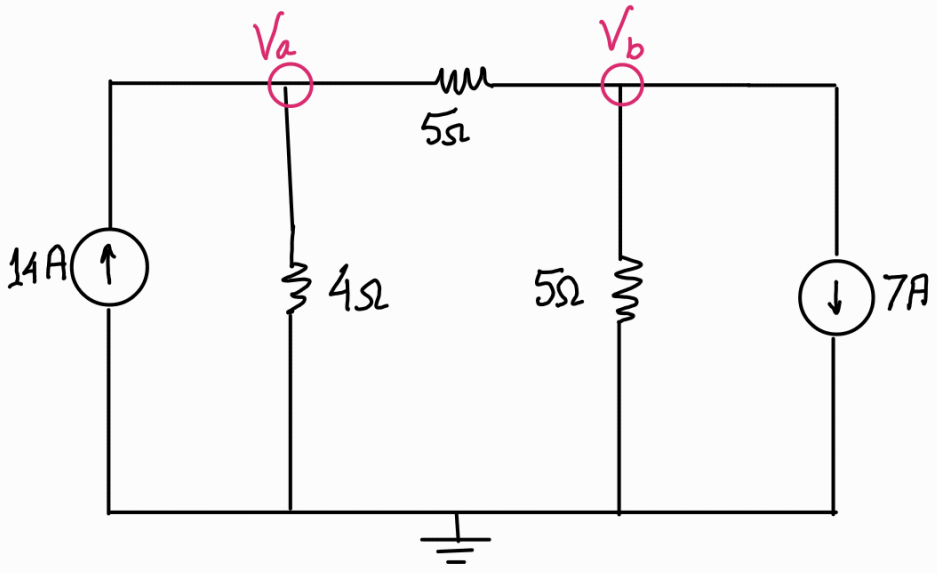
Problem 3



Find all node voltages

KCL @ a —

$$-14 + \frac{V_a}{4} + \frac{V_a - V_b}{5} = 0$$

$$\Rightarrow 5V_a + 4V_a - 4V_b = 280$$

$$\Rightarrow 9V_a - 4V_b = 280 \quad \text{——— (i)}$$

KCL @ b ———

$$7 + \frac{V_b}{5} + \frac{V_b - V_a}{5} = 0$$

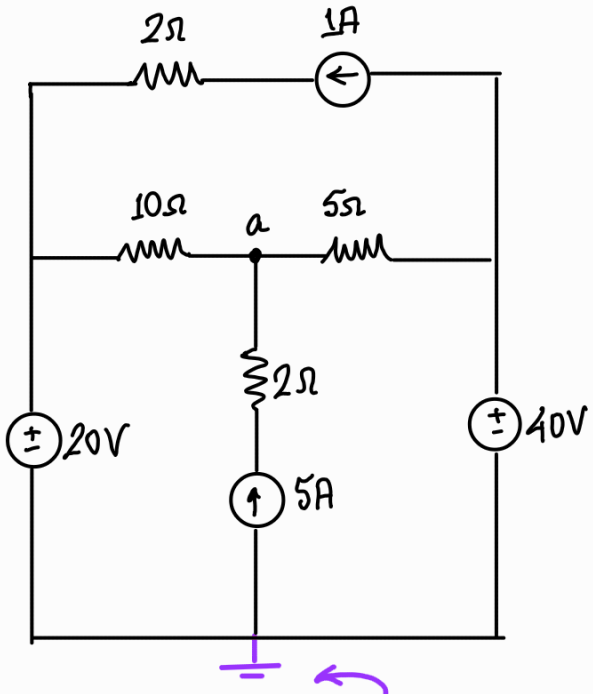
$$\Rightarrow V_a - 2V_b = 35 \quad \text{——— (ii)}$$

Solving (i) & (ii) — $V_a = 30V$

$V_b = -2.5V$

Answer - 0V, 30V, -2.5V

Problem 4



Find V_a

This is an example of a problem that will never come in the central exam (probably)

Since there is no fixed ground location, you can literally place a ground on the node a and write the answer as 0V.

If you place a ground here ⟶ the equation @ a (KCL) —

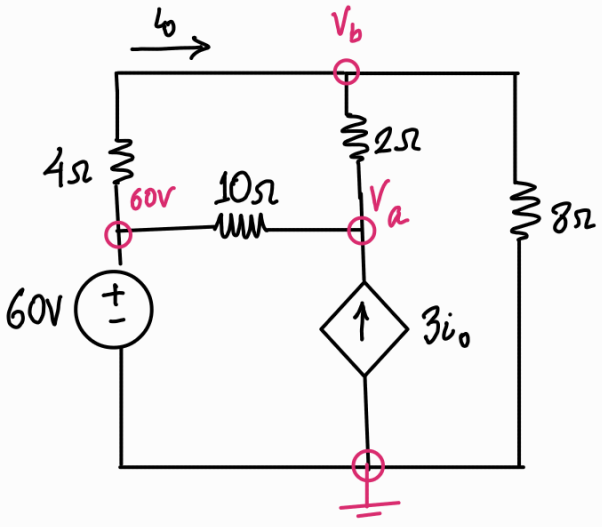
$$\frac{V_a - 20}{10} + \frac{V_a - 40}{5} - 5 = 0$$

$$\Rightarrow V_a - 20 + 2V_a - 80 = 50$$

$$\Rightarrow 3V_a = 150$$

$$\Rightarrow V_a = 50$$

Problem 5



Find i_o using nodal analysis. Determine the current supplied by the 60V source.

KCL @ a —

$$-3i_o + \frac{V_a - 60}{10} + \frac{V_a - V_b}{2} = 0$$

$$\Rightarrow V_a - 60 + 5V_a - 5V_b - 30i_o = 0$$

$$\Rightarrow 6V_a - 5V_b - 60 - 30i_o = 0 \quad \text{——} \quad (i)$$

From the ckt —

$$i_o = \frac{60 - V_b}{4} = 15 - \frac{V_b}{4}$$

$$\therefore (i) \Rightarrow 6V_a - 5V_b - 450 - 60 + \frac{15V_b}{2} = 0$$

$$\Rightarrow 12V_a + 5V_b = 1020 \quad \text{——} \quad (iii)$$

KCL @ b —

$$\frac{V_b - 60}{4} + \frac{V_b - V_a}{2} + \frac{V_b}{8} = 0$$

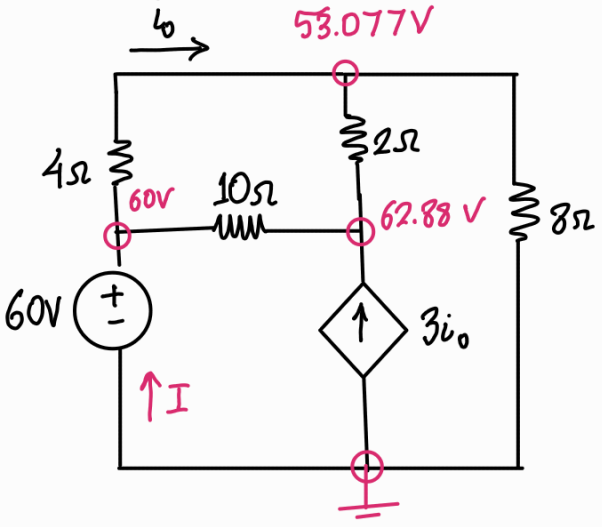
$$\Rightarrow 2V_b - 120 + 4V_b - 4V_a + V_b = 0$$

