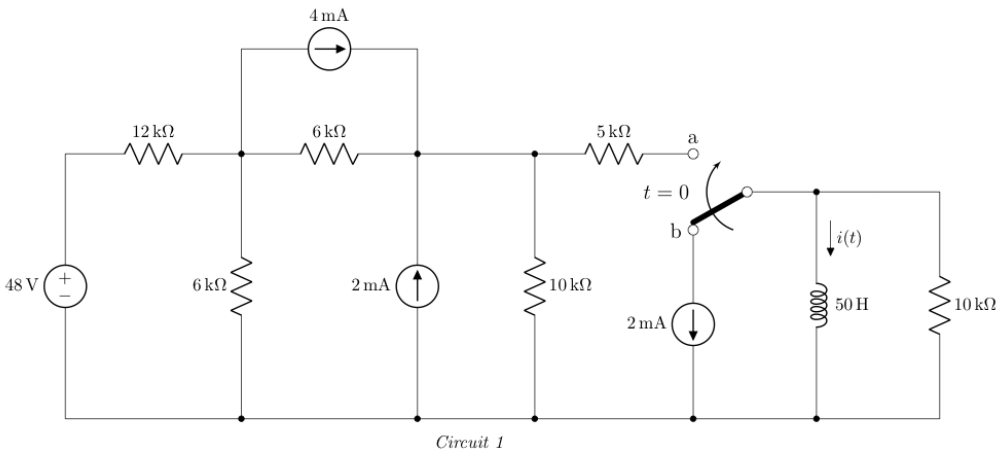
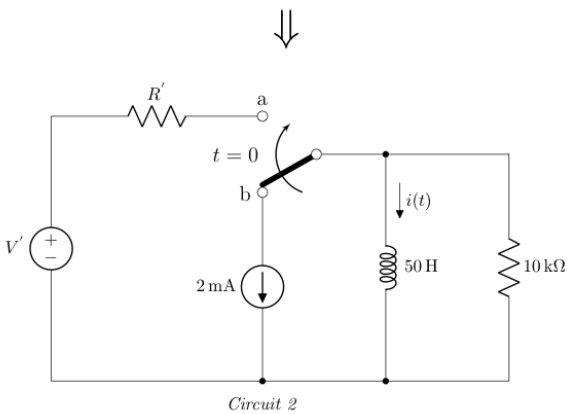
■ Question 1 of 3 *[CO2, CO3] [20 marks]*

Consider the the following circuits which are equivalent to each other.



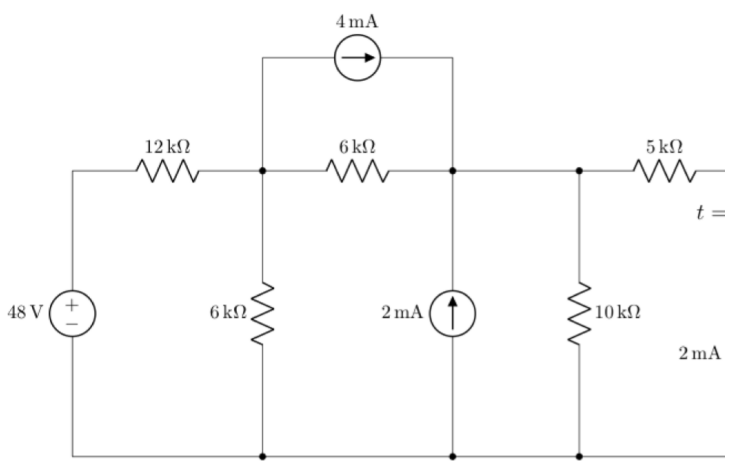
Circuit 1



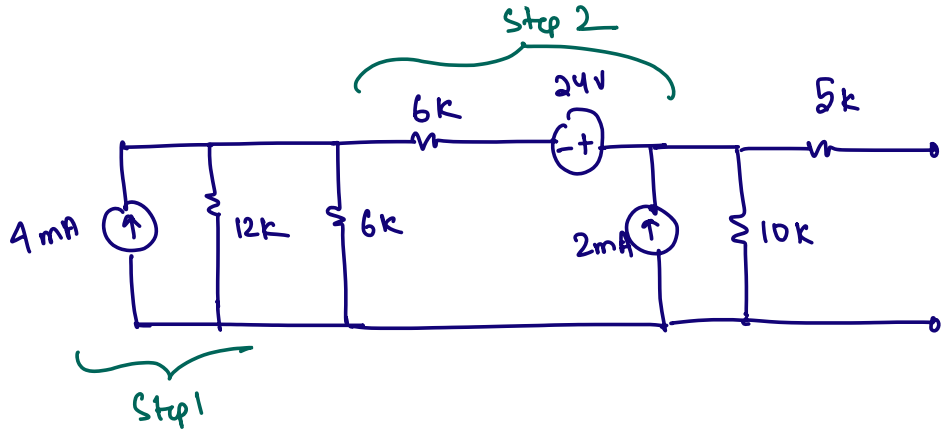
Circuit 2

(a) ***[7 marks]*** **Derive** *Circuit 2* from *Circuit 1*. What are the values of V' and R'?

