Fall 2023

CSE250 - Circuits and Electronics

Source Transformation

BRAC UNIVERSITY
Inspiring Excellence

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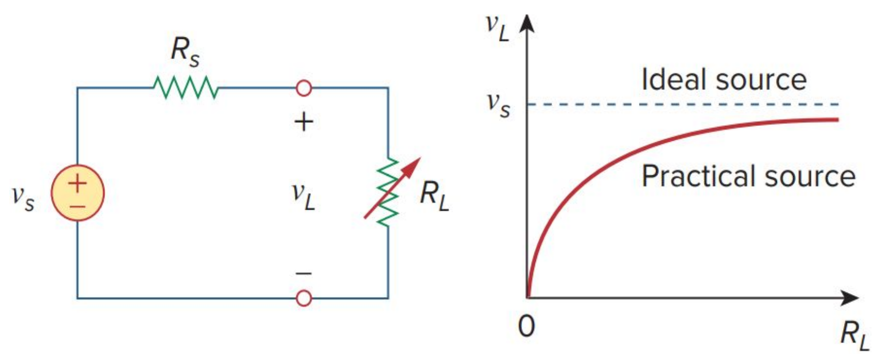
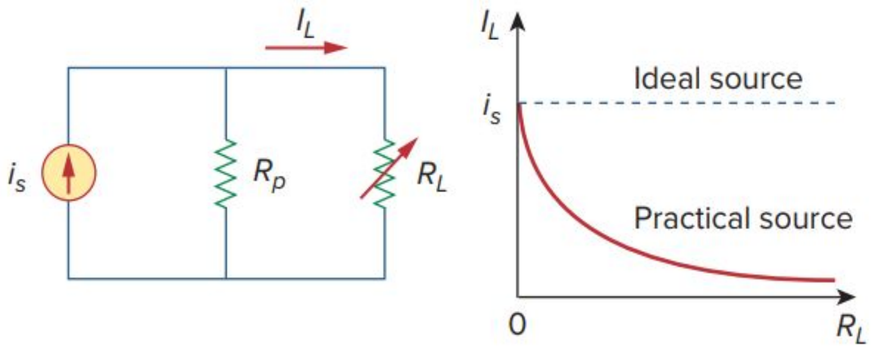
Ideal and non-ideal sources

- An *ideal voltage source* provides a constant voltage irrespective of the current drawn by the load, while an *ideal current source* supplies a constant current regardless of the load voltage.
- *Practical* voltage and current sources are not ideal, due to their *internal resistances* or *source resistances* R_s and R_p. They become ideal as $R_s \to 0$ and $R_p \to \infty$.

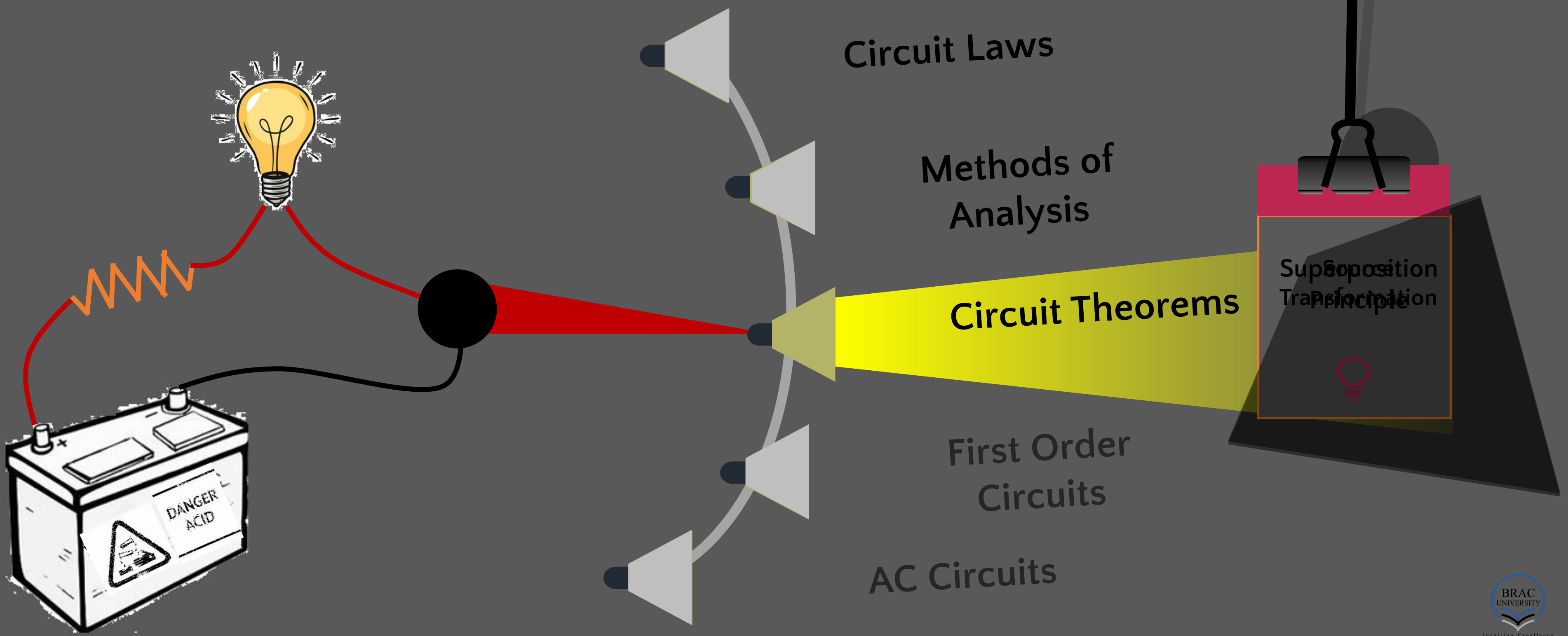
$$v_L = \frac{R_L}{R_s+R_L} v_s, \text{ if } R_s \ll R_L \text{ or } R_L = \infty, v_L \to v_s$$



$$i_L = \frac{R_p}{R_p+R_L} i_s, \text{ if } R_p \gg R_L \text{ or } R_p \to \infty, i_L \to i_s$$

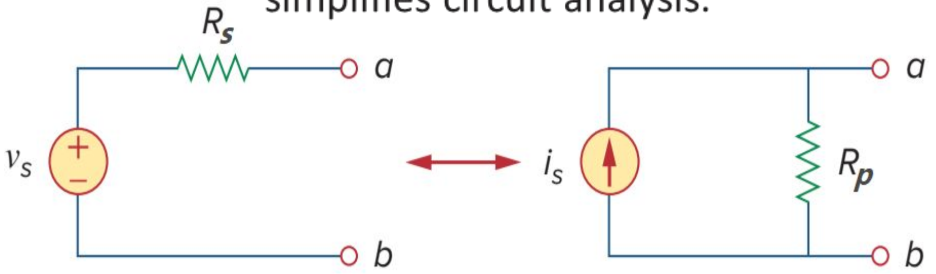


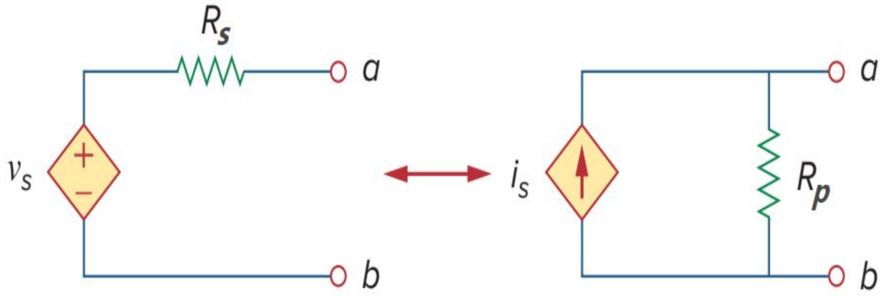
Course Outline: broad themes



Source Transformation

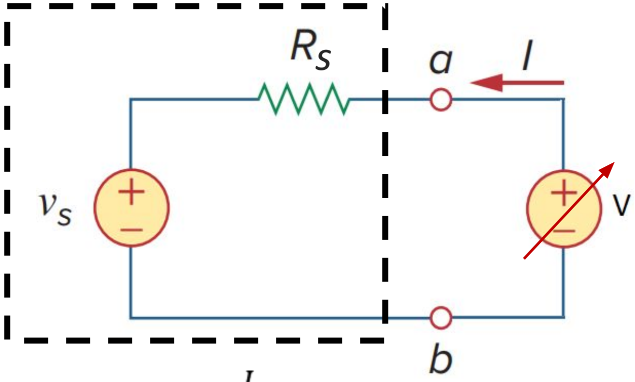
- A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_p in parallel with a resistor R, or vice versa.
- The transformation does not affect the remaining part of the circuit but greatly simplifies circuit analysis.





- Note that the arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when $R = 0$ and $R = \infty$ (see next slide), which are the cases with an ideal voltage and current source respectively. However, for a practical, nonideal voltage source, $R \neq 0$, and for a practical, nonideal current source, $R \neq \infty$.

V in series with a R

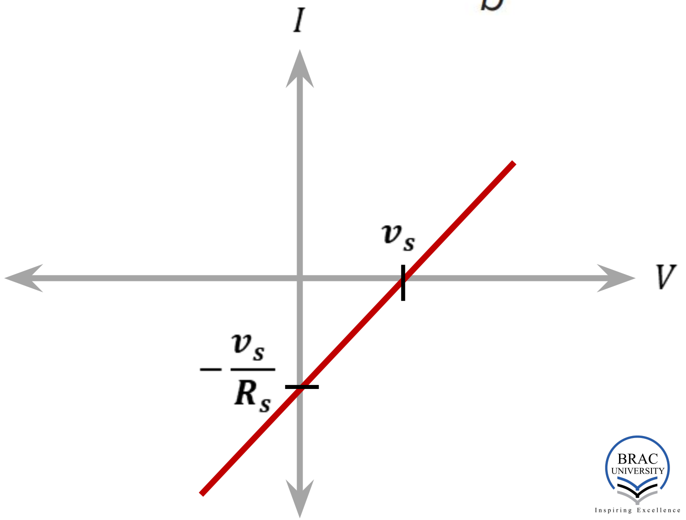


- We recall that *an equivalent circuit is one whose $I - V$ characteristics are identical with the original circuit*. Let's see what conditions make the two circuits to have the same $I - V$ relations at terminals $a - b$.