$$\Rightarrow 4V_a - 7V_b = -120 \quad \text{——} \quad (ii)$$

Solving (ii) & (iii) —

$$V_a = 62.88\,V \quad \& \quad V_b = 53.077\,V$$

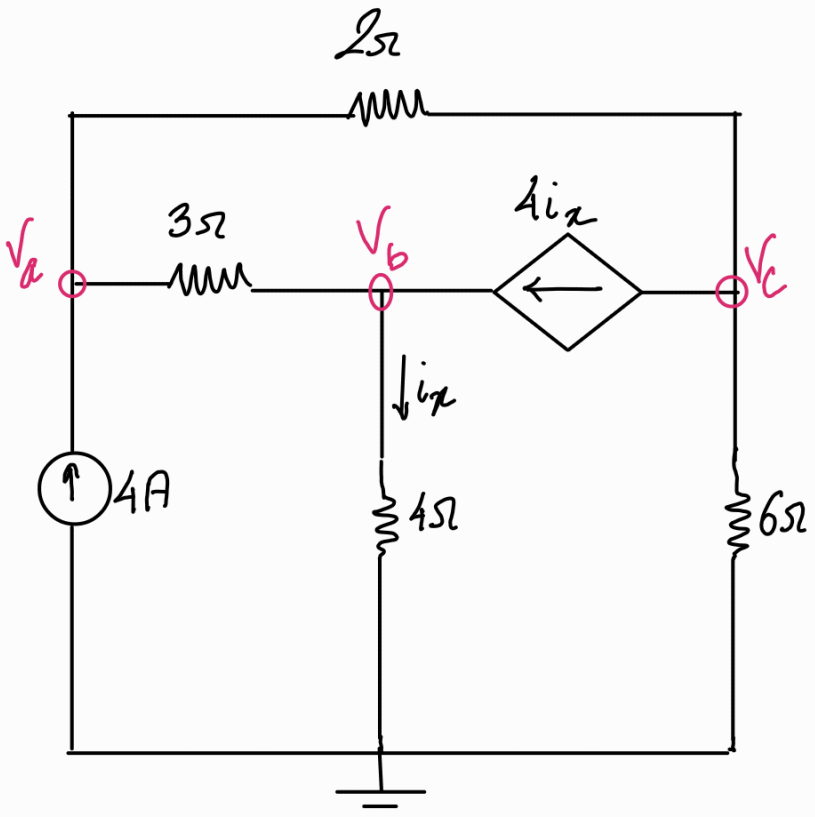
$$\therefore i_o = \frac{60 - 53.077}{4}\,A = 1.731\,A$$



$$I = i_o + \frac{60 - 62.88}{10} = 1.731 - 0.288$$

$$= 1.443\,A$$ (Wrong answer in the slide)

