CSE251: Electronic Devices and Circuits

Lecture 5:

Closed Loop Op-amp configurations

Prepared By:

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Outline

- **Closed Loop Operational Amplifier: Introduction**

  - Open Loop VS Closed Loop Gain

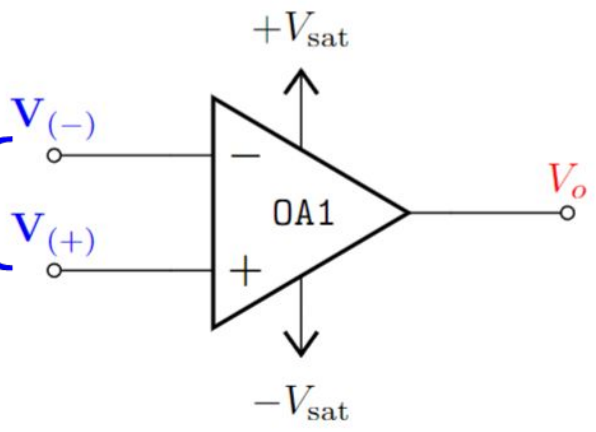
  - Closed Loop Configuration: Negative Feedback

  - Solving Closed Loop Circuit

Open Loop Gain VS Closed Loop Gain

Open Loop (OL) Configuration

$$V_d = V_{(+)} - V_{(-)}$$



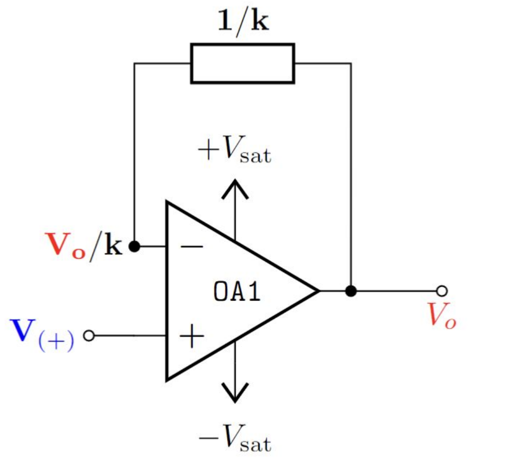
Input Voltage: V_d

Output Voltage: V_O

$\therefore$ Voltage Gain: $\dfrac{V_O}{V_d} = A \text{ or } K$

OL Gain	CL Gain

With "**Negative Feedback**": Closed Loop (CL) Configuration



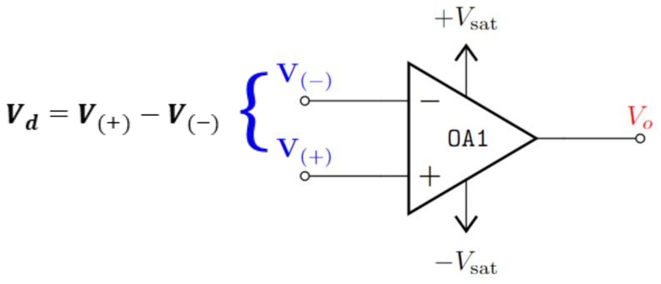
Input Voltage: $V_{(+)}$

Output Voltage: V_O

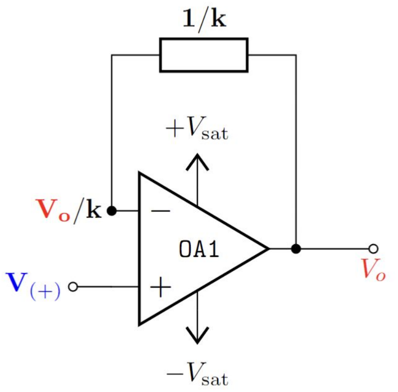
$\therefore$ Voltage Gain: $\dfrac{V_O}{V_{(+)}} = k$

Open Loop Gain VS Closed Loop Gain

Open Loop (OL) Configuration



Closed Loop (CL) Configuration



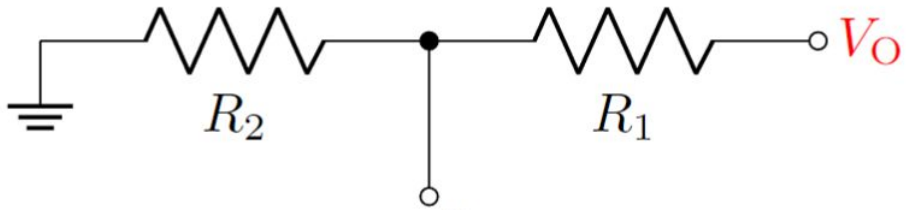
OL Gain	CL Gain
Can't be controlled	Can be controlled by the feedback element
*Used as "**Comparator**"*	*Used as "**Linear Amplifier**"*

Negative Feedback in Op-Amp circuit

The **output voltage** is transformed in the following way:

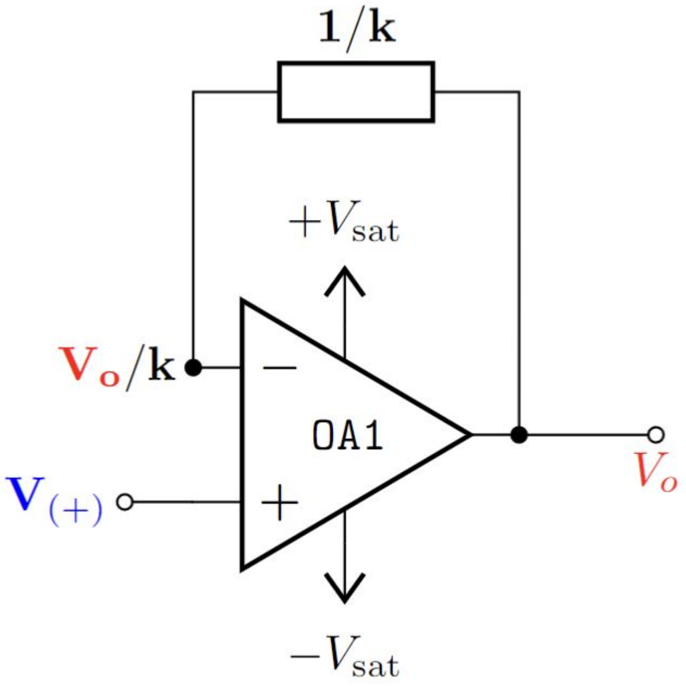
$$V_{(-)} = \frac{\mathbf{1}}{\boldsymbol{k}} \cdot V_O$$

This factor of $\mathbf{1}/\boldsymbol{k}$ can be achieved with a voltage divider network.



$$V_{(-)} = \frac{R_2}{R_2+R_1} \cdot V_O$$

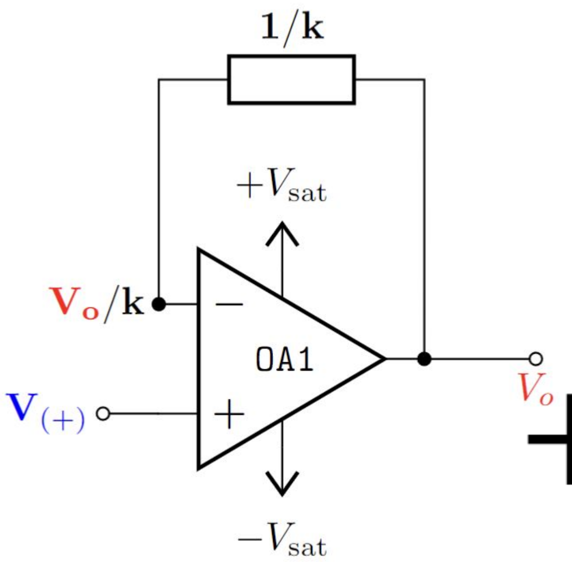
$$\boxed{\frac{\mathbf{1}}{\boldsymbol{k}} = \frac{R_2}{R_1 + R_2}}$$



A voltage divider can act as a **multiplier/factor** in the feedback branch

Negative Feedback in Op-Amp circuit

A voltage divider can act as a **multiplier/factor** in the **feedback** branch



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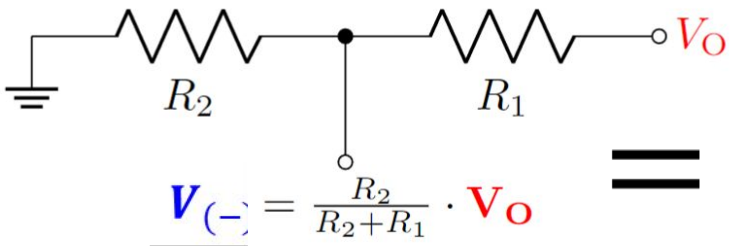
$$\frac{R_2}{R_1+R_2}=\frac{\mathbf{1}}{\boldsymbol{k}}$$



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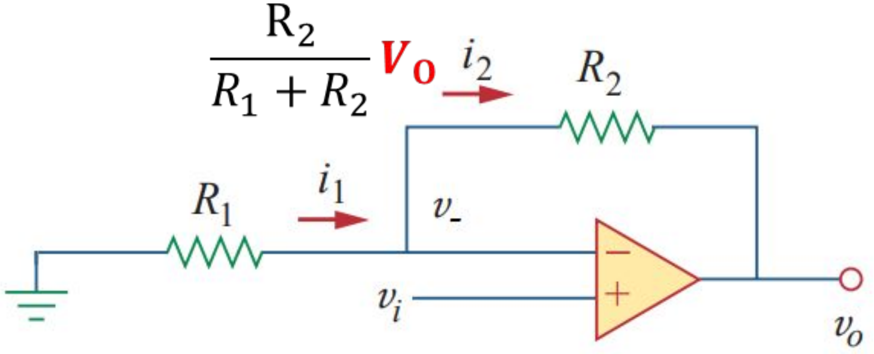
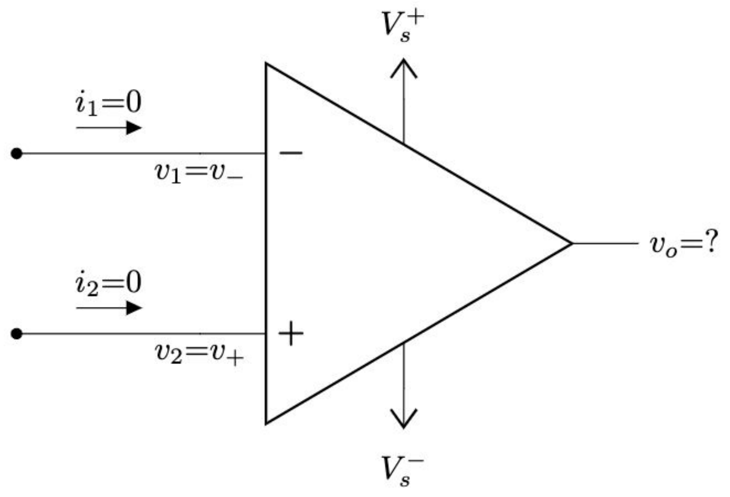


If $\boldsymbol{k}$ = 10 (meaning we feed back one tenth of the output to negative input), we will get $\boldsymbol{v_o}$=10*$\boldsymbol{v_i}$.
that is 10-fold gain.

