Department of Computer Science and Engineering (CSE)
BRAC University

Lecture 12

CSE250 - Circuits and Electronics

AC Fundamentals, AC Circuits, and AC Power

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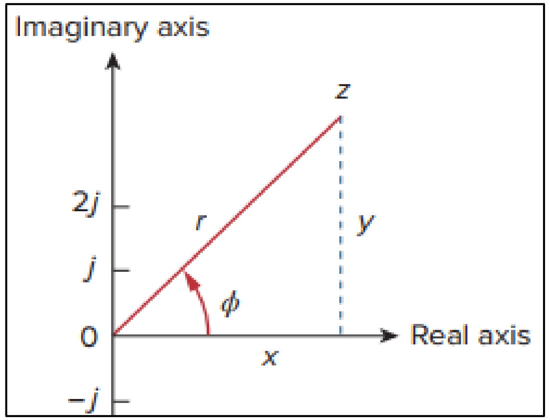
Complex Number

- A complex number z can be written as,
- $z = x + jy$ Rectangular form
- $z = r\angle\phi$ Polar form $j = \sqrt{-1} = -\frac{1}{j}$
- $z = re^{j\phi}$ Exponential form

The relationship between the rectangular and the polar form can be written from the figure as,

- $r = \sqrt{x^2 + y^2}$
- $\varphi = \tan^{-1}\frac{y}{x}$
- $x = rcos\varphi$
- $y = rsin\varphi$



The complex exponential can be expanded by a Taylor's series expansion as,

- $$e^{j\varphi} = 1 + j\varphi + \frac{(j\varphi)^2}{2!} + \frac{(j\varphi)^3}{\left(1-\frac{\varphi^2}{2!}+\frac{\varphi^4}{4!}-\cdots\right)3!} + \frac{(j\varphi)^4}{4!} + \cdots$$

- Separating the real and imaginary parts,

$$\Rightarrow \quad e^{j\varphi} = \left(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \cdots\right) + j\left(1 - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \cdots\right)$$

The real and imaginary parts are the Taylor's series expansion of a cosine and a sine respectively.

$$\Rightarrow \quad e^{j\varphi} = cos\varphi + jsin\varphi$$

$$\Rightarrow \quad re^{j\varphi} = r(cos\varphi + jsin\varphi)$$

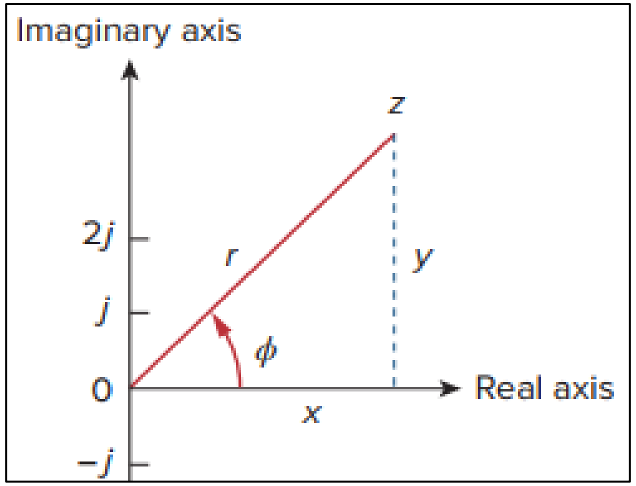
So, $\boldsymbol{z = re^{j\varphi} = x + jy}$

r is called Modulus *φ is called argument*

Operations of Complex Numbers

- Rectangular form $z = x + jy$
- Polar form $z = r\angle\phi$ where, $j = \sqrt{-1} = -\frac{1}{j}$
- Exponential form $z = re^{j\phi}$

- Let, $z_1 = x_1 + jy_1 = r_1\angle\varphi_1 = r_1e^{j\varphi_1}$ and $z_1 = x_2 + jy_2 = r_2\angle\varphi_2 = r_2e^{j\varphi_2}$
- Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
- Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
- Multiplication: $z_1z_2 = r_1r_2\angle(\varphi_1 + \varphi_2)$
- Division: $z_1/z_2 = \frac{r_1}{r_2}\angle(\varphi_1 - \varphi_2)$
- Reciprocal: $\frac{1}{z} = \frac{1}{r}\angle-\varphi$
- Square root: $\sqrt{z} = \sqrt{r}\angle\frac{\varphi}{2}$



- Complex Conjugate: $z^* = x - jy = r\angle{-\phi} = re^{-j\phi}$

Problem 1

- Determine the polar and rectangular form of the following quantities:

a) $$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)}$$

b) $$5\angle 81.87° \left(4 - j3 + \frac{3\sqrt{2}\angle -45°}{7-j1}\right)$$

c) $$\left[5^{-1} + 10^{-1} + (j24)^{-1} + (1\angle -90°)^{-1}\right]^{-1}$$

d) $$\frac{\left[5^{-1}+(j12)^{-1}+10^{-1}\right]^{-1}}{\left[5^{-1}+(j12)^{-1}+10^{-1}\right]^{-1} + (-j2)} \times 10\angle 60°$$

<u>Ans</u>: $I.\ 200\angle -16.2° = 192.0587 - j55.7982$

$II.\ 76.2520 + j44.2506 = 88.162\angle 30.127°$

$III.\ 1\angle -72.62° = -0.297 + j0.95$

$IV.\ 9.738\angle 95.75° = -0.976 + j9.689$

Solution to problem a

$$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)}$$

$$= \frac{2000\angle -16.2°}{10}$$

$$= 200\angle -16.2°$$

$$\boxed{= 192.0587 - j55.7982}$$

Solution to problem b

$$5\angle 81.87°\left(4 - j3 + \frac{3\sqrt{2}\angle -45°}{7 - j1}\right)$$

$$= 5\angle 81.87°(4 - j3 + 0.48 - j0.36)$$

$$= 5\angle 81.87°(4.48 - j3.36)$$

$$= 5\angle 81.87°(5.6\angle -36.87°)$$

$$\boxed{= 19.80 + j19.80°}$$

Solution to problem c

$$[5^{-1} + 10^{-1} + (j24)^{-1} + (1\angle -90°)^{-1}]^{-1}$$

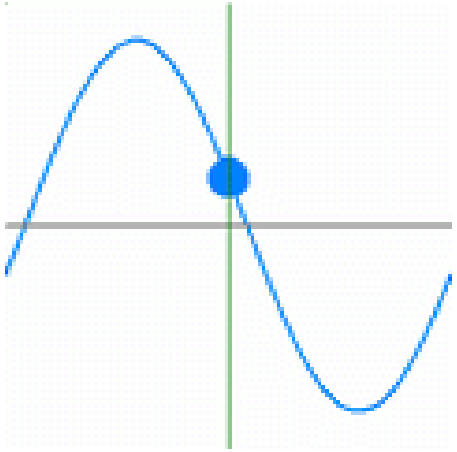
$$= \left[\frac{1}{5} + \frac{1}{10} - \frac{j}{24} + j\right]^{-1}$$

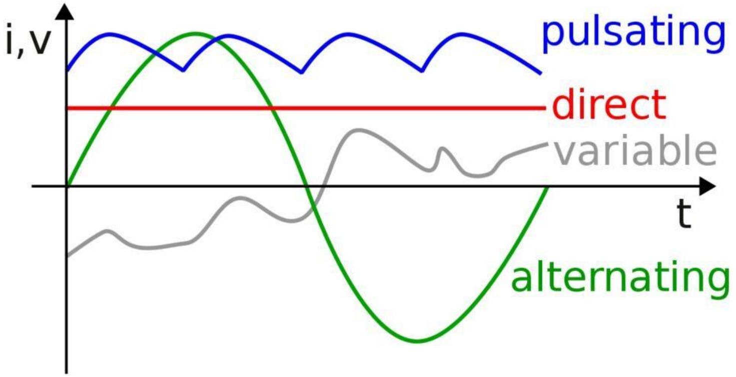
$$\boxed{= 0.29 - j0.95}$$

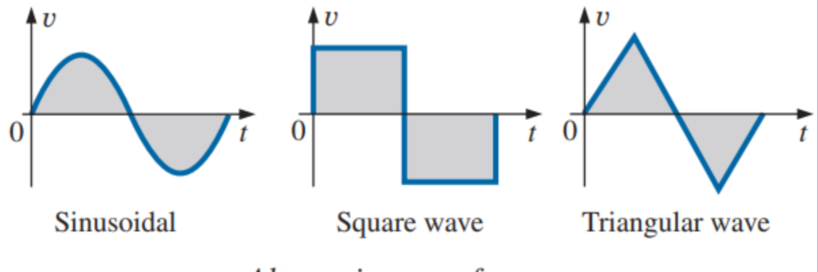
Do the problem d by yourself …

Alternating Current or Voltage

- *Alternating current (AC)* is a flow of electric charge that periodically reverses its direction, in contrast to direct current (DC) which only flows in a single direction. It starts, say, from zero, grows to a maximum, decreases to zero, reverses, reaches a maximum in the opposite direction, returns again to the original value, and repeats this cycle indefinitely.







Alternating waveforms.

Sinusoid

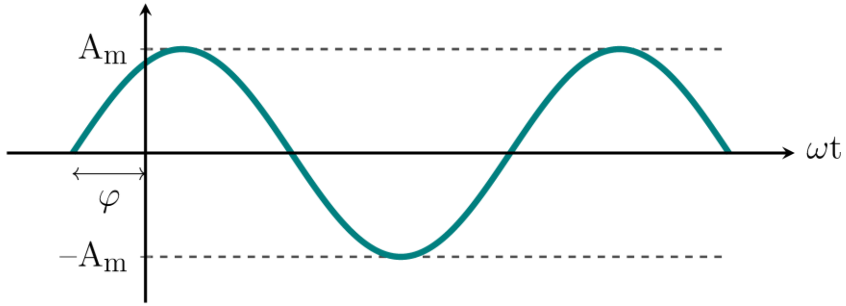
Among different types of ac waveforms, the pattern of particular interest is the sinusoidal because

1. It is the voltage generated by utilities throughout the world and supplied to homes, factories, laboratories, and so on
2. Any periodic signal can be represented as a series of summation of sine and cosine.

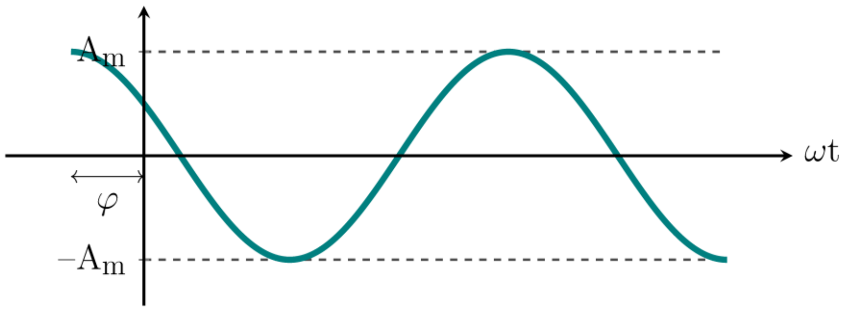
A *sinusoid* is a signal that has the form of the sine or cosine function. A sinusoidal current is usually referred to as *alternating current (ac)*. Circuits driven by sinusoidal current sources or voltage sources are called *ac circuits*.

- The basic mathematical format for the sinusoidal waveform is

$A_m\,sin(\omega t + \phi)$

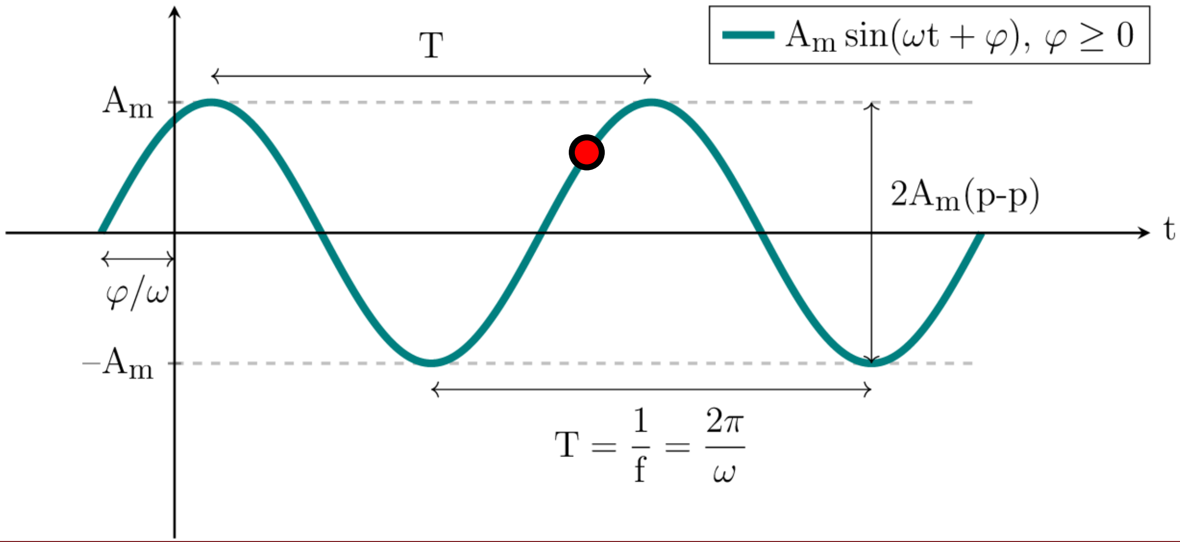


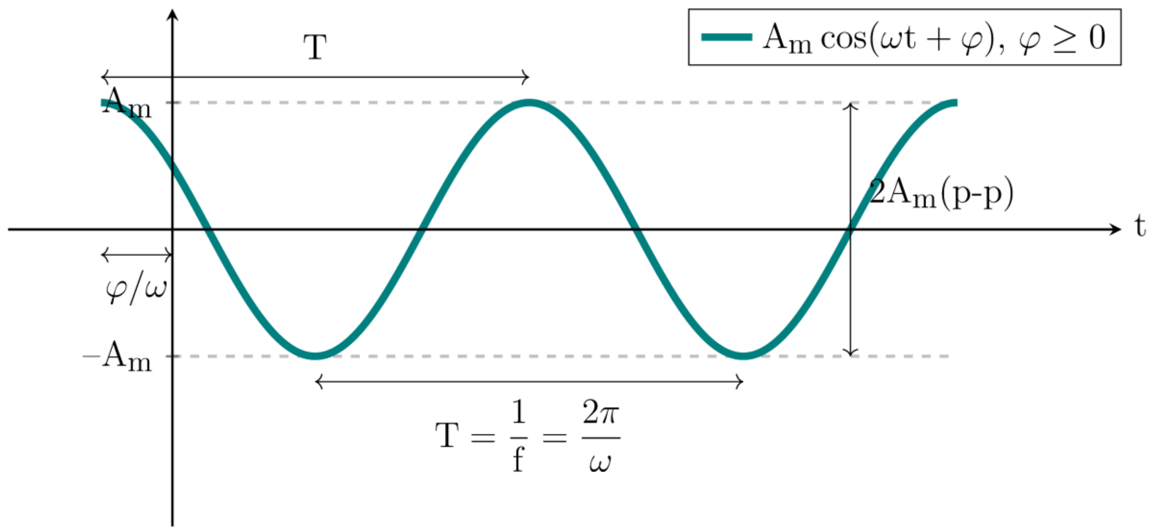
$A_m\,cos(\omega t + \phi)$



Sinusoid: basic terms

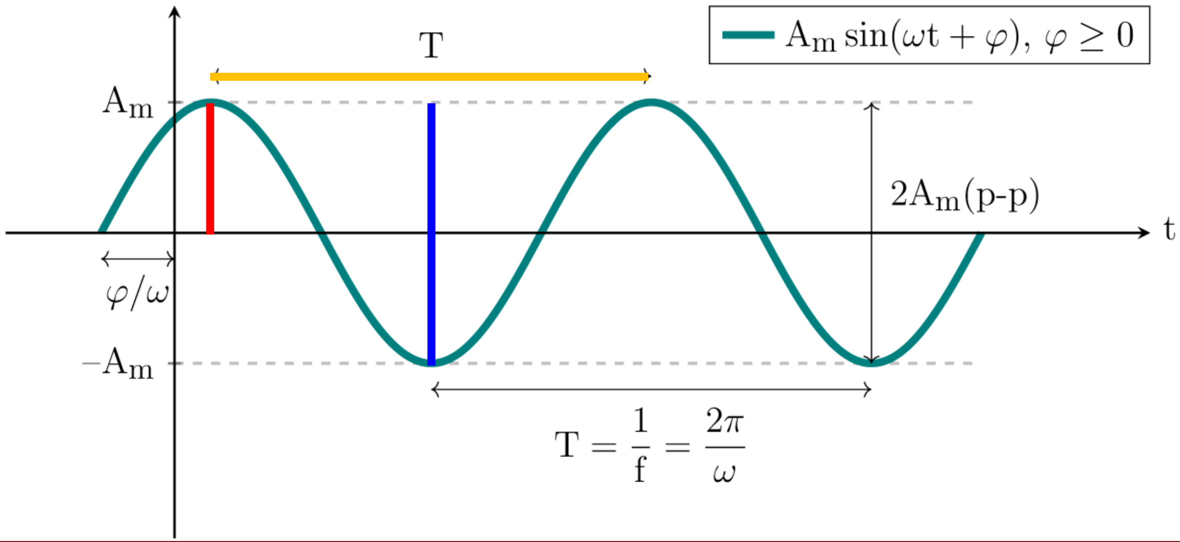
- It is necessary to define a few basic terms for a sinusoid of the form $A_m\,cos(\omega t + \phi)$ or $A_m\,sin(\omega t + \phi)$. These terms, however, can be applied to any alternating waveform.
- *Waveform:* The path traced by a quantity plotted as a function of some variable, such as time, position, degrees, radians, temperature, and so on.
- *Periodic Waveform:* A waveform that continually repeats itself after the same time interval. The waveforms shown are periodic waveforms.
- *Instantaneous Value:* The magnitude of a waveform at any instant of time.