Problem 6



Find the node voltages

KCL @ a —

$$-4 + \frac{V_a - V_b}{3} + \frac{V_a - V_c}{2} = 0$$

$$\Rightarrow 2V_a - 2V_b + 3V_a - 3V_c = 24$$

$$\Rightarrow 5V_a - 2V_b - 3V_c = 24 \quad \text{— (i)}$$

KCL @ b —

$$\frac{V_b - V_a}{3} + \frac{V_b}{4} - 4i_x = 0$$

$$\Rightarrow 4V_b - 4V_a + 3V_b - 12V_b = 0 \quad [\text{From (ii)}]$$

$$\Rightarrow 4V_a + 5V_b = 0 \quad \text{— (iii)}$$

KCL @ c —

$$\frac{V_c - V_a}{2} + 4i_x + \frac{V_c}{6} = 0$$

$$\Rightarrow 3V_c - 3V_a + 6V_b + V_c = 0 \quad [\text{From (ii)}]$$

$$\Rightarrow 3V_a - 6V_b - 4V_c = 0 \quad \text{— (iv)}$$

$$i_x = \frac{V_b}{4} \quad \text{— (ii)}$$

Solving (i), (iii) & (iv) —

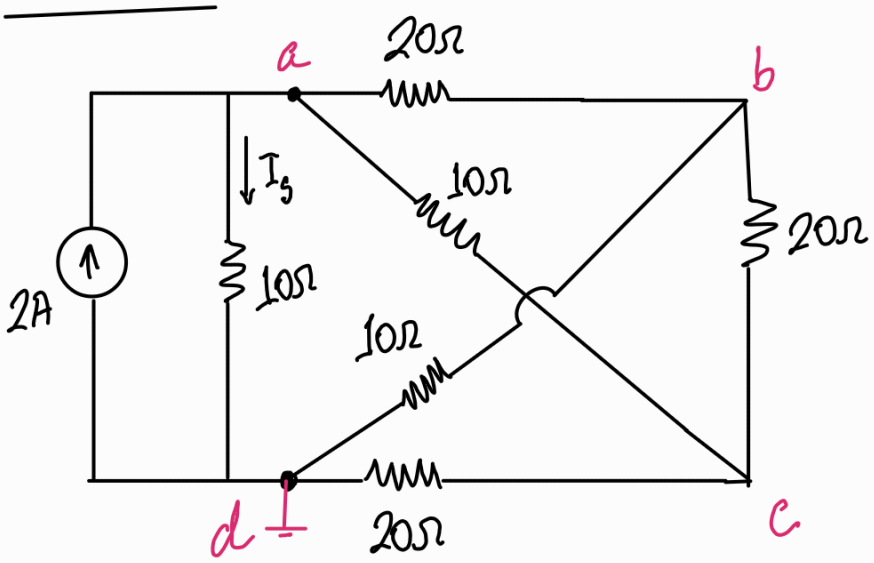
$V_a = 32\,V$

$V_b = -25.6\,V$

$V_c = 62.4\,V$

and $V_{ref} = 0\,V$

Problem 7



Find I_s using nodal analysis

KCL @ a —

$$-2+\frac{V_a}{10}+\frac{V_a-V_c}{10}+\frac{V_a-V_b}{20}=0$$

$$\Rightarrow 2V_a+2V_a-2V_c+V_a-V_b=40$$

$$\Rightarrow 5V_a-V_b-2V_c=40 \quad \text{— (i)}$$

KCL @ b —

$$\frac{V_b-V_a}{20}+\frac{V_b}{10}+\frac{V_b-V_c}{20}=0$$

$$\Rightarrow V_b-V_a+2V_b+V_b-V_c=0$$

$$\Rightarrow -V_a+4V_b-V_c=0 \quad \text{— (ii)}$$

KCL @ c —

$$\frac{V_c-V_a}{10}+\frac{V_c-V_b}{20}+\frac{V_c}{20}=0$$

$$\Rightarrow 2V_c-2V_a+V_c-V_b+V_c=0$$

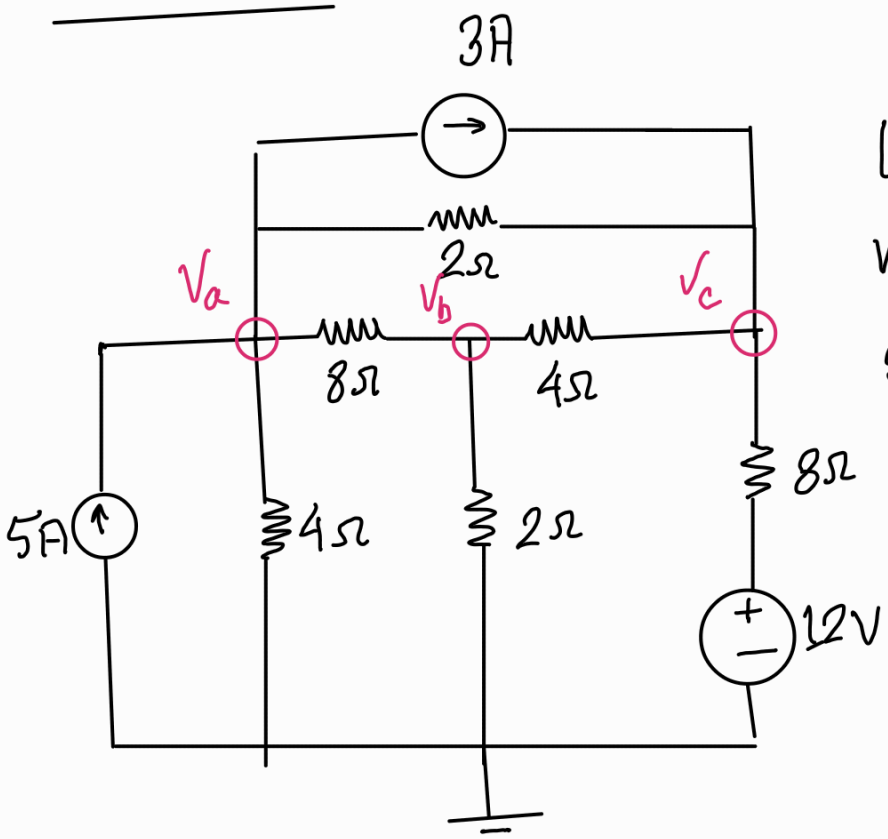
$$\Rightarrow -2V_a-V_b+4V_c=0 \quad \text{— (iii)}$$

Solving (i), (ii) & (iii) —

$V_a = 11.76\ V$ $V_b = 4.71\ V$ $V_c = 7.06\ V$

$$\therefore I_s=\frac{V_a}{10}=1.18\ A$$

Problem 8



Use nodal analysis to determine voltage across the 3A current source. Also find the power of the source. Is it absorbing or supplying?

KCL @ a —

$$-5 + \frac{V_a}{4} + \frac{V_a - V_b}{8} + \frac{V_a - V_c}{2} + 3 = 0$$

$$\Rightarrow 2V_a + V_a - V_b + 4V_a - 4V_c = 16$$

$$\Rightarrow 7V_a - V_b - 4V_c = 16 \quad \text{(i)}$$

KCL @ b —

$$\frac{V_b - V_a}{8} + \frac{V_b}{2} + \frac{V_b - V_c}{4} = 0$$

$$\Rightarrow V_b - V_a + 4V_b + 2V_b - 2V_c = 0$$

$$\Rightarrow -V_a + 7V_b - 2V_c = 0 \quad \text{(ii)}$$

KCL @ c —

$$-3 + \frac{V_c - V_a}{2} + \frac{V_c - V_b}{4} + \frac{V_c - 12}{8} = 0$$

$$\Rightarrow 4V_c - 4V_a + 2V_c - 2V_b + V_c - 12 = 24$$

$$\Rightarrow -4V_a - 2V_b + 7V_c = 36 \quad \text{— (iii)}$$

Solving (i), (ii) & (iii) —

$V_a = 10\,V$

$V_b = 4.933\,V$

$V_c = 12.267\,V$

Here, $V_{3A} = \pm(V_a - V_c)$ ← ± because no polarity specified

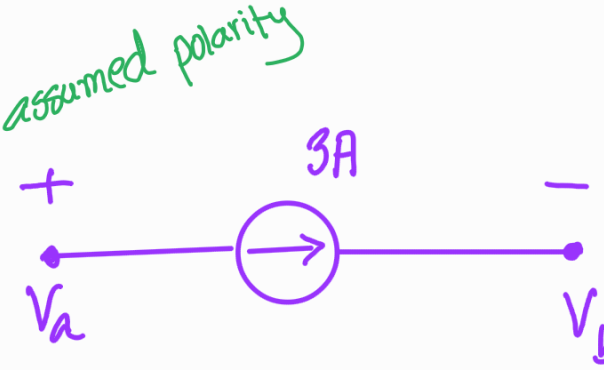
$= \pm 2.267\,V$

Here $P_{3A} = \pm V_{3A} \times 3$

$= +(V_a - V_c) \times 3$ ← Current entering +ve polarity

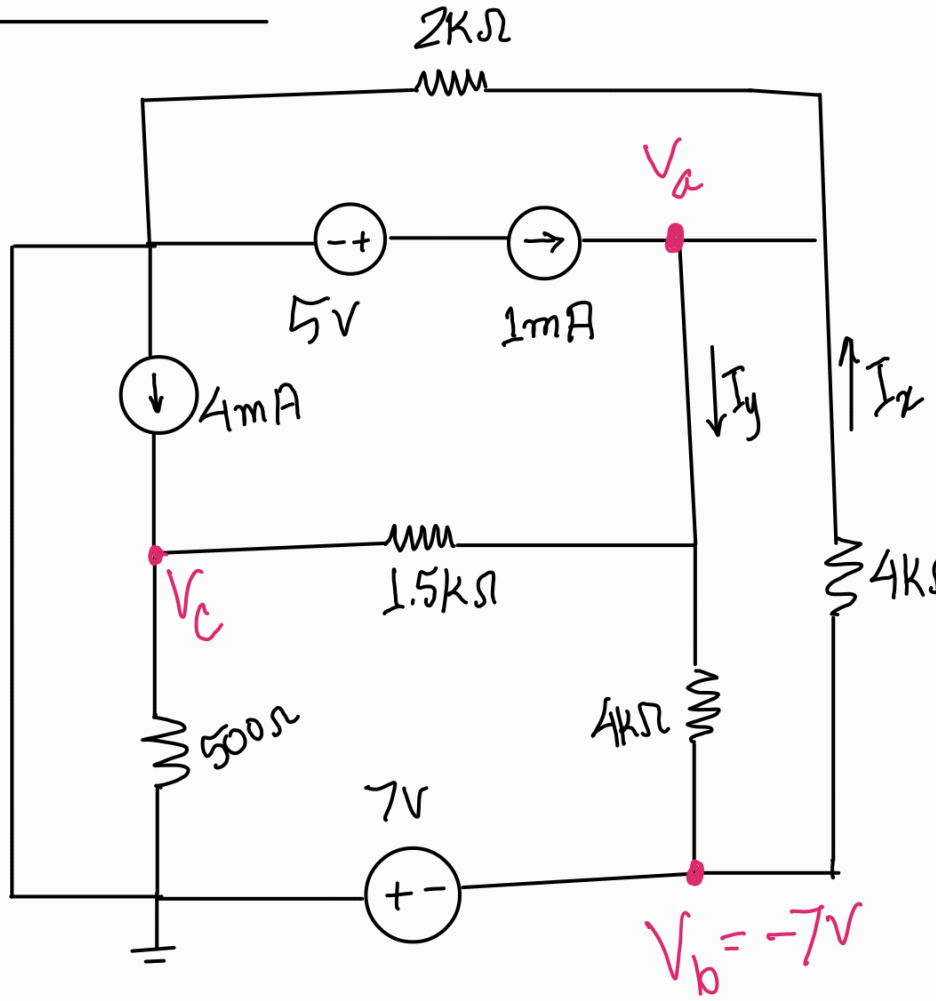
$= -2.267 \times 3$

$= -6.8\,W$ [Supplying]



You can consider opposite polarity and still reach the same result.

