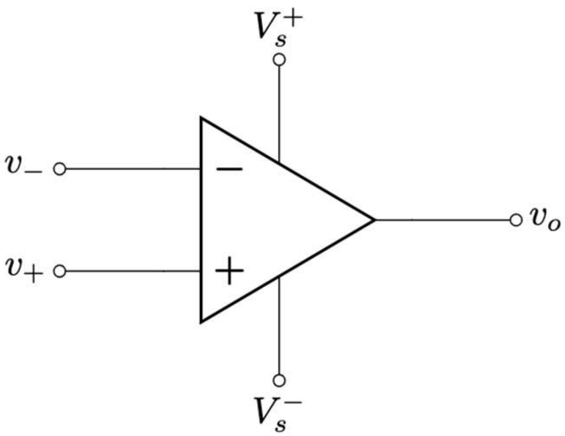
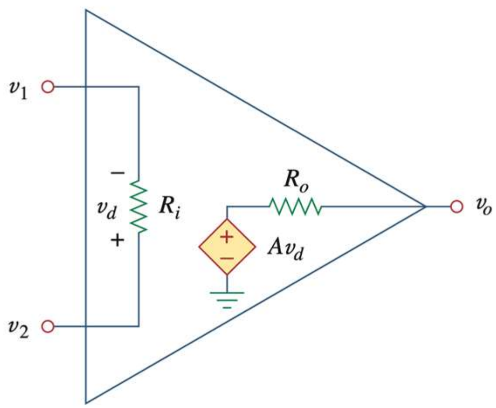
Lecture 4 & 5

Op Amp – Part 2 & 3

Review – Operational Amplifier





$v_1 = v_- =$ voltage of inverting terminal

$v_2 = v_+ =$ voltage of noninverting terminal

$v_d = v_+ - v_- = v_2 - v_1$
$= $ differential input voltage for VCVS

$A =$ Open loop gain

$R_i =$ Input resistance

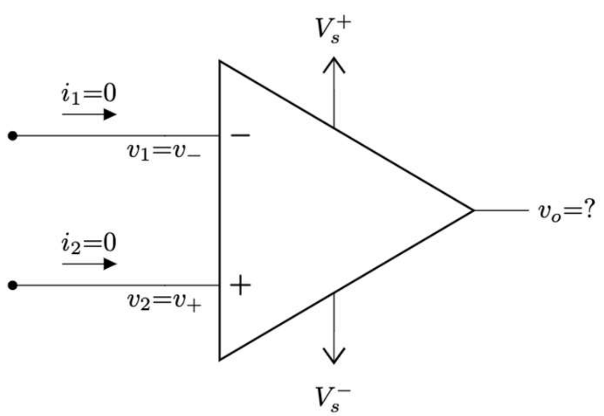
$R_o =$ Output resistance

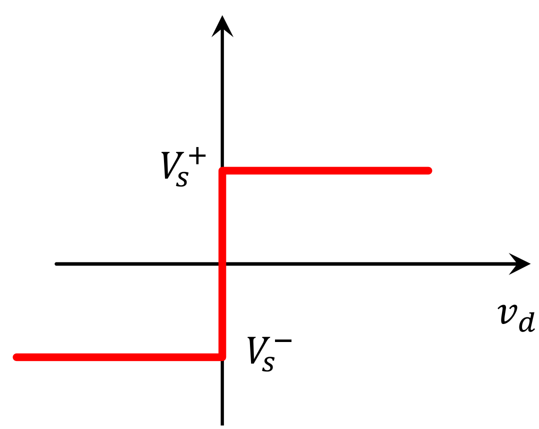
Differential amplifier ⇒ amplifies the difference

$$v_o = Av_d = A(v_2 - v_1) = A(v_+ - v_-)$$

Review – Ideal Op Amp

- Infinite open-loop gain, $A = \infty$
- Infinite input resistance, $R_i = \infty = \text{open circuit}$
- Zero output resistance, $R_o = 0 = \text{short circuit}$
- As $R_i = \infty$ (open circuit), $i_1 = i_2 = 0$. **Therefore, <u>circuit solving become much simpler</u>**





VTC of ideal op amp

$$v_o = \begin{cases} V_s^+ \ if \ v_d > 0 \Rightarrow v_2 > v_1 \\ V_s^- \ if \ v_d < 0 \Rightarrow v_2 < v_1 \end{cases}$$

Application of Ideal Op Amp - Comparator

- A comparator compares two voltages to determine which is larger.
- The comparator is essentially an op-amp operated in an open-loop configuration
- Two types –
    - (1) **Non-inverting**: outputs a positive voltage ($V_H = V_s^+$) when input is greater than reference
    - (2) **Inverting**: outputs a negative voltage ($V_L = V_s^-$) when input is greater than reference
- Application – smoke detector, turning AC on/off automatically, etc

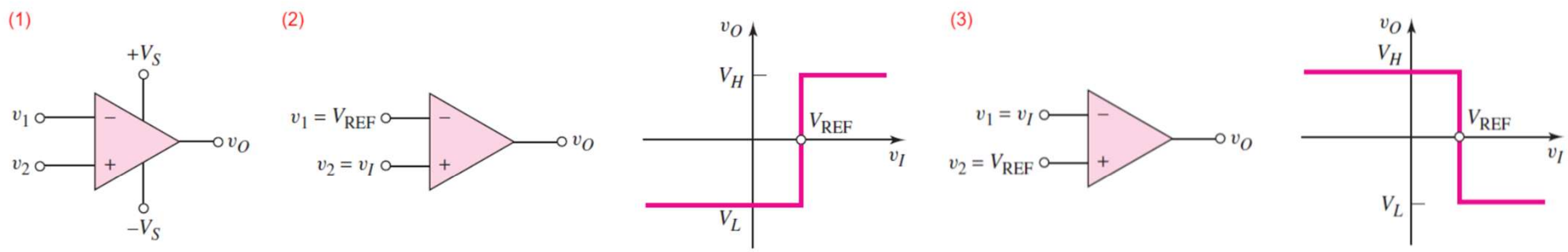
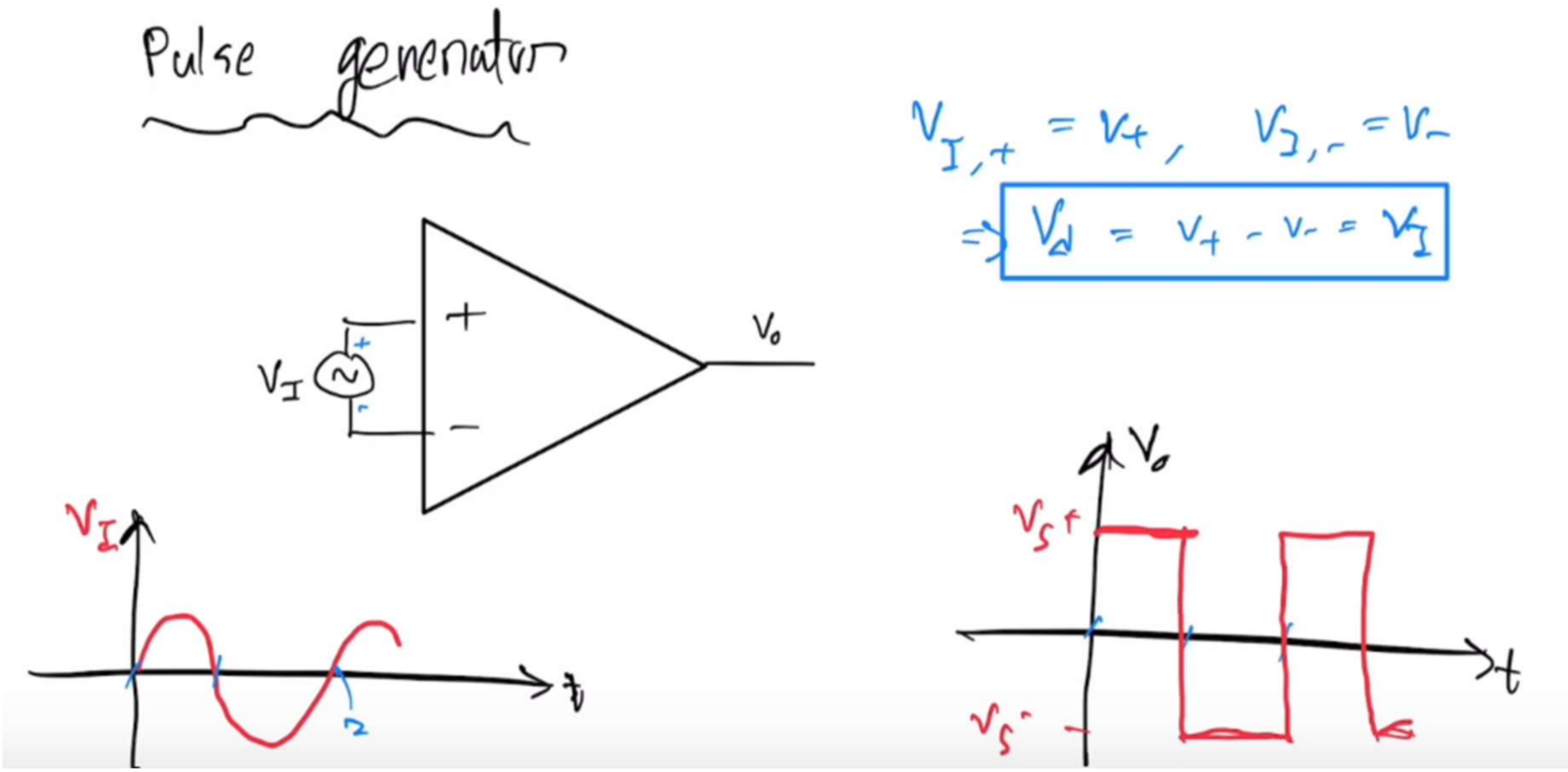


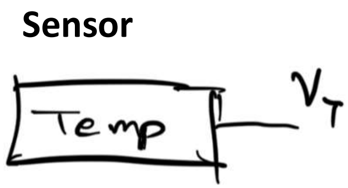
Figure 2: (1) Op-Amp Comparator (2) Noninverting Circuit (3) Inverting Circuit