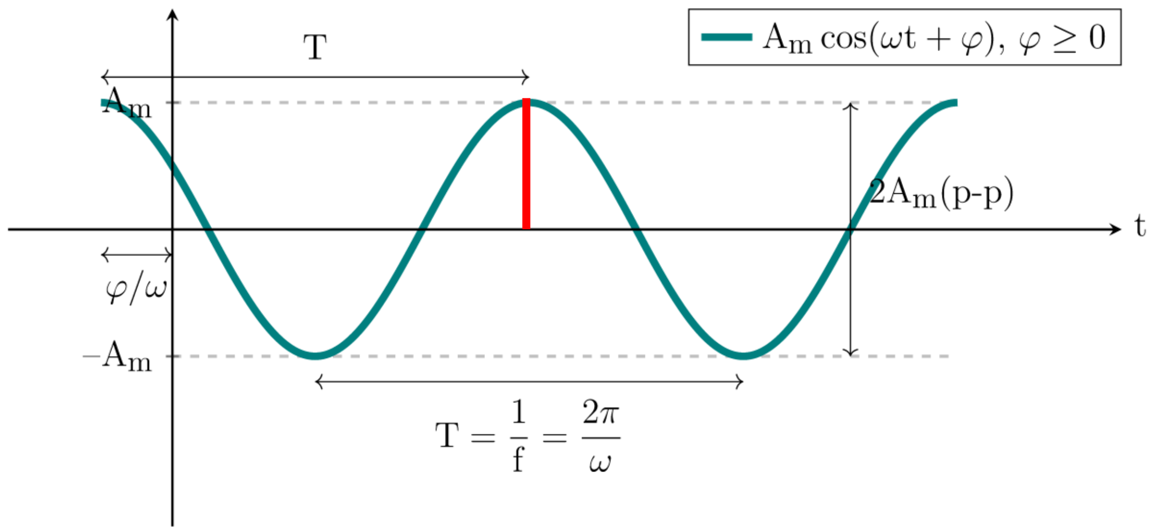




Sinusoid: basic terms (2)

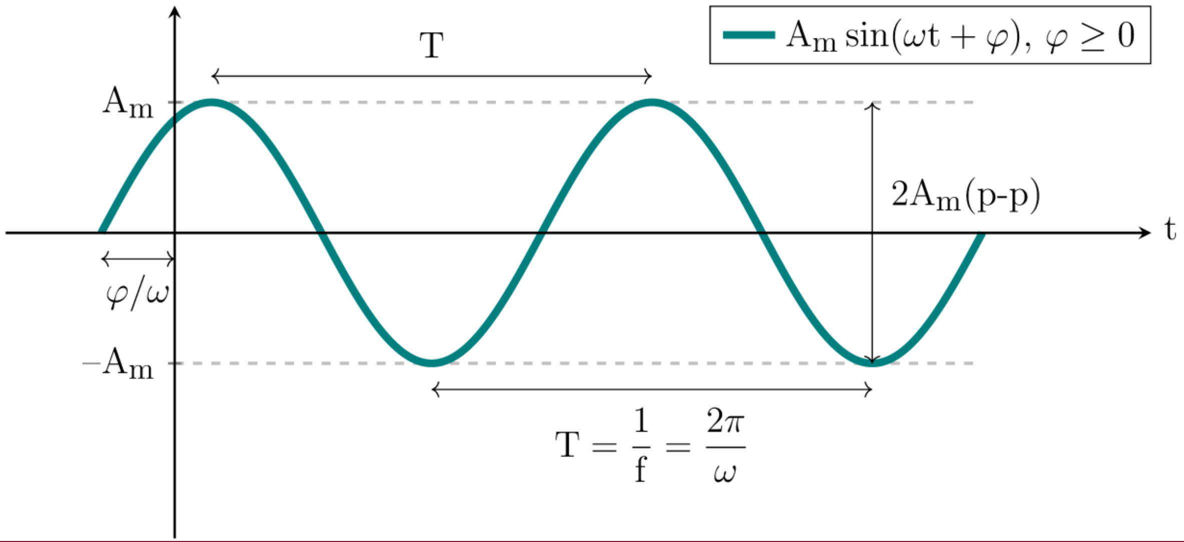
- *Peak amplitude or Amplitude:* The maximum value (A_m) of a waveform as measured from its average, or mean value.
- *Peak value or Magnitude:* The maximum instantaneous value of a function as measured from the zero volt level. For the waveforms shown, the peak amplitude and peak value are the same (A_m) since the average value of the function is zero volts.
- *Peak-to-peak value:* Denoted by $p-p$, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.
- *Period (T):* The time interval of a periodic waveform after which the waveform repeats.

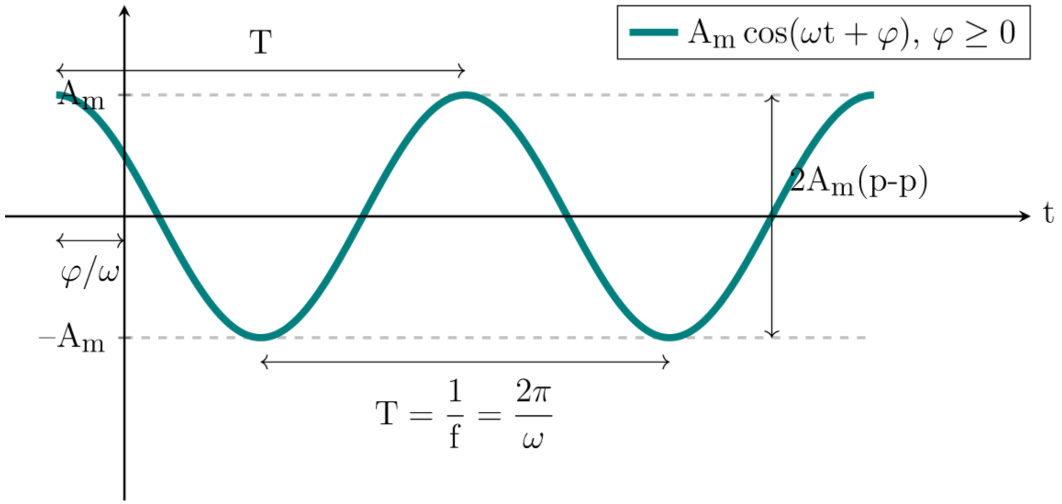




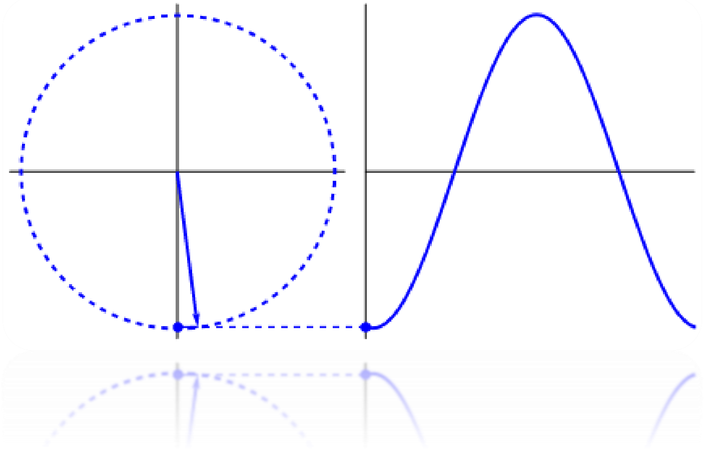
Sinusoid: basic terms (3)

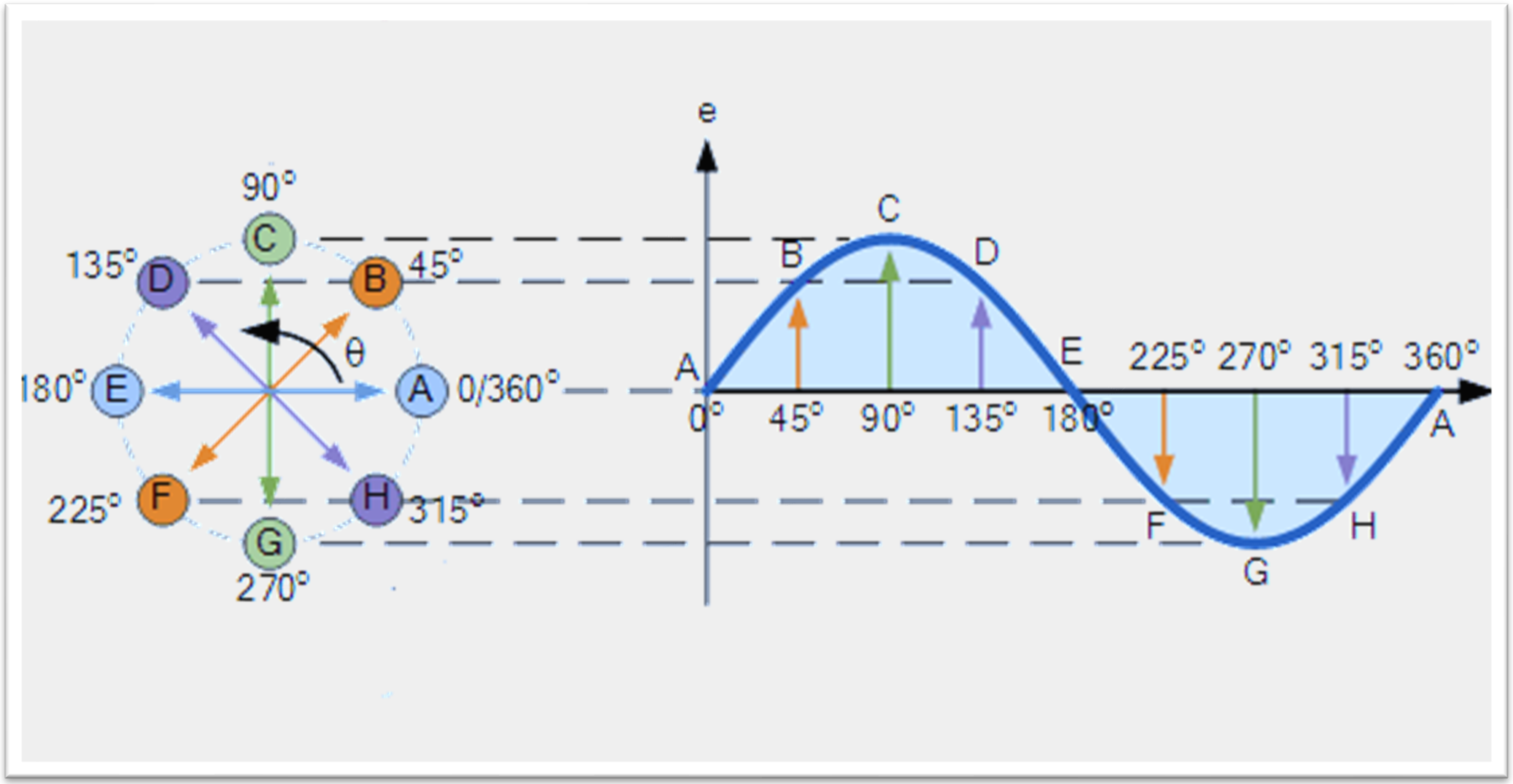
- *Cycle:* The portion of a waveform contained in one period of time.
- *Natural Frequency* (f)*:* The number of cycles that occur in 1 s. The unit of measure for frequency is the $hertz\ (Hz)$, where $1\ hertz\ (Hz) = 1\ cycle\ per\ second\ (cps)$.
- *Angular frequency* (ω)*:* Measure of angular displacement per unit of time. The unit of measure is $radian\ per\ second$ or $rads^{-1}$.
- *Instantaneous phase* $(\omega t + \varphi)$*:* Phase of a sine or cosine at any instant of time.
- *Initial phase* (φ)*:* Phase of a sinusoid at $t = 0$.



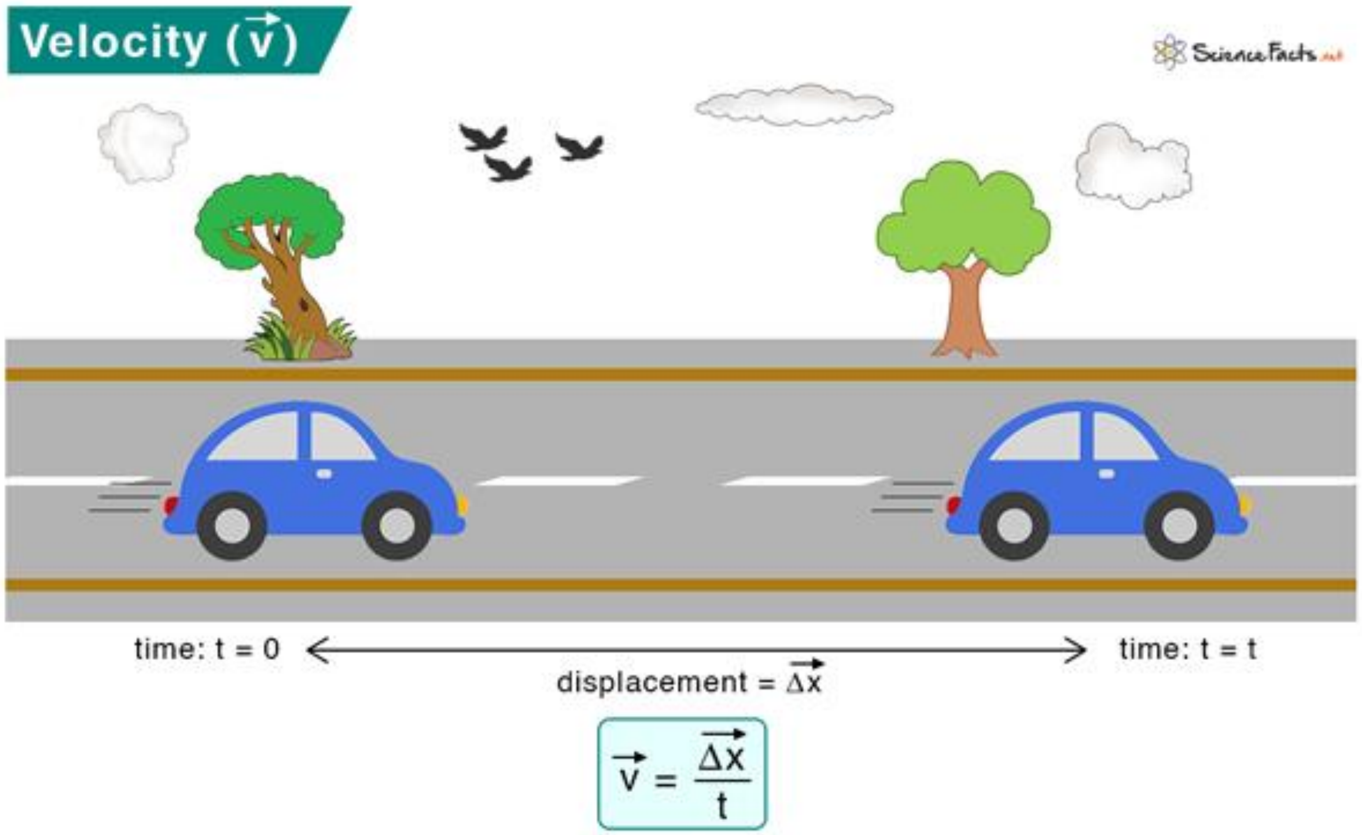


When an object moves in a circle, we can project its position onto a linear axis (like the x-axis or y-axis) to see how its position changes over time. The result of this projection is a sinusoidal wave.

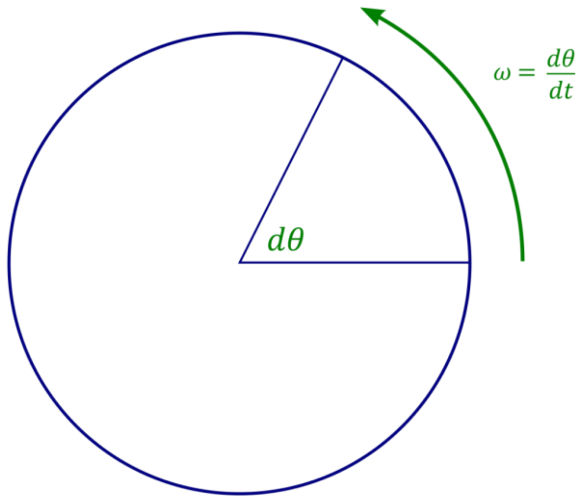




Linear Velocity (v)



Angular Velocity Or Angular Frequency (ω)



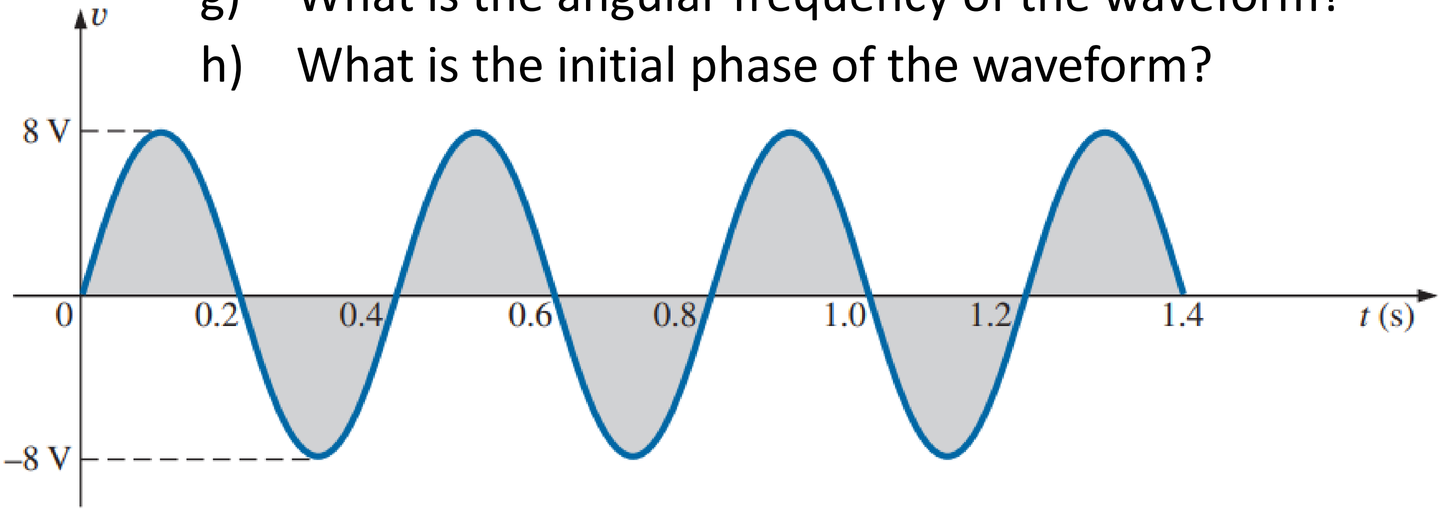
If θ is in radian and time is in seconds, then the unit of ω is $rads^{-1}$

$$\boldsymbol{\omega = 2\pi f = \frac{2\pi}{T}}$$

N.B. The term **angular velocity** is often used in mechanics to describe rotational motion of objects, while **angular frequency** is more common in wave and oscillation contexts. Mathematically, they are identical (ω).

Problem 2

- For the sinusoidal waveform shown below,
  a) What is the peak value?
  b) What is the instantaneous value at $0.3s$ and $0.6s$?
  c) What is the peak-to-peak value of the waveform?
  d) What is the period of the waveform?
  e) How many cycles are shown?
  f) What is the natural frequency of the waveform?
  g) What is the angular frequency of the waveform?
  h) What is the initial phase of the waveform?