Solving Closed Loop Op-Amp Circuit

- For "**ideal**" op-amp
    - **Infinite** input resistance, $\boldsymbol{R_i} = \infty =$ open circuit
    - **Zero** output resistance, $\boldsymbol{R_o} = \mathbf{0} =$ short circuit
    - $\boldsymbol{i_i} = \mathbf{0}$ and $\boldsymbol{i_+} = \mathbf{0}$

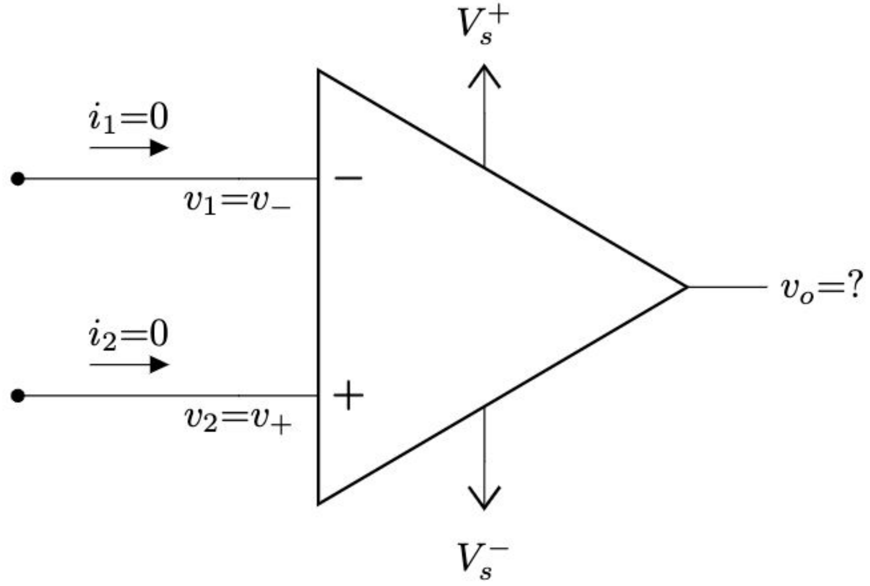


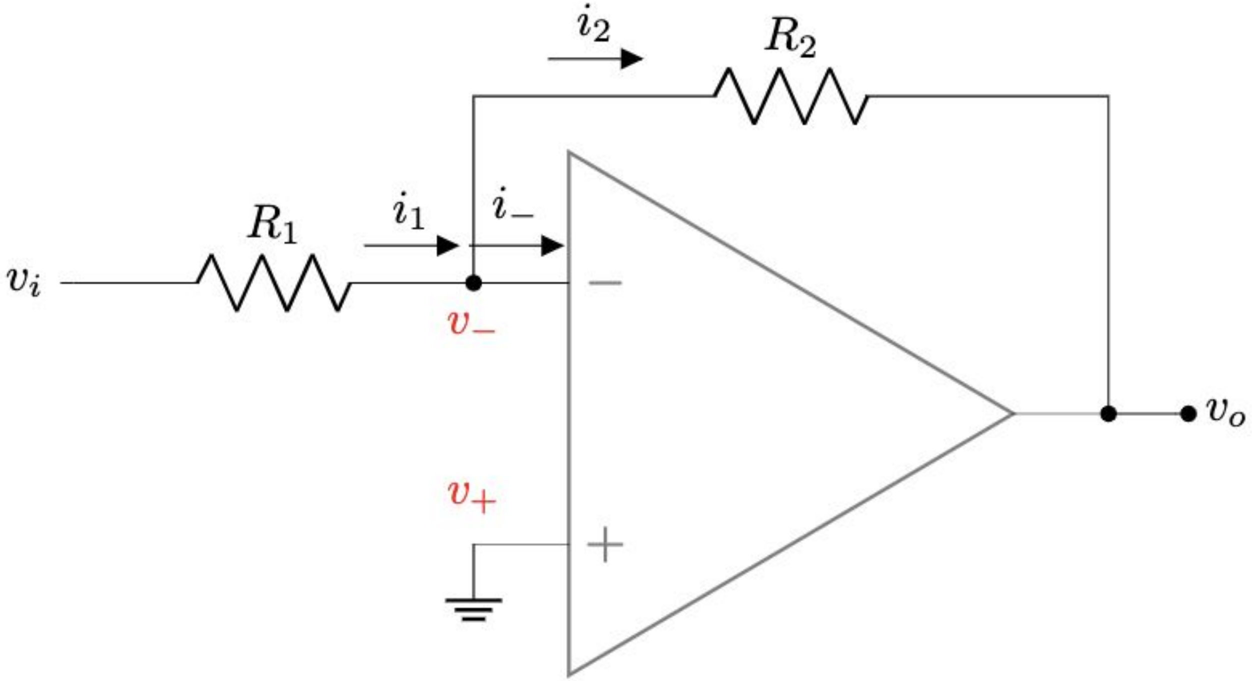
- ***<u>When there is negative feedback</u>***,
    - In an ideal op-amp, "$\boldsymbol{A}$" (or $\boldsymbol{K}$) is **infinitely high**. Thus, for a finite output voltage $\boldsymbol{v_o}$:

$$\frac{v_o}{A} = v_d \to 0 \Rightarrow v_+ = v_-.$$

- This is called **virtual short circuit**

Solving Closed Loop Op-Amp Circuit

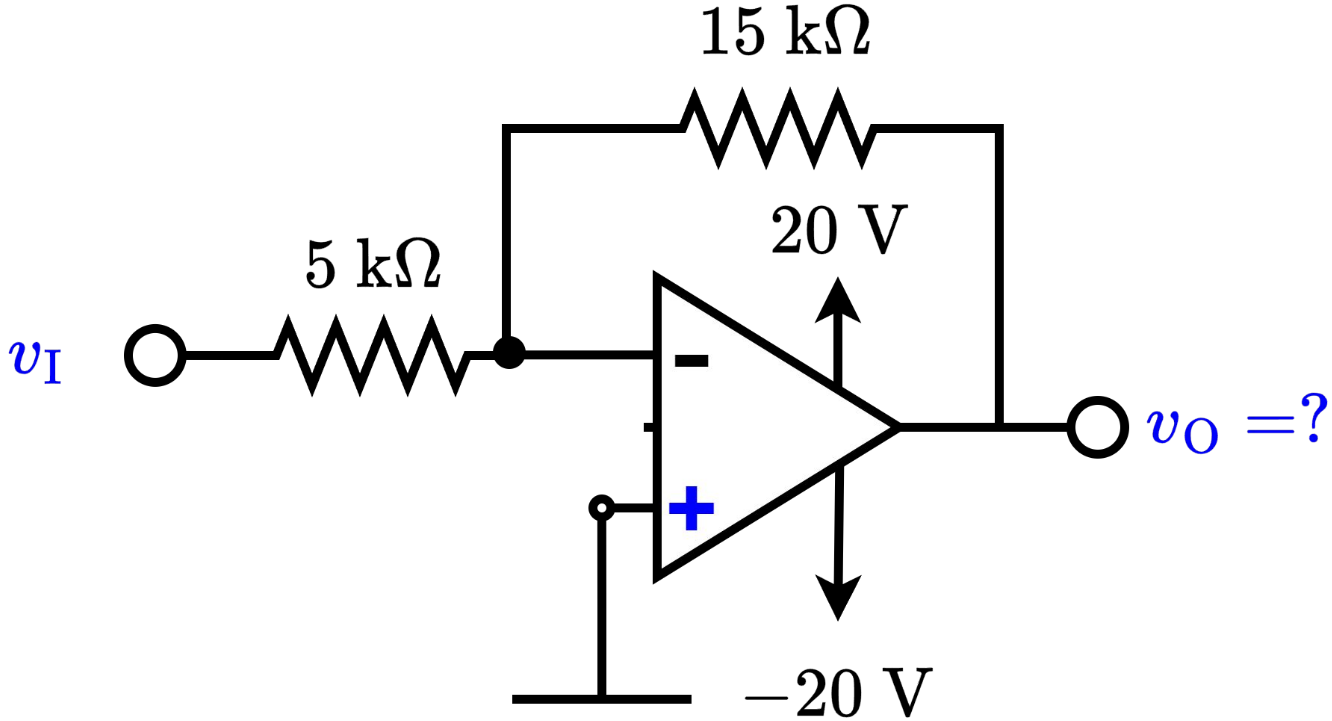




Two Rules:

1. Virtual Shorting: $\boldsymbol{v_+ = v_-}$
2. Zero input bias current: $\boldsymbol{i_- = i_+ = 0}$

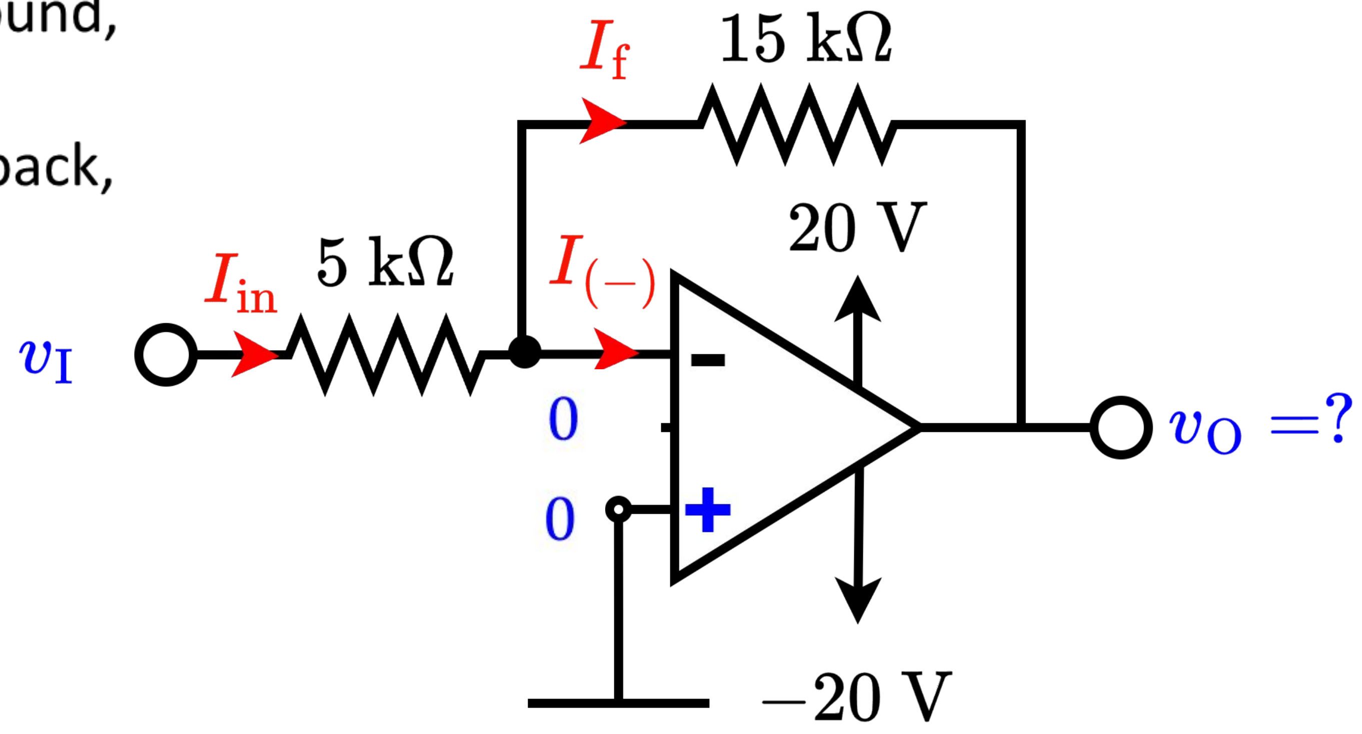
Solving Closed Loop Op-Amp Circuit



Solving Closed Loop Op-Amp Circuit

1. Since $\boldsymbol{v_+}$ is connected to ground,
$$\boldsymbol{v_+ = 0\ \mathrm{V}}$$
2. Since there is negative feedback, from virtual short,
$$\boldsymbol{v_- = v_+ = 0\ \mathrm{V}}$$

