Lecture 7

CSE250 - Circuits and Electronics

MESH ANALYSIS

BRAC UNIVERSITY
Inspiring Excellence

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Mesh Analysis

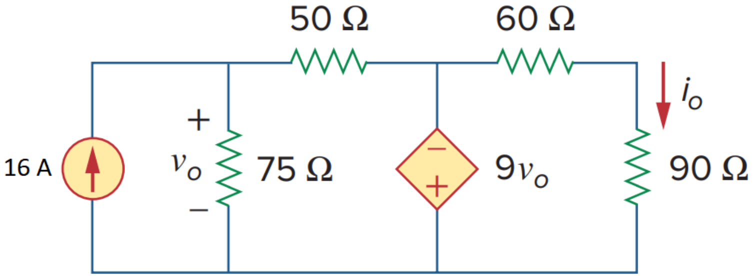
- *Mesh analysis* provides another general procedure for analysing circuits, using mesh currents as the circuit variables. Mesh analysis applies KVL to find unknown currents in a given circuit.
- A *mesh* is a loop that does not contain any other loops within it.
- *Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. Nonplanar circuits cannot be handled with mesh analysis.*
- *A nonplanar circuit is one that has branches that cross each other and cannot be redrew without doing so.*

Steps to Determine Mesh Currents:

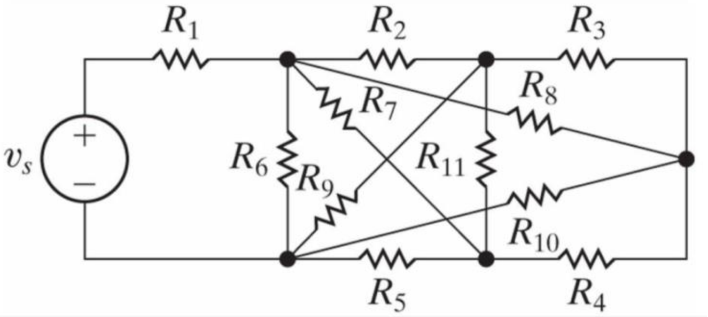
1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Planer vs Non Planer Circuit

A **planar** circuit is a circuit that can be drawn on a flat surface without any wires crossing each other.

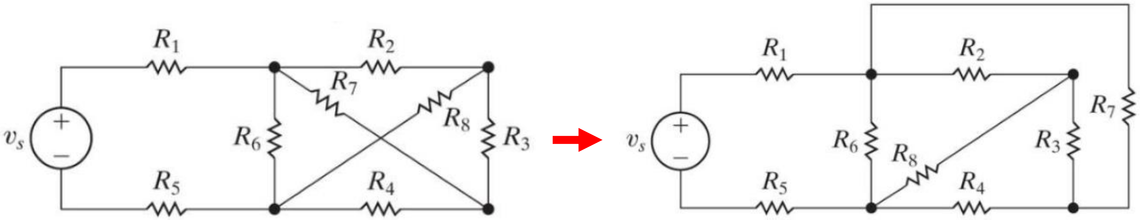


A **non-planar** circuit is a circuit that cannot be drawn on a flat surface without any wires crossing each other.



How to Identify Planar and Non-Planar Circuits?

- If the circuit can be redrawn without any wires crossing each other, then it is planar.
- If the circuit cannot be redrawn without any wires crossing each other, then it is non-planar.



Mesh Analysis: steps

Start

Step 1: Identify all the meshes and assign mesh variables

Recall that, 'Mesh' is a loop which does not contain any other loops within it. The mesh currents can be taken CW or CCW or a mix of both. It is better to follow any one.

Step 2: Write equations for all the current sources

If a current source exists in a mesh which is not part of any other mesh, the current of that mesh is equal to the value of the current source. If a current exists between two loops, it will form a Supermesh, however, we can also write the equations for such current sources.

Step 3: Apply KVL

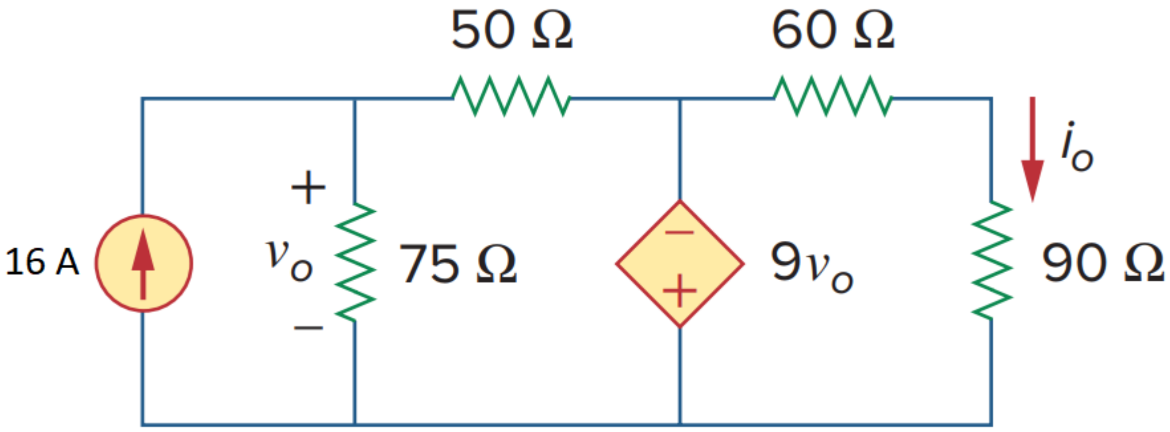
No need to apply KVL to those meshes where mesh currents are already known from Step 3.

Step 4: Solve the simultaneous equations

You should have a number of independent equations equal to the number of meshes.

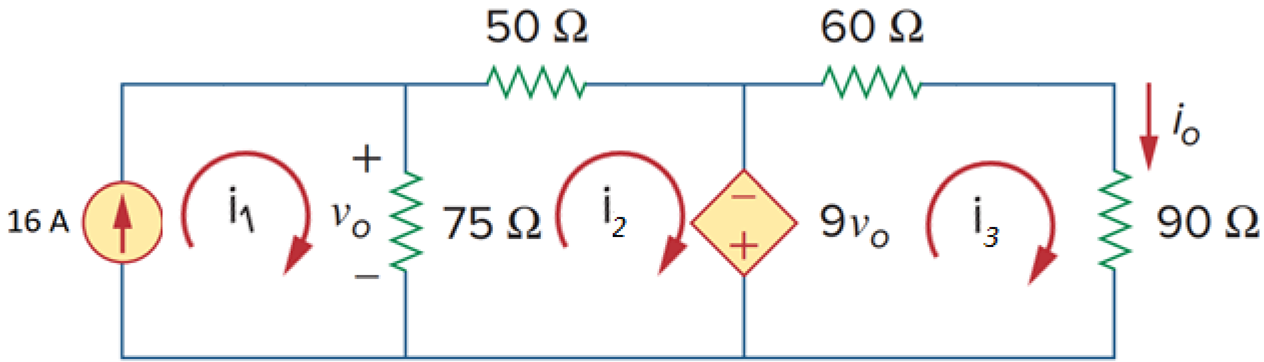
Example 1 - 1/7

Use mesh analysis, determine v_0. What is the current supplied by the dependent voltage source? What is the power of it? Is it absorbing or supplying?



Before solving the circuit using mesh analysis, recall that, "*For passive elements, current enters through the positive terminal of the voltage drop across it*." This is according to the *passive sign convention*, current must always flow from a higher potential to a lower potential through a passive element that is absorbing power.

Example 1 - 2/7

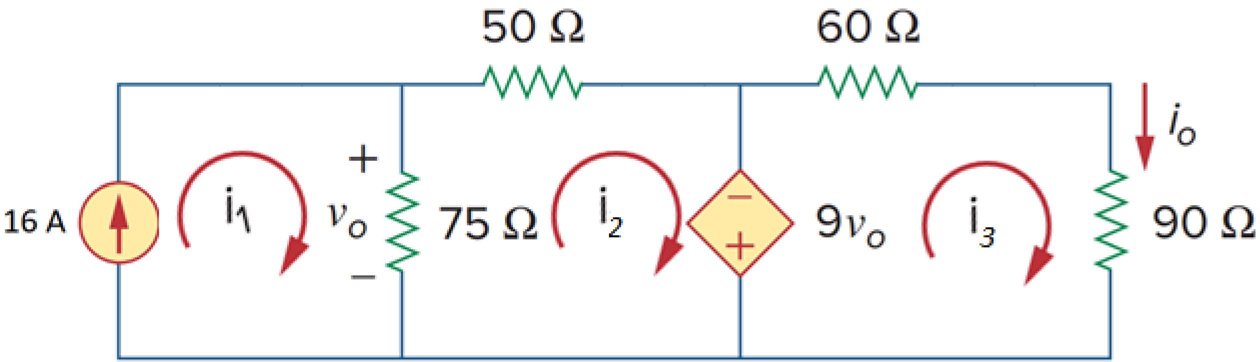


👉 First identify all the meshes (independent loops) in this circuit.

There are 3 meshes as identified in the circuit.

👉 Assign mesh currents $(i_1, i_2, and\ i_3)$ to all the meshes. The assigned currents can be clockwise, anti-clockwise, or a combination of the two.

Example 1 - 3/7



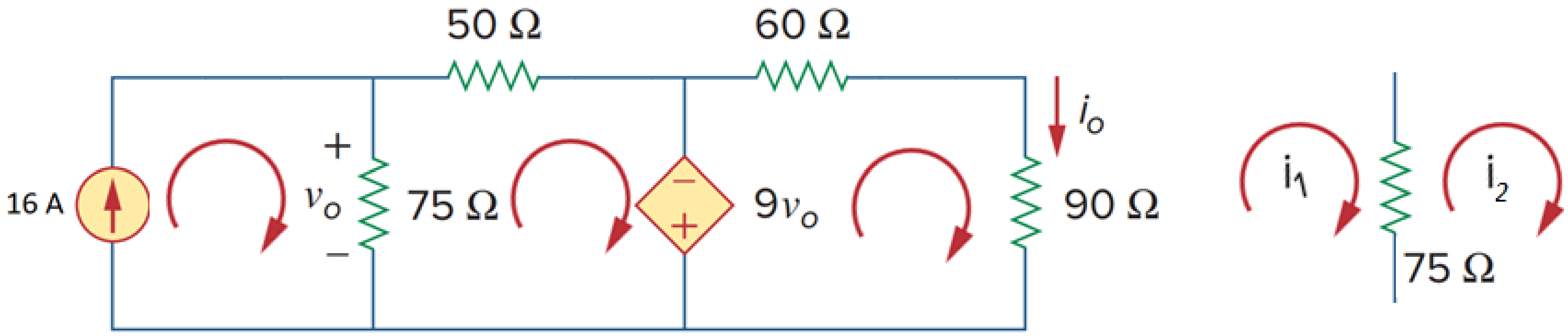
👉 The 2nd step is to apply KVL to each mesh.

Note that, we already know the mesh 1 current. i_1 and the 16 A current flow through the same wire in the same direction. We can write directly,

$$i_1 = 16\ A ---- -(i)$$

For meshes whose mesh currents are already known, we don't need to apply KVL.

Example 1 - 4/7

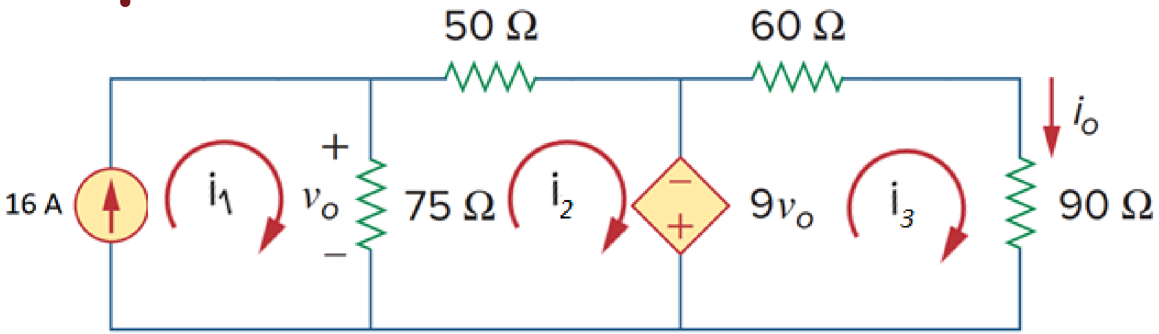


Next, apply KVL to mesh 2.

$$75\,(i_2 - i_1)\; + 50 i_2 - 9v_0 = 0$$

Notice that, the two mesh currents ($i_1$and i_2) overlap through the 75 Ω. As there can be no more than a current in a wire, the resulting current through the 75 Ω will be either $i_1 - i_2$ or $i_2 - i_1$. But we won't know exactly before solving. As we are moving in the direction of i_2, we take $i_2 - i_1$ as the resulting current and the KVL equation is written accordingly.