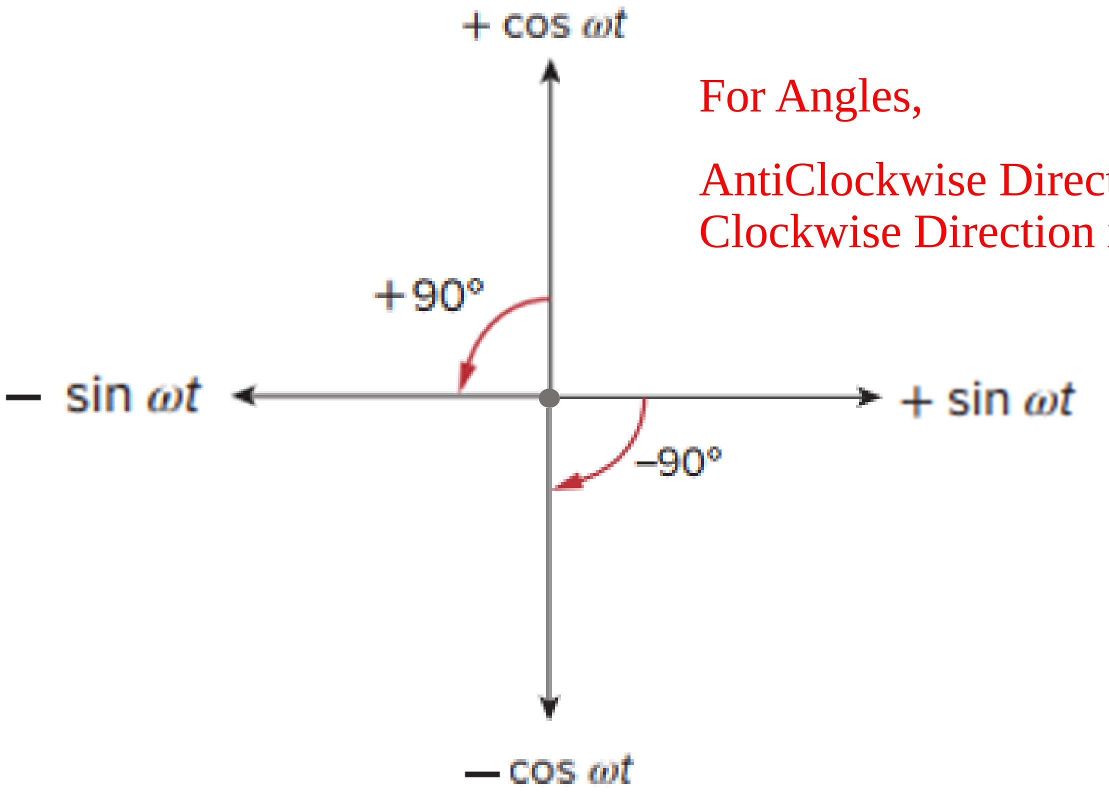
Ans: Try yourself

Sine-Cosine Conversion

A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is suitable to express both as either sine or cosine with **positive amplitudes.**

- The following trigonometric identities can be used to convert from sine to cosine or vice versa.

- $\sin(\omega t \pm 180°) = -\,sin\omega t$
- $\cos(\omega t \pm 180°) = -cos\omega t$
- $\sin(\omega t + 90°) = \ cos\omega t$
- $\sin(\omega t - 90°) = -cos\omega t$
- $\cos(\omega t + \ 90°) = -sin\omega t$
- $\cos(\omega t \ - \ 90°) = \ sin\omega t$



For Angles,

AntiClockwise Direction is Positive
Clockwise Direction is Negative

Problem 3

- Calculate amplitude, initial phase ($-180° \le \varphi \le 180°$), angular frequency, period, and frequency for the following sinusoids. What are the values of V_s and I_s at $t = 20\,ms$? ***[Both sin and cosine forms can be employed]***

I. $V_s = 45\cos(5\pi t + 36°)\ (V)$

II. $I_s = 15\cos(25\pi t + 25°)\ (A)$

III. $I_s = -20\cos(314t - 30°)\ (A)$

IV. $V_s = -4\sin(628t + 55°)\ (V)$

See the next page before solving

Ans: $I.\ 45\,V;\ \mathbf{36°}\ or\ \mathbf{126°};\ 5\pi\ {}^{rad}/_{s};\ 0.4\,s;\ 2.5\,Hz;\ \mathbf{26.45\,V}$

$II.\ 15\,A;\ \mathbf{25°}\ or\ \mathbf{115°};\ 25\pi\ {}^{rad}/_{s};\ 80\,ms;\ 12.5\,Hz;\ \mathbf{-6.34\,A}$

$III.\ 20\,A;\ \mathbf{150°}\ or\ \mathbf{60°};\ 314\ {}^{rad}/_{s};\ 20\,ms;\ 50\,Hz;\ \mathbf{-17.29\,A}$

$IV.\ 4\,V;\ \mathbf{-125°}\ or\ \mathbf{60°};\ 628\ {}^{rad}/_{s};\ 10\,ms;\ 100\,Hz;\ \mathbf{-3.26\,V}$

Initial phase from, **Cos equation** and **Sin Equation**

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MUST REMEMBER The terms inside the cosine or sine function must be of same unit. If not, you have to convert them into same unit before doing calculations. For example, for the 1st problem. If we want to calculate V_s at t = 20 ms. The equation will be,

$$V_s\,(t) = 45\cos(5\pi t + 36°)\,(V)$$

$$V_s\,(20 \times 10^{-3}) = 45\cos\left(\mathbf{5 \times \pi} \times \frac{180}{\pi} \times 20 \times 10^{-3} + \mathbf{36°}\right)(V)$$

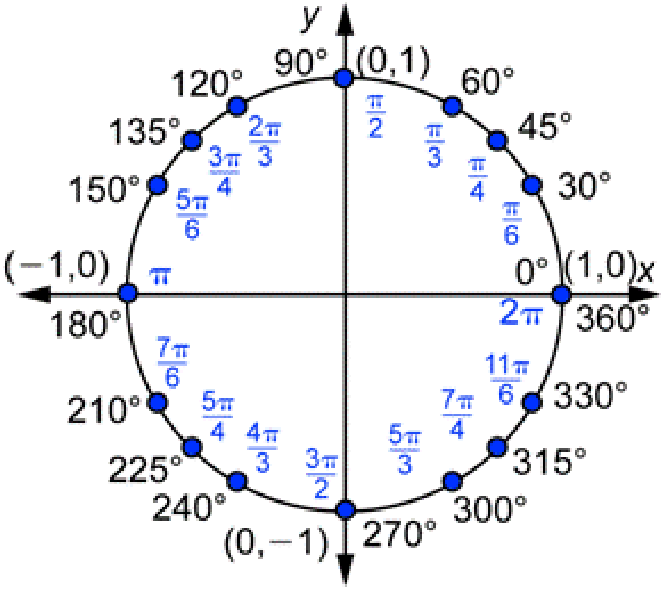
See here, we multiplied the **radian term** with $\frac{180°}{\pi}$, Why? Because the **initial phase is given in degree**. And to sum/subtract, we need to have units inside the sinusoidal term. So, we converted the first term in degree as well.

From a circle,

$$2\pi = 360°$$

The relation between radian and degree is,

$$radian \times \frac{180}{\pi} = degree$$



I. $V_s = 45\cos(5\pi t + 36°)\ (V)$

Amplitude,

$$A_m = 45\ V$$

Initial Phase,

$$\varphi = 36°$$

Angular Frequency,

$$\omega = 5\pi\ rads^{-1}$$

Frequency:

$$f = \frac{\omega}{2\pi}$$

$$\Rightarrow f = \frac{5\pi}{2\pi}$$

$$\Rightarrow f = \frac{5}{2}\ Hz$$

Time Period,

$$T = \frac{1}{f} = \frac{2}{5} = 0.4s$$

At time t = 20 ms,

$$V_s(20 \times 10^{-3}) = 45\cos(5 \times \pi \times \frac{180}{\pi} \times 20 \times 10^{-3} + 36°)\ (V)$$

$$\Rightarrow V_s(20 \times 10^{-3}) = 26.45\ V$$

II. $I_s = 15\cos(25\pi t + 25°)\ (A)$

Amplitude,

$$A_m = 15\ A$$

Initial Phase,

$$\varphi = 25°$$

Angular Frequency,

$$\omega = 25\pi\ rads^{-1}$$

Frequency:

$$f = \frac{\omega}{2\pi}$$

$$\Rightarrow f = \frac{25\pi}{2\pi}$$

$$\Rightarrow f = 12.5\ Hz$$

Time Period,

$$T = \frac{1}{f} = \frac{1}{12.5} = 0.08s = 80\ ms$$

At time t = 20 ms,

$$I_s\,(20 \times 10^{-3}) = 15\cos(\mathbf{25} \times \boldsymbol{\pi} \times \frac{180}{\pi} \times 20 \times 10^{-3} + \mathbf{25}°)\ (V)$$

$$\Rightarrow Is\,(20 \times 10^{-3}) = -6.34\ A$$

III. $I_s = -20\cos(314t - 30°)(A)$

It will more suitable if we convert this negative cos into positive cos or sin. It will make calculations easier.

$$I_s = -20\cos(314t - 30°) = 20\cos(314t - 30° + 180°)$$
$$I_s = 20\cos(314t - 30° + 180°)$$

Why did we add 180° instead of subtract? Because we want the initial phase term to be $\varphi \leq |180°|$

Amplitude,

$$A_m = 20\ A$$

Initial Phase,

$$\varphi = 180° - 30° = 150°$$

Angular Frequency,

$$\omega = 314\ rads^{-1}$$

Frequency:

$$f = \frac{\omega}{2\pi}$$
$$\Rightarrow f = \frac{314}{2\pi}$$
$$\Rightarrow f = 49.97\ Hz$$

Time Period,

$$T = \frac{1}{f} = \frac{1}{49.97} = 0.02s = 20\ ms$$

At time t = 20 ms,

$$I_s\,(20 \times 10^{-3}) = 20\cos(314 \times \frac{180}{\pi} \times 20 \times 10^{-3} - 30° + 180°)\ (V)$$
$$\Rightarrow Is\,(20 \times 10^{-3}) = -17.29\ A$$

$IV.\ Vs = -4\,sin(628t + 55°)\ (V)$

It will more suitable if we convert this negative cos into positive cos or sin. It will make calculations easier.

$$V_s = -4\,sin(628t + 55°) = 4\sin(628t + 55° - 180°)$$
$$I_s = 4\sin(628t + 55° - 180°)$$

Why did we subtract 180° instead of add? Because we want the initial phase term to be $\varphi \leq |180°|$

Amplitude,

$$A_m = 4\ V$$

Initial Phase,

$$\varphi = 55° - 180° = -125°$$

Angular Frequency,

$$\omega = 628\ rads^{-1}$$

Frequency:

$$f = \frac{\omega}{2\pi}$$
$$\Rightarrow\ f = \frac{628}{2\pi}$$
$$\Rightarrow\ f = 100\ Hz$$

Time Period,

$$T = \frac{1}{f} = \frac{1}{100} = 0.01s = 10\ ms$$

At time t = 20 ms,

$$V_s\,(20 \times 10^{-3}) = 4\,sin(628 \times \frac{180}{\pi} \times 20 \times 10^{-3} + 55° - 180°)\ (V)$$
$$\Rightarrow Vs\,(20 \times 10^{-3}) = -3.26\ V$$

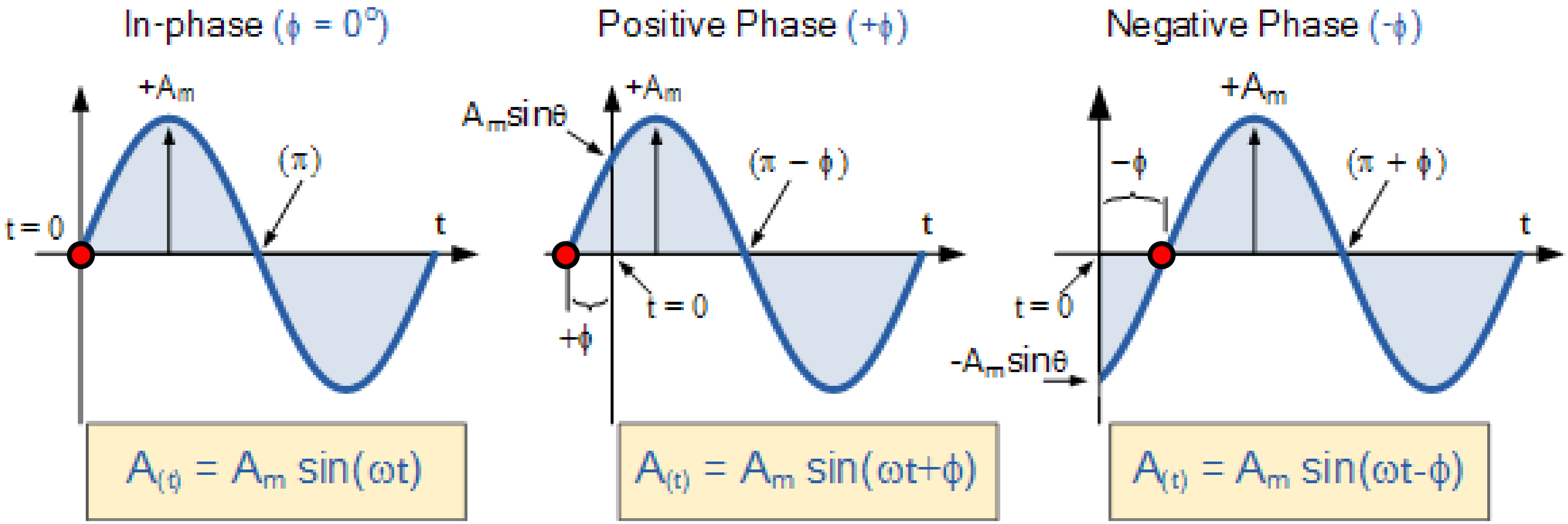
To find if the phase is positive or negative

If it is a sin (or Cos) wave, I will observe the starting point of the sin (or Cos) wave.

If the starting point is at the point of t=0, then the value of φ is zero, so Phase is zero

If the starting point is to the left of t=0, then the value of φ is positive, so Phase is positive

If the starting point is to the right of t=0, then the value of φ is negative, so Phase is negative

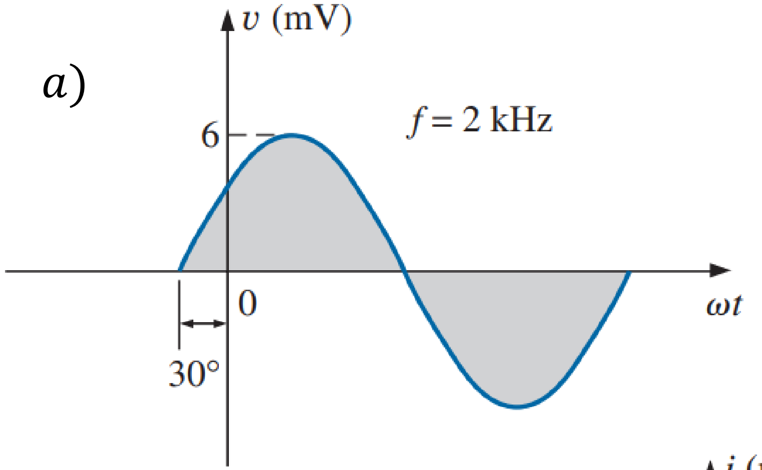


Problem 4

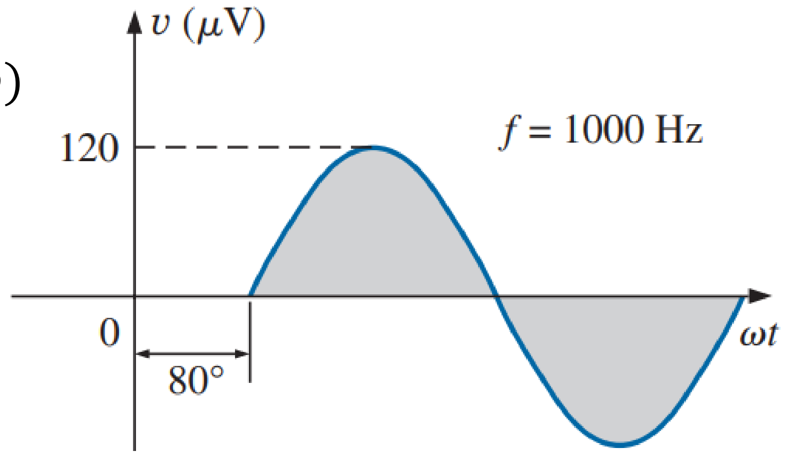
If the sin wave seems inverted, look at the starting point of the non inverted sin wave. And calculate the phase for that wave

- Write analytical expressions for the waveforms with the initial phase in degrees.

