal analysis to determine the current i.



<u>Step 1</u>: Identify all the nodes and label them (with ground being the 0th node).

<u>Step 2</u>: Write the component equations for all the voltage sources (voltage difference = labeled variable).

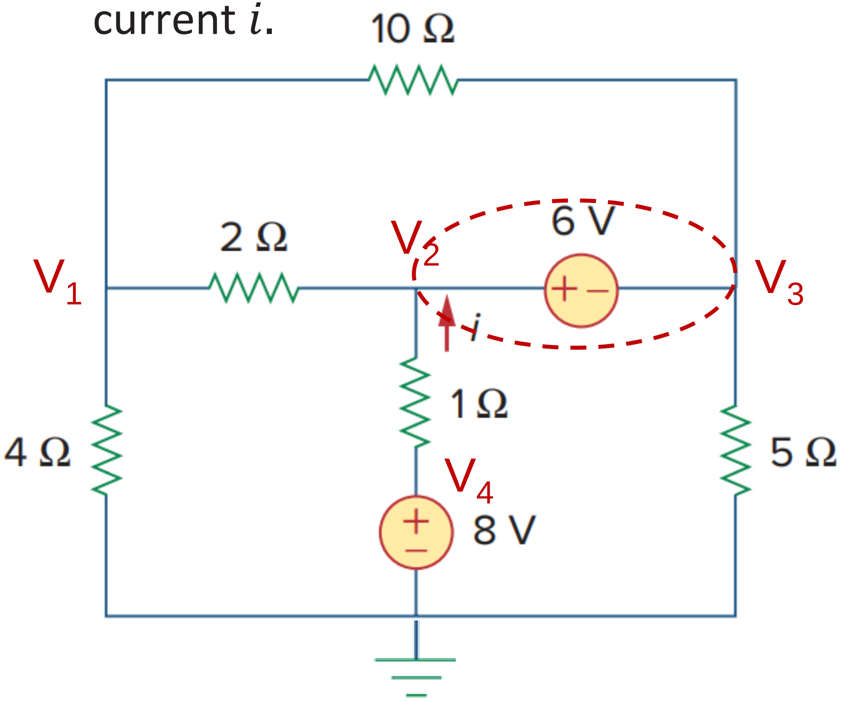
$$V_4 = 8\,V ----- (i)$$

Check for supernodes. Check if a voltage source (dependent or independent) is connected between two nonreference nodes. There can be multiple supernodes in a circuit.

In this circuit, the $6\,V$ voltage source forms a supernode between nodes 2 and 3.

Example 3 - 6/8

Use nodal analysis to determine the current i.



Step 3: Node equation formation.

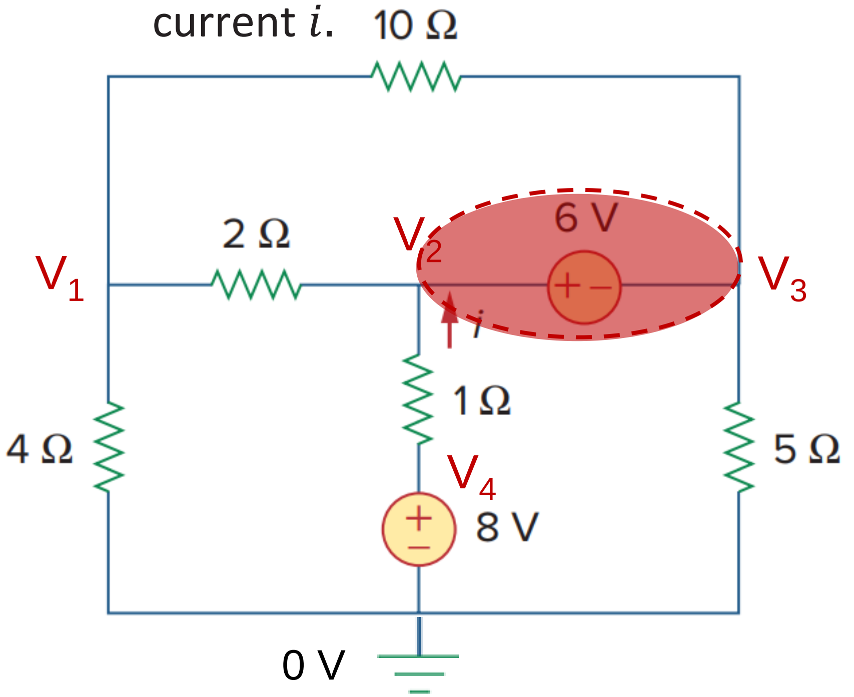
Node 1, V_1: $4\,\Omega, 2\,\Omega,$ and $10\,\Omega$ resistors are connected between V_1 and *ground*, V_1 and V_2, and V_1 and V_3 respectively. We write,

$$V_1\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{10}\right)-\frac{0}{4}-\frac{V_2}{2}-\frac{V_3}{10}=0$$

$$\Rightarrow \quad \frac{17V_1}{20}-\frac{V_2}{2}-\frac{V_3}{10}=0 \text{ ------ } -(i)$$

Example 3 - 7/8

Use nodal analysis to determine the current i.



Node 2 & 3 (Supernode): Now we will apply the same but together in both the nodes 2 & 3.

The 1 Ω and 2 Ω resistors are connected between V_2 and V_4, and V_2 and V_1 respectively. Again, the 10 Ω and 5 Ω resistors are connected between V_3 and $ground$, and V_3 and V_1 respectively. So,

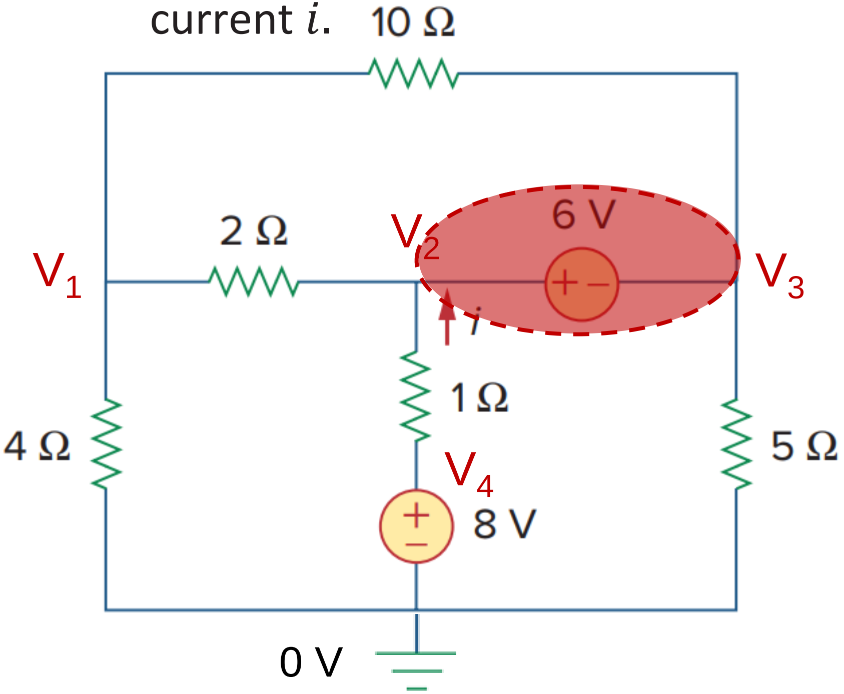
$$V_2\left(\frac{1}{1}+\frac{1}{2}\right)-\frac{V_4}{1}-\frac{V_1}{2}+V_3\left(\frac{1}{10}+\frac{1}{5}\right)-\frac{V_1}{10}-\frac{0}{5}=0$$

With $V_4=8\,V$,

$$\Rightarrow \quad \frac{3V_1}{5}-\frac{3V_2}{2}-\frac{3V_3}{10}+8=0 -----(ii)$$

Example 3 - 8/8

Use nodal analysis to determine the current i.