Example 1 - 5/7



$$75\,(i_2 - i_1)\; + 50i_2 - 9v_0 = 0\;[from\ the\ previous\ slide]$$

Now we have to replace v_0 in terms of the mesh currents as the mesh equations should not contain unknowns other than the mesh currents.

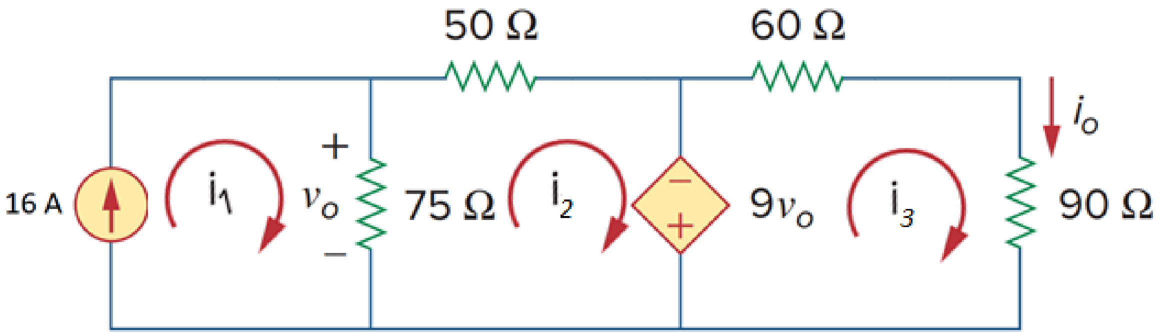
v_0 is the voltage drop across the 75 Ω resistor. With the polarity of v_0 given,

$$v_0 = 75\,(i_1 - i_2)$$

Substituting, $$75\,(i_2 - i_1)\; + 50i_2 - 9 \times 75\,(i_1 - i_2) = 0$$

$$750\,i_1 - 800i_2 = 0 ----(ii)$$

Example 1 - 6/7



👉 Next, apply KVL to mesh 3.

$$9v_0 + 60i_3 + 90i_3 = 0$$

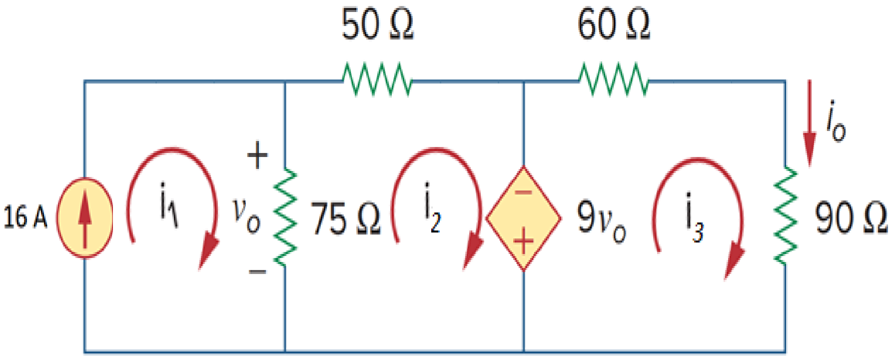
Substituting $v_0 = 75\ (i_1 - i_2)$ for v_0,

$$9 \times 75\ (i_1 - i_2) + 60i_3 + 90i_3 = 0$$

After simplifying,

$$675\ i_1 - 675i_2 + 150i_3 = 0 ---- -(iii)$$

Example 1 - 7/7



We have derived the three mesh equations,

$$i_1 = 16\ A$$

$$750\ i_1 - 800 i_2 = 0$$

$$675\ i_1 - 675 i_2 - 150 i_3 = 0$$

Solving … …,

$$i_1 = 16\ A; \quad i_2 = 15\ A; \quad i_3 = -\,4.5\ A;$$

So,

$$v_0 = 75(i_1 - i_2) = 75\,(16 - 15) = 75\ V$$

Current supplied (entering into the –ve terminal) by the dependent source is,

$$i_2 - i_3 = 15 - (-4.5) = 19.5\ A$$

Power supplied by the dependent source is thus,

$$p = -\,vi = 9v_0 \times 19.5$$

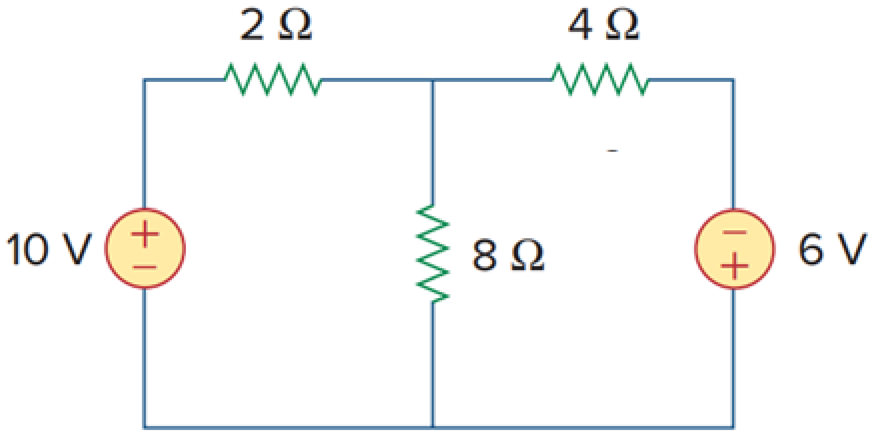
$$= 9 \times 75 \times 19.5$$

$$= 13162.5\ W$$

Problem 1

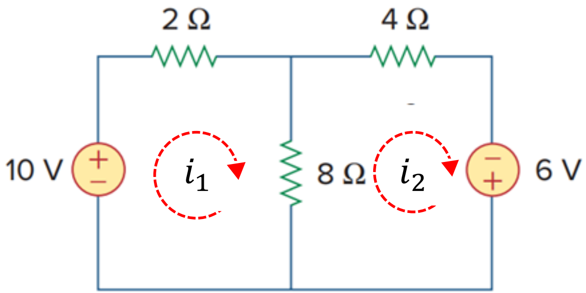
i. Perform *branch current analysis* to determine the current absorbed by the $6\ V$ source in the following circuit.

ii. Perform *mesh analysis* to determine the current absorbed by the $6\ V$ source in the following circuit.



Ans: $-2.5\ A$

Solution to Problem 1



Applying KVL in mesh 1,

$$-10 + 2i_1 + 8(i_1 - i_2) = 0$$

$$\Rightarrow -10 + 2i_1 + 8i_1 - 8i_2 = 0$$

$$\Rightarrow 10i_1 - 8i_2 = 10 \quad \text{........ (i)}$$

Applying KVL in mesh 2,

$$8(i_2 - i_1) + 4i_2 - 6 = 0$$

$$\Rightarrow 8i_2 - 8i_1 + 4i_2 - 6 = 0$$

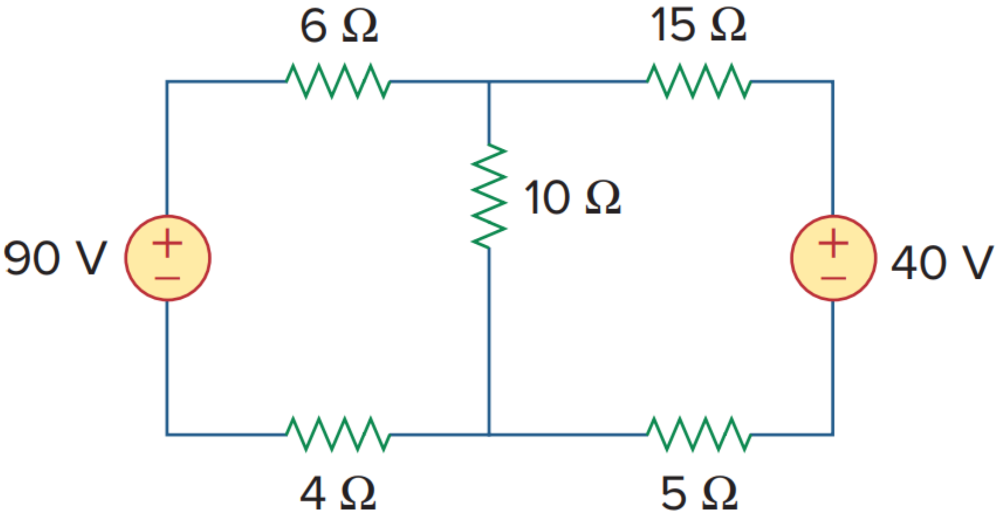
$$\Rightarrow -8i_1 + 12i_2 = 6 \quad \text{........ (ii)}$$

Solving (i) and (ii),

$$\boxed{\begin{aligned} i_1 &= 3\ A \\ i_2 &= 2.5\ A \end{aligned}}$$

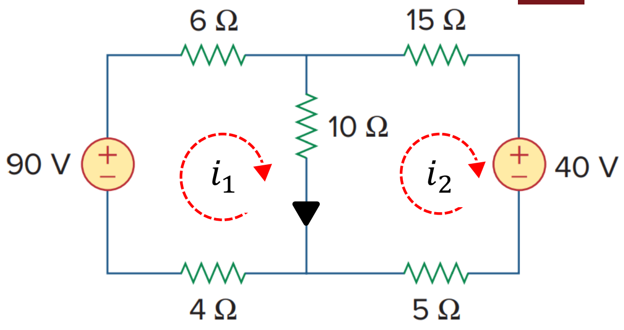
Problem 2

- Calculate the current through the 10 Ω resistor using mesh analysis.



Ans: $I_{10\Omega} = 4.4\ A$

Solution to Problem 2



Applying KVL in mesh 1,

$$-90 + 6i_1 + 10(i_1 - i_2) + 4i_1 = 0$$

$$\Rightarrow -90 + 6i_1 + 10i_1 - 10i_2 + 4i_1 = 0$$

$$\Rightarrow 20i_1 - 10i_2 = 90 \quad \text{........ (i)}$$

Applying KVL in mesh 2,

$$10(i_2 - i_1) + 15i_2 + 40 + 5i_2 = 0$$

$$\Rightarrow 10i_2 - 10i_1 + 15i_2 + 40 + 5i_2 = 0$$

$$\Rightarrow -10i_1 + 30i_2 = -40 \quad \text{........ (ii)}$$

Solving (i) and (ii),

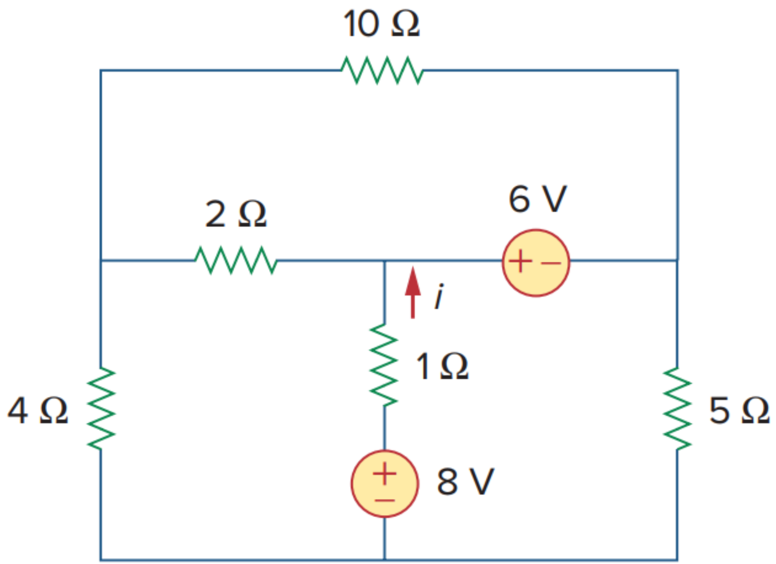
$$\boxed{\begin{aligned} i_1 &= 4.6\ A \\ i_2 &= 0.2\ A \end{aligned}}$$

Current through the 10 Ω resistor

$$i_1 - i_2 = 4.6 - 0.2 = \mathbf{4.4\ A}$$

Problem 3

- Calculate the current i using mesh analysis.



