Department of Computer Science and Engineering (CSE)
BRAC University

Lecture 4

CSE250 - Circuits and Electronics

I-V OF LINEAR CIRCUITS

BRAC UNIVERSITY
Inspiring Excellence

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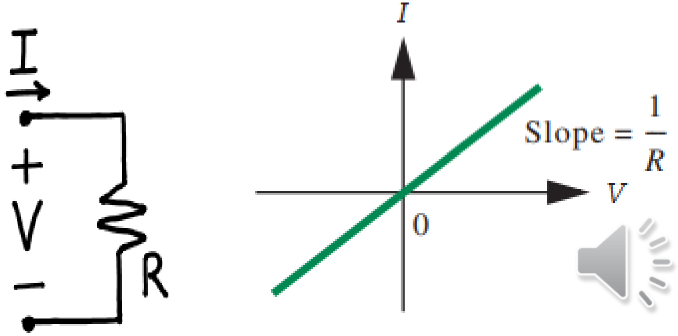
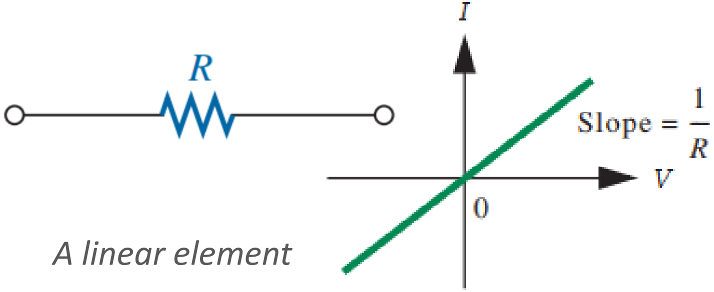
I-V Characteristics

- The *current-voltage characteristics* or the *$I - V$ characteristics* is a relationship, typically expressed graphically, between the electrical current flowing through an element, circuit, device, or material and the corresponding voltage across it.
- From the viewpoint of circuit analysis, $I - V$ the most important characteristics of a two-terminal element, also called *Element Law*.
- So far, we have seen that the current voltage relationship for a resistor follows Ohm's Law, that is, $V = IR$. In an I vs. V plot it is a straight line with slope equal to $\frac{1}{R}$ that goes through the origin.
- It is important to note the direction of current to be plotted. Generally, the current plotted along the y-axis is the current drawn by the element, circuit, or device.

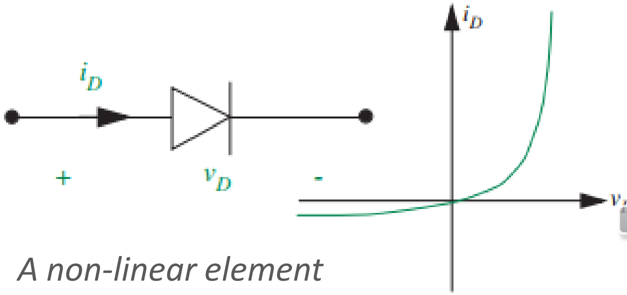


Linear vs. Non-linear Elements

- We can distinguish between the linear and non-linear circuit components from $I-V$ point of view.
- A two-terminal circuit element is said to be *linear* if it constitutes a linear (straight line) relationship between the current through and the voltage across it. Examples of linear circuit elements are ideal resistor, ideal voltage source and current source, open circuit, short circuit, capacitors, inductors etc. A circuit constructed with linear circuit elements is called a *linear circuit*.
- *Non-linear* devices, on the other hand, have a $I-V$ curve that is not straight line. Examples of non-linear elements include diodes, transistors, and nonlinear capacitors.



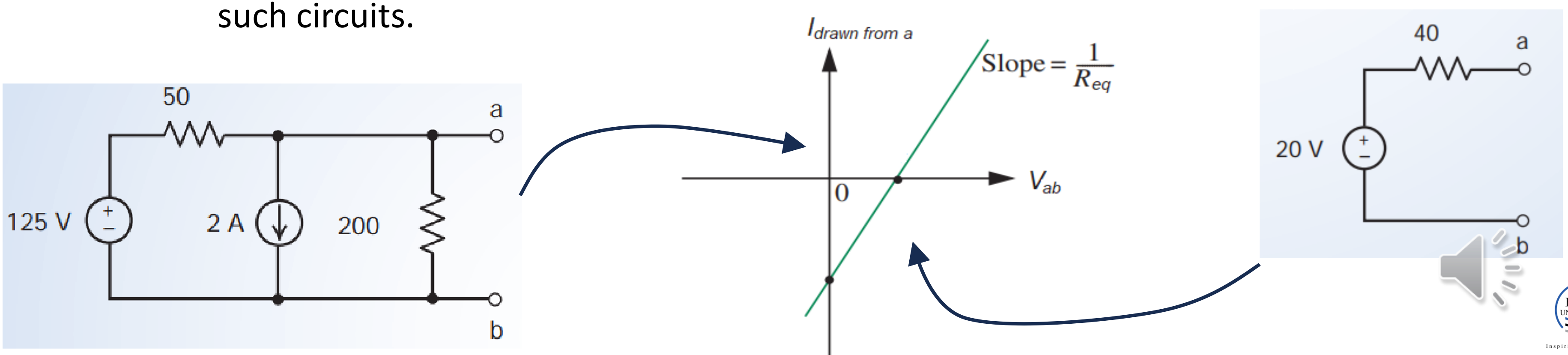
A linear element



A non-linear element

Circuit Equivalence

- Two circuits are said to be equivalent with respect to two particular terminals (or node) if they have *identical $I - V$ relationships* between those terminals.
- For example, a resistive series network can only be replaced with their equivalent resistance if the $I - V$ line remains identical after replacement. This requires the relation $R_{eq} = R_1 + R_2 + \cdots + R_N$ to be followed.
- Similarly, the following two linear circuits are equivalent as they have the identical $I - V$ relation between terminals $a - b$ as shown. Let's see how to derive $I - V$ plots for such circuits.



I-V: theoretically

Method 1

Assume V and I variables
Assume a voltage variable between the terminals to be considered (let's say $a - b$) and a current variable directed outward from the '+' of V. The direction ensures the current is drawn by the circuit.

↓

Derive an equation
Apply circuit laws or other solving methods to derive a relation between the variables I and V. I and V are the only variable of the equation.

↓

Plot the relation
In a I vs. V grid, plot the equation derived in step 2.

Valid only for linear circuits whose $I - V$ is a straight line.

Method 2

Apply a known voltage
Apply a voltage source between the terminals with any arbitrary value.

↓

Solve for current
Solve the circuit and determine the current supplied by the voltage source.

↓

Repeat the previous steps
Again, apply another known voltage and solve for the same current.

↓

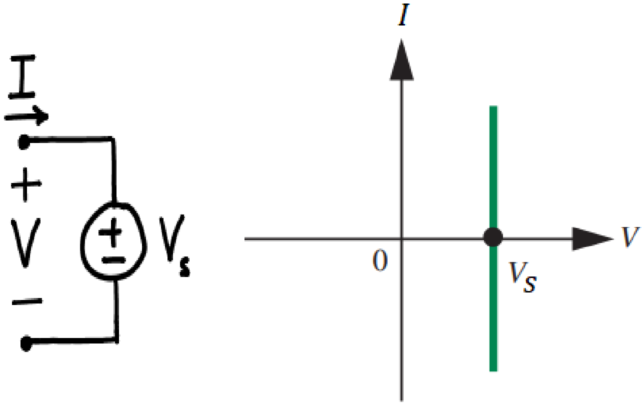
Connect the data points
Place the two data points (v_1, i_1) and (v_2, i_2) in a grid and connect them with a line.

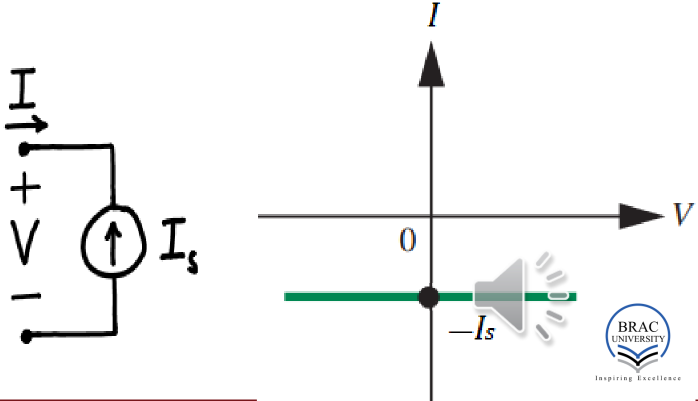
I-V of Sources

- An ideal *independent voltage source* always holds a constant potential difference between its terminals irrespective of the current drawn from it.
- So, the constituent relation for an independent voltage source supplying a voltage of V_s is,

$$V = V_s$$

- This is a straight line that is parallel to the I-axis and intersects the V-axis at V_s.
- Similarly, an ideal *independent current source* always supplies a constant current to the wire it is connected irrespective of the voltage across it.
- The constituent relation is then $I = -I_s$, which is a straight line parallel to V-axis.

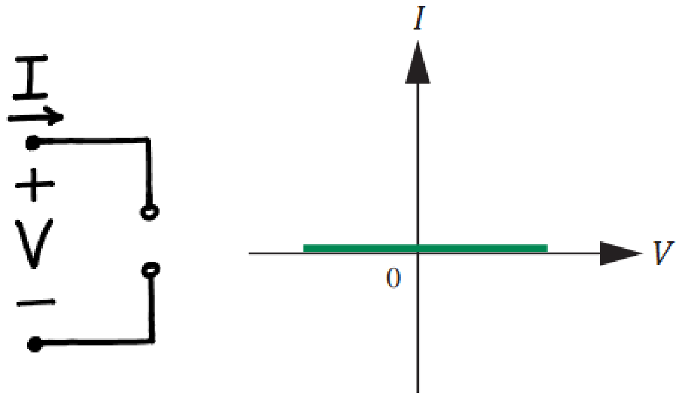




I-V of OC and SC

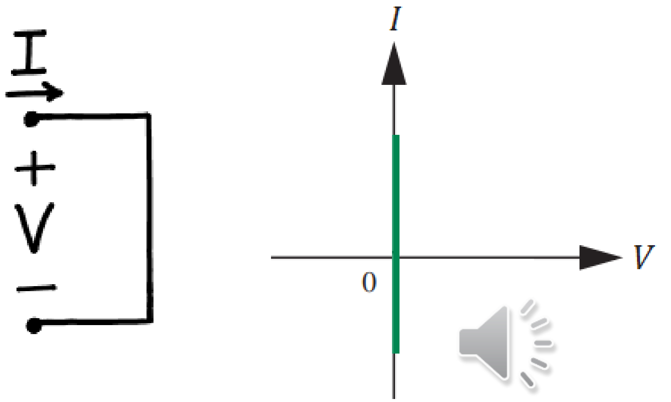
- Recall that, an *ideal open circuit* is the limiting case of a resistor where the resistance approaches infinite.
- As infinite resistance means zero current according to the Ohm's law, the constituent relation for an open circuit is,

$$I = 0$$



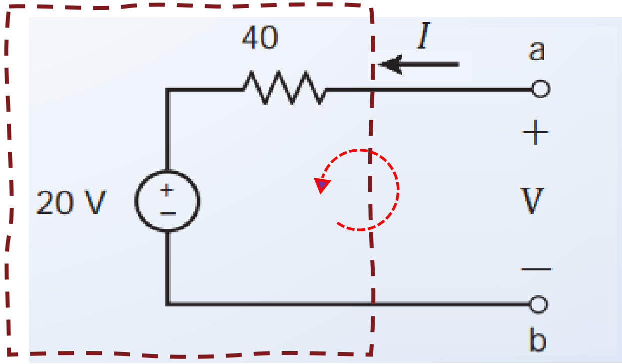
- An *ideal short circuit (or a wire)* is the limiting case of a resistor where the resistance approaches zero.
- As zero resistance means there can be no voltage difference according to the Ohm's law, the constituent relation for a short circuit is,

$$V = 0$$



Example 1

- Derive and plot the $I - V$ relationships of the following configurations: a $20\ V$ voltage source in series with a $40\ \Omega$ resistor.

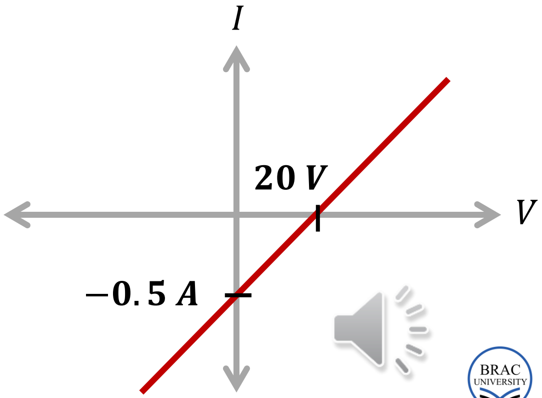


- $20\ V$ voltage source is in series with a $40\ \Omega$ resistor between terminals $a - b$ as shown.
- Applying KVL to the loop yields,

$$-V + 40I + 20 = 0$$

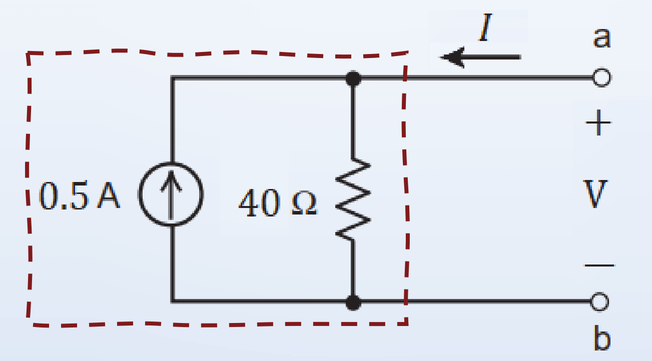
$$\Rightarrow \ \boldsymbol{I = \frac{1}{40}V - 0.5}$$

- This is straight line that intersects the current and voltage axes at $(\mathbf{20\ V},\ \mathbf{0})$ **and** $(\mathbf{0},\ \mathbf{-0.5\ A})$ respectively.
- **It is important to notice here that, I is the current resulting from the application of a bias V. One must not interpret the a − b terminals as open circuit with 0 current in this case. Think V as a applied voltage source connected between a and b.**



Example 2

- Derive and plot the $I-V$ relationships of the following configurations: a $0.5\ A$ current source in parallel with a $40\ \Omega$ resistor.