Finally, applying KVL to the supernode yields.

$$V_2 - V_3 = 6 ----- -(iii)$$

We get the three equations,

$$\frac{17V_1}{20} - \frac{V_2}{2} - \frac{V_3}{10} = 0$$

$$\frac{3V_1}{5} - \frac{3V_2}{2} - \frac{3V_3}{10} + 8 = 0$$

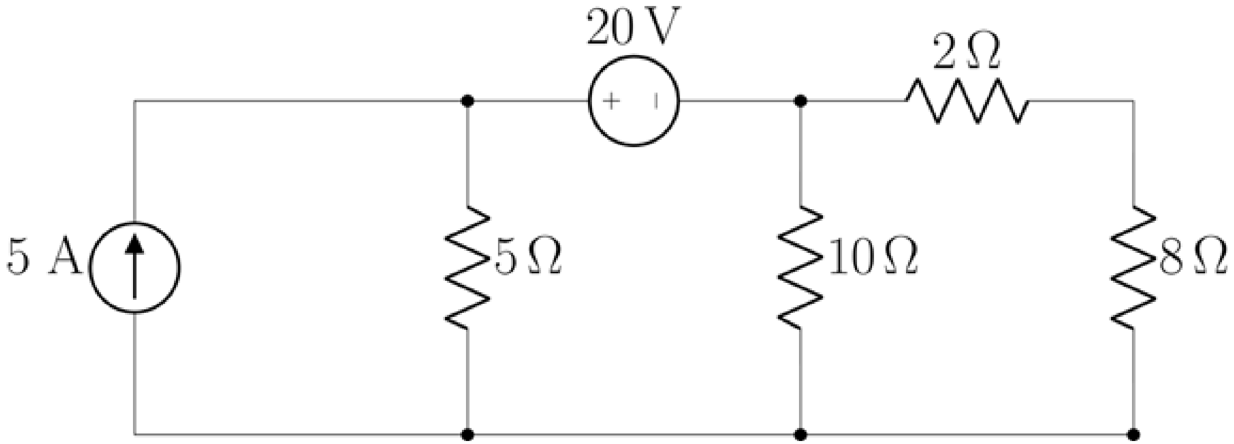
$$V_2 - V_3 = 6$$

Solving …

$V_1 = 4.1\,V;$ $V_2 = 6.8\,V;$ $V_3 = 0.8\,V$

Problem 10

- Use nodal analysis to find the node voltages. Use the node voltages to find the voltage across the $8\ \Omega$ resistor. Don't use Source Transformation.



<u>Ans</u>: With the ground placed at the bottom-most node, $\mathbf{0\ V\ (GND)},\ \boldsymbol{22.5\ V},\ \boldsymbol{2.5\ V}; \boldsymbol{V_{8\Omega} = 2\ V}$

Applying KCL at supernode between 1 & 2,

$$\frac{V_1}{5} - 5 + \frac{V_2}{10} + \frac{V_2}{10} = 0$$

$$\Rightarrow \frac{1}{5}V_1 + \frac{1}{5}V_2 = 5 \quad \text{........ (i)}$$

Applying KVL between node 1 & 2,

$$V_1 - V_2 = 20 \quad \text{........ (ii)}$$

Solving (i) and (ii),

$$\boxed{\begin{aligned} V_1 &= 22.5\,V \\ V_2 &= 2.5\,V \end{aligned}}$$

Now, voltage across the right 2 Ω and 8 Ω is V_2

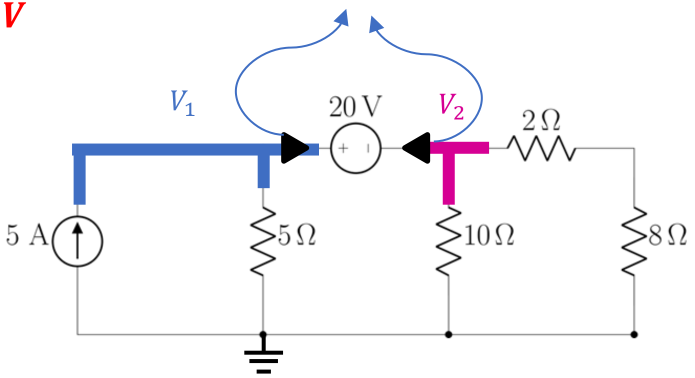
Applying voltage divider rule,

$$v_{8\Omega} = \frac{8}{8+2} \times V_2$$

$$\Rightarrow v_{8\Omega} = \frac{8}{8+2} \times 2.5$$

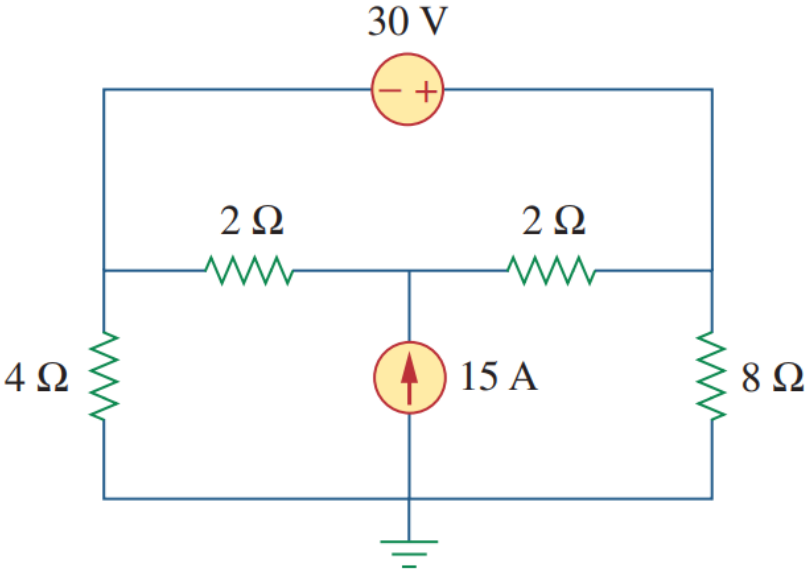
$$\Rightarrow \boldsymbol{v_{8\Omega} = 2\,V}$$

These two currents cancel each other. Hence, we do not need to consider these 2 currents in supernode equation. (Applicable to all supernodes from now)



Problem 11

- Use nodal analysis, determine the current through the $2\ \Omega$ resistance in the right. Determine the current supplied by the $30\ V$ source.