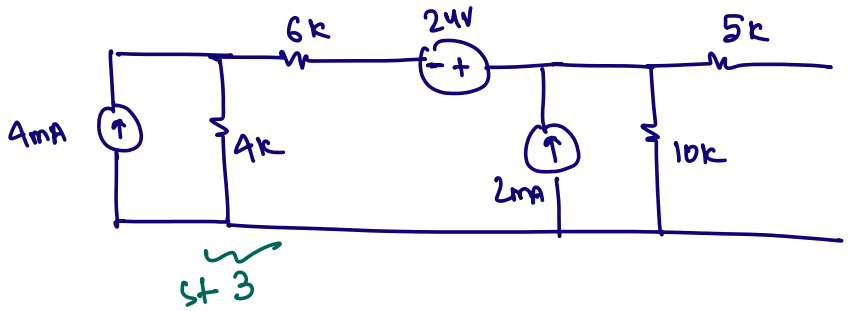
Ans:



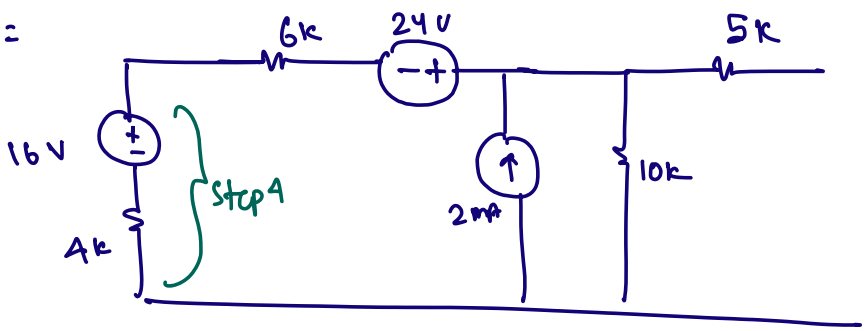
Circuit 1


Step 2
6K
24V
5K
4mA
12K
6K
2mA
10K
Step1

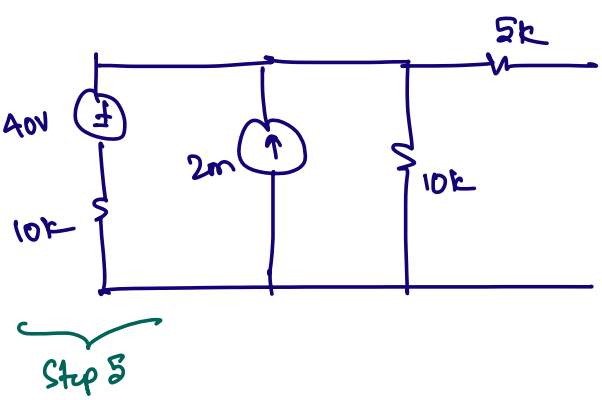
=

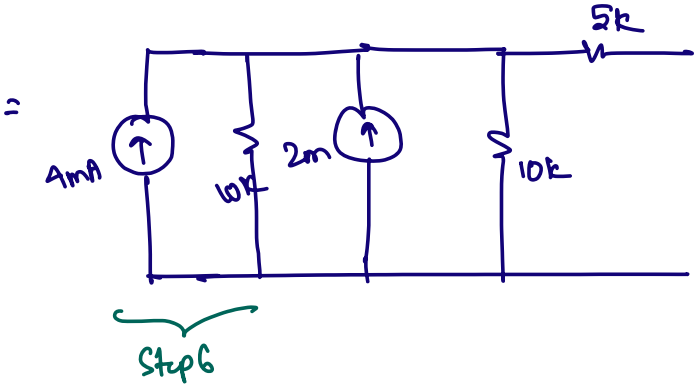

6K
24V
5K
4mA
4K
2mA
10K
st 3

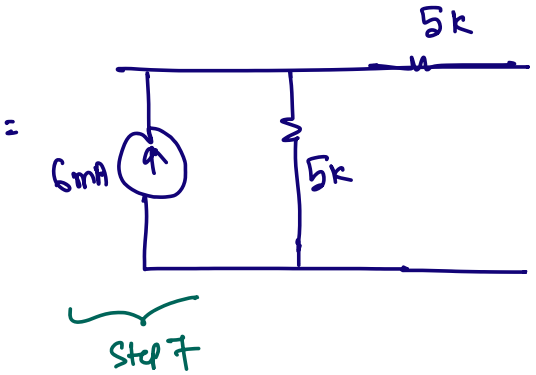
=

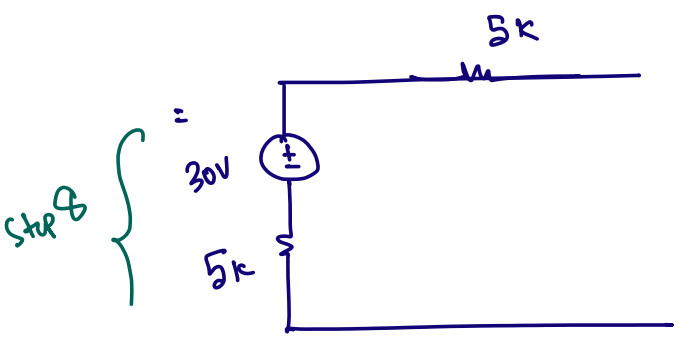

6K
24V
5K
16V
Step4
4K
2mA
10K

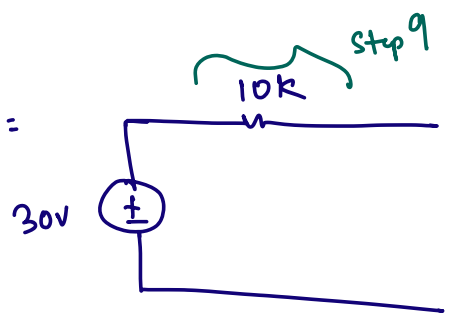
=


5K
40V
2m
10K
10K
Step 3





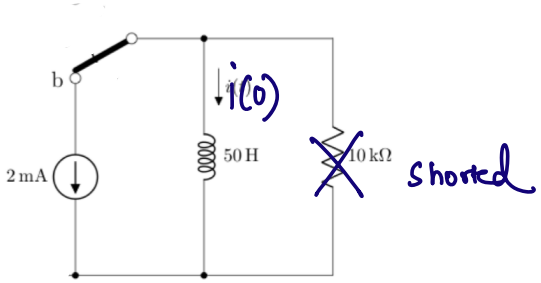




$V' = 30V$	$R' = 10k\Omega$

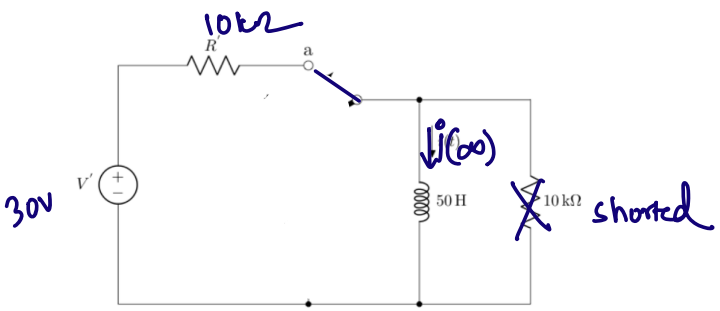
(b) *[9 marks]* Now, **analyze** the transient behavior of the circuit assuming that the switch moves from position b to position a at t = 0. Determine $i(t)$ for t > 0.