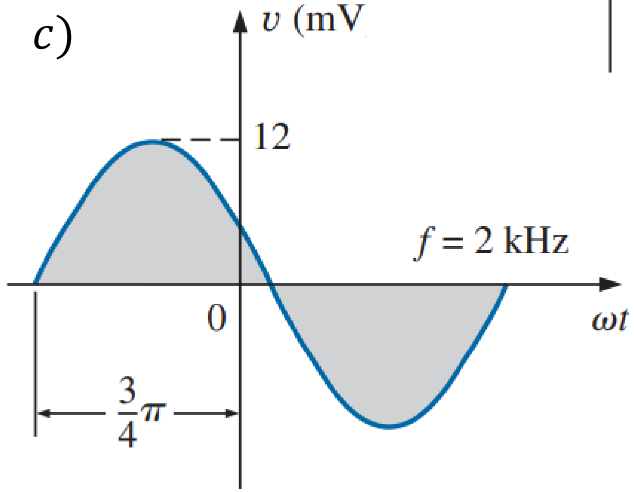
a)



b)

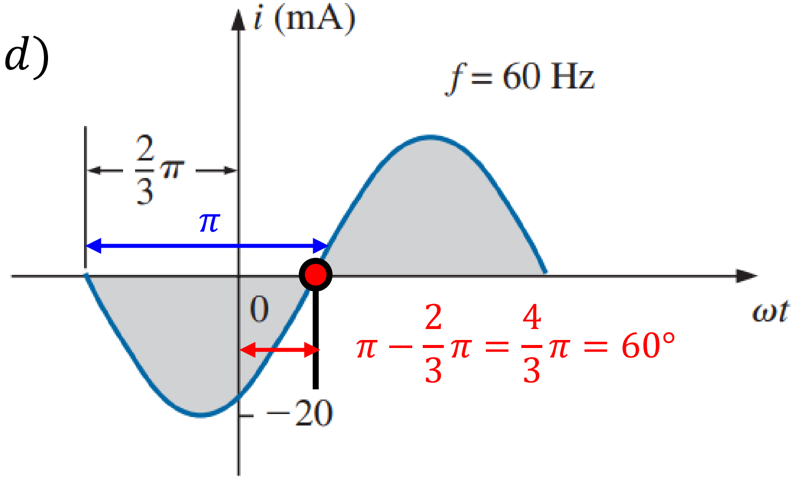


c)



If the phase is in radian, convert it in degree

d)



<u>Ans</u>:

a) $6 \times 10^{-3} \sin(2\pi 2000t + 30°)\ V$

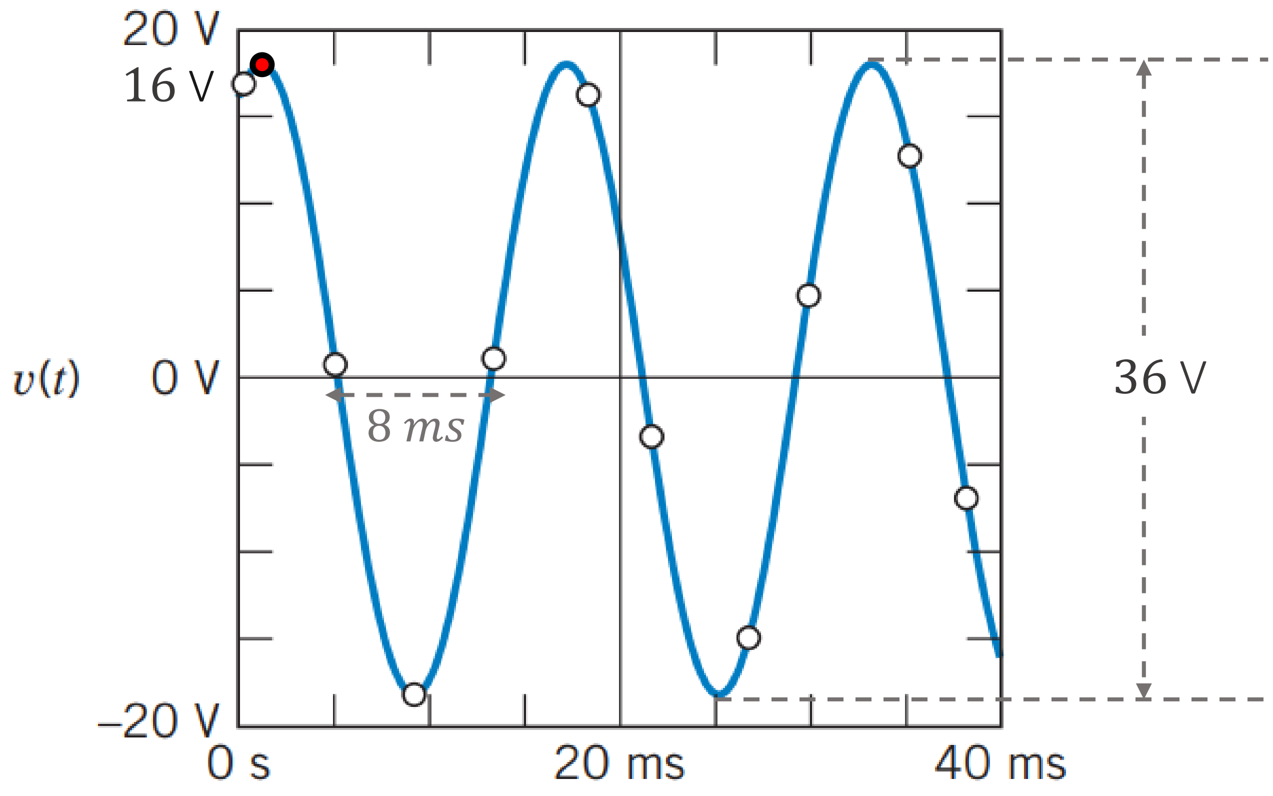
b) $120 \times 10^{-6} \sin(2\pi 1000t - 80°)\ V$

c) $12 \times 10^{-3} \sin(2\pi 2000t + 135°)\ V$

d) $20 \times 10^{-3} \sin(2\pi 60t - 60°)\ A$

Problem 5

- The following figure shows a sinusoidal voltage $v(t)$, plotted as a function of time t. Represent $v(t)$ by a function of the form $A_m\cos(\omega t + \varphi)$ with φ in degrees.



Solution:

Peak to peak voltage,

$$V_{p-p} = 36\ V$$

Amplitude

$$A_m = \frac{V_{p-p}}{2} = \frac{36}{2} = 18\ \mathrm{V}$$

Half period

$$\frac{T}{2} = 8\ ms$$

Period

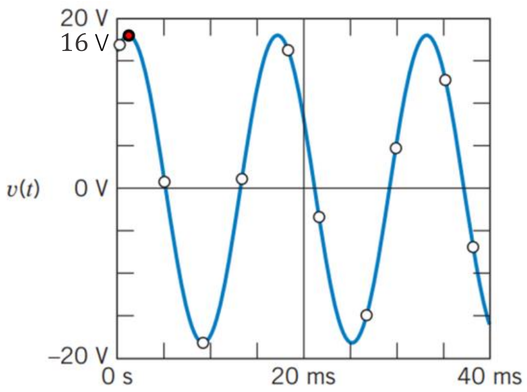
$$T = 16\ ms$$

Frequency

$$f = \frac{1}{T} = \frac{1}{16ms} = 62.5\ Hz$$

Angular Frequency,

$$\omega = 2\pi f = 125\pi\ rads^{-1}$$



The equation becomes,

$$v(t) = A_m cos(\omega t + \varphi) = 18\ cos(125\pi t + \varphi)\ V$$

We can find the initial phase (φ) at t = 0

From the graph, at t = 0,

$$v(0) = 16 = 18\ cos(125\pi \times 0 + \varphi)$$

$$16 = 18\cos(\varphi)$$

$$\varphi = \cos^{-1}\left(\frac{16}{18}\right)$$

$$\varphi = 27.266°$$

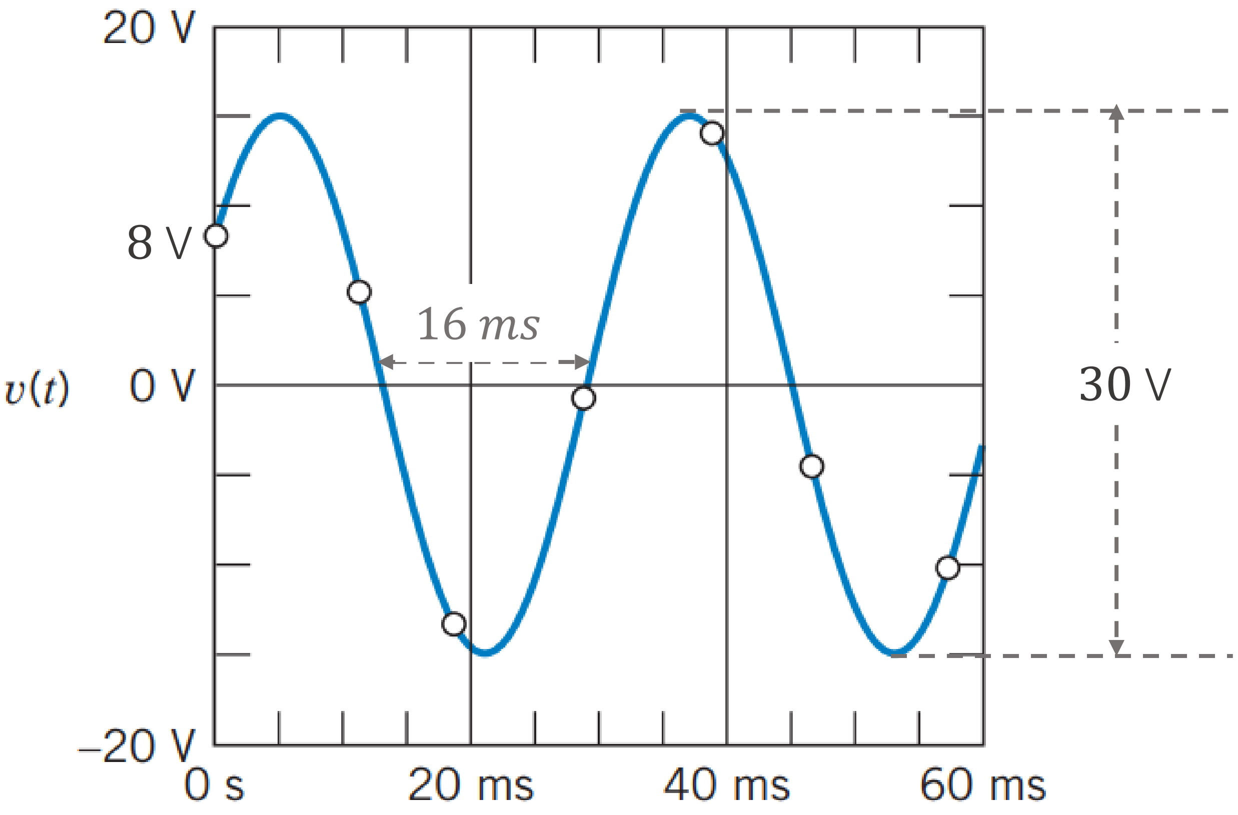
The initial phase for cos term will always yield positive value due to calculations. However, the initial phase can be positive or negative. We can determine if the phase is positive or negative from the initial point of cosine term in graph (If it is left of t = 0 or right of t = 0). From the graph, the initial point is right of t = 0, hence the initial phase will be negative. The initial point is marked by red point in the graph.

So, the final equation becomes

$$v(t) = A_m cos(\omega t + \varphi) = 18\ cos(125\pi t - 27.266°)\ V$$

Problem 6

- The following figure shows a sinusoidal voltage $v(t)$, plotted as a function of time t. Represent $v(t)$ by a function of the form $A_m \sin(\omega t + \varphi)$ with φ in degrees.



Solution:

Peak to peak voltage,

$$V_{p-p} = 30\ V$$

Amplitude

$$A_m = \frac{V_{p-p}}{2} = \frac{30}{2} = 15\ \mathrm{V}$$

Half period

$$\frac{T}{2} = 16\ ms$$

Period

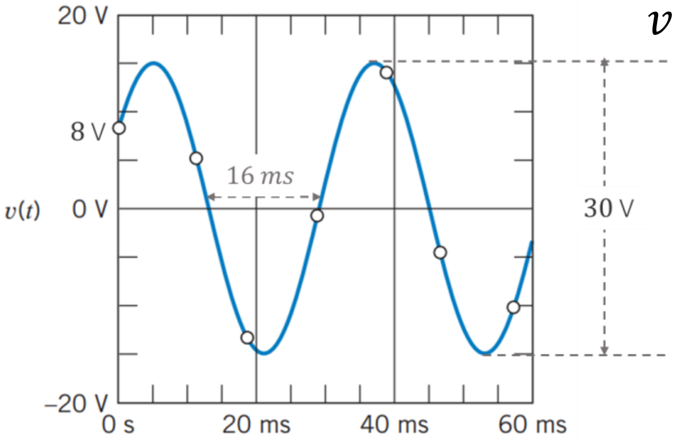
$$T = 32\ ms$$

Frequency

$$f = \frac{1}{T} = \frac{1}{32ms} = 31.25\ Hz$$

Angular Frequency,

$$\omega = 2\pi f = 62.5\pi\ rads^{-1}$$



The equation becomes,

$$v(t) = A_m sin(\omega t + \varphi) = 15\ sin(62.5\pi t + \varphi)\ V$$

We can find the initial phase (φ) at t = 0

From the graph, at t = 0,

$$v(0) = 8 = 15\sin(62.5\pi \times 0 + \varphi)$$

$$8 = 15\sin(\varphi)$$

$$\varphi = \sin^{-1}\left(\frac{8}{15}\right)$$

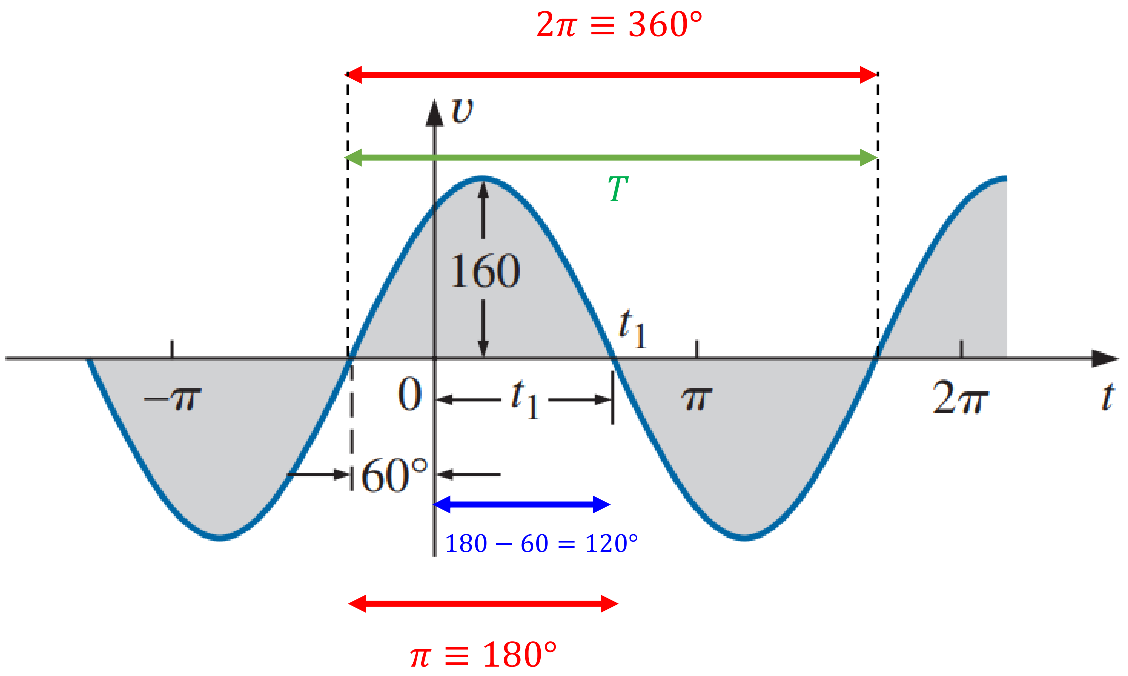
$$\varphi = 32.23°$$

So, the final equation becomes

$$v(t) = A_m sin(\omega t + \varphi) = 15\ sin(62.5\pi t + 32.23°)\ V$$

Problem 7

- A sinusoidal voltage $v(t) = 160\sin(2\pi 1000t + 60°)$ is plotted as a function of time t below. Determine the time t_1 when the waveform crosses the axis.



$\boxed{\underline{\text{Ans}}: t_1 = 0.333\,ms}$

From the equation,

$$\omega = 2\pi 1000$$

As,

$$\omega = \frac{2\pi}{T} = 2\pi 1000$$

$$T = 1\,ms$$

From concept of Sinusoidal,

$$360° \equiv T \equiv 1\,ms$$

To find t_1,

$$120° \equiv \frac{1\,ms}{360°} \times 120° = 0.333\,ms$$

Leading and Lagging Sinusoids

- The equation for a sinusoid in more general form,

$$v(t) = V_m \, sin(\omega t + \phi)$$

where $(\omega t + \varphi)$ is the argument and φ is the initial phase. Both argument and phase can be in radians or degrees.

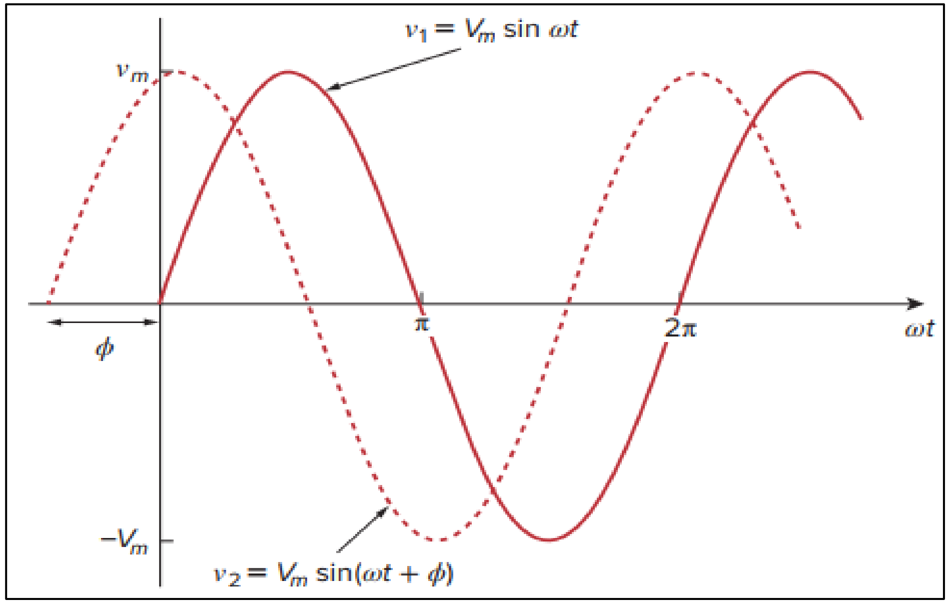
- Let's examine two sinusoids $v_1(t) = V_m sin\omega t$ and $v_2(t) = V_m sin(\omega t + \phi)$

- The starting point of v_2 occurs first in time. **Or v_2 passes the zero-crossing line first** if compared between two same phase points of v_1 and v_2.

- Therefore, we say that v_2 leads v_1 by φ or that v_1 lags v_2 by φ.

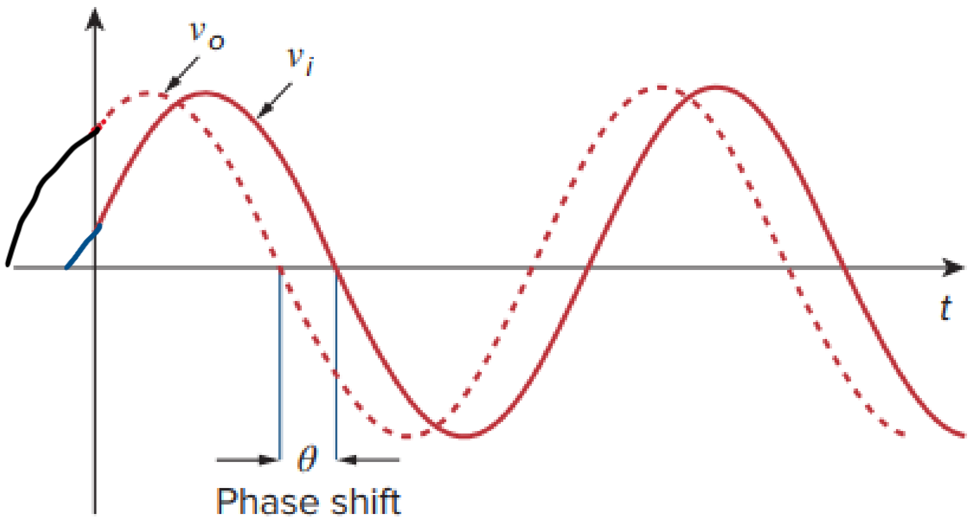
- If $\varphi \neq 0$, v_1 and v_2 are out of phase.