3. Ohm's law for **5 kΩ**:
$$I_{\mathrm{in}} = \frac{v_{\mathrm{I}} - 0}{5} = \frac{v_{\mathrm{I}}}{5}$$
4. Ohm's law for **15 kΩ**:
$$I_{\mathrm{f}} = \frac{0 - v_O}{15} = -\frac{v_{\mathrm{O}}}{15}$$



Solving Closed Loop Op-Amp Circuit

3. Ohm's law for $\mathbf{5\ k\Omega}$:

$$I_{\text{in}} = \frac{v_I - 0}{5} = \frac{v_I}{5}$$

4. Ohm's law for $\mathbf{15\ k\Omega}$:

$$I_f = \frac{0 - v_O}{15} = -\frac{v_O}{15}$$

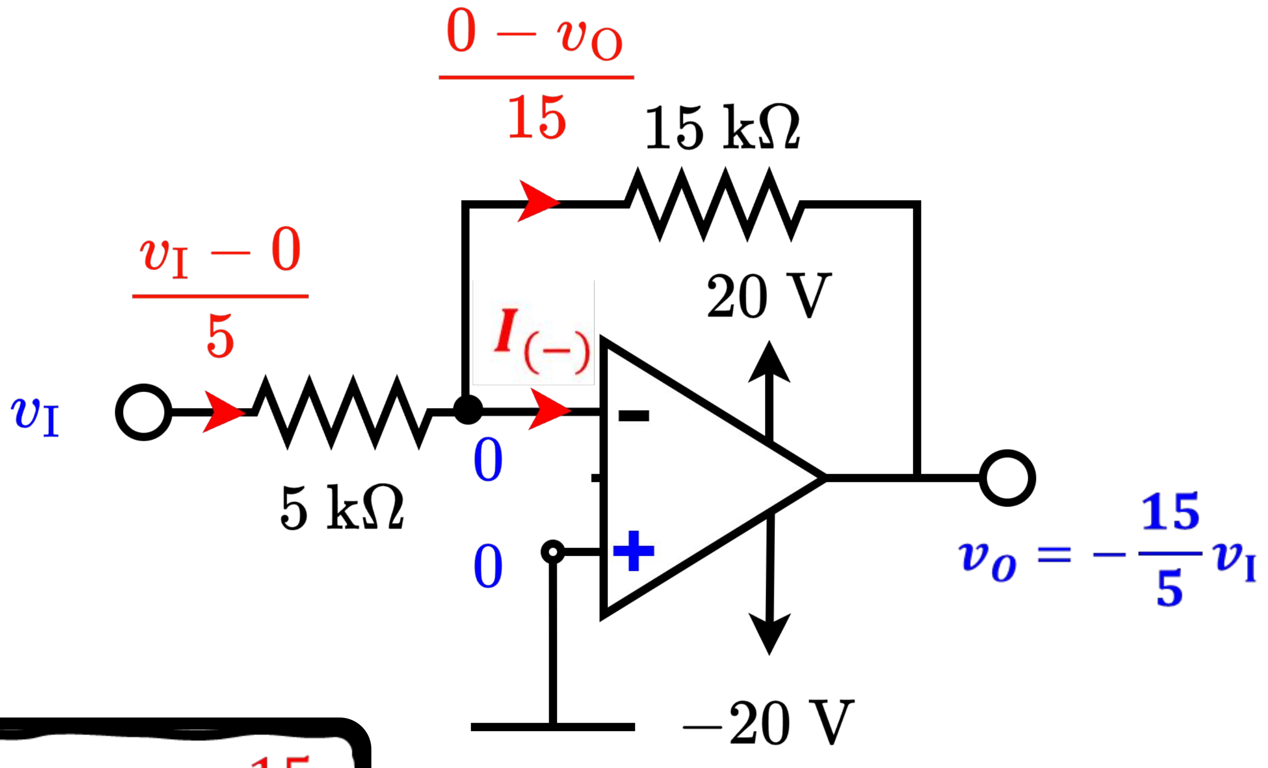
5. For "**ideal**" op-amp,

$$\boldsymbol{I}_{(-)} = \boldsymbol{I}_{(+)} = 0$$

6. So, $\boldsymbol{I}_{\text{in}} = \boldsymbol{I}_{\text{f}}$

$$-\frac{v_O}{15} = \frac{v_I}{5}$$

$$\boxed{\Rightarrow \frac{v_O}{v_I} = -\frac{15}{5}}$$



Solving Closed Loop Op-Amp Circuit

3. Ohm's law for **5 kΩ**:

$$I_{\text{in}} = \frac{v_{\text{I}} - 0}{R_{\text{I}}} = \frac{v_{\text{I}}}{R_{\text{I}}}$$

4. Ohm's law for **15 kΩ**:

$$I_{\text{f}} = \frac{0 - v_O}{R_f} = -\frac{v_{\text{O}}}{R_f}$$

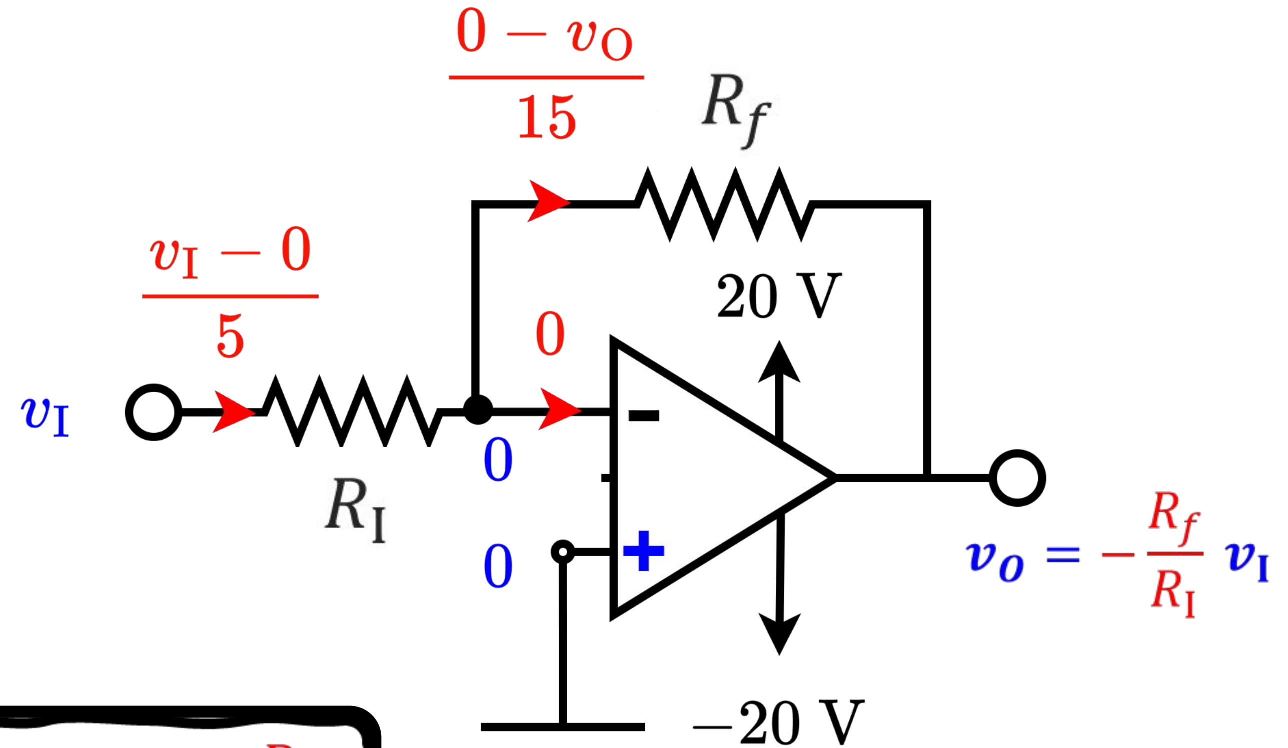
5. For "**ideal**" op-amp,

$$\boldsymbol{I}_{(-)} = \boldsymbol{I}_{(+)} = 0$$

6. So, $\boldsymbol{I}_{\text{in}} = \boldsymbol{I}_{\text{f}}$

$$-\frac{v_{\text{O}}}{R_f} = \frac{v_{\text{I}}}{R_{\text{I}}}$$

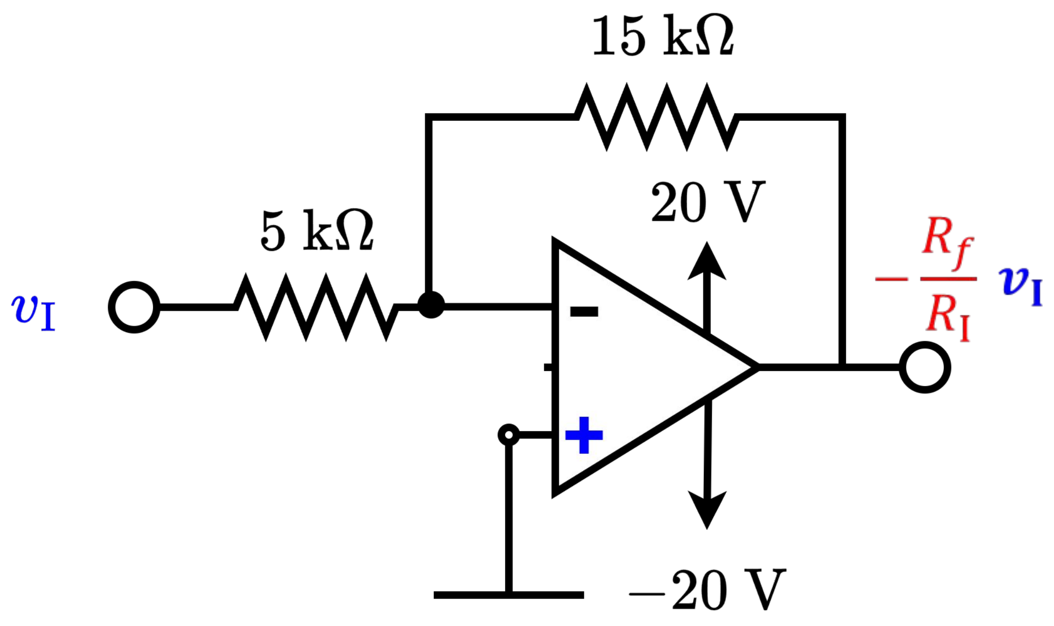
$$\boxed{\Rightarrow \frac{v_{\text{O}}}{v_{\text{I}}} = -\frac{R_f}{R_{\text{I}}}}$$