Ans: $\mathbf{0}\ \boldsymbol{A}; \mathbf{7.5}\ \boldsymbol{A}$

Solution to Problem 11

Applying KCL at node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} - 15 = 0$$

$$\Rightarrow -\frac{V_1}{2} + \frac{V_2}{2} + \frac{V_2}{2} - \frac{V_3}{2} - 15 = 0$$

$$\Rightarrow -\frac{1}{2}V_1 + V_2 - \frac{1}{2}V_3 = 15 \quad \ldots\ldots\ldots \text{(i)}$$

Applying KCL at supernode between node 1 & 3,

$$\frac{V_1 - V_2}{2} + \frac{V_1}{4} + \frac{V_3 - V_2}{2} + \frac{V_3}{8} = 0$$

$$\Rightarrow \frac{V_1}{2} + \frac{V_1}{4} - \frac{V_2}{2} - \frac{V_2}{2} + \frac{V_3}{2} + \frac{V_3}{8} = 0$$

$$\Rightarrow \frac{3}{4}V_1 - V_2 + \frac{5}{8}V_3 = 0 \quad \ldots\ldots\ldots \text{(ii)}$$

Applying KVL between node 1 & 3,

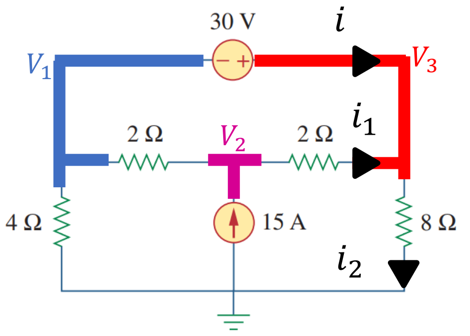
$$V_1 - V_3 = -30 \ldots\ldots\ldots\ldots \text{(iii)}$$

Solving (i), (ii) and (iii),

$$V_1 = 30\ V$$
$$V_2 = 60\ V$$
$$V_3 = 60\ V$$

Current through the 2 Ω at the right,

$$i_1 = \frac{V_2 - V_3}{2} = \mathbf{0}$$



To find the current i supplied by the 30 V voltage source. Let us find i_2 as,

$$i = i_1 + i_2$$

Now,

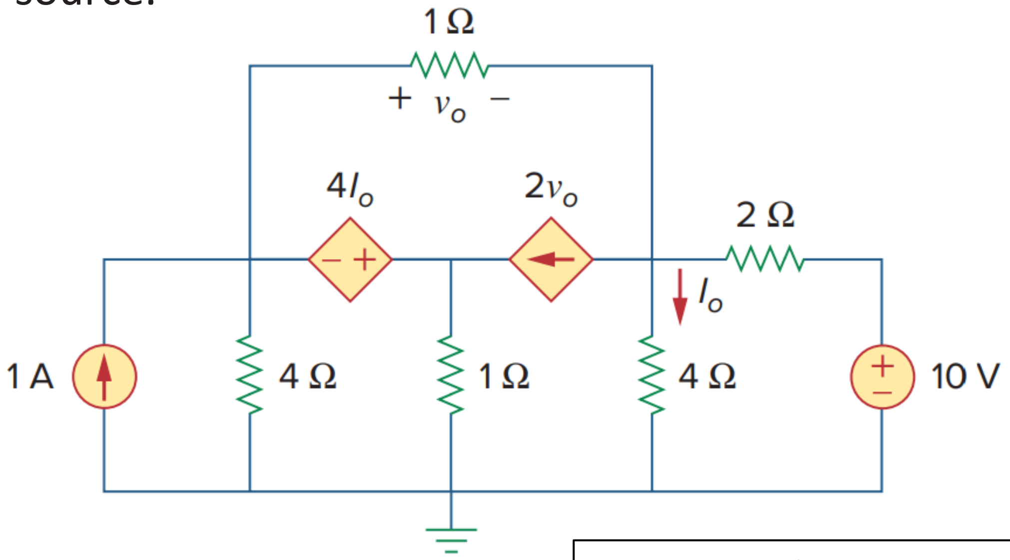
$$i_2 = \frac{V_3}{8} = \frac{60}{8} = 7.5\ A$$

Finally

$$i = 0 + 7.5 = \mathbf{7.5\ A}$$

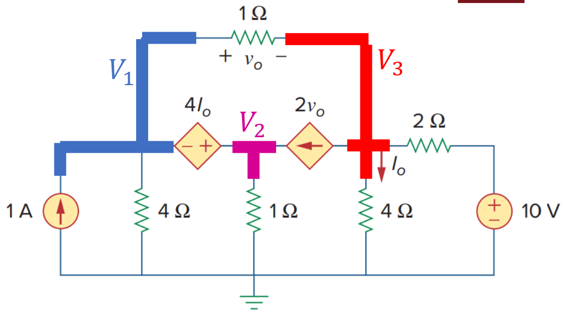
Problem 12

- Use nodal analysis to determine the current through the dependent voltage source.



Ans: Node voltages = $\mathbf{0\ V; 10\ V;\ 4.97\ V;\ 4.85\ V; -0.12\ V;}$
Current through the $4I_0$ source = $\pm\mathbf{5.33\ A}$

Solution to Problem 12



From the circuit,

$$v_o = \frac{V_1 - V_3}{1} = V_1 - V_3$$

$$i_o = \frac{V_3}{4}$$

Applying KCL at node 3,

$$\frac{V_3 - V_1}{1} + \frac{V_3 - 10}{2} + \frac{V_3}{4} + 2v_o = 0$$

$$\Rightarrow \frac{V_3 - V_1}{1} + \frac{V_3 - 10}{2} + \frac{V_3}{4} + 2(V_1 - V_3) = 0$$

$$\Rightarrow -V_1 + 2V_1 + V_3 + \frac{V_3}{2} + \frac{V_3}{4} - 2V_3 - \frac{10}{2} = 0$$

$$\Rightarrow V_1 - \frac{1}{4}V_3 = 5 \quad \text{........ (i)}$$

Applying KCL at supernode between node 1 & 2,

$$\frac{V_1 - V_3}{1} + \frac{V_1}{4} - 1 + \frac{V_2}{1} - 2v_o = 0$$

$$\Rightarrow \frac{V_1 - V_3}{1} + \frac{V_1}{4} - 1 + \frac{V_2}{1} - 2(V_1 - V_3) = 0$$

$$\Rightarrow V_1 + \frac{V_1}{4} - 2V_1 + V_2 - V_3 + 2V_3 - 1 = 0$$

$$\Rightarrow -\frac{3}{4}V_1 + V_2 + V_3 = 1 \quad \text{........ (ii)}$$

Applying KVL between node 1 & 2,

$$V_1 - V_2 = -4i_o$$

$$\Rightarrow V_1 - V_2 = -\frac{4V_3}{4}$$

$$\Rightarrow V_1 - V_2 + V_3 = 0 \quad \text{........... (iii)}$$

Solving (i), (ii) and (iii),

$$V_1 = 4.97\ V$$

$$V_2 = 4.85\ V$$

$$V_3 = -0.12\ V$$

Solution to Problem 12 (Continued)

To find the current through the dependent voltage source, let us apply KCL at node 2