- If $\varphi = 0$, v_1 and v_2 are in phase



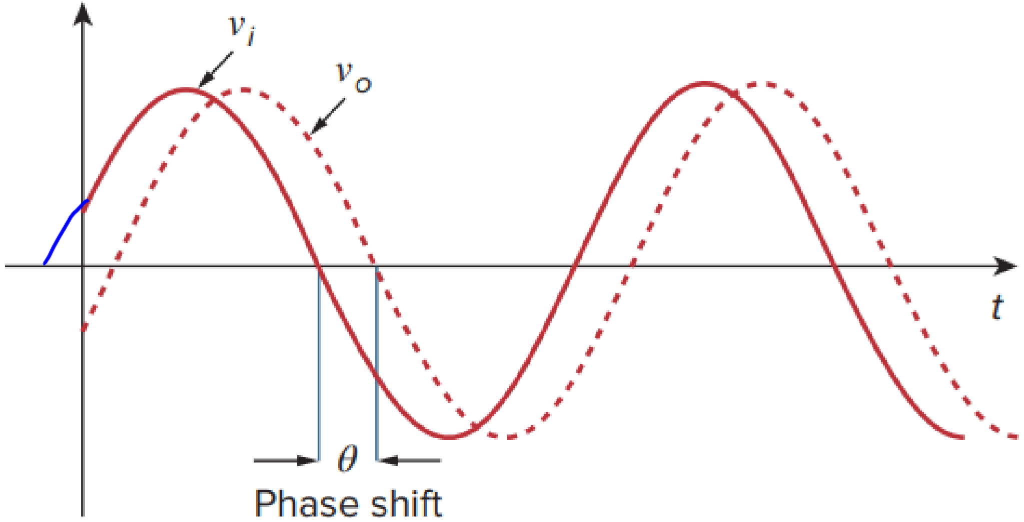
Problem 8

- Determine for each of the plots, which one is leading/lagging.



(a)

v_0 leading



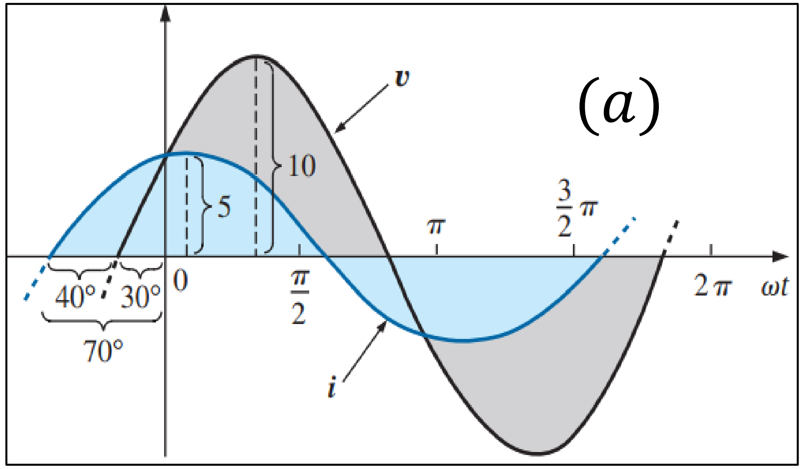
(b)

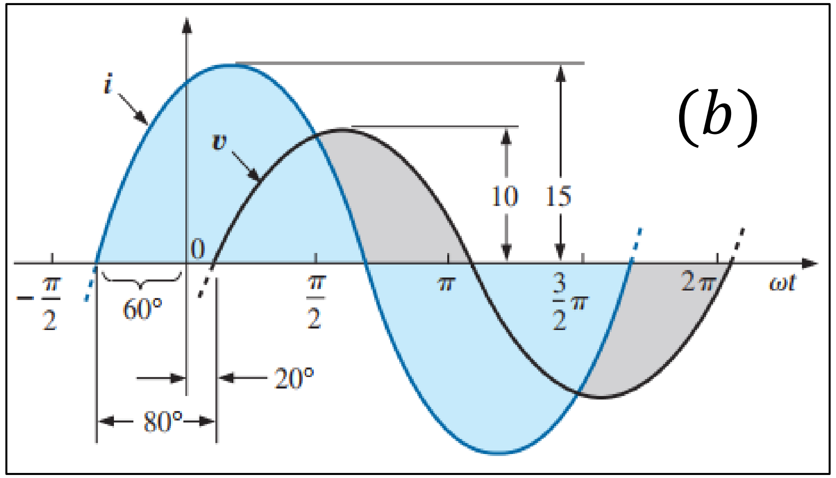
v_i leading

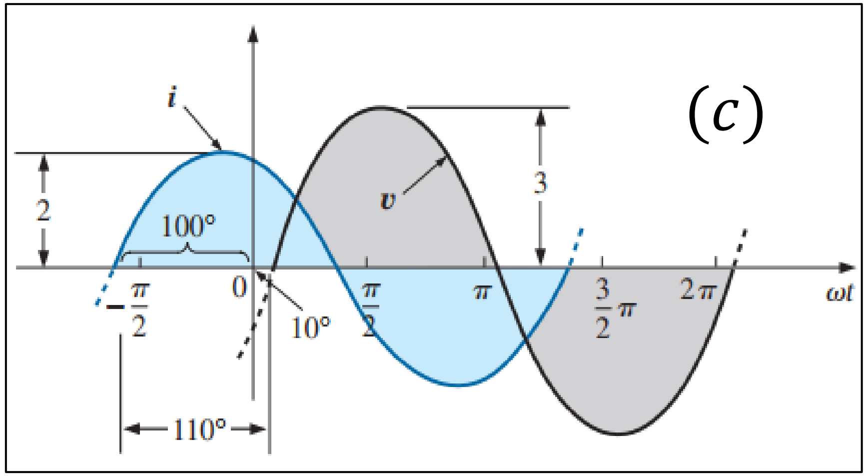
Problem 9

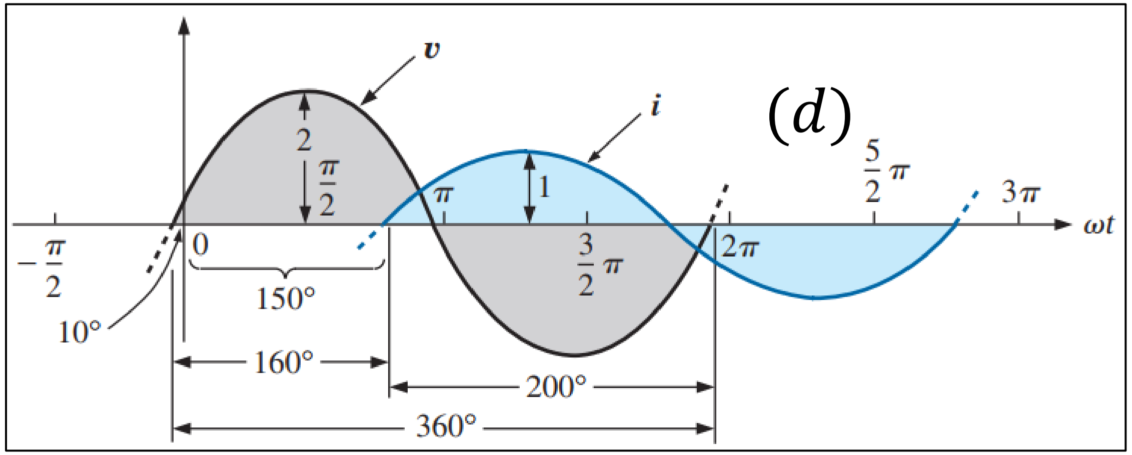
<u>Ans</u>: (a) i leads v by 40°; (b) i leads v by 80°; (c) i leads v by 110°; (d) v leads i by 160°; (e) v and i are in phase

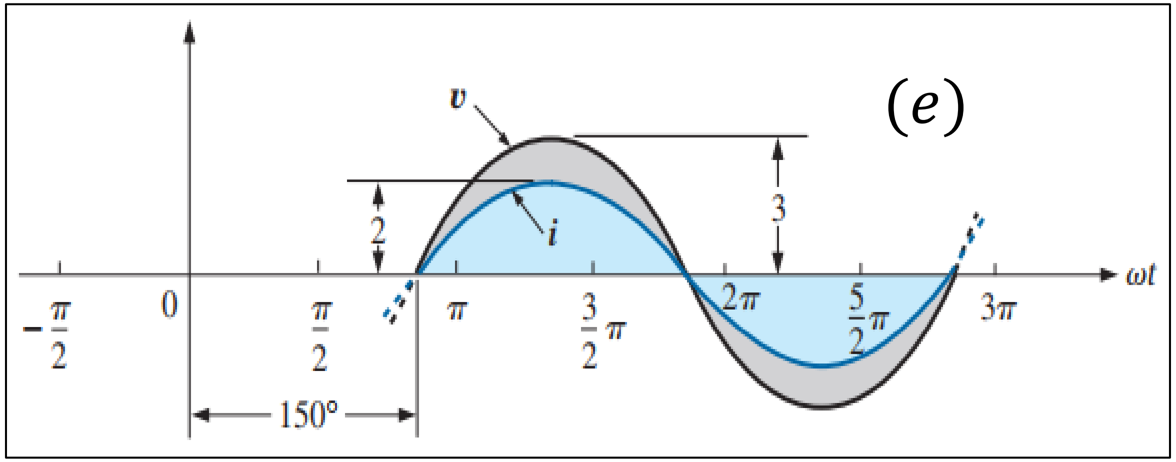
- Determine for each of the plots, which one (v or i) is leading/lagging and by how much.











Problem 10

To solve this type of problem,
- Convert both terms into sin form.
- Make both term positive

- For the following pairs of sinusoids, determine which one leads and by how much $(0 \leq \theta \leq 180°)$.

I. $v(t) = 10\cos(4t - 60°)\ \&\ i(t) = 4\sin(4t + 50°)$

II. $v_1(t) = 4\cos(377t + 10°)\ \&\ v_2(t) = -20\cos 377t$

III. $v_1(t) = 45\sin(\omega t + 30°)\,V\ \&\ v_2(t) = 50\cos(\omega t - 30°)$

IV. $i_1(t) = -4\sin(377t + 55°)\ \&\ i_2(t) = 5\cos(377t - 65°)$

V. $x(t) = \underbrace{(13\cos 2t + 5\sin 2t)}\ \&\ y(t) = 15\cos(2t - 11.8°)$

Hint: convert both the sinusoids into sine or cosine form → convert them into phasors → add them in frequency domain → convert them back in the time domain and compare

Ans: I. i leads v by 20°

II. v_2 leads v_1 by 170°

III. v_2 leads v_1 by 30°

IV. i_2 leads i_1 by 150°

V. y leads x by 9.24°

I. $v(t) = 10\cos(4t - 60°)\ \&\ i(t) = 4\sin(4t + 50°)$

Converting the $v(t)$ into sin,

$$v(t) = 10\sin(4t - 60° + 90°)$$
$$v(t) = 10\sin(4t + 30°)$$

Analyzing $v(t)\ and\ i(t)$,

$i(t)\ leads\ v(t)$ by

$$50° - 30° = 20°$$

II. $v_1(t) = 4\cos(377t + 10°)\ \&\ v_2(t) = -20\cos 377t$

Converting the $v_1(t)$ into sin,

$$v_1(t) = 4\sin(377t + 10° + 90)$$
$$v_1(t) = 4\sin(377t + 100°)$$

Converting the $v_2(t)$ into positive sin,

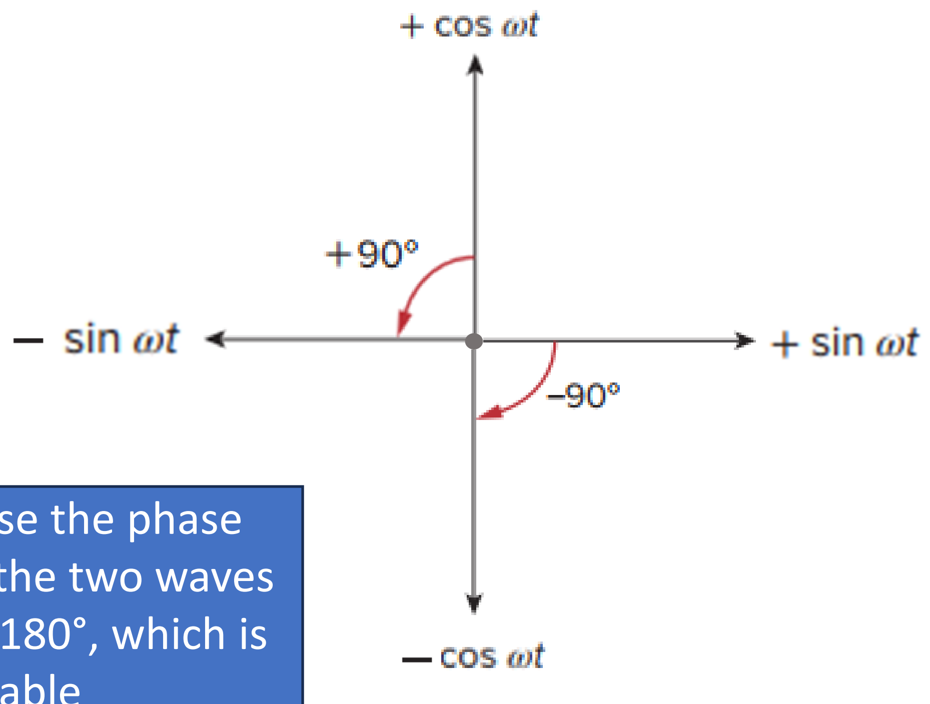
$$v_2(t) = 20\sin(377t - 90°)$$
$$v_2(t) = 20\sin(377t - 90° + 360°) = 20\sin(377t + 270°)$$

Did this because else the phase difference between the two waves will be greater than 180°, which is not acceptable

Analyzing $v_2(t)\ and\ v_1(t)$,

$v_2(t)\ leads\ v_1(t)$ by

$$270° - 100° = 170°$$



III. $v_1(t) = 45\sin(\omega t + 30°)\,V \;\&\; v_2(t) = 50\cos(\omega t - 30°)$

Converting the $v_2(t)$ into sin,

$$v_2(t) = 50\sin(\omega t - 30° + 90°)$$
$$v_2(t) = 50\sin(\omega t + 60°)$$

Analyzing $v_1(t)$ *and* $v_2(t)$,

$v_2(t)$ *leads* $v_1(t)$ by

$$60° - 30° = 30°$$

IV. $i_1(t) = -4\sin(377t + 55°) \;\&\; i_2(t) = 5\cos(377t - 65°)$

Converting the $i_1(t)$ into positive sin,

$$v_1(t) = 4\sin(377t + 55° - 180°)$$
$$v_1(t) = 4\sin(377t - 125°)$$

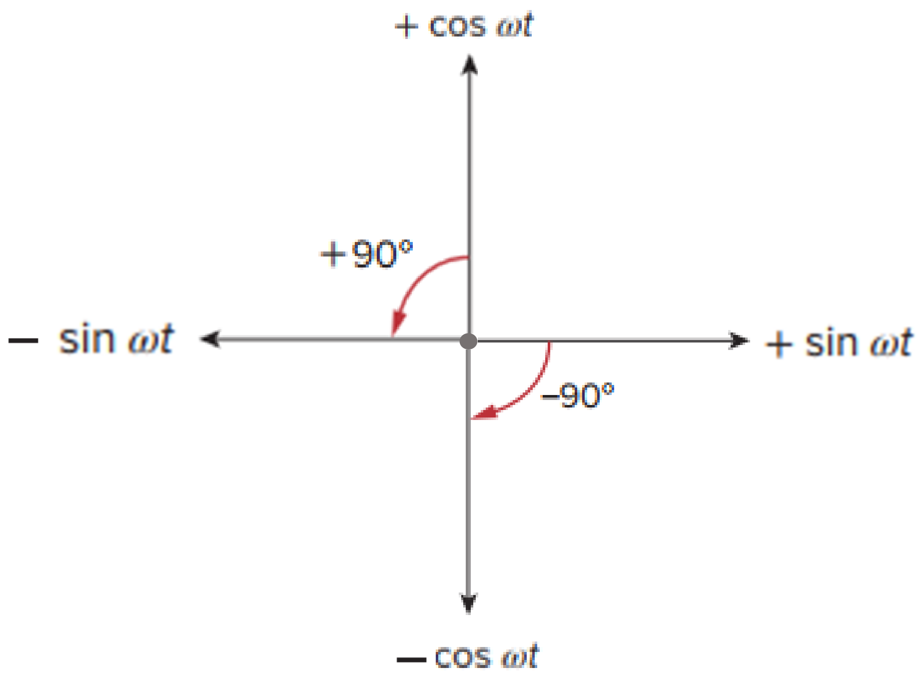
Converting the $i_2(t)$ into sin,

$$i_2(t) = 5\sin(377t - 65° + 90°)$$
$$i_2(t) = 5\sin(377t + 25°)$$

Analyzing $i_2(t)$ *and* $i_1(t)$,

$i_2(t)$ *leads* $i_1(t)$ by

$$25° - (-125)° = 150°$$



Problem 11

- Consider the sinusoidal voltage $v(t) = 80\, cos(1000\pi t - 30°)\, V$.
  a) What is the first time after $t = 0$ that $v(t) = 80\, V$?
  b) If the sinusoidal voltage is shifted $^2/_3\, ms$ to the left along the time axis, what will be the expression for $v(t)$?
  c) What is the minimum number of microseconds that the function must be shifted to the (right/left?) if the expression for $v(t)$ is $80\sin(1000\pi t)\, V$?

<u>Ans</u>:

(a) $t = 166.67\ \mu s$

(b) $80\cos(1000\pi t - 150°)\ V$

(c) $t = 333.33\ \mu s$

Solution Problem 11

(a) What is the first time after $t = 0$ that $v(t) = 80\ V$?

$$v(t) = 80\ cos(1000\pi t - 30°)\ V$$

For $v(t) = 80\ V$,

$$80 = 80\ cos(1000\pi t - 30°)\ V$$

$$\Rightarrow 80 = 80\cos\left(\frac{180}{\pi}1000\pi t - 30\right)$$

$$\Rightarrow 1 = \cos(180000t - 30)$$

$$\Rightarrow \cos^{-1}(1) = 180000t - 30$$

$$\Rightarrow 0 = 180000t - 30$$

$$\Rightarrow t = 1.667 \times 10^{-4}\ s = \mathbf{0.1667\ ms}$$

(b) If the sinusoidal voltage is shifted $2/3\ ms$ to the **left** along the time axis, what will be the expression for $v(t)$?

For this, let us find out the frequency from the equation

$$v(t) = 80\ cos(1000\pi t - 30°)\ V$$

$$f = \frac{1000\pi}{2\pi} = 500\ Hz$$

$$T = \frac{1}{f} = 2 \times 10^{-3}\ s = 2\ ms$$

Now, let's find the change of theta corresponding to the 2/3 ms shift

$$2\ ms \equiv 360°$$

$$\frac{2}{3}ms \equiv \frac{360}{2} \times \frac{2}{3} = 120°$$

$$v(t) = 80\ cos(1000\pi t - 30° - 120°)\ V$$

$$\boldsymbol{v(t) = 80\ cos(1000\pi t - 150°)\ V}$$

Why -120°? Because the graph shifted to the left. Hence it is leading more.

Solution Problem 11 (Continued)

What is the minimum number of microseconds that the function must be shifted to the (right/left?) if the expression for $v(t)$ is 80 sin($1000\pi t$) V?