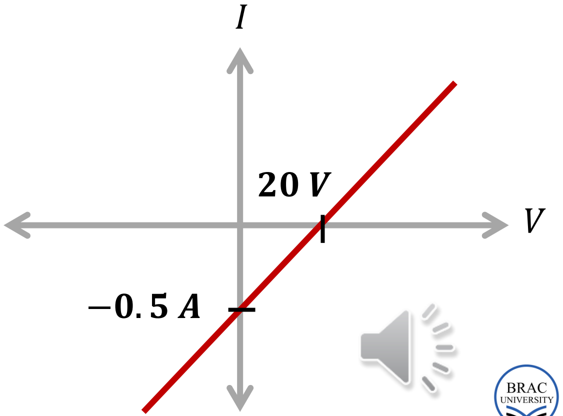


- $0.5\ A$ current source in parallel with a $40\ \Omega$ resistor between terminals $a-b$ as shown.
- Applying KCL to the node a yields,

$$0.5 + I - \frac{V}{40} = 0$$

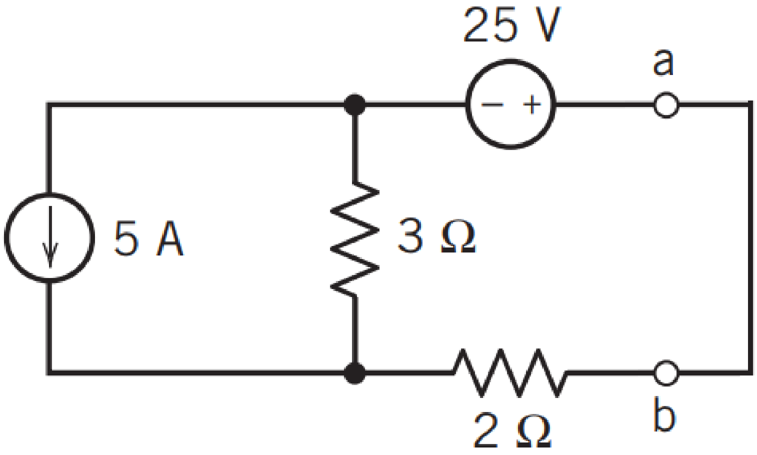
$$\Rightarrow\ \boldsymbol{I = \frac{1}{40}V - 0.5}$$

- This is straight line that intersects the current and voltage axes at $(20\ V,\ 0)$ and $(0,\ -0.5\ A)$ respectively.
- Notice that the $\mathrm{I-V}$ curve is identical to that derived in Example 1. Thus, the two circuits are equivalent to each other.



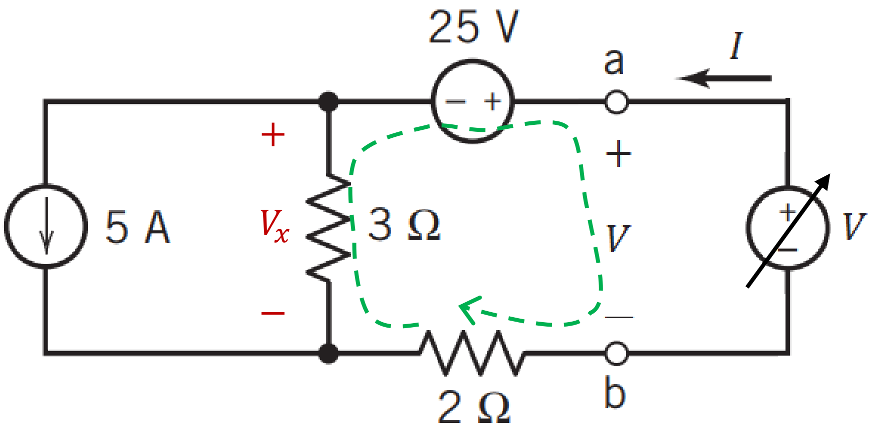
Example 3 - 1/2

- Derive and plot the $I - V$ relationship of the left portion of $a - b$ in the following circuit.



- The first step is to consider only the left portion of the terminal $a - b$ disconnect anything connected to the right (a short circuit in this case).
- Then we have to apply a voltage (taken as a variable V) between terminals $a - b$ and determine the current supplied by V (denoted as variable I).

Example 3 - 2/2



- The arrow symbol with a voltage source means it is a voltage to be varied, variable in our case.
- To solve the circuit using KVL, KCL, and Ohm's law, let the voltage across the $3\ \Omega$ resistor be V_x.

- Applying KCL at the positive node of V_x,

$$5 + \frac{V_x}{3} - I = 0$$

$$\Rightarrow \quad V_x = 3I - 15$$

- Now, for the $25\ V$ source, we can write using KVL to the loop consisting of $3\ \Omega$, $25\ V$, $2\ \Omega$, and V as shown by the dashed arrow,

$$-V_x - 25 + V - 2I = 0$$

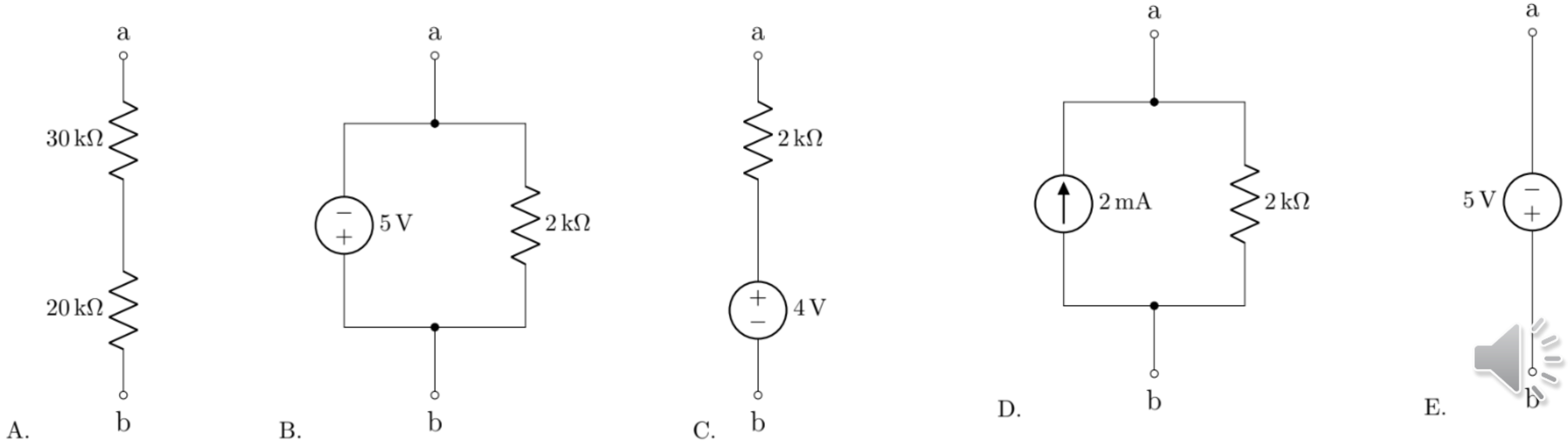
$$\Rightarrow \quad -(3I - 15) - 25 + V - 2I = 0$$

$$\Rightarrow \quad I = \frac{1}{5}V - 2$$

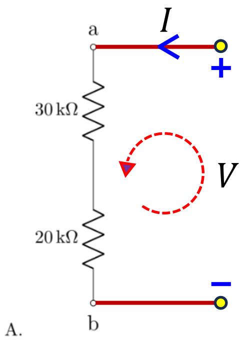
- This constitutes a straight line of slope $^1/_5\ (\Omega^{-1})$ that intersects the axes at $(10\ V,\ 0)$ and $(0,\ -2\ A)$.

Problem 1

- Write $I - V$ characteristic equation between terminals a and b for each of the circuits shown below. Plot the I-V characteristic graphs for each of the equations.
- Are there any equivalent circuit pairs among them? Find all such sets of equivalent circuits.



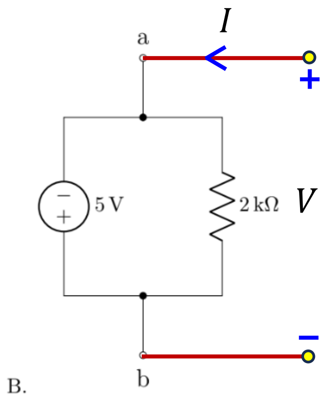
Solution to Problem 1



Applying KVL in the mesh,

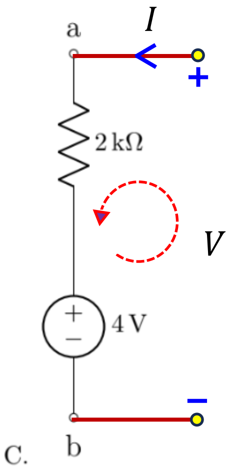
$$-V + 30I + 20I = 0$$

$$\Rightarrow \boldsymbol{I = \frac{1}{50}V}$$



There is a voltage source across the ab terminal. So,

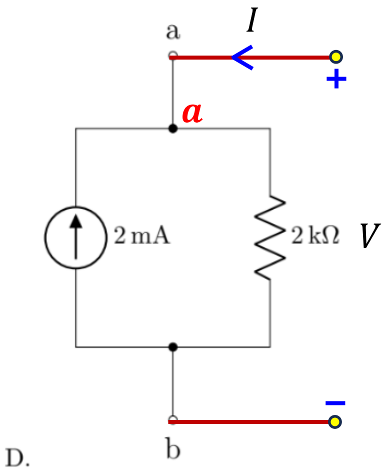
$$\boldsymbol{V = -5}$$



Applying KVL in the mesh,

$$-V + 2I + 4 = 0$$

$$\boxed{\Rightarrow \boldsymbol{I = \frac{1}{2}V - 2}}$$



Applying KCL at node **a**,

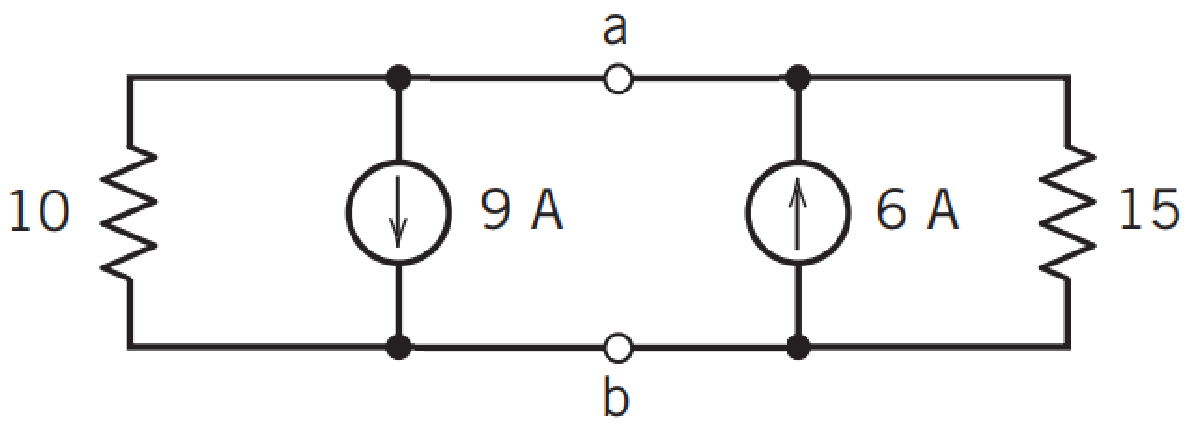
$$I + 2 = \frac{V}{2}$$

$$\boxed{\Rightarrow \boldsymbol{I = \frac{1}{2}V - 2}}$$

IV equations are same. So, the circuits are equivalent pairs

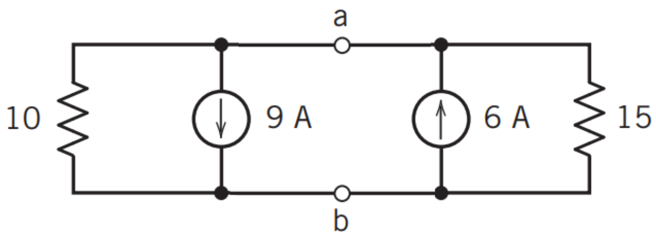
Problem 2

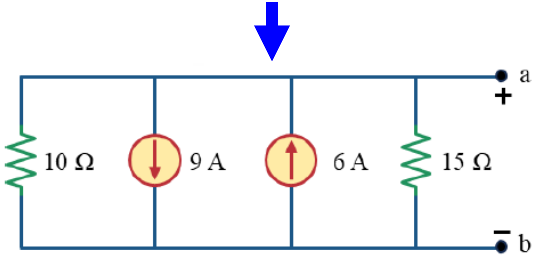
- Derive the $I - V$ characteristics of the following circuit with respect to the terminals $a - b$.



Ans: $I = \frac{1}{6}V + 3$

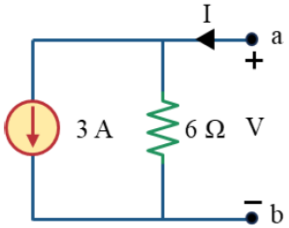
Solution to Problem 2





Did nothing but to bring point a and b at the right side for better visualization

Calculating the equivalent current source and equivalent resistance in the circuit, we get the next circuit. We also defined V and I in the circuit.



