Lecture 9

CSE250 - Circuits and Electronics

THEVENIN'S AND NORTON'S THEOREM

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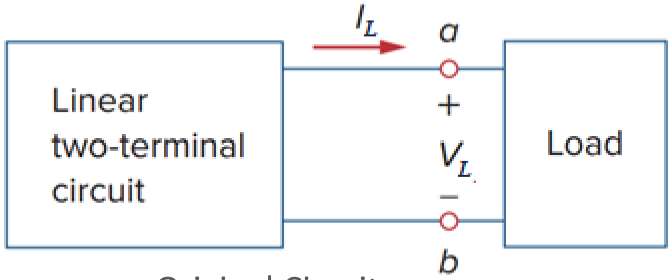
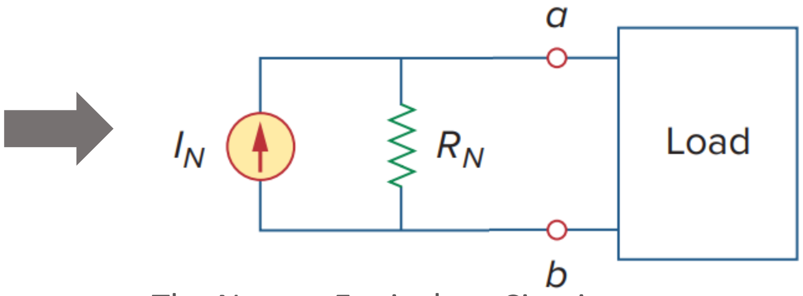
Norton's Theorem

- *Norton's theorem* states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and I_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



Original Circuit



The Norton Equivalent Circuit

- Two circuits are said to be equivalent if they have the same $I - V$ characteristics at their terminals.
- Let's find out what will make the two circuits equivalent!

I-V of Norton Equivalent

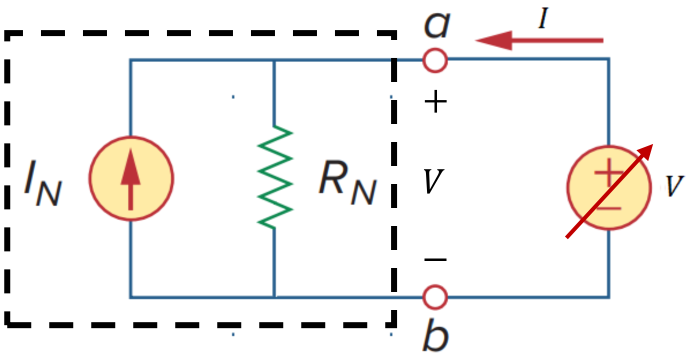
- We can derive the *$I-V$ characteristics of the Norton equivalent* in a similar way as we did in for Thevenin.
- The configuration is a current source (I_N) in series with a resistor (R_N). To determine the configuration's $I-V$ characteristics, if applying a voltage V gives rise to a current i_x through the resistor, we can write using KCL,

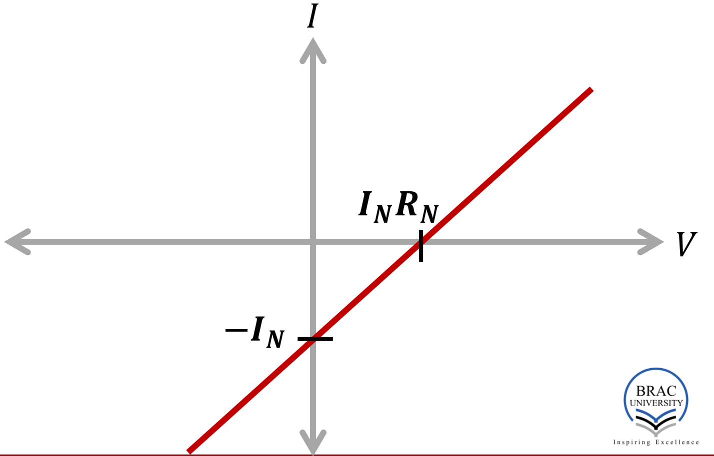
$$i_x = I_N + I$$

- So, voltage across the resistor can be written as,

$$V = i_x R_N = (I_N + I)R_N$$
$$\Rightarrow I = \frac{1}{R_N}V - I_N$$

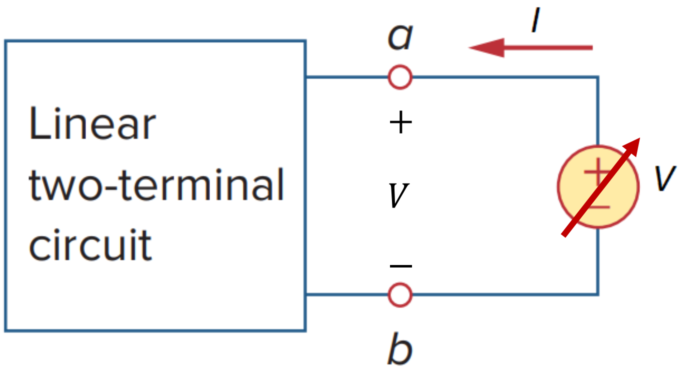
- The equation results in a linear I vs V plot that intersects the axes at $I_N R_N$ and $-I_N$





I-V of Actual Circuit

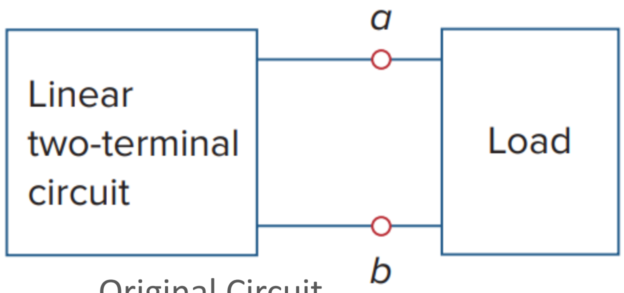
- The procedure to derive the *$I - V$ characteristics of the original circuit* is exactly the same as done in the Thevenin part. This is described here again.
- To theoretically derive exactly the relation between I and V it is required to know the actual circuitry. As the circuit is linear, the $I - V$ characteristic will be a straight line and the line can be drawn if minimum two points on the line are known.



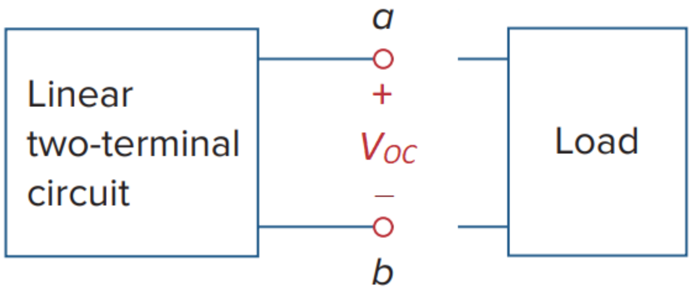
- The two points we can get are the intersecting points of x and y axis.
- To get the intersecting location on the voltage axis, current (I) at the terminals should be made equal to 0. That is, the terminals $a - b$ must be open circuited.
- Similarly, for the intersecting location on current axis, $V_{ab} = V = 0$. That is, the terminals $a - b$ must be shorted.