Comparator: Pulse Generator



Comparator Application – Automatic AC

Auto AC ON / OFF



$V_T \propto T$

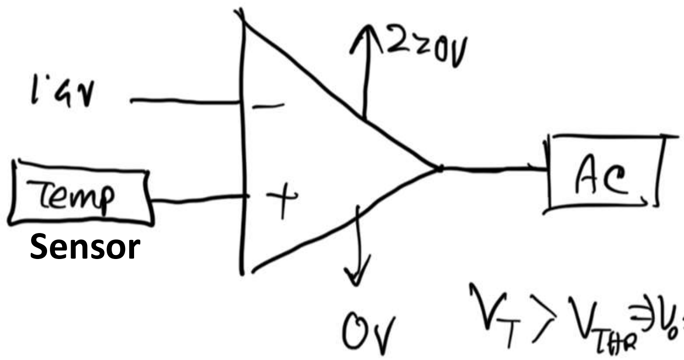
23°, $V_T = 1.2V$

$\boxed{24°, \quad V_T = 1.4V}$

25°, $V_T = 1.6V$

AC should be on if

$T > 24°C$



$V_{THR} = 1.4V$

$V_T > V_{THR} \Rightarrow V_0 = V_S^+$

$V_T < V_{THR} \Rightarrow V_S^-$

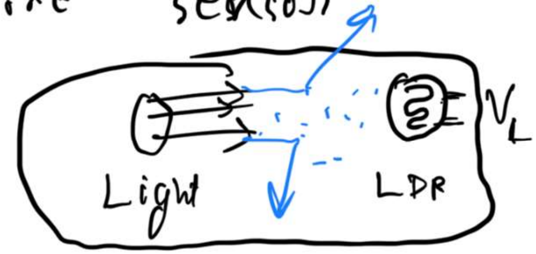
Smoke Detector

③ Smoke Detector

No smoke, V_L = small = 2V

Smoke, V_L = high = 4V

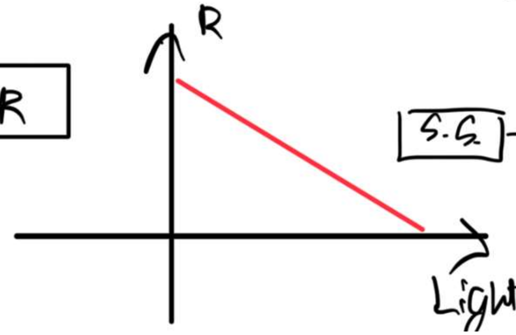
Smoke sensor

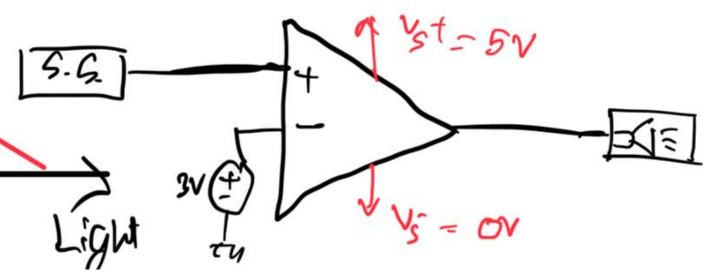


$V_L > 3V \Rightarrow$ ALarm ON

$V_L < 3V \Rightarrow$ " OFF

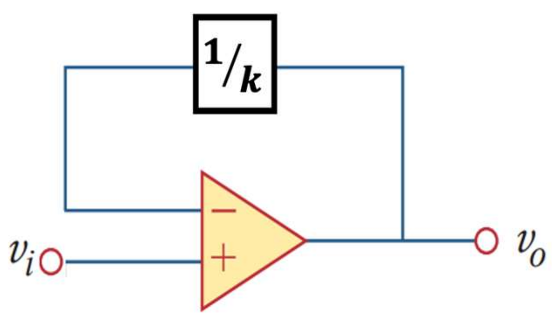
$\boxed{V_L \propto R}$





Introducing Negative Feedback

- The gain (A) of an ideal op amp is infinity, practically extremely large.
- The power supply (+Vs and –Vs) limits the op amp's output.
- We require a method to have a finite gain. That is what negative feedback does.
- Negative feedback: feeding back **a portion** of ***<u>output</u>*** to inverting ***<u>input</u>***
- Idea – the output will become stable due to a self-correcting mechanism

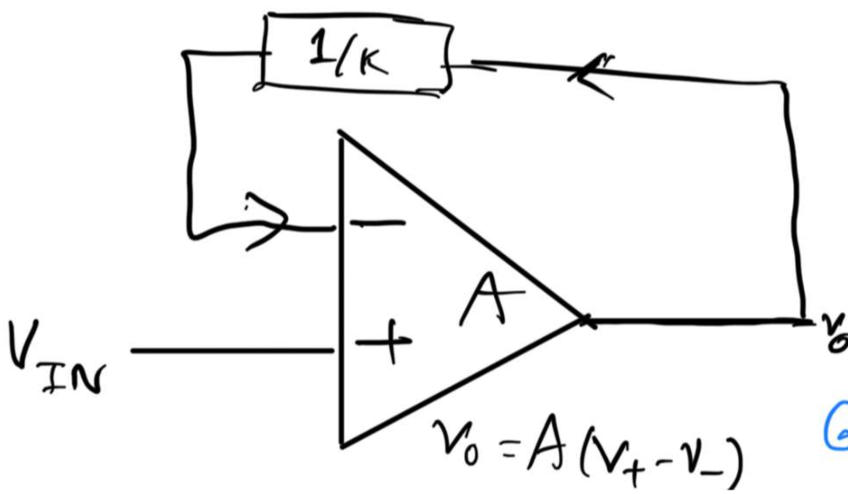


For example, her, $v_- = \text{one k'th part of ouput} = \frac{1}{k}$

If v_o increases, v_- will increase, hence v_d will decrease, eventtually v_o decreases

If v_o decreases, v_- will decrease, hence v_d will increase, eventtually v_o increase

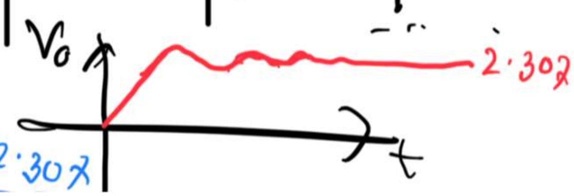
Negative Feedback – Numerical Example



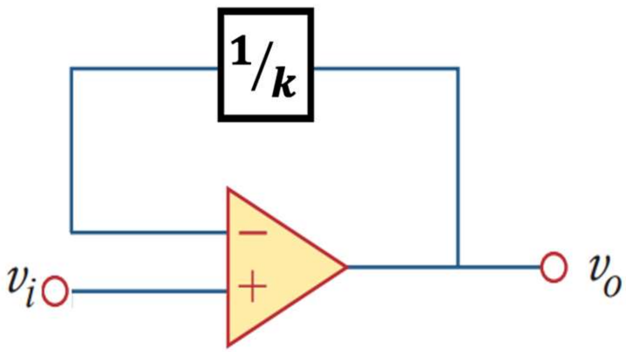
$V_{IN} = 1V$, $K = 10$, $A = 3$

$V_o = 0V$	$V_o = 3V$	$V_o = 2.1V$	$V_o = 2.307$
$V_- = 0V$	$V_- = 0.3V$	$V_- = 0.21V$	$V_- = 0.2307$
$V_d = 1V$	$V_d = 0.7$	$V_d = 0.79$	$V_d = 0.7693$
$\therefore V_o = 3V$	$V_o = 2.1V$	$V_o = 2.37$	$V_o = 2.307$

$Gain = \frac{OUT}{IN} = 2.307$



Negative Feedback – Derivation of Gain



Here, $v_- = \frac{v_0}{k}$

We know, $v_o = Av_d$

or, $v_o = A(v_+ - v_-)$

$= A(v_i - \frac{v_0}{k})$

$= Av_i - \frac{A}{k}v_0$

or, $v_o(1 + \frac{A}{k}) = Av_i$

So, $v_o = \frac{Av_i}{1+\frac{A}{k}}$

or, $v_o = \frac{v_i}{\frac{1}{A}+\frac{1}{k}}$

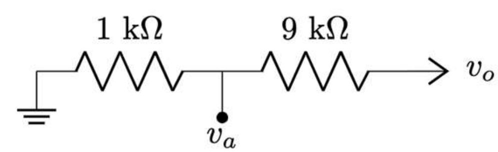
A is extremely large,

so, $\frac{1}{A} \approx 0$

$$v_o = \frac{v_i}{\frac{1}{k}} = kv_i$$

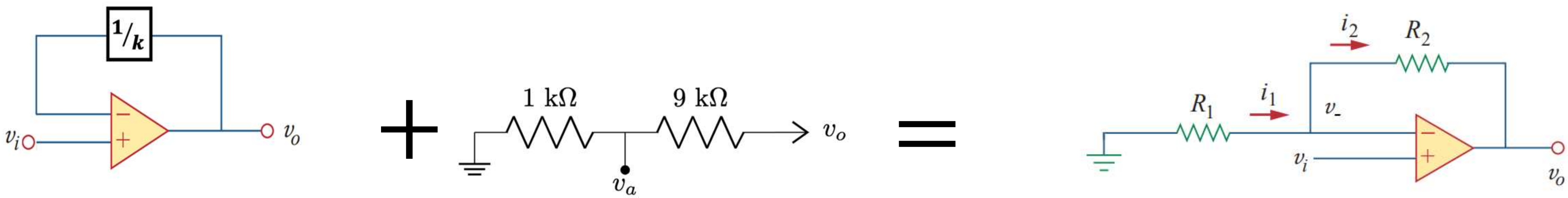
If k = 10 (meaning we feed back one tenth of the output to negative input), we will get v_o=10*v_i. that is 10 fold gain.

How to get 1/k of output to input? Voltage dividers!



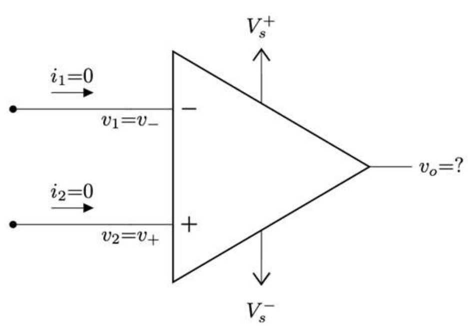
$$v_a = \frac{1\ k\Omega}{1\ k\Omega + 9\ k\Omega} \times v_o = \frac{v_o}{10}$$

Inverting Amplifier

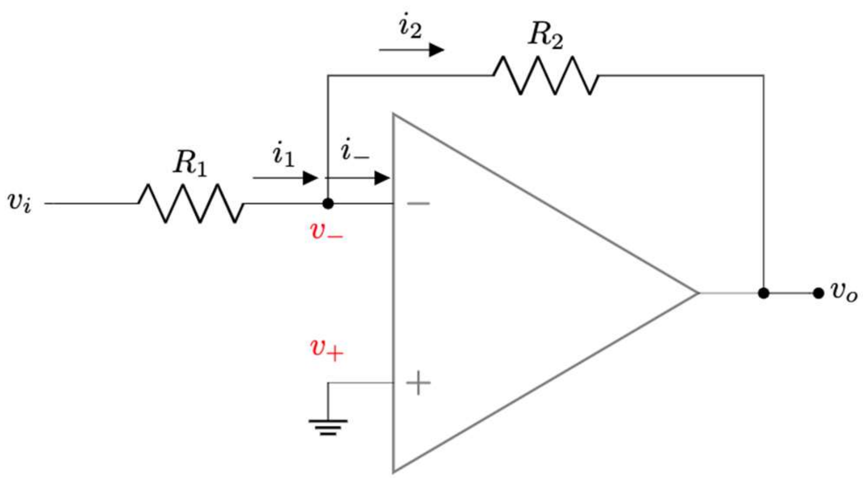


Solving Circuit with Ideal Op Amp + NF

- For ideal op-amp
  - Infinite input resistance, $R_i = \infty = \text{open circuit}$
  - Zero output resistance, $R_o = 0 = \text{short circuit}$
  - $i_i = 0$ and $i_+ = 0$
- ***<u>When there is negative feedback</u>***, For ideal A as is infinitely high, for a finite output voltage v_o, $\frac{v_o}{A} = v_d = 0 \Rightarrow v_+ = v_-$. This is called **virtual short circuit**
- Because of these, solving ideal op-amp circuit with negative feedback is very simple