Solving Closed Loop Op-Amp Circuit

7. Check whether the amplified voltage exceeds **saturation limit.**

$$-\frac{R_f}{R_\mathrm{I}}v_\mathrm{I} = -\frac{15}{5}v_\mathrm{I} = -3v_\mathrm{I}$$



If $v_I = 3$ V:

$$-3v_\mathrm{I} = -3 \times 3 > -20\ \mathrm{V}$$

$$\therefore\ v_O = -9\ \mathrm{V}$$

If $|v_\mathrm{I}| > 6.67$ V:

Op-amp goes into saturation as $-3v_\mathrm{I} = -3 \times 6.67 < -20$

$$\therefore\ v_O = -20\ \mathrm{V}$$

Part 1 ends here

Outline

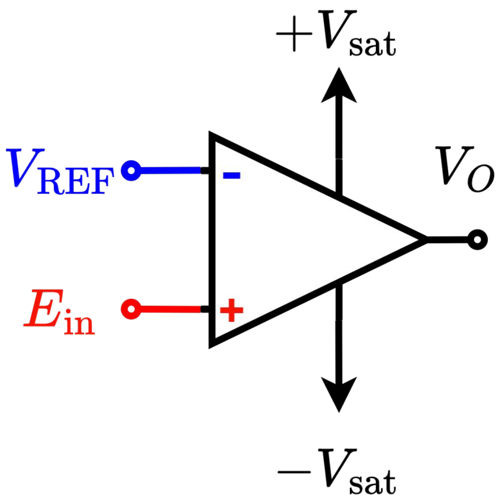
- **Closed Loop Operational Amplifier: Configurations**

  - Basic Op-Amp Configurations: OL vs CL

  - Closed Loop Configurations: Formulas and Examples

Basic Op-Amp Configurations

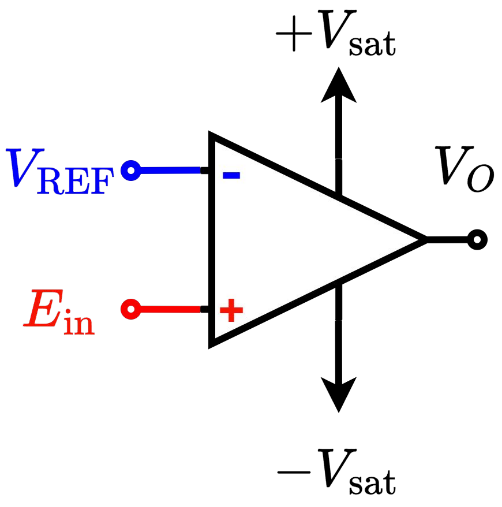
- **Open-loop Configurations**
  1. Comparator / Voltage Level Detectors



- **Closed Loop Configurations**
  1. Voltage Follower
  2. Inverting Amplifier
  3. Inverting Summer
  4. Non-Inverting Amplifier
  5. Weighted Subtractor
  6. Inverting Integrator
  7. Inverting Differentiator
  8. Exponential Converter
  9. Logarithmic Converter
  10. Multiplier
  11. Divider

Basic Op-Amp Configurations

- **Open-loop Configurations**
  1. Comparator / Voltage Level Detectors



- **Closed Loop Configurations**
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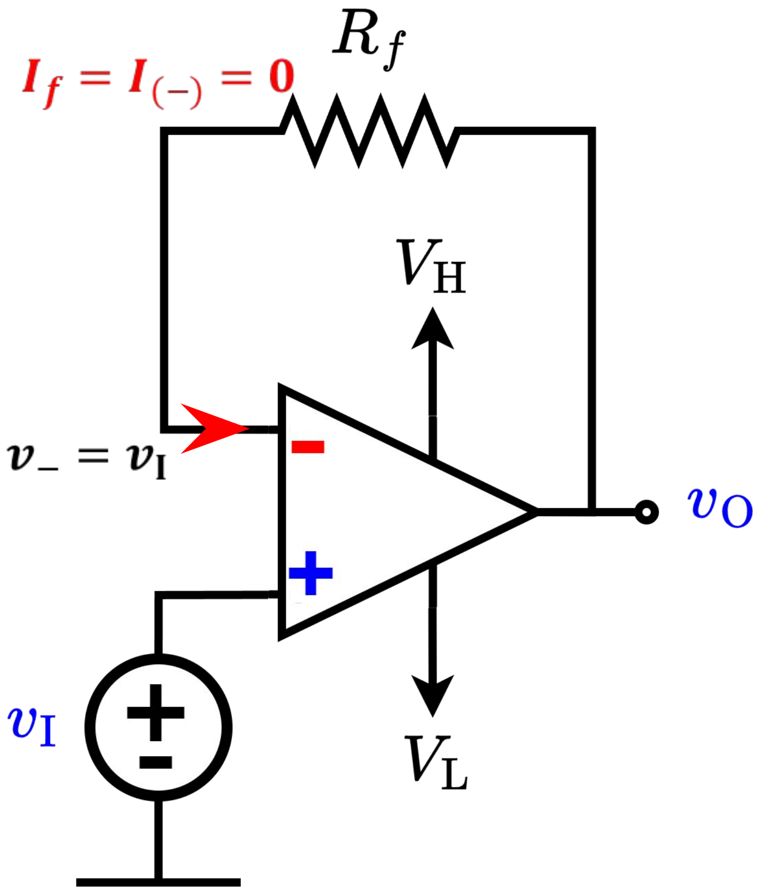
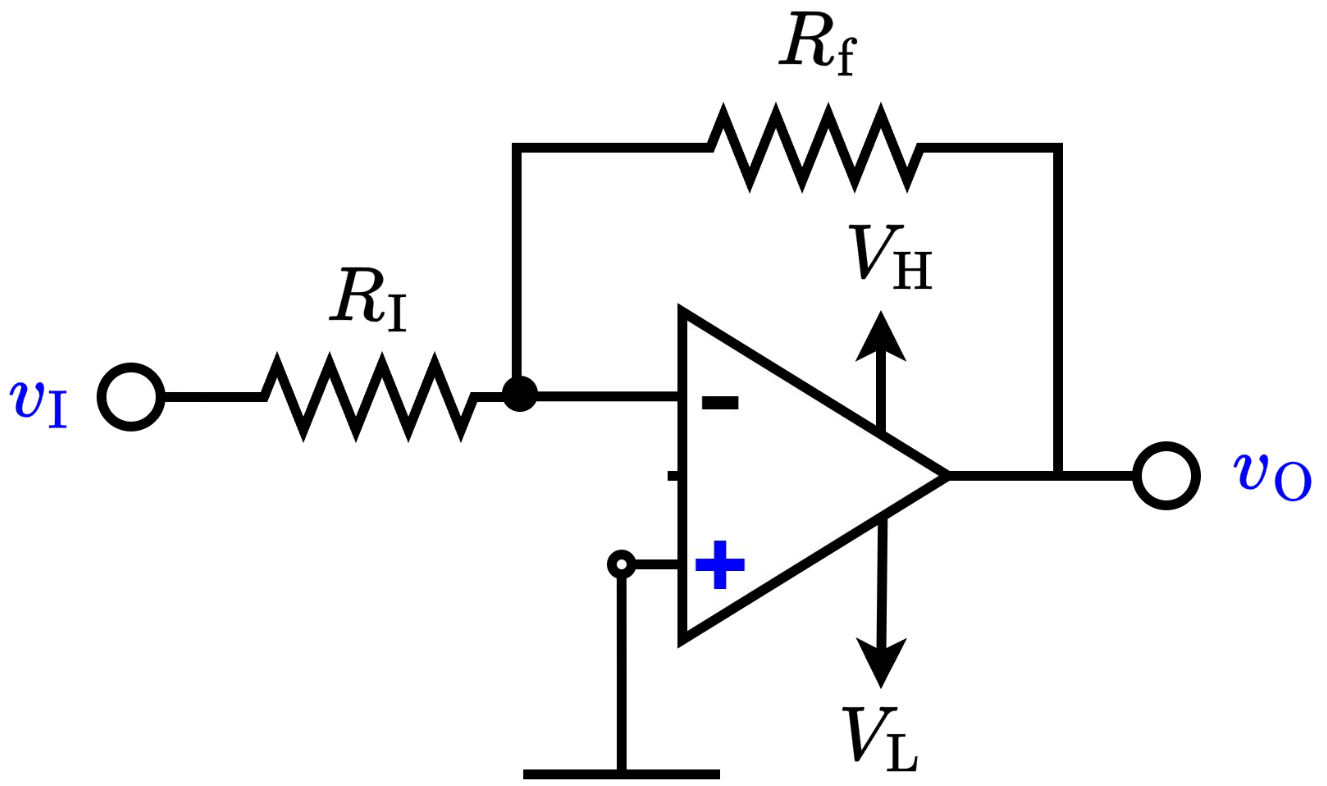
Voltage Follower / Buffer:



$$v_O = v_I$$

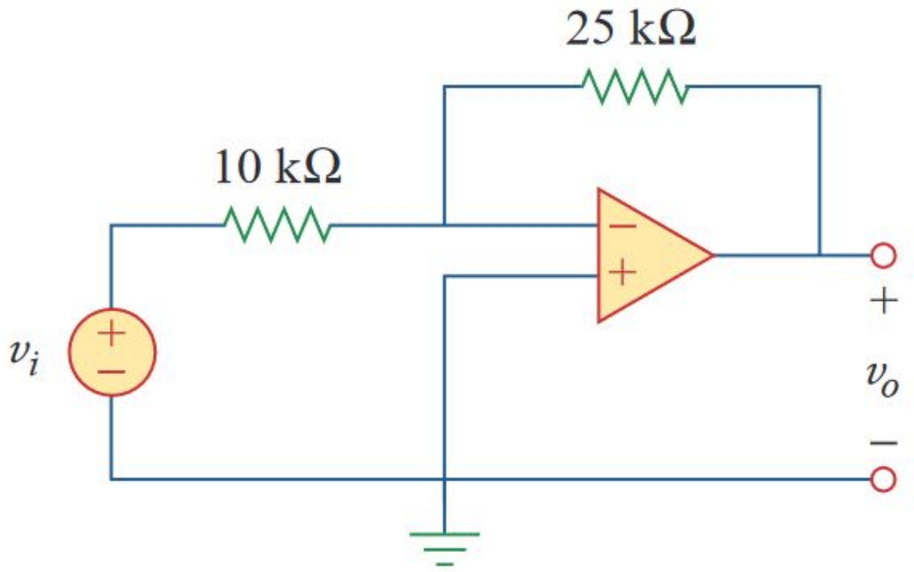
Regardless of the value of R_f

Inverting Amplifier



$$v_{\mathrm{O}} = \begin{cases} V_{\mathrm{H}}, & \text{if } v_O \geq V_{\mathrm{H}} \\ -v_I \cdot \frac{R_f}{R_i}, & \text{if } V_{\mathrm{L}} \leq v_O \leq V_{\mathrm{H}} \\ V_{\mathrm{L}}, & \text{if } v_O \leq V_{\mathrm{L}} \end{cases}$$

Example - 1



If $\boldsymbol{v_i} = \mathbf{0.5}$ V, calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Current in the $\mathbf{10\ k\Omega}$ resistor.