Problem 9



Find I_x & I_y

KCL @ a —

$$-1 + \frac{V_a}{2} + \frac{V_a+7}{4} \times 2 + \frac{V_a - V_c}{1.5} = 0$$

$$\Rightarrow 3V_a + 3V_a + 21 + 4V_a - 4V_c = 6$$

$$\Rightarrow 10V_a - 4V_c = -15 \quad \text{— (i)}$$

KCL @ c —

$$-4 + \frac{V_c}{0.5} + \frac{V_c - V_a}{1.5} = 0$$

$$\Rightarrow -12 + 6V_c + 2V_c - 2V_a = 0$$

$$\Rightarrow -2V_a + 8V_c = 12 \quad \text{— (ii)}$$

Solving (i) and (ii) —

$V_a = -1V$

$V_c = 1.25V$

$$\therefore I_x = \frac{-7 - V_a}{4}$$

$$= \frac{-7+1}{4}$$

$$= -1.5 \text{ mA}$$

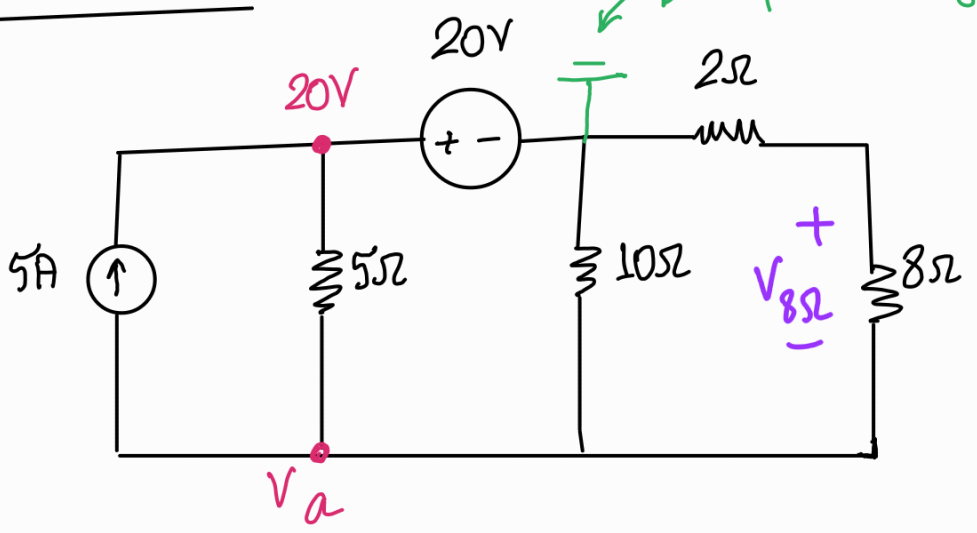
KCL @ a —

$$I_y = 1 - 1.5 + \frac{-V_a}{2}$$

$$= 0 \text{ mA}$$

Note – make sure units are consistent
If resistances are in kΩ and voltages are in V, current will be in mA

Problem 10



Find voltage across the 8Ω resistor

KCL @ a ——

$$5 + \frac{V_a - 20}{5} + \frac{V_a}{10} + \frac{V_a}{10} = 0$$

$$\Rightarrow \frac{2V_a - 20}{5} = -5$$

$$\Rightarrow 2V_a = -25 + 20$$

$$\Rightarrow V_a = -2.5\,V$$

Applying voltage divider—

$$V_{8\Omega} = \frac{0 - V_a}{10} \times 8$$

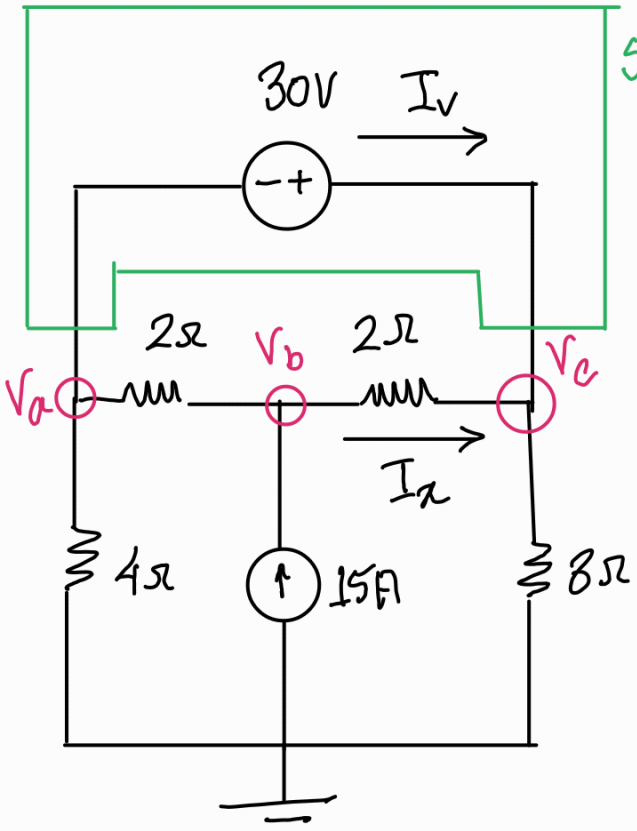
$$= \frac{2.5}{5} \times 4$$

$$= 2V$$

but since polarity isn't mentioned,

$$V_{8\Omega} = \pm 2V$$

Problem 11



Find I_x and I_v using nodal analysis.

@ Supernode 1 —

$$V_a - V_b = -30 \quad \text{——} (i)$$

& $$\frac{V_a}{4} + \frac{V_a - V_b}{2} + \frac{V_c - V_b}{2} + \frac{V_c}{8} = 0$$