- Let's say we have a configuration of a voltage source (v_s) in series with a resistor (R_s) between terminals a and b. To determine the configuration's $I - V$ characteristics, if applying a voltage V gives rise to a current I, we can write,

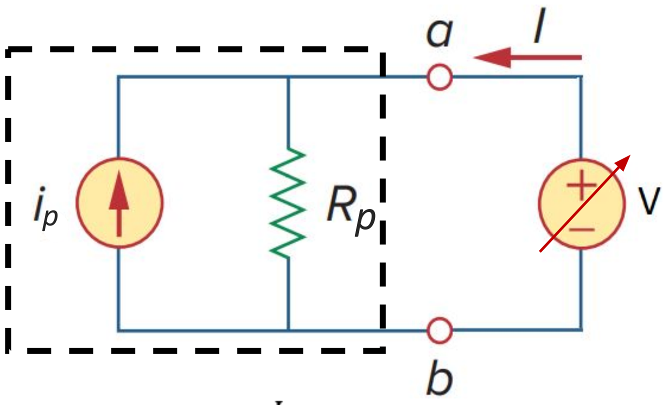
$$V = v_s + IR_s$$

$$\Rightarrow I = \frac{1}{R_s}V - \frac{v_s}{R_s}$$



- The equation results in a linear I vs V plot that intersects the axes at v_s and $-\frac{v_s}{R_s}$.

I in parallel with a R



- For the other configuration: a current source (i_p) in parallel with a resistor (R_p) between terminals a and b, if applying a voltage V gives rise to a current I, using KCL the current through the resistor is,

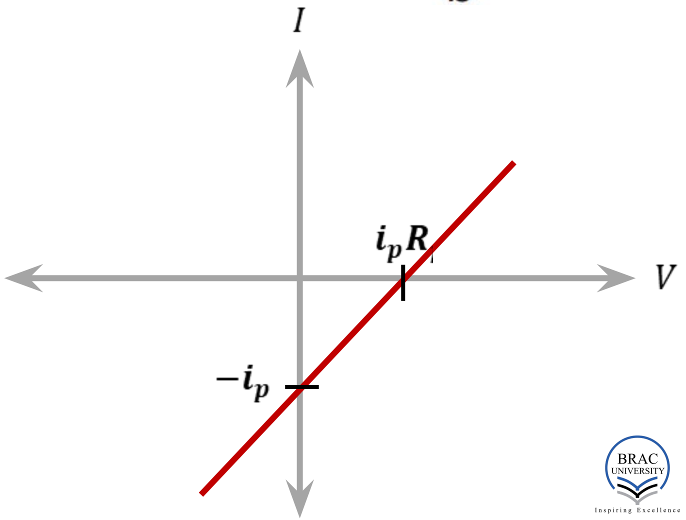
$$I + i_p$$

- So, the voltage across the resistor can be written as,

$$\left(I + i_p\right) R_p = V$$

$$\Rightarrow I = \frac{1}{R_p} V - i_p$$

- The equation results in a linear I vs V plot that intersects the axes at $i_p R_p$ and $-i_p$.

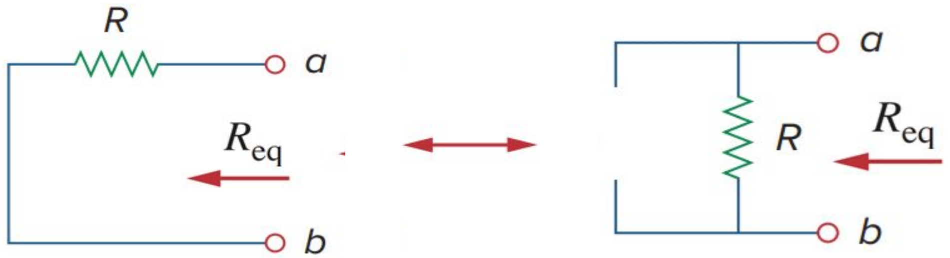


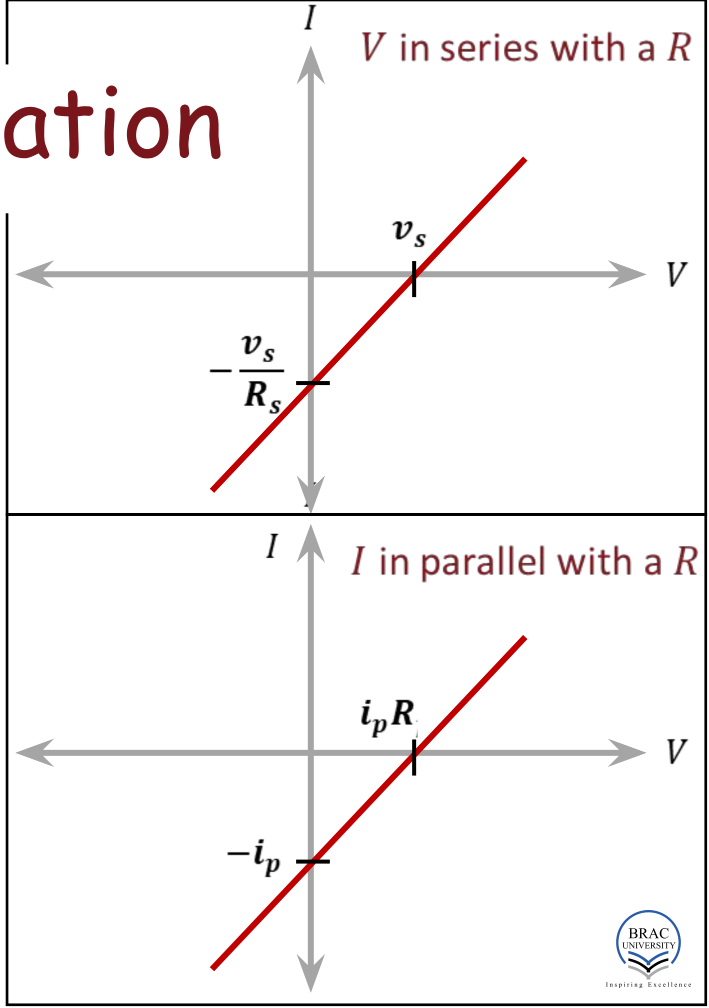
Conditions for transformation

- The two configurations will be equivalent to each other if their $I - V$ characteristics are similar. It can be said by looking at the two plots, they will indeed be similar if the intersecting points are same, that is, if $v_s = i_p R_p$ and $-\frac{v_s}{R_s} = -i_p$. This requires $\boxed{R_s = R_p} = R$. Both the equations result in an ohmic relation,

$$\boxed{v_s = i_p R \text{ or } i_p = \frac{v_s}{R}}$$

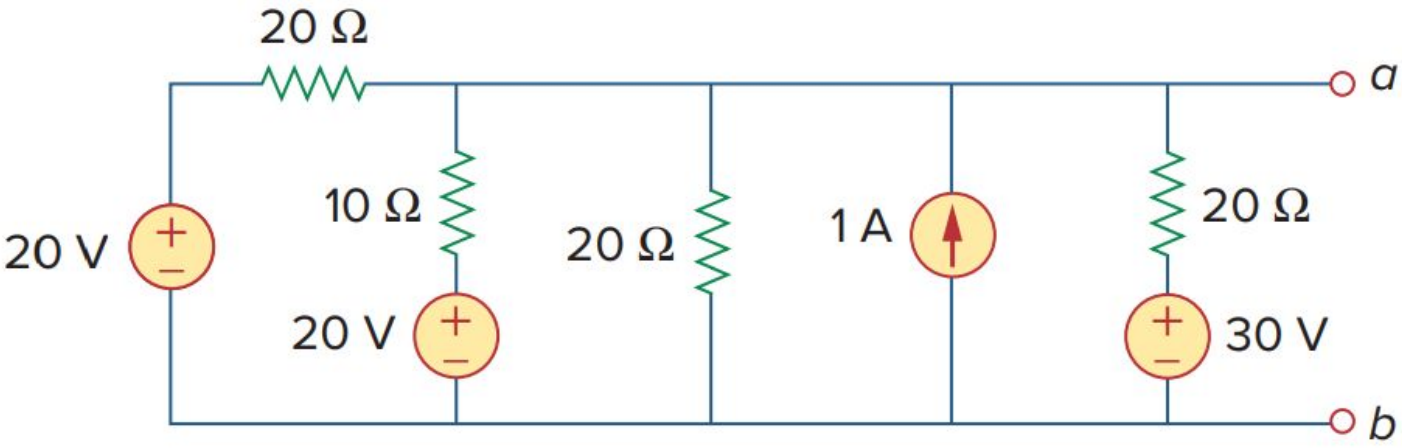
- So, if the sources are turned off, the equivalent resistance at terminals $a - b$ in both circuits is R.