OC Voltage & SC Current

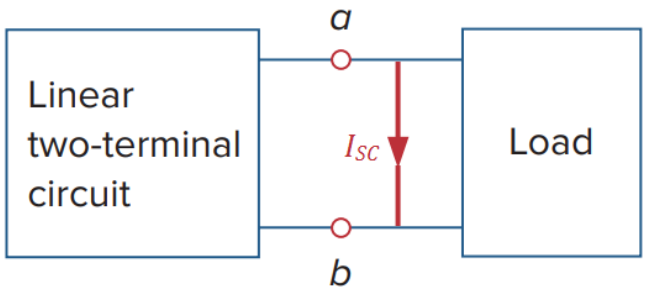
- Let's denote V_{oc} be the voltage at the open terminals upon disconnecting the load and I_{sc} be the current through the shorted terminals upon short circuiting the load.



Original Circuit



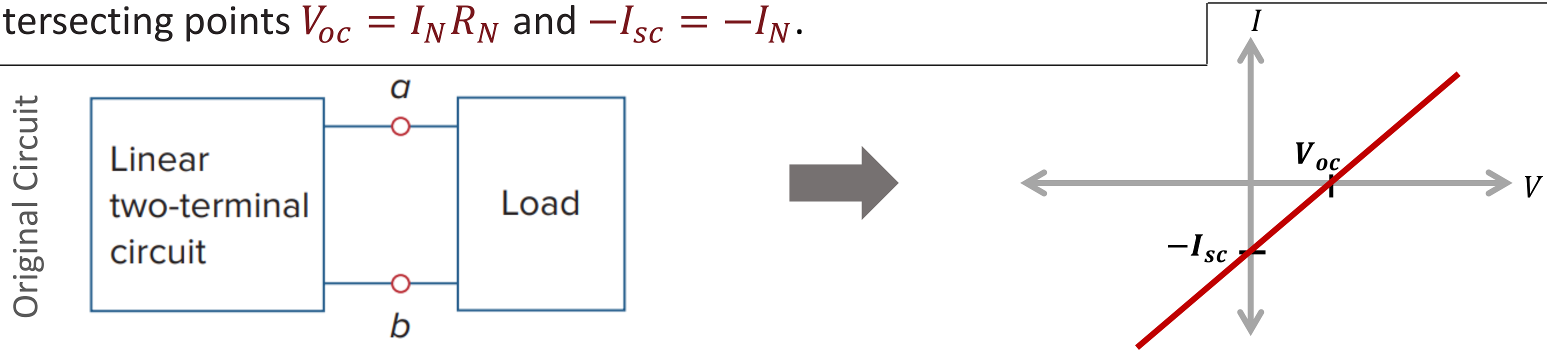
Open Circuited at the terminals

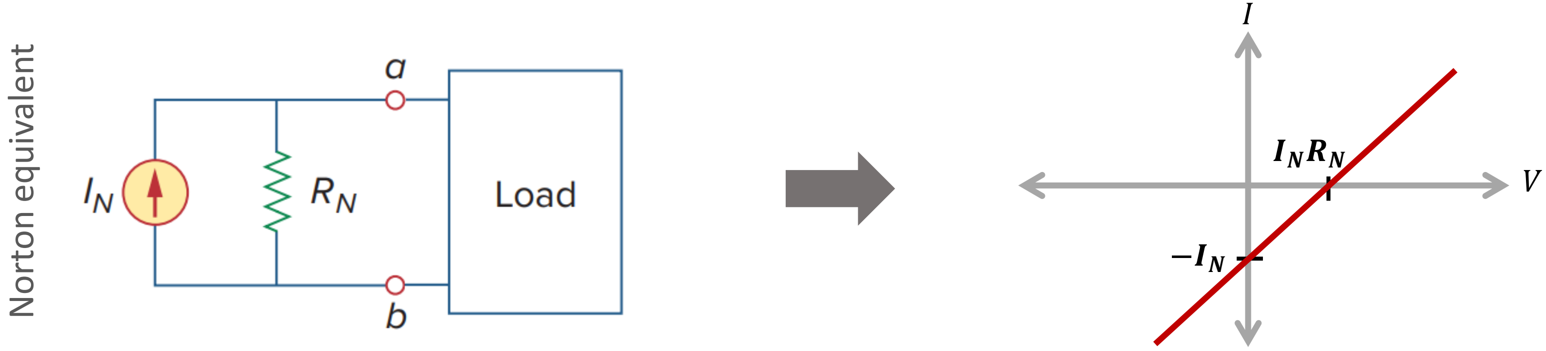


So, the $I - V$ characteristic should be the straight line passing through the points $(V_{oc}, 0)$ and $(0, -I_{sc})$. The reason for the negative sign is that I_{sc} is opposite to the current (I) plotted along the y-axis.

Circuit Equivalence

- The original circuit and the reduced Norton equivalent circuit will be equivalent to each other if the $I - V$ characteristics of the two are identical. They will indeed be identical if the intersecting points $V_{oc} = I_N R_N$ and $-I_{sc} = -I_N$.





How to determine R_N?

- Refer to the previous slides, Norton's conversion is valid if

  i. $V_{oc} = I_N R_N \ or \ I_N = \frac{V_{oc}}{R_N}$

  ii. $-I_N = -I_{sc} \ or \ \frac{V_{oc}}{R_N} = I_{sc}$

- For the linear $I - V$ characteristic, R_N is the inverse of the slope of the straight line passing through the points $(I_N R_N,\ 0)$ and $(0,\ -I_N)$. That is,
- $Slope = \frac{\Delta I}{\Delta V} = \frac{0-(-I_N)}{I_N R_N - 0} = \frac{1}{R_N}$
- Thus, R_N may be found from the open circuit voltage V_{oc} and the Norton current I_N.
- The undefined scenario that occurs when determining R_{Th} when V_{Th} is zero (see here) also occurs when determining R_N when $I_{sc} = 0$. In that situation, the *Universal Rule* used to derive R_{Th} applies exactly to R_N.