t<0,



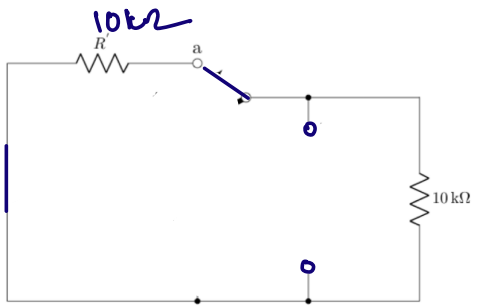
i(0) : − 2mA.

t>0,



$$i(\infty) = \frac{30\,V}{10\,k\Omega} = 3\,mA$$

Req:



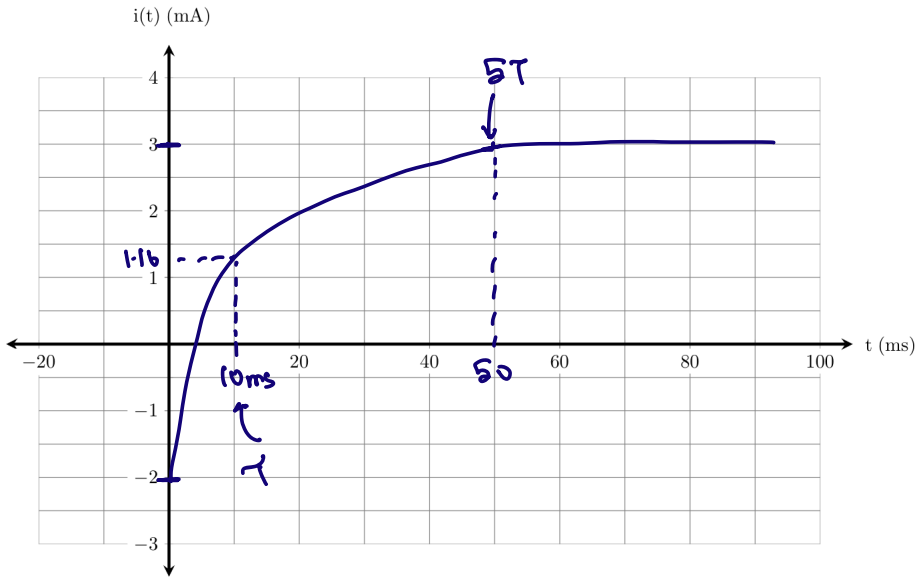
Req : 10k || 10k = 5kΩ

$$\tau = \frac{L}{R} = \frac{50}{5k} = 0.01s$$

$t > 0$, $i(t) : \quad 3 + \left(-2 - 3\right) e^{-t/0.01}$ mA

$$\boxed{i(t) = 3 - 5e^{-t/0.01} \text{ mA}}$$

(c) ***[4 marks]*** Based on your answer in (b), does the inductor get charged or discharged? In the following grid, draw the current $i(t)$ found in (b) as a function of time. Mark the time where the inductor is fully charged or discharged.