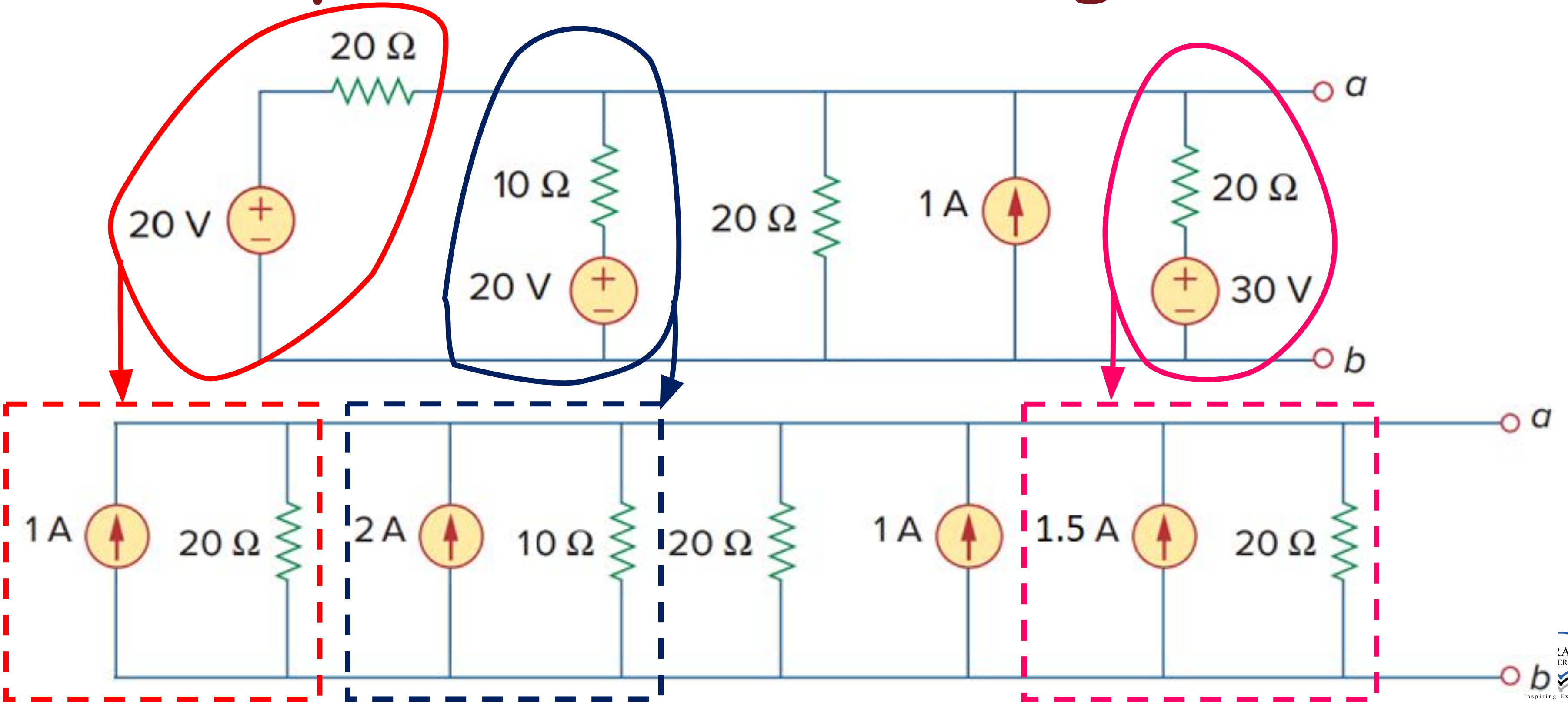


Example 1

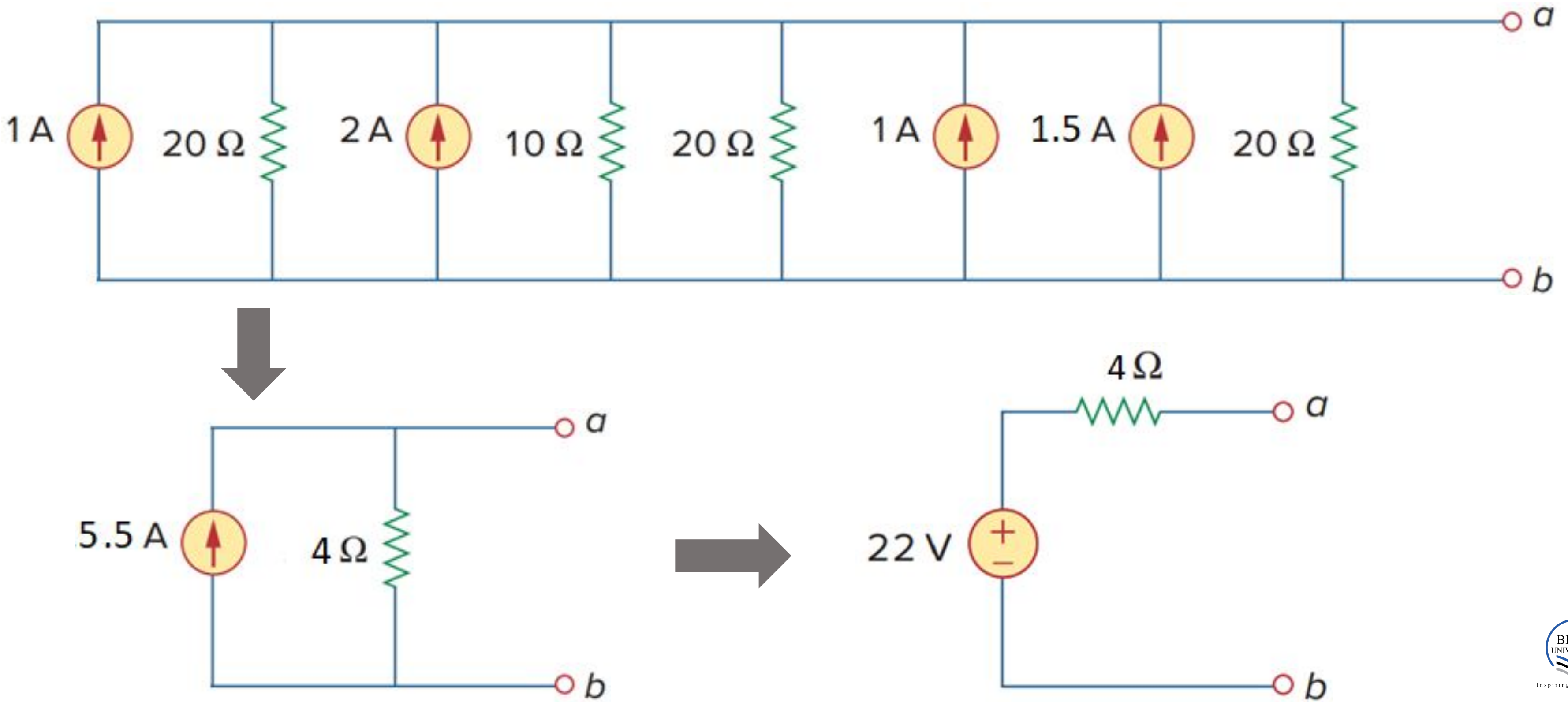
- Use source transformation to reduce the circuit between terminals *a* and *b* shown to a single voltage source in series with a single resistor.



Example 1: transforming sources

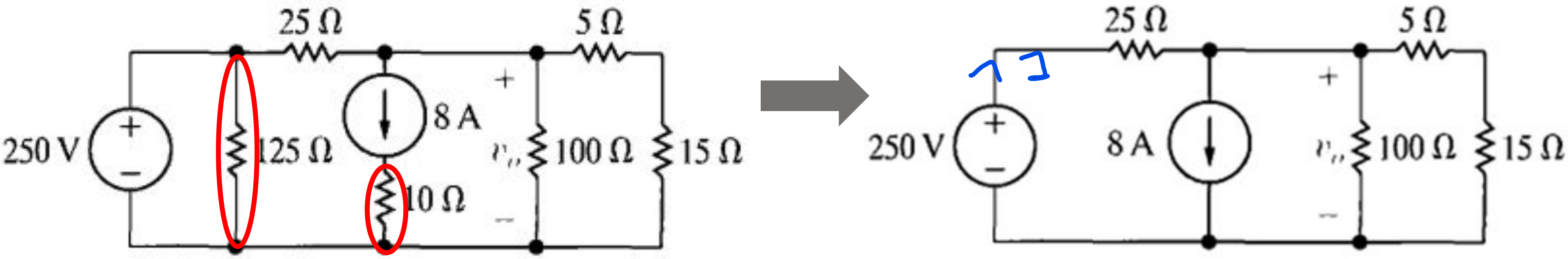


Example 1 (contd … 2)



Problem 1

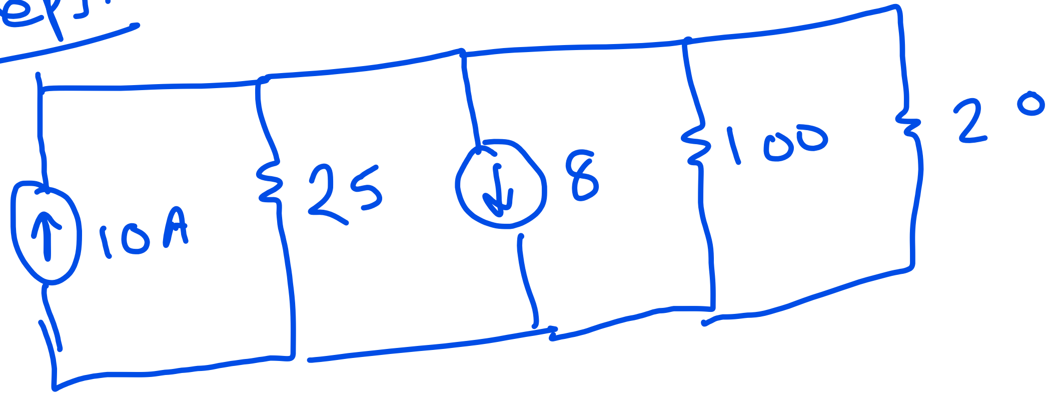
- Use Source Transformation to find the voltage v_0. Find the power developed by the $250\ V$ source and $8\ A$ source.

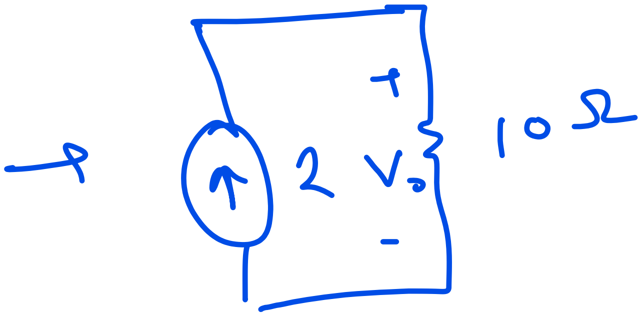


A resistor in series with a current source is redundant, as is a resistor in parallel with a voltage source. We can remove them; this will have no effect on the circuit <u>except for the sources</u>. Opening a resistor parallel to a voltage source will ***reduce*** the current supplied by the source. Similarly, shorting a resistor in series with a current source ***increases*** the voltage across the current source. We have to keep in mind those facts while calculating parameters for the sources.

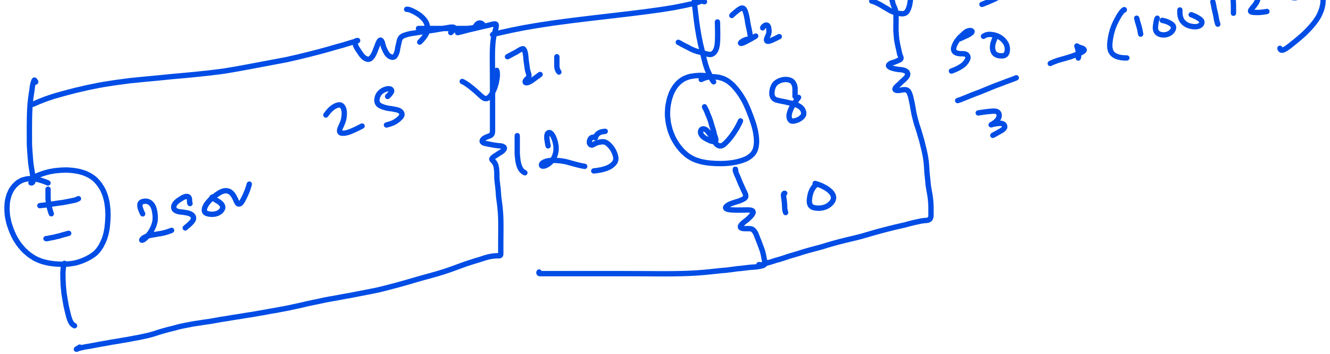
<u>Ans</u>: $\boldsymbol{v_0 = 20\ V;\ P_{250V} = -2.8\ kW;\ P_{8A} = +480\ W}$

Step 1:





So $V_o = 20V$



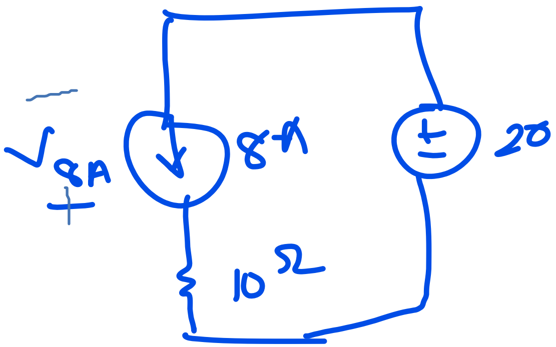
$I = I_1 + I_2 + I_3$

$= \frac{250}{125} + 8 + \frac{v_o}{25/3} = 11.2A$

So, $P_{250} = -11.2 \times 250$

$= -2.8kW$

$\underline{\underline{8A}}$

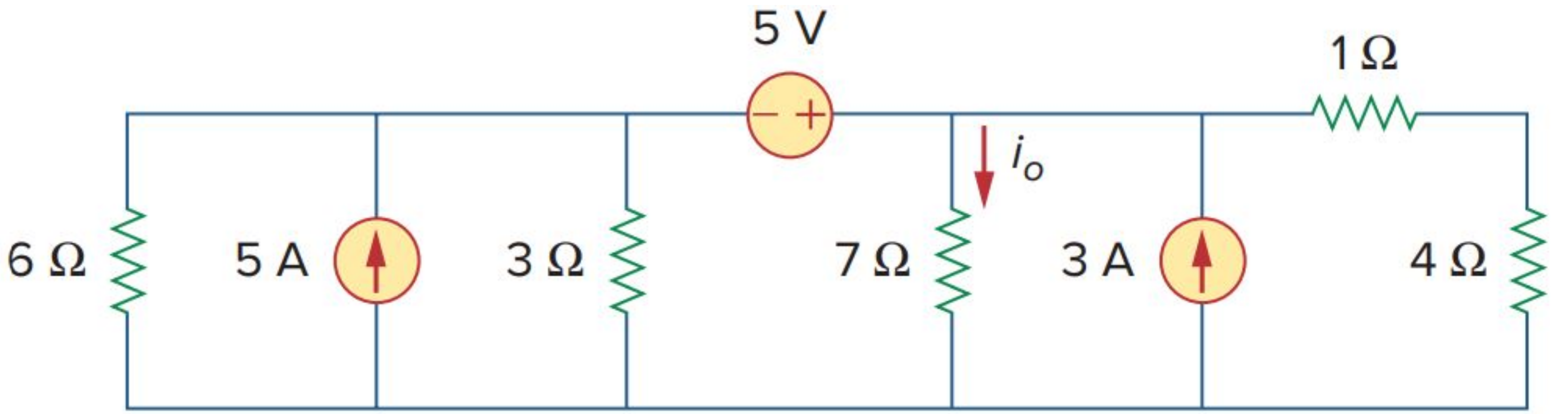


$V_{8A} = 20 + 8 \times 10 = 60$

$P_{8A} = -(60 \times 8) = -480W$

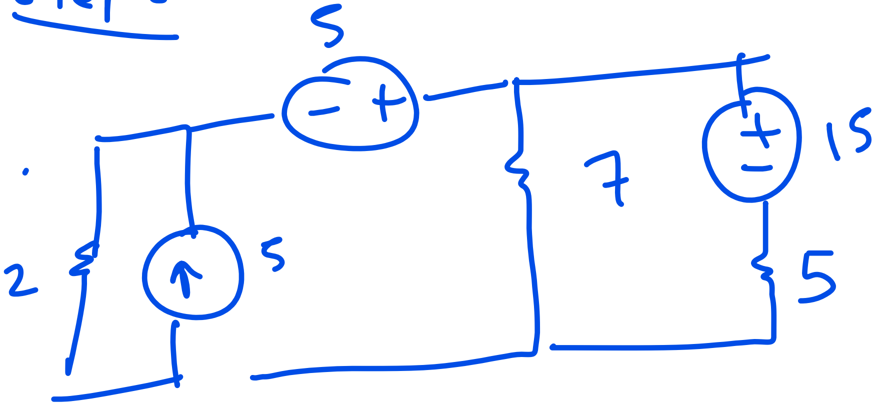
Problem 2

- Find i_o in the circuit using Source Transformation.

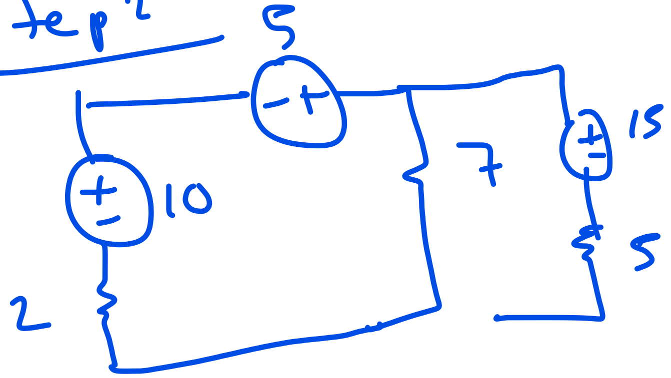


Ans: $\boldsymbol{i_0 = 1.78\ A}$

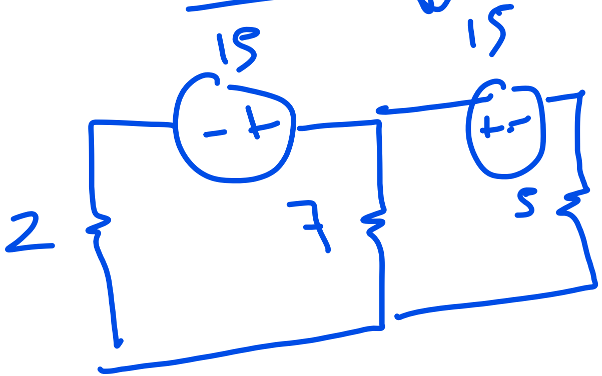
Step 1

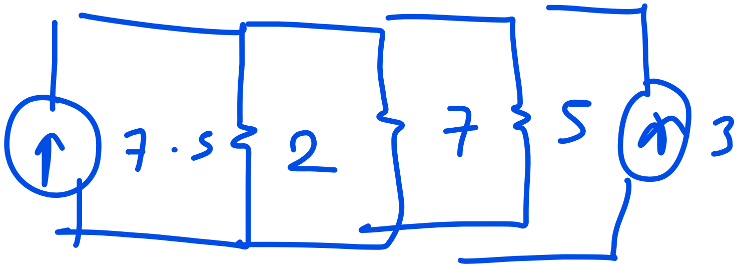


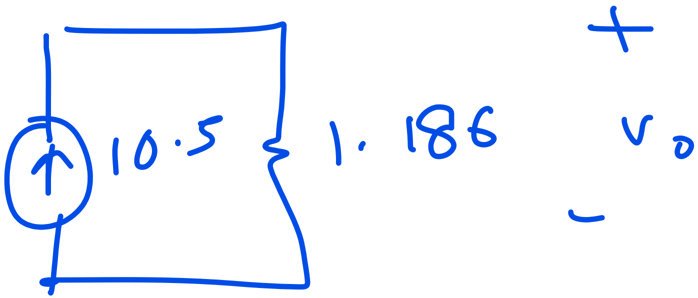
Step 2'



Step 3'





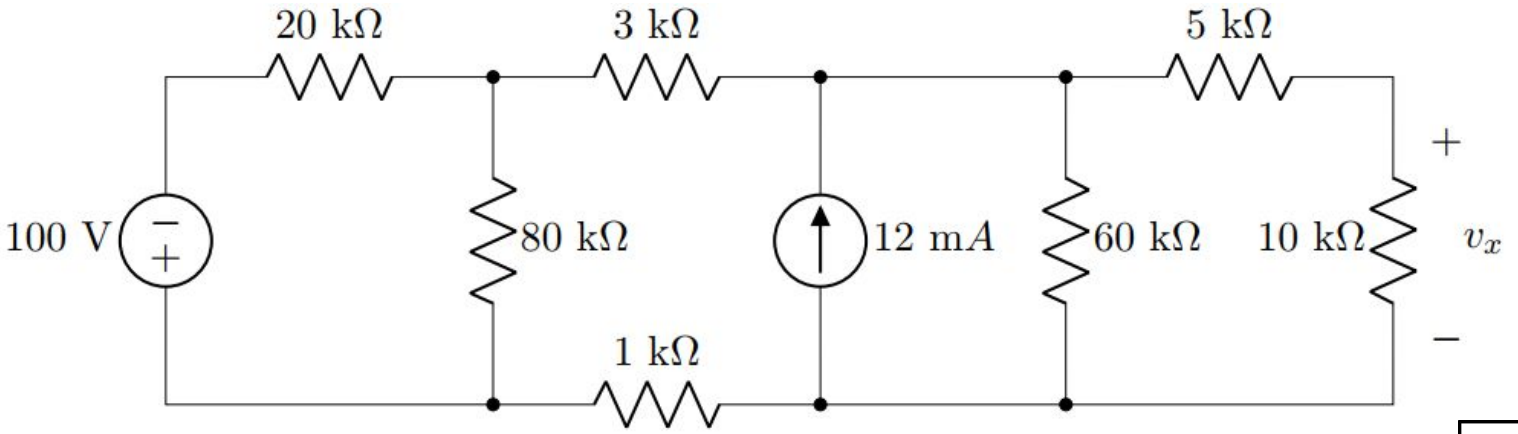


$$V_0 = 1.186 \times 10.5$$

$$\text{So } I_{7\Omega} = \frac{V_0}{7} = 1.78A$$

Problem 3

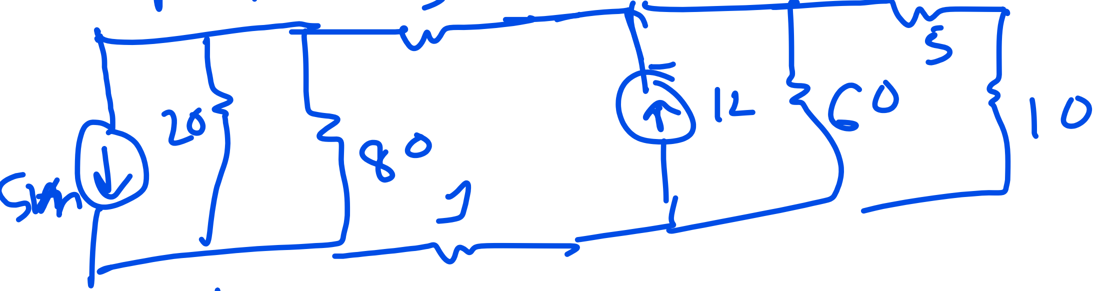
- Determine the voltage v_x across the $10\ k\Omega$ resistor by performing a succession of appropriate Source Transformations.