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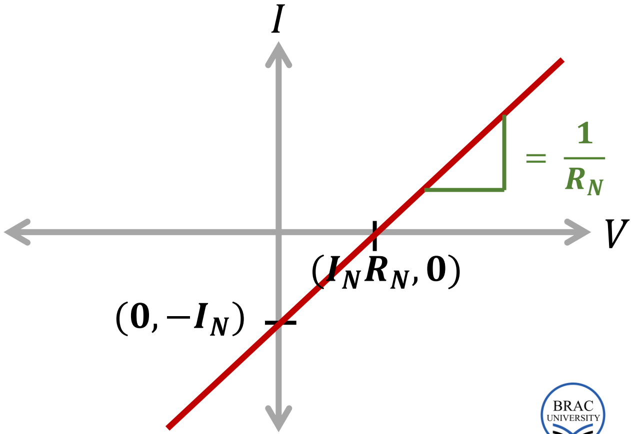
  ii. $-I_N = -I_{sc} \ or \ \frac{V_{oc}}{R_N} = I_{sc}$

- For the linear $I - V$ characteristic, R_N is the inverse of the slope of the straight line passing through the points $(I_N R_N,\ 0)$ and $(0,\ -I_N)$. That is,

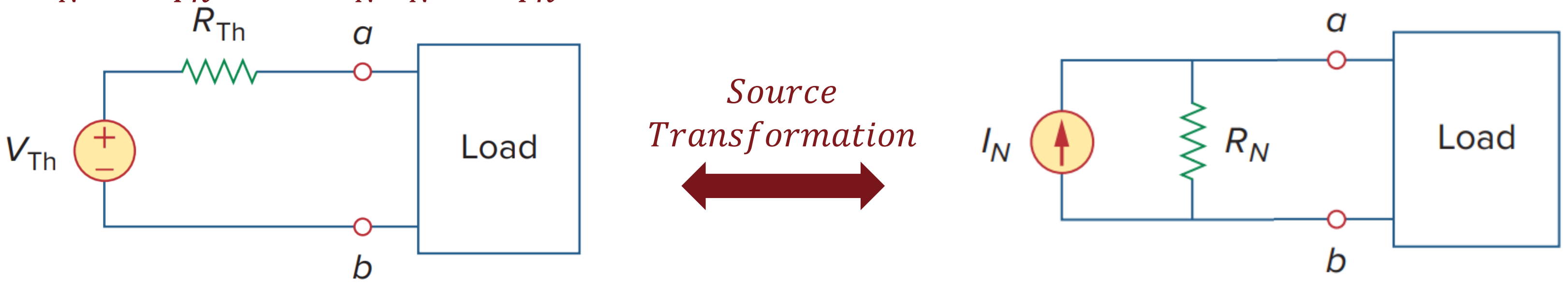
  $Slope = \frac{\Delta I}{\Delta V} = \frac{0-(-I_N)}{I_N R_N - 0} = \frac{1}{R_N}$

- Thus, R_N may be found from the open circuit voltage V_{oc} and the Norton current I_N.
- The undefined scenario that occurs when determining R_{Th} when V_{Th} is zero (see here) also occurs when determining R_N when $I_{sc} = 0$. In that situation, the *Universal Rule* used to derive R_{Th} applies exactly to R_N.



Thevenin ↔ Norton

- As you may have already noticed, Norton equivalent of a circuit can be derived from the Thevenin equivalent (or vice versa) of the same circuit by performing a source transformation.
- The requirement is that the two must have the same $I - V$ characteristics.
- From the conditions for which source transformation is valid (shown in slide 7 of Source Transformation) or by comparing the $I - V$ characteristics of the two, it can be seen that the conversion is valid if and only if,
- $R_N = R_{Th}$ & $I_N R_N = V_{Th}$



Methods in a nutshell

Method to determine I_N

Short the load terminals

Determine the current through the short circuit $(I_{sc} = I_N)$

Methods to determine R_N

Valid only if $I_N \neq 0$

Open the load terminals

Determine the voltage at the open terminals (V_{oc})

$R_N = \frac{V_{oc}}{I_N}$

Universal Rule

Kill all the independent sources

Is there any dependent source(s)?

No

Use series-parallel combinations of resistors to calculate $R_{eq} = R_N$

Yes

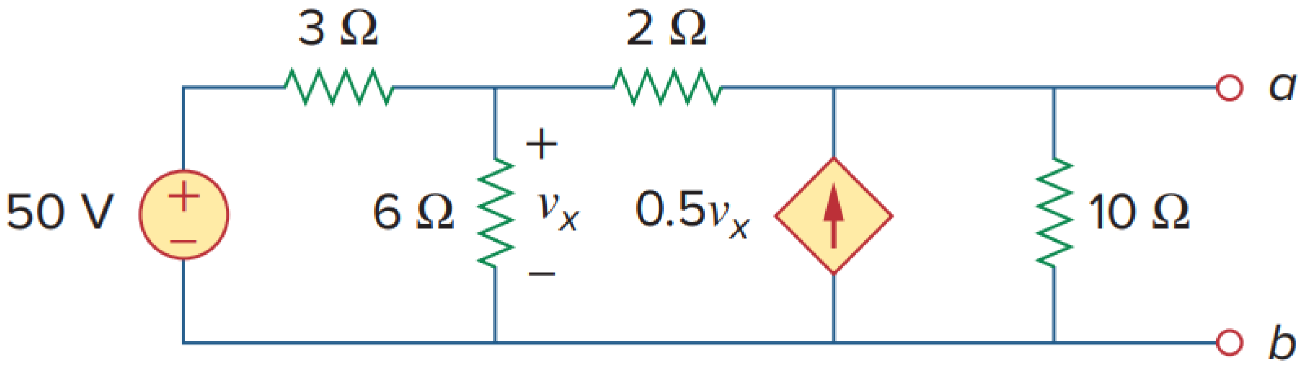
Add a dummy voltage or current source to the load terminals

Calculate the current (i_0) supplied or voltage (v_0) across the voltage or current source respectively.

Use Ohm's Law to calculate $R_N = \frac{v_0}{i_0}$

Example 4

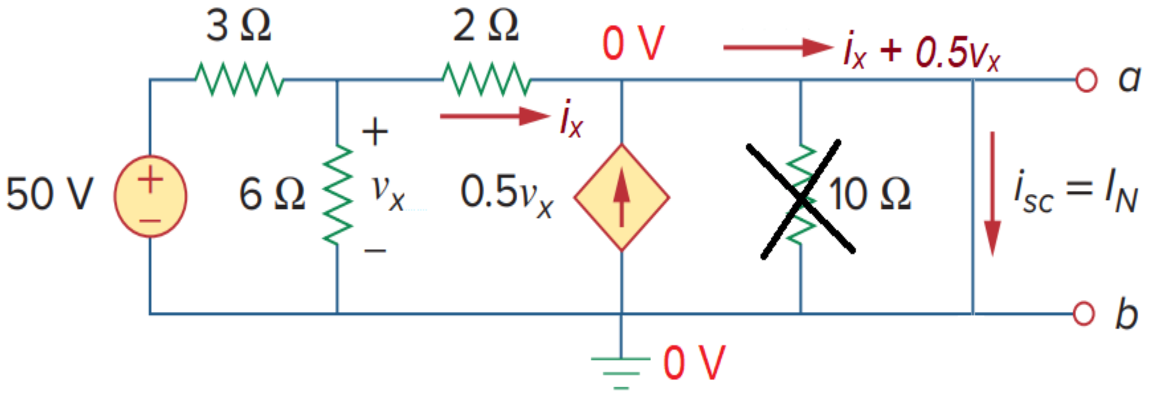
- Obtain the Norton equivalent circuit at terminals $a - b$.



Ans: $\boldsymbol{R_N = 10\ \Omega;\ I_N = 16.667\ A}$

** See solution in the next slide if necessary*

Example 4: finding I_N



The 1st step is to disconnect the load and short the terminals.