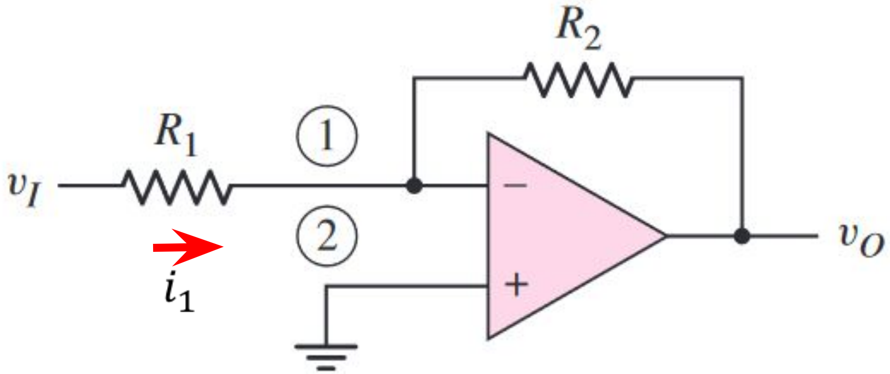
(a)

$$\boldsymbol{v_o = -\frac{R_f}{R_i} \cdot v_i = -2.5 v_i = -1.25\ \mathrm{V}}$$

(b) Current through the $\mathbf{10\ k\Omega}$ resistor is

$$i = \frac{v_i}{R_i} = \frac{0.5}{10}\ \mathrm{mA} = \mathbf{50\ \mu A}$$

Example - 2



Design the circuit such that the closed loop voltage gain is $\boldsymbol{A_{\mathrm{CL}}} = \mathbf{-5}$. Assume the op-amp is driven by an ideal sinusoidal source, $\boldsymbol{v_I} = \mathbf{0.1\,sin}\,(\boldsymbol{\omega t})\,(\boldsymbol{V})$, that can supply a maximum current of $\mathbf{5\ \mu A}$.

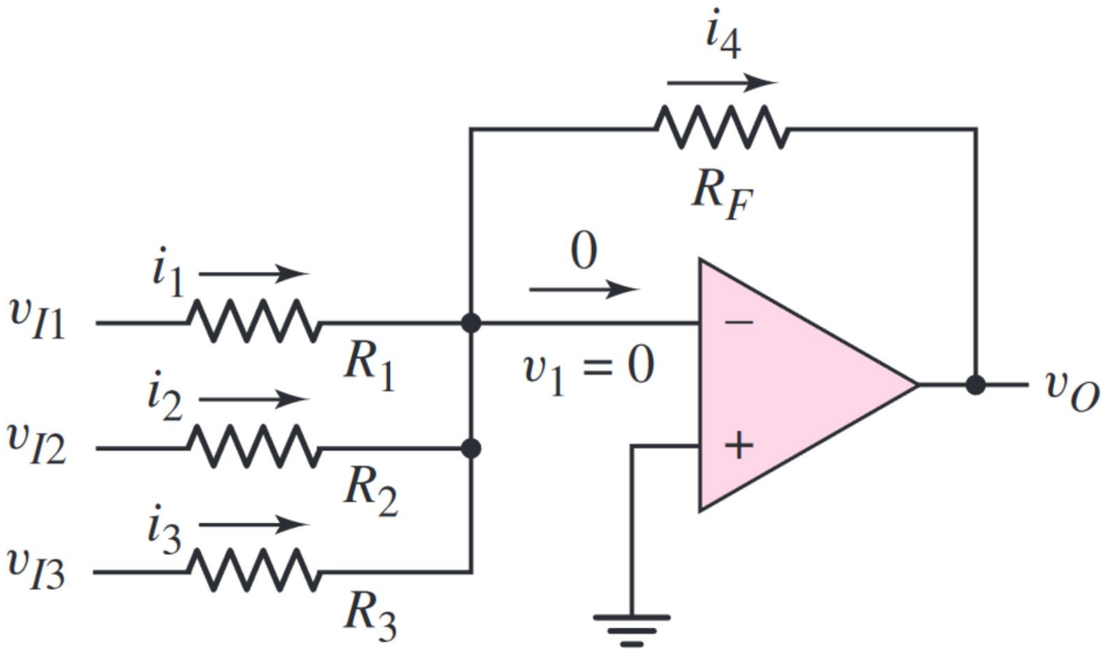
$$\boldsymbol{i_1} = \frac{\boldsymbol{v_I}}{\boldsymbol{R_1}}$$

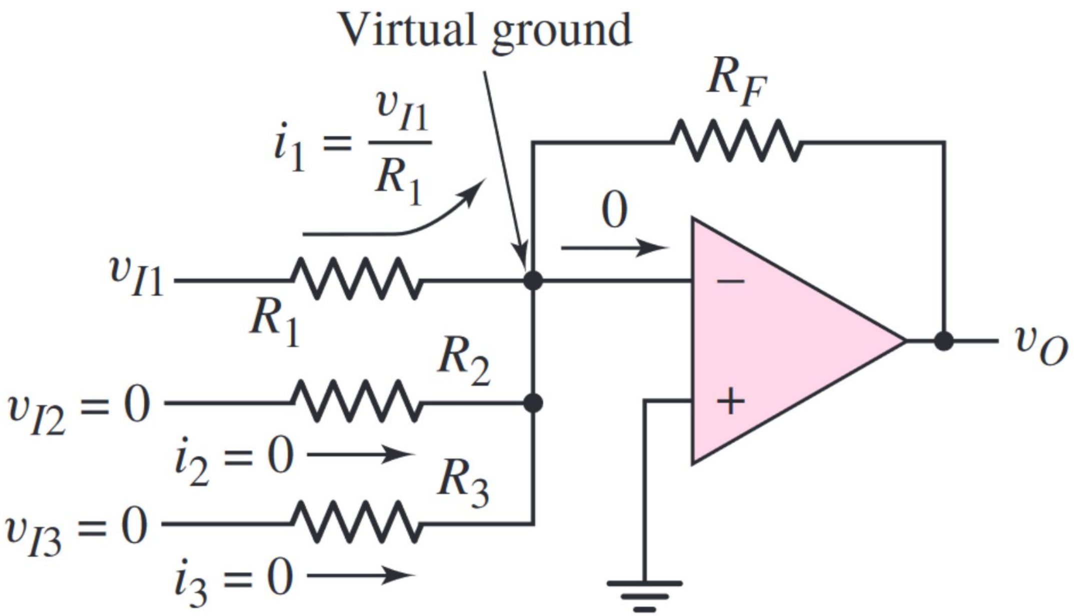
$$\boldsymbol{R_1} = \frac{v_I(\max)}{i_1(\max)} = \frac{0.1}{5\times10^{-3}} = \mathbf{20\ k\Omega}$$

$$\boldsymbol{R_2} = -A_{\mathrm{CL}}\cdot R_1 = 5\times20 = \mathbf{100\ k\Omega}$$

Inverting Summer

- Multichannel Amplifier

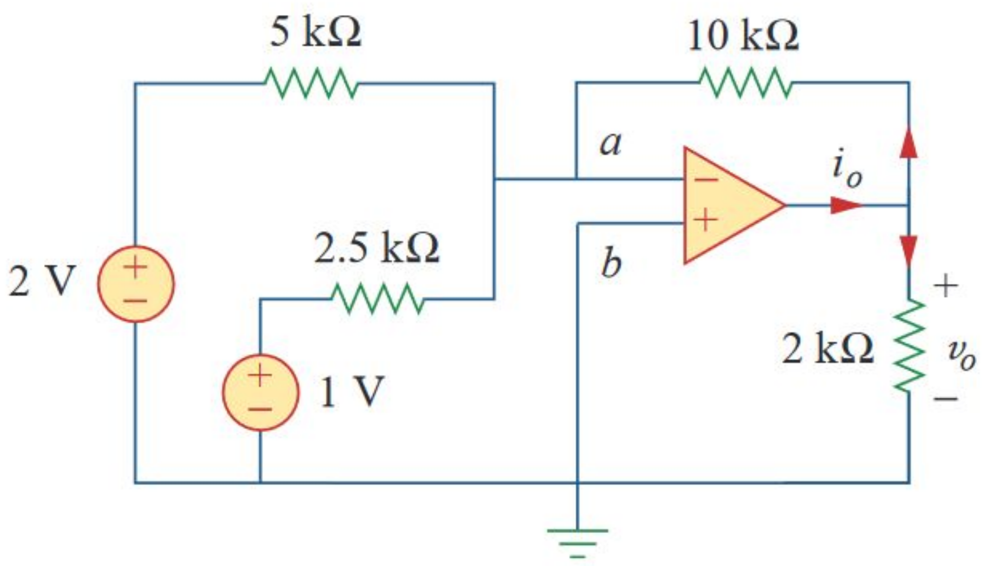




$$v_O(v_{I1}) = -i_1 R_F = -\left(\frac{R_F}{R_1}\right) v_{I1}$$

$$v_O = -\left(\frac{R_F}{R_1} v_{I1} + \frac{R_F}{R_2} v_{I2} + \frac{R_F}{R_3} v_{I3}\right)$$

Example - 3



Calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Output current $\boldsymbol{i_o}$.