Ans: $i = 1.188\ A$

Solution to Problem 3

Applying KVL in mesh 1,

$$4i_1 + 2(i_1 - i_3) + 1(i_1 - i_2) + 8 = 0$$

$$\Rightarrow 4i_1 + 2i_1 - 2i_3 + i_1 - i_2 + 8 = 0$$

$$\Rightarrow 7i_1 - i_2 - 2i_3 = -8 \quad \ldots\ldots\ldots \text{(i)}$$

Applying KVL in mesh 2,

$$-8 + 1(i_2 - i_1) + 6 + 5i_2 = 0$$

$$\Rightarrow -8 + i_2 - i_1 + 6 + 5i_2 = 0$$

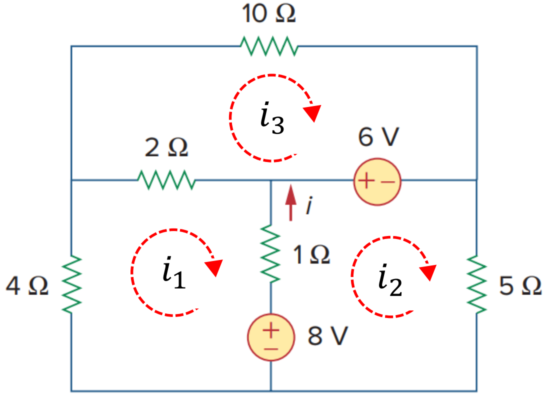
$$\Rightarrow -i_1 + 6i_2 = 2 \quad \ldots\ldots\ldots \text{(iii)}$$

Applying KVL in mesh 3,

$$10i_3 - 6 + 2(i_3 - i_1) = 0$$

$$\Rightarrow 10i_3 - 6 + 2i_3 - 2i_1 = 0$$

$$\Rightarrow -2i_1 + 12i_3 = 6 \quad \ldots\ldots\ldots \text{(ii)}$$



Solving (i) and (ii),

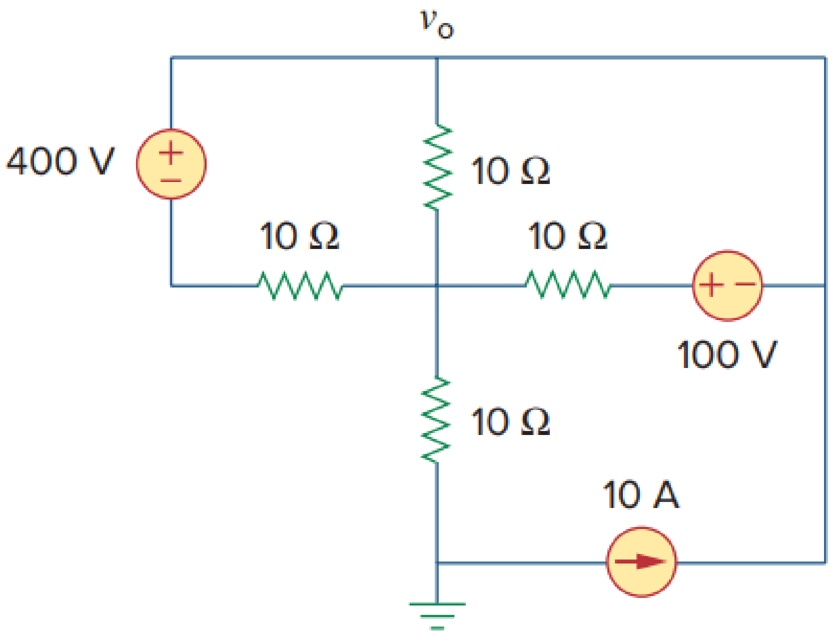
$$\boxed{\begin{aligned} i_1 &= -1.1\ A \\ i_2 &= 0.15\ A \\ i_3 &= 0.32\ A \end{aligned}}$$

The value of current i

$$i = i_2 - i_1 = 0.15 - (-1.1) = \mathbf{1.25\ A}$$

Problem 4

- Apply mesh analysis to find v_o in the following circuit.



Ans: $v_0 = 233.3\ V$

Solution to Problem 4

From mesh 3,

$$i_3 = -10\ A \quad \ldots\ldots\ldots \text{(i)}$$

Applying KVL in mesh 1,

$$-400 + 10(i_1 - i_2) + 10i_1 = 0$$

$$\Rightarrow -400 + 10i_1 - 10i_2 + 10i_1 = 0$$

$$\Rightarrow 20i_1 - 10i_2 = 400 \ \ldots\ldots\ldots \text{(ii)}$$

Applying KVL in mesh 2,

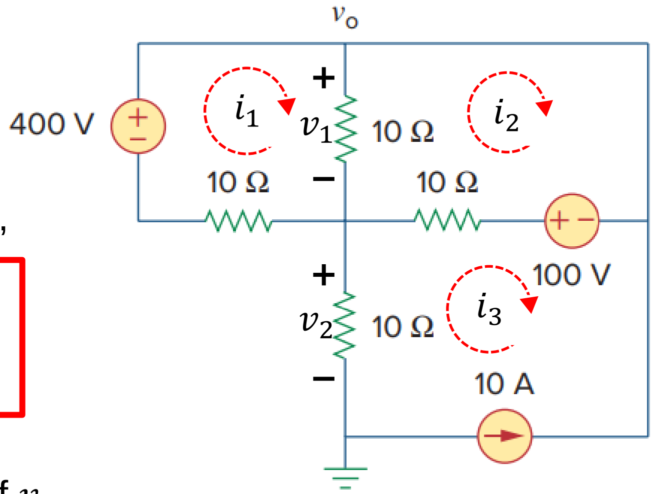
$$10(i_2 - i_1) - 100 + 10(i_2 - i_3) = 0$$

$$\Rightarrow 10i_2 - 10i_1 - 100 + 10i_2 - 10i_3 = 0$$

$$\Rightarrow -10i_1 + 20i_2 - 10i_3 = 100$$

$$\Rightarrow -10i_1 + 20i_2 - 10 \times (-10) = 100$$

$$\Rightarrow -10i_1 + 20i_2 = 0 \ \ldots\ldots\ldots \text{(iii)}$$



Solving (ii) and (iii),

$$\boxed{\begin{aligned} i_1 &= 26.67\ A \\ i_2 &= 13.33\ A \end{aligned}}$$

To find the value of v_o
We can see from the circuit

$$v_o = v_1 + v_2$$

So,

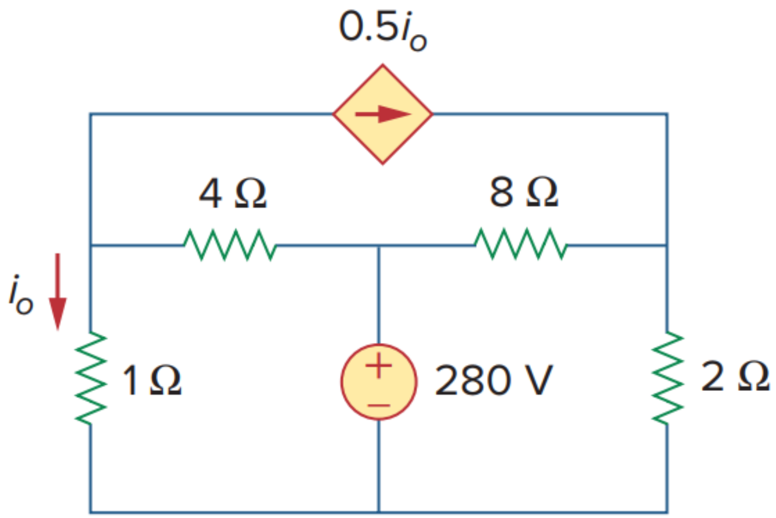
$$v_1 = 10(i_1 - i_2) = 10(26.67 - 13.33) = 133.4\ V$$

$$v_2 = -10i_3 = -10 \times (-10) = 100\ V$$

$$v_o = 133.4 + 100 = \mathbf{233.4}$$

Problem 5

- Find i_0 using mesh analysis. What is the voltage across the $0.5i_0$ source?



Ans: $\boldsymbol{i_0 = 40\ A;\ \pm 48\ V}$

Solution to Problem 5

From the circuit,

$$i_o = -i_2 \quad \text{........ (i)}$$

From mesh 1,

$$i_1 = 0.5 i_o$$

$$\Rightarrow i_1 = 0.5(-i_2)$$

$$\Rightarrow i_1 + 0.5 i_2 = 0 \quad \text{........ (ii)}$$

Applying KVL in mesh 2,

$$1 i_2 + 4(i_2 - i_1) + 280 = 0$$

$$\Rightarrow i_2 + 4 i_2 - 4 i_1 + 280 = 0$$

$$\Rightarrow -4 i_1 + 5 i_2 = -280 \quad \text{........ (ii)}$$

Applying KVL in mesh 3,

$$-280 + 8(i_3 - i_1) + 2 i_3 = 0$$

$$\Rightarrow -280 + 8 i_3 - 8 i_1 + 2 i_3 = 0$$

$$\Rightarrow -8 i_1 + 10 i_3 = 280 \quad \text{........ (iii)}$$

Solving (i), (ii) and (iii),

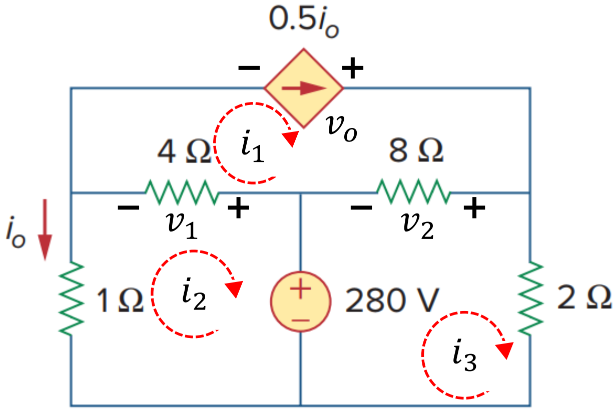
$$\boxed{\begin{aligned} i_1 &= 20\ A \\ i_2 &= -40\ A \\ i_3 &= 44\ A \end{aligned}}$$

Let the voltage across the dependent Source is v_o
We can see from the circuit

$$v_o = v_1 + v_2$$

$$v_1 = 4(i_1 - i_2) = 4(20 - (-40)) = 240\ V$$

$$v_2 = 8(i_1 - i_3) = 4(20 - 44) = -192\ V$$



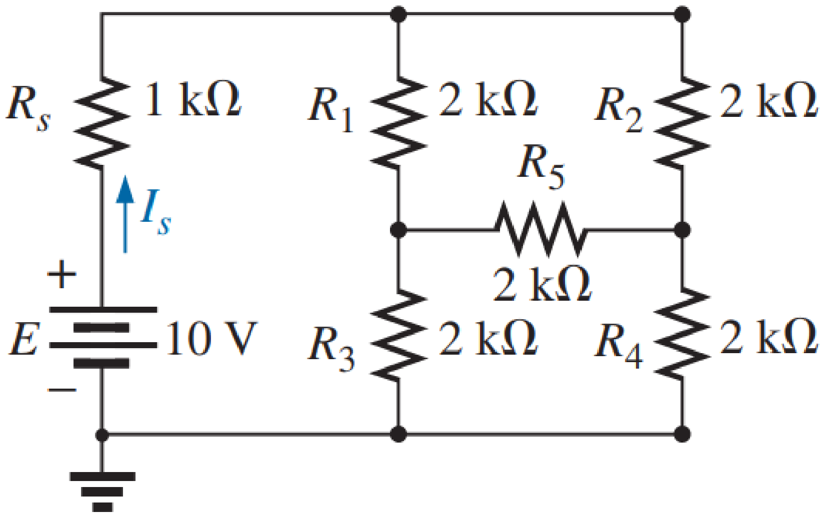
$$v_o = 240 - 192$$

$$v_o = \mathbf{48\ V}$$

In fact, you can find v_o by applying KVL in mesh 1 after finding out all the currents

Problem 6

- Determine the current through the source resistor R_s using mesh analysis.