Upon short circuiting the terminals a-b, the $10\,\Omega$ is shorted out. The whole circuit to the left of the dependent source is shorted with respect to it. As a result, the $0.5v_x$ current supplied by the dependent source will only flow through the short circuit.

Let's use nodal analysis to solve for the current i_x going towards the short circuit though the 2 Ω resistor.

KCL at v_x

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x}{2} = 0$$

$$\Rightarrow \quad v_x = 16.667\ V$$

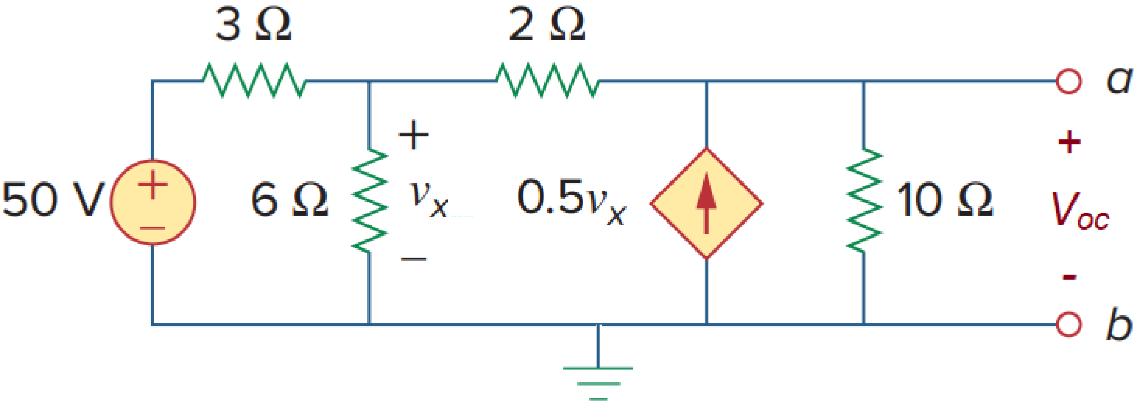
Now,

$$i_x = \frac{v_x - 0}{2} = 8.334\ A$$

So,

$$I_N = i_x + 0.5v_x = 16.667\ A$$

Example 4: finding R_N



R_N can be found by (i) determining V_{oc} and then using $R_N = \frac{V_{oc}}{I_N}$ (as $I_N \neq 0$) or (ii) first turning off all the independent sources and determining the R_{eq} at the terminals.

Let's employ the first method here.

Nodal analysis:

KCL at v_x,

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x - V_{oc}}{2} = 0$$

$$\Rightarrow \quad 6v_x - 3V_{oc} = 100 \;\; ----(i)$$

KCL at V_{oc},

$$\frac{V_{oc} - v_x}{2} + \frac{V_{oc}}{10} = 0.5v_x$$

$$\Rightarrow \quad 10v_x - \;\; 6V_{oc} = 0 \;\; ----(ii)$$

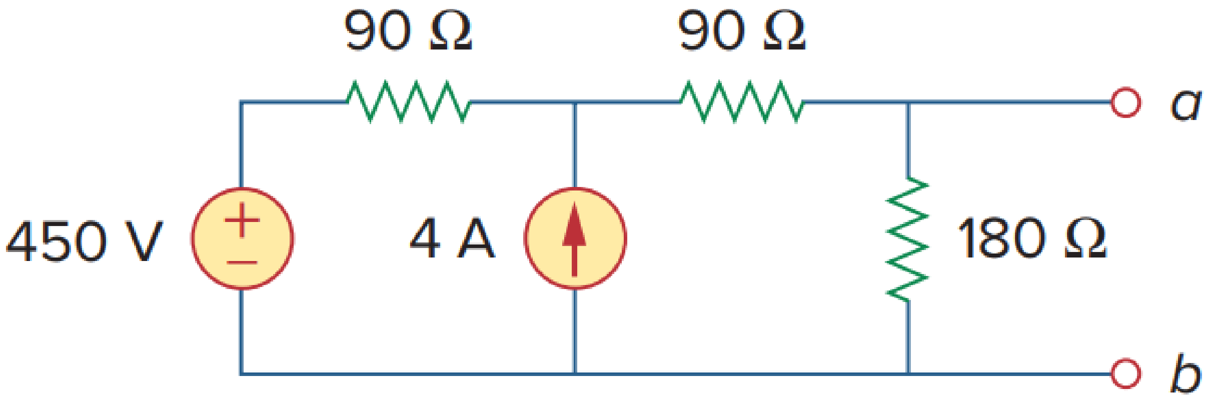
Solving (i) and (ii),

$$V_{oc} = 166.667\ V$$

$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{166.667}{16.667} = 10\ \Omega$$

Problem 11

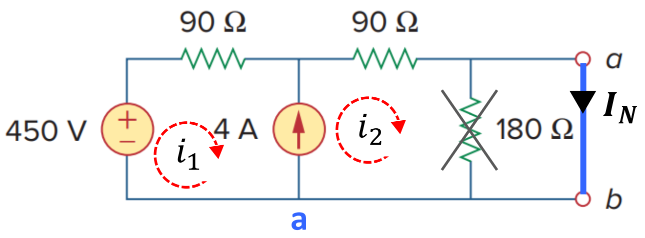
- Find the Norton equivalent circuit for the circuit at terminals $a - b$.



Ans: $I_N = 4.5\ A$; $R_N = 90\ \Omega$

Solution to Problem 11

Finding I_N



Let's use mesh analysis to find the I_N
From the circuit,

$$i_2 = I_N$$

Applying KVL at supermesh between 1 and 2,

$$-450 + 90i_1 + 90I_N = 0$$

$$\Rightarrow 90i_1 + 90I_N = 450 \;\text{...........}\; \text{(i)}$$

Applying KCL at node a,

$$i_1 + 4 = I_N$$

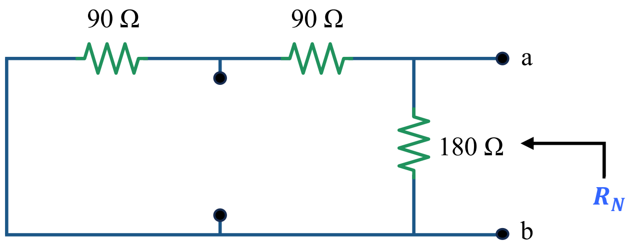
$$i_1 - I_N = -4 \;\text{...........}\; \text{(ii)}$$

Solving (i) and (ii),

$$\boxed{\begin{aligned} i_1 &= 0.5\,A \\ I_N &= 4.5\,A \end{aligned}}$$

Finding R_N

At first, let's deactivate all the Independent Sources. As there is no dependent sources, we simply use series-parallel combination to find the equivalent resistance seen from the load terminal.