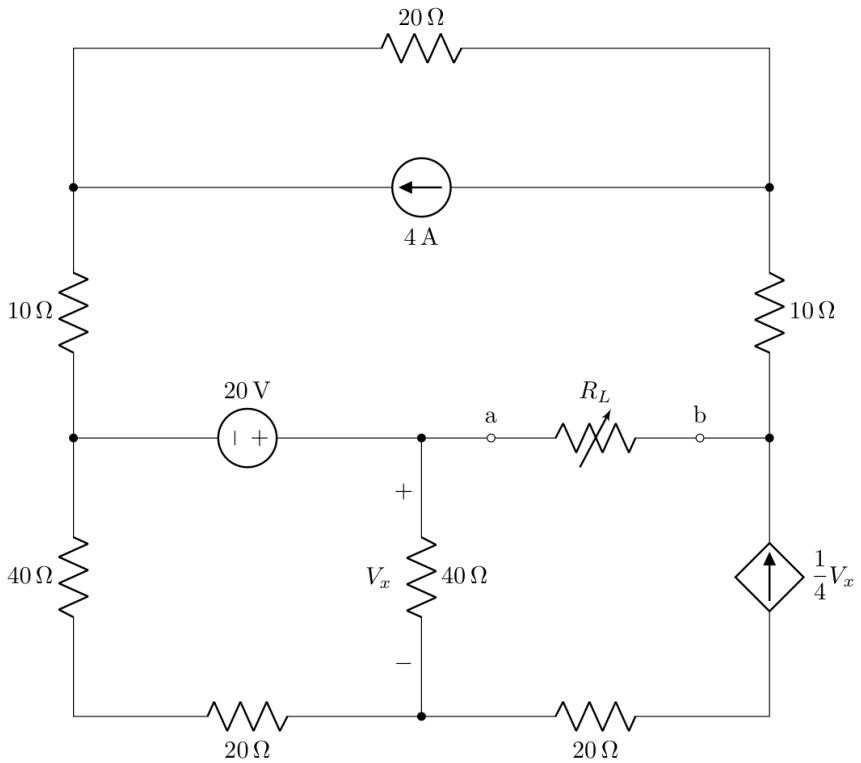


$i(\infty) > i(0)$ ∴ charged

$5\tau = 5 \times 0.01 = 50\text{ ms}$

■ Question 2 of 3 *[CO2] [15 marks]*

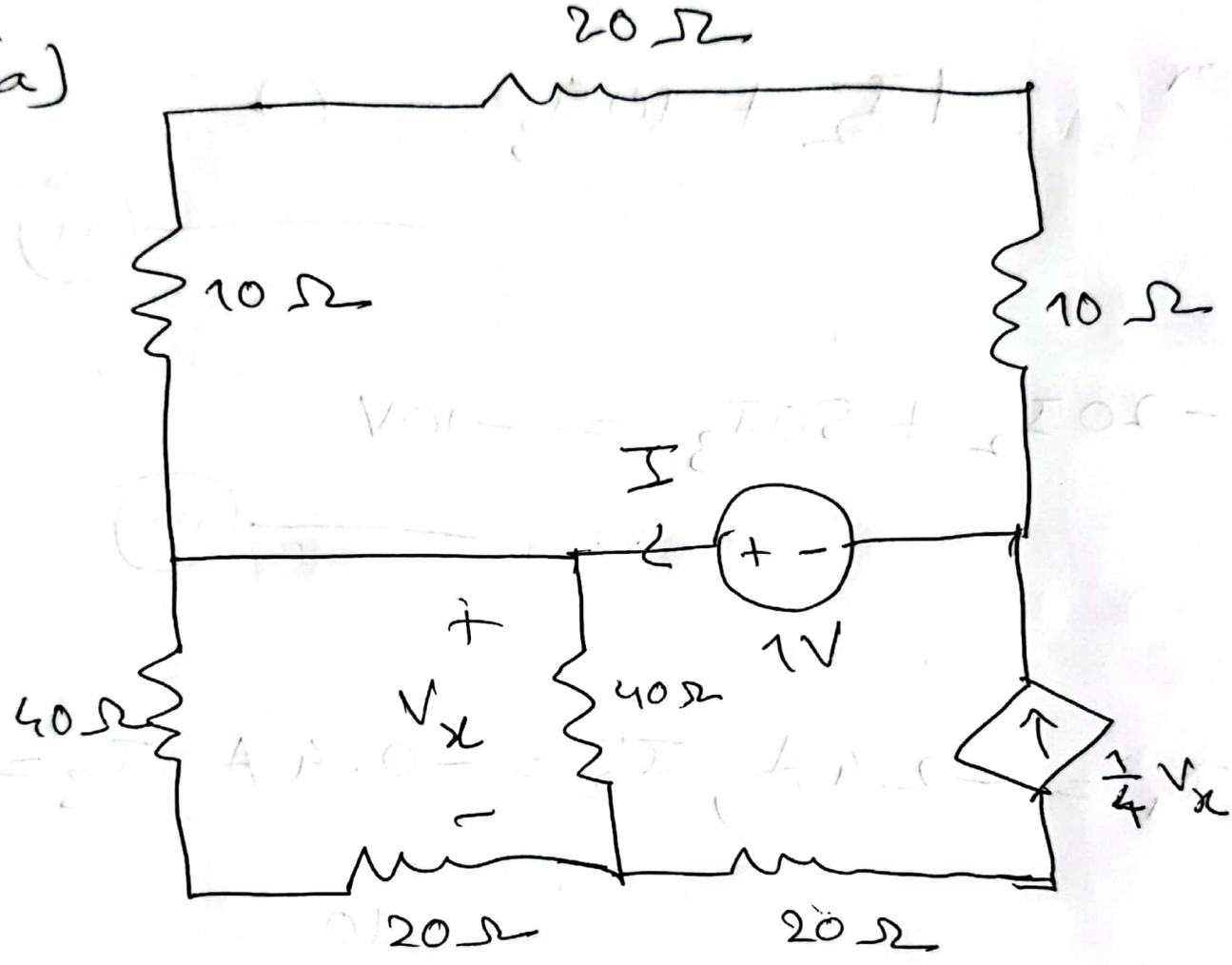