Ans: $i_s = 3.33\ mA$

Solution to Problem 6

Applying KVL in mesh 1,

$$-10 + i_1 + 2(i_1 - i_2) + 2(i_1 - i_3) = 0$$

$$\Rightarrow -10 + i_1 + 2i_1 - 2i_2 + 2i_1 - 2i_3 = 0$$

$$\Rightarrow 5i_1 - 2i_2 - 2i_3 = 10 \text{ …….. (i)}$$

Applying KVL in mesh 2,

$$2(i_2 - i_1) + 2i_2 + 2(i_2 - i_3) = 0$$

$$\Rightarrow 2i_2 - 2i_1 + 2i_2 + 2i_2 - 2i_3 = 0$$

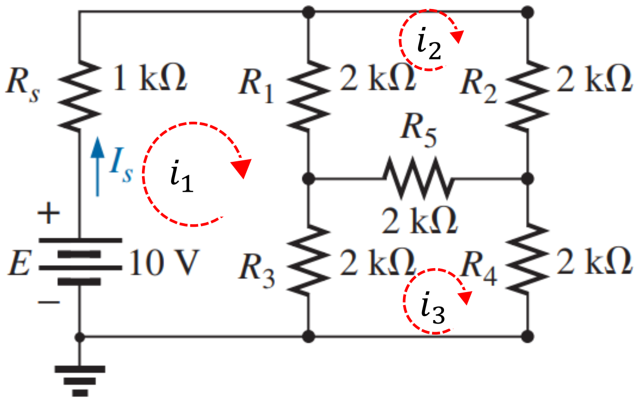
$$\Rightarrow -2i_1 + 6i_2 - 2i_3 = 0 \text{ …….. (iii)}$$

Applying KVL in mesh 3,

$$2(i_3 - i_1) + 2(i_3 - i_2) + 2i_3 = 0$$

$$\Rightarrow 2i_3 - 2i_1 + 2i_3 - 2i_2 + 2i_3 = 0$$

$$\Rightarrow -2i_1 - 2i_2 + 6i_3 = 0 \text{ …….. (ii)}$$



Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= 3.33\ A \\ i_2 &= 1.67\ A \\ i_3 &= 1.67\ A \end{aligned}}$$

The value of current I_s

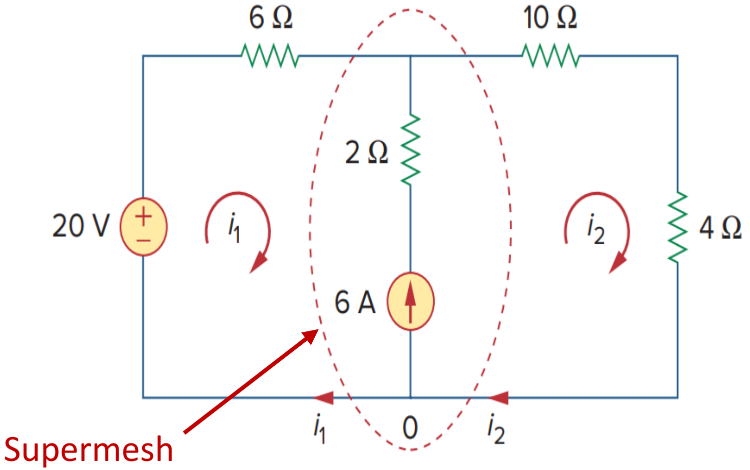
$$I_s = i_1 = \mathbf{3.33\ A}$$

Analysis with current source bet^n loops

- **CASE 1** When a current source (dependent or independent) exists only in one mesh, we simply set the current at that mesh equal to the current of the current source. (We have already seen this in example 1).

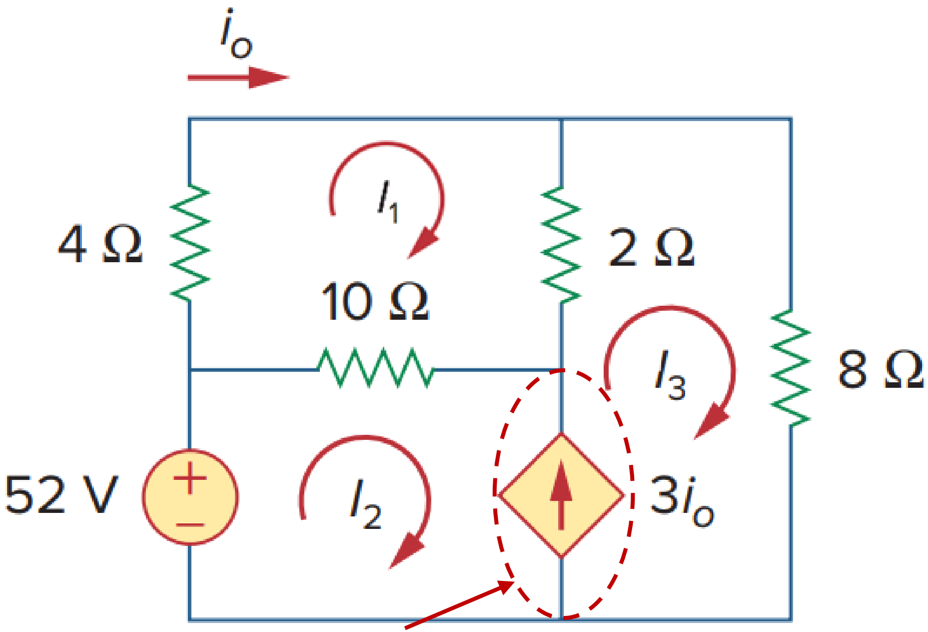
- **CASE 2** When a current source (dependent or independent) exists between two meshes, the two meshes form a generalized mesh or supermesh.

In other words, a *supermesh* results when two meshes have a (dependent or independent) current source in common.



Example 2 - 1/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Step 1: Identify all the meshes and assign mesh variables to each of the meshes.

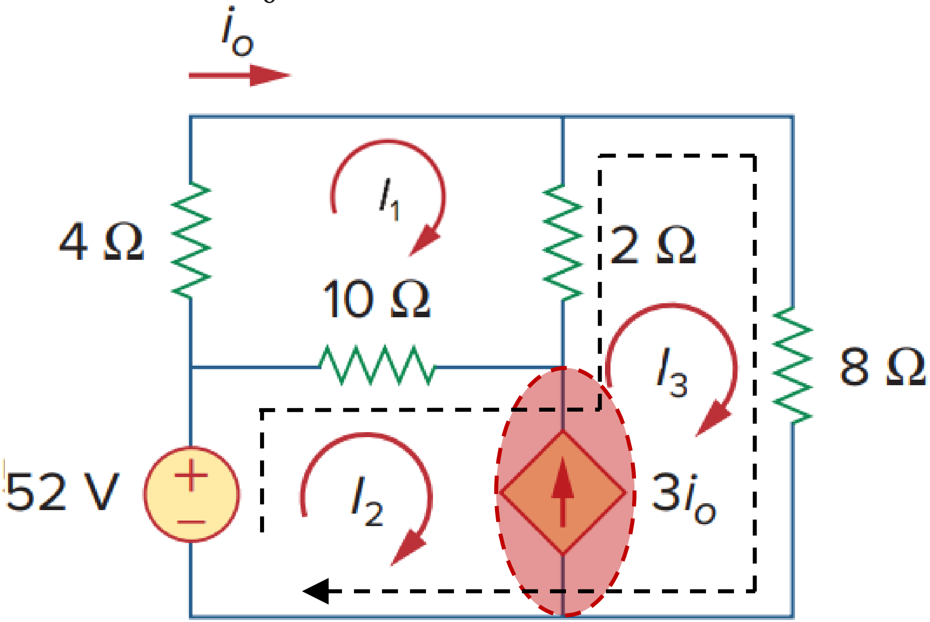
Check for supermeshes. Check if a current source (dependent or independent) is connected between two meshes. There can be multiple supermeshes in a circuit.

In this circuit, the $3i_0$ current source forms a supermesh between meshes 2 and 3.

We need to handle such conditions differently because there is no way to know the voltage across a current source in advance.

Example 2 - 2/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Step 2: Apply KVL to each of the meshes.

KVL to the mesh 1,

$$4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$$

$$\Rightarrow \quad 16i_1 - 10i_2 - 2i_3 = 0 ---- -(i)$$

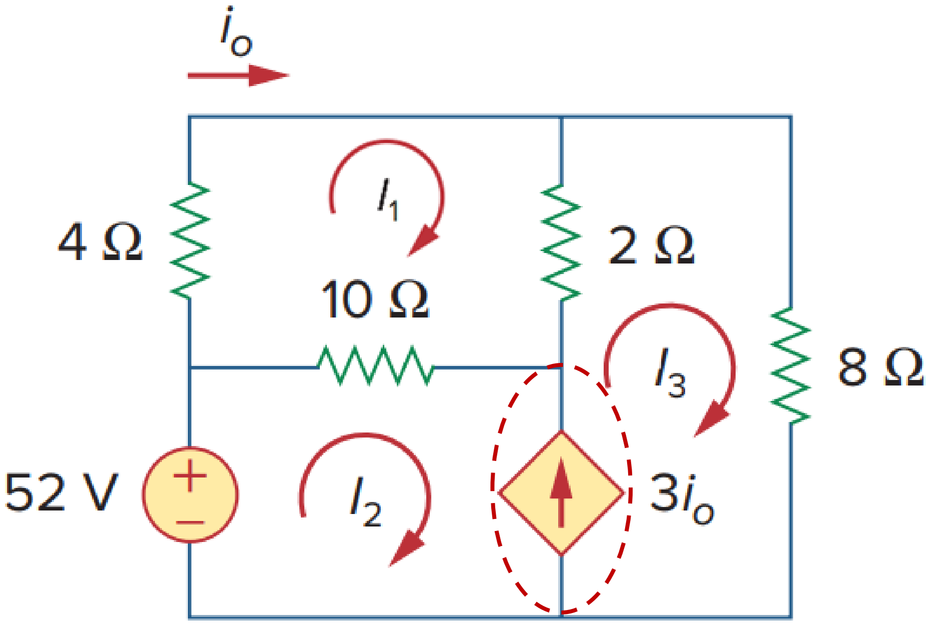
Next, ignore the current source that forms the supermesh and apply KVL to the corresponding meshes together. Careful with the current notations. Applying KVL to the supermesh along the black dotted line shown in the figure,

$$-52 + 10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 0$$

$$\Rightarrow \quad 12i_1 - 10i_2 - 10i_3 = -52 ---- -(ii)$$

Example 2 - 3/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



We have 2 equations, 3 variables, and no remaining mesh for KVL.