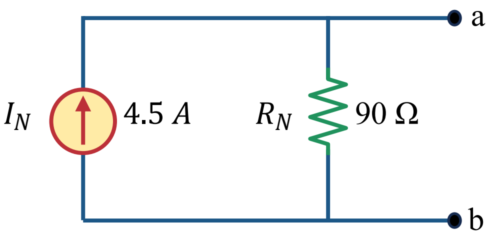


$$R_N = (90 + 90) \,||\, 180$$

$$\Rightarrow R_N = 180 \,||\, 180$$

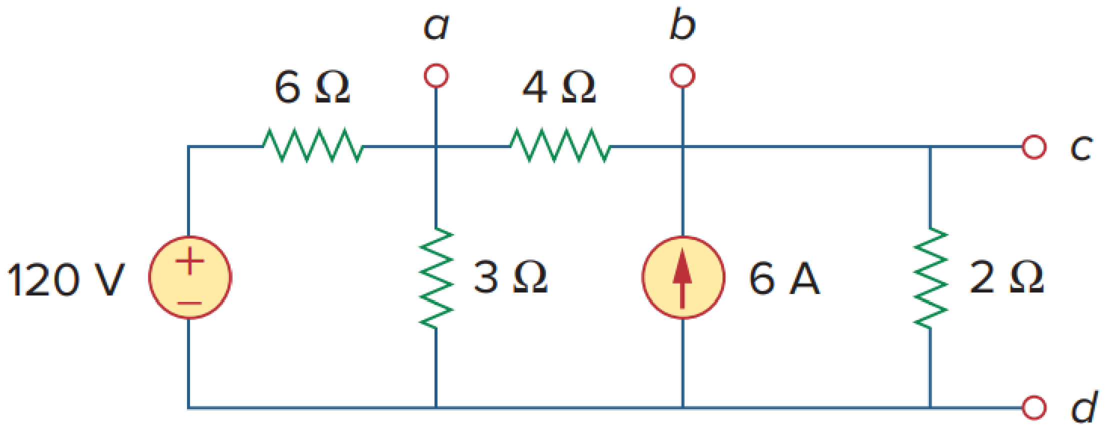
$$\Rightarrow R_N = \frac{180 \times 180}{180 + 180} = \mathbf{90\ \Omega}$$

Norton equivalent circuit at terminals a–b



Problem 12

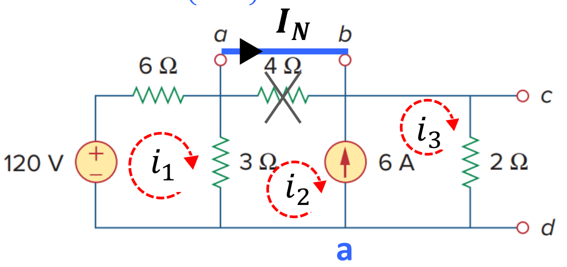
- Find the Norton equivalent circuit for the circuit at terminals $(i)\ a - b$ *and* $(ii)\ c - d$.



Ans: $(i)\ I_N = 7\ A;\ R_N = 2\ \Omega$
$(ii)\ I_N = 12.667\ A;\ R_N = 1.5\ \Omega$

Solution to Problem 12

Finding $I_{N(a-b)}$



Let's use mesh analysis to find the I_N
From the circuit,

$$i_2 = I_N$$

Applying KVL at mesh 1,

$$-120 + 6i_1 + 3(i_1 - I_N) = 0$$

$$\Rightarrow 120 + 6i_1 + 3i_1 - 3I_N = 0$$

$$\Rightarrow 9i_1 - 3I_N = 120 \quad \text{.......... (i)}$$

Applying KVL at supermesh (2 & 3),

$$3(I_N - i_1) + 2i_3 = 0$$

$$3I_N - 3i_1 + 2i_3 = 0$$

$$\Rightarrow -3i_1 + 3I_N + 2i_3 = 0 \quad \text{.......... (ii)}$$

Applying KCL at node a,

$$I_N + 6 = i_3$$

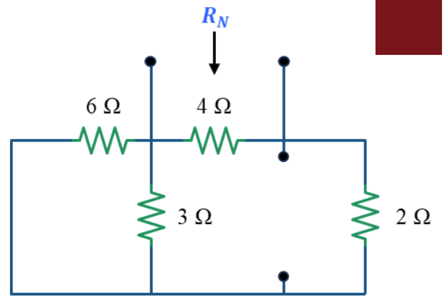
$$\Rightarrow I_N - i_3 = -6 \quad \text{.......... (iii)}$$

Solving (i), (ii) and (iii),

$$\boxed{\begin{aligned} i_1 &= 15.67\ A \\ I_N &= 7\ A \\ i_3 &= 13\ A \end{aligned}}$$

Finding $R_{N(a-b)}$

At first, let's deactivate all the Independent Sources. As there is no dependent sources, we simply use series-parallel combination to find the equivalent resistance seen from the load terminal.

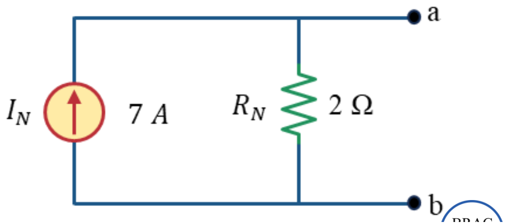


$$R_N = 4 \,||\, [(6 \,||\, 3) + 2]$$

$$\Rightarrow R_N = 4 \,||\, [2 + 2]$$

$$\Rightarrow R_N = 2\ \Omega$$

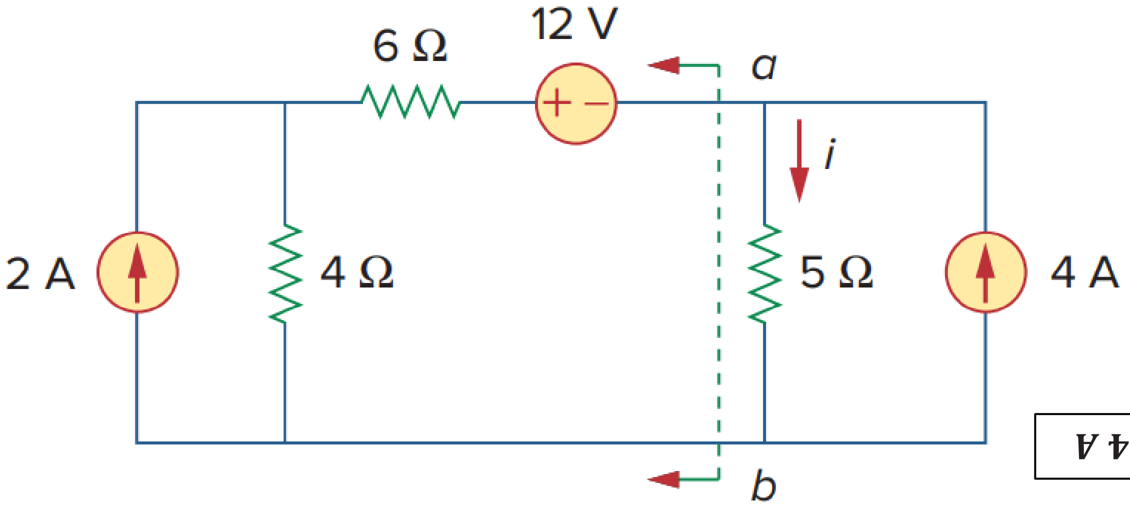
Norton equivalent circuit at terminals a–b



Try the terminal b-c yourself

Problem 13

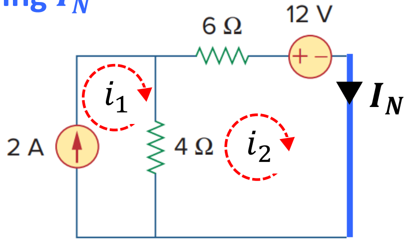
- Obtain the Thevenin/Norton equivalent of the circuit to the left of terminals $a - b$. Use the result to find current i.