Consider the following circuit with a load R_L connected between terminals a and b.

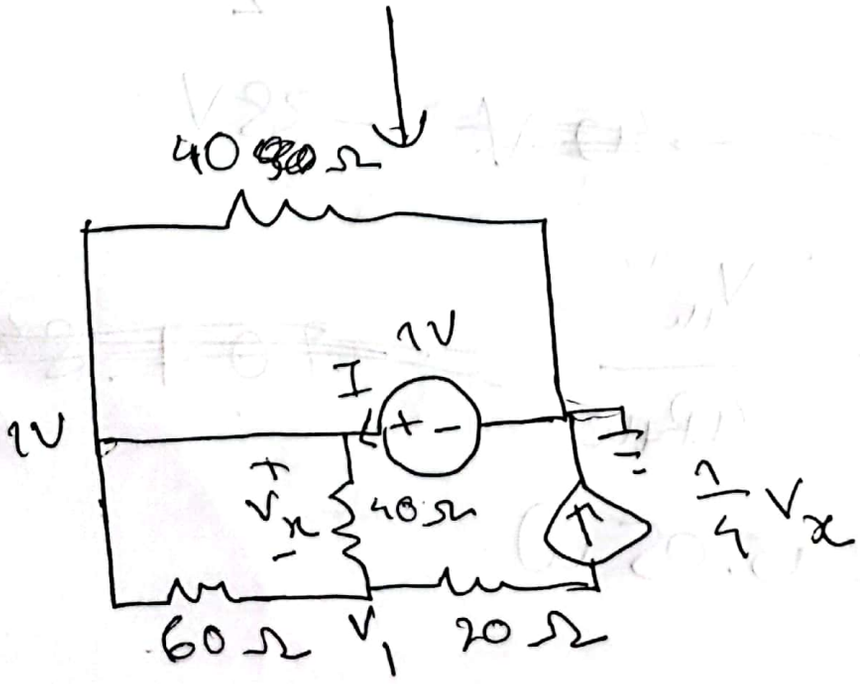


(a) ***[8 marks]*** **Determine** the value of R_L that will draw the maximum power from the circuit.

Set - A

2 (a)