(a)

$$\boldsymbol{v_o} = -\left(\frac{\mathbf{10}}{\mathbf{5}}\cdot \mathbf{2} + \frac{\mathbf{10}}{\mathbf{2.5}}\cdot \mathbf{1}\right) = \mathbf{-8\ V}$$

(b)

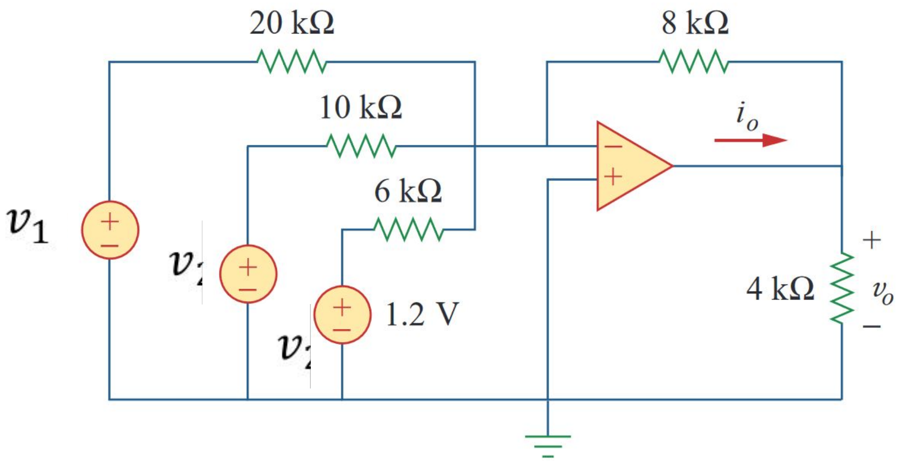
$$i = \frac{v_o}{10} + \frac{v_o}{2} = (-0.8 - 4\ )\ \mathrm{mA} = \mathbf{-4.8\ mA}$$

Part 2 ends here

Example 4

Design an op-amp circuit with inputs $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ and $\boldsymbol{v_3}$ such that, output voltage $\boldsymbol{v_o}$:

$$\boldsymbol{v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3}$$



Solution:

The given function can be achieved by an **inverting summing amplifier**. Having the voltage transfer formula as:

$$v_o = \left(-\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2 - \cdots \frac{R_f}{R_n}v_n\right)$$

Here, the numerators of all the coefficients of input voltages are same ($\boldsymbol{R_f}$). As per the given problem, this can be achieved by setting the numerator to the LCM of 2 and 4 (i.e., to **8**).

$$\boldsymbol{v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3}$$

$$\boldsymbol{= -\frac{8}{20}v_1 - \frac{8}{10}v_2 - \frac{8}{6}v_3}$$

Example 4

Design an op-amp circuit with inputs v_1, v_2 and v_3 such that, output voltage v_o:

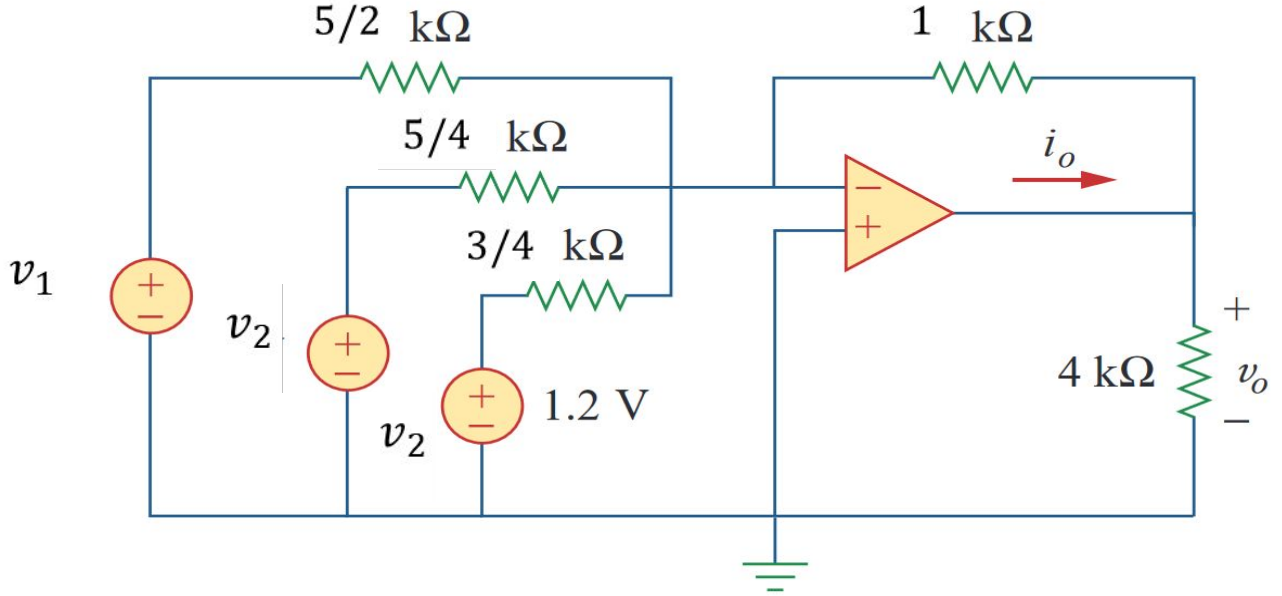
$$v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3$$

Easier Solution:

$$v_o = -\frac{2}{5}v_1 - \frac{4}{5}v_2 - \frac{4}{3}v_3$$

$$\cancel{= -\frac{8}{20}v_1 - \frac{8}{10}v_2 - \frac{8}{6}v_3}$$

$$= -\frac{1}{5/2}v_1 - \frac{1}{5/4}v_2 - \frac{1}{3/4}v_3$$

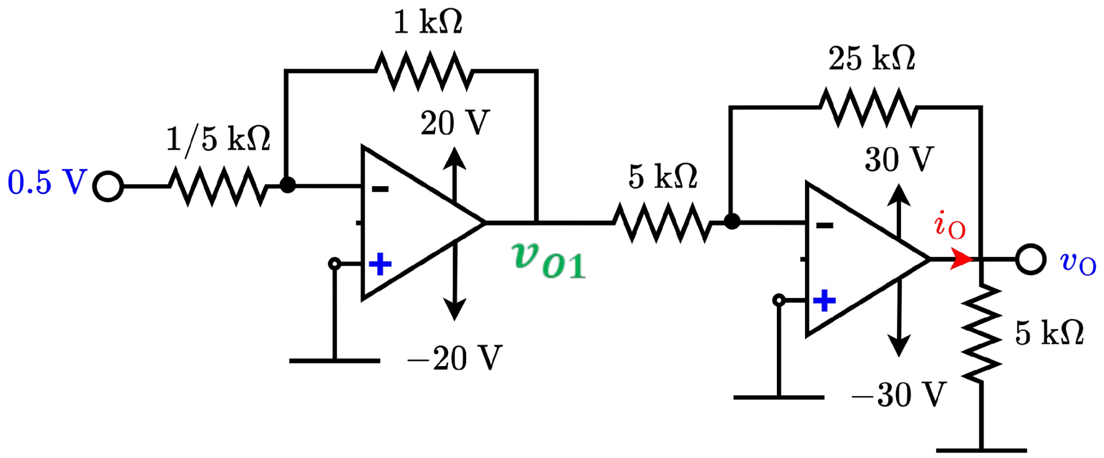


Example - 5

$v_I = 0.5$ V. Calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Output current $\boldsymbol{i_o}$.



(a)

$$\boldsymbol{v_{O1} = -\frac{1}{1/5} \times 0.5\ \mathrm{V} = -2.5\ \mathrm{V}}$$

$$\boldsymbol{v_o = -\frac{25}{5} \cdot v_{O1} = 12.5\ \mathrm{V}}$$

$$\boldsymbol{v_o = \left(-\frac{1}{1/5}\right) \cdot \left(-\frac{25}{5}\right) \cdot 0.5 = 12.5\ \mathrm{V}}$$

(b)

$$i = \frac{v_o}{5} + \frac{v_o}{25} = (2.5 + 0.5\ )\ \mathrm{mA} = \mathbf{3\ mA}$$

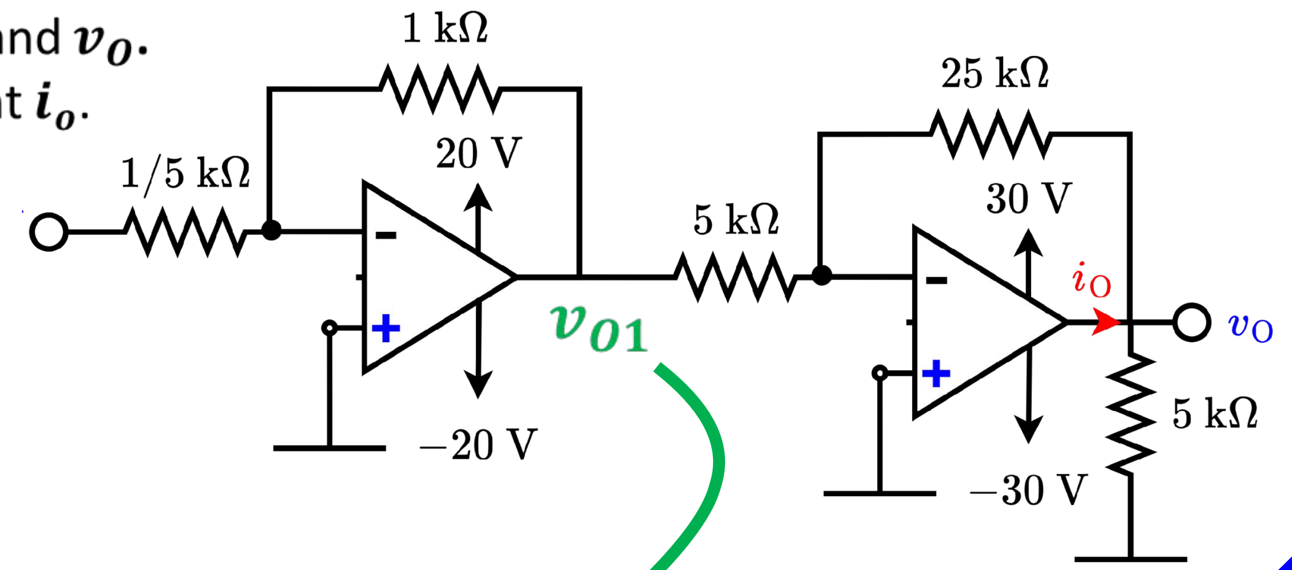
Example - 6

Calculate:

(a) Voltages $\boldsymbol{v_{o1}}$ and $\boldsymbol{v_O}$.

(b) Output current $\boldsymbol{i_o}$.