Inverting Amplifier-General



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $\mathrm{R}_1 \Rightarrow i_1 = \frac{v_i - 0V}{R_1} = v_i/R_1$

Since ideal op-amp, $i_- = i_+ = 0$

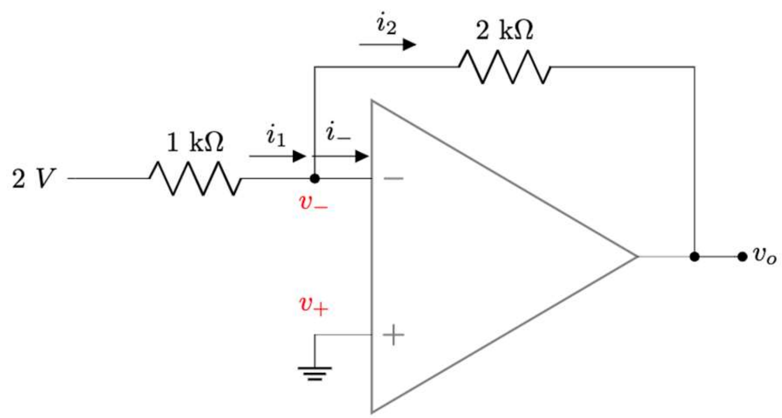
From KCL at v_-, $i_1 = i_- + i_2 \Rightarrow i_1 = i_2 = v_i/R_1$

From Ohm's law for $R_2 \Rightarrow i_2 = \frac{v_- - v_0}{R_2} = \frac{v_i}{R_1} \Rightarrow v_o = -i_2 \times R_2 \Rightarrow v_o = -\frac{R_2}{R_1} v_i$ [ANS]

$$\text{Gain} = -\frac{R_2}{R_1}$$

Example – Inverting Amplifier

Solve the ciruit to find v_o



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $1\ k\Omega \Rightarrow i_1 = \frac{2V - 0V}{1\ k\Omega} = 2mA$

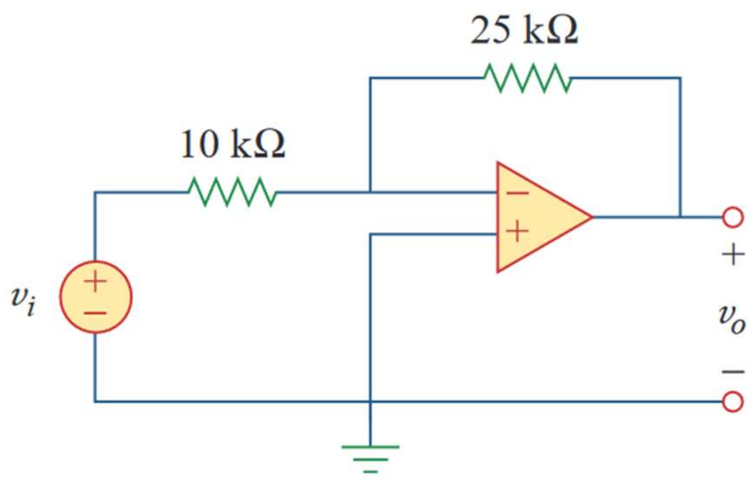
Since ideal op-amp, $i_- = i_+ = 0$

From KCL at v_-, $i_1 = i_- + i_2 \Rightarrow i_1 = i_2 = 2\ mA$

From Ohm's law for $2\ k\Omega \Rightarrow i_2 = \frac{v_- - v_0}{2\ k\Omega} = 2mA \Rightarrow v_o = -i_2 \times 2 = -4V$ [ANS]

Gain = $-\frac{4V}{2V} = -2$ (hence **inverting**)

Example – Inverting Amplifier



If $\boldsymbol{v_i} = \mathbf{0.5}$ V, calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Current in the $\mathbf{10\ k\Omega}$ resistor.

(a)

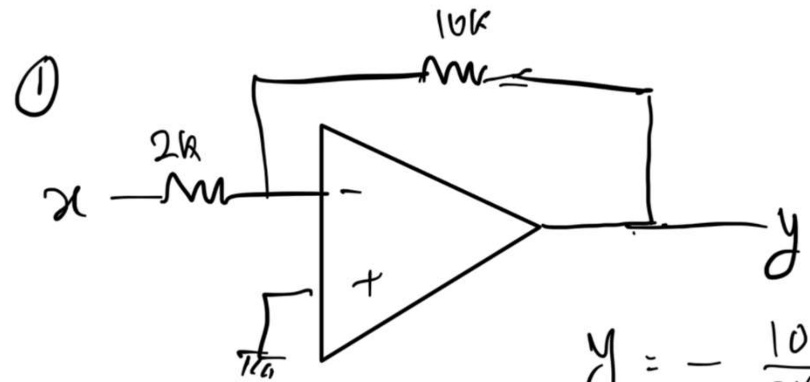
$$\boldsymbol{v_o = -\frac{R_f}{R_i} \cdot v_i = -2.5v_i = -1.25\ \mathrm{V}}$$

(b) Current through the $\mathbf{10\ k\Omega}$ resistor is

$$i = \frac{v_i}{R_i} = \frac{0.5}{10}\ \mathrm{mA} = \mathbf{50\ \mu A}$$

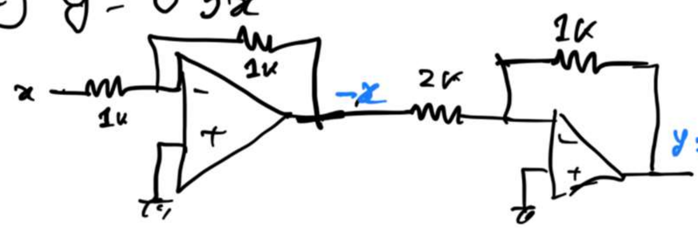
Example

①



$$y = -\frac{10K}{2K}x \Rightarrow \boxed{y = -5x}$$

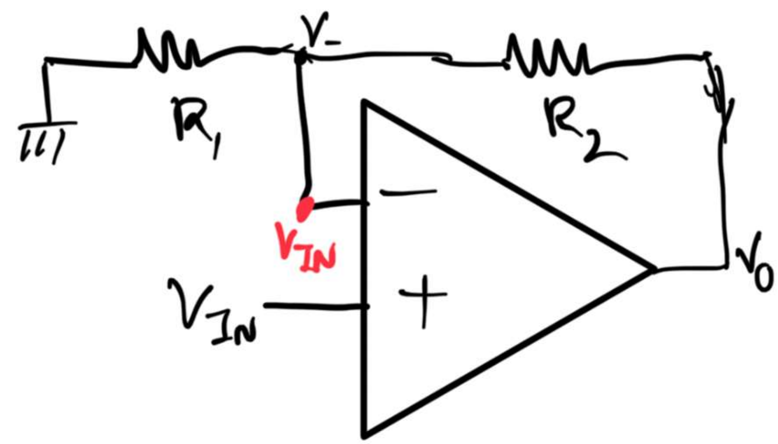
② $y = 0.5x$



$$y = \left(-\frac{1}{2}\right)(-x) = 0.5x$$

Non-Inverting Amplifier

Gain > 1

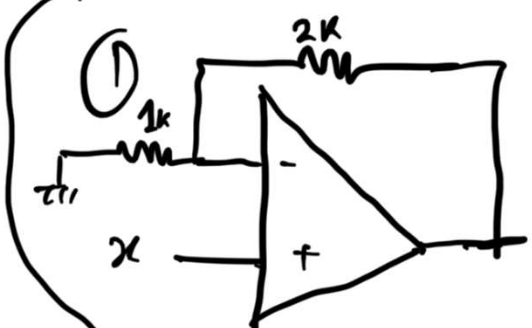


$$V_- = \frac{R_1}{R_1 + R_2} V_0 = \frac{1}{1 + \frac{R_2}{R_1}} V_0$$

K

$$\boxed{\therefore V_0 = \left(1 + \frac{R_2}{R_1}\right) V_{IN}}$$

①



$$y = \left(1 + \frac{2}{1}\right) x$$

$$\Rightarrow y = 3x$$

② Design $y = 4x$

$$1 + \frac{R_2}{R_1} = 4 \Rightarrow \frac{R_2}{R_1} = 3$$

$$y = 4x$$

Inverting Adder

Consider v_1 first, and deactivate other (v_2, v_3, v_4) sources.

It is nothing but an inverting amplifier.

So, $v_{o1} = -\frac{R_f}{R_1} v_1$

Similarly, if we active one source and deactivate others, we will get:

$v_{o2} = -\frac{R_f}{R_2} v_2$, $v_{o3} = -\frac{R_f}{R_3} v_3$, $v_{o4} = -\frac{R_f}{R_4} v_4$

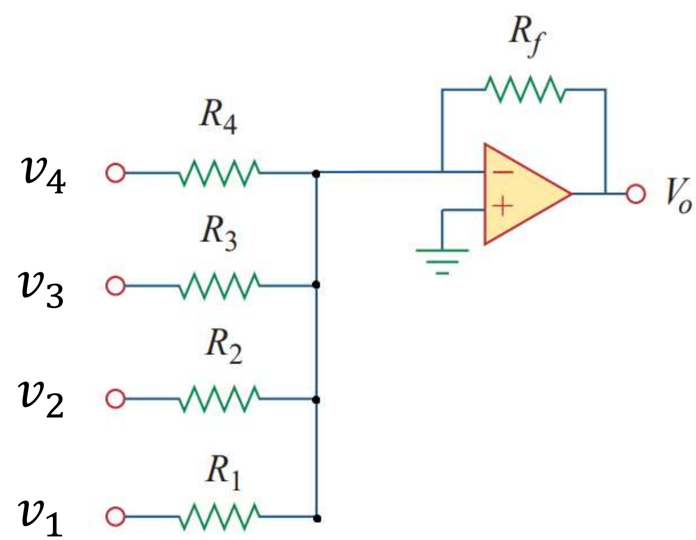
Now, using **superposition principle**,

$v_o = v_{o1} + v_{o2} + v_{o3} + v_{o4}$

So, $v_o = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 - \frac{R_f}{R_3} v_3 - \frac{R_f}{R_4} v_4$

Or, $v_o = -(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 + \frac{R_f}{R_4} v_4)$

We can use this circuit to add any ′n′ number of inputs!



Example

Implement the following function using op-amps:

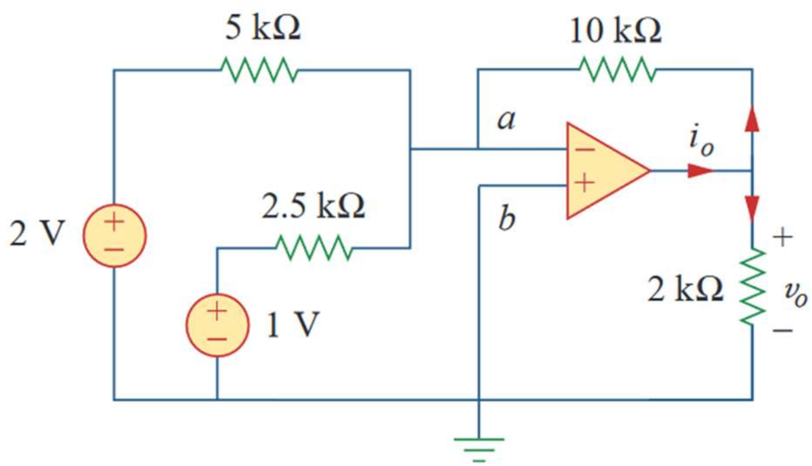
$$v_0 = -(v_1 + 0 \cdot 5v_2 + v_{\mathbf{3}})$$

Solution:

Here, $R_f/R_1 = 1$, $R_f/R_2 = 0.5$, $R_f/R_3 = 1$

If $R_f = 1\ \mathrm{k\Omega}$, $R_2 = 2\ \mathrm{k\Omega}$, $R_3 = 1\ \mathrm{k\Omega}$

Example-Inverting Adder



Calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Output current $\boldsymbol{i_o}$.