The 3rd equation required, can be found by applying KCL to the supermesh.

$$i_3 - i_2 = 3i_0$$

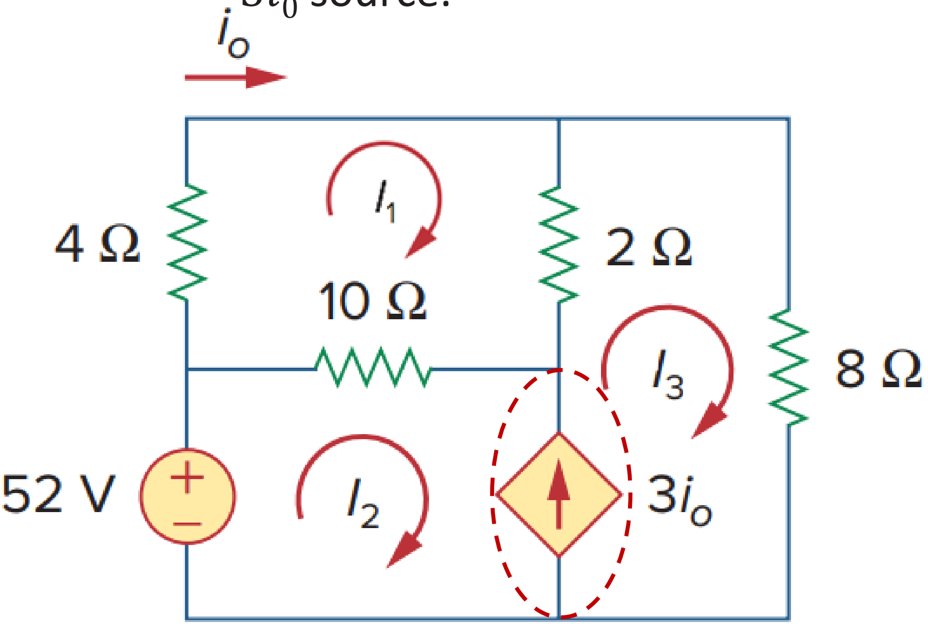
Now replace i_0 in terms of the mesh currents. It can be seen from the figure that, $i_0 = i_1$. Substituting,

$$i_3 - i_2 = 3i_1$$

$$\Rightarrow \quad 3i_1 + i_2 - i_3 = 0 ----(iii)$$

Example 2 - 4/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



We have derived the three equations,

$$16i_1 - 10i_2 - 2i_3 = 0$$
$$12i_1 - 10i_2 - 10i_3 = -52$$
$$3i_1 + i_2 - i_3 = 0$$

Solving … …,

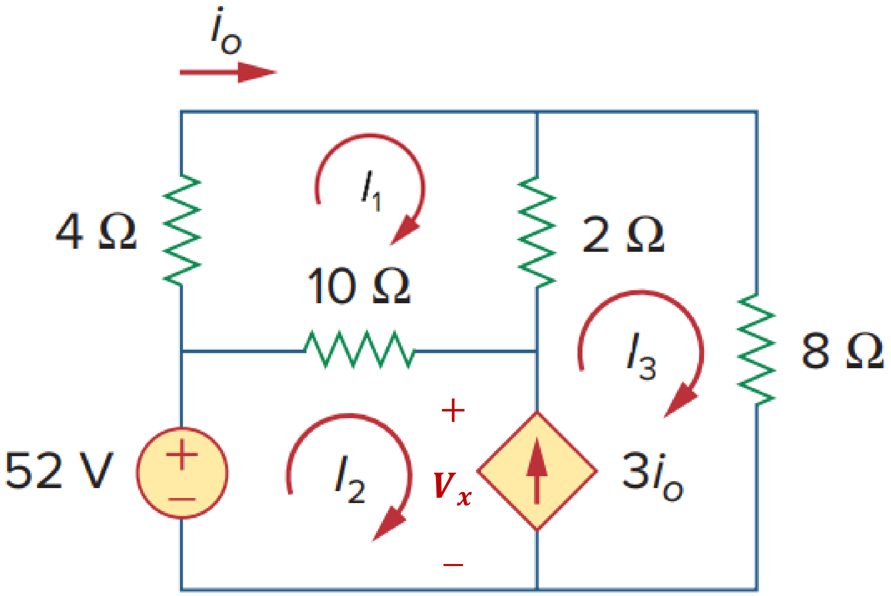
$$i_1 = 1.5\,A; \quad i_2 = 1.25\,A; \quad i_3 = 5.75\,A$$

So, $\quad i_0 = i_1 = 1.5\,A$

To calculate the voltage across the $3i_0$ dependent source, we have to apply KVL to either loop 2 or loop 3.

Example 2 - 5/5

- Find i_0 using mesh analysis. Also, calculate the voltage across the $3i_0$ source.



Let the voltage across the $3i_0$ source is V_x as indicated in the figure.

Applying KVL to the loop 2,

$$-52 + 10(i_2 - i_1) + V_x = 0$$

$$\Rightarrow \quad V_x = 52 - 10(1.25 - 1.5) = 54.5\ V$$

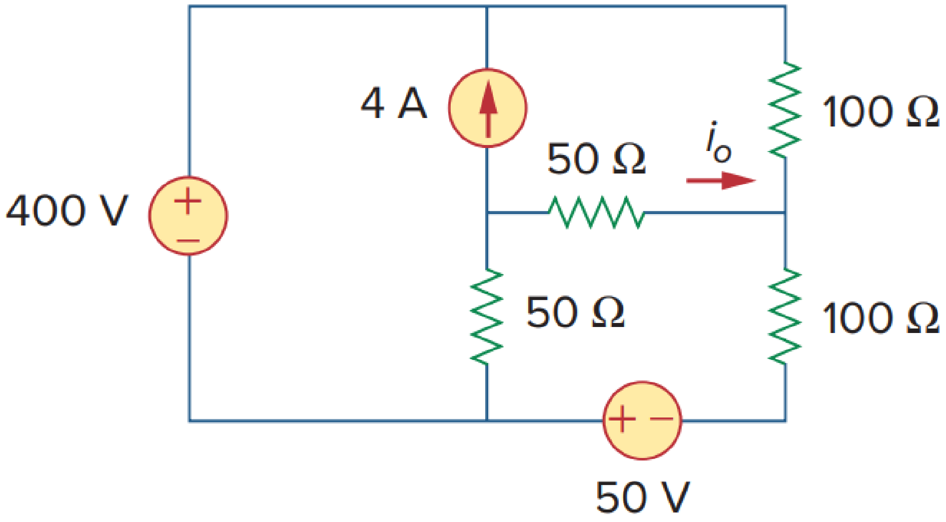
As observed by the polarities of voltage and current, the dependent source is supplying power.

$$p = +vi = 54.5 \times 3i_0 = 54.5 \times 3i_1$$

$$\Rightarrow \quad p = 54.5 \times 3 \times 1.5 = 245.25\ W$$

Problem 7

- Find i_0 using mesh analysis.



Ans: $\boldsymbol{i_0 = -2.5\ A}$

Solution to Problem 7

Applying KVL in mesh 3,

$$-50 + 50(i_3 - i_1) + 50(i_3 - i_2) + 100i_3 = 0$$

$$\Rightarrow -50 + 50i_3 - 50i_1 + 50i_3 - 50i_2 + 100i_3 = 0$$

$$\Rightarrow -50i_1 - 50i_2 + 200i_3 = 50 \quad \text{........ (i)}$$

mesh 1 and mesh 2 make supermesh
Applying KVL in supermesh of 1 and 2,

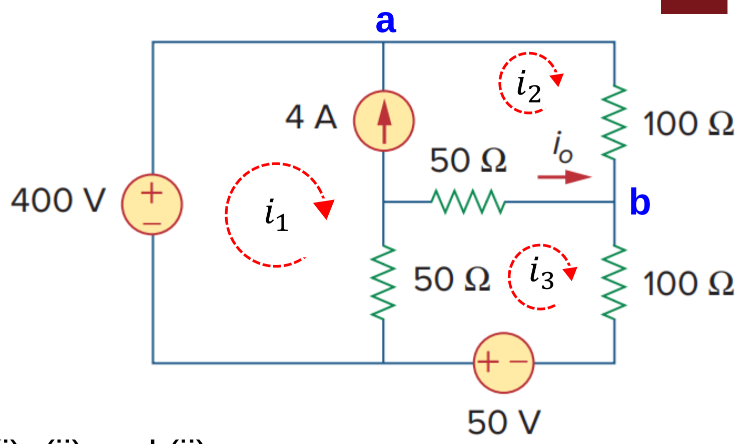
$$-400 + 100i_2 + 50(i_2 - i_3) + 50(i_1 - i_3) = 0$$

$$\Rightarrow -400 + 100i_2 + 50i_2 - 50i_3 + 50i_1 - 50i_3 = 0$$

$$\Rightarrow 50i_1 + 150i_2 - 100i_3 = 400 \quad \text{........ (ii)}$$

Applying KCL in node "**a**",

$$i_1 - i_2 = -4 \quad \text{........ (iii)}$$



Solving (i), (ii) and (ii),

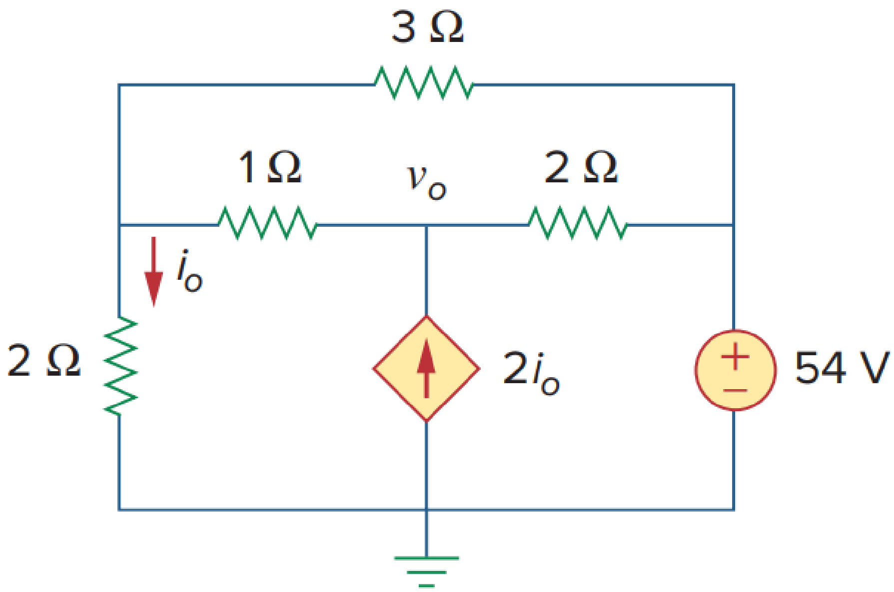
$$\boxed{\begin{aligned} i_1 &= -0.5\ A \\ i_2 &= 3.5\ A \\ i_3 &= 1\ A \end{aligned}}$$

The value of current i_o (Applying KCL at node "**b**")

$$i_o = i_3 - i_2 = 1 - 3.5 = \mathbf{-2.5\ A}$$

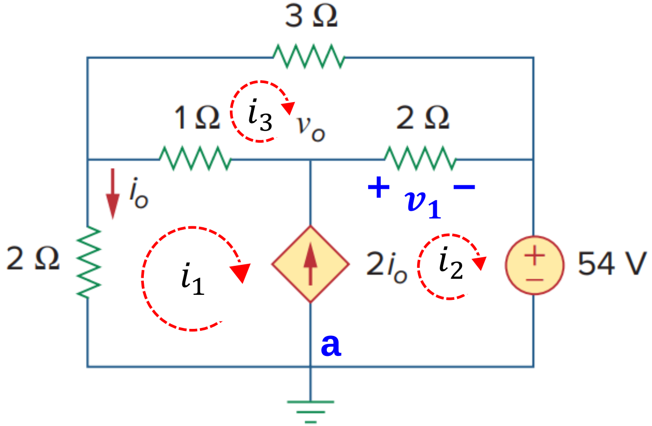
Problem 8

- Find i_0 using mesh analysis. Determine the node voltage v_0.