Ans: $I_N = -0.4\,A$; $R_N = 10\,\Omega$; $i = 2.4\,A$

Solution to Problem 13

Finding I_N



Let's use mesh analysis to find the I_N
From the circuit,

$$i_2 = I_N$$

Applying KVL at mesh 1,

$$i_1 = 2A \quad \text{.......... (i)}$$

Applying KVL at mesh 2,

$$4(I_N - i_1) + 6I_N + 12 = 0$$

$$\Rightarrow 4I_N - 4i_1 + 6I_N + 12 = 0$$

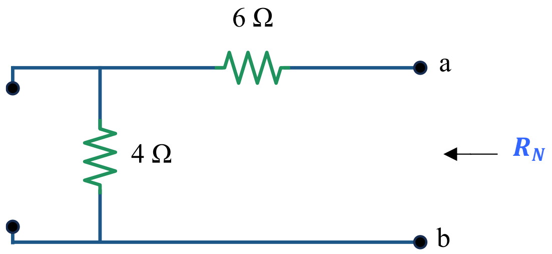
$$\Rightarrow -4i_1 + 10I_N = -12 \quad \text{.......... (ii)}$$

Solving (i) and (ii),

$$\boxed{\begin{aligned} i_1 &= 2\ A \\ I_N &= -0.4\ A \end{aligned}}$$

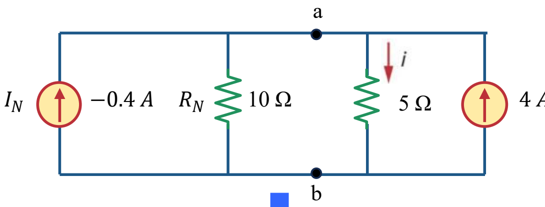
Finding R_N

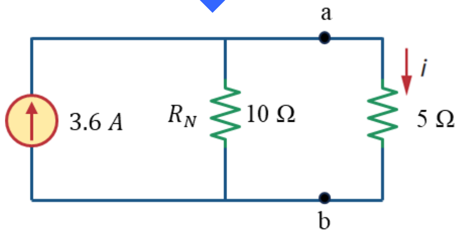
At first, let's deactivate all the Independent Sources. As there is no dependent sources, we simply use series-parallel combination to find the equivalent resistance seen from the load terminal.



$$R_N = 6 + 10 = 10\ \Omega$$

Norton equivalent circuit at terminals a–b



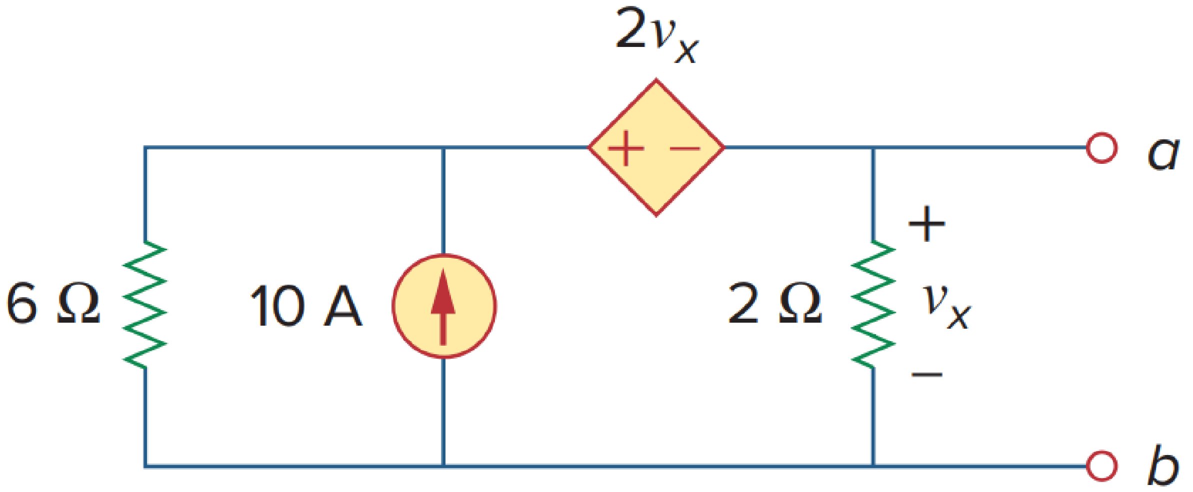


To find i, applying current divider law,

$$i = \frac{5\ ||\ 10}{5} \times 3.6 = 2.4\ A$$

Problem 14

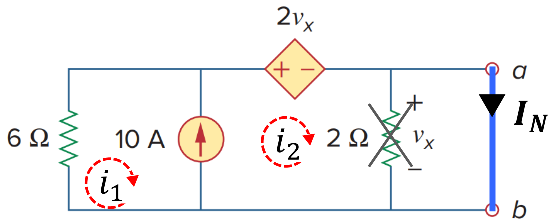
- Find the Norton equivalent circuit for the circuit at terminals $a - b$.



Ans: $I_N = 10\ A$; $R_N = 1\ \Omega$

Solution to Problem 14

Finding I_N



Let's use mesh analysis to find the I_N
From the circuit,

$$i_2 = I_N$$

$$v_x = 0\ (Since\ short\ circuit)$$

Applying KVL at supermesh (1 & 2)

$$6i_1 + 2v_x = 0$$

$$\Rightarrow 6i_1 + 0 = 0$$

$$\Rightarrow i_1 = 0 \quad \text{..........} \ (i)$$

If i_1 is 0, then the 10 A current source contributes fully to I_N

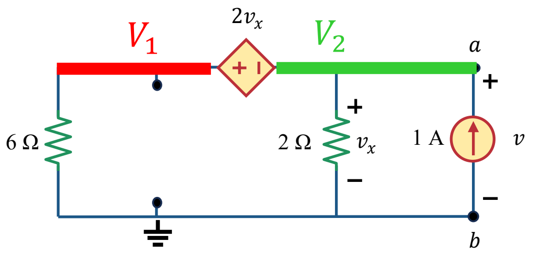
$$\boldsymbol{I_N = 10\ A}$$

Finding R_N

At first, let's deactivate all the Independent Sources. As there is dependent source. We need to use a known voltage source across the terminal a-b and find out the current through the node a-b. Alternatively, we can use a known current source across the terminal a-b and find out the voltage across the terminal.