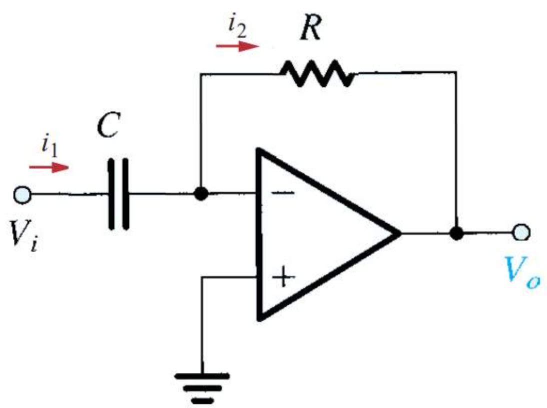
(a)

$$\boldsymbol{v_o = -\left(\frac{10}{5} \cdot 2 + \frac{10}{2.5} \cdot 1\right) = -8\ \mathrm{V}}$$

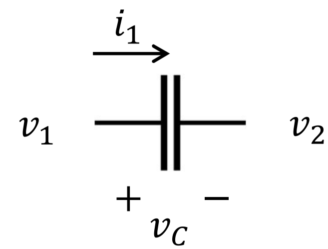
(b)

$$i = \frac{v_o}{10} + \frac{v_o}{2} = (-0.8 - 4\,)\ \mathrm{mA} = \boldsymbol{-4.8\ \mathrm{mA}}$$

Op Amp as Differentiator



Review – Capacitor



$$i_1 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

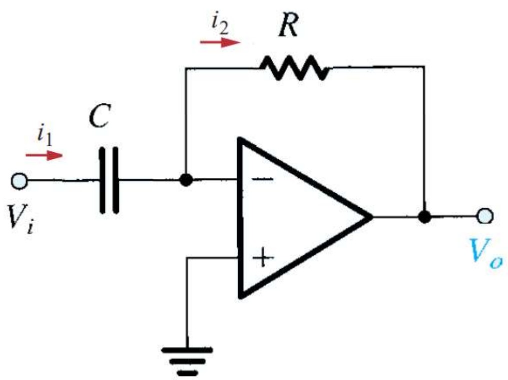
For the capacitor C, $\Rightarrow i_1 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_i - v_-)}{dt} = C\frac{dv_i}{dt}$

From Ohm's law for $R \Rightarrow i_2 = \frac{v_- - v_0}{R} = -\frac{v_o}{R}$

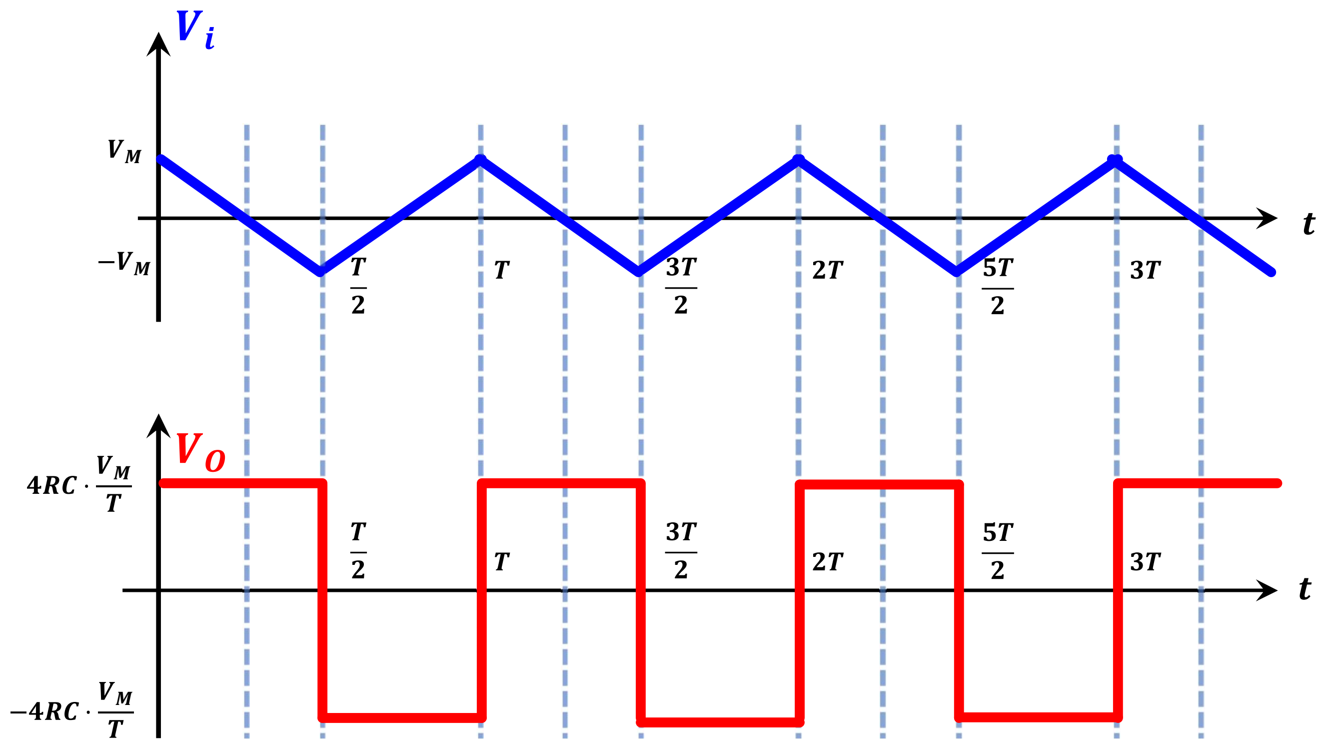
Since ideal op-amp, $i_- = i_+ = 0$, so $i_1 = i_2$

$$\Rightarrow -\frac{v_o}{R} = C\frac{dv_i}{dt} \Rightarrow v_o = -RC\frac{dv_i}{dt}$$ [Ans.]

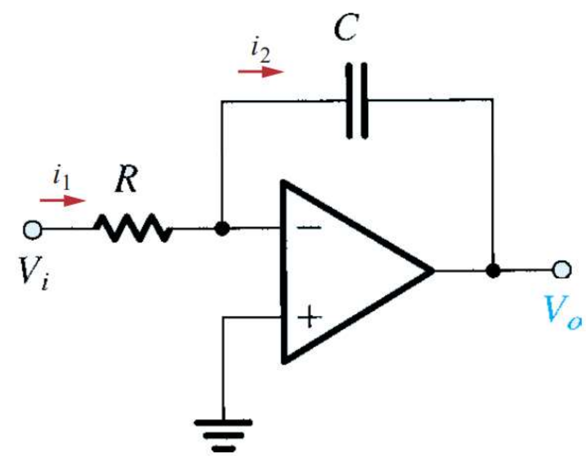
Example



Slope: $\left|\frac{\mathrm{d}v}{\mathrm{d}t}\right| = \frac{V_M-(-V_M)}{T/2} = \frac{4V_M}{T}$



Op Amp as Integrator



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $R \Rightarrow i_1 = \frac{v_i - v_-}{R} = \frac{v_i}{R}$

For the capacitor C, $\Rightarrow i_2 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_- - v_o)}{dt} = -C\frac{dv_o}{dt}$

Since ideal op-amp, $i_- = i_+ = 0$, so $i_1 = i_2$

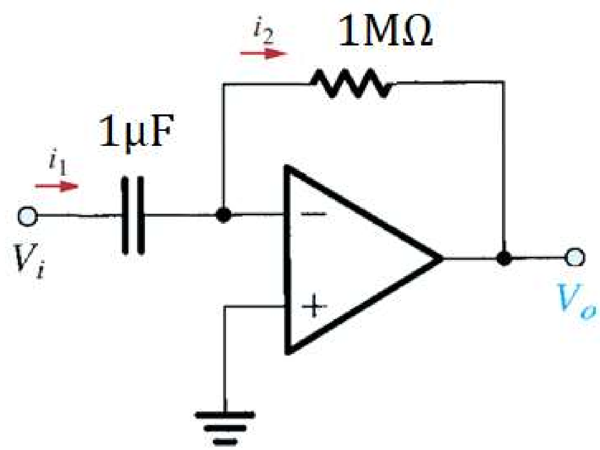
$$\Rightarrow \frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$= -RC\frac{dv_o}{dt}$$

$$\Rightarrow v_o = -\frac{1}{RC}\int v_i\, dt$$

Example

Observe the following Figure. If v_i = $5\sin 6t$, Find the value of $\boldsymbol{v_0}$.



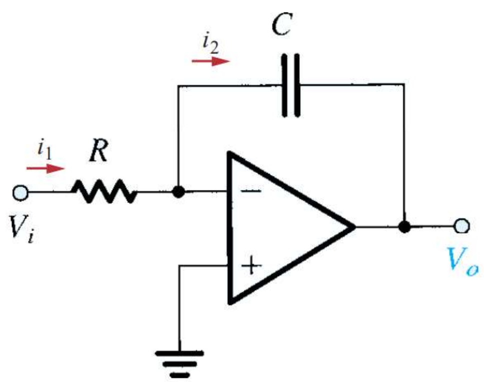
Solution:

This is a **differentiator**.