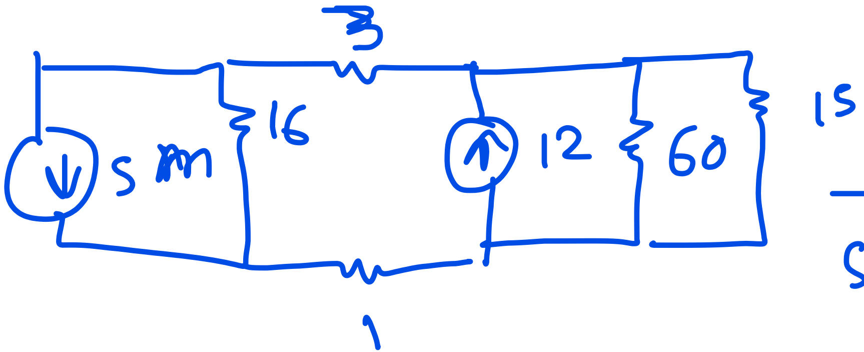
Ans: $v_x = 40\ V$

Fall'22

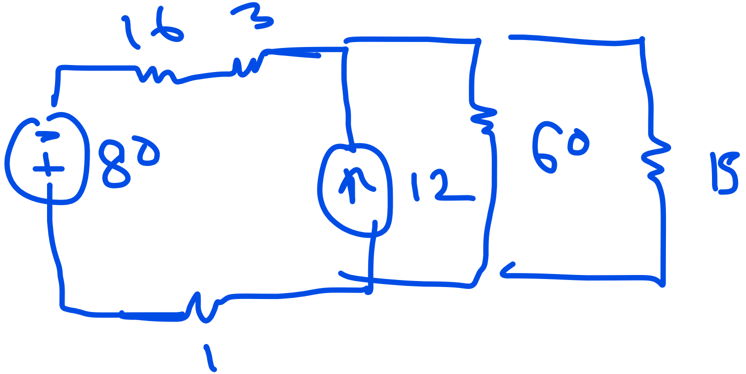
Step 1:



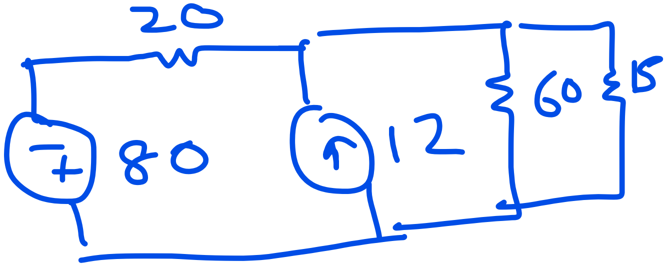
step 2



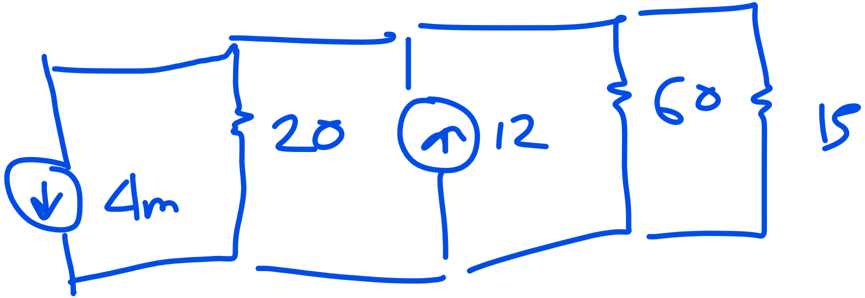
Step 3



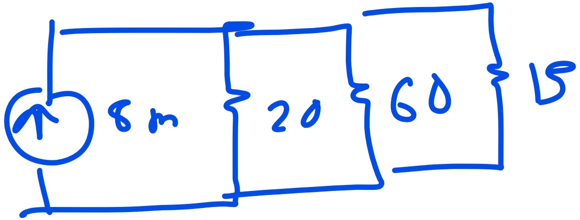
Step 4



Step 5:



Step 6:



Step 7:

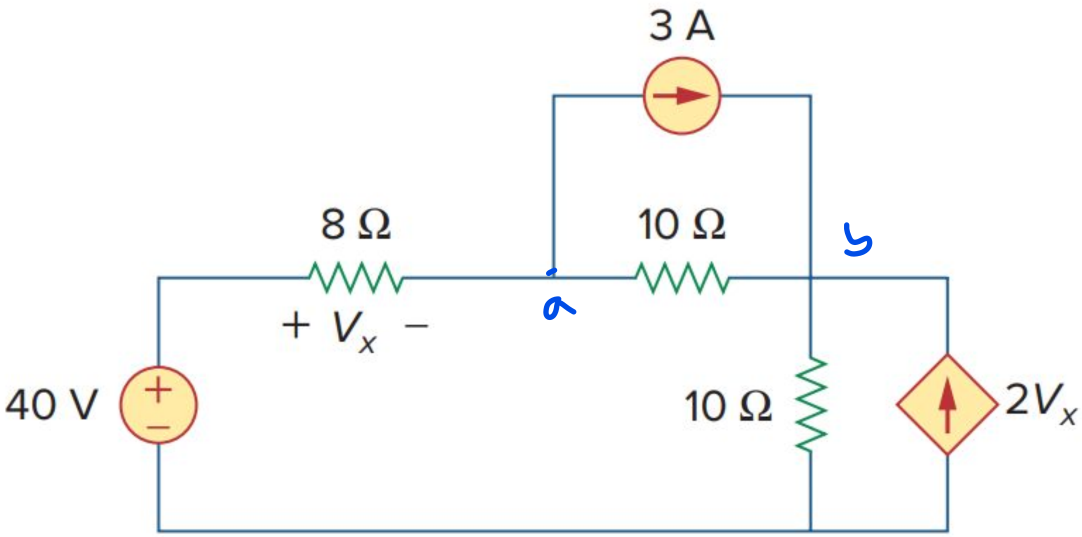
$$V_x = 10 \times I_{15}$$

$$= 10 \times \frac{15^{-1} \times 8}{15^{-1} + 60^{-1} + 20^{-1}}$$

$$= 40V$$

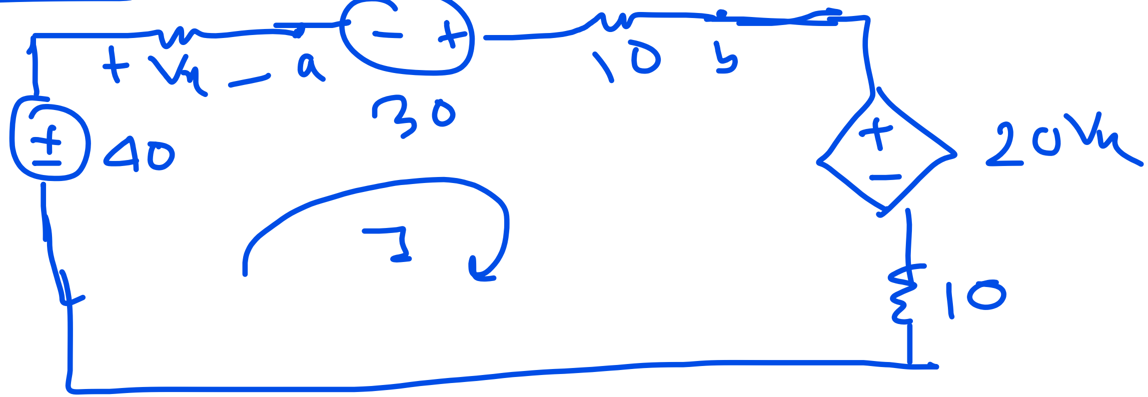
Problem 4

- Use Source Transformation to find V_x.



Ans: $V_x = 2.98\ V$

Step 1:



$188I = 70$

$I = \frac{70}{188}$

$\therefore V_x = 2.98$

Step 2:

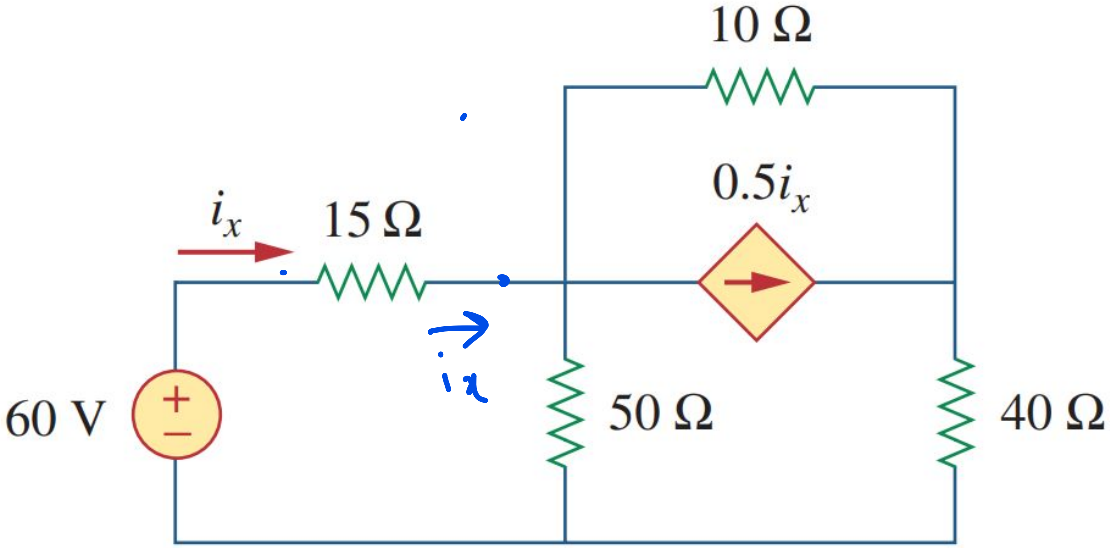
$V_x = 8I$

$-40 + 8I - 30 + 10I + 20V_x + 10I = 0$

$-40 + 8I - 30 + 10I + 160I + 10I = 0$

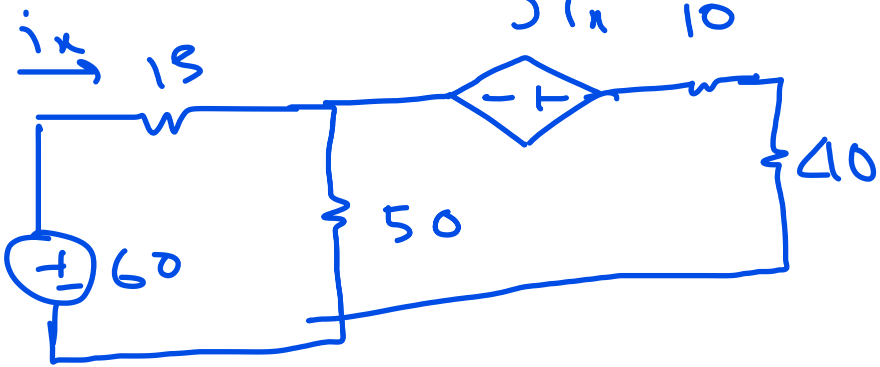
Problem 5

- Use Source Transformation to find i_x in the following circuit.