$$\Rightarrow 2V_a + 4V_a - 4V_b + 4V_c - 4V_b + V_c = 0$$

$$\Rightarrow 6V_a - 8V_b + 5V_c = 0 \quad \text{——} (ii)$$

KCL @ b —

$$\frac{V_b - V_a}{2} + \frac{V_b - V_c}{2} = 15$$

$$\Rightarrow -V_a + 2V_b - V_c = 30 \quad \text{——} (iii)$$

Solving (i), (ii), (iii) —

$V_a = 30V$

$V_b = 60V$

$V_c = 60V$

$$I_x = \frac{V_b - V_c}{2}$$

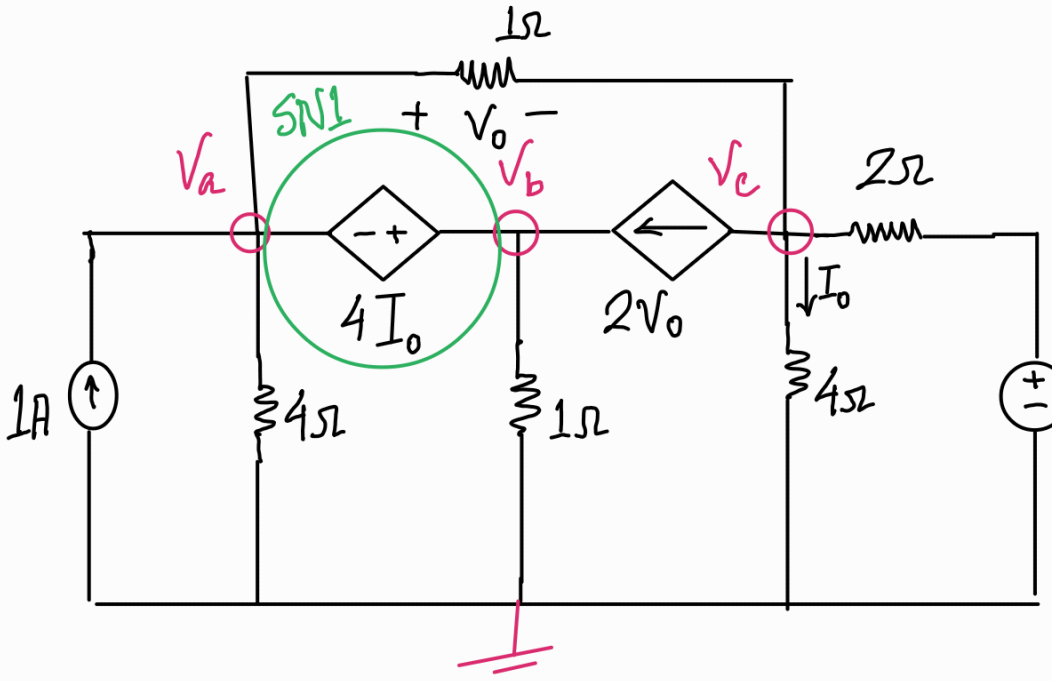
$$= 0A$$

@ C —

$$I_v + I_x = \frac{V_c}{8}$$

$$\Rightarrow I_v = \frac{60}{8} = 7.5A$$

Problem 12



Use nodal analysis to find current through the dependent voltage source

Voltage relation @ Supernode 1 (SN1) —

$$V_a - V_b = -4I_o$$

$$\Rightarrow V_a - V_b = -V_c \quad [\text{From } (1)]$$

$$\Rightarrow V_a - V_b + V_c = 0 \quad \text{——} (i)$$

$$I_o = \frac{V_c}{4} \quad \text{——} (1)$$

$$V_o = V_a - V_c \quad \text{——} (2)$$

KCL @ SN1 —

$$-1 + \frac{V_a - V_c}{1} + \frac{V_a}{4} - 2V_o + \frac{V_b}{1} = 0$$

$$\Rightarrow -4 + 4V_a - 4V_c + V_a - 8V_o + 4V_b = 0$$

$$\Rightarrow 5V_a + 4V_b - 4V_c - 8V_o = 4$$

$$\Rightarrow 5V_a + 4V_b - 4V_c - 8V_a + 8V_c = 4 \quad [\text{From } (2)]$$

$\Rightarrow -3V_a + 4V_b + 4V_c = 4$ —— (ii)

KCL @ C —

$$2V_o + \frac{V_c - V_a}{1} + \frac{V_c}{4} + \frac{V_c - 10}{2} = 0$$

$\Rightarrow 8V_o + 4V_c - 4V_a + V_c + 2V_c - 20 = 0$

$\Rightarrow -4V_a + 7V_c + 8V_o = 20$

$\Rightarrow -4V_a + 7V_c + 8V_a - 8V_b = 20$

$\Rightarrow 4V_a - V_c = 20$ —— (iii)

Solving (i), (ii) & (iii) ——

$V_a = 4.97\,V$

$V_b = 4.85\,V$

$V_c = -0.12\,V$

Current through $4I_o$ — KCL @ b —

$$2V_o - V_b = \pm I_{4I_o}$$

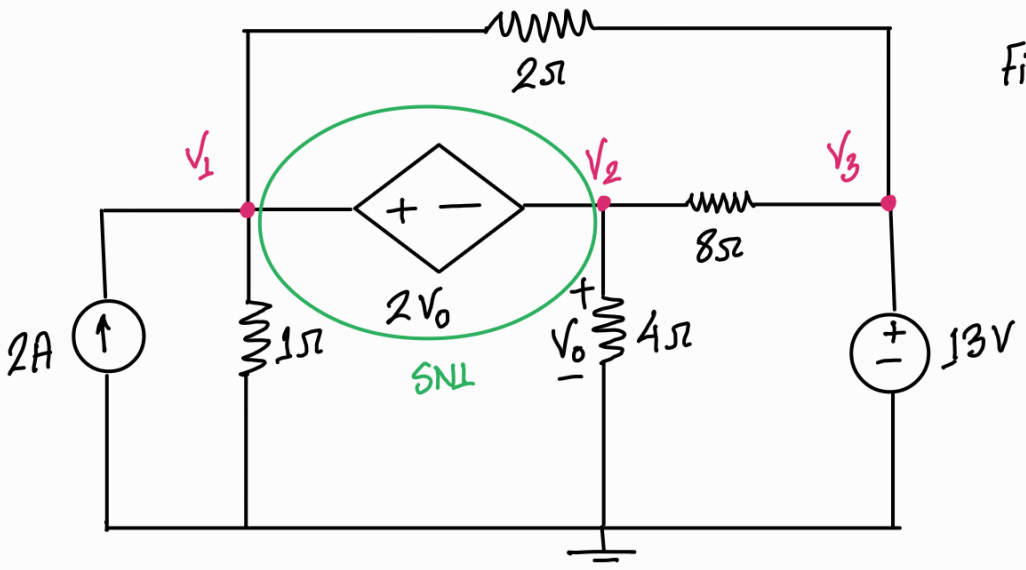
$$\Rightarrow \pm I_{4I_o} = 2V_a - 2V_c - V_b$$

$$= 5.33\,A$$

$$\Rightarrow I_{4I_o} = \pm 5.33\,A$$

direction not specified

Problem 13



Find V_1, V_2 & V_3

$V_0 = V_2$ —— (1)

$V_3 = 13$ —— (2)

Voltage relation at SNI —

$$V_1 - V_2 = 2V_0$$

$$\Rightarrow V_1 - 3V_2 = 0 \quad \text{—— (i) [From (1)]}$$

KCL @ SNI —

$$-2 + \frac{V_1}{1} + \frac{V_1 - 13}{2} + \frac{V_2}{4} + \frac{V_2 - 13}{8} = 0$$