Applying KCL at node a,

$$I = \frac{V}{6} + 3$$

To draw the graph, let's find two points (because it is a linear circuit and if we can find 2 points, we can draw a straight line through them to draw the IV characteristics)

From the equation,

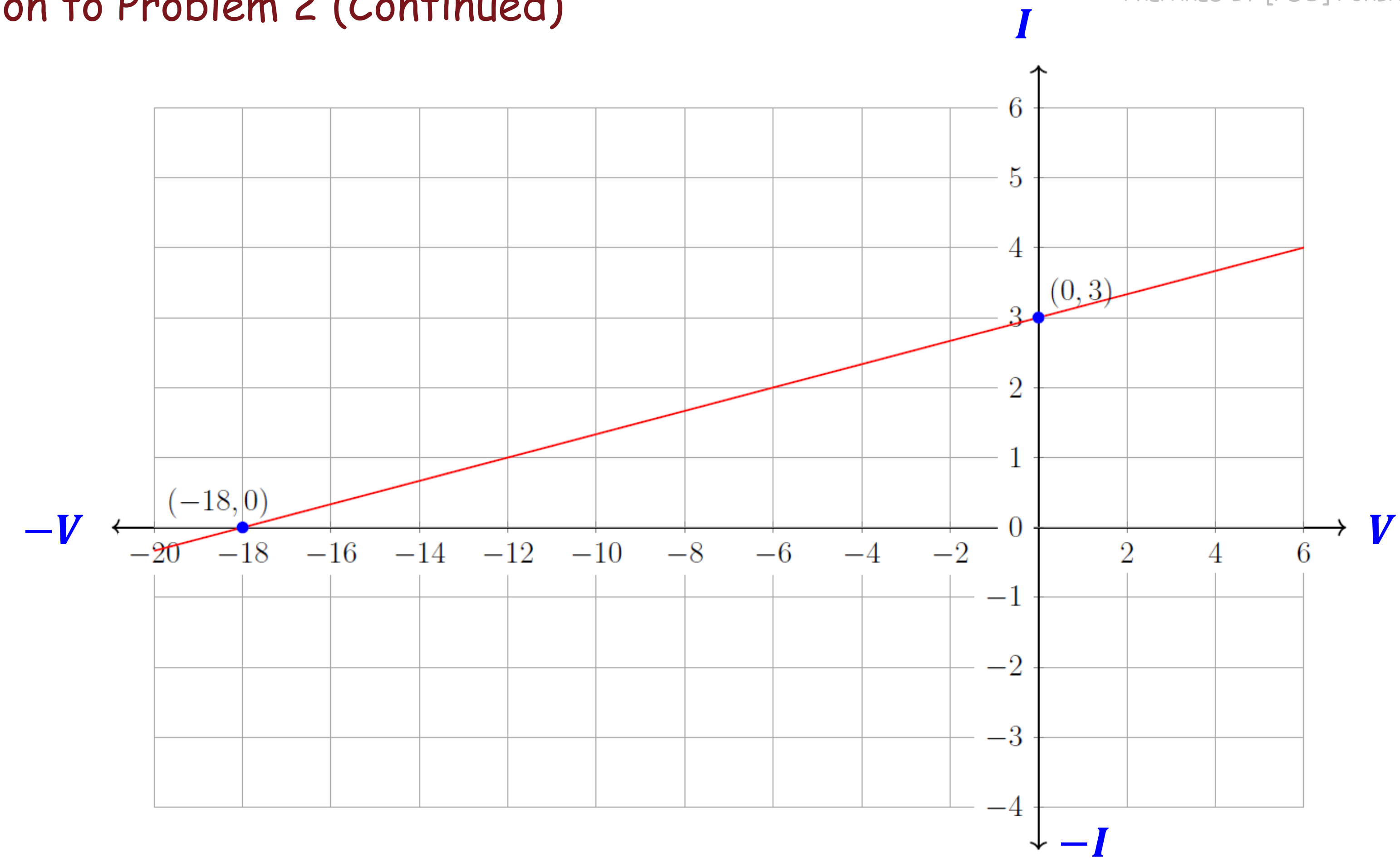
$$if, I = 0, \quad V = -18$$

$$if, V = 0, \quad I = 3$$

So, we get two points, (V_1, I_1) ≡ (-18, 0) and (V_2, I_2) ≡ (0, 3).

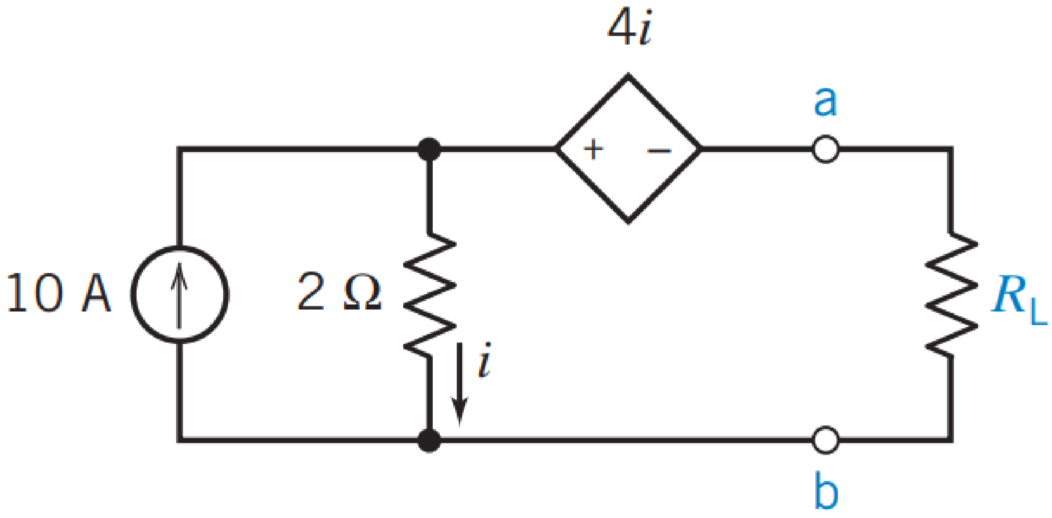
See the graph in the next page

Solution to Problem 2 (Continued)



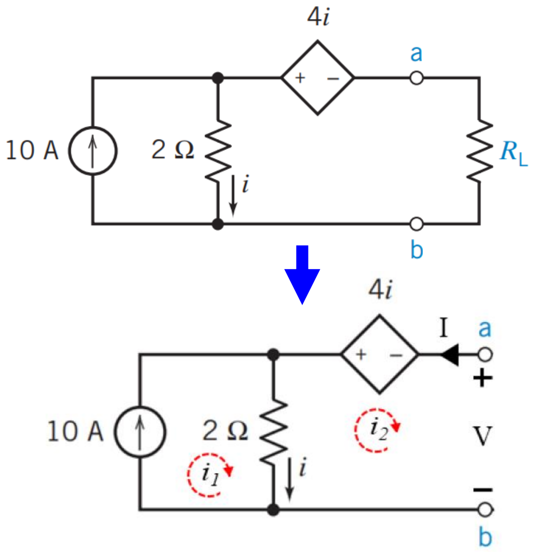
Problem 3

- Derive the $I - V$ characteristics of the portion left to the terminals $a - b$.



Ans: $\boldsymbol{I = -\frac{1}{2}V - 10}$

Solution to Problem 3



We can see that there are 2 meshes. From the circuit,

$$i_2 = -I$$

$$i = i_1 - i_2$$

Applying KVL in mesh 1,

$$i_1 = 10\ A$$

Applying KVL in mesh 2,

$$2(i_2 - i_1) + 4i + V = 0$$

$$\Rightarrow 2(i_2 - i_1) + 4(i_1 - i_2) + V = 0$$

$$\Rightarrow 2i_2 - 2i_1 + 4i_1 - 4i_2 + V = 0$$

$$\Rightarrow 2i_1 - 2i_2 + V = 0$$

$$\Rightarrow 2 \times 10 - 2(-I) + V = 0$$

$$\Rightarrow 20 + 2I + V = 0$$

$$\Rightarrow \boldsymbol{I = -\frac{V}{2} - 10}$$

To draw the graph, let's find two points (because it is a linear circuit and if we can find 2 points, we can draw a straight line through them to draw the IV characteristics)

From the equation,

$$if, I = 0,$$

$$V = -20$$

$$if, V = 0,$$

$$I = -10$$

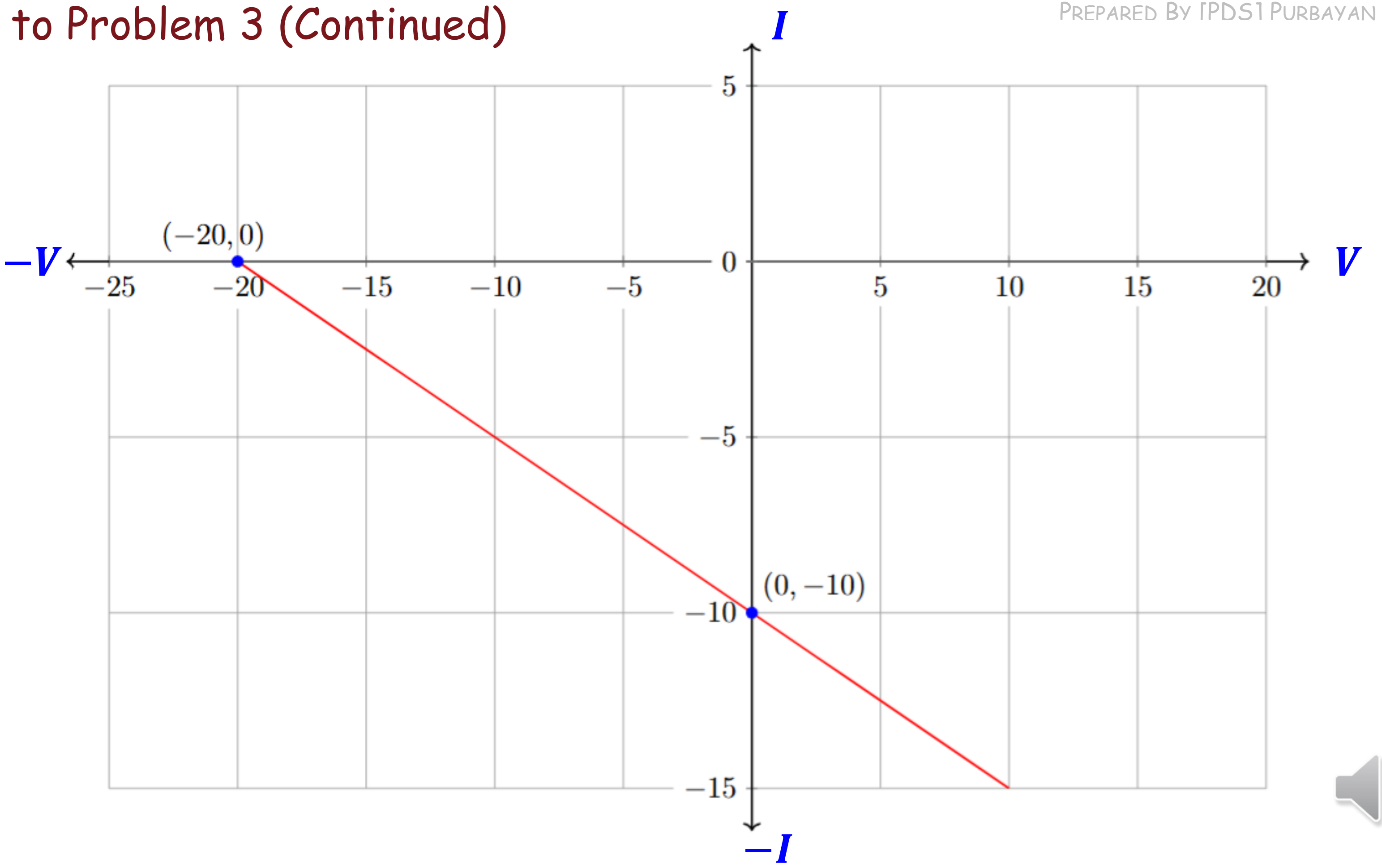
So, we get two points,

$(V_1, I_1) \equiv (-20, 0)$

$(V_2, I_2) \equiv (0, -10)$.

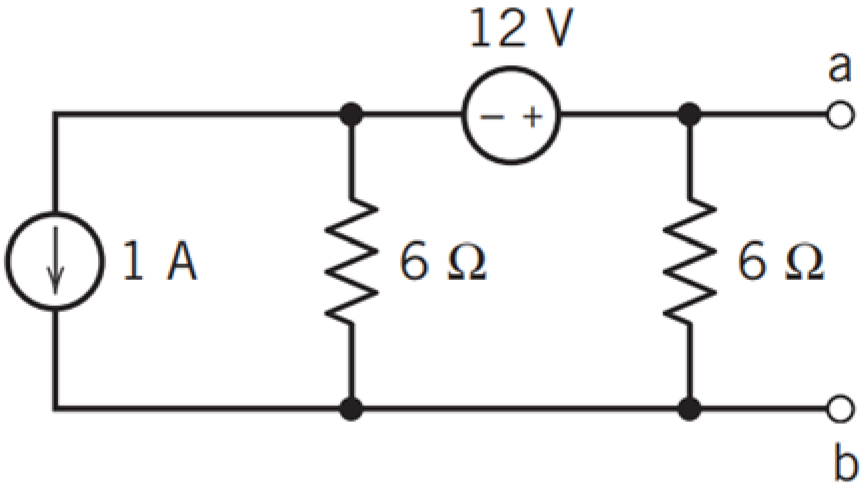
See the graph in the next page

Solution to Problem 3 (Continued)



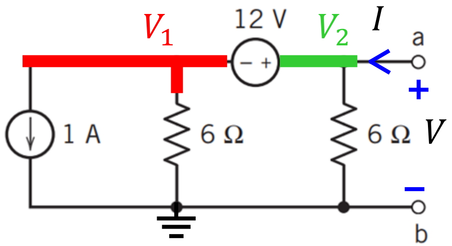
Problem 4

- From the following circuit, derive the current–voltage characteristics equation between the terminals $a - b$.



Ans: $I = \frac{1}{3}V - 1$

Solution to Problem 4



To find the IV of this circuit, let us implement nodal analysis.