20 Ω
10 Ω
10 Ω
I
+ −
1V
+
V_x
−
40 Ω
40 Ω
$\frac{1}{4} V_x$
20 Ω
20 Ω


40 Ω
1V
I
+ −
1V
+
V_x
−
40 Ω
$\frac{1}{4} V_x$
60 Ω
V_1
20 Ω

$V_x = 1 - v_1$

$$\frac{v_1 - 1}{60} + \frac{v_1 - 1}{40} + \frac{1}{4}(1 - v_1) = 0$$

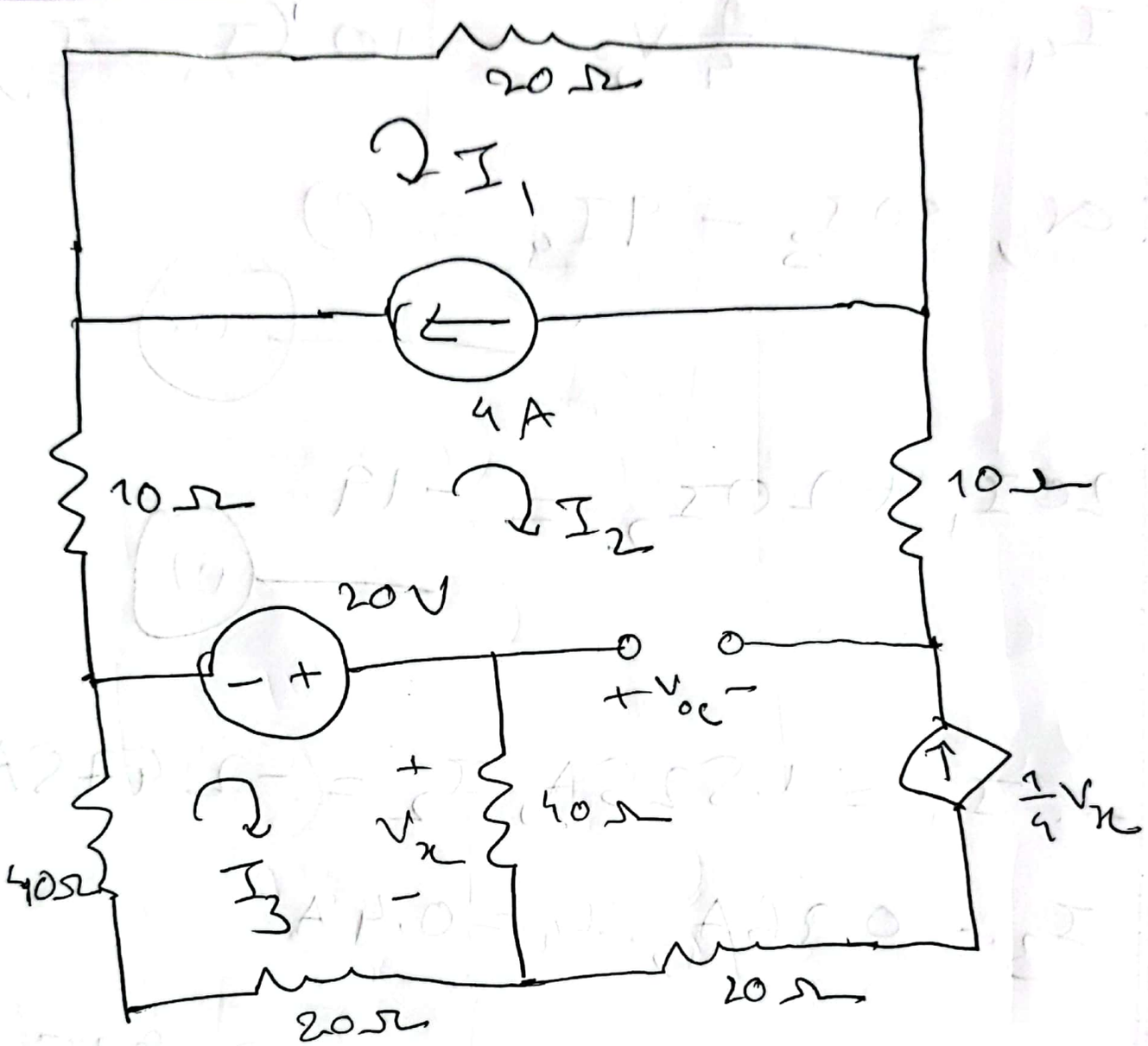
or, $v_1 = 1V$

$$I = \frac{1}{40} + \frac{1}{4} V_x$$

$$= \frac{1}{40}$$

$\therefore R_{th} = 40\ \Omega$

(b)