$$\Rightarrow 8V_1 + 4V_1 - 52 + 2V_2 + V_2 - 13 = 16$$

$$\Rightarrow 12V_1 + 3V_2 = 81 \quad \text{—— (ii)}$$

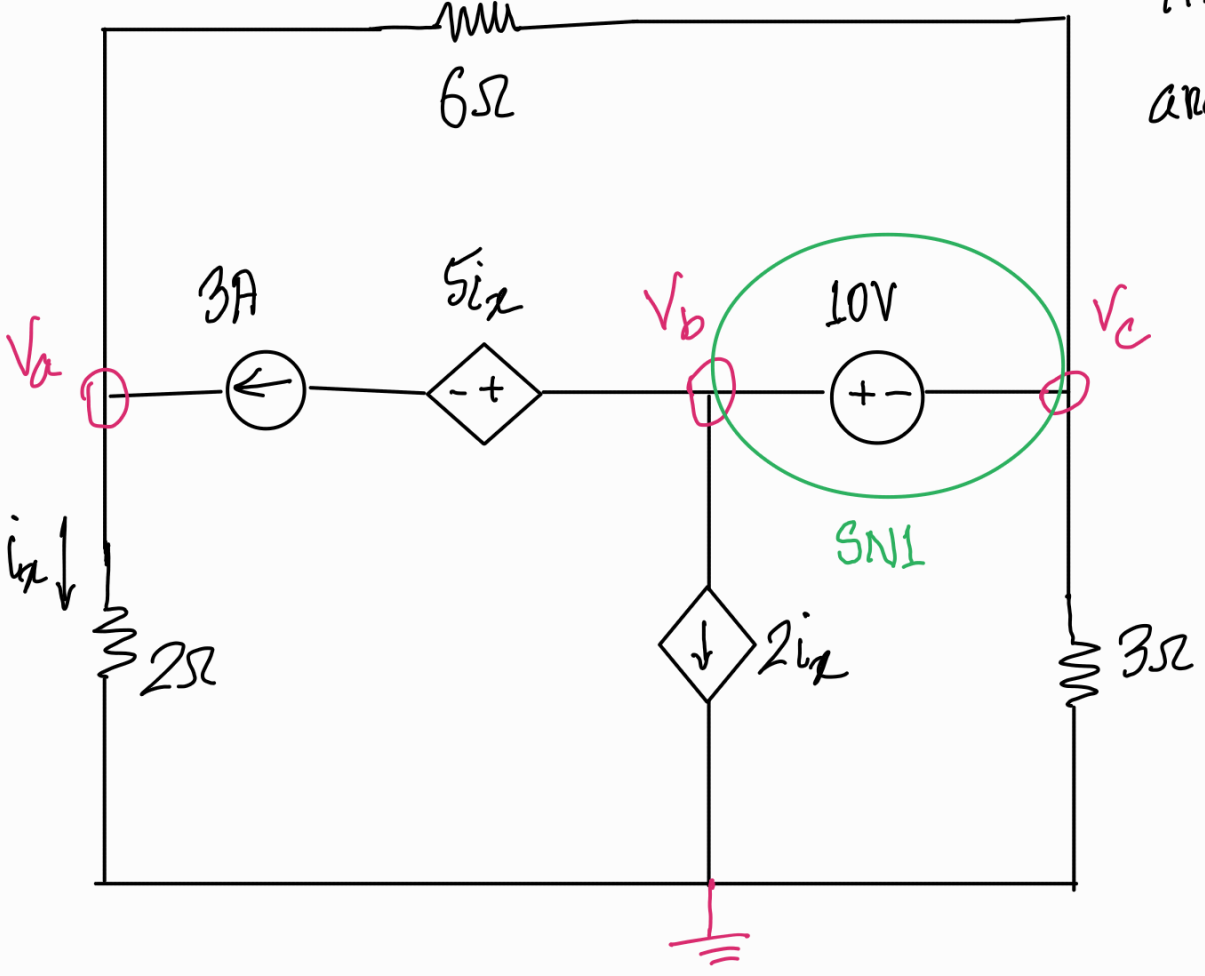
Solving (i) & (ii) —

$V_1 = 6.23\,V$

$V_2 = 2.08\,V$

Problem 14

Find i_x, V_{2i_x}, and i_{20V}



KCL @ a ——

$$\frac{V_a}{2} - 3 + \frac{V_a - V_c}{6} = 0$$

$$\Rightarrow 3V_a - 18 + V_a - V_c = 0$$

$$\Rightarrow 4V_a - V_c = 18 \quad \text{——} (i)$$

$$i_x = \frac{V_a}{2} \quad \text{——} (1)$$

Voltage relation @ SN1 —

$V_b - V_c = 10$ ——— (ii)

KCL @ SN1 ———

$$3 + 2i_x + \frac{V_c}{3} + \frac{V_c - V_a}{6} = 0$$

$$\Rightarrow 18 + 12i_x + 2V_c + V_c - V_a = 0$$

$$\Rightarrow -V_a + 3V_c + 12i_x = -18$$

$$\Rightarrow 5V_a + 3V_c = -18 \quad [\text{From } (1)] \text{ ——— (iii)}$$

Solving (i), (ii), (iii) —

$V_a = 2.12\ V$

$V_b = 0.47\ V$

$V_c = -9.53\ V$

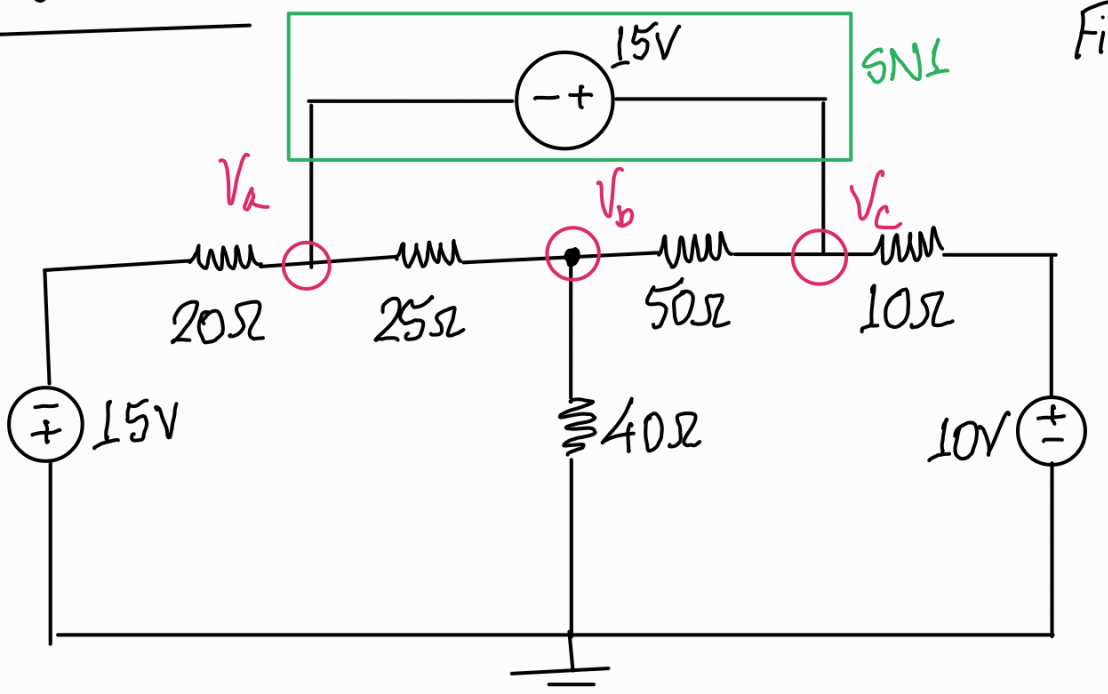
$$\therefore i_x = \frac{V_a}{2} = 1.06\ A$$