$$i + i_1 + 2v_o = 0$$

Now,

$$i_1 = \frac{0 - V_2}{1} = \frac{0 - 4.85}{1} = -4.85\ A$$

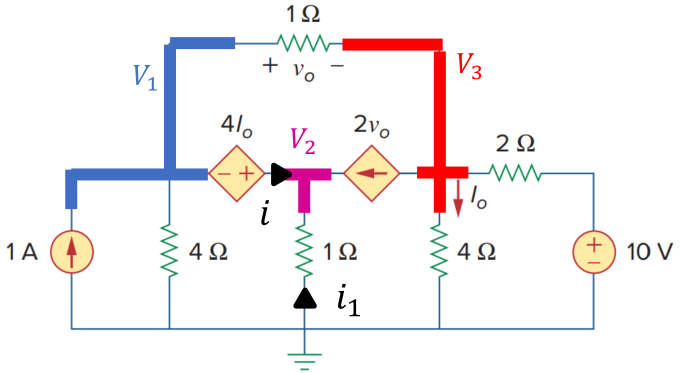
Also,

$$2v_o = 2(V_1 - V_3) = 2(4.97 - (-0.12) = 10.18\ A$$

Finally,

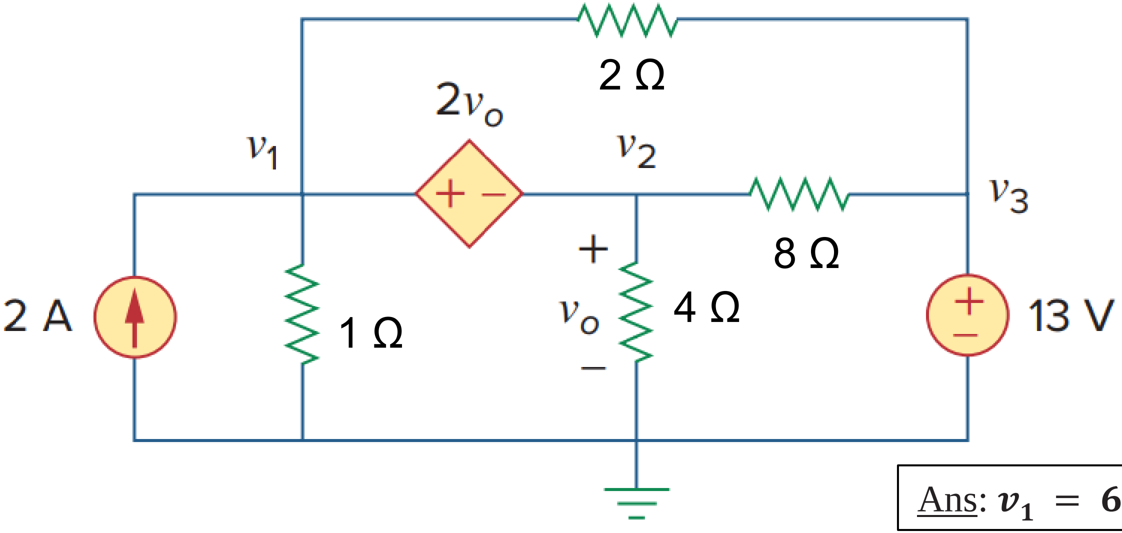
$$i - 4.85 + 10.18 = 0$$

$$\Rightarrow i = \mathbf{-5.33\ A}$$



Problem 13

- Determine voltages v_1 through v_3 in the circuit using nodal analysis.



Ans: $v_1 = \mathbf{6.23}\ V;\ v_2 = \mathbf{2.08}\ V;\ v_3 = \mathbf{13}\ V$

Solution to Problem 13

From the circuit,

$$v_o = V_2$$

$$V_3 = 13\ V$$

Applying KCL at supernode between node 1 & 2,

$$\frac{V_1 - 13}{2} + \frac{V_1}{1} - 2 + \frac{V_2 - 13}{8} + \frac{V_2}{4} = 0$$

$$\Rightarrow \frac{V_1}{2} + V_1 + \frac{V_2}{8} + \frac{V_2}{4} - \frac{13}{2} - 2 - \frac{13}{8} = 0$$

$$\Rightarrow \frac{3}{2}V_1 + \frac{3}{8}V_2 = \frac{81}{8} \quad \text{........ (i)}$$

Applying KVL between node 1 & 2,

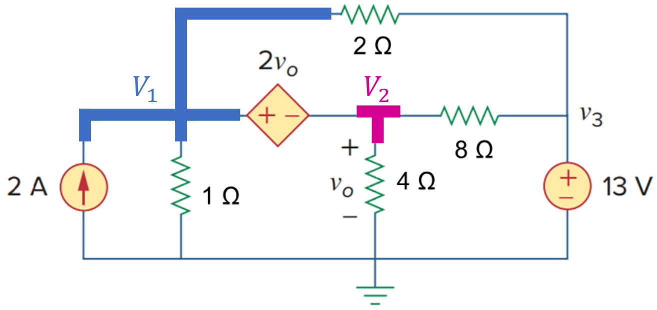
$$V_1 - V_2 = 2v_o$$

$$\Rightarrow V_1 - V_2 = 2V_2$$

$$\Rightarrow V_1 - 3V_2 = 0 \quad \text{........ (ii)}$$

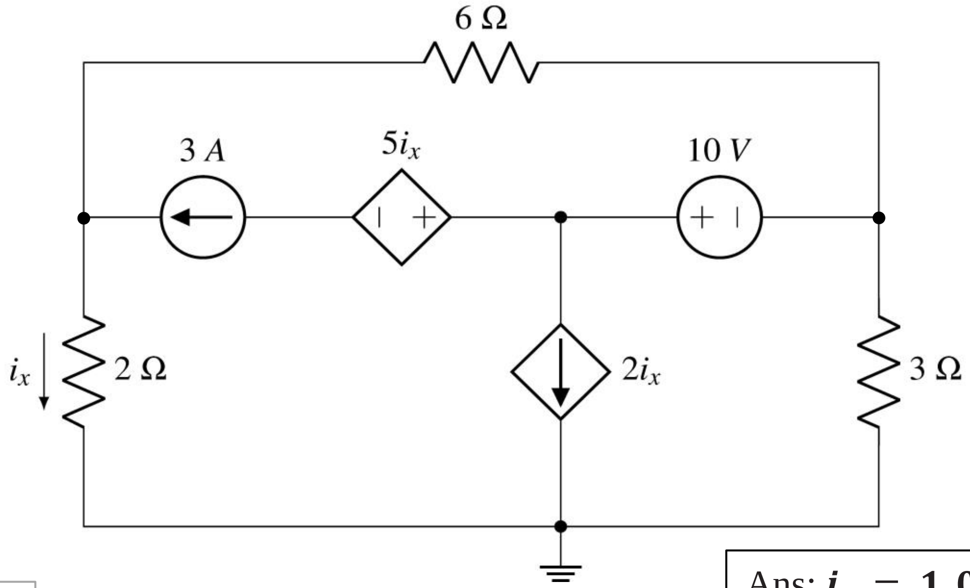
Solving (i) & (ii),

$$\boxed{\begin{aligned} V_1 &= 6.23\ V \\ V_2 &= 2.08\ V \end{aligned}}$$



Problem 14

- Use nodal analysis to find i_x. What is the voltage across the dependent current source? Find the current through the $10\ V$ source.



Ans: $\boldsymbol{i_x = 1.059\ A;\ v_{2i_x} = 0.47\ V;\ i_{10\ V} = \pm 5.12\ A}$

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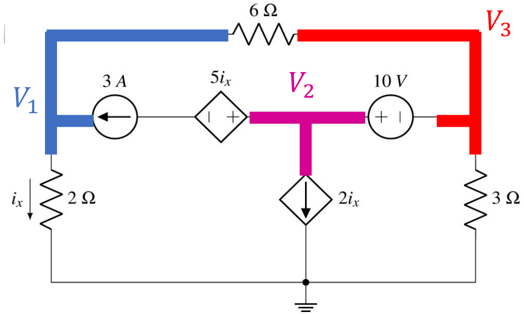
Solution to Problem 14



From the circuit,

$$i_x = \frac{V_1}{2}$$

Applying KCL at node 1,

$$\frac{V_1 - V_3}{6} + \frac{V_1}{2} - 3 = 0$$

$$\Rightarrow \frac{V_1}{6} + \frac{V_1}{2} - \frac{V_3}{6} - 3 = 0$$

$$\Rightarrow \frac{2}{3}V_1 - \frac{1}{3}V_3 = 3 \quad \text{........ (i)}$$

Applying KCL at supernode between node 2 & 3,

$$3 + 2i_x + \frac{V_3 - V_1}{6} + \frac{V_3}{3} = 0$$

$$\Rightarrow 3 + \frac{2V_1}{2} + \frac{V_3 - V_1}{6} + \frac{V_3}{3} = 0$$

$$\Rightarrow V_1 - \frac{V_1}{6} + \frac{V_3}{6} + \frac{V_3}{3} + 3 = 0$$

$$\Rightarrow \frac{5}{6}V_1 + \frac{1}{2}V_1 = -3 \quad \text{........ (ii)}$$

Applying KVL between node 2 & 3,

$$V_2 - V_3 = 10 \quad \text{........... (iii)}$$

Solving (i), (ii) and (iii),

$$V_1 = 2.12\ V$$

$$V_2 = 0.47\ V$$

$$V_3 = -9.53\ V$$

Value of i_x

$$i_x = \frac{2.12}{2} = \mathbf{1.06\ A}$$

Voltage across the dependent current source

$$V_{2ix} = V_2 = \mathbf{0.47\ V}$$

Current through the 10 V voltage source.