From the circuit,

$$V_2 = V$$

We can see that there is a supernode between node 1 and 2. Applying KCL at supernode,

$$\frac{V_1}{6} + 1 + \frac{V}{6} - I = 0 \quad \text{........... (i)}$$

Applying KVL in supernode,

$$V_1 - V = -12$$

$$\Rightarrow V_1 = -12 + V \quad \text{........... (ii)}$$

(ii) In (i),

$$\frac{-12 + V}{6} + 1 + \frac{V}{6} - I = 0$$

$$\Rightarrow -2 + \frac{V}{6} + 1 + \frac{V}{6} - I = 0$$

$$\Rightarrow \boldsymbol{I = \frac{V}{3} - 1}$$

To draw the graph, let's find two points (because it is a linear circuit and if we can find 2 points, we can draw a straight line through them to draw the IV characteristics)

From the equation,

$$if, I = 0,$$

$$V = 3$$

$$if, V = 0,$$

$$I = -1$$

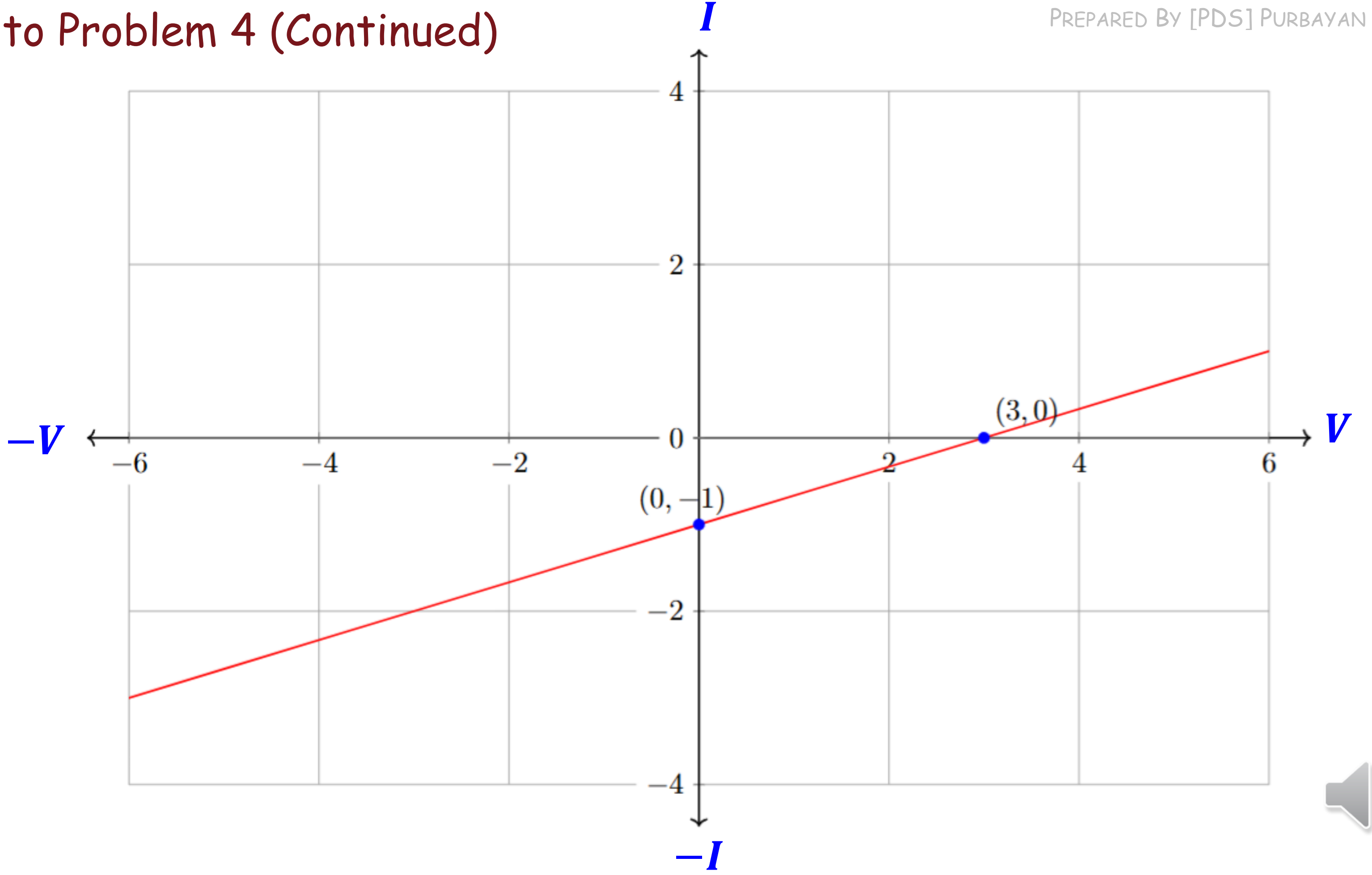
So, we get two points,

$(V_1, I_1) \equiv (3, 0)$

$(V_2, I_2) \equiv (0, -1)$.

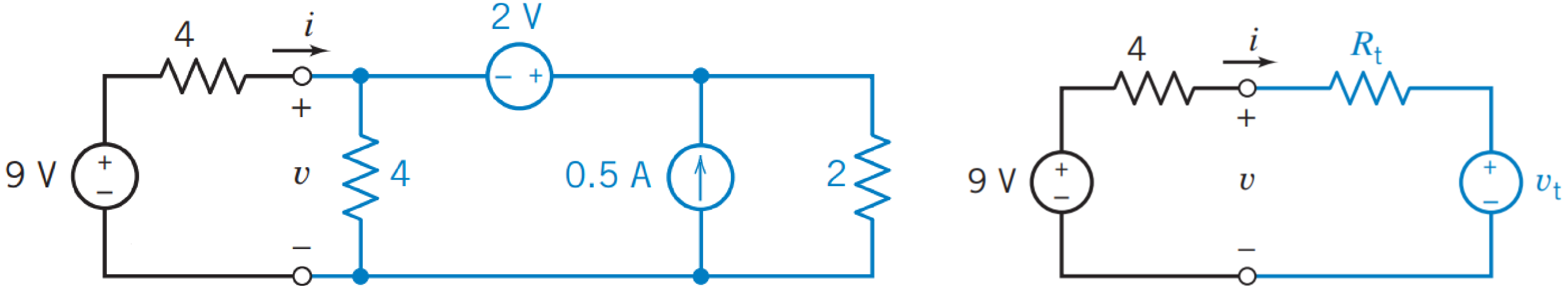
See the graph in the next page

Solution to Problem 4 (Continued)



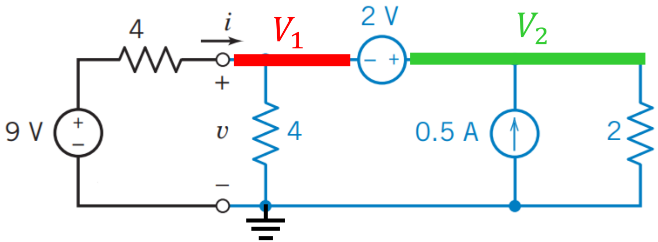
Problem 5

- Determine the values of R_t and v_t, if the following two circuits are equivalent to each other.



Ans: $v_t = -{}^{2}/_{3}\,V,\ \ R_t = {}^{4}/_{3}\ \Omega$

Solution to Problem 5



In this problem, we have 2 circuits. One in the left has values of all the resistors and sources and the right circuit is missing value for R_t and v_t. As it is said that both the circuits are equivalent, the IV characteristics will be same (for blue marked portion for both circuits). So, we will find the IV characteristics (equation) **from the left circuit** and using that, will find the unknown parameters in right circuit.

From the above circuit,

$$V_1 = v$$

We can see that there is a supernode between node 1 and 2. Applying KCL at supernode,

$$\frac{V_1}{4} - i + \frac{V_2}{2} - 0.5 = 0$$

$$\Rightarrow \frac{v}{4} - i + \frac{V_2}{2} - 0.5 = 0 \quad \text{.......... (i)}$$

Applying KVL at supernode,

$$V_1 - V_2 = -2$$

$$\Rightarrow v - V_2 = -2$$

$$\Rightarrow V_2 = v + 2 \quad \text{.......... (ii)}$$

(ii) In (i),

$$\frac{v}{4} - i + \frac{v+2}{2} - 0.5 = 0$$

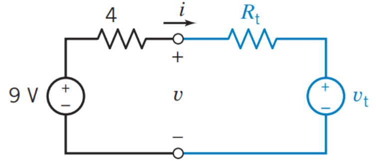
$$\Rightarrow \frac{v}{4} - i + \frac{v}{2} + 1 - 0.5 = 0$$

$$\Rightarrow \boldsymbol{i = \frac{3}{4}v + \frac{1}{2}}$$

This IV equation is in KCL form (**Current term has coefficient 1**). To find out the unknown parameters in the right circuit, we need to make this equation into KVL form (**Voltage term has coefficient 1**). So

$$v - \frac{4}{3}i + \frac{2}{3} = 0 \quad \text{.......... (iii)}$$

Applying KVL in the right circuit,



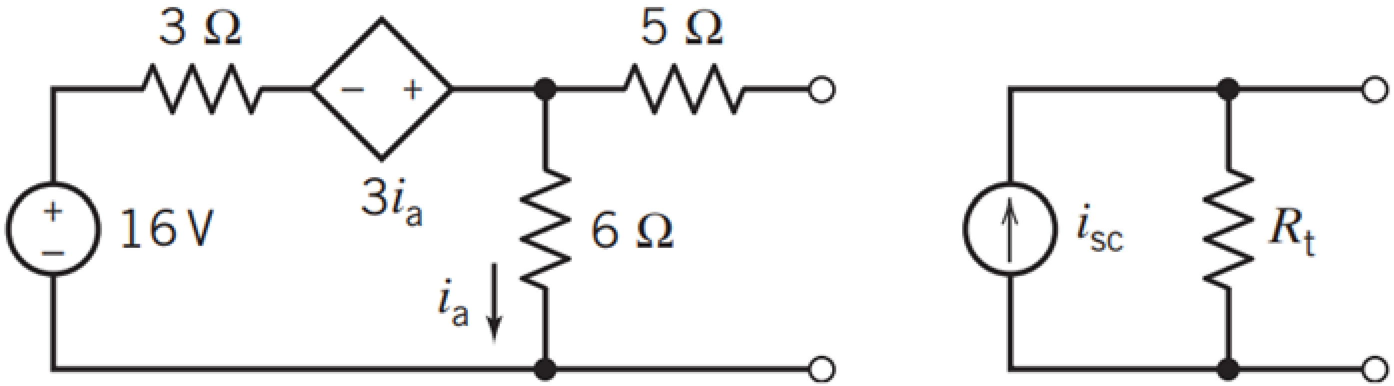
$$v - R_t i - v_t = 0 \quad \text{.......... (iv)}$$

Comparing (iii) and (iv),

$$\boldsymbol{R_t = \frac{4}{3}\ \Omega} \qquad \boldsymbol{v_t = -\frac{2}{3}\text{V}}$$

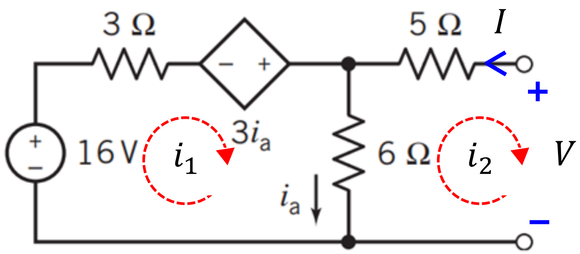
Problem 6

- Determine the values of R_t and i_{sc}, if the following two circuits are equivalent to each other.



Ans: $\boldsymbol{i_{sc} = 2\ A,\ \ R_t = 8\,\Omega}$

Solution to Problem 5



In this problem, we have 2 circuits. One in the left has values of all the resistors and sources and the right circuit is missing value for R_t and i_{sc}. As it is said that both the circuits are equivalent, the IV characteristics will be same So, we will find the IV characteristics (equation) from the left circuit and using that, will find the unknown parameters in right circuit.

From the above circuit,

$$i_a = i_1 - i_2$$

$$i_2 = -I$$

Applying KVL in mesh 1

$$16 + 3i_1 - 3i_a + 6(i_1 - i_2) = 0$$

$$\Rightarrow -16 + 3i_1 - 3(i_1 - i_2) + 6(i_1 - i_2) = 0$$

$$\Rightarrow -16 + 3i_1 + 3i_1 - 3i_2 = 0$$

$$\Rightarrow 6i_1 - 3i_2 = 16$$

$$\Rightarrow i_1 = \frac{1}{2}i_2 + \frac{16}{6} \text{ .... (i)}$$

Applying KVL in mesh 2

$$6(i_2 - i_1) + 5i_2 + V = 0 \text{ .... (ii)}$$

(i) In (ii),

$$6\left(i_2 - \frac{1}{2}i_2 - \frac{16}{6}\right) + 5i_2 + V = 0$$

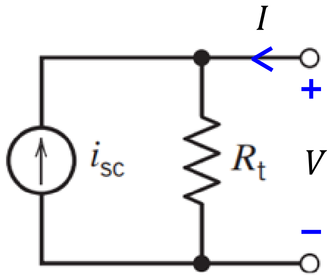
$$\Rightarrow 3i_2 - 16 + 5i_2 + V = 0$$

$$\Rightarrow 8i_2 = -V + 16$$

$$\Rightarrow i_2 = -\frac{V}{8} + 2 \Rightarrow \boldsymbol{I = \frac{V}{8} - 2} \text{ .... (iii)}$$

This IV equation is in KCL form (**Current term has coefficient 1**).