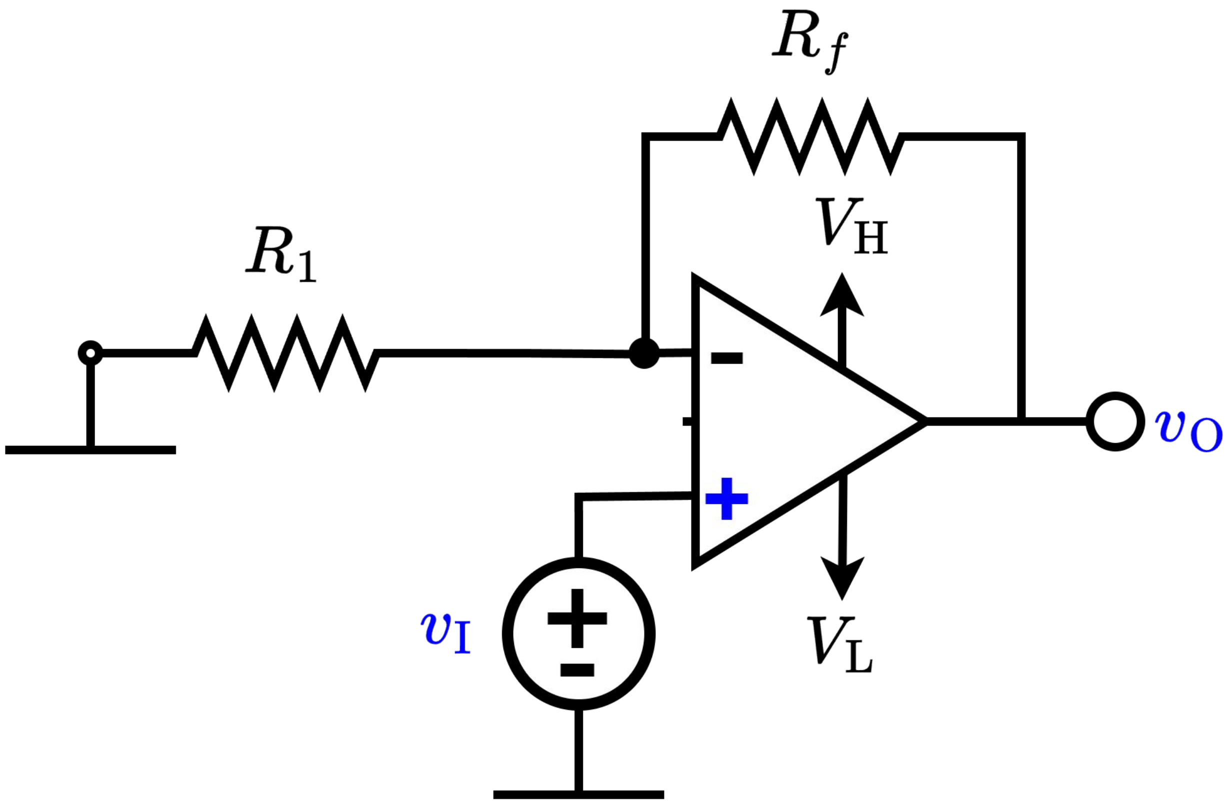
$v_I = 0.5\sin(120\pi t)$



$v_{O1} = 2.5\sin(120\pi t)$

$v_O = 12.5\sin(120\pi t)$

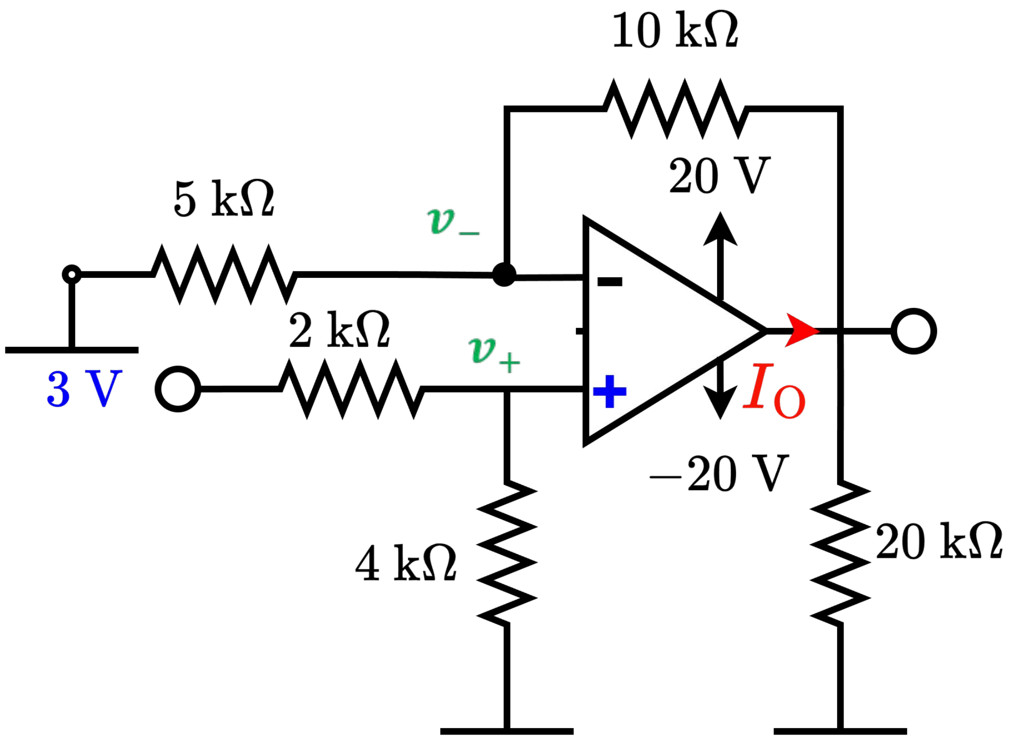
Non-Inverting Amplifier



$$v_O = \begin{cases} V_H, & \text{if } v_O \geq V_H \\ v_I \cdot (1 + \frac{R_f}{R_I}), & \text{if } V_L \leq v_O \leq V_H \\ V_L, & \text{if } v_O \leq V_L \end{cases}$$

$$v_O > v_I$$

Non-Inverting Amplifier: Example 7



Calculate:

(a) Output voltage $\boldsymbol{v_o}$.

(b) Output current $\boldsymbol{I_o}$.

(a)

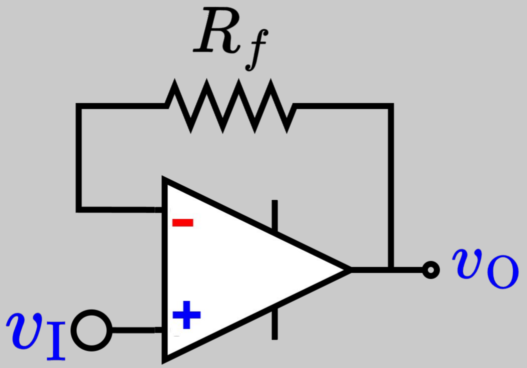
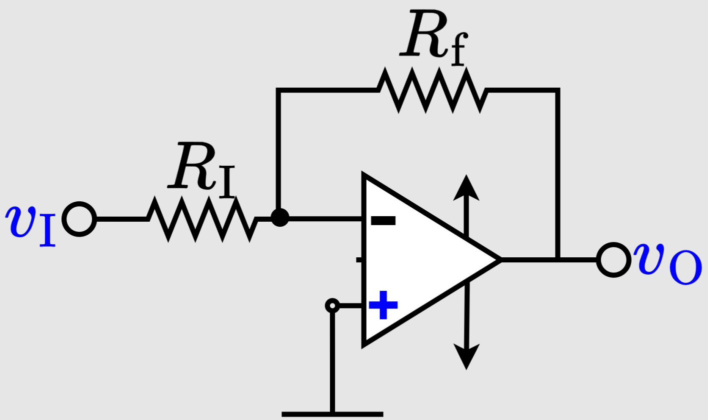
$$v_+ = \frac{4}{2+4} \times 3\ \mathrm{V} = 2\ \mathrm{V}$$

$$v_o = \left(1 + \frac{10}{5}\right) \cdot v_+ = 6\ \mathrm{V}$$

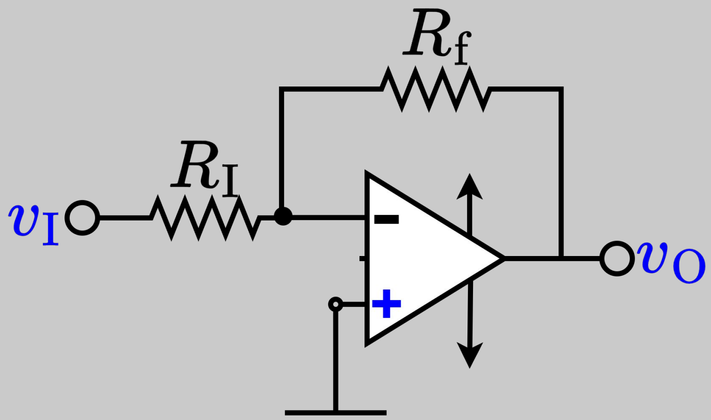
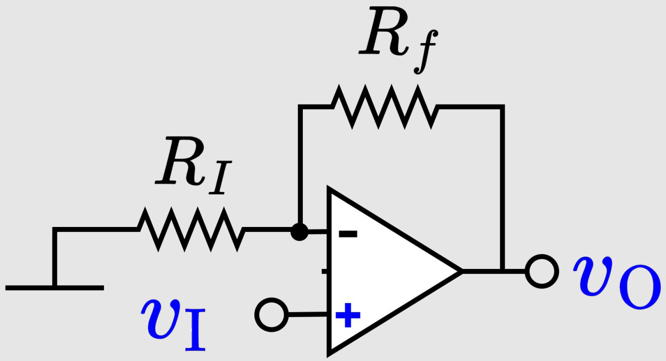
(b)

$$i = \frac{v_o}{20} + \frac{v_o}{10} = (0.3 + 0.6\,)\ \mathrm{mA} = 0.9\ \mathbf{mA}$$

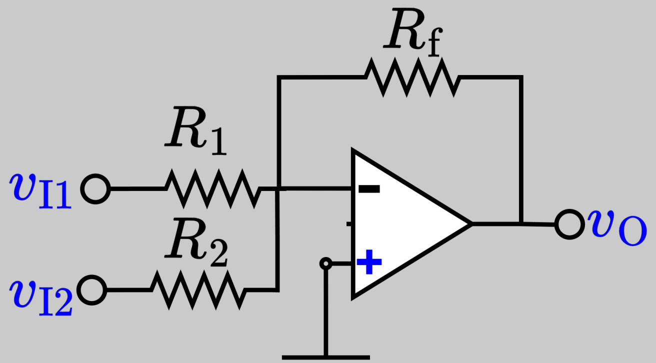
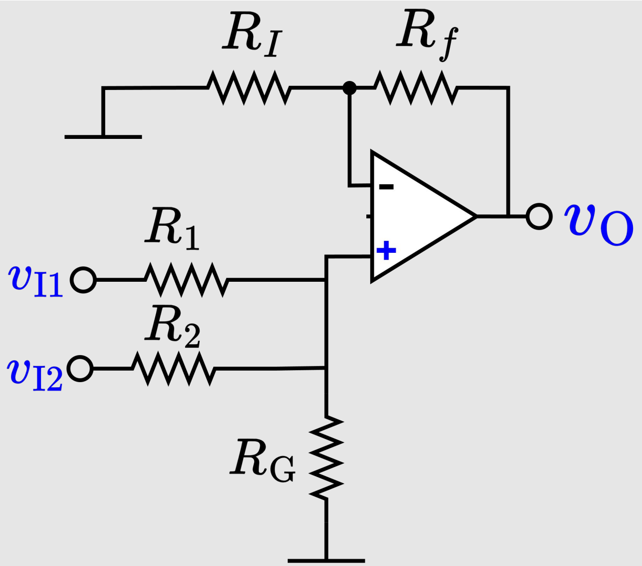
Summary so far