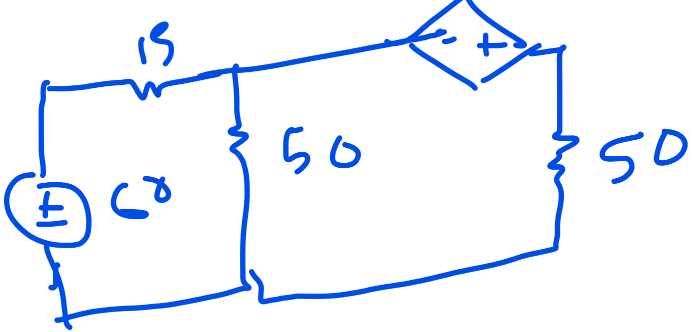


Ans: $\boldsymbol{i_x = 1.6\ A}$

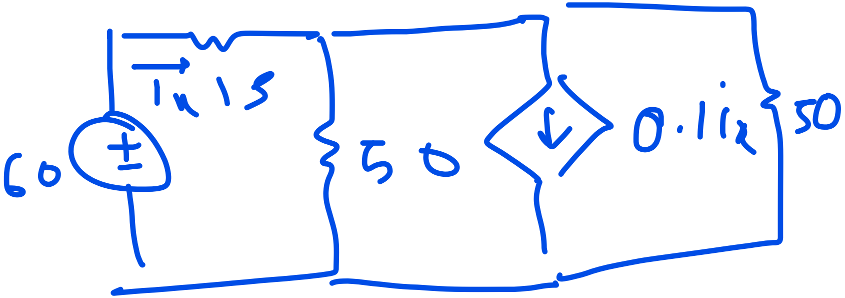
Step 1:



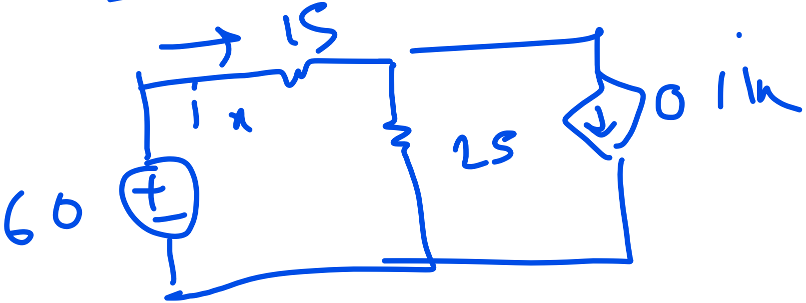
step 2:



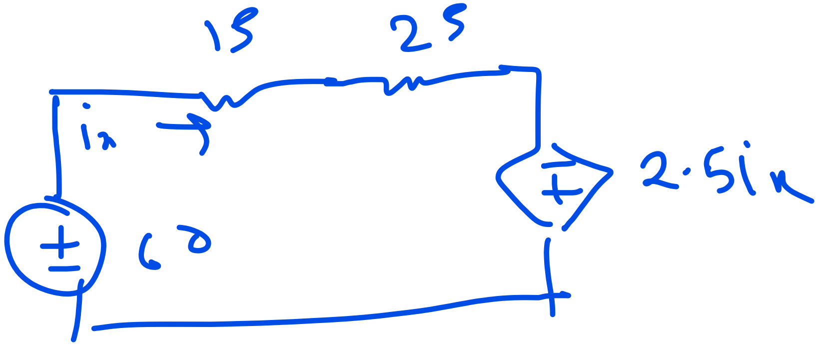
Step 3:



Step 4:



Step 5 :

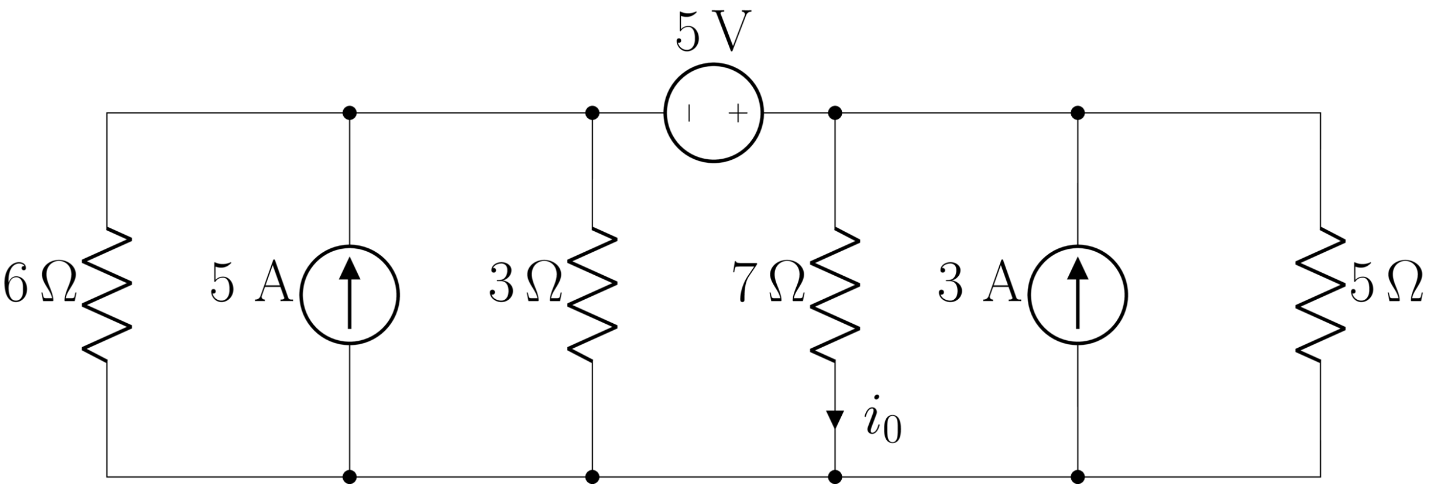


So, $-60 + 15 i_x + 25 i_x - 2.5 i_x = 0$

$i_x = 1.6A$

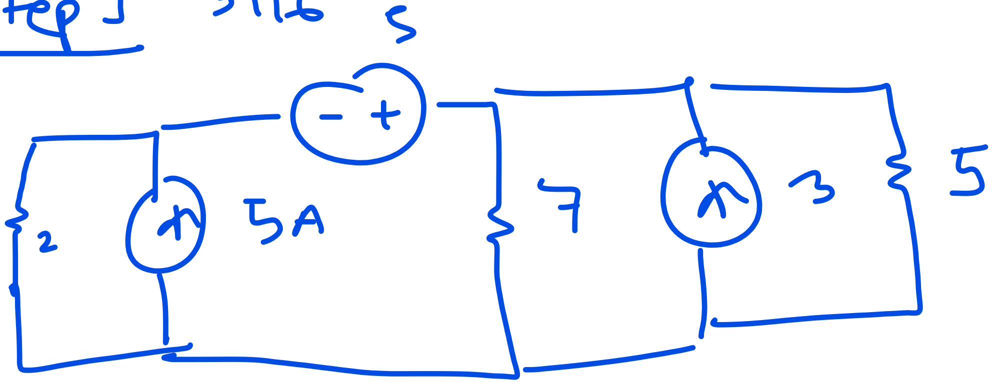
Problem 6

- Reduce the circuit to a single loop using Source Transformation. Then determine i_0.

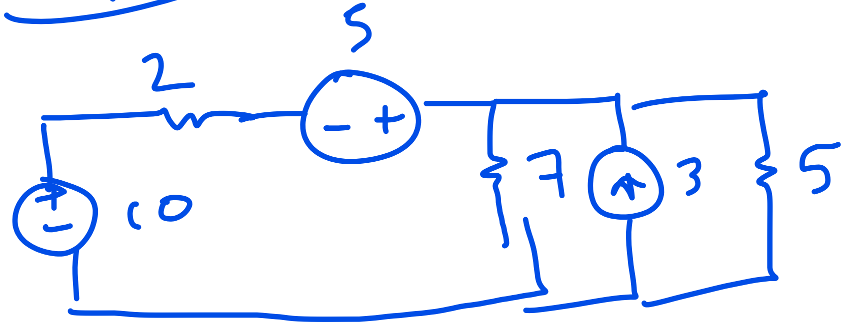


Ans: $\boldsymbol{i_0 = 1.78\ A}$

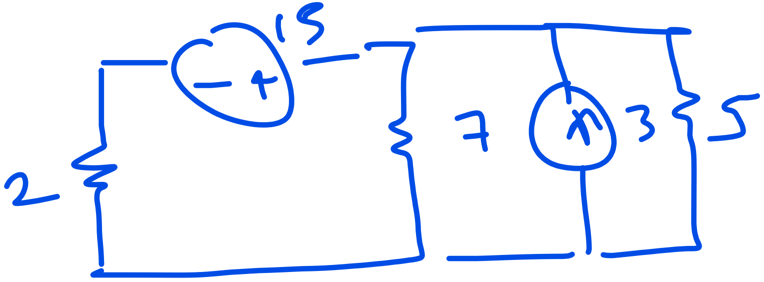
Step 1: 3||6



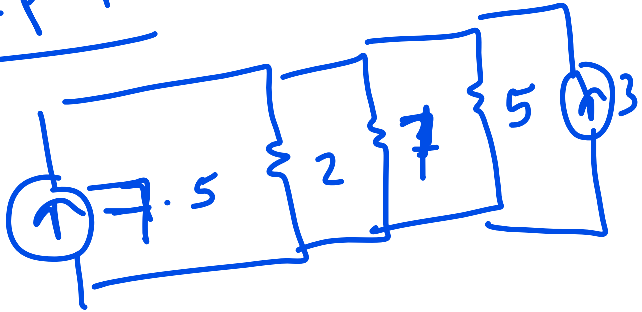
Step 2.



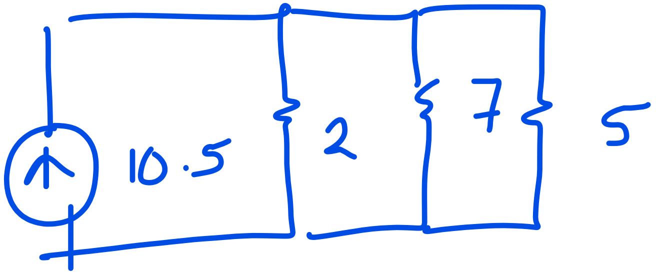
Step 3: 10V + 5V = 15



Step 4.