Applying KCL in the right circuit,



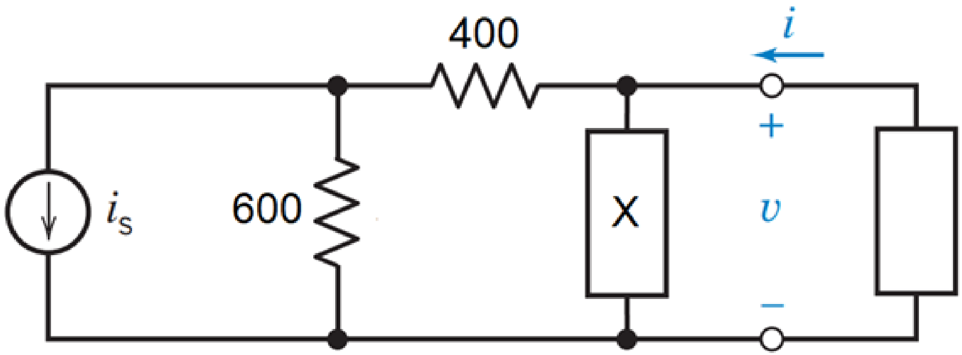
$$I = \frac{V}{R_t} - i_{sc} \text{ ..... (iv)}$$

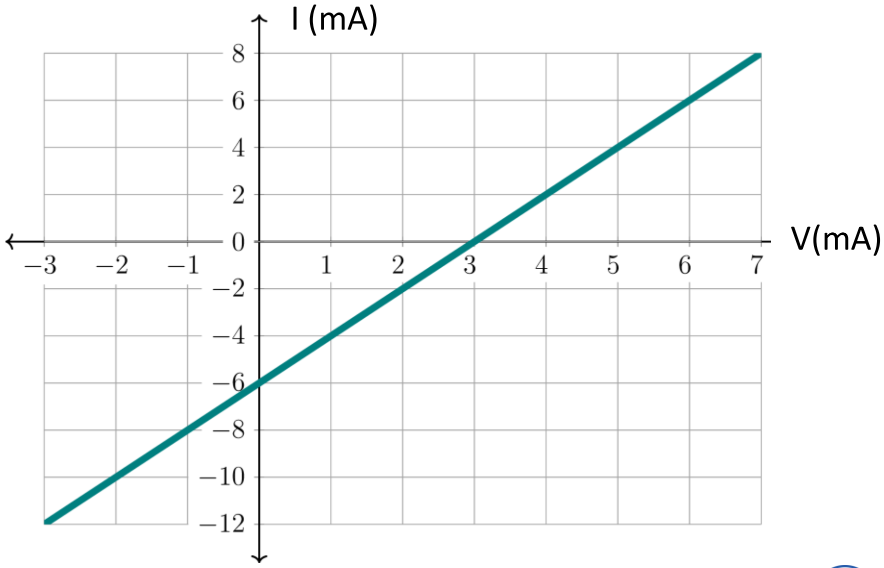
Comparing (iii) and (iv),

$$\boldsymbol{R_t = 8\ \Omega} \qquad \boldsymbol{i_{sc} = 2\ A}$$

Problem 7

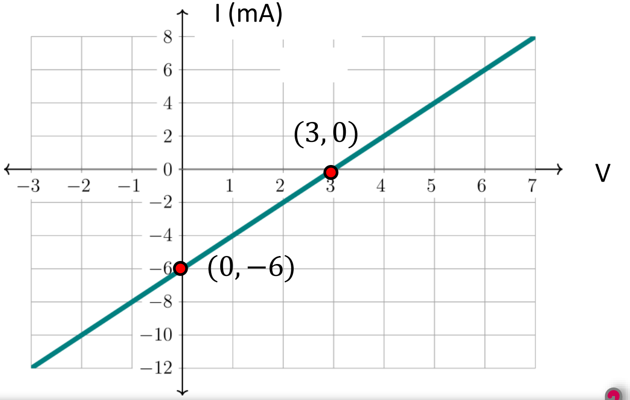
- If the voltage v vs. current i has the following relationship expressed graphically, determine the resistance of the circuitry X.





Ans: $1\ k\Omega$

Solution to Problem 5



Do not worry about current being in the x axis and voltage being in the y axis. I know it is unconventional. But if you keep the calculation according to the graph, it will be okay !

From the graph, we can find the IV characteristics (equation) and using that equation we can find the value of Rx

The equation of the IV characteristics can be found using the 2 points where the line intersects the x and y axis. Those points are:

$$(v_1, i_1) \equiv (3, 0)\ and\ (v_2, i_2) \equiv (0, -6)$$

The equation of the line

$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

$$\Rightarrow \frac{v - 3}{i - 0} = \frac{3 - 0}{0 - (-6)}$$

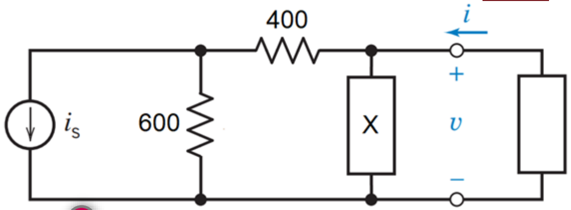
$$\Rightarrow i = 2v - 6 \quad \text{..........} \text{ (i)}$$

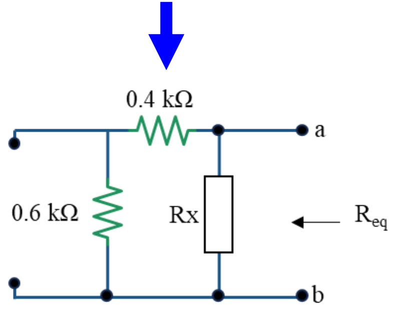
Converted the resistance unit to Kilo Ohm due to the current in the graph being in mA unit

We can find the Req of the circuit from equation (i)

$$\mathrm{R_{eq}} = R_{ab} = \frac{1}{2}\ \mathrm{k\Omega} \quad \text{..........} \text{ (ii)}$$

To find the resistance Rx, we need to find the Req from the circuit. For this, the ideal sources must be replaced by their ideal internal resistances. (Voltage source with short circuit, current source with open circuit)





$$R_{eq} = (0.4 + 0.6) \mid\mid R_x$$

$$\Rightarrow R_{eq} = \frac{1R_x}{1 + R_x} \quad \text{..........} \text{ (iii)}$$

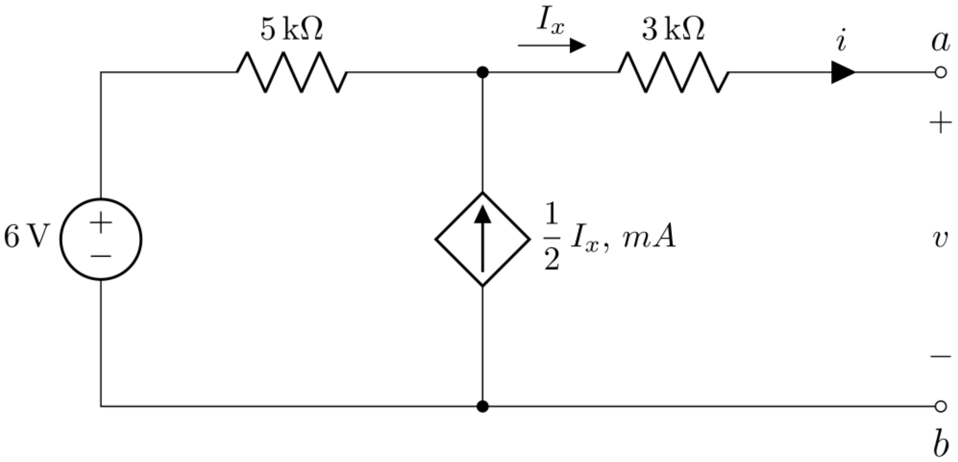
Comparing (ii) and (iii),

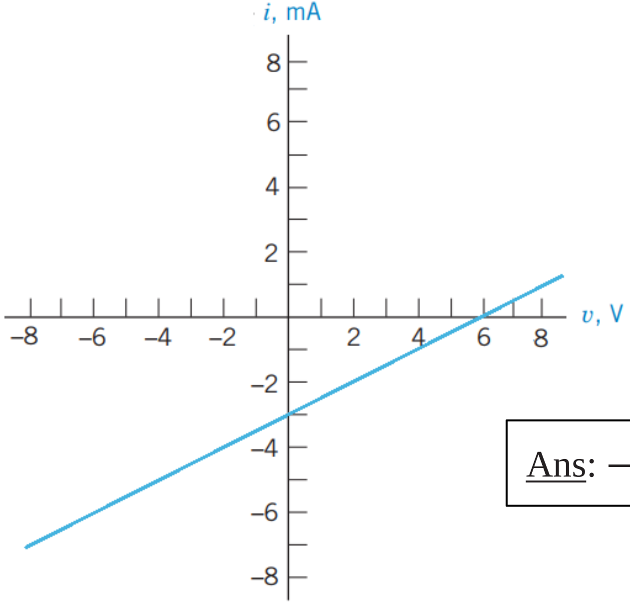
$$\frac{1}{2} = \frac{1R_x}{1 + R_x} \Rightarrow \mathbf{R_x = 1\ k\Omega}$$

Inspiring Excellence

Problem 8

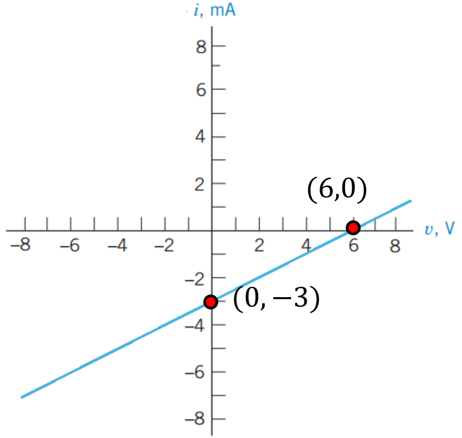
- The $I - V$ characteristic of the following circuit with respect to terminals $a - b$ is plotted below. Determine the resistance contributed by the dependent source. *[Hint: an ideal independent voltage source has zero resistance]*





Ans: $-\frac{5}{6}$ $k\Omega$

Solution to Problem 8



From the graph, we can find the IV characteristics (equation) and using that equation we can find the resistance of the dependent source

The equation of the IV characteristics can be found using the 2 points where the line intersects the x and y axis. Those points are:

$$(i_1, v_1) \equiv (6, 0) \; and \; (i_2, v_2) \equiv (0, -3)$$

The equation of the line

$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

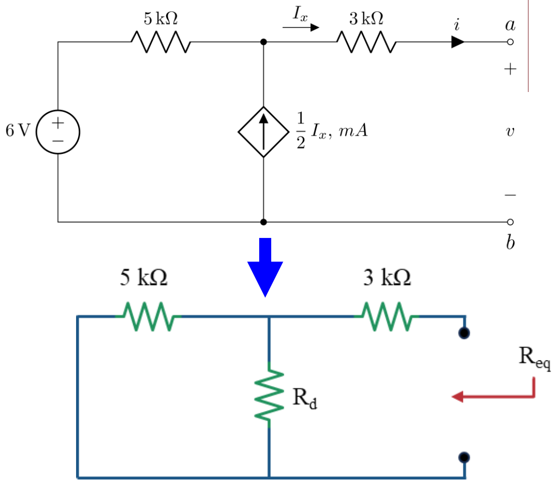
$$\Rightarrow \frac{v - 6}{i - 0} = \frac{6 - 0}{0 - (-3)}$$

$$\Rightarrow i = \frac{1}{2} v - 3 \quad \text{..........} \; \text{(i)}$$

We can find the Req of the circuit from equation (i)

$$\mathrm{R_{eq}} = R_{ab} = 2 \text{ k}\Omega \quad \text{..........} \; \text{(ii)}$$

To find the dependent source resistance (let's say R_d), we need to find out the Req of the whole circuit by replacing the ideal sources with their internal resistance (voltage source with short circuit, current source with open circuit). We will replace the dependent source with R_d.



$$R_{eq} = (5 \,||\, R_d) + 3$$

$$\Rightarrow R_{eq} = \frac{5 \times R_d}{5 + R_d} + 3 \quad \text{..........} \; \text{(iii)}$$

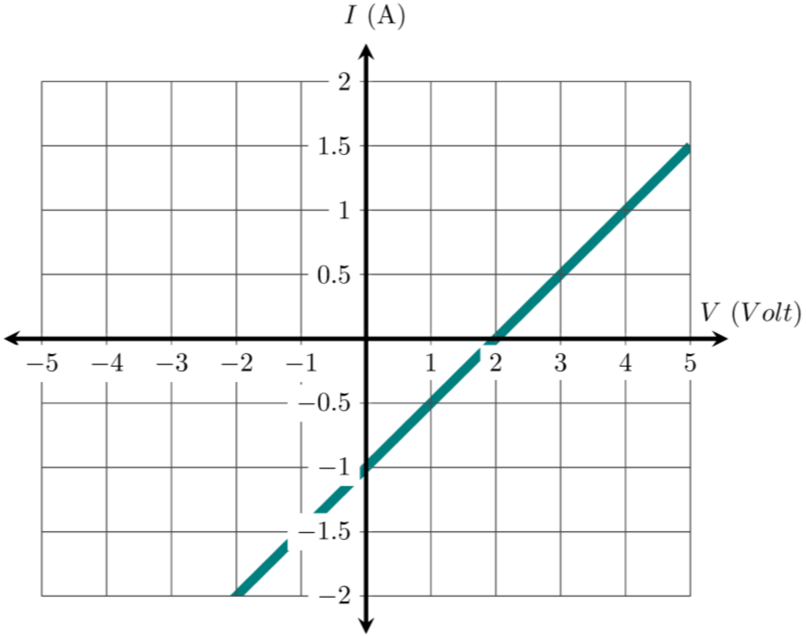
Comparing (ii) and (iii),

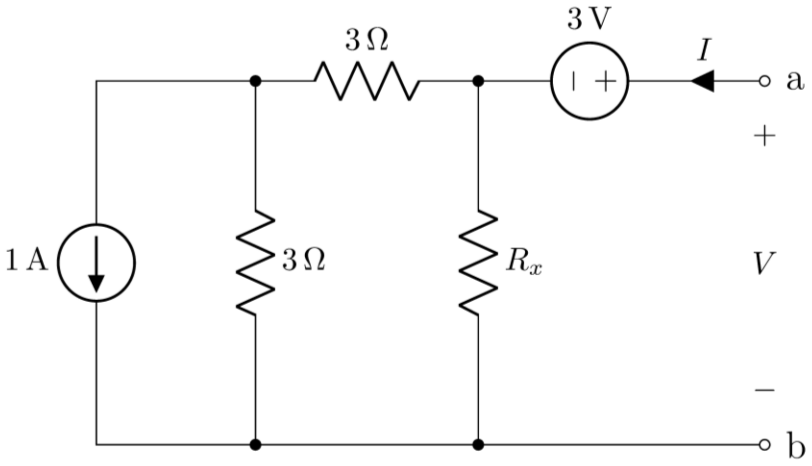
$$2 = \frac{5 \times R_d}{5 + R_d} + 3 \Rightarrow \mathbf{R_d = 0.83 \; k\Omega}$$

Ignoring the minus sign

Problem 9

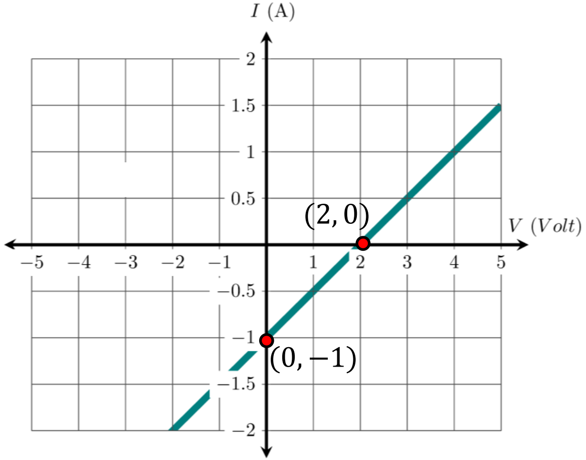
- The circuit below has the following $I-V$ characteristic with respect to terminals a and b. Determine the unknown resistance R_x.