Let's go to the next page ….

Solution to Problem 14 (Continued)

Current through the 10 V voltage source.
Let the current be i

Applying KCL at node 2,

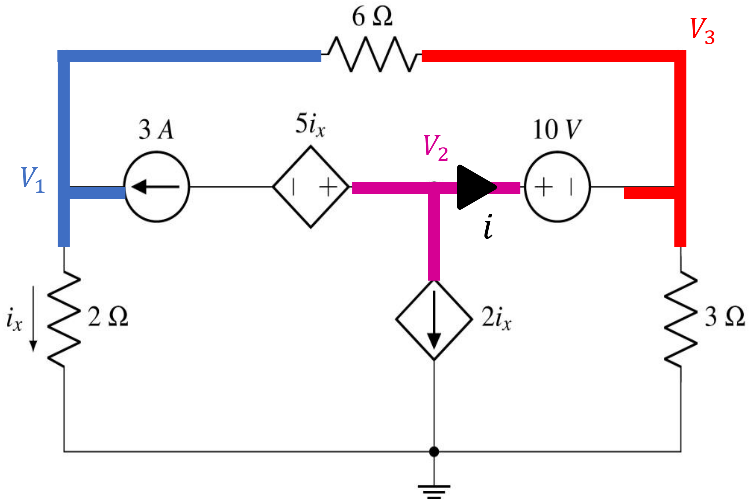
$$3 + 2i_x + i = 0$$

$$\Rightarrow 3 + 2 \times 1.06 + i = 0$$

$$\Rightarrow i = \mathbf{-5.12\ A}$$

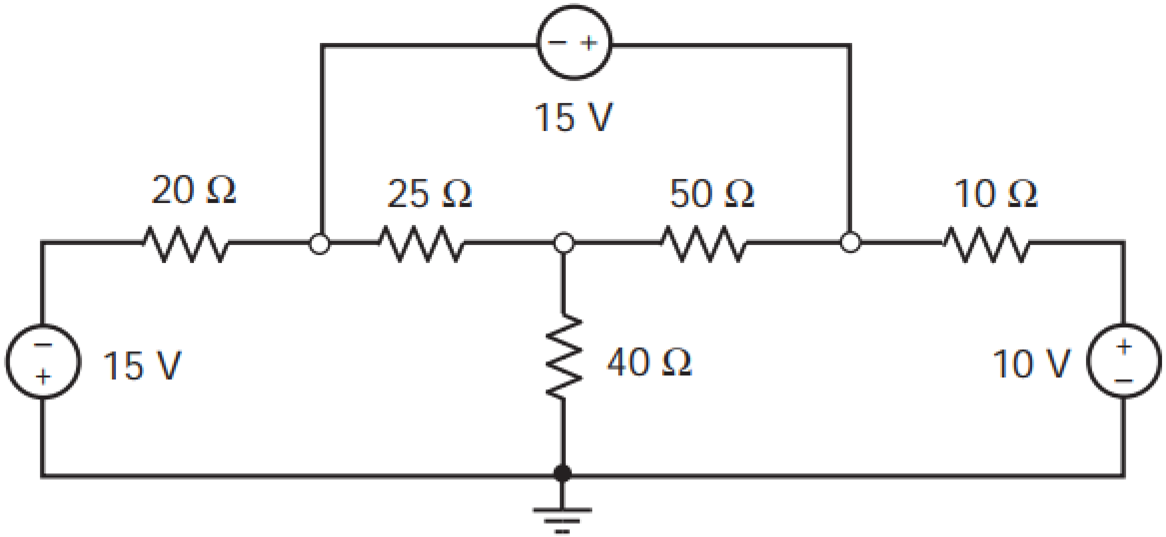
Wondering why the value of i is given as $\pm 5.12\ A$?

It is because of the direction chosen by the solver. If I chose the direction of i to be opposite of the current direction. The value of i would have been $+\,5.12\ A$



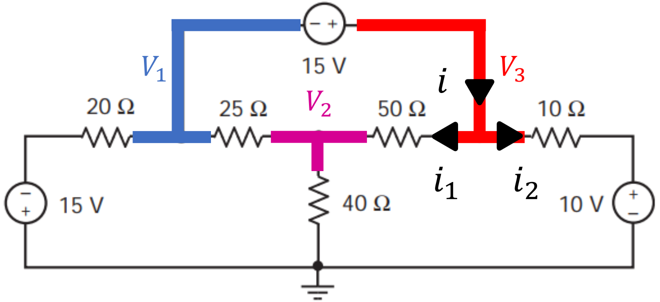
Problem 15

- Determine current through the $15\ V$ source using nodal analysis.



Ans: Node voltages $= \mathbf{0\ V\ (GND),\ -15\ V,\ 10\ V,\ -7.9825\ V,\ -2.1053\ V,\ 7.0175\ V;\ I_{15V} = \pm\,0.12\ A}$

Solution to Problem 15



Applying KCL at node 2,

$$\frac{V_2 - V_1}{25} + \frac{V_2 - V_3}{50} + \frac{V_2}{40} = 0$$

$$\Rightarrow -\frac{V_1}{25} + \frac{V_2}{25} + \frac{V_2}{50} + \frac{V_2}{40} - \frac{V_3}{50} = 0$$

$$\Rightarrow -\frac{1}{25}V_1 + \frac{17}{200}V_2 - \frac{1}{50}V_3 = 0$$

.... (i)

Applying KCL at supernode between node 1 & 3,

$$\frac{V_1 - V_2}{25} + \frac{V_1 - (-15)}{20} + \frac{V_3 - V_2}{50} + \frac{V_3 - 10}{10} = 0$$

$$\Rightarrow \frac{V_1}{25} + \frac{V_1}{20} - \frac{V_2}{25} - \frac{V_2}{50} + \frac{V_3}{50} + \frac{V_3}{10} + \frac{15}{20} - \frac{10}{10} = 0$$

$$\Rightarrow \frac{9}{100}V_1 - \frac{3}{50}V_2 + \frac{3}{25}V_3 = \frac{1}{4}$$

.... (ii)

Applying KVL between node 1 & 3,

$$V_1 - V_3 = -15 \quad \text{.......... (iii)}$$

Solving (i), (ii) and (iii),

$$V_1 = -7.98\ V$$
$$V_2 = -2.10\ V$$
$$V_3 = 7.02\ V$$

Current through the 15 V voltage source.