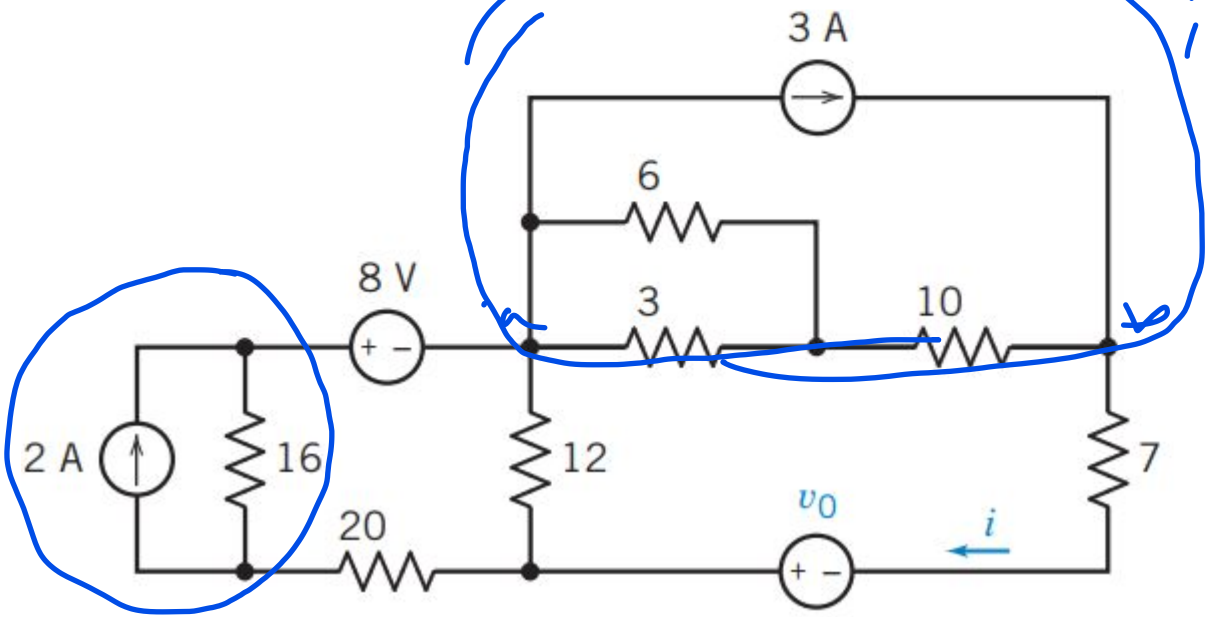
Step 5:



$$i_o = \frac{7^{-1}}{2^{-1}+5^{-1}+7^{-1}} \times 10.5 = 1.78\,A$$

Problem 7

- Reduce the circuit to a single loop. If $i = 2.5\ A$, determine v_0.

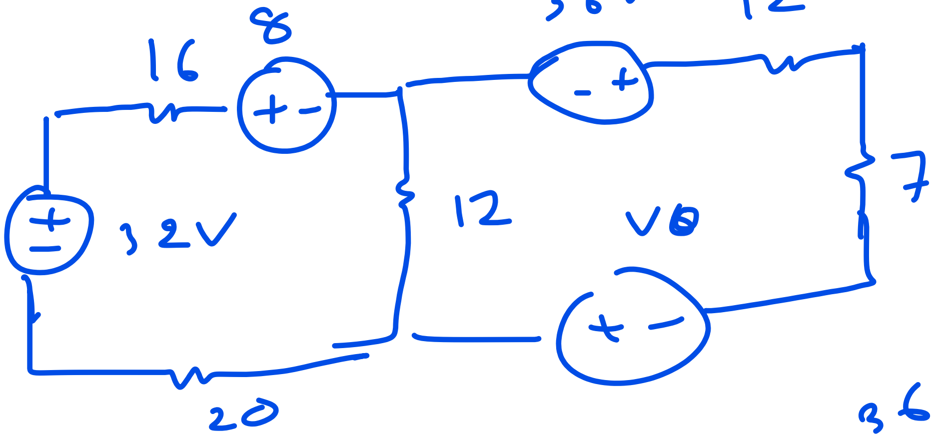


in a,b node

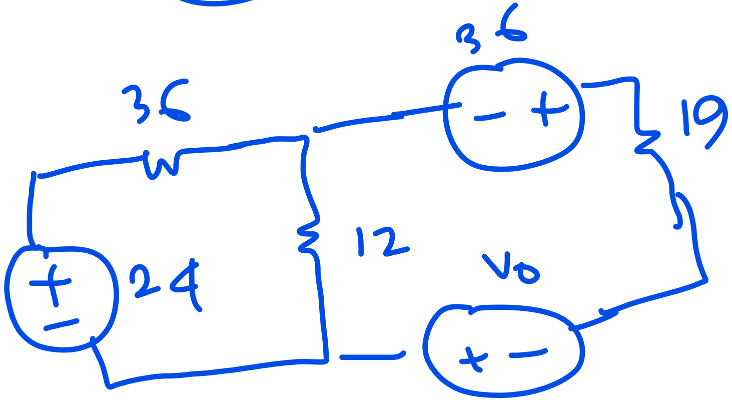
$(6\|3) + 10 = 12\Omega$

Ans: $v_0 = 28\ V$

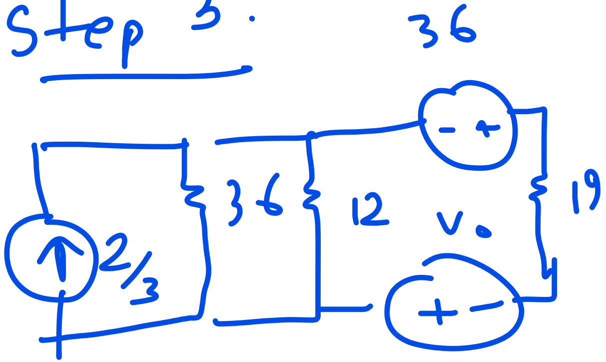
Step 1:



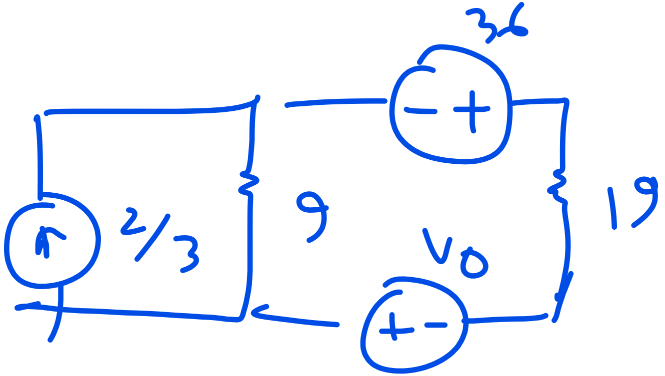
Step 2:



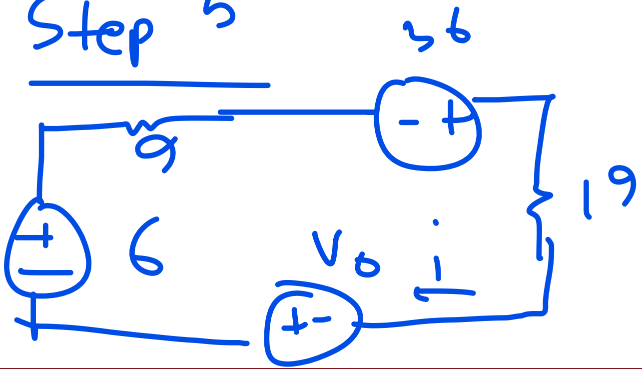
Step 3:



Step 4: $(3\Omega | R)$



Step 5:



Step 6.

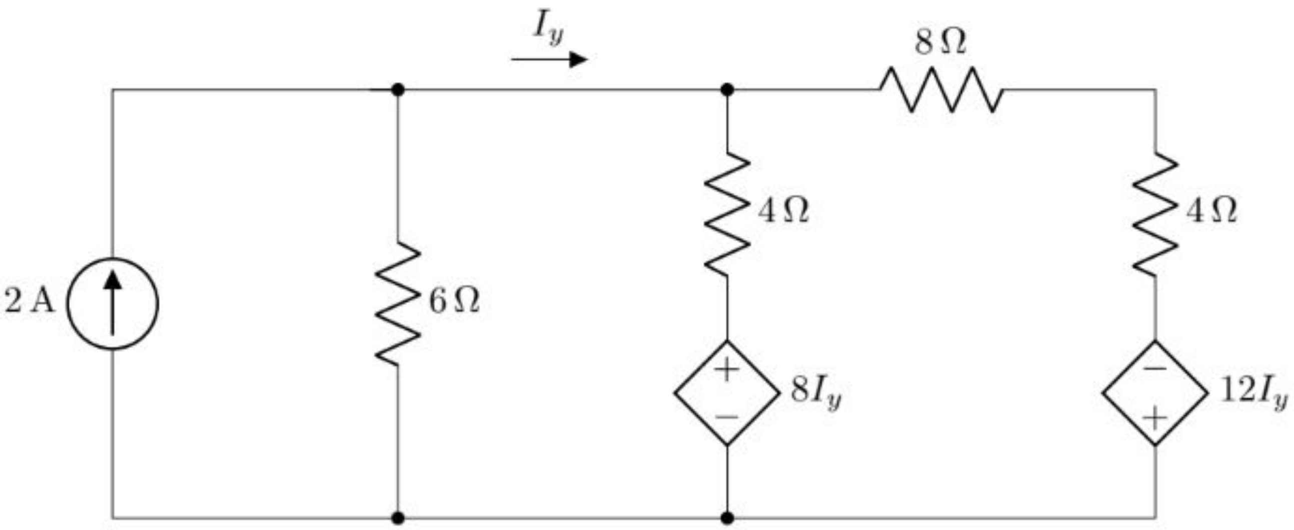
$$-6+9i-36+19i-V_o=0$$

$I = 2.5A$

So, $V_o = 28V$

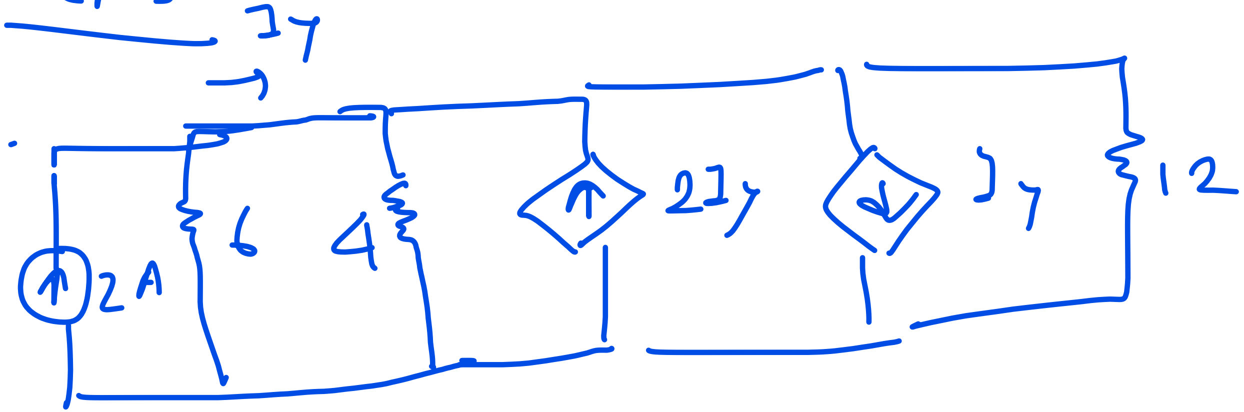
Problem 8

- Reduce the circuit to a single loop using Source Transformation. Then determine I_y.