Ans: $\boldsymbol{R = 3\ \Omega}$

Solution to Problem 9



From the graph, we can find the IV characteristics (equation) and using that equation we can find the resistor Rx

The equation of the IV characteristics can be found using the 2 points where the line intersects the x and y axis. Those points are:

$$(v_1, i_1) \equiv (2,0)\ and\ (v_2, i_2) \equiv (0,-1)$$

The equation of the line

$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

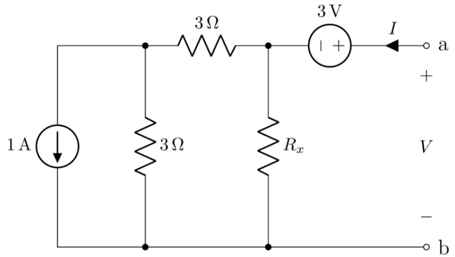
$$\Rightarrow \frac{v - 2}{i - 0} = \frac{2 - 0}{0 - (-1)}$$

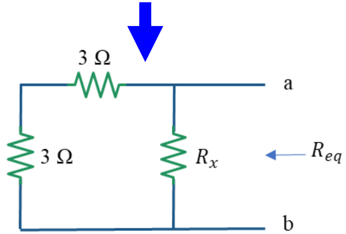
$$\Rightarrow i = \frac{1}{2}v - 1 \quad \text{.......... (i)}$$

We can find the Req of the circuit from equation (i)

$$\mathrm{R_{eq}} = R_{ab} = 2\ \mathrm{k\Omega} \quad \text{.......... (ii)}$$

To find the resistance Rx, we need to find the Req from the circuit. For this, the ideal sources must be replaced by their ideal internal resistances. (Voltage source with short circuit, current source with open circuit)





$$R_{eq} = R_x \,||\, (3 + 3)$$

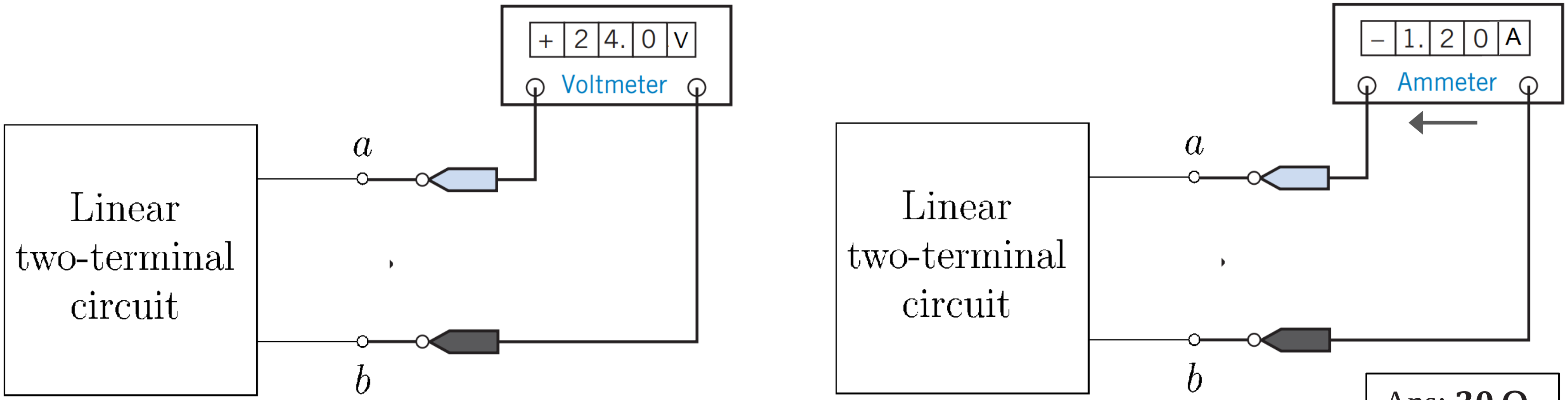
$$\Rightarrow R_{eq} = \frac{6 \times R_x}{6 + R_x} \quad \text{.......... (iii)}$$

Comparing (ii) and (iii),

$$2 = \frac{6 \times R_x}{6 + R_x} \Rightarrow \mathbf{R_x = 3\ \Omega}$$

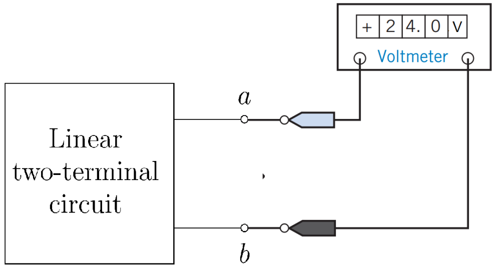
Problem 10

- Connecting a voltmeter and ammeter with a linear two terminal circuit shows the following measurement data. Determine the equivalent resistance of the circuit with respect to the corresponding terminals. Consider the meters ideal. *[Hint: an ideal voltmeter and an ideal ammeter have infinite and zero resistances respectively.]*



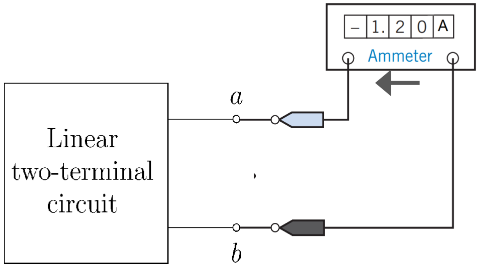
Ans: **20 Ω**

Solution to Problem 10



From voltmeter,

$$V = 24\ V$$



From ammeter,

$$I = 1.2\ A$$

Using Ohm's law,

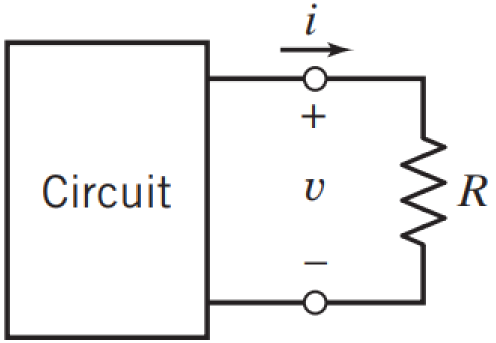
$$R = \frac{V}{I}$$

$$\Rightarrow R = \frac{24}{1.2}$$

$$\Rightarrow \boldsymbol{R = 20\Omega}$$

Problem 11

- A resistor, R, was connected to a circuit box as shown below. The current i was measured. The resistance was changed, and the current was measured again. The results are shown in the table.
  i. Plot the relationship between i and v.
  ii. Draw a circuit diagram with minimum number of circuit elements that can give rise to the same $i - v$ curve derived in i.



R	i
2 kΩ	4 mA
4 kΩ	3 mA

$\boxed{\underline{\text{Ans}}: \boldsymbol{i} = -\frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{v} + \mathbf{6}}$

Solution to Problem 11

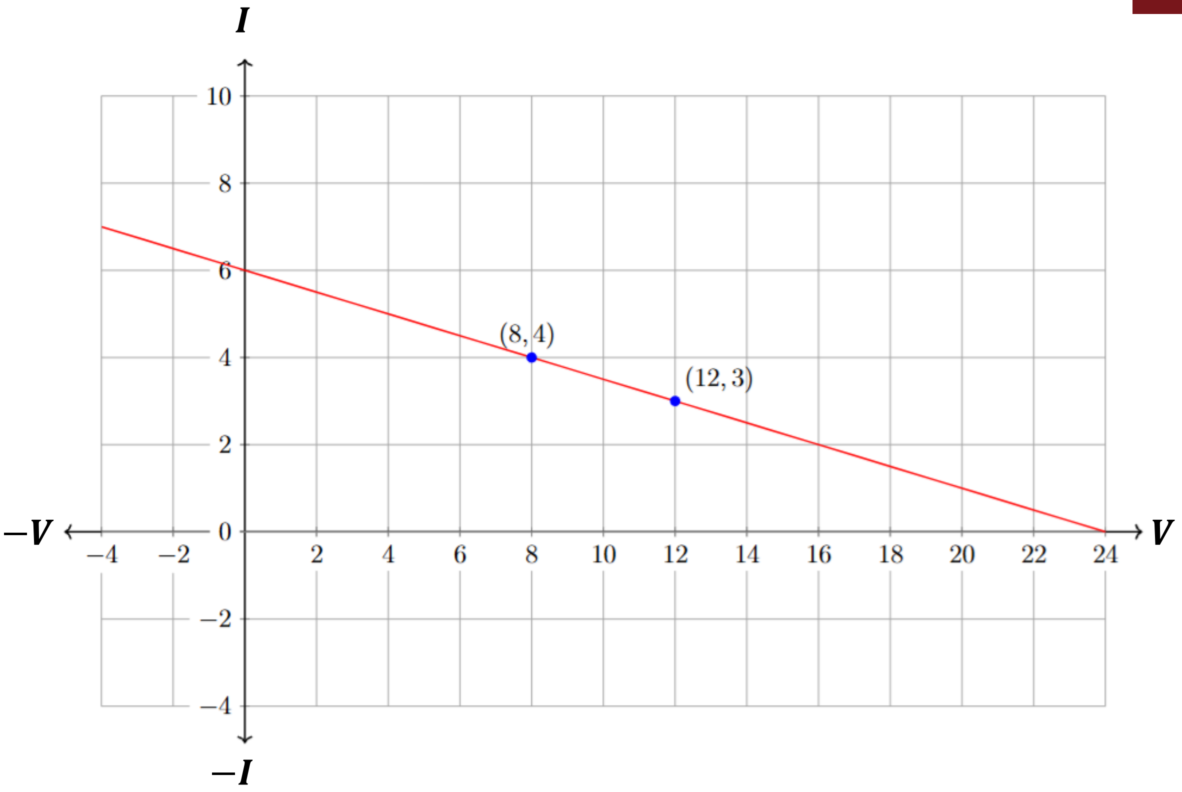
Let us complete the table first by finding out the value of voltages using Ohm's law,

$$v = iR$$

R	i	v
2 kΩ	4 mA	8 V
4 kΩ	3 mA	12 V

i) From the table, we get two points,

$$(v_1, i_1) \equiv (8,4) \; and \; (v_2, i_2) \equiv (12,3)$$



Solution to Problem 11 (Continued)

ii) From the table, we get two points,

$$(v_1, i_1) \equiv (8,4) \; and \; (v_2, i_2) \equiv (12,3)$$

The equation of the line

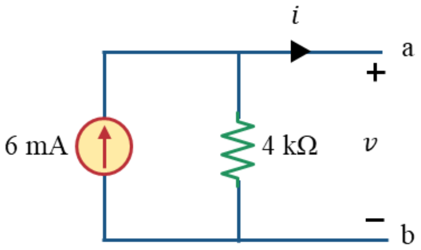
$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

$$\Rightarrow \frac{v - 8}{i - 4} = \frac{8 - 12}{4 - 3}$$

$$\Rightarrow \boldsymbol{i = -\frac{1}{4}v + 6} \quad \text{..........} \; \text{(i)}$$

The equation (i) is a KCL equation. We can draw the circuit from KCL equation or KVL equation (your wish). Let us first draw the circuit from KCL equation. How do we know it is a KCL equation? Because the current term's coefficient is 1. And in a KCL equation, any constant means current source. So, 6 is a current source.

The circuit from KCL equation,



I have adjusted the polarity of the resistor, direction of current source and the current direction according to the equation (i)

To draw a KVL circuit, we should express the equation in Kirchhoff's Voltage Law (KVL) form, where the voltage coefficients are 1, and any constant term represents a voltage source.