Let the current be i

Applying KCL at node 3,

$$i = i_1 + i_2$$

$$\Rightarrow i = \frac{V_3 - V_2}{50} + \frac{V_3 - 10}{10}$$

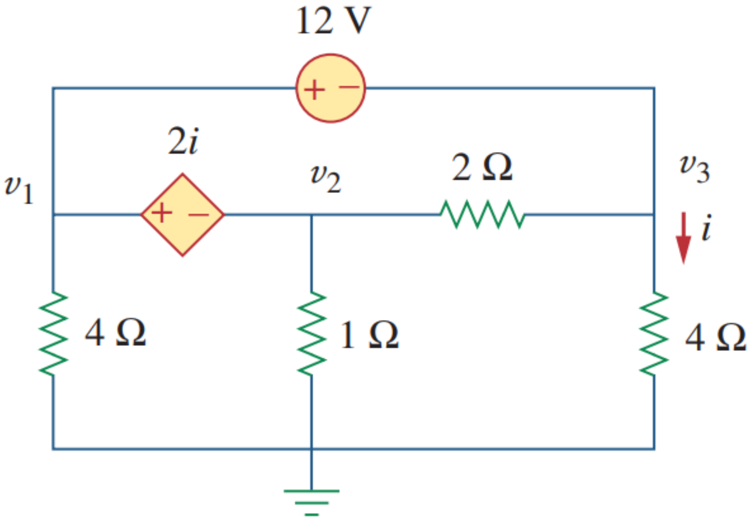
$$\Rightarrow i = \frac{7.02 - (-2.10)}{50} + \frac{7.02 - 10}{10}$$

$$\Rightarrow i = \frac{7.02 - (-2.10)}{50} + \frac{7.02 - 10}{10}$$

$$\Rightarrow i \approx \mathbf{0.12\ A}$$

Problem 16

- Find v_1, v_2, and v_3 using nodal analysis. Determine the currents supplied by the $12\ V$ and the $2i$ source?



Ans: $\boldsymbol{v_1 = -3\ V;\ v_2 = 4.5\ V;\ v_3 = -15\ V;\ I_{12\ V} = 23.25\ A; I_{2i,} = -14.25\ A}$

Solution to Problem 16

From the circuit,

$$i = \frac{v_3}{4}$$

There is voltage source between node 1 and 2. There is also voltage source between node 1 and 3. So we can apply KCL at the supernode between node 1, 2 and 3

$$\frac{v_1}{4} + \frac{v_2 - v_3}{2} + \frac{v_2}{1} + \frac{v_3 - v_2}{2} + \frac{v_3}{4} = 0$$

$$\Rightarrow \frac{1}{4}v_1 + v_2 + \frac{1}{4}v_3 = 0 \quad \text{........ (i)}$$

Applying KVL between node 1 & 3,

$$v_1 - v_3 = 12 \quad \text{........ (ii)}$$

Applying KVL between node 1 & 2,

$$v_1 - v_2 = 2i$$

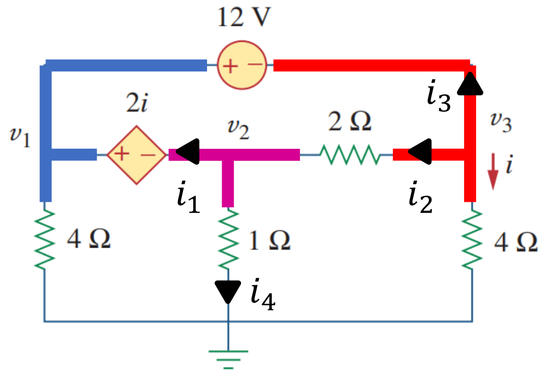
$$\Rightarrow v_1 - v_2 = \frac{2v_3}{4}$$

$$\Rightarrow v_1 - v_2 - \frac{1}{2}v_3 = 0 \quad \text{........ (iii)}$$

Solving (i), (ii) and (iii),

$$\boxed{\begin{aligned} V_1 &= -3\ V \\ V_2 &= 4.5\ V \\ V_3 &= -15\ V \end{aligned}}$$

Let the current supplied by the 12 V and 2i source is i_3 and i_1



Applying KCL at node 2,

$$i_1 = i_2 - i_4$$

$$\Rightarrow i_1 = \frac{v_3 - v_2}{2} - \frac{v_2}{1}$$

$$\Rightarrow i_1 = \frac{-15 - 4.5}{2} - \frac{4.5}{1} = \mathbf{-14.25\ A}$$

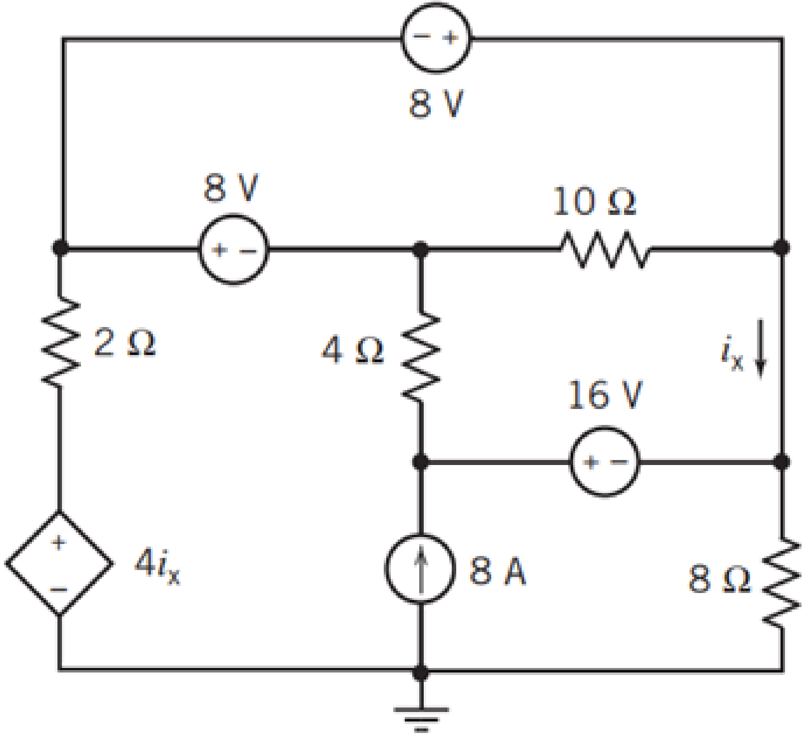
Applying KCL at node 3,

$$i_3 = -i - i_2 = \frac{v_3}{4} - \frac{v_3 - v_2}{2}$$

$$= -\frac{-15}{4} - \frac{-15 - 4.5}{2} = \mathbf{13.5\ A}$$

Problem 17

- Use nodal analysis and determine I_x.

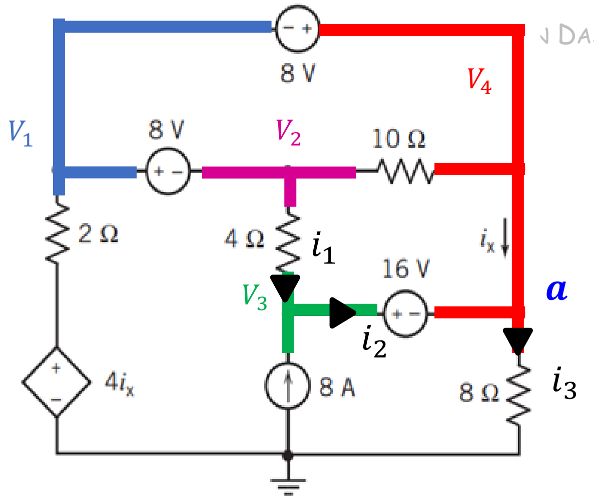


Ans: Node voltages $= \mathbf{0\ V\ (GND),\ 16\ V,\ 24\ V,\ 48\ V,\ 16\ V,\ 32\ V;} \boldsymbol{I_x = 4\ A}$