Ans: $\boldsymbol{i_0 = 36\ A; v_0 = 114\ V}$

Solution to Problem 8



From the figure,

$$i_o = -i_1$$

Applying KVL in mesh 3,

$$\Rightarrow i_3 - i_1 + 3i_3 + 2i_3 - 2i_2$$

$$= 01(i_3 - i_1) + 3i_3 + 2(i_3 - i_2) = 0$$

…….. (i)

$$\Rightarrow -i_1 - 2i_2 + 6i_3 = 0$$

Applying KVL in supermesh of 1 and 2,

$$2i_1 + i(i_1 - i_3) + 2(i_2 - i_3) + 54 = 0$$

$$\Rightarrow 2i_1 + i_1 - i_3 + 2i_2 - 2i_3 + 54 = 0$$

$$\Rightarrow 3i_1 + 2i_2 - 3i_3 = -54 \quad \text{…….. (ii)}$$

Applying KCL in node "**a**",

$$i_1 + 2i_o = i_2$$

$$\Rightarrow i_1 + 2(-i_1) = i_2$$

$$\Rightarrow i_1 + i_2 = 0 \quad \text{…….. (iii)}$$

Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= -36\ A \\ i_2 &= 36\ A \\ i_3 &= 6\ A \end{aligned}}$$

The value of current i_o

$$i_o = -i_1 = \mathbf{36\ A}$$

To find out node voltage v_o
Let us define

$$v_o = v_1 + 54$$

Now,

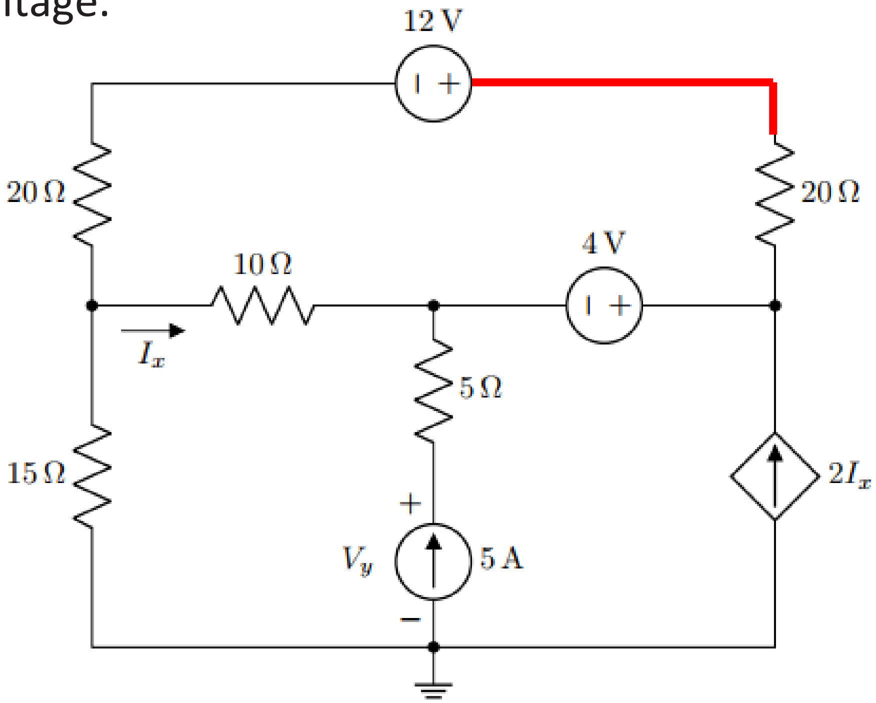
$$v_1 = 2(i_2 - i_3) = 2(36 - 6) = 60\ V$$

Finally,

$$v_o = 60 + 54 = \mathbf{114\ V}$$

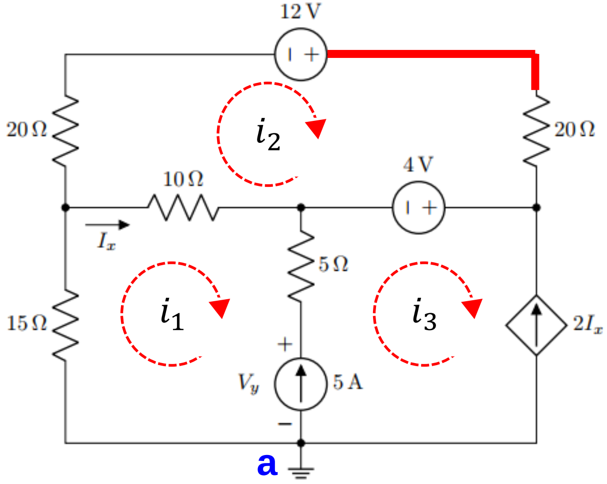
Problem 9

- Use Mesh Analysis to analyze the circuit. Find V_y. Determine the red colored node voltage.



Ans: $V_y = 68\ V; V_{red} = 43\ V$

Solution to Problem 9



From the figure,

$$I_x = i_1 - i_2$$

Applying KVL in mesh 3,

$$i_3 = -2I_x$$

$$\Rightarrow i_3 = -2(i_1 - i_2)$$

$$\Rightarrow i_3 = -2i_1 + 2i_2$$

$$\Rightarrow 2i_1 - 2i_2 + i_3 = 0 \quad \ldots\ldots\ldots \text{(i)}$$

Applying KVL in mesh 2,

$$20i_2 - 12 + 20i_2 + 4 + 10(i_2 - i_1) = 0$$

$$\Rightarrow 20i_2 - 12 + 20i_2 + 4 + 10i_2 - 10i_1 = 0$$

$$\Rightarrow -10i_1 + 50i_2 = 8$$

Applying KCL in node "**a**",

$$i_1 - i_3 = -5 \quad \ldots\ldots\ldots \text{(iii)}$$

Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= -1.8\ A \\ i_2 &= -0.2\ A \\ i_3 &= 3.2\ A \end{aligned}}$$

To find V_y, let us apply KVL in mesh 1

$$15i_1 + 10(i_1 - i_2) + 5(i_1 - i_3) + V_y = 0$$

$$\Rightarrow 15 \times (-1.8) + 10\big((-1.8) - (-0.2)\big) + 5\big((-1.8) - 3.2\big) + V_y = 0$$

$$\Rightarrow V_y = \mathbf{68\ V}$$

Solution to Problem 9 (Continued)

To find the node voltage of the red line, let us denote the node voltage as v_o

From the figure we can write,

$$v_o = v_1 + v_2$$

Now,

$$v_1 = 20i_2 = 20 \times (-0.2) = -4\ V$$

To find v_2, let us apply KVL in mesh 3,

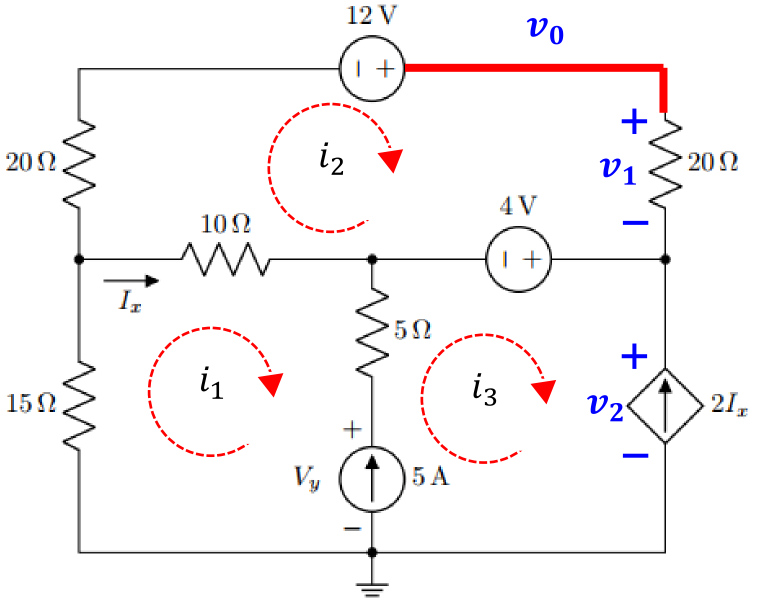
$$-V_y + 5(i_3 - i_1) - 4 + v_2 = 0$$

$$\Rightarrow -68 + 5\big(3.2 - (-1.8)\big) - 4 + v_2 = 0$$

$$\Rightarrow v_2 = 47\ V$$

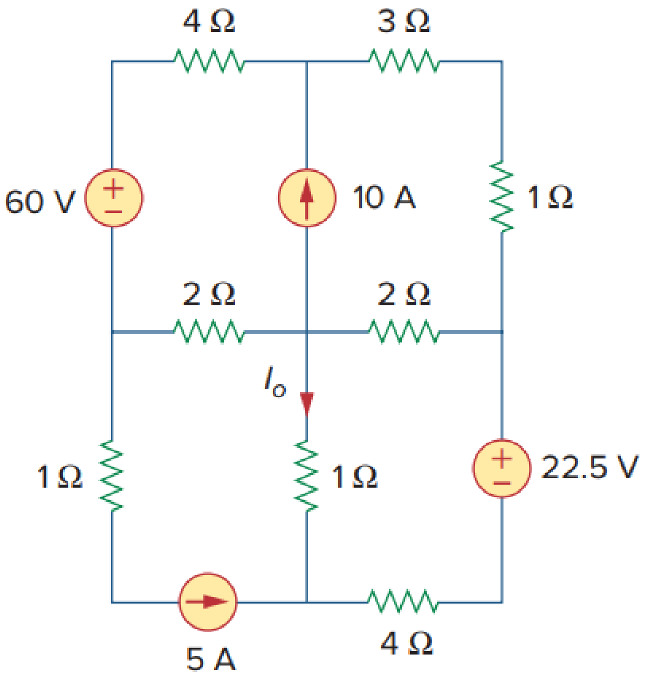
Finally,

$$v = -4 + 47 = \mathbf{43\ V}$$



Problem 10

- Derive the mesh equations for the following circuit. Determine i_0.



Ans: $i_0 = -3.62\ A$

Solution to Problem 10

From mesh 3,

$$i_3 = -5\ A$$

Applying KVL in mesh 4,

$$1(i_4 - i_3) + 2(i_4 - i_2) + 22.5 + 4i_4 = 0$$

$$\Rightarrow i_4 - i_3 + 2i_4 - 2i_2 + 22.5 + 4i_4 = 0$$

$$\Rightarrow -2i_2 - i_3 + 7i_4 = -22.5$$

$$\Rightarrow -2i_2 - (-5) + 7i_4 = -22.5$$

$$\Rightarrow -2i_2 + 7i_4 = -27.5 \quad \text{........ (i)}$$

Applying KVL in supermesh of 1 and 2,

$$-60 + 4i_1 + 3i_2 + 1i_2 + 2(i_2 - i_4) + 2(i_1 - i_3) = 0$$

$$\Rightarrow -60 + 4i_1 + 3i_2 + 1i_2 + 2i_2 - 2i_4 + 2i_1 - 2i_3 = 0$$

$$\Rightarrow 6i_1 + 6i_2 - 2i_3 - 2i_4 = 60$$

$$\Rightarrow 6i_1 + 6i_2 - 2(-5) - 2i_4 = 60$$

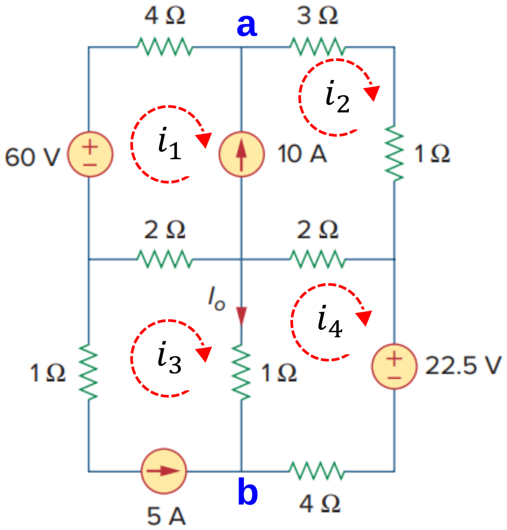
$$\Rightarrow 6i_1 + 6i_2 - 2i_4 = 50 \quad \text{........ (ii)}$$

Applying KCL in node "a",

$$i_1 - i_2 = -10 \quad \text{........ (iii)}$$

Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= -1.06\ A \\ i_2 &= 8.94\ A \\ i_4 &= -1.375\ A \end{aligned}}$$



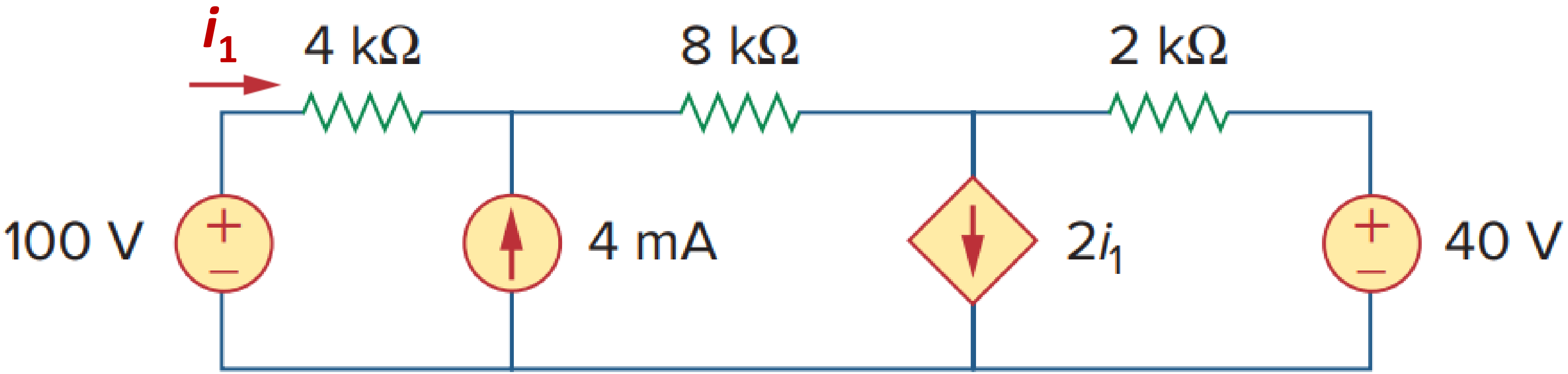
To find the value of i_o

Applying KCL at node "b"

$$i_o = -i_4 - 5 = -(-1.375) - 5 = \mathbf{-3.625\ A}$$

Problem 11

- Find the mesh currents.