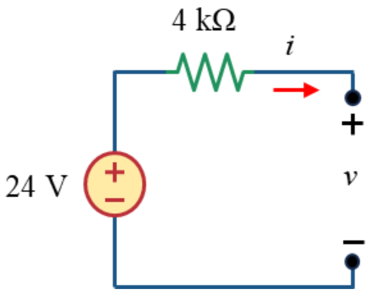
From equation (i),

$$i = -\frac{1}{4}v + 6$$

$$\Rightarrow 4i = -v + 24$$

$$\Rightarrow \boldsymbol{v + 4i - 24 = 0} \quad \text{..........} \; \text{(ii)}$$

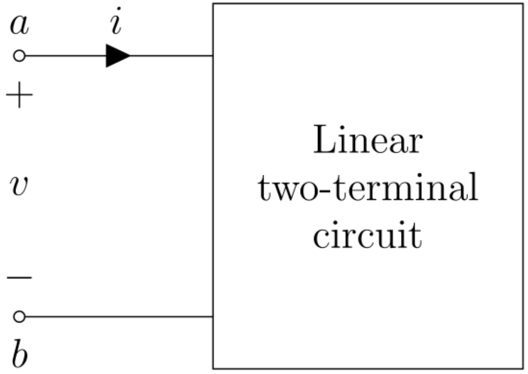
The circuit from KVL equation,

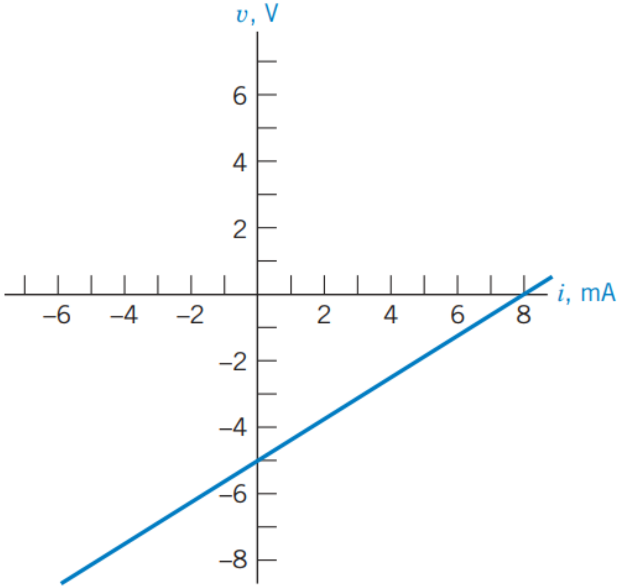


I have adjusted the polarity of the voltage source and the current direction according to the equation (ii)

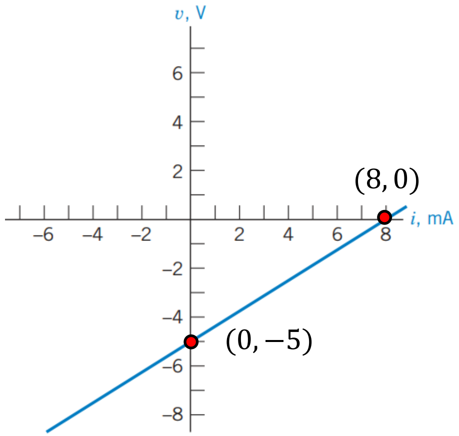
Problem 12

- The $V - I$ characteristic line of a linear circuit with respect to the nodes a and b are plotted below. Derive an equivalent version of the circuit with a minimum number of circuit elements so that it will give rise to the same $V - I$.





Solution to Problem 12



From the graph, we can find the IV characteristics (equation) and using that we can draw the required circuit

The equation of the IV characteristics can be found using the 2 points where the line intersects the x and y axis. Those points are:

$$(i_1, v_1) \equiv (8, 0)\ and\ (i_2, v_2) \equiv (0, -5)$$

The equation of the line

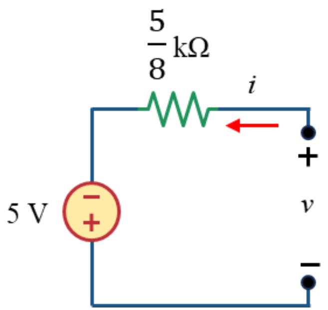
$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

$$\Rightarrow \frac{v - 0}{i - 8} = \frac{0 - (-5)}{8 - 0}$$

$$\Rightarrow \boldsymbol{v - \frac{5}{8}i + 5 = 0} \quad \text{..........} \ (i)$$

This is a KVL equation

The circuit from KVL equation,

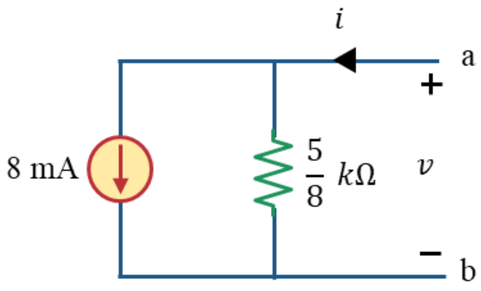


Let us convert the equation (i) into KCL equation

$$v - \frac{5}{8}i + 5 = 0$$

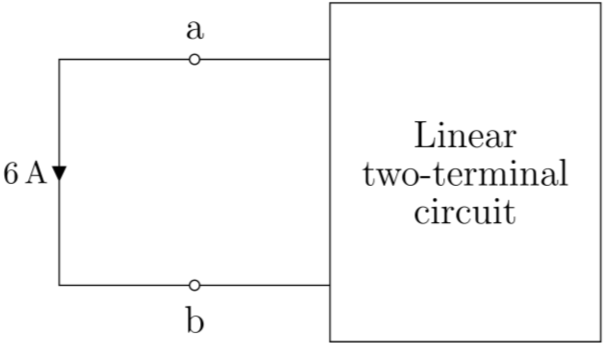
$$\Rightarrow \boldsymbol{i = \frac{v}{\frac{5}{8}} + 8} \quad \text{..........} \ (ii)$$

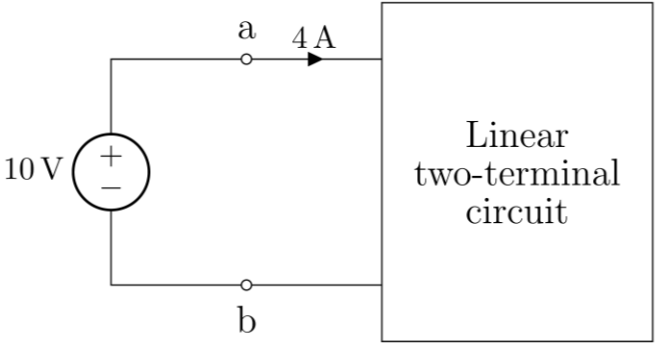
The circuit from KCL equation,



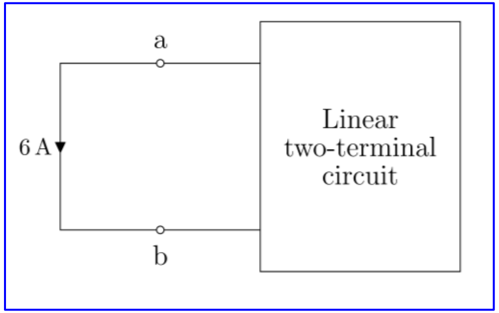
Problem 13

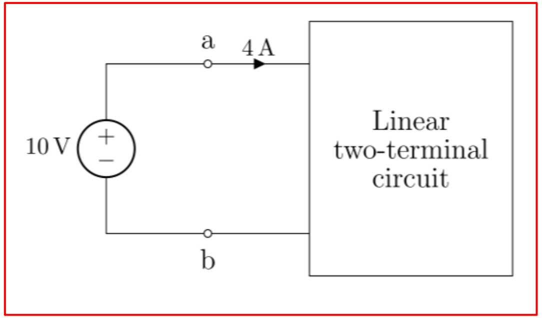
- For an unknown linear two terminal circuit as shown below, if the terminals $a-b$ are shorted, $6\ A$ current flows through the short circuit. When $10\ V$ is applied between the terminals $a-b$, the circuit draws a current equal to $4\ A$. Derive two equivalent versions of the circuit with a minimum number of circuit elements for each.





Solution to Problem 13





From the circuits, we get two data points for I-V characteristics,

$$(v_1, i_1) \equiv (0, -6) \; and \; (v_2, i_2) \equiv (10, 4)$$

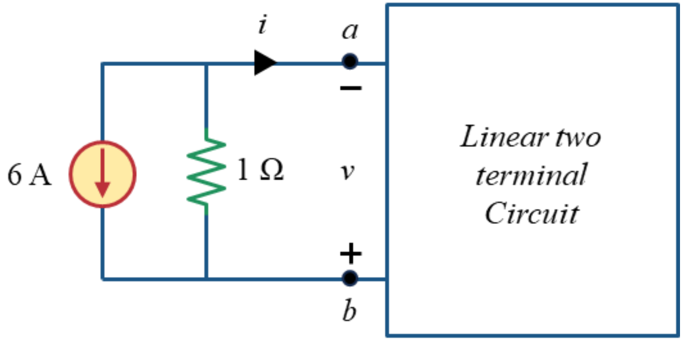
So, IV equation, $\dfrac{v - v_1}{i - i_1} = \dfrac{v_1 - v_2}{i_1 - i_2}$

$$\Rightarrow \frac{v - 0}{i - (-6)} = \frac{0 - 10}{-6 - 4}$$

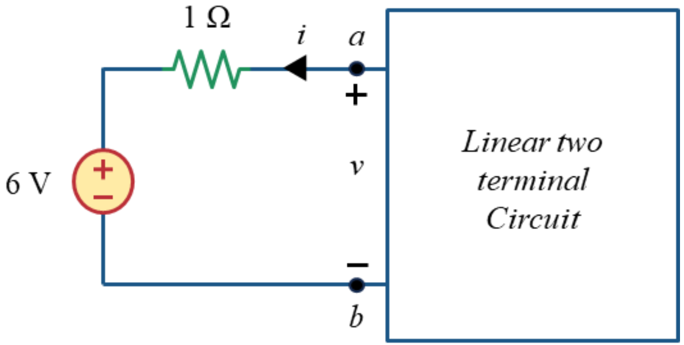
$$\Rightarrow \boldsymbol{i = v - 6}$$

In this equation both the coefficient of v and i are 1. So, we can consider the equation as KCL or KVL as our wish. We can construct two circuits from two points of views (KCL and KVL equation).

The circuit from KCL equation,

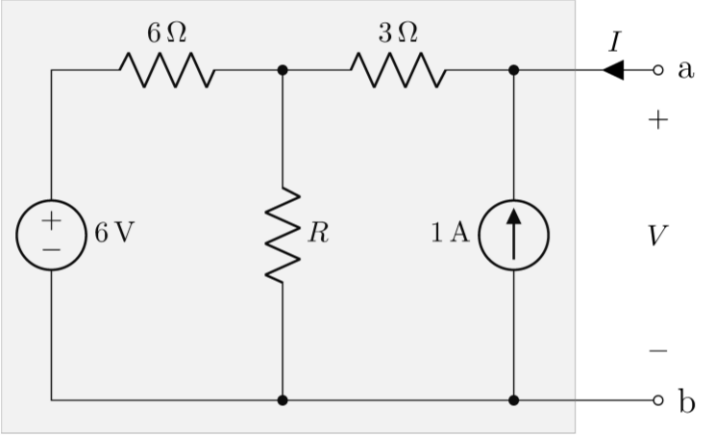


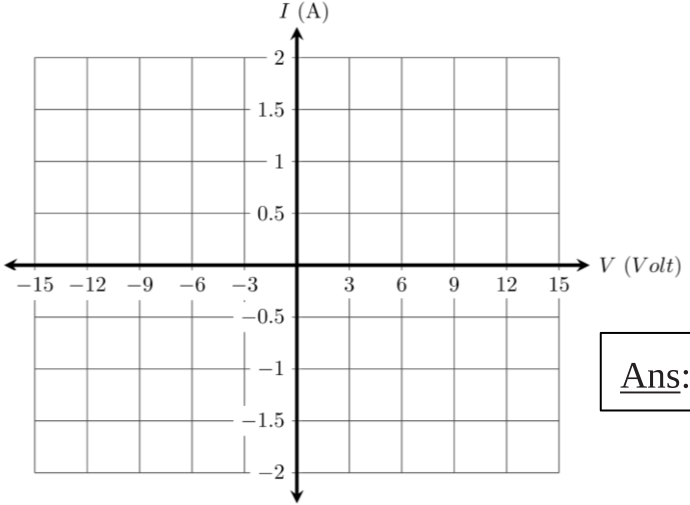
The circuit from KVL equation,



Problem 14

- If a voltage $V = 3\ V$ is applied between terminals a and b, the shaded portion of the circuit draws a current $I = -1\ A$ and when $9\ V$ is applied, it draws no current.
  a) Plot the $I - V$ characteristics of the circuit with respect to the terminals.
  b) Determine the resistance R.
  c) Draw an equivalent version of the circuit which can produce the same $I - V$ plotted in (a).





Ans: $R = 6\ \Omega$

Solution to Problem 14

(a) If a voltage $V = 3\,V$ is applied between terminals a and b, the shaded portion of the circuit **draws** a current $I = -1\,A$ and when $9\,V$ is applied, it draws no current.