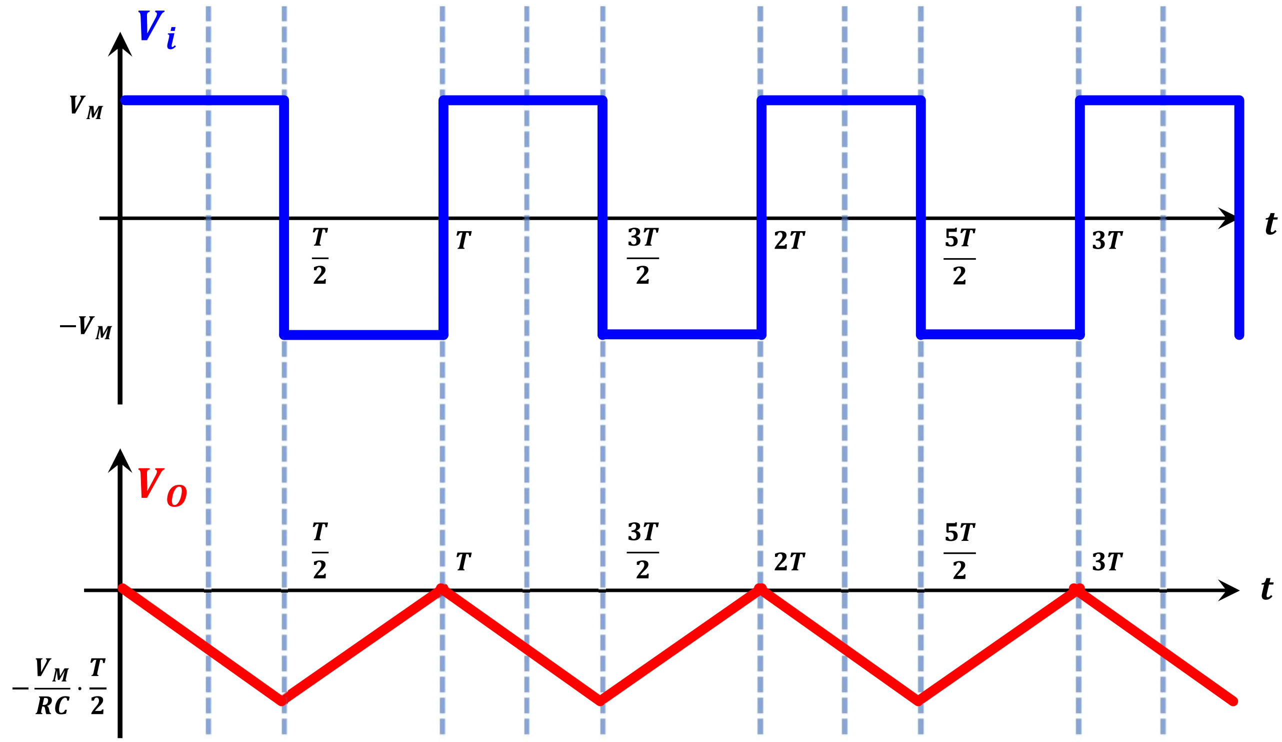
$$So, v_o = -RC\frac{dv_i}{dt} = -1 \times 10^6 \times 1 \times 10^{-6} \times \frac{d(5\sin 6t)}{dt}$$

$$\Rightarrow v_o = -1 \times (5 \times 6cos6t) = -30cos6t \text{ [Ans.]}$$

Example



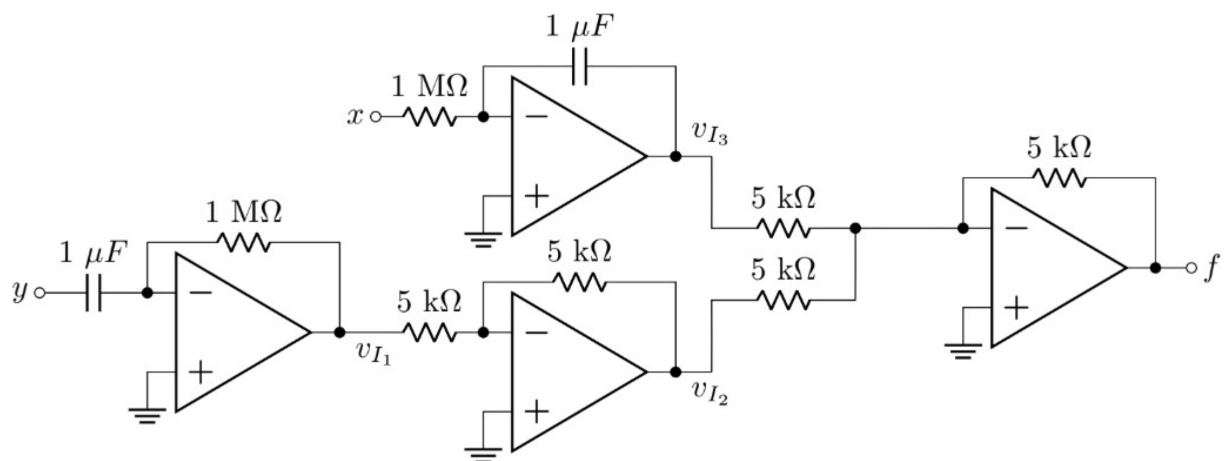
i_2
C
i_1
R
V_i
V_o



V_i
V_M
$-V_M$
t
$\frac{T}{2}$
T
$\frac{3T}{2}$
$2T$
$\frac{5T}{2}$
$3T$
V_O
$-\frac{V_M}{RC}\cdot\frac{T}{2}$

Example

Analyze the circuit below to **find** an expression of f in terms of inputs x and y.



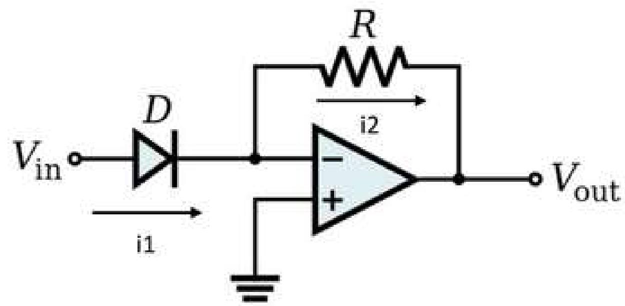
Solution:

$$v_{I1} = -\frac{dy}{dt}\,;\; v_{I3} = -\frac{1}{RC}\int x\,dt\,;\; v_{I2} = -v_{I1} = \frac{dy}{dt};\;\; v_o = f = -(v_{I2}+v_{I3})$$

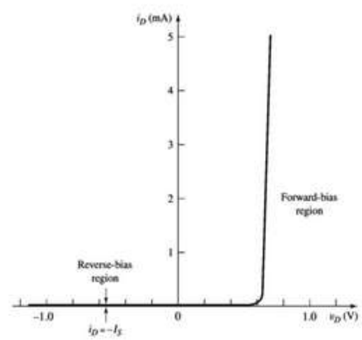
Exponential Amplifier

$V_+ = V_- = 0V$
$i_1 = i_2$
$\Rightarrow I_s\, e^{Vd/VT} = (0 - V_{out})/R$
$\Rightarrow V_{out} = -I_sR\, e^{Vd/VT}$

$V_d = V_{in} - 0 = V_{in}$
Considering $I_sR = 1$, $V_T = 1$
$V_{out} = -e^{Vin}$



Real diode



I-V characteristics of a real diode

Relation between diode current and diode voltage:

$$i_D = I_S\left(e^{\frac{v_D}{\eta V_T}} - 1\right)$$

where $v_D\ (= v_A - v_C)$ is the voltage across the diode, i_D is the current through the diode (from anode to cathode) and V_T, called the thermal voltage, is a temperature dependent constant. For temperature $T = 300K$, $V_T = 25\ mV$.

η is called the ideality factor (try to recall, you measured this in the lab!)

Logarithmic Amplifier

$V_+ = V_- = 0V$

$i_1 = i_f$

$\Rightarrow (V_i - 0)/R = I_s\, e^{Vd/VT}$

$\Rightarrow V_i = -I_s R\, e^{Vd/VT}$

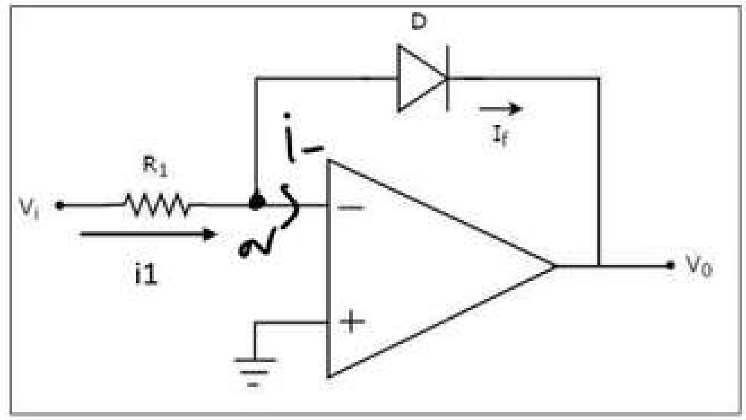
$V_d = 0 - V_o = -V_o$

Considering $I_s R = 1$, $V_T = 1$

$V_i = e^{-Vo}$

$\Rightarrow -V_o = \ln(V_i)$

$\Rightarrow V_o = -\ln(V_i)$



Example Problem

Design the following function using op-amps:

- $f = -\frac{1}{3}\int x \cdot dt + 2\ln y + 4z$

Example Problem

Design the following function using op-amps:

- $f = -3\frac{dx}{dt} + 2\exp(y) + 4z$

Example Problems

- Design f= x*y by using op-amps

$$xy = e^{\ln(xy)} = e^{\ln x +}$$

x → Logarithmic → -ln(x)

y → Logarithmic → -ln(y)

Inverting Adder → ln(x)+ln(y)

Exponential op-amp

$-e^{(\ln x + \ln y)}$

=- xy

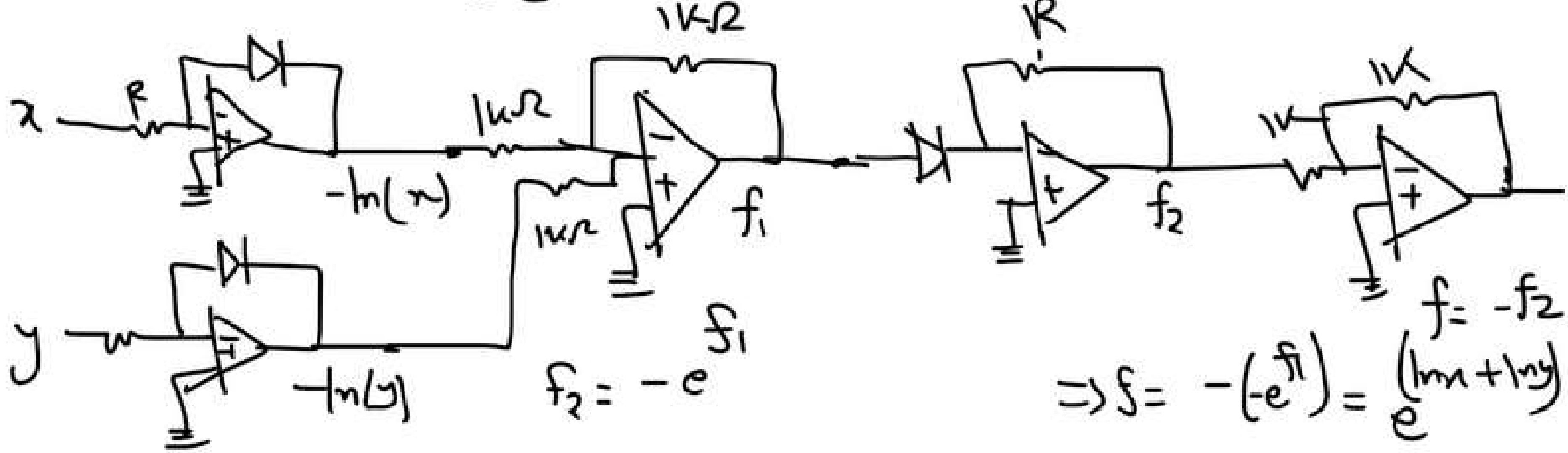
Inverting op-amp

xy

Example Problems

$f_1 = -(-\ln x - \ln y) = \ln x + \ln y$

- f= x*y $= e^{\ln(xy)} = e^{(\ln x + \ln y)}$



$f_2 = -e^{f_1}$

$f = -f_2$

$\Rightarrow f = -(-e^{f_1}) = e^{(\ln x + \ln y)}$

$f = x*y$

Practice Problem:

Design the following function using op-amps:

- $f = xy/z$

Hint: $f = xy/z$

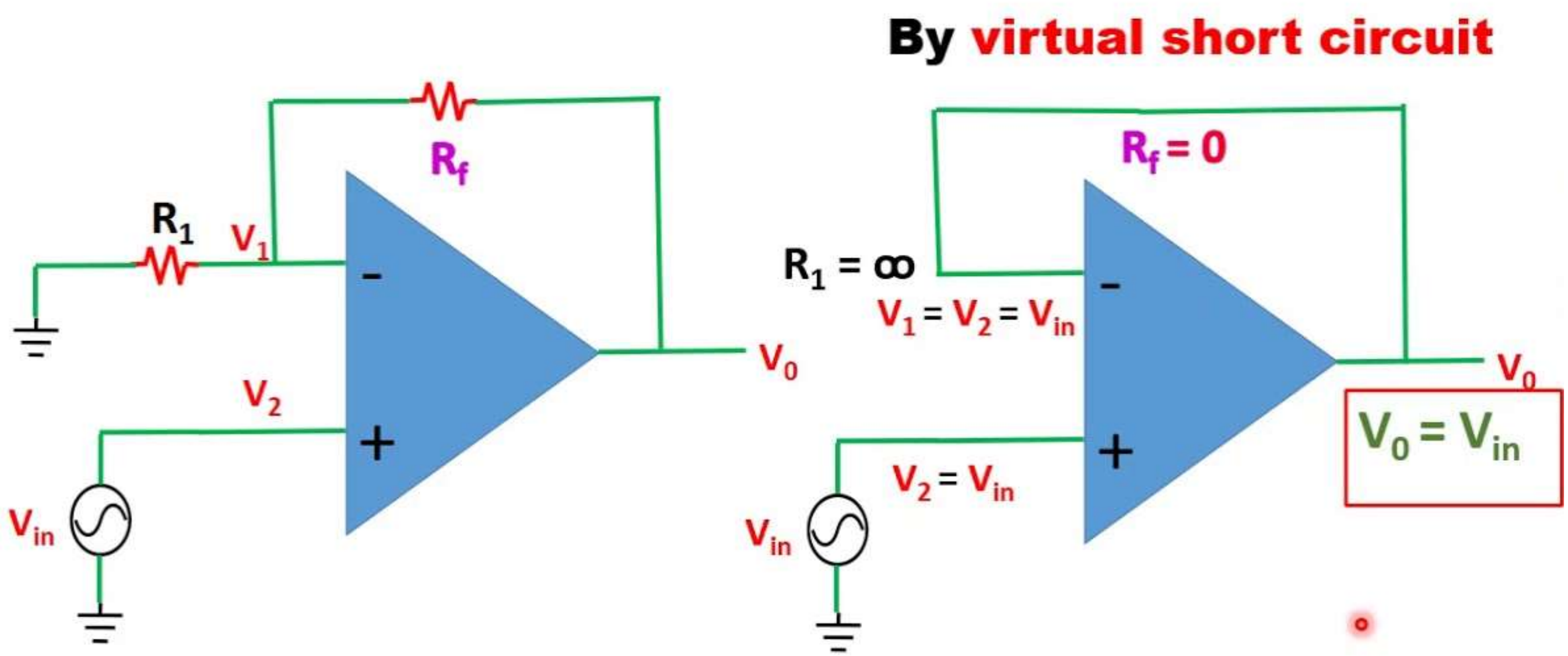
$$\ln(f) = \ln(xy/z) = \ln(x) + \ln(y) - \ln(z)$$

$$f = \exp(\ln(x) + \ln(y) - \ln(z))$$

So,

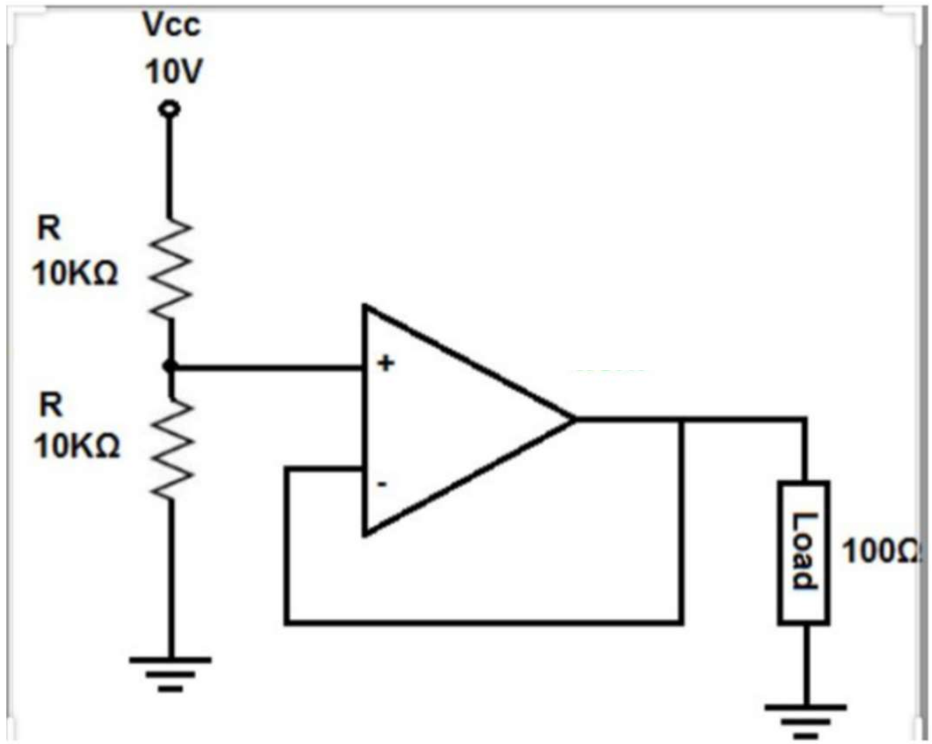
$f = \exp(z)$ where $z = \ln(x) + \ln(y) - \ln(z)$

Voltage follower or Buffer amplifier



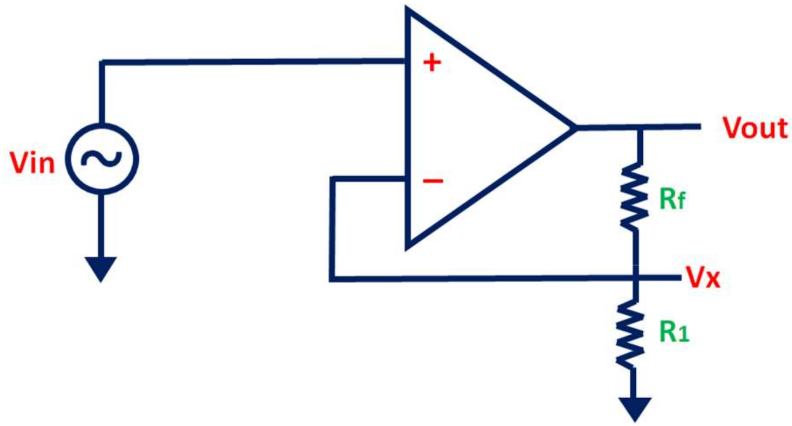
The output of the op-amp follows the input signal. In this configuration, the **gain of the op-amp is unity.**

Buffer Amplifier

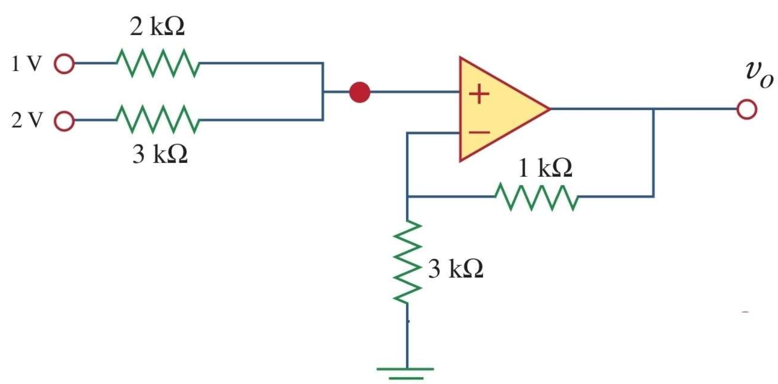


As we use the op amp buffer between , we can isolate the two stages completely , thus ensuring that we get 10/2 =5V at our 100Ohm load which won't be the case if we connect it directly in parallel

Example Problems



What type of amplifier is this?



- Find out Vo

  $I_+ = 0mA$

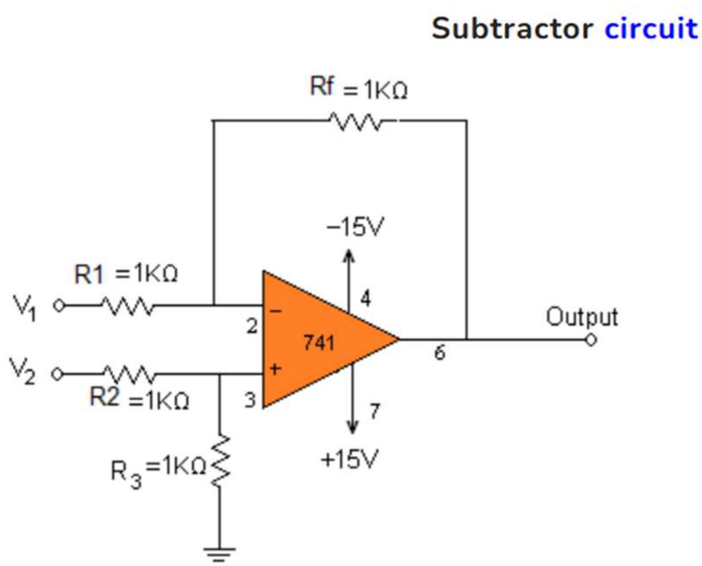
  $(1-V_+)/2+(1-V_+)/3= I_+ =0$

  $V_+= V_-= 1.4V$

  $Vo= (1+1/3)*V_+$

  $= 1.8667\ V$

Example Problems



❖ **Find output**

$V_+ = V_2 * R3/(R3+R2) = V_2/2$

Superposition theorem,

$Vo1 = -(Rf/R1) * V_1 = -V_1$

$Vo2 = (1+Rf/R1) * V_+ = 2 * V_2/2 = V_2$

$Vo = Vo1 + Vo2 = -V_1 + V_2 = V_2 - V_1$

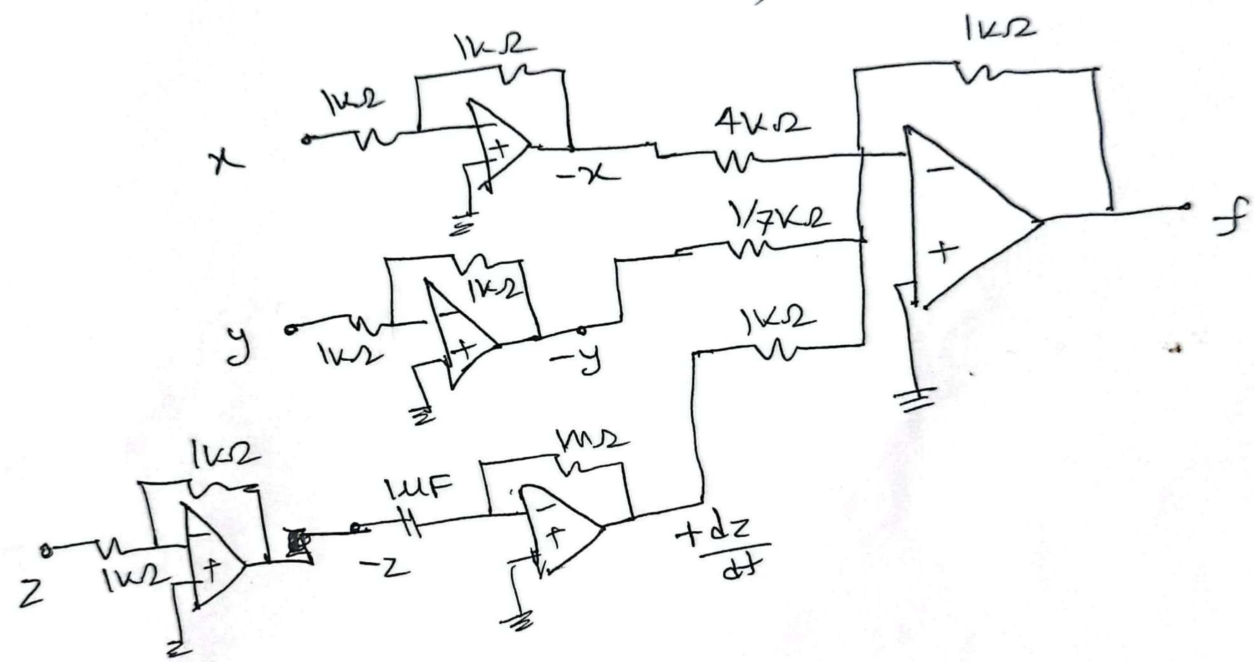
Example Problems

Design a circuit using Op-Amps to implement the following expression:

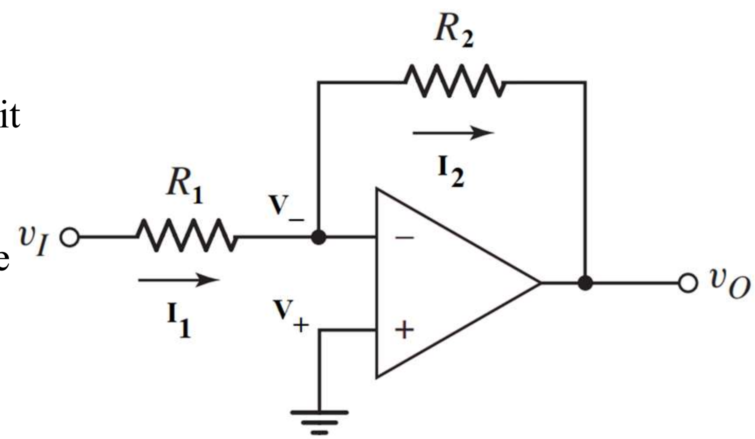
$f = \frac{1}{4}x + 7y - \frac{d}{dt}z$

$$f = \frac{1}{4}x + 7y - \frac{dz}{dt}$$

$$\Rightarrow f = -\left(-\frac{1}{4}x - 7y + \frac{dz}{dt}\right)$$



Example Problems



A. Design an inverting amplifier (i.e., find the values of R_1 and R_2 of the circuit shown in the Figure above) in such a way that the voltage gain is **−5**.

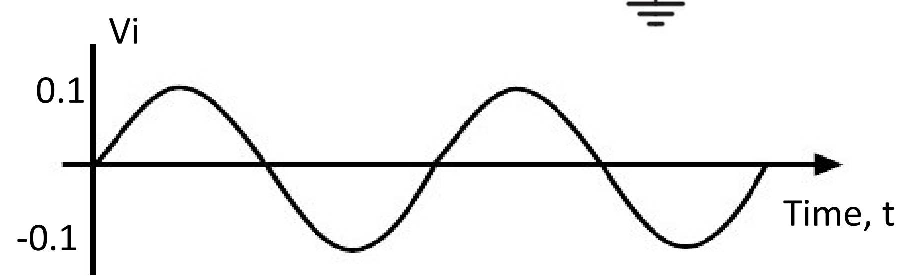
B. Consider the circuit you drew in (a) again. Assume the input $v_i = 0.1 \sin\omega t$ (V) has a maximum current rating of 5 μA. What design changes, if any, are required for this input, if the voltage gain remains the same?

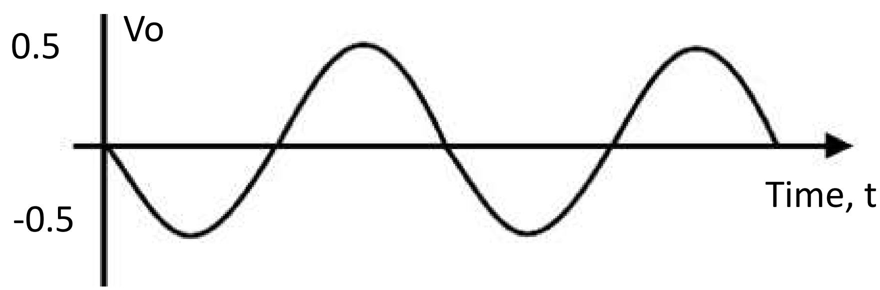
C. Draw the input and output waveforms of the circuit.

A. $-R2/R1 = -5$
Assuming R1=1 kΩ so, R2= 5kΩ

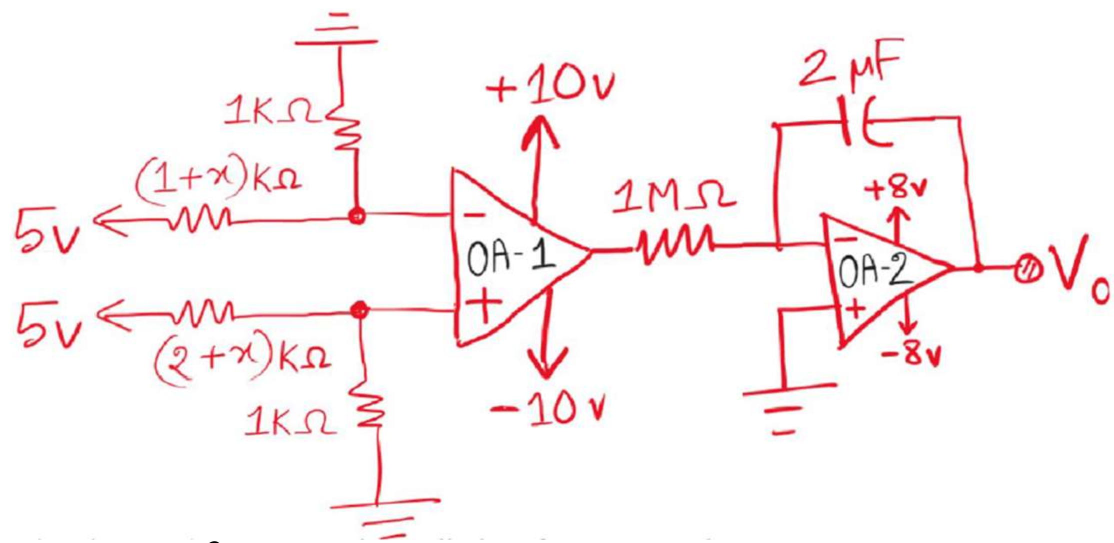
B. $I_{1,\,max} = 5$ uA
So, $R_{1,min} = |vi| / I_{1,\,max} = 0.1V/5uA = 20k\Omega$
But in A we assumed R1= 1kΩ ($< R_{1,min}$)
Now, assuming new value of R1= 25kΩ
Therefore, R2= 5*25 kΩ = 100kΩ

C. Vo= gain*Vi= -5*0.1 sinωt = -0.5 sinωt





Example



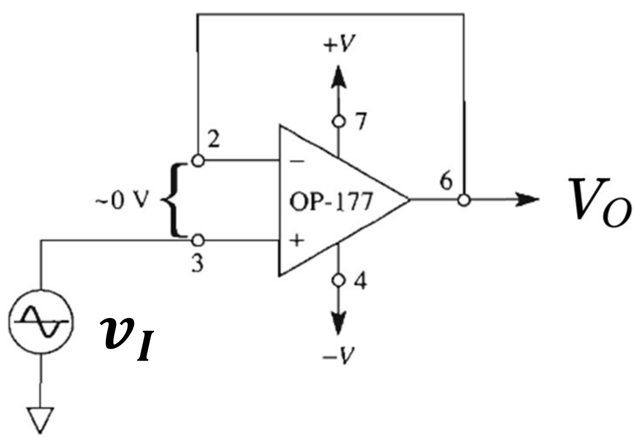
In the circuit given above, $\boldsymbol{x = 0}$

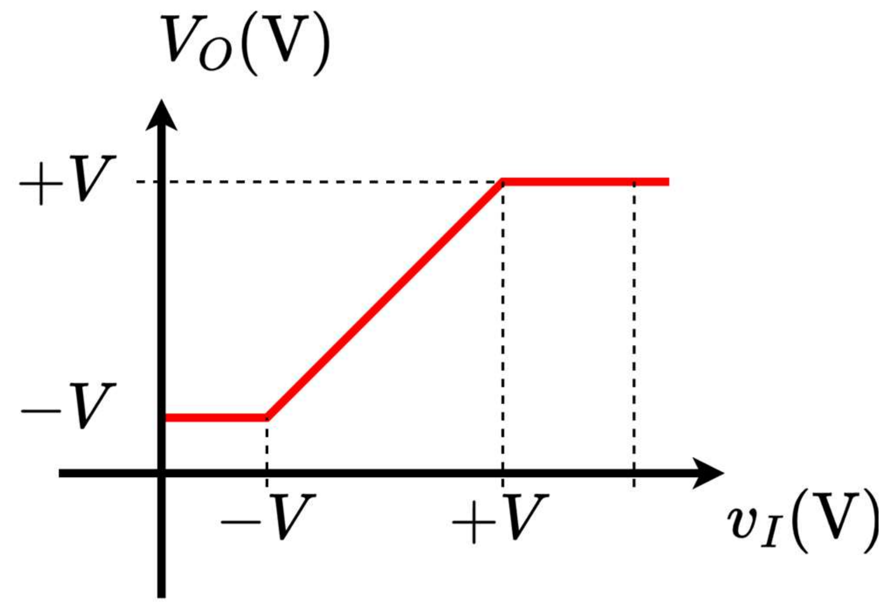
(a) Analyze the circuit and **determine** the voltage at the inverting, non-inverting terminals and the output of OA-1.

(b) Determine the highest V_O you can get from this circuit. **Explain** briefly.