Ans: $\pm 2\ mA;\ \pm 6\ mA;\ \pm 2\ mA$

mesh 1 and mesh 2 make supermesh. Also, mesh 2 and mesh 3 make super mesh. So, Applying KVL in supermesh of 1, 2 and 3.

$$-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$$

$$\Rightarrow 4i_1 + 8i_2 + 2i_3 = 60 \quad \ldots\ldots\ldots \text{(i)}$$

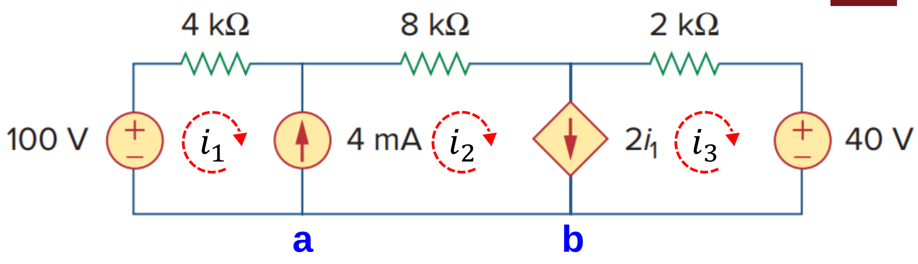
Applying KCL in node "**a**",

$$i_1 - i_2 = -4 \quad \ldots\ldots\ldots \text{(ii)}$$

Applying KCL in node "**b**",

$$i_2 - i_3 = 2i_1$$

$$\Rightarrow 2i_1 - i_2 + i_3 = 0 \quad \ldots\ldots\ldots \text{(ii)}$$

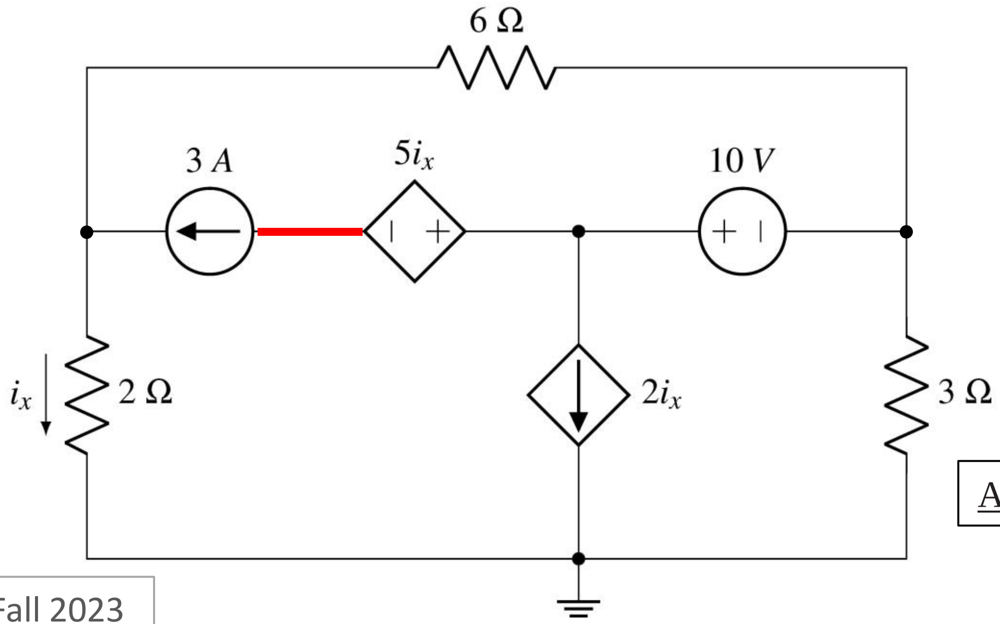


Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= 2\,mA \\ i_2 &= 6\,mA \\ i_3 &= 2\,mA \end{aligned}}$$

Problem 12

Use mesh analysis to find i_x. Determine the voltage of the red colored node.



Ans: $\boldsymbol{i_x = 1.059\ A; v_{3A} = \pm 6.94\ V; v_{red} = -4.82\ V}$

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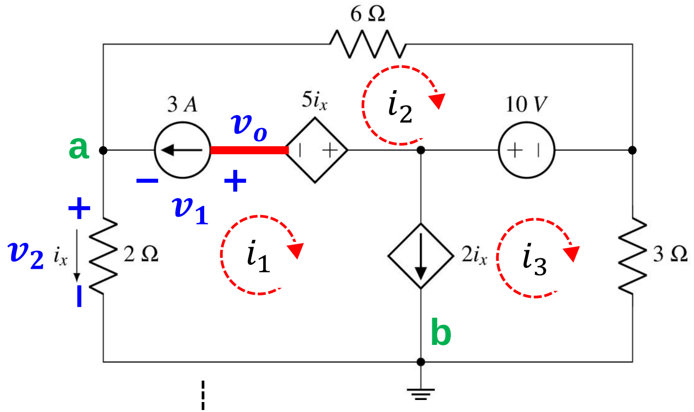
Solution to Problem 9



From the figure,

$$i_x = -i_1$$

mesh 1 and mesh 2 make supermesh. Also, mesh 2 and mesh 3 make super mesh. So, Applying KVL in supermesh of 1, 2 and 3.

$$2i_1 + 6i_2 + 3i_3 = 0 \;\;\ldots\ldots\ldots \text{(i)}$$

Applying KCL in node "a",

$$i_1 - i_2 = -3 \;\;\ldots\ldots\ldots \text{(ii)}$$

Applying KCL in node "b",

$$i_1 - i_3 = 2i_x$$

$$\Rightarrow i_1 - i_3 = 2(-i_1)$$

$$\Rightarrow 3i_1 - i_3 = 0 \;\ldots\ldots\ldots \text{(iii)}$$

Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= -1.059\ A \\ i_2 &= 1.94\ A \\ i_3 &= -3.176\ A \end{aligned}}$$

Value of i_x

$$i_x = -i_1 = -(-1.059) = \mathbf{1.059\ A}$$

Node voltage of the red node is

$$v_o = v_1 + v_2$$

To find, v_1 let us apply KVL in mesh 2

$$6i_2 - 10 + 5i_x + v_1 = 0$$

$$\Rightarrow 6 \times 1.94 - 10 + 5 \times 1.059 + v_1 = 0$$

$$\Rightarrow v_1 = -6.935\ V$$

To find, v_2

$$v_2 = -2i_1$$

$$\Rightarrow v_2 = -2(-1.059)$$

$$\Rightarrow v_2 = 2.118\ V$$

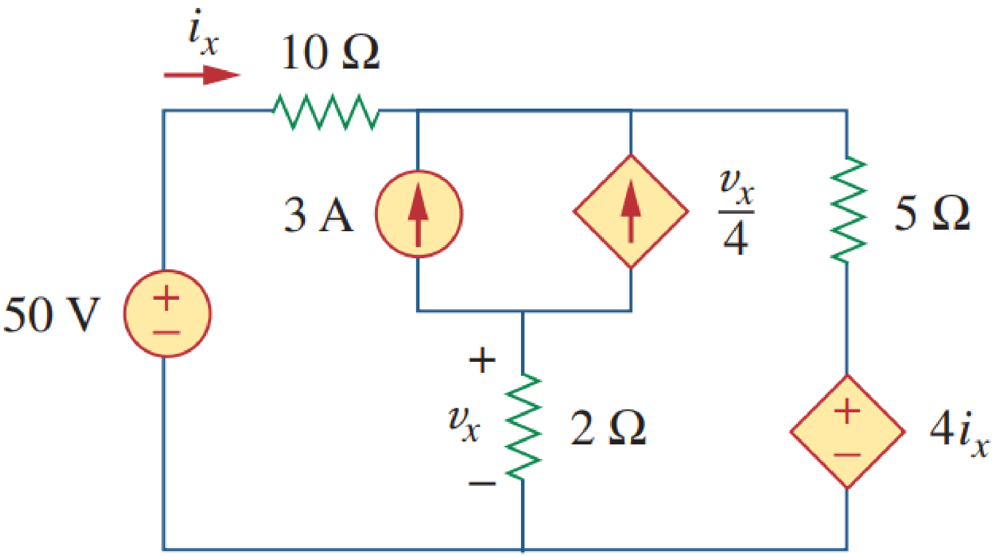
Finally,

$$v_o = -6.935 + 2.118$$

$$\Rightarrow v_o = \mathbf{-4.817\ V}$$

Problem 14

- Use mesh analysis to determine v_x and i_x. What is the voltage across the 3 A source?



Ans: $v_x = -4\ V$; $i_x = 2.105\ A$

Solution to Problem 13

From the figure,

$$i_x = i_1$$

$$v_x = 2(i_1 - i_3)$$

mesh 1 and mesh 2 make supermesh. Also, mesh 2 and mesh 3 make super mesh. So, Applying KVL in supermesh of 1, 2 and 3.

$$-50 + 10i_1 + 5i_3 = 0$$

$$\Rightarrow 10i_1 + 5i_3 = 50 \;\ldots\ldots\ldots \text{(i)}$$

Applying KCL in node "a",

$$i_1 - i_2 = -3 \;\ldots\ldots\ldots \text{(ii)}$$

Applying KCL in node "b",

$$i_2 + \frac{v_x}{4} = i_3$$

$$i_2 + \frac{2(i_1 - i_3)}{4} = i_3$$

$$\Rightarrow i_2 + \frac{1}{2}i_1 - \frac{1}{2}i_3 = i_3$$

$$\Rightarrow \frac{1}{2}i_1 + i_2 - \frac{3}{2}i_3 = 0 \;\ldots\ldots\ldots \text{(iii)}$$