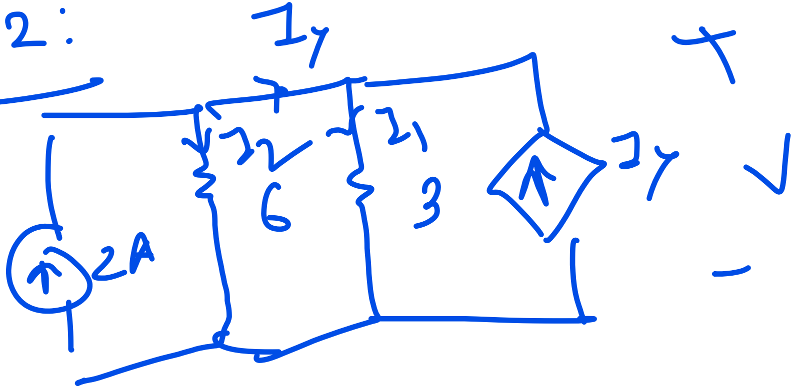


Ans: $\boldsymbol{I_y = 1\ A}$

Step 1



Step 2:



Step 3:

$I_y + I_y = I_1$

$2I_y = \frac{V}{3}$

$V = 6I_y$

So, $2 = I_1 + I_y$

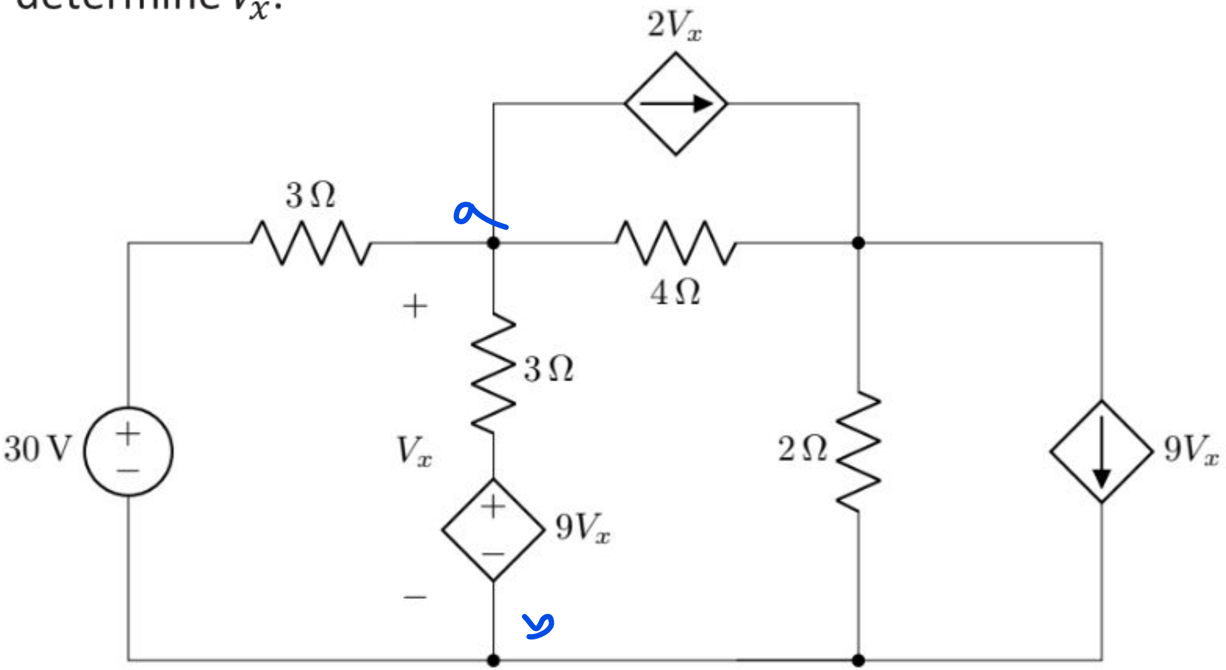
$2 = \frac{V}{6} + I_y$

$2 = \frac{6I_y}{6} + I_y$

$\therefore I_y = 1\,A$

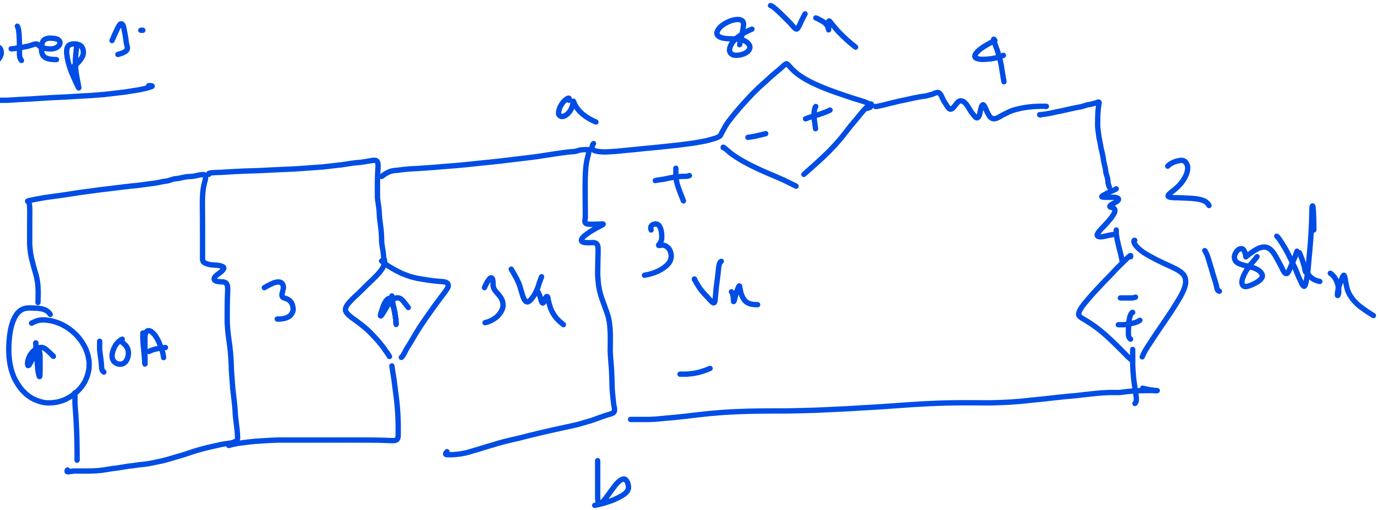
Problem 9

- Reduce the circuit to a single loop using Source Transformation. Then determine V_x.

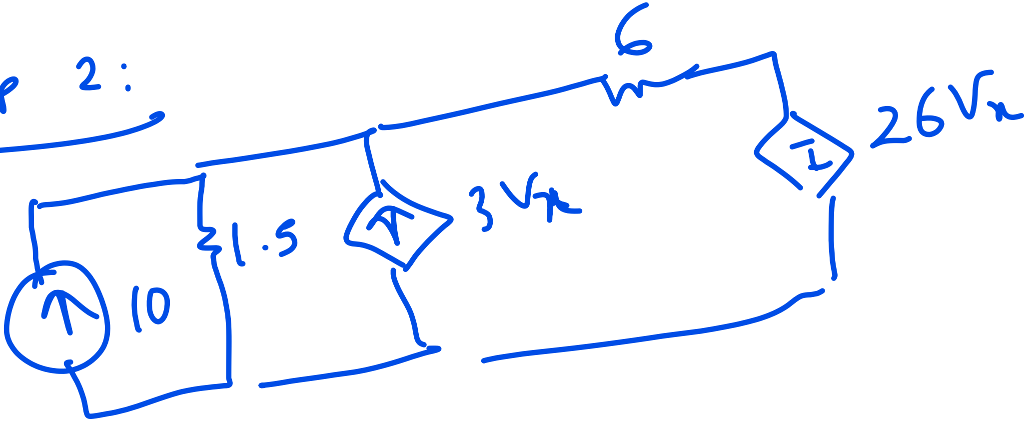


Ans: $V_x = 4.62\ V$

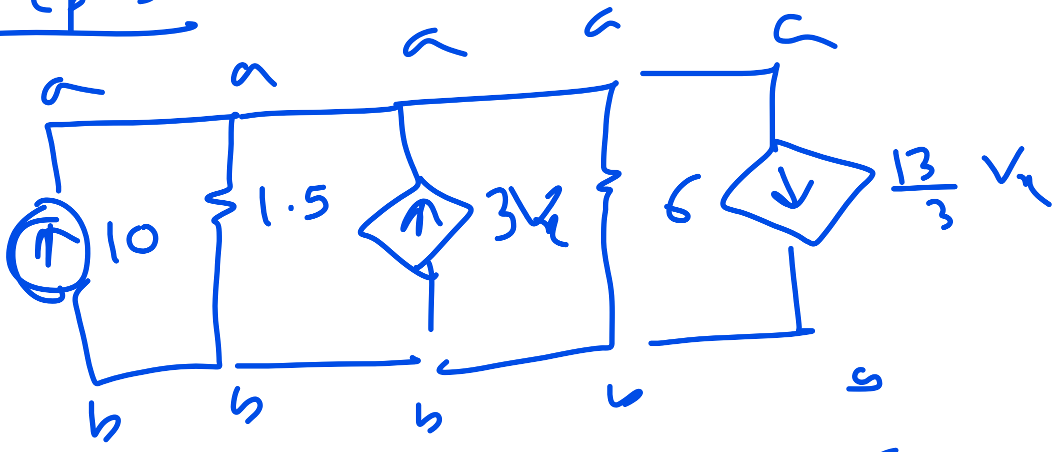
Step 1:



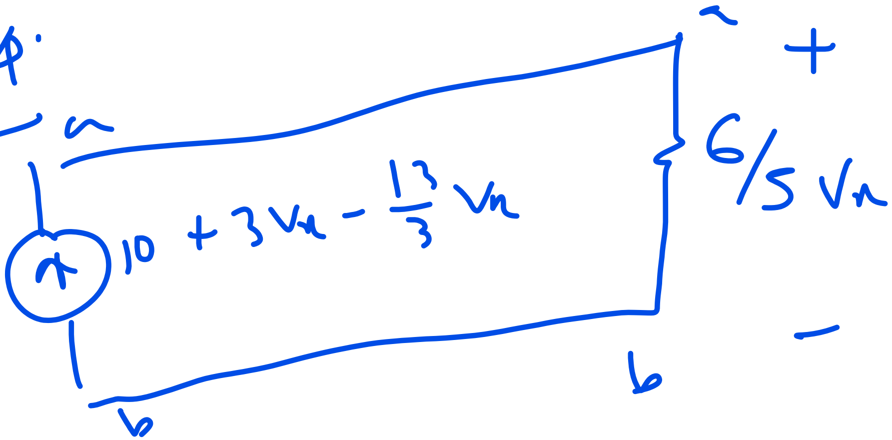
Step 2:



Step 3:



Step 4:



So, $V_{ab} = V_x$

$$V_x = \frac{6}{5}\left(10 - \frac{4}{3}V_x\right)$$

$$V_x = 12 - \frac{8}{5}V_x$$

$$\frac{13}{5}V_x = 12$$

$$V_x = 4.62\ V$$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here

Thank you for your attention

Course Outline: broad themes