Solution to Problem 17

There is voltage source between node 1 and 2. There is also voltage source between node 1 and 4. There is also voltage source between node 3 and 4. So, we can apply KCL at the supernode between node 1, 2, 3 and 4.

$$\frac{V_1 - 4i_x}{2} + \frac{V_2 - V_3}{4} + \frac{V_2 - V_4}{10} + \frac{V_3 - V_2}{4} - 8 + \frac{V_4 - V_2}{10} + \frac{V_4}{8} = 0 \quad \text{........ (i)}$$



Let us find the value of i_x first

Applying KCL at node 3

$$i_2 = i_1 + 8$$

Applying KCL at junction a

$$i_x + i_2 = i_3$$

$$\Rightarrow i_x + (i_1 + 8) = i_3$$

$$\Rightarrow i_x = -i_1 + i_3 - 8$$

$$\Rightarrow \; i_x = -\frac{V_2 - V_3}{4} + \frac{V_4}{8} - 8 \quad \text{........ (ii)}$$

Putting (ii) in (i),

$$\frac{V_1 - 4i_x}{2} - 8 + \frac{V_4}{8} = 0$$

$$\Rightarrow \frac{V_1}{2} - 2i_x - 8 + \frac{V_4}{8} = 0$$

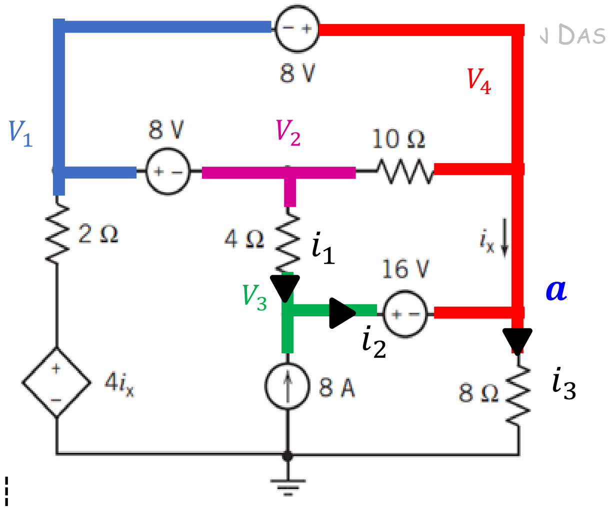
$$\Rightarrow \frac{V_1}{2} - 2\left(-\frac{V_2 - V_3}{4} + \frac{V_4}{8} - 8\right) - 8 + \frac{V_4}{8} = 0$$

$$\Rightarrow \frac{V_1}{2} + \frac{V_2}{2} - \frac{V_3}{2} - \frac{V_4}{4} + 16 + \frac{V_4}{8} - 8 = 0$$

Solution to Problem 17 (Continued)

$$\Rightarrow \frac{V_1}{2} + \frac{V_2}{2} - \frac{V_3}{2} - \frac{V_4}{4} + \frac{V_4}{8} + 8 = 0$$

$$\Rightarrow \frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{2}V_3 - \frac{1}{8}V_4 = -8 \quad \text{........ (iii)}$$



Applying KVL between node 1 and 2,

$$V_1 - V_2 = 8 \quad \text{........ (iv)}$$

Applying KVL between node 1 and 4,

$$V_1 - V_4 = -8 \quad \text{........ (v)}$$

Applying KVL between node 3 and 4,

$$V_3 - V_4 = 16 \quad \text{........ (vi)}$$

Solving (iii), (iv), (v) and (vi),

$V_1 = 24\ V$

$V_2 = 16\ V$

$V_3 = 48\ V$

$V_4 = 32\ V$

Value of i_x

$$i_x = -\frac{16-48}{4} + \frac{32}{8} - 8 = \mathbf{4A}$$

But, how did we calculate four (04) variables? Surely not everyone's calculator is not able to calculate 4 simultaneous equations. Let us find out the process in the next page

Solution to Problem 17 (Continued) (How to calculate 4 variables)

We have 4 equations,

$$\frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{2}V_3 - \frac{1}{8}V_4 = -8 \quad \ldots\ldots\ldots \text{(iii)}$$

$$V_1 - V_2 = 8 \quad \ldots\ldots\ldots \text{(iv)}$$

$$V_1 - V_4 = -8 \quad \ldots\ldots\ldots \text{(v)}$$

$$V_3 - V_4 = 16 \quad \ldots\ldots\ldots \text{(vi)}$$

If you have a calculator which can solve 4 simultaneous equations for 4 unknown variables, then you can get the answer by putting the equations in the calculator. However, if you do not have a calculator which can solve only 3 simultaneous equations, then we have to convert 4 equations in 3 equations

From the equation (vi)

$$V_3 - V_4 = 16$$

$$\Rightarrow V_3 = V_4 + 16 \quad \ldots\ldots\ldots \text{(vii)}$$

Putting the value of V_4 in equation (iii)

$$\frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{2}(V_4 + 16) - \frac{1}{8}V_4 = -8$$

$$\Rightarrow \frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{2}V_4 - 8 - \frac{1}{8}V_4 = -8$$

$$\Rightarrow \frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{5}{8}V_4 = 0 \quad \ldots\ldots\ldots \text{(viii)}$$

Now, we have 3 equations.

$$\frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{5}{8}V_4 = 0 \quad \ldots\ldots\ldots \text{(viii)}$$

$$V_1 - V_2 = 8 \quad \ldots\ldots\ldots \text{(iv)}$$

$$V_1 - V_4 = -8 \quad \ldots\ldots\ldots \text{(iii)}$$

Solving (iii), (iv). and (viii)

$$\boxed{\begin{aligned} V_1 &= 24\ V \\ V_2 &= 16\ V \\ V_4 &= 32\ V \end{aligned}}$$

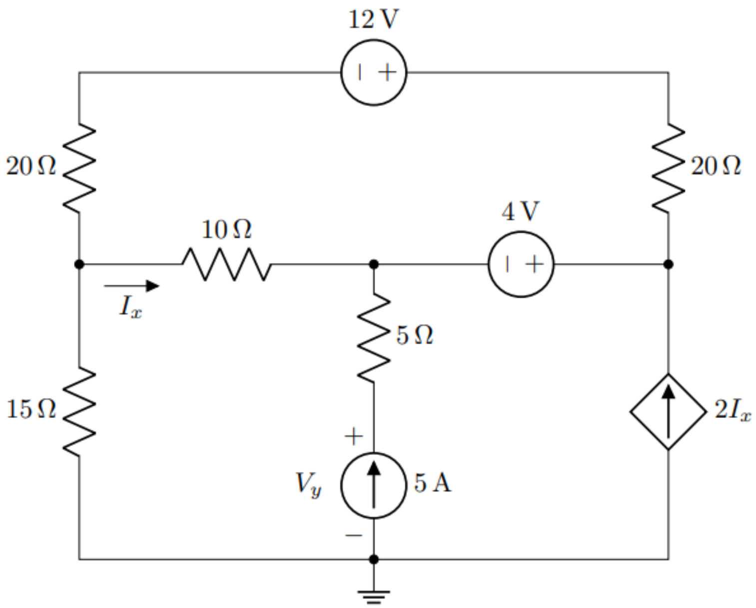
Putting the value of V_4 in equation (vii) to find V_3