Let's do the second type and apply $1\ A$ at terminal a-b

Solution to Problem 14 (Continued)



We need to find the voltage v

Let's use Nodal analysis to find the v

From the circuit,

$$V_2 = v_x = v$$

Applying KCL at supernode (1 & 2)

$$\frac{V_1}{6} + \frac{v}{2} - 1 = 0$$

$$\Rightarrow \frac{1}{6}V_1 + \frac{1}{2}v = 1 \qquad \text{..........} \text{ (iii)}$$

Applying KVL at supernode,

$$V_1 - v = 2v_x$$

$$\Rightarrow V_1 - v = 2v$$

$$\Rightarrow V_1 - 3v = 0 \qquad \text{..........} \text{ (iv)}$$

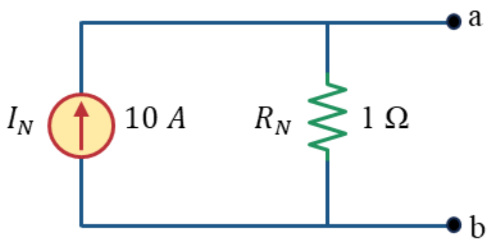
Solving (iii) and (iv),

$$\boxed{\begin{aligned} V_1 &= 3\ V \\ v &= 1\ V \end{aligned}}$$

So,

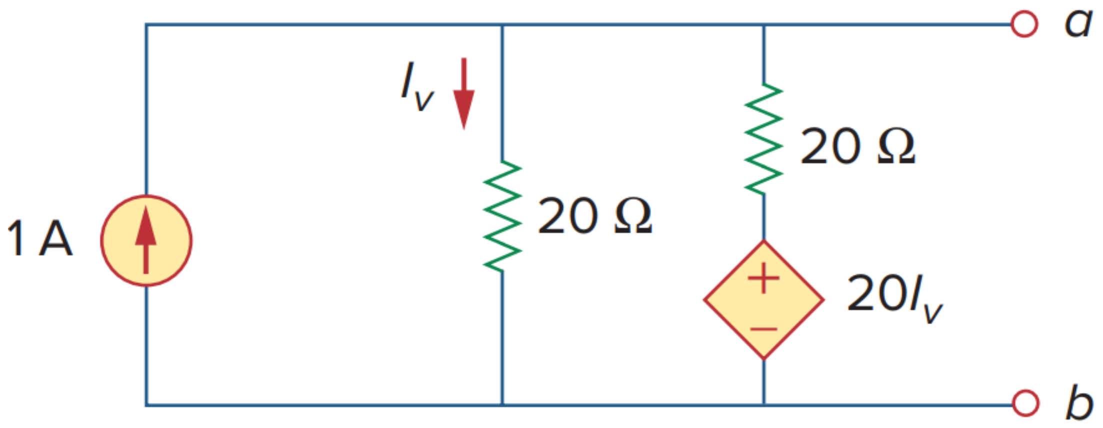
$$R_{Th} = \frac{v}{1\ A} = \frac{1}{1} = \mathbf{1\ \Omega}$$

Norton equivalent circuit at terminals a–b



Problem 15

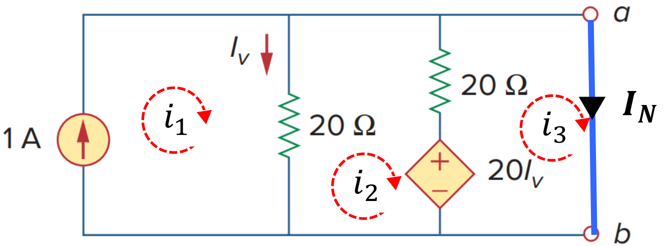
- Obtain the Norton equivalent circuit with respect to terminals a and b.



Ans: $\boldsymbol{I_N = 1\,A;\ R_N = 20\,\Omega}$

Solution to Problem 15

Finding $I_{N(a-b)}$



Let's use mesh analysis to find the I_N
From the circuit,

$$i_3 = I_N$$
$$I_v = i_1 - i_2$$

Applying KVL at mesh 1,

$$i_1 = 1\ A \text{ .... (i)}$$

Applying KVL at mesh 2,

$$20(i_2 - i_1) + 20(i_2 - I_N) + 20I_v = 0$$
$$\Rightarrow 20i_2 - 20i_1 + 20i_2 - 20I_N + 20(i_1 - i_2) = 0$$

$$\Rightarrow 20i_2 - 20i_1 + 20i_2 - 20I_N + 20(i_1 - i_2) = 0$$
$$\Rightarrow 20i_2 - 20I_N = 0 \text{ .... (ii)}$$

Applying KVL at mesh 3,

$$-20I_v + 20(I_N - i_2) = 0$$
$$\Rightarrow -20(i_1 - i_2) + 20I_N - 20i_2 = 0$$
$$\Rightarrow -20i_1 + 20i_2 + 20I_N - 20i_2 = 0$$
$$\Rightarrow -20i_1 + 20I_N = 0 \text{ .... (iii)}$$

Solving (i), (ii) and (iii),

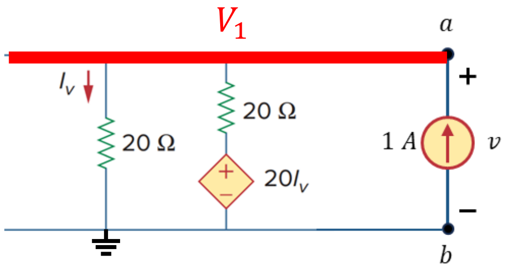
$$\boxed{\begin{aligned} i_1 &= 1\ A \\ I_N &= 1\ A \\ i_3 &= 1\ A \end{aligned}}$$

Finding R_N

At first, let's deactivate all the Independent Sources. As there is dependent source. We need to use a known voltage source across the terminal a-b and find out the current through the node a-b. Alternatively, we can use a known current source across the terminal a-b and find out the voltage across the terminal.

Let's do the second type and apply $1\ A$ at terminal a-b

Solution to Problem 15 (Continued)