$V_{2i_x} = \pm V_b = \pm 0.47\ V$ ← no assigned direction/polarity

$i_{10V} = \pm (3 + 2i_x) = \pm 5.12\ A$

Problem 15

Find I_{15V}



KCL @ SNI —

$$\frac{V_a+15}{20} + \frac{V_a-V_b}{25} + \frac{V_c-V_b}{50} + \frac{V_c-10}{10} = 0$$

$$\Rightarrow 5V_a + 75 + 4V_a - 4V_b + 2V_c - 2V_b + 10V_c - 100 = 0$$

$$\Rightarrow 9V_a - 6V_b + 12V_c = 25 \quad \text{——} (i)$$

Voltage relation @ SNI —

$$-V_a + V_c = 15 \quad \text{——} (ii)$$

KCL @ b —

$$\frac{V_b-V_a}{25} + \frac{V_b}{40} + \frac{V_b-V_c}{50} = 0$$

$$\Rightarrow 8V_b - 8V_a + 5V_b + 4V_b - 4V_c = 0$$

$$\Rightarrow -8V_a + 17V_b - 4V_c = 0 \quad \text{——} \quad \text{(iii)}$$

Solving (i), (ii) (iii) —

$$V_a = -7.982\ V$$

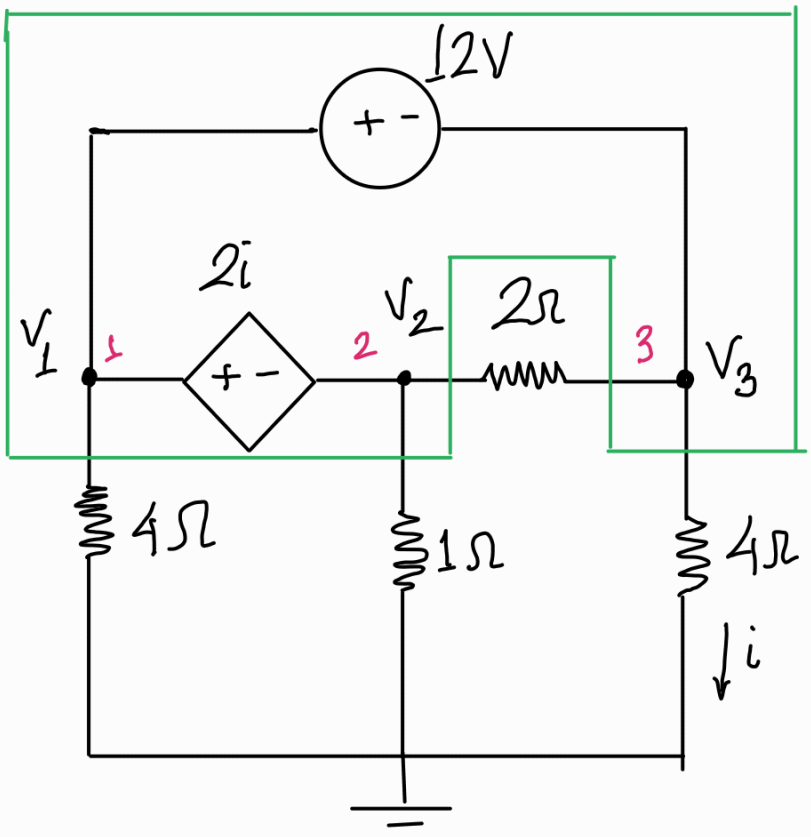
$$V_b = -2.105\ V$$

$$V_c = 7.017\ V$$

$$\therefore I_{15V} = \pm\left(\frac{V_c - V_b}{50} + \frac{V_c - 10}{10}\right)$$

$$= \pm 0.116A$$

Problem 16



SN1

Find v_1, v_2 & v_3. Find current supplied by 12V & 2i source.

$$i = \frac{V_3}{4} \quad \text{——} (1)$$

Voltage relationships at SN1 —

$$V_1 - V_2 = 2i$$

$$\Rightarrow V_1 - V_2 = 2 \cdot \frac{V_3}{4} \quad [\text{From } (1)]$$

$$\Rightarrow 2V_1 - 2V_2 - V_3 = 0 \quad \text{——} (i)$$

&

$$V_1 - V_3 = 12 \quad \text{——} (ii)$$

KCL @ SN1 —

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \quad [2\Omega \text{ is shorted}]$$

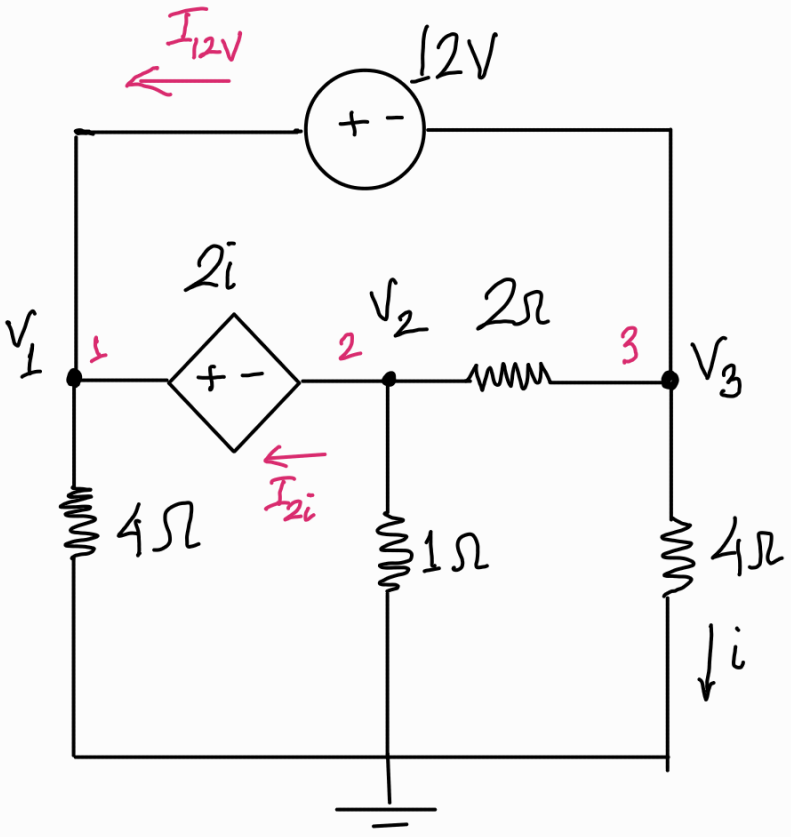
$$\Rightarrow V_1 + 4V_2 + V_3 = 0 \quad \text{——} (iii)$$

Solving (i), (ii) & (iii) —

$V_1 = -3V$

$V_2 = 4.5V$

$V_3 = -15V$



Here, KCL @ 3 —

$$I_{12V} = \frac{V_2 - V_3}{2} + \frac{0 - V_3}{4}$$

$$= 9.75 + 3.75$$

$$= 13.5\,A$$ (Wrong in slide)