$$I_1 - I_2 = 4 \quad \text{---(1)}$$

$$V_x = 40(I_3 - I_2)$$

$$I_2 = \text{(2)} \quad -\frac{1}{4}V_x = 10(I_2 - I_3)$$

$$\text{or, } 9I_2 - 10I_3 = 0 \quad \text{---(11)}$$

$$-40I_2 + 100I_3 = 20 \quad \text{---(111)}$$

$$\therefore I_1 = 4.4A,\ I_2 = 0.4A,\ I_3 = 0.36A$$

$$\therefore V_{oc} = V_{th} = 20I_1 + 10I_2 + 20 + 10I_2$$

$$= 116V$$

$$\therefore P_{max} = \frac{V_{th}^2}{4R_{th}}$$

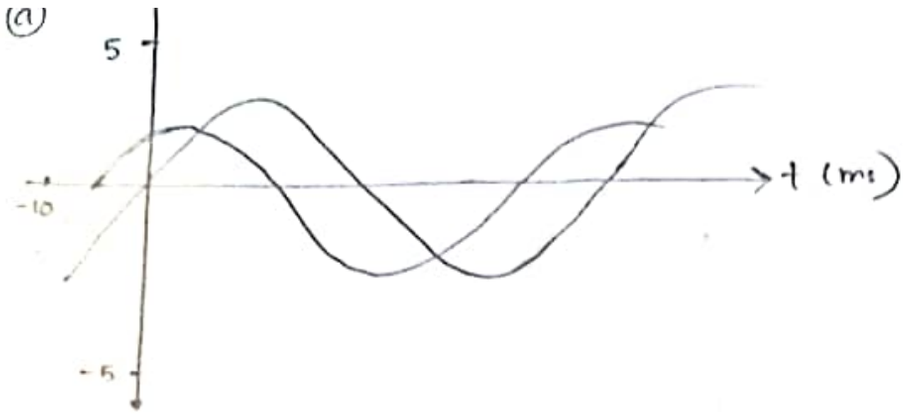
~~$= 1611.43\ \Omega$~~

~~$= 1.6\ k\Omega$~~

$$= 84.1\ \Omega$$

■ Question 3 of 3 *[CO3] [20 marks]*

(a) ***[4 marks]*** The input $v_{in}(t)$ and output $v_{out}(t)$ voltage waveforms of a two terminal ac circuit are plotted as a function of time below. **Determine** mathematically the phase difference between the two and specify which one is leading.



v_{out} → leading

$f = \frac{1}{T} = \frac{1}{40\times10^{-3}} = 25\,Hz$

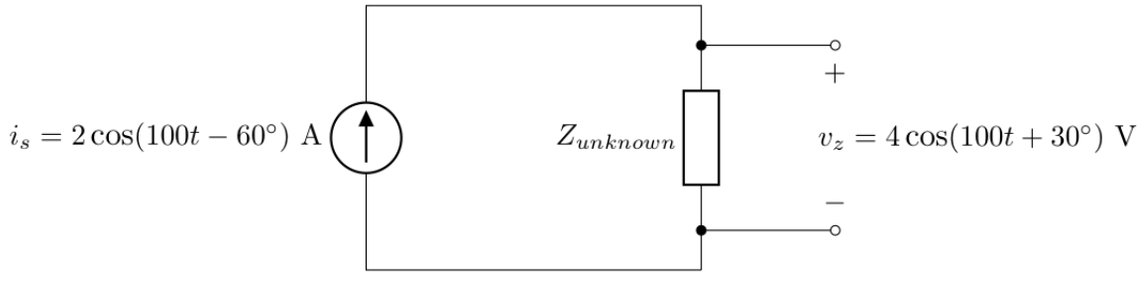
$\theta = 360° \times f \times dt$

$\Delta t = 5\times10^{-3}$

$= 360° \times \frac{1}{40\times10^{-3}} \times 5\times10^{-3}$

$= 45°$

(b) When a current of $i_s = 2\cos(100t - 60°)$ A passes through an unknown circuit element with an impedance of $Z_{unknown}$, it causes a voltage drop of $v_z = 4\cos(100t + 30°)$ V across it as shown below.



(i) ***[1 mark]*** Does the voltage (v_z) lead or lag the current (i_s)?

(ii) ***[1 mark]*** Determine the value of the impedance $Z_{unknown}$.

(iii) ***[2 marks]*** Based on your answer in (ii), guess the circuit element and **determine** the value of it with appropriate units.