We need to find the voltage v

Let's use Nodal analysis to find the v

From the circuit,

$$V_1 = v$$

$$I_v = \frac{V_1}{20} = \frac{v}{20}$$

Applying KVL at node 1,

$$\frac{v}{20} + \frac{v - 20I_v}{20} - 1 = 0$$

$$\Rightarrow \frac{v}{20} + \frac{v}{20} - I_v - 1 = 0$$

$$\Rightarrow \frac{v}{20} + \frac{v}{20} - \frac{v}{20} - 1 = 0$$

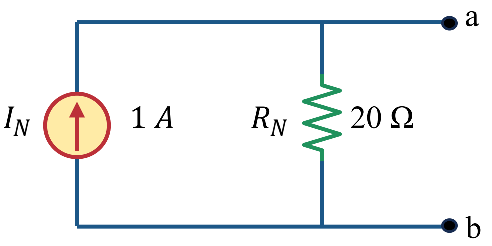
$$\Rightarrow \frac{v}{20} = 1$$

$$\Rightarrow v = 20\ V$$

So,

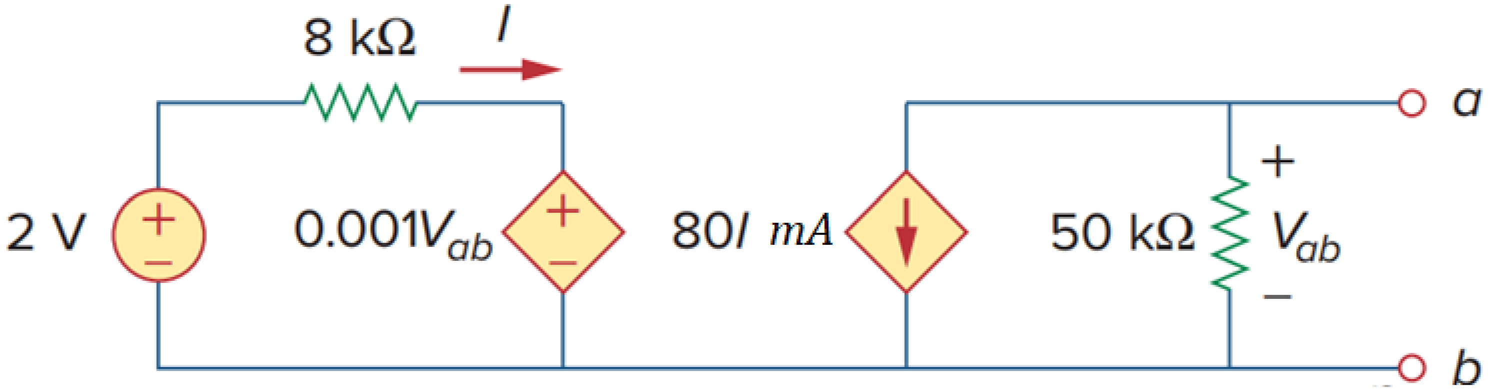
$$R_{Th} = \frac{v}{1\ A} = \frac{20}{1} = \mathbf{20\ \Omega}$$

Norton equivalent circuit at terminals a–b



Problem 16

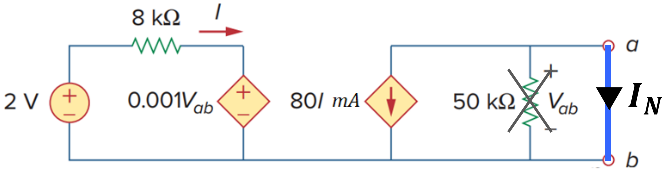
- Obtain the Thevenin/Norton equivalent of the circuit to the left of terminals $a - b$.



Ans: $V_{Th} = -2000\ V;\ I_N = -20\ mA;\ R_{Th} = R_N = 100\ k\Omega$

Solution to Problem 16

Finding I_N



From the circuit,

$$V_{ab} = 0\,V\ (Short\ circuit)$$

Applying KVL at the left portion,

$$-2 + 8I + 0.001V_{ab} = 0$$

$$\Rightarrow -2 + 8I = 0$$

$$\Rightarrow I = 0.25\,mA$$

From the right portion,

$$80I = -I_N$$

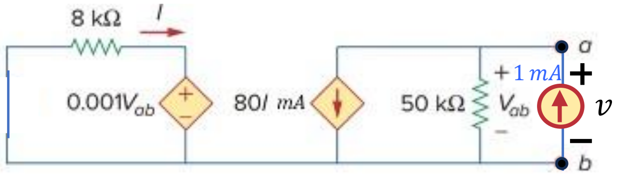
$$\Rightarrow 80 \times 0.25 = -I_N$$

$$\Rightarrow \boldsymbol{I_N = -20\,mA}$$

Finding R_{Th}

At first, let's deactivate all the Independent Sources. As there is dependent source, we need to use a known current source across the terminal a-b and find out the voltage across the terminal.

Let's do the second type and apply $1\,A$ at terminal a-b



From the circuit,

$$V_{ab} = v$$

Applying KVL at the left portion,

$$8I + 0.001V_{ab} = 0$$

$$8I + 0.001v = 0 \quad .... \text{(i)}$$

Applying KCL at node a,

$$80I + \frac{v}{50} = 1 \quad .... \text{(ii)}$$

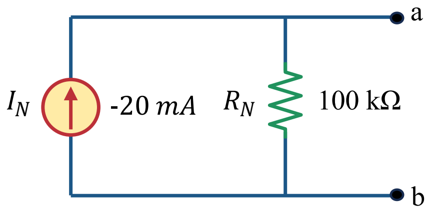
Solving (i) and (ii),

$$\boxed{\begin{aligned} I &= -80\,mA \\ v &= 100\,V \end{aligned}}$$

So,

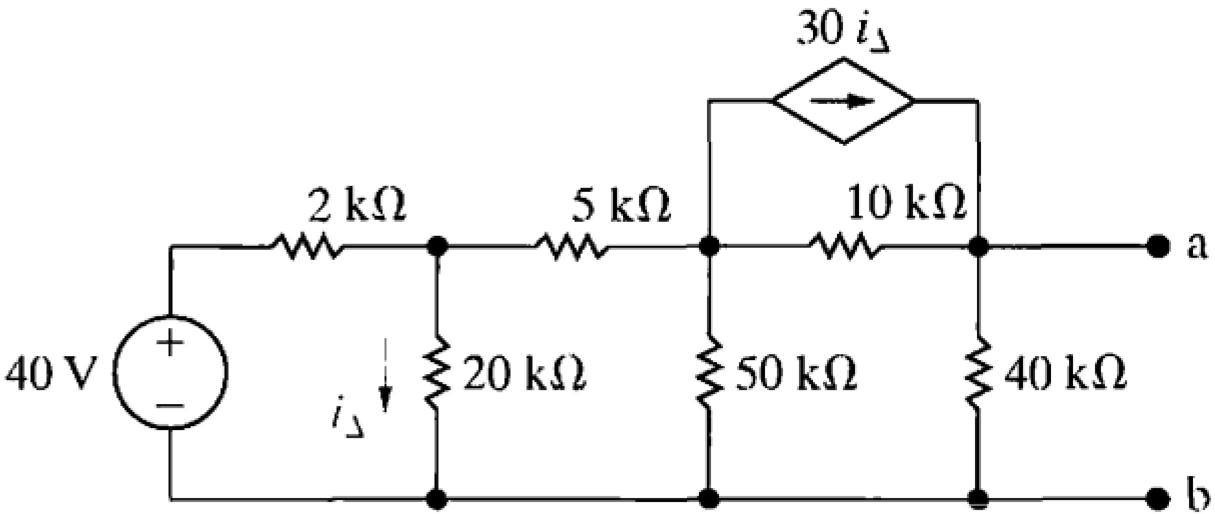
$$R_{Th} = \frac{v}{1\,mA} = \frac{100}{1} = \boldsymbol{100\,k\Omega}$$

Norton equivalent circuit at terminals a–b



Problem 17

- Obtain the Norton equivalent circuit with respect to terminals a and b.



Ans: $I_N = 14\,mA;\ R_N = 20\,k\Omega$

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the textbook: here