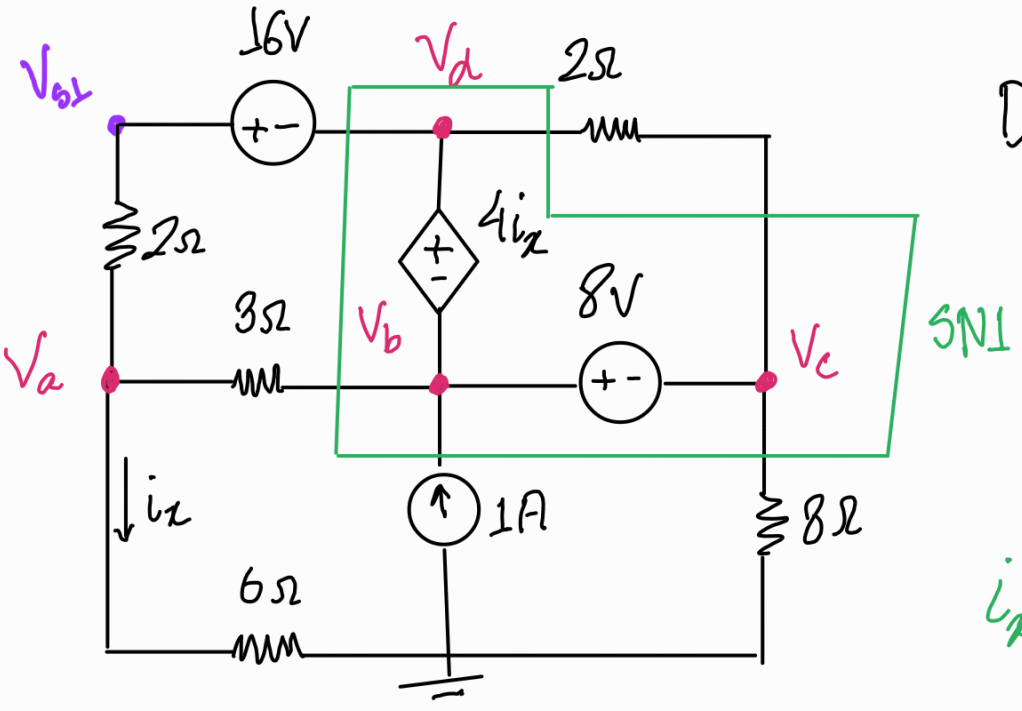
KCL @ 1 —

$$I_{2i} + I_{12V} = \frac{V_1}{4}$$

$$\Rightarrow I_{2i} = \frac{-3}{4} - 13.5$$

$$= -14.25\,A$$

Problem 17



Determine node voltages.

$$i_x = \frac{V_a}{6} \quad \text{—— (1)}$$

Voltage relationships at SN1—

$$V_d - V_b = 4i_x$$

$$\Rightarrow V_d - V_b = \frac{2V_a}{3} \quad [\text{From (1)}]$$

$$\Rightarrow 2V_a + 3V_b - 3V_d = 0 \quad \text{—— (i)}$$

&

$$V_b - V_c = 8 \quad \text{—— (ii)}$$

KCL @ SN1—

$$\frac{V_d + 16 - V_a}{2} + \frac{V_b - V_a}{3} - 1 + \frac{V_c}{8} = 0 \quad [2\Omega \text{ shorted}]$$

$$\Rightarrow 12V_d - 12V_a + 192 + 8V_b - 8V_a - 24 + 3V_c = 0$$

$$\Rightarrow -20V_a + 8V_b + 3V_c + 12V_d = -168 \quad \text{---(iii)}$$

KCL @ a ——

$$\frac{V_a - V_d - 16}{2} + \frac{V_a - V_b}{3} + \frac{V_a}{6} = 0$$

$$\Rightarrow 3V_a - 3V_d - 48 + 2V_a - 2V_b + V_a = 0$$

$$\Rightarrow 6V_a - 2V_b - 3V_d = 48 \quad \text{---(iv)}$$

Solving (i), (ii), (iii) & (iv) ——

$V_a = 12V$

$V_b = 0V$

$V_c = -8V$

$V_d = 8V$

$\therefore V_{s1} = V_d + 16 = 24V$

If you do not have an EX calculator, replace 1 variable in all equations.

$2V_a + 3V_b - 3V_d = 0$

$V_b - V_c = 8 \Rightarrow V_b = 8 + V_c$

$-20V_a + 8V_b + 3V_c + 12V_d = -168$

$6V_a - 2V_b - 3V_d = 48$

$2V_a + 3V_c - 3V_d = -24$

$-20V_a + 11V_c + 12V_d = -232$

$6V_a - 2V_c - 3V_d = 64$

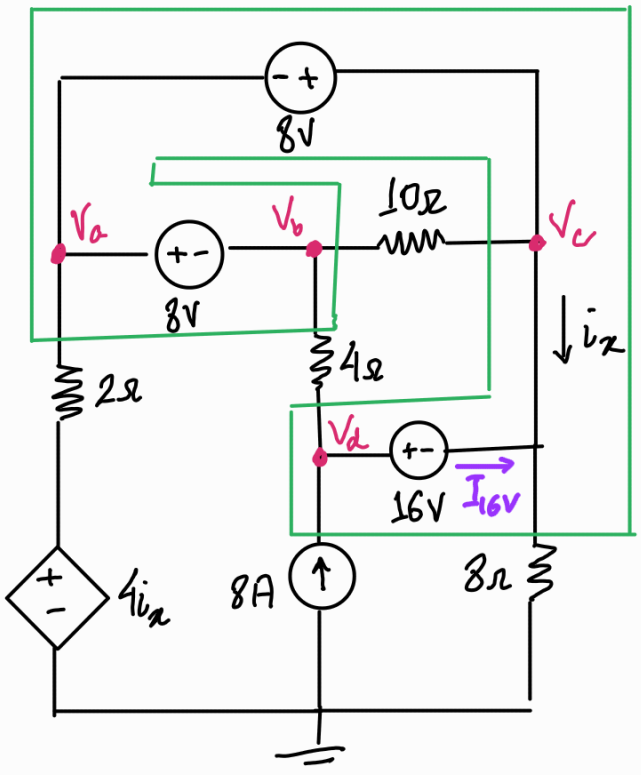
$V_a = 12V$

$V_c = -8V$

$V_d = 8V$

$\therefore V_b = 0V$

Problem 18



Determine i_x

Voltage relations @ SN1 —

$V_a - V_b = 8$ —— (i)

$-V_a + V_c = 8$ —— (ii)

$-V_c + V_d = 16$ —— (iii)

KCL @ SN1 —

$\frac{V_a - 4i_x}{2} - 8 + \frac{V_c}{8} = 0$ [4Ω & 10Ω shorted]

$\Rightarrow 4V_a - 16i_x + V_c = 64$

$\Rightarrow 4V_a - 2V_c + 4V_b - 4V_d + 128 + V_c = 64$ [From 3]

$\Rightarrow 4V_a + 4V_b - V_c - 4V_d = -64$ —— (iv)

KCL @ C —

$i_x + I_{16V} = \frac{V_c}{8}$ —— (1)

KCL @ d —

$8 + \frac{V_b - V_d}{4} = I_{16V}$ —— (2)

$\therefore$ (1) $\longrightarrow i_x = \frac{V_c}{8} - \frac{V_b - V_d}{4} - 8$ —— (3)

Solving (i), (ii), (iii), (iv) —

$V_a = 24\,V$

$V_b = 16\,V$

$V_c = 32\,V$

$V_d = 48\,V$

$$\therefore i_a = \frac{V_c}{8} - \frac{V_b - V_d}{4} - 8$$

$$= \frac{32}{8} - \frac{16-48}{4} - 8$$

$$= 4 - 4 + 12 - 8$$

$$= 4A$$