(c) Analyze the circuit to **determine** the output voltage, VO of OA-2 and **plot** VO vs. time. **Label** the plot appropriately. [at t = 0, $V_O = 0$]

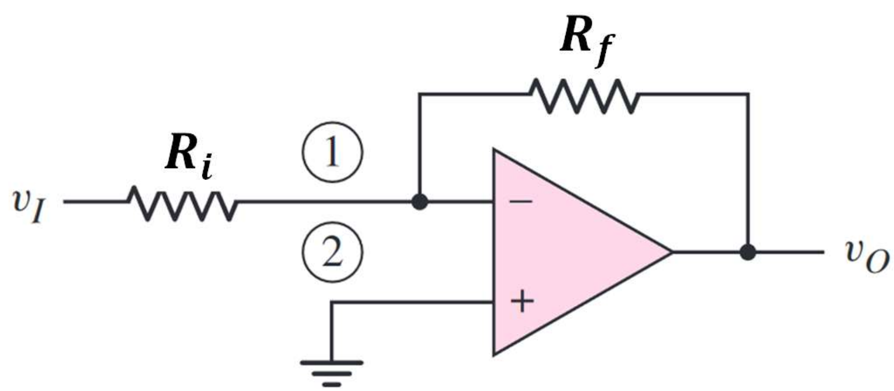
Voltage Follower – VTC



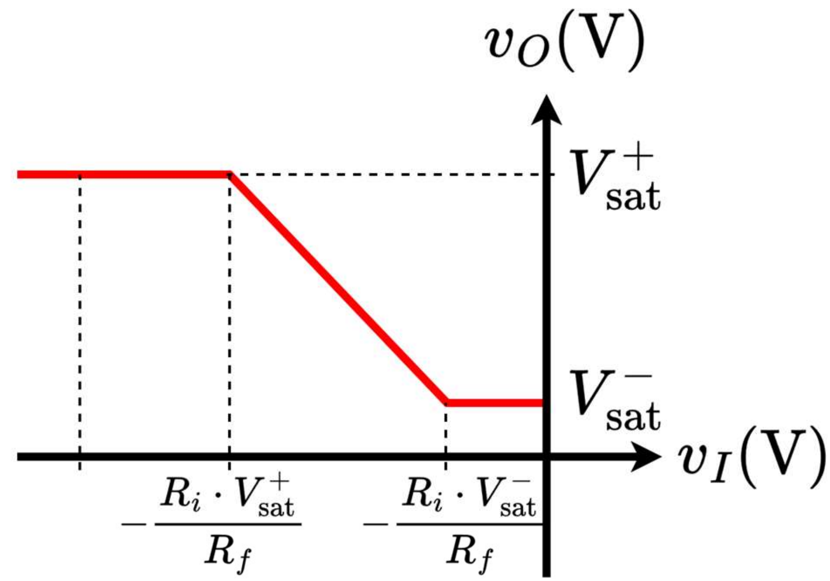


$$V_O = \begin{cases} +V, & \text{if } v_I \geq +V \\ v_I, & \text{if } -V \leq v_I \leq +V \\ -V, & \text{if } v_I \leq -V \end{cases}$$

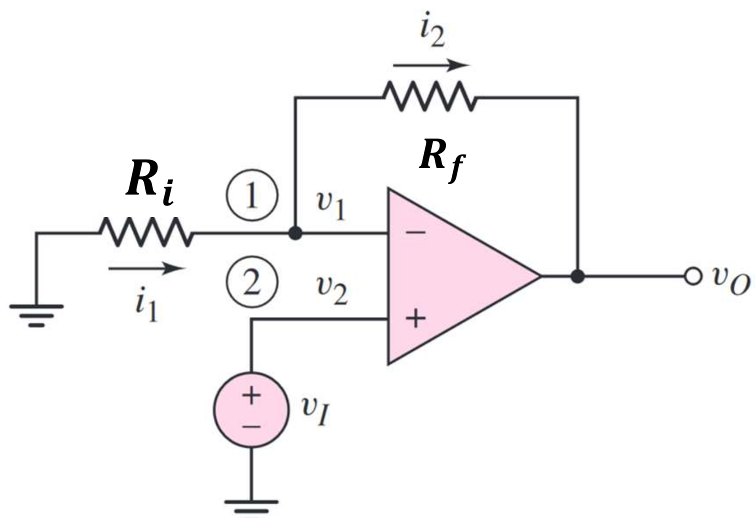
Inverting Amplifier – VTC

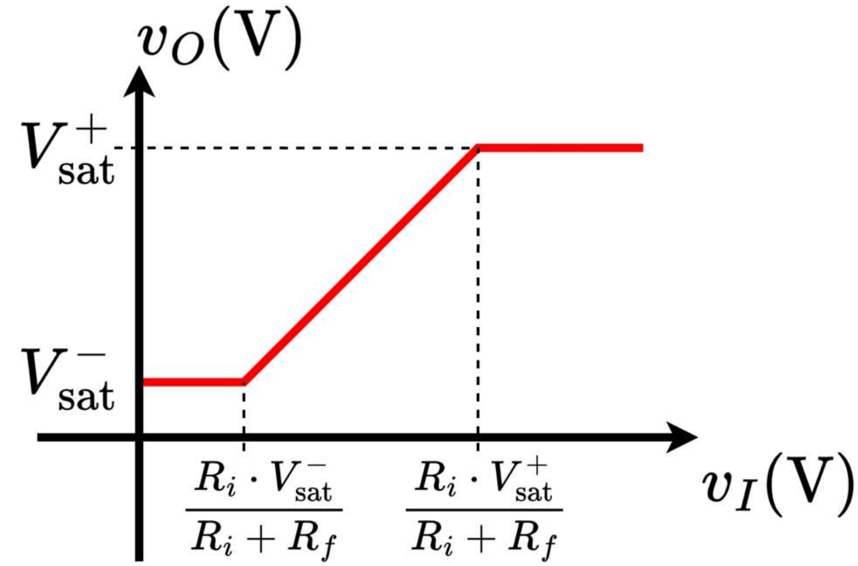


$$v_O = \begin{cases} V_{\text{sat}}^{+}, & \text{if } v_O \geq V_{\text{sat}}^{+} \\ -v_I \cdot \frac{R_f}{R_i}, & \text{if } V_{\text{sat}}^{-} \leq v_O \leq V_{\text{sat}}^{+} \\ V_{\text{sat}}^{-}, & \text{if } v_O \leq V_{\text{sat}}^{-} \end{cases}$$



Non-Inverting Amplifier – VTC

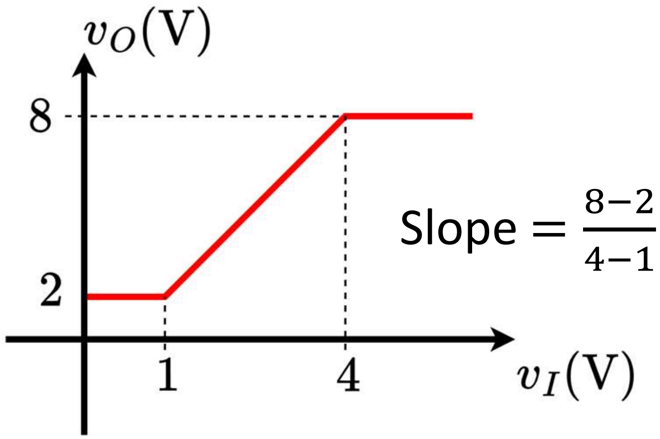




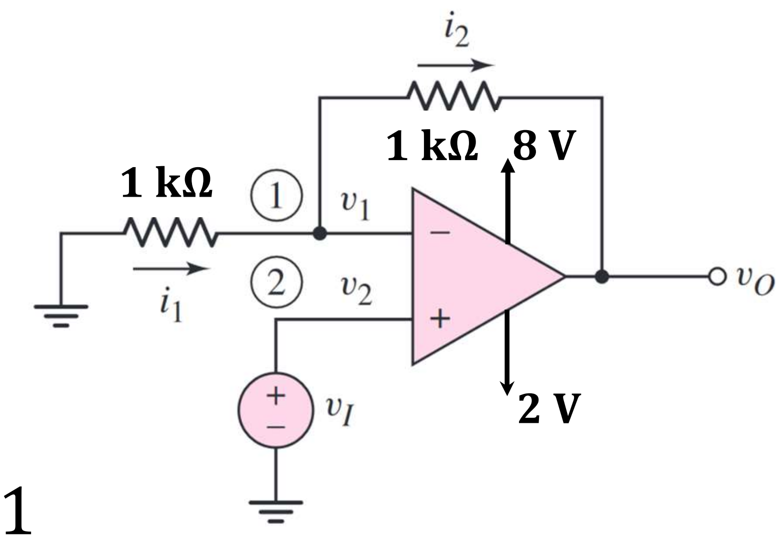
$$v_O = \begin{cases} V_{\text{sat}}^{+}, & \text{if } v_O \geq V_{\text{sat}}^{+} \\ v_I \cdot (1 + \frac{R_f}{R_i}), & \text{if } V_{\text{sat}}^{-} \leq v_O \leq V_{\text{sat}}^{+} \\ V_{\text{sat}}^{-}, & \text{if } v_O \leq V_{\text{sat}}^{-} \end{cases}$$

Non-Inverting Amplifier – VTC

Draw an Op-Amp Circuit with the following VTC



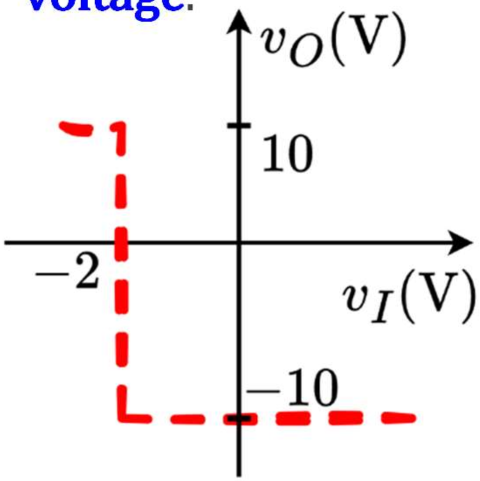
$$\text{Slope} = \frac{8-2}{4-1} = 2 = \left(1 + \frac{R_f}{R_i}\right)$$



$$\left(1 + \frac{R_f}{R_i}\right) = 2 \qquad \Rightarrow \frac{R_f}{R_i} = 1$$

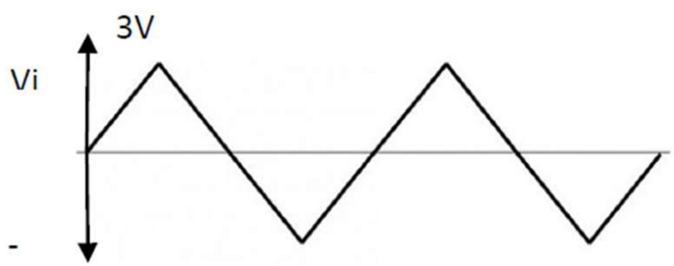
VTC Problems

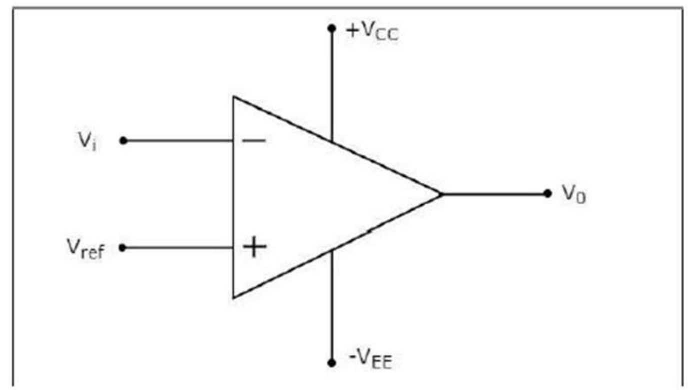
Design a circuit using **op-amp** that has the voltage transfer characteristics as shown in the figure below. v_0(V) is the **output voltage** and v_I(V) is the **input voltage**.



VTC Examples

Q1: V_{CC}= 15V= V_{EE}, Vref= 1V, Vi is a 6V p-p triangular signal as shown below
Draw output Vo for the following op-amp circuit.





Thank You