(b) Impedance = $\frac{4\angle 30^\circ}{2\angle -60^\circ} = 2j$ → inductor

So, $j\omega L = 2j$

$\omega = 100$

$100 \times L = 2$

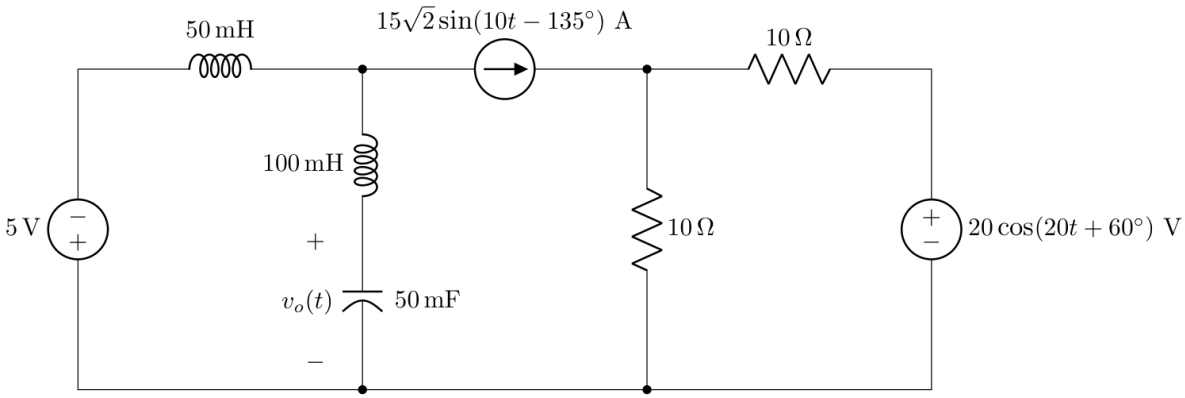
$L = \frac{2}{100} = 0.02\,H$

(i) V_2 leads i_6

(ii) $Z_{unknown} = 0j$

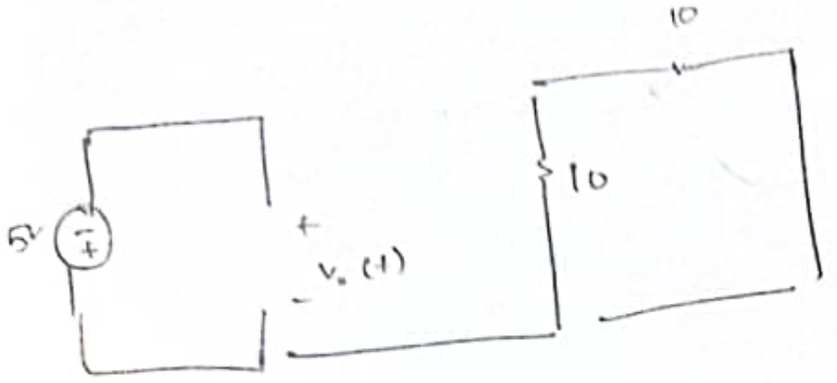
(iii) $L = 0.02\,H$

(c) ***[12 marks]*** For the circuit shown below, **determine** $v_o(t)$, the voltage across the capacitor.



c/

DC source active:



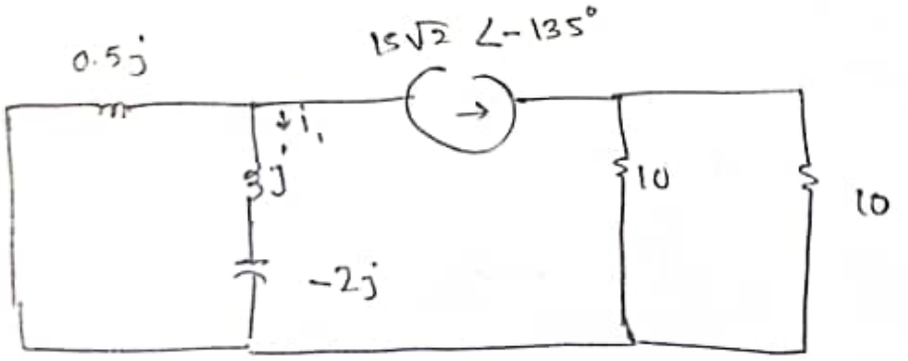
$V_o(t) = -5V$

AC Current: $\omega = 10$

$Z_1 = j\omega_1 L_1 = j \times 10 \times 50 \times 10^{-3} = 0.5j$

$Z_2 = j\omega_2 L_2 = j \times 10 \times 100 \times 10^{-3} = 1j$

$Z_3 = \frac{1}{j \times 10 \times 50 \times 10^{-3}} = -2j$



$$i_1 = \frac{-\frac{1}{j-2j}}{\frac{1}{0.5j} + \frac{1}{j-2j}} \times 15\sqrt{2} \angle -135°$$

$i_1 = ~~20.8 \angle 33.69°~~ \quad 15\sqrt{2} \angle -135°$

$V_{o_2} = (-2j \times i_1) = ~~41.60 \angle 123.7°~~$

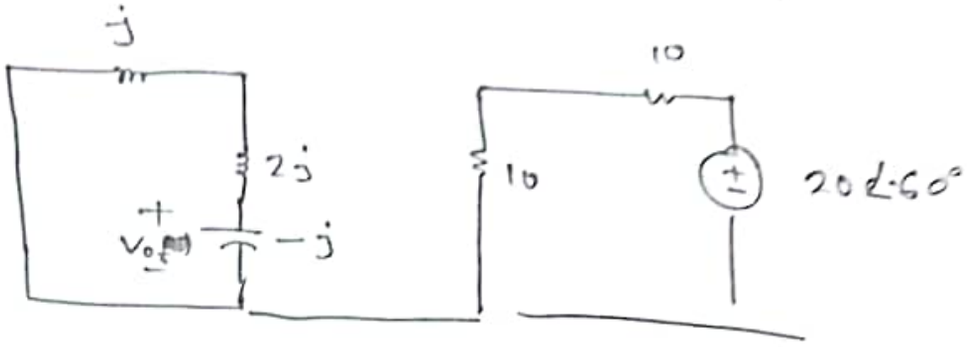
$= 30\sqrt{2} \angle 135°$

AC voltage: $\omega = 2$ rad/s

$Z_1 = j\omega_1 L_1 = j$

$Z_2 = 2j$

$Z_3 = -j$



$V_{o_3} = 0\,V$

So, $V_o = V_{o_1} + V_{o_2} + V_{o_3}$

~~$= -5 + 41.6 \sin(10t - 135°) + 0$~~

~~$= -5 + 41.6 \sin(10t + 135° - 31°)$~~

$= -5 + 30\sqrt{2}\sin(10t + 135°)\ V$