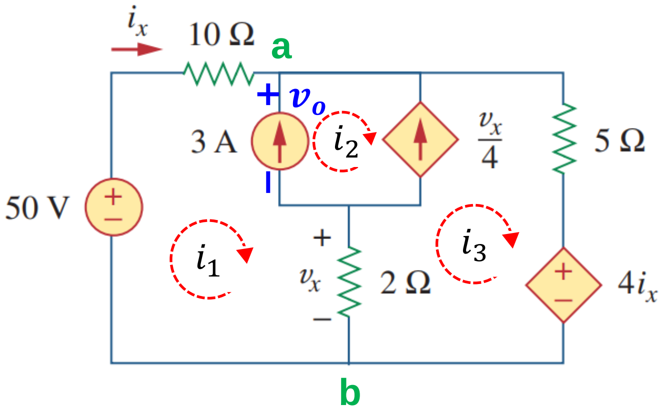
Solving (i), (ii) and (ii),

$$\boxed{\begin{aligned} i_1 &= 2.105\,A \\ i_2 &= 5.105\,A \\ i_3 &= 4.105\,A \end{aligned}}$$

$$i_x = \mathbf{2.105\,A}$$

$$v_x = 2(2.105 - 4.105)$$

$$\Rightarrow v_x = \mathbf{-4\,V}$$



Let, voltage across 3 A source is v_o

Applying KVL in mesh 1

$$-50 + 10i_1 + v_o + 2(i_1 - i_3) = 0$$

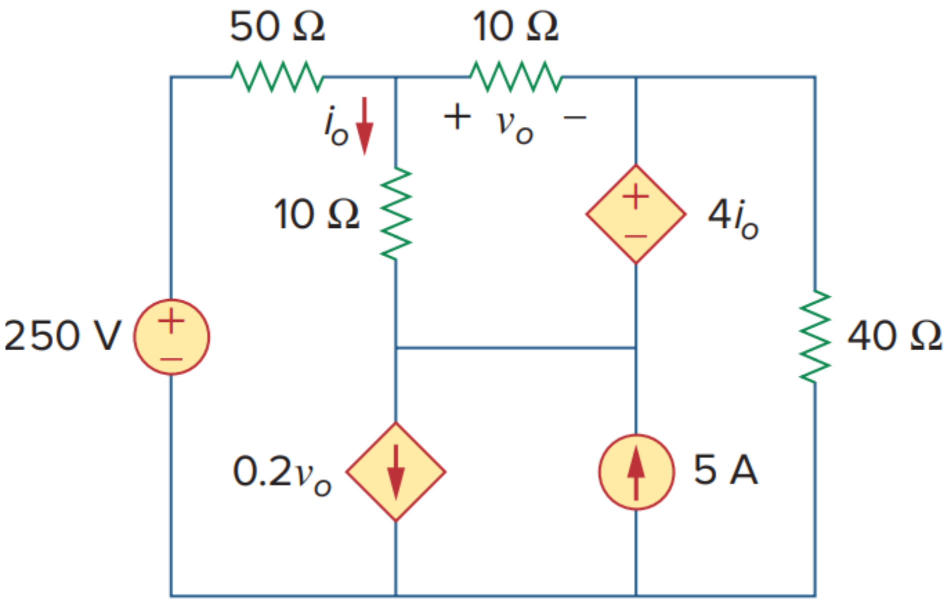
$$\Rightarrow -50 + 10 \times 2.105 + v_o$$

$$+ 2(2.105 - 4.105) = 0$$

$$\Rightarrow v_o = \mathbf{32.95\,V}$$

Problem 14

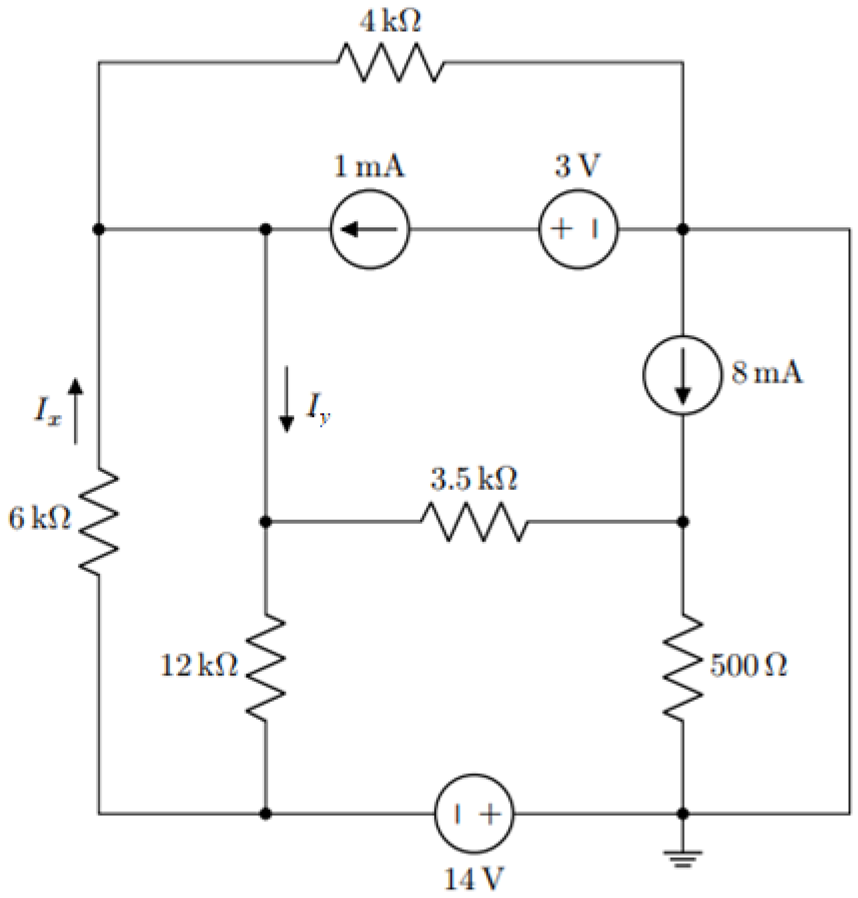
- Use mesh analysis to determine v_0 and i_0. What is the voltage across the $5\ A$ source?



Ans: $\boldsymbol{v_0 = 2.941\ V;\ i_0 = 0.49\ A}$

Problem 15

- Use mesh analysis to analyze the circuit. Find I_x.
- Determine the current I_y.



Ans: $I_x = -2\ mA$; $I_y = -0.5\ mA$

Nodal vs Mesh Analysis

- Given a network to be analysed, how do we know which method is better or more efficient? *The choice of the better method is dictated by two factors:*

■ Nature of the network

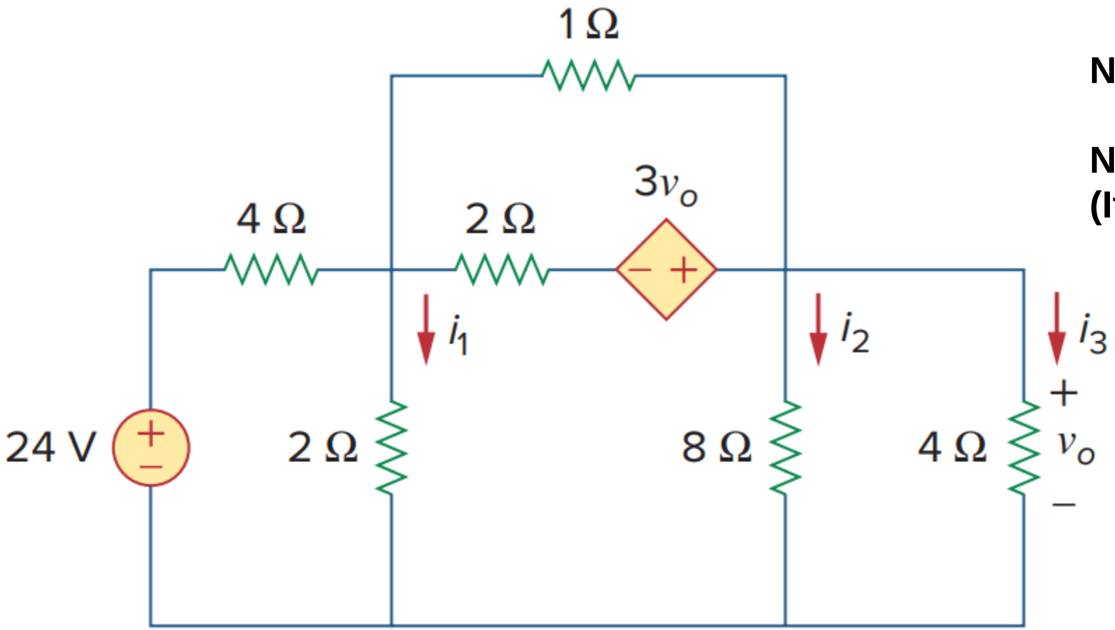
Mesh analysis is easier for networks that contain many series-connected elements, voltage sources, or supermeshes	Nodal analysis is easier for networks with parallel connected elements, current sources, or supernodes.
A circuit with fewer nodes than meshes is better analysed using nodal analysis, and **A circuit with fewer meshes than nodes is better analysed using mesh analysis**. The key is to select the method that results in the smaller number of equations.	

■ Information required

Mesh analysis is easier if branch or mesh currents are required. **However, Mesh analysis is easier if node voltage (a node with 2 branches connected) is required)**	Nodal analysis is easier if node voltages are required

Problem 16

- Which method, nodal or mesh, is more convenient for solving the circuit? Derive the equations that correspond to the convenient one.



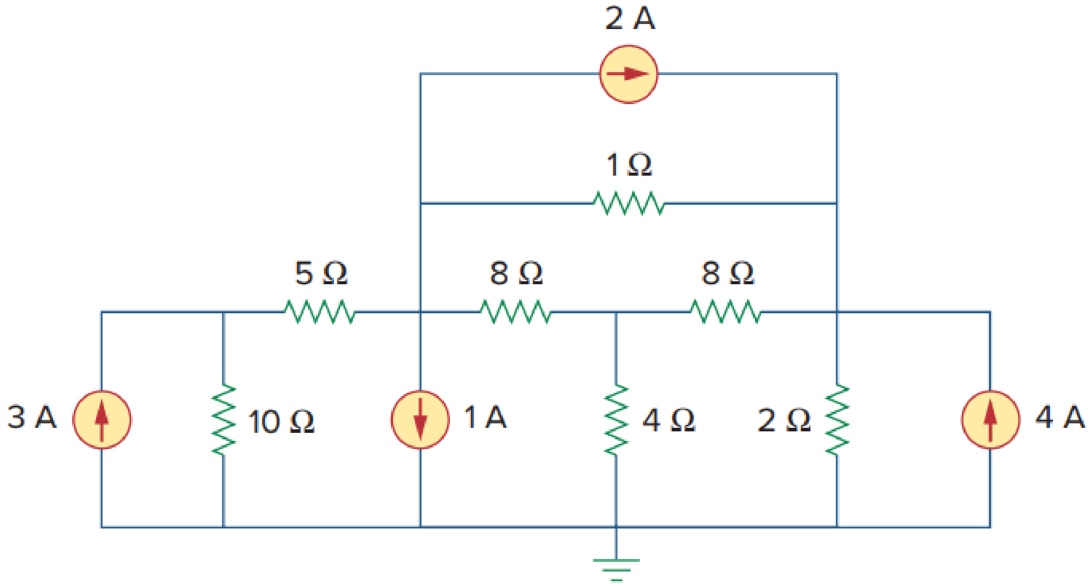
Number of mesh = 4

Number of unknown node variables = 2
(If ground is placed at the bottom node)

So, USE Nodal Analysis

Problem 17

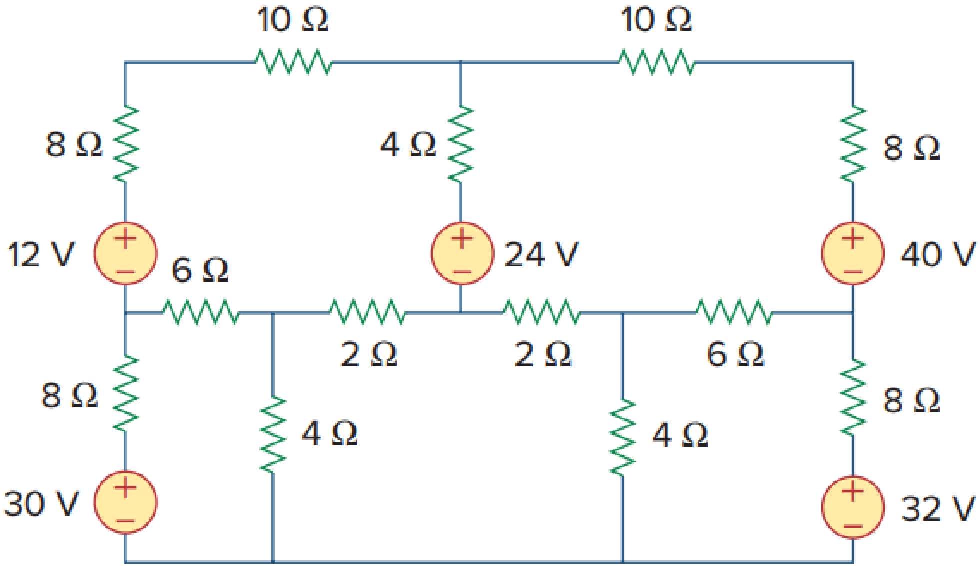
- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.



Ans:
- **# of nodes = 4;**
- **# of meshes = 7;**
- **minimum # of variables for nodal analysis = 4;**
- **minimum # of variables for mesh analysis = 7.**

Problem 18

- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.



Ans:
- **# of nodes = 14;**
- **# of meshes = 5;**
- **minimum # of variables for nodal analysis = 6;**
- **minimum # of variables for mesh analysis = 5.**

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here