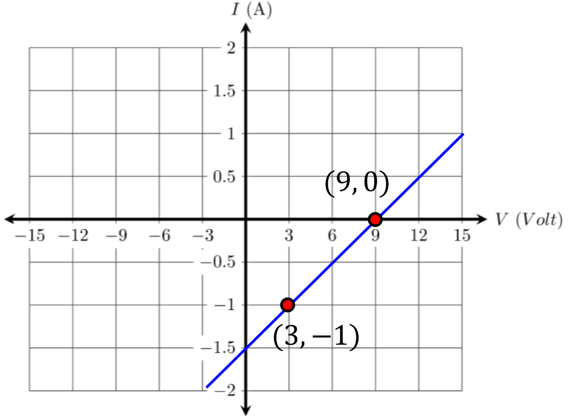
$$(v_1, i_1) \equiv (3, -1)\ and\ (v_2, i_2) \equiv (9, 0)$$

The equation of the line

$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

$$\Rightarrow \frac{v - 3}{i - (-1)} = \frac{3 - 9}{-1 - 0}$$

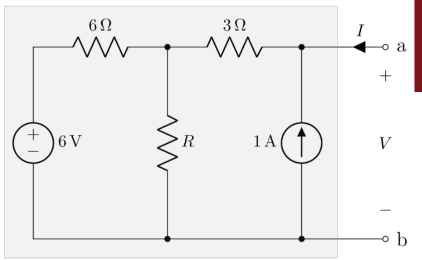
$$\Rightarrow \boldsymbol{i = \frac{1}{6} v - \frac{9}{6}} \quad \text{.......... (i)}$$

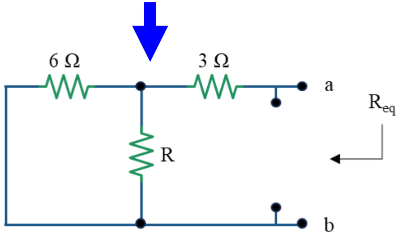


(b) We can find the Req of the circuit from equation (i)

$$\mathrm{R_{eq}} = R_{ab} = 6\ \Omega \quad \text{.......... (ii)}$$

To find the resistance R, we need to find the Req from the circuit. For this, the ideal sources must be replaced by their ideal internal resistances. (Voltage source with short circuit, current source with open circuit)





$$R_{eq} = (R\,||\,6) + 3$$

$$\Rightarrow R_{eq} = \frac{6 \times R}{6 + R} + 3 \quad \text{.......... (iii)}$$

Comparing (ii) and (iii),

$$6 = \frac{6 \times R}{6 + R} + 3 \Rightarrow \boldsymbol{R = 6\ \Omega}$$

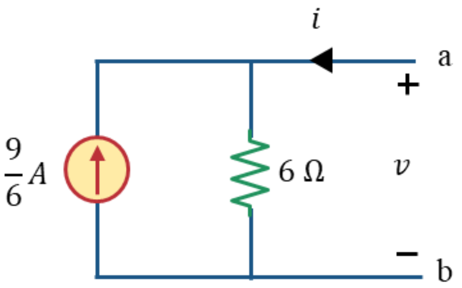
Solution to Problem 9 (Continued)

c) The IV equation

$$i = \frac{1}{6}v - \frac{9}{6} \qquad \text{..........} \text{ (i)}$$

This is a KCL equation

The circuit from KCL equation,



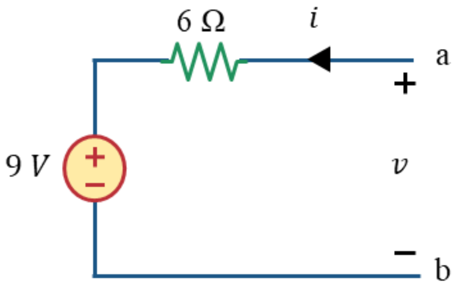
Let us convert the equation (i) into KVL equation

$$i = \frac{1}{6}v - \frac{9}{6}$$

$$\Rightarrow 6i = v - 9$$

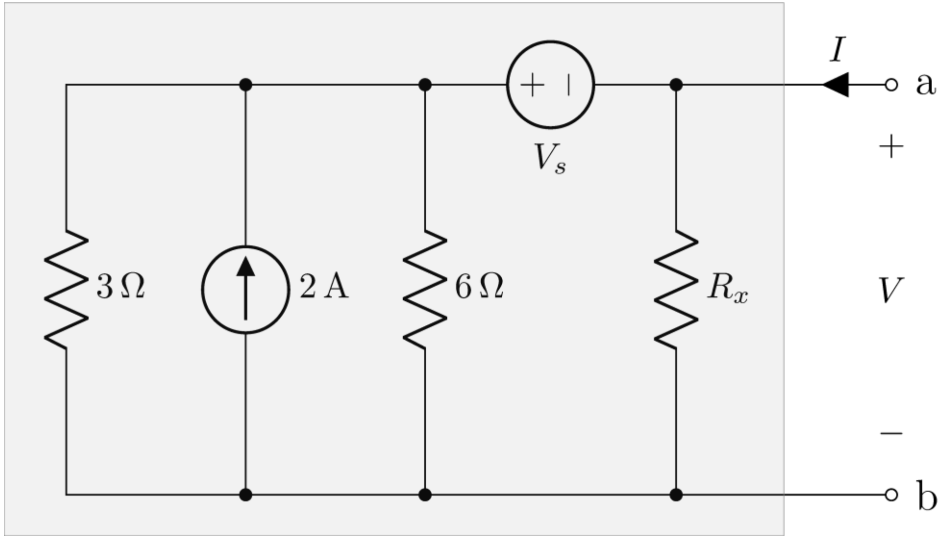
$$\boldsymbol{\Rightarrow v - 6i - 9 = 0} \qquad \text{..........} \text{ (iv)}$$

The circuit from KVL equation,



Problem 15

- If a voltage $V = 1\,V$ is applied between terminals a and b, the shaded portion of the circuit draws a current $I = 4\,A$ and when $0\,V$ is applied, it draws $3\,A$ current.
  a) Determine the unknown resistance R_x.
  b) Determine the unknown voltage V_s.



Ans: $\boldsymbol{R_x = 2\ \Omega, V_s = 10\ V}$

Solution to Problem 15

(a) If a voltage $V = 1\,V$ is applied between terminals a and b, the shaded portion of the circuit **draws** a current $I = 4\,A$ and when $0\,V$ is applied, it draws $3\,A$ current.

$$(v_1, i_1) \equiv (1, 4) \; and \; (v_2, i_2) \equiv (0, 3)$$

The equation of the line

$$\frac{v - v_1}{i - i_1} = \frac{v_1 - v_2}{i_1 - i_2}$$

$$\Rightarrow \frac{v - 1}{i - 4} = \frac{1 - 0}{4 - 3}$$

$$\Rightarrow \boldsymbol{i = v + 3} \quad \text{..........} \text{ (i)}$$

We can find the Req of the circuit from equation (i)

$$\mathrm{R_{eq}} = R_{ab} = 1\,\Omega \quad \text{..........} \text{ (ii)}$$

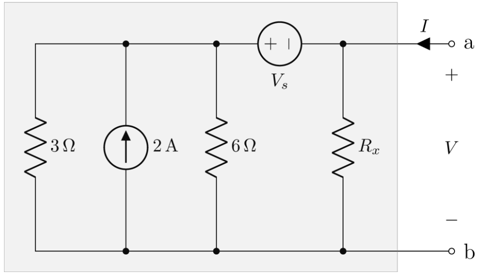
To find the resistance Rx, we need to find the Req from the circuit. For this, the ideal sources must be replaced by their ideal internal resistances. (Voltage source with short circuit, current source with open circuit)

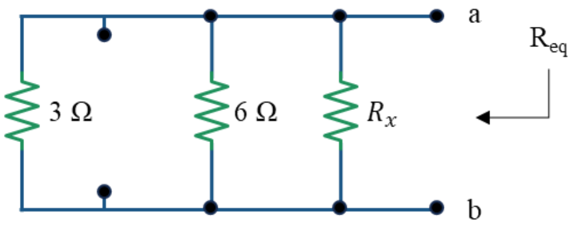
$$R_{eq} = (R_x \,||\, 6 \,||\, 3)$$

$$\Rightarrow R_{eq} = \frac{18 + 9R_x}{18R_x} \quad \text{..........} \text{ (iii)}$$

Comparing (ii) and (iii),

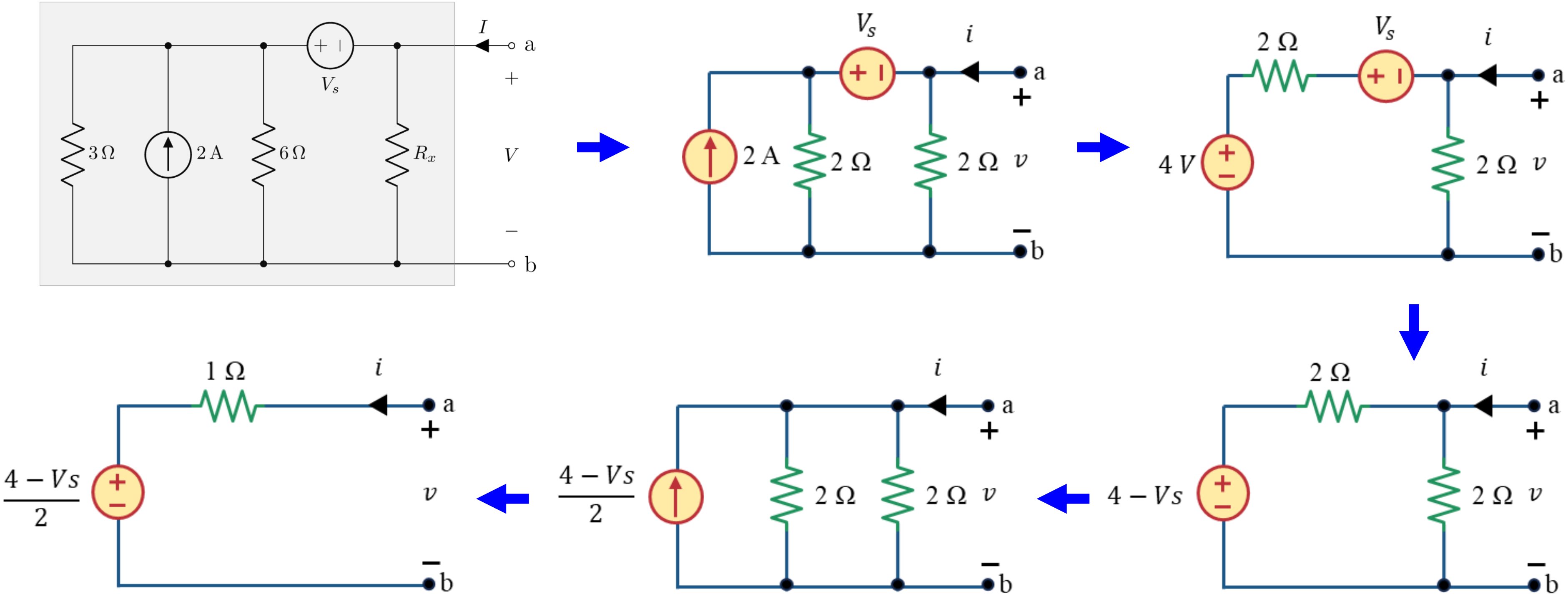
$$1 = \frac{18 + 9R_x}{18R_x} \Rightarrow \boldsymbol{R_x = 2\,\Omega}$$



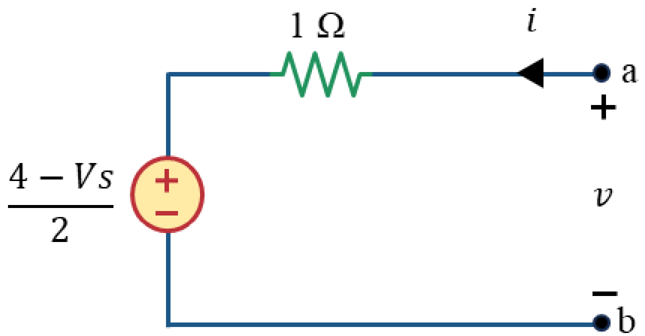


Solution to Problem 15 (Continued)

(b) To determine V_s let us first simplify the circuit using source transformation to make the circuit into single mesh.



Solution to Problem 15 (Continued)



Applying KVL in the last circuit,

$$\boldsymbol{v} - \frac{4-V_s}{2} - i = 0$$

$$\Rightarrow i = v - \frac{4-V_s}{2} \quad \text{..........} \text{ (iv)}$$

Comparing (i) and (iv),

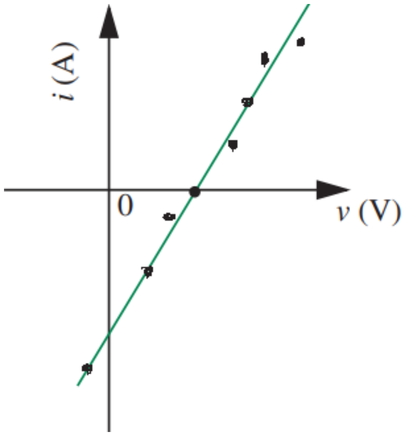
$$3 = -\frac{4-V_s}{2}$$

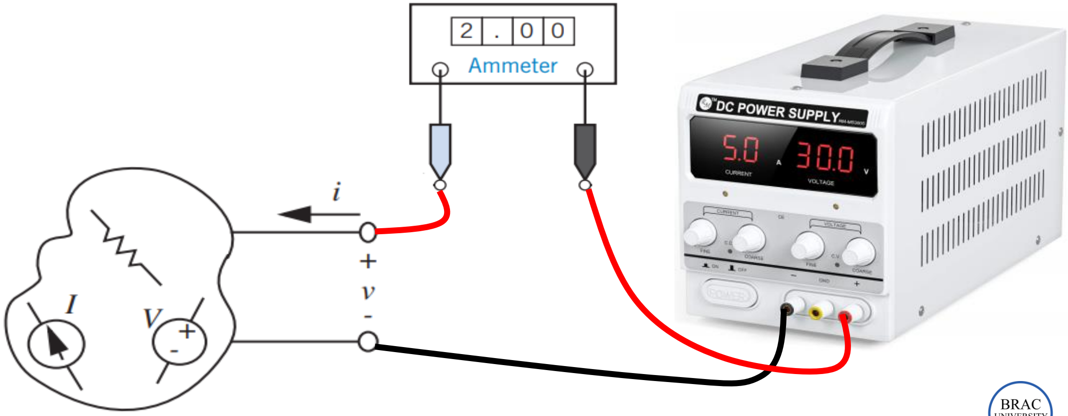
$$\Rightarrow 6 = -4 + V_s$$

$$\Rightarrow \boldsymbol{V_s = 10\ V}$$

I-V: experimentally

- To determine the $I - V$ graph of a circuit experimentally, connect and vary a voltage source between the terminals where current and voltage are to be plotted.
- The varying voltages can be measured with a parallel voltmeter (or from the dc source's display), and the corresponding currents drawn by the circuitry can be measured with an ammeter in series. The more data points we collect, the more accurate the $I - V$, particularly for non-linear devices.





Thank you for your attention