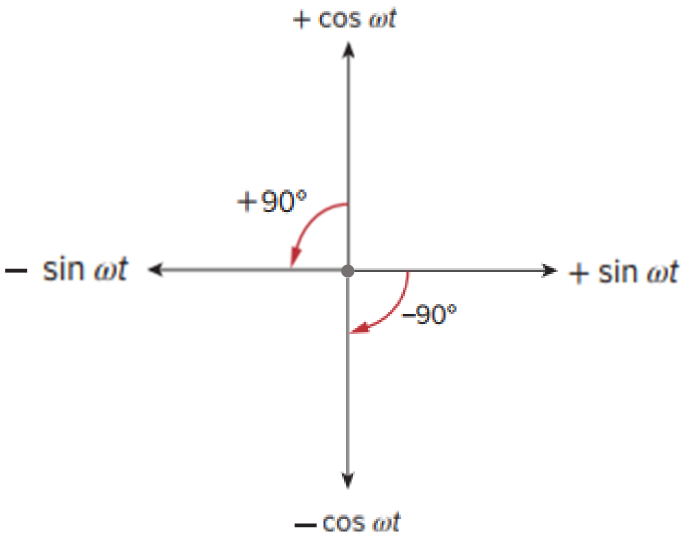
$$v(t) = 80\, cos(1000\pi t - 30°)\, V$$

To convert this equation into sin form, we need -90 term inside the cos. As we can see that there is -30° already there. We can subtract 60° to make the term -90. This means shifting to the left (due to minus)

$$v(t) = 80\, cos(1000\pi t - 30° - 60°)\, V$$

$$v(t) = 80\, cos(1000\pi t - 90°)\, V$$

$$v(t) = 80\, sin(1000\pi t)\, V$$



So, we shifted the function to the left by 60 °. So,

$$360° \equiv 2\, ms\ (Time\ period)$$

$$60° \equiv \frac{2}{360°} 60°\, ms = \mathbf{0.333\, ms}$$

Problem 12

- At $t = -2\ ms$, a sinusoidal voltage $v(t)$ is known to be zero and going positive. The voltage is next zero at $t = 8\ ms$. It is also known that the voltage is $10\ V$ at $t = 0$. What is the expression fot $v(t)$?

Solution:

$$v(t) = A\ sin(2\pi f t + \theta)\ V$$

Amplitude Frequency

Initial phase

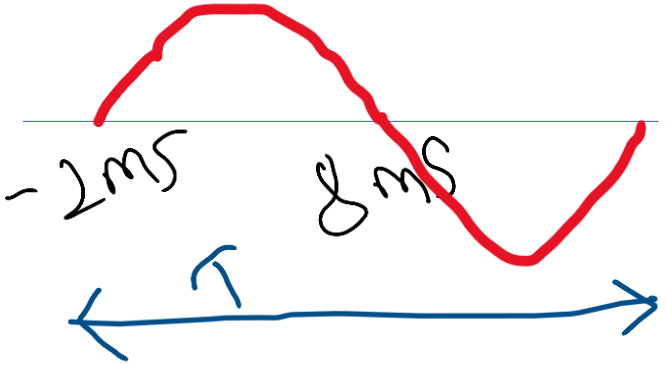
To find the expression for v(t), we need to find these 3 values

At $t = -2\ ms$, a sinusoidal voltage $v(t)$ is known to be zero and going positive. The voltage is next zero at $t = 8\ ms$.

$$\frac{T}{2} = 8 - (-2) = 10\ ms$$

$$\Rightarrow T = 20\ ms = 20 \times 10^{-3} s$$

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = \mathbf{50\ Hz}$$



Solution Problem 12 (Continued)

It is said that at $t = -2\ ms$, a sinusoidal voltage $v(t)$ is known to be zero

$$v(t) = A\ sin(2\pi f t + \theta)\ V$$

$$0 = A\ sin(2\pi 50(-2 \times 10^{-3}) + \theta)$$

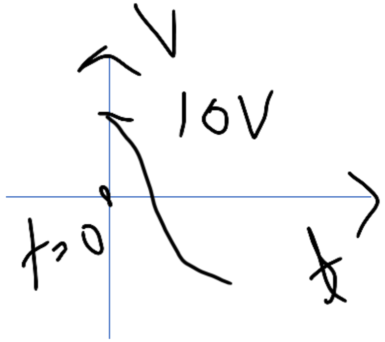
$$\Rightarrow \sin^{-1} 0 = 2\pi \times \frac{180}{\pi} 50(-2 \times 10^{-3}) + \theta$$

$$\Rightarrow 0 = -36 + \theta$$

$$\Rightarrow \boldsymbol{\theta = 36°}$$

It is also said that, voltage is $10\ V$ at $t = 0$.

$$v(t) = A\ sin(2\pi f t + \theta)\ V$$



$$v(t) = A\ sin(2\pi 50 t + 36°)$$

$$\Rightarrow 10 = A\ sin(2\pi 50 \times 0 + 36°)$$

$$\Rightarrow 10 = A\ sin(36°)$$

$$\Rightarrow \boldsymbol{A = 17\ V}$$

$$\boldsymbol{v(t) = 17\ sin(2\pi 50 t + 36°)\ V}$$

Problem 13

- At $t = 6\ ms$, a sinusoidal voltage $v(t)$ is known to be zero and going negative. At $t = 2\ ms$, the voltage reaches its first peak after zero. It is also known that the voltage is $-9.9\ V$ at $t = 0$. What is the expression fot $v(t)$?

TRY Yourself

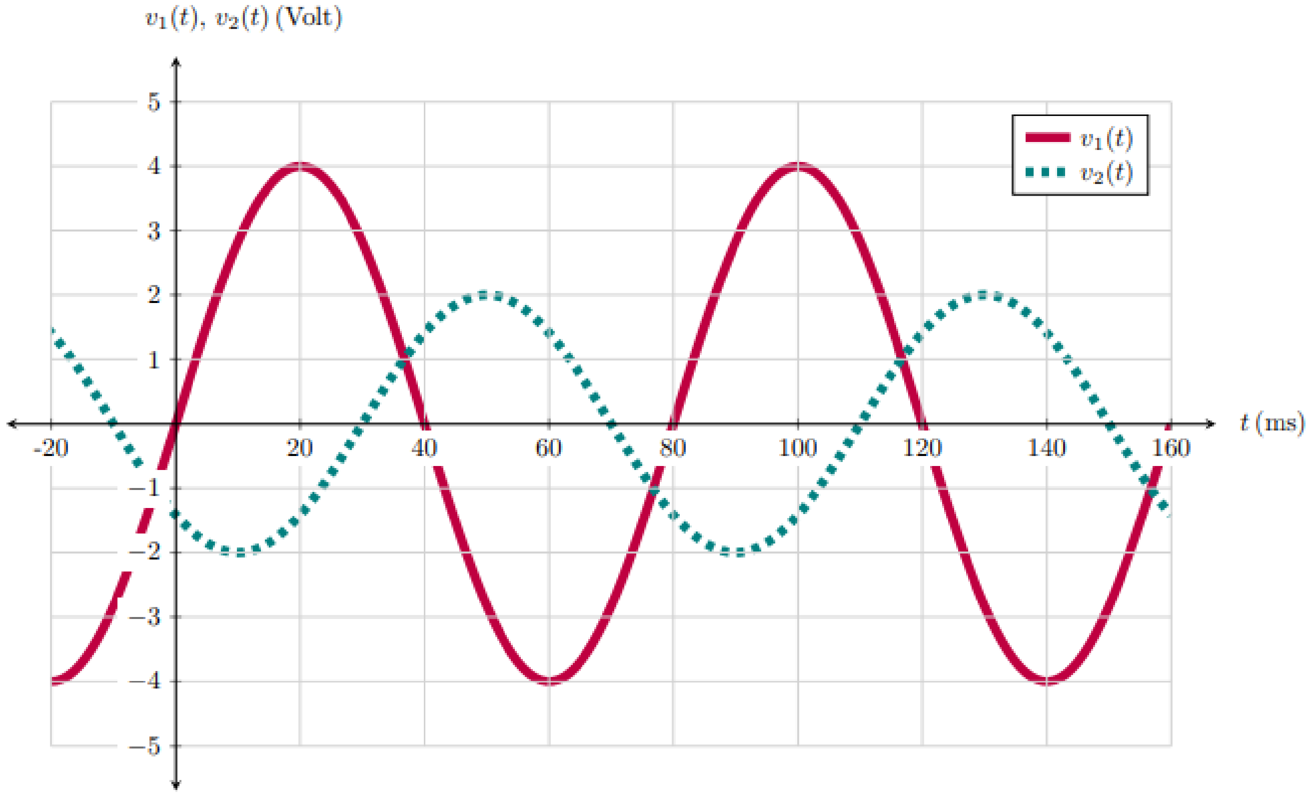
Ans: $v(t) = 14\ sin(125\pi t - 135°)\ V$ or $14\ cos(125\pi t - 165°)\ V$

Problem 14

Two ac voltages $v_1(t)$ and $v_2(t)$ from an ac circuit are plotted below.

a) Which one is leading and by how much in degrees?

b) Write analytical expressions for $v_1(t)$ and $v_2(t)$.



Summer'24

Ans:

(a) $\Delta\varphi = 135^o$

(b) $v_1(t) = 4\sin(25\pi t)\ V$

$v_2(t) = 4\sin(25\pi t - 135^o)\ V$

a) For both waves,

$$Time\ period, T = 80\ ms$$

Time difference between waves (Consider both the waves sin wave and consider the starting point of the sin wave. The starting point should be close to the origin. We are considering the blue dots as the starting points for the waves)

$$Time\ difference = 30\ ms$$

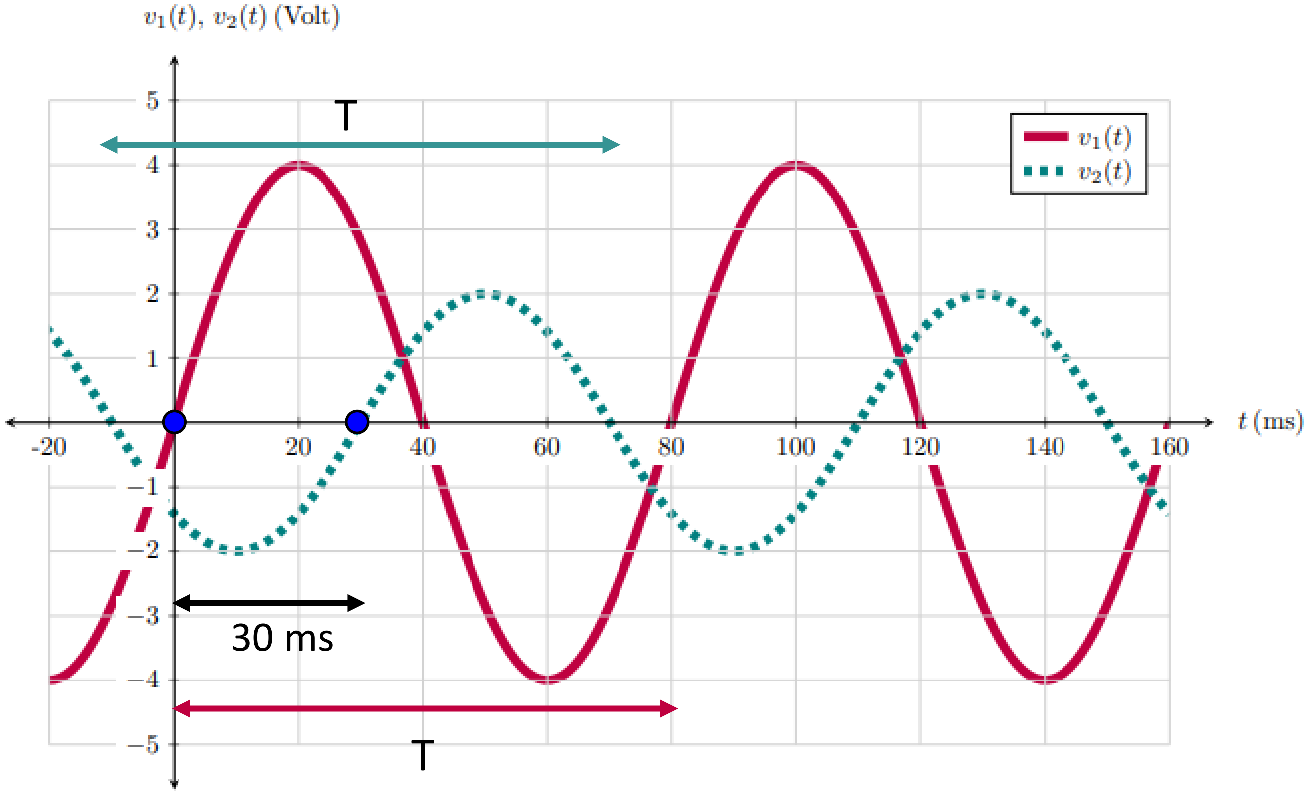
Let's convert this time into phase

$$80\ ms \equiv 360°$$

$$80\ ms \equiv 360° \frac{30}{80} = 135°$$

Starting point of $v_1(t)$ is to the left of the starting point of $v_2(t)$.

So, $v_1(t)$ is leading $v_2(t)$ by 135°



b) For both waves,

$$Time\ period, T = 80\ ms$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{80 \times 10^{-3}} = 25\pi$$

For $v_1(t)$

Amplitude: 4 V

$initial\ phase$: 0°

For $v_2(t)$

Amplitude: 2V

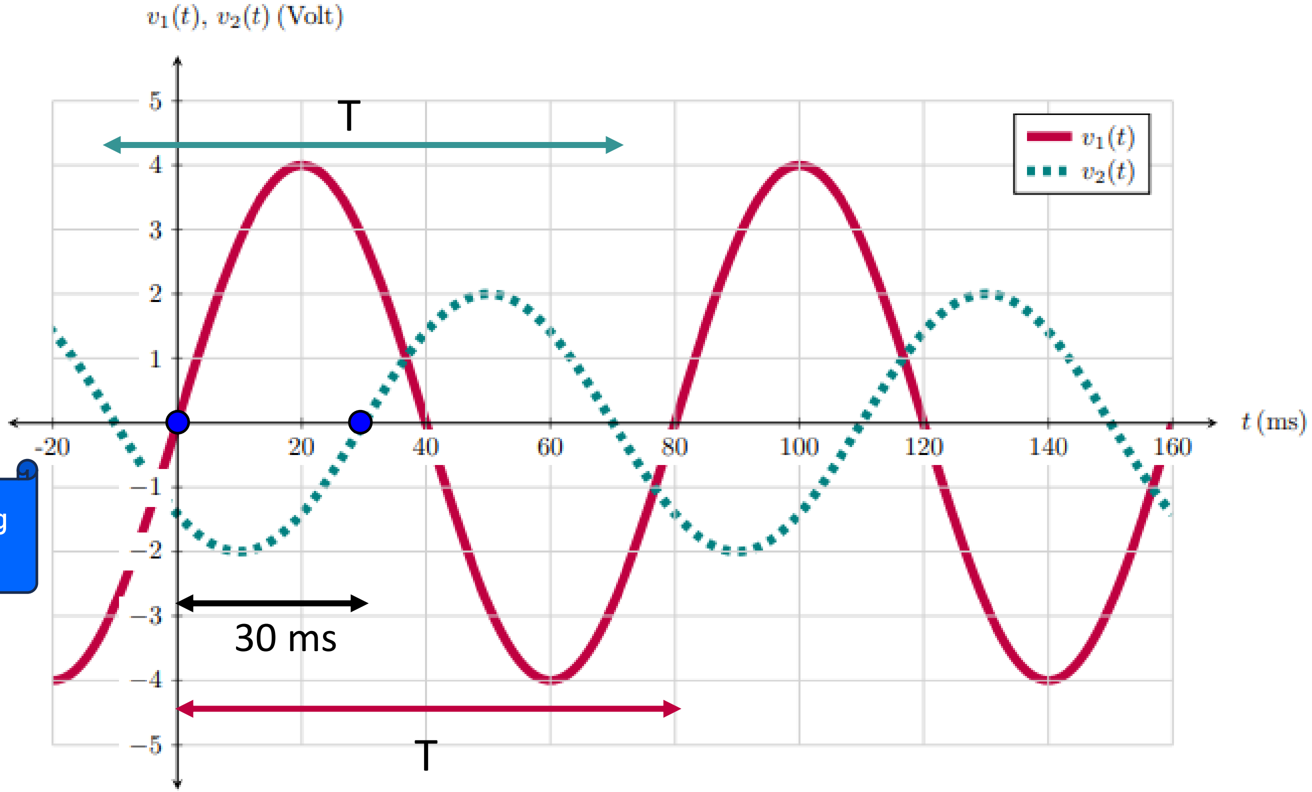
- Sign because starting point is right of origin

$$initial\ phase: \quad -30\ ms \equiv 360° \frac{-30}{80} = -135°$$

Expressions:

$$v(t) = 4\sin(25\pi t)\ V$$

$$v_2(t) = 2\sin(25\pi t - 135^o)\ V$$

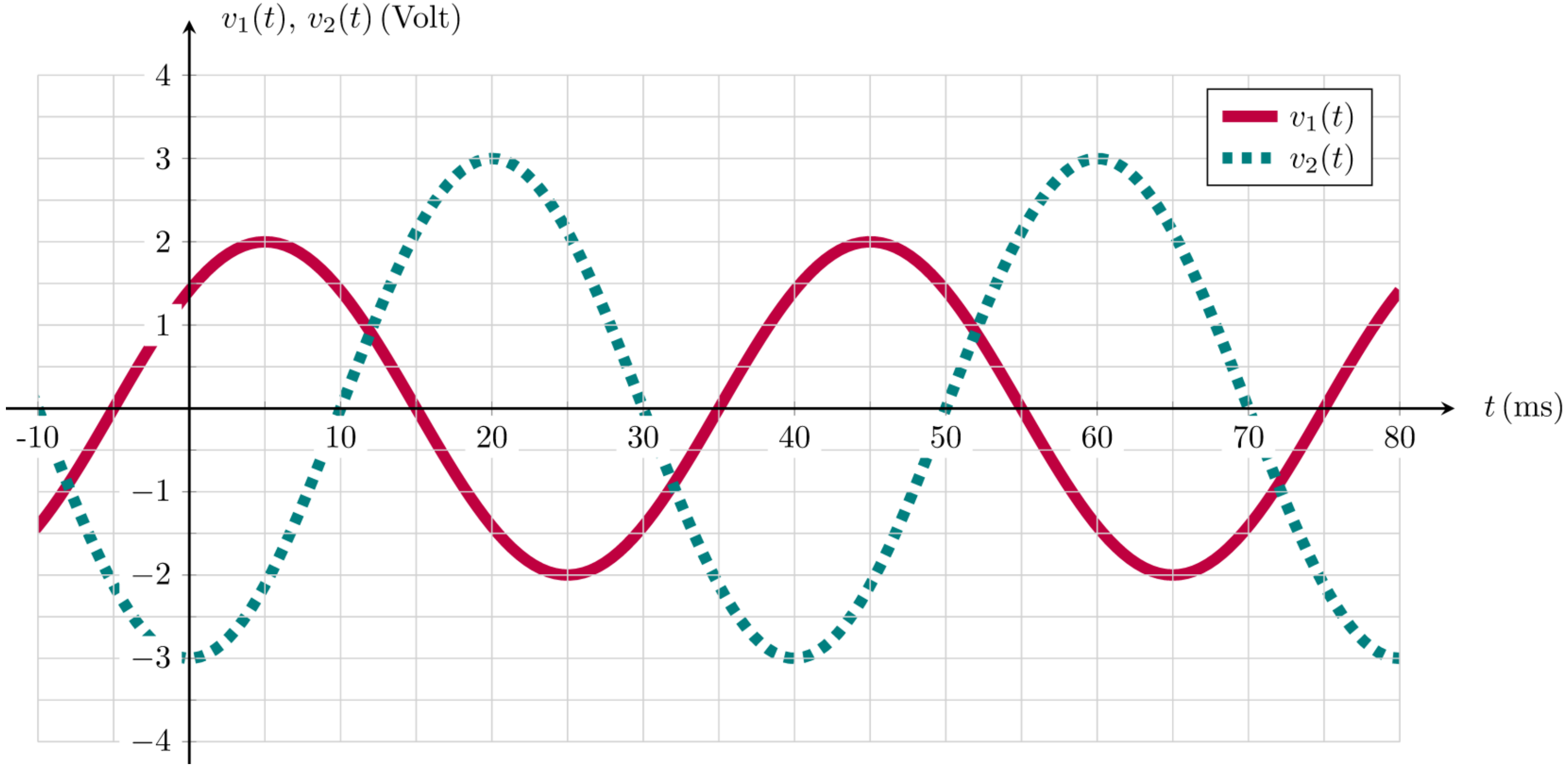


Problem 15

Two ac voltages $v_1(t)$ and $v_2(t)$ from an ac circuit are plotted below.

a) Which one is leading and by how much in degrees?

b) Write analytical expressions for $v_1(t)$ and $v_2(t)$ in the format $A_m\, cos(\omega t \pm \varphi)$.