$$V_3 = 32 + 16$$

$$\Rightarrow V_3 = \mathbf{48\ V}$$

Problem 18

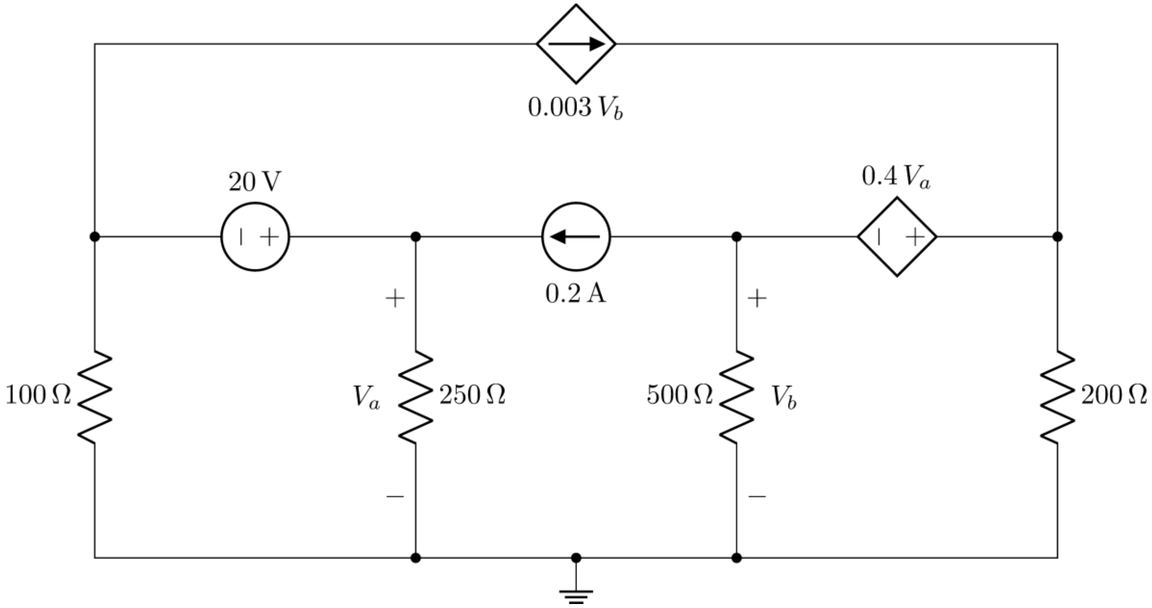
- Find all the node mesh currents in the circuit.
- Find Vy, the voltage across the 5 A current source.
- How much power is the 5 A current source consuming/supplying to the circuit? Also mention whether the source is supplying or consuming power.



Ans: Node voltages = $\mathbf{0\ V\ (GND),\ 24\ V,\ 44\ V,\ -72\ V,\ 54.4\ V}$

Problem 19

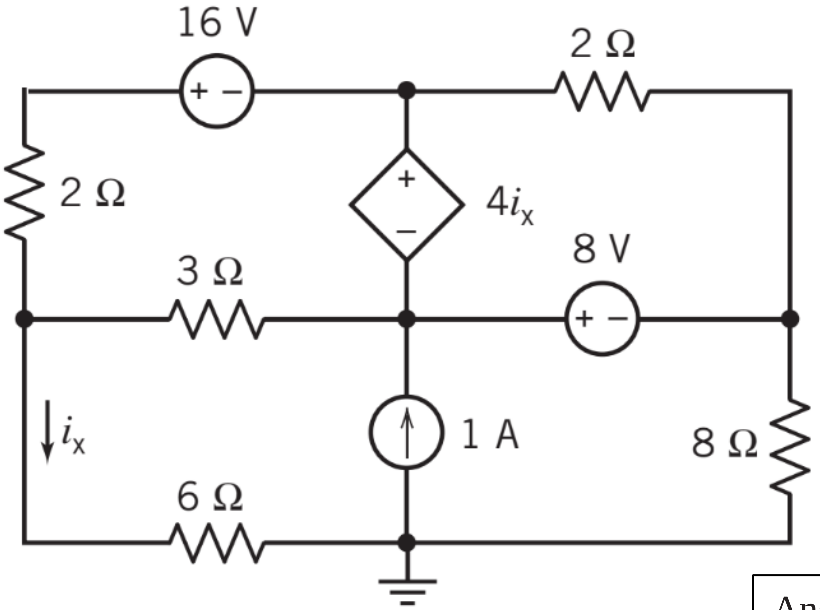
- Use nodal analysis and determine all the node voltages in the following circuit.



Ans: Node voltages $= \mathbf{0\ V\ (GND),\ 24\ V,\ 44\ V,\ -72\ V,\ 54.4\ V}$

Problem 20

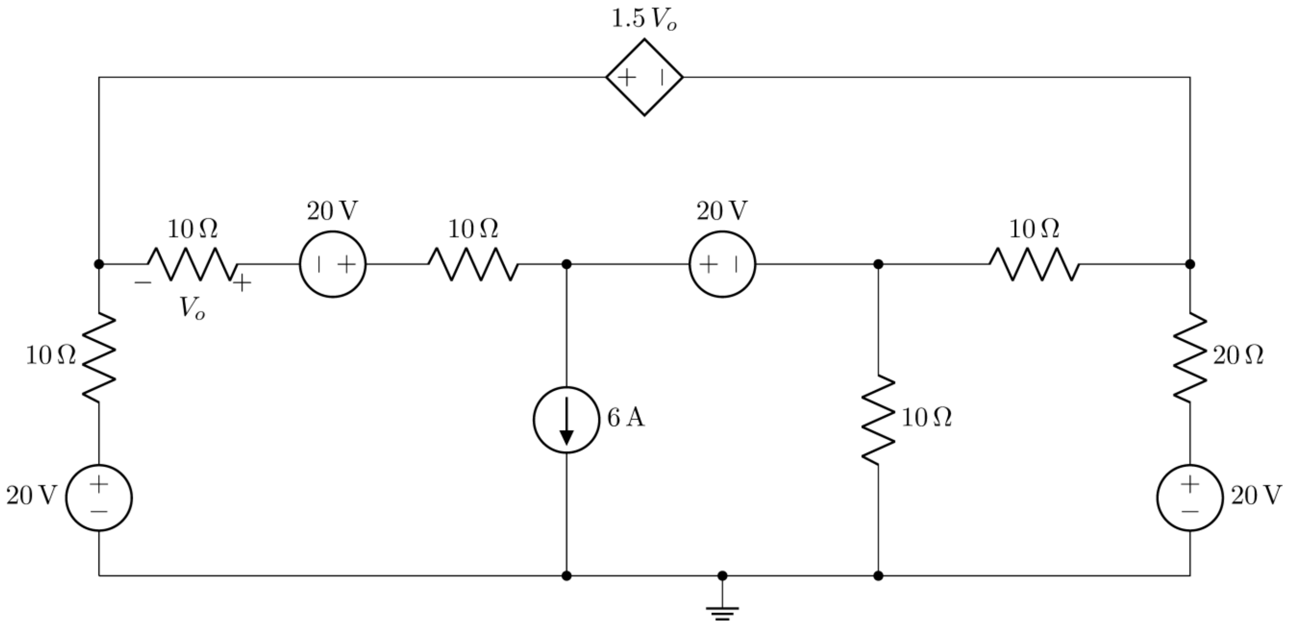
- Determine all the node voltages.



Ans: $\mathbf{0\ V\ (GND); 8\ V,\ 0\ V,\ -8\ V,\ 12\ V,\ 24\ V}$

Problem 21

- Use nodal analysis and determine all the node voltages in the following circuit.



Ans: Node voltages $= \mathbf{0\,V\,(GND),\ -8\,V,\ -16\,V,\ 4\,V,\ -4\,V,\ -24\,V,\ \ 4\,V}$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Thank you for your attention

Course Outline: broad themes