<u>Ans</u>:

(a) $\Delta\varphi = 135^o$

(b) $v_1(t) = 2\cos(50\pi t - 45°)\ V$

$v_2(t) = 3\cos(50\pi t - 180°)\ V$

Solution Problem 15 (Continued)

Though, in the problem it said the waves are to be represented with cos. However, we will consider them sin wave, do all the calculations and convert them to cos at the last. Making calculations considering sin wave is easier

a) For both waves,

$$Time\ period, T = 40\ ms$$

Time difference between waves (Consider both the waves sin wave and consider the starting point of the sin wave. The starting point should be close to the origin. We are considering the blue dots as the starting points for the waves)

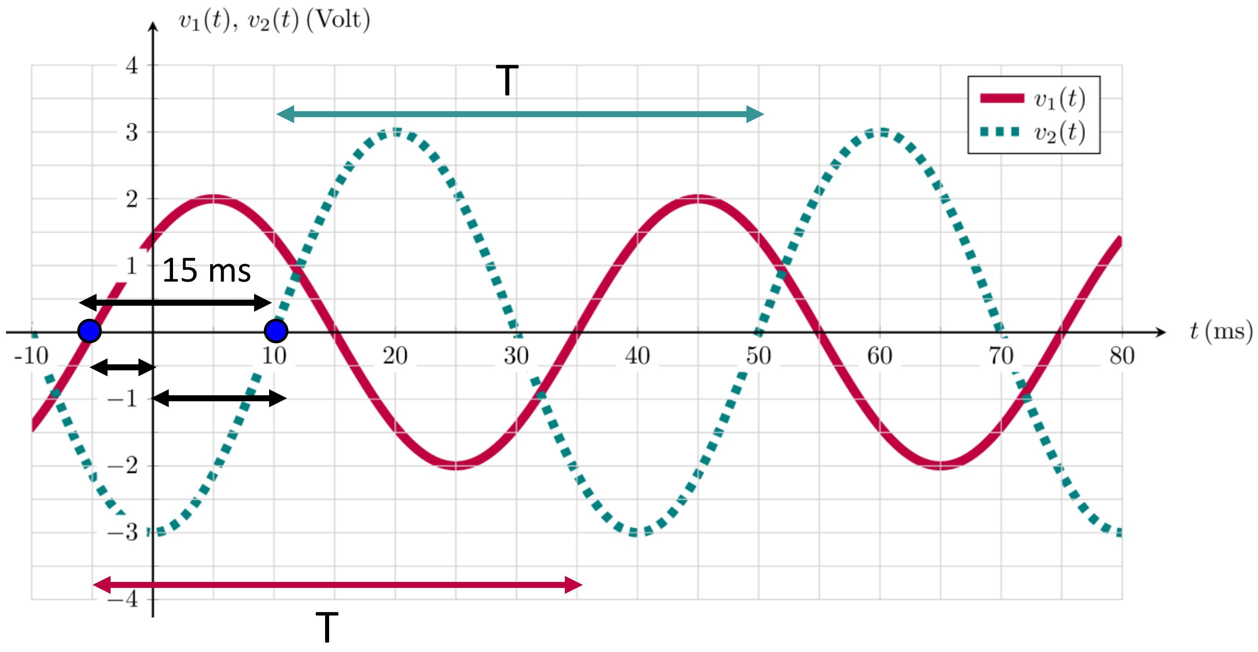
$$Time\ difference = 15\ ms$$

Let's convert this time into phase

$$40\ ms \equiv 360°$$
$$15\ ms \equiv 360° \frac{15}{40} = 135°$$

Starting point of $v_1(t)$ is to the left of the starting point of $v_2(t)$.

So, $v_1(t)$ is leading $v_2(t)$ by 135°



Solution Problem 15 (Continued)

b) For both waves,

$$Time\ period, T = 40\ ms$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{40 \times 10^{-3}} = 50\pi$$

For $v_1(t)$

Amplitude: 2 V

+ Sign because starting point is left of origin

$$initial\ phase: \quad 5\ ms \equiv 360° \frac{5}{40} = +45°$$

For $v_2(t)$

Amplitude: 3 V

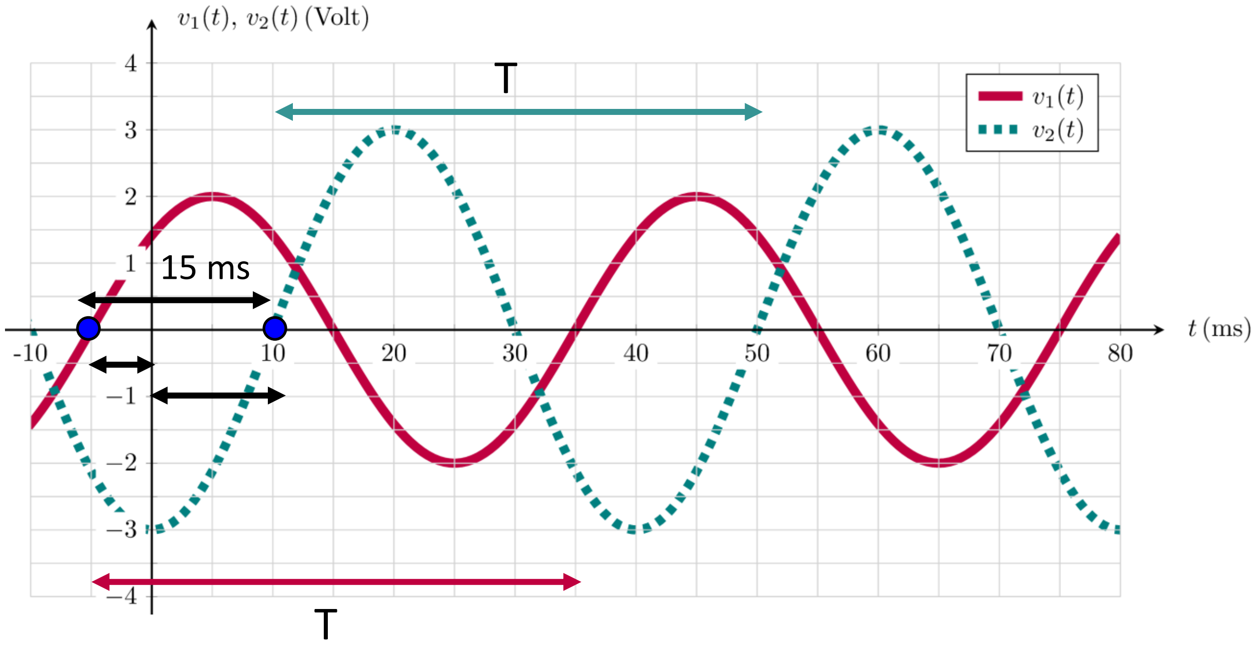
– Sign because starting point is right of origin

$$initial\ phase: \quad -10\ ms \equiv 360° \frac{-10}{40} = -90°$$

Expressions:

$$v_1(t) = 2 \sin(50\pi t + 45°)\ V$$

$$v_2(t) = 3 \sin(50\pi t - 90°)\ V$$



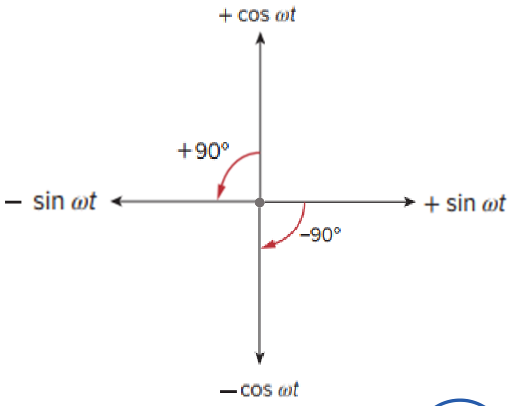
Converting sin wave equation to cos wave

$$v_1(t) = 2 \cos(50\pi t + 45° - 90°)\ V$$

$$\Rightarrow \boldsymbol{v_1\ (t) = 2\cos(50\pi t - 45°)\ V}$$

$$v_2(t) = 3 \cos(50\pi t - 90° - 90°)\ V$$

$$\Rightarrow \boldsymbol{v_2(t) = 3\cos(50\pi t - 180°)\ V}$$

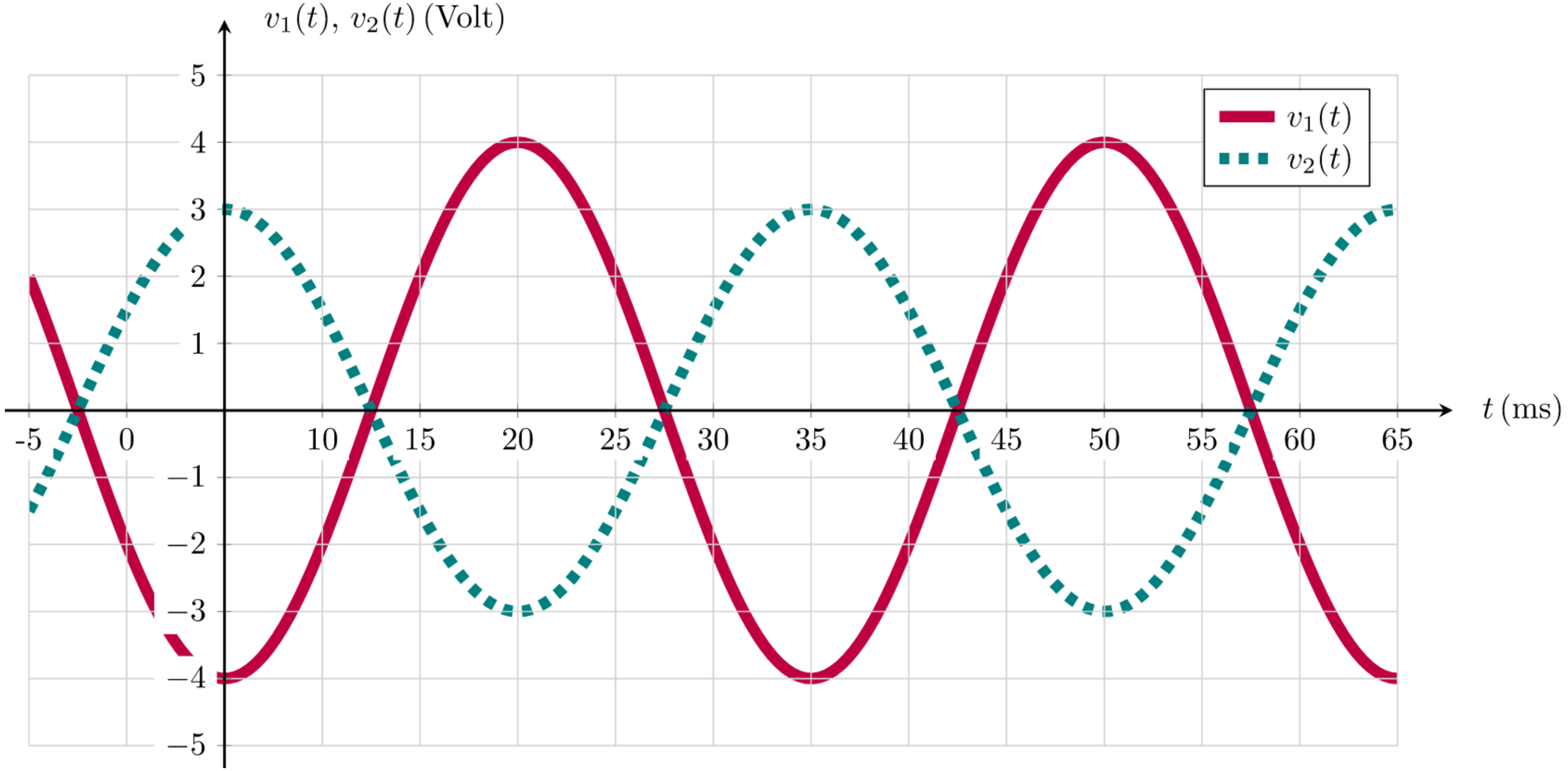


Problem 16

Two ac voltages $v_1(t)$ and $v_2(t)$ from an ac circuit are plotted below.

a) Which one is leading and by how much in degrees?

b) Write analytical expressions for $v_1(t)$ and $v_2(t)$ in the format $A_m\, sin(\omega t \pm \varphi)$.



<u>Ans</u>:

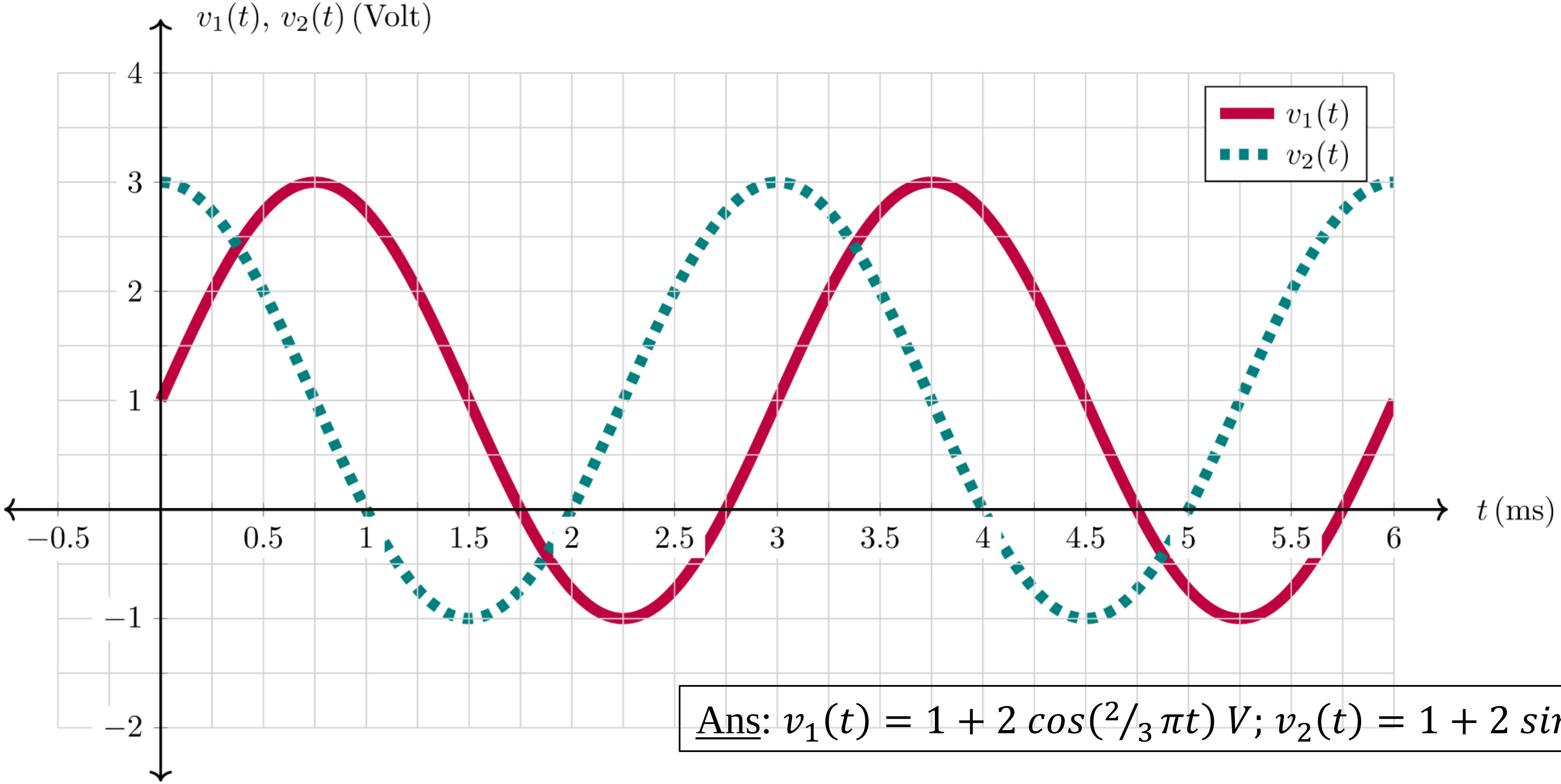
(a) $\Delta\varphi = 180^o$

(b) $v_1(t) = 4\sin(57\pi t - 90°)\ V$

$v_2(t) = 3\sin(57\pi t + 90°)\ V$

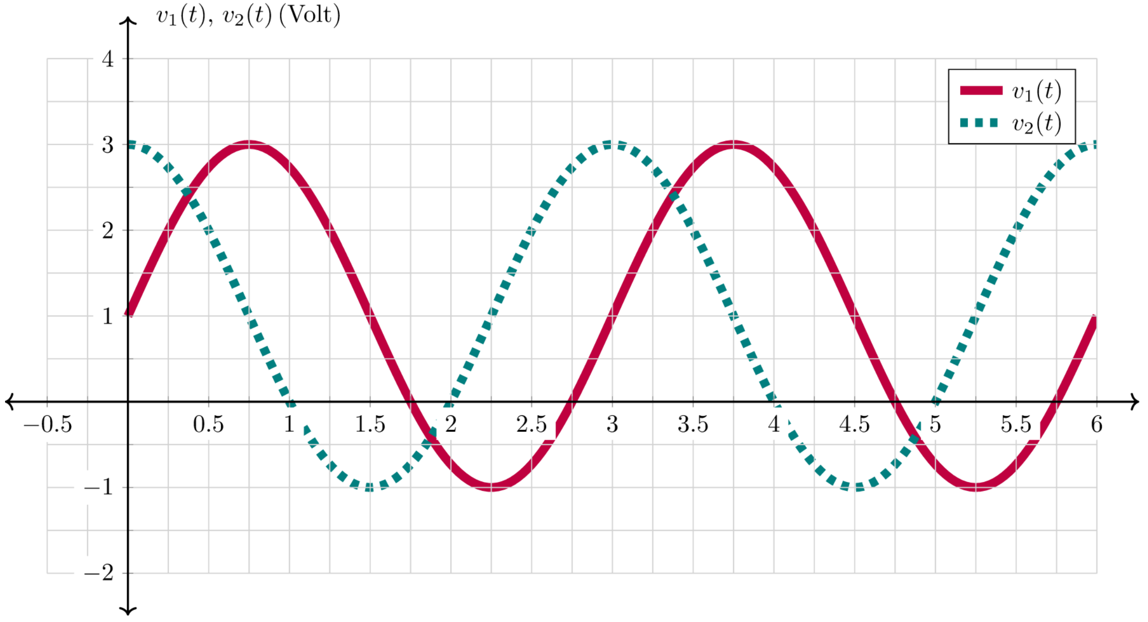
Problem 17

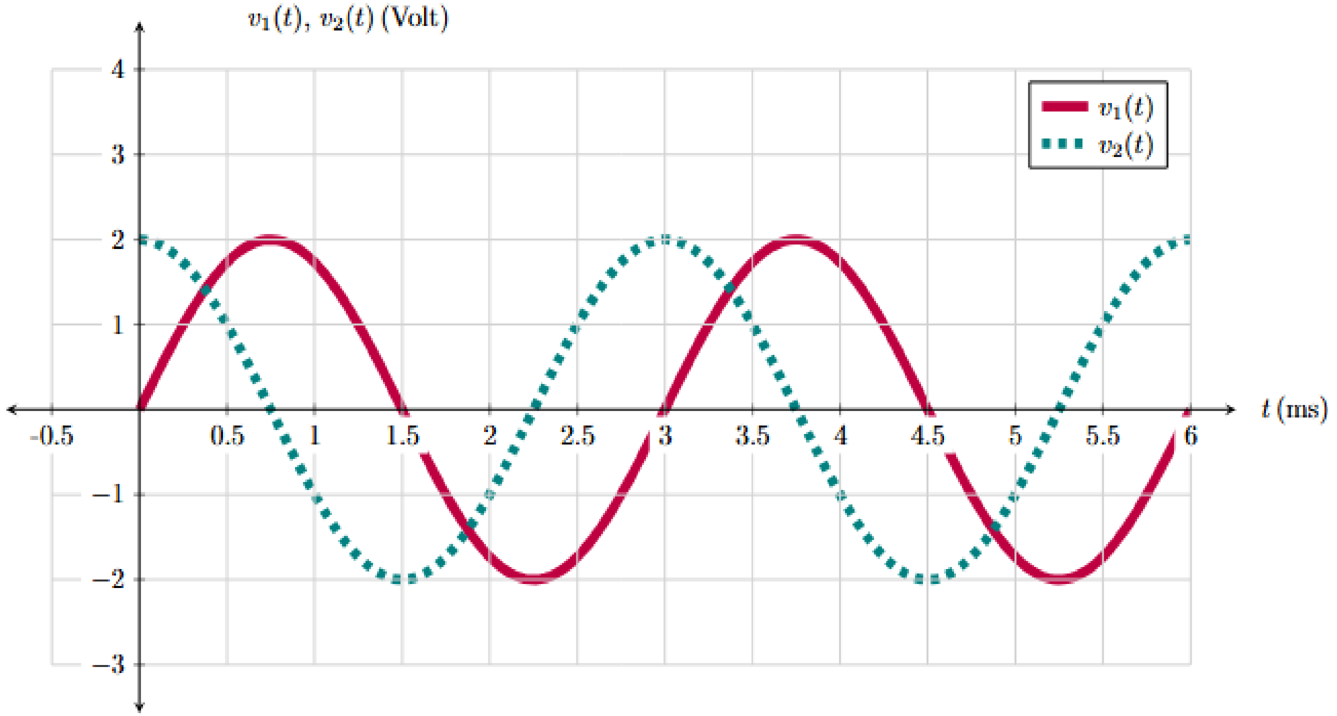
- Write analytical expressions, as a function of t, for v_1 and v_2.



Ans: $v_1(t) = 1 + 2\,cos(^2/_3\,\pi t)\ V$; $v_2(t) = 1 + 2\,sin(^2/_3\,\pi t)\ V$

Solution Problem 17 (Continued)



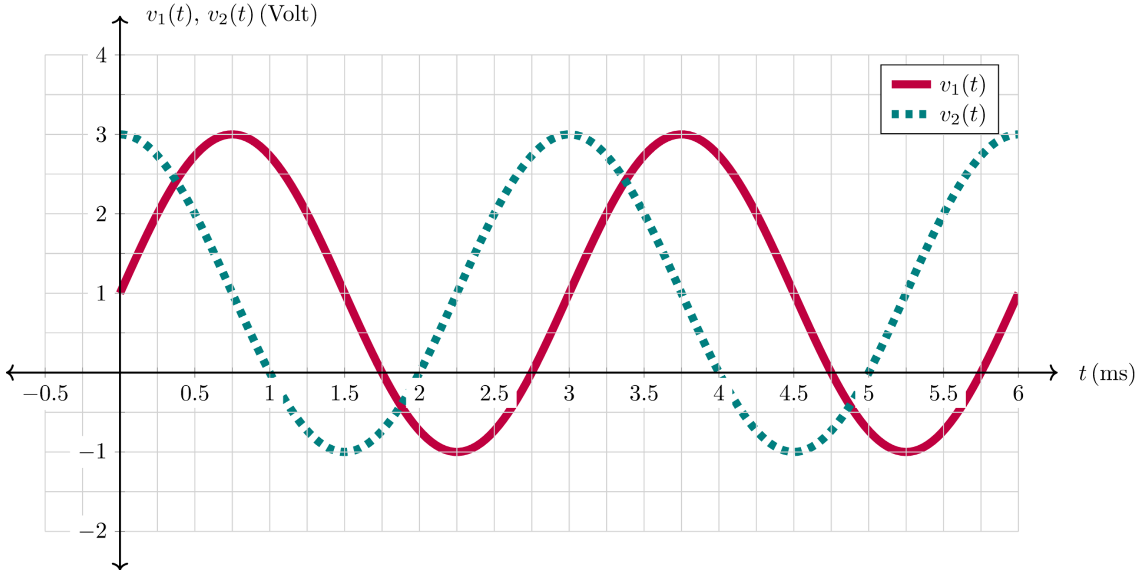


Both of the graphs are shifted upward. Because they are not equally divided by the x axis line. Meaning that, there is DC offset.

This is what the non shifted version of the graph would look like

Solution Problem 17 (Continued)

To find the equations of the waveform. First forget there is any DC offset and imagine (try to draw) nonshifted version of the graphs like the one below.



For both $v_1(t)$ and $v_2(t)$, **Time period:**

$$T = 3\,ms = 3 \times 10^{-3}\,s$$

$$f = \frac{1}{T}$$

Amplitude:

$$V_1 = V_2 = 2\,V$$

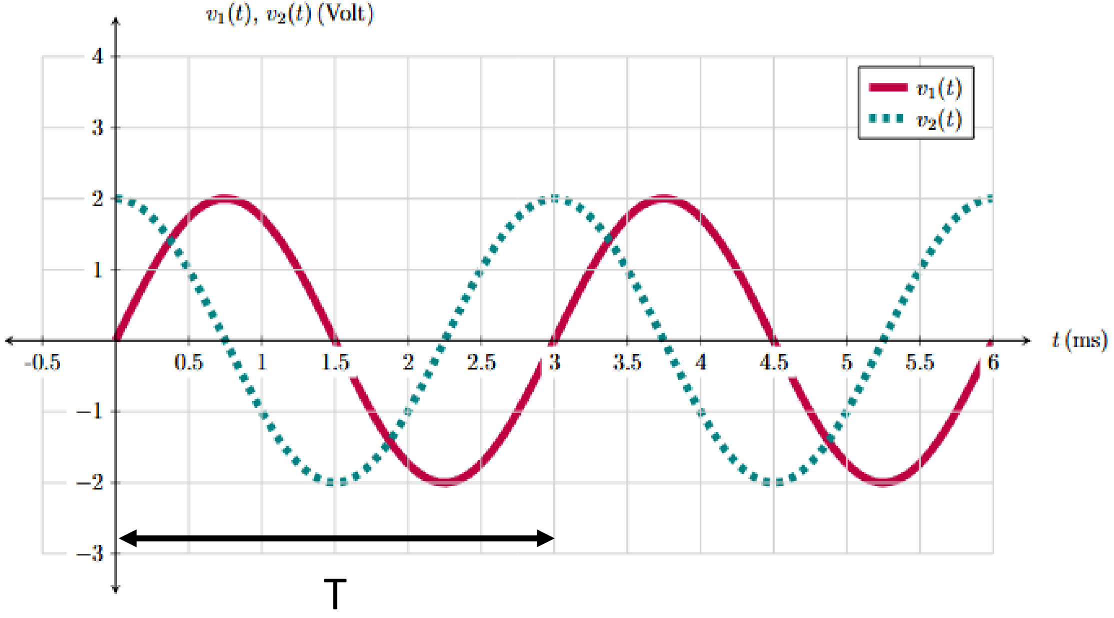
Phase shift:

If we consider $v_1(t)$ sin wave,

$$\theta_1 = 0$$

If we consider $v_2(t)$ cos wave,

$$\theta_2 = 0$$



Solution Problem 17 (Continued)

What about DC offset?

For both $v_1(t)$ and $v_2(t)$

DC Offset:

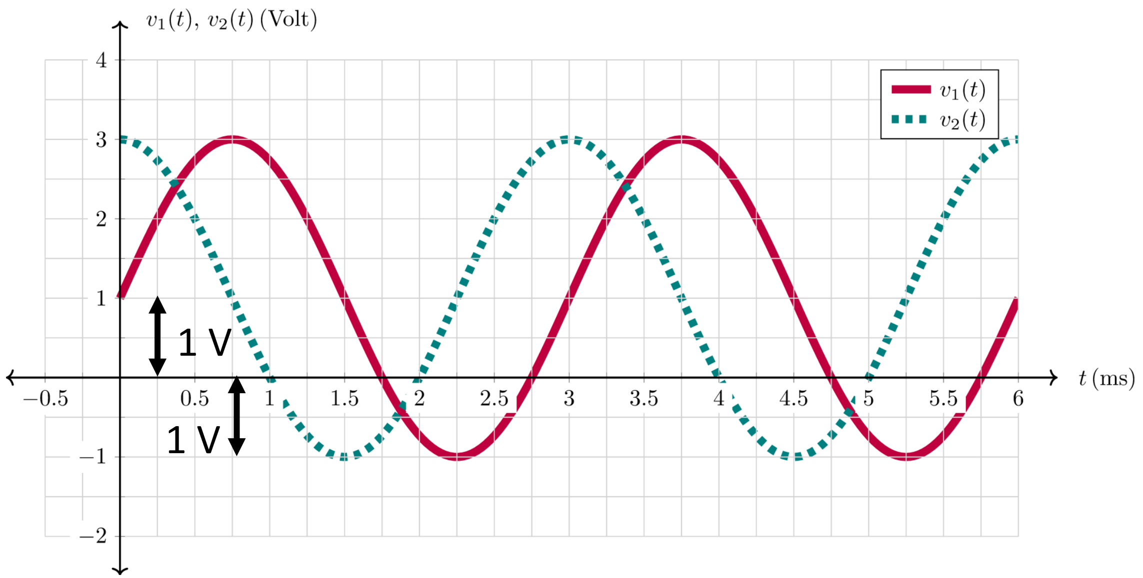
$$+1\,V$$

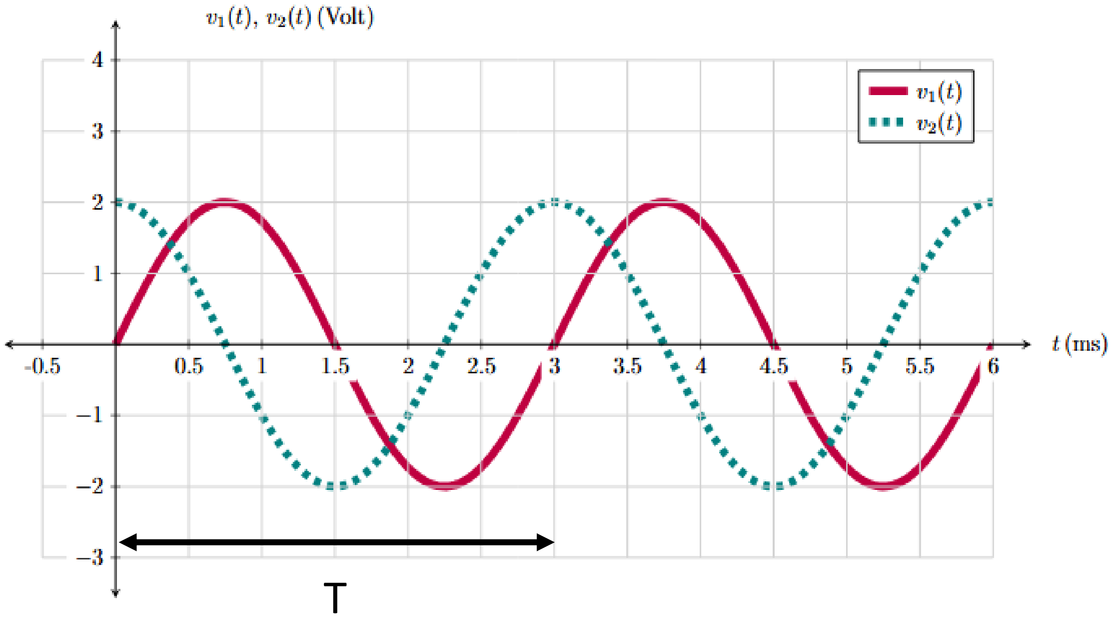
Final Equation

$$v_1(t) = 1 + 2\,sin(2\pi 333.33t)\,V$$

$$v_2(t) = 1 + 2\,cos(2\pi 333.33t)\,V$$

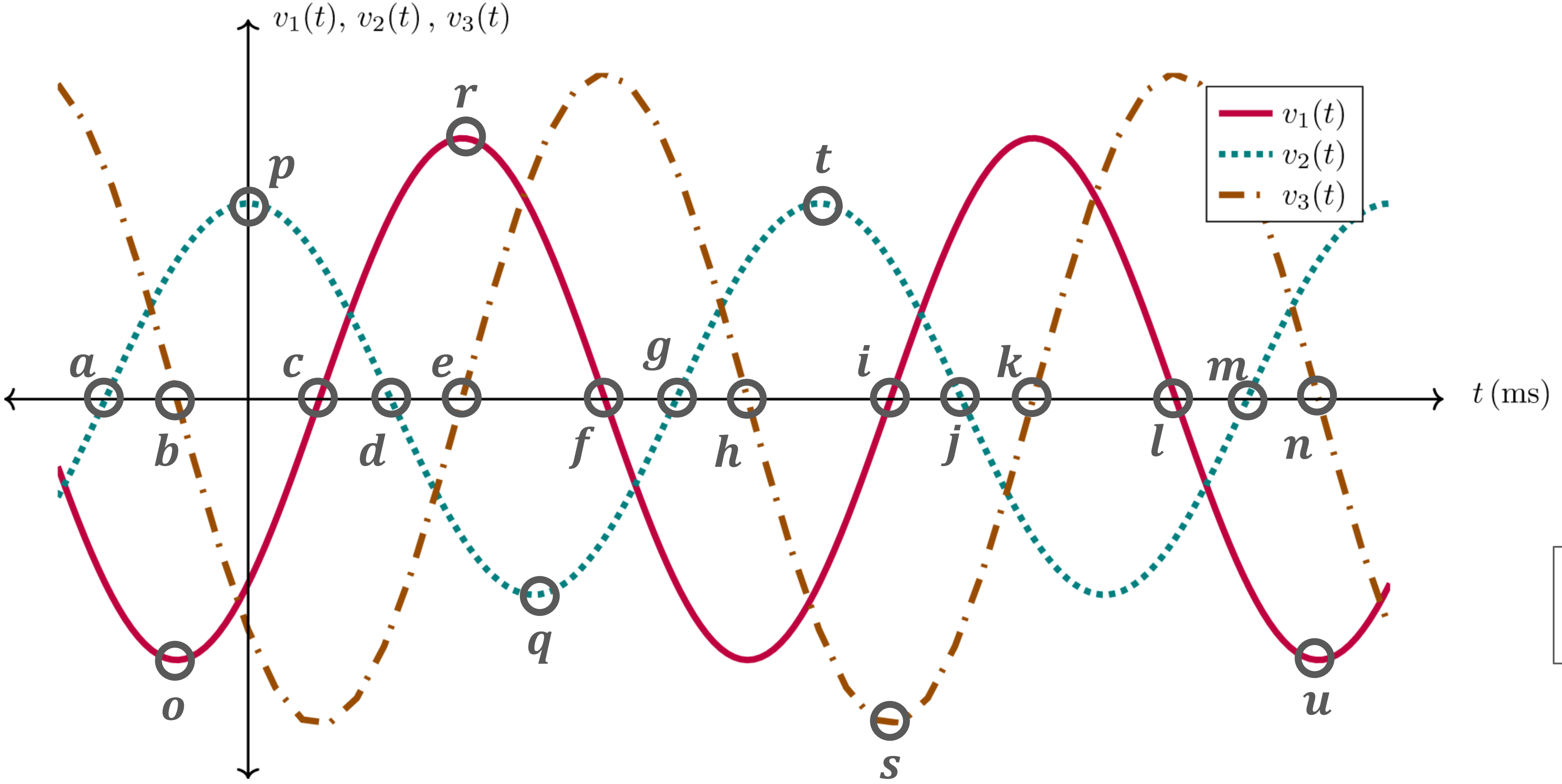
where t is in units of seconds





Problem 18

- In the following figure, certain locations on the plots are marked with circles and labelled from a to u. Create sets by grouping the labels that correspond to the same phase.



Hint: There will be 4 such sets

Practice

Books: Introductory Circuit Analysis (11th edition)

Chapter 13

Link: https://drive.google.com/drive/folders/1k7JLZnhEzP2cMwrjG9ggIHJxUCUYcZXM?usp=sharing

Example:

13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7

Exercise:

1, 2, 3, 25, 26, 27, 28, 34, 35