Configuration	Circuit diagram	Amplification Formula
Voltage Follower		
Inverting Amplifier		

Summary so far

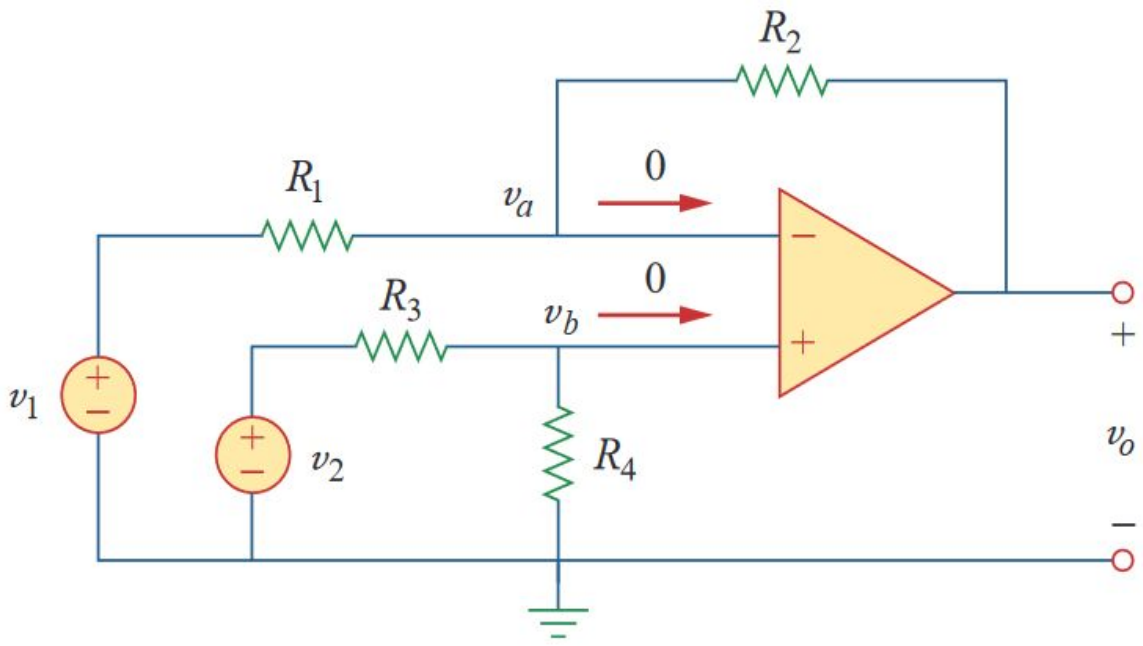
Configuration	Circuit diagram	Amplification Formula
Inverting Amplifier		
Non-Inverting Amplifier		

Summary so far

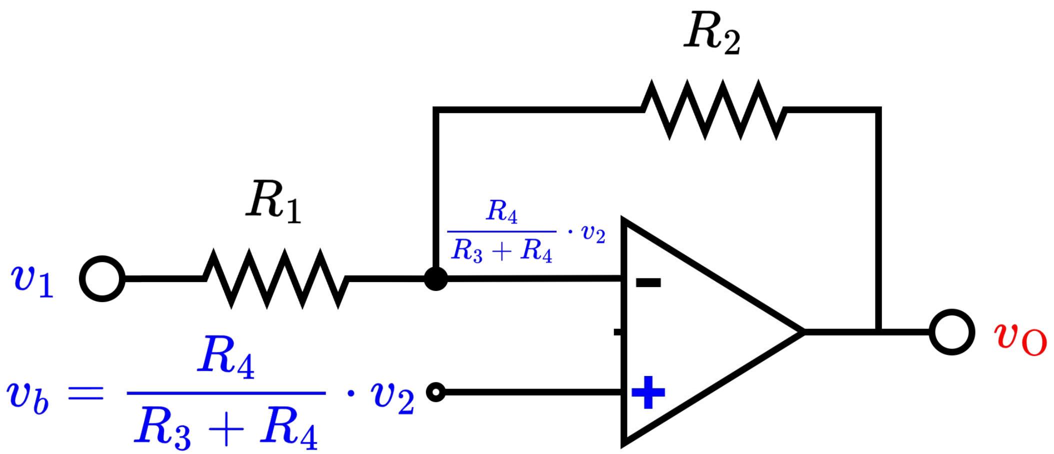
Configuration	Circuit diagram	Formula
Inverting ~~Amplifier~~ Adder		
Non-Inverting ~~Amplifier~~ Adder		

Part 3 ends here

Difference Amplifier



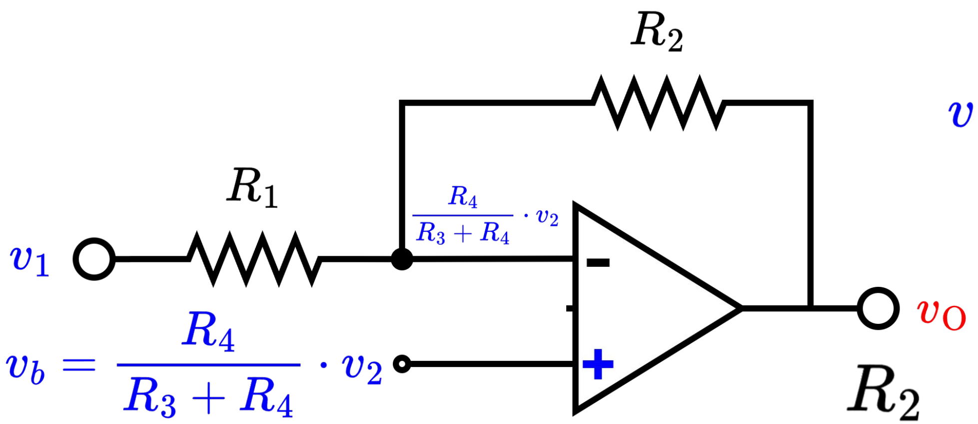
$$v_b = v_a = \frac{R_4}{R_3 + R_4} v_2$$



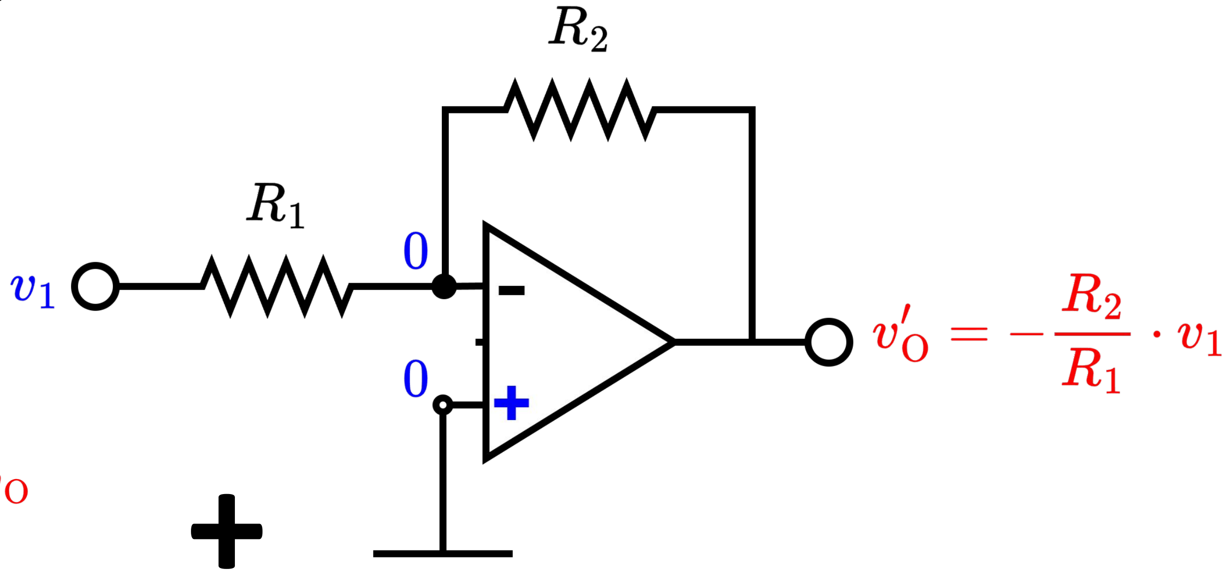
Simplified Circuit

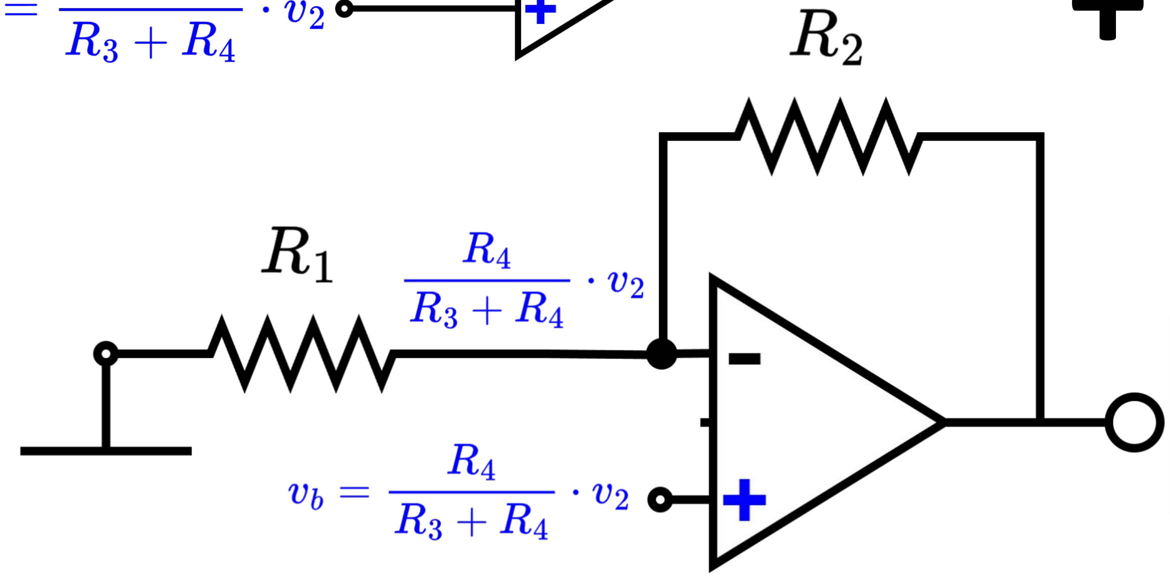
Difference Amplifier

Apply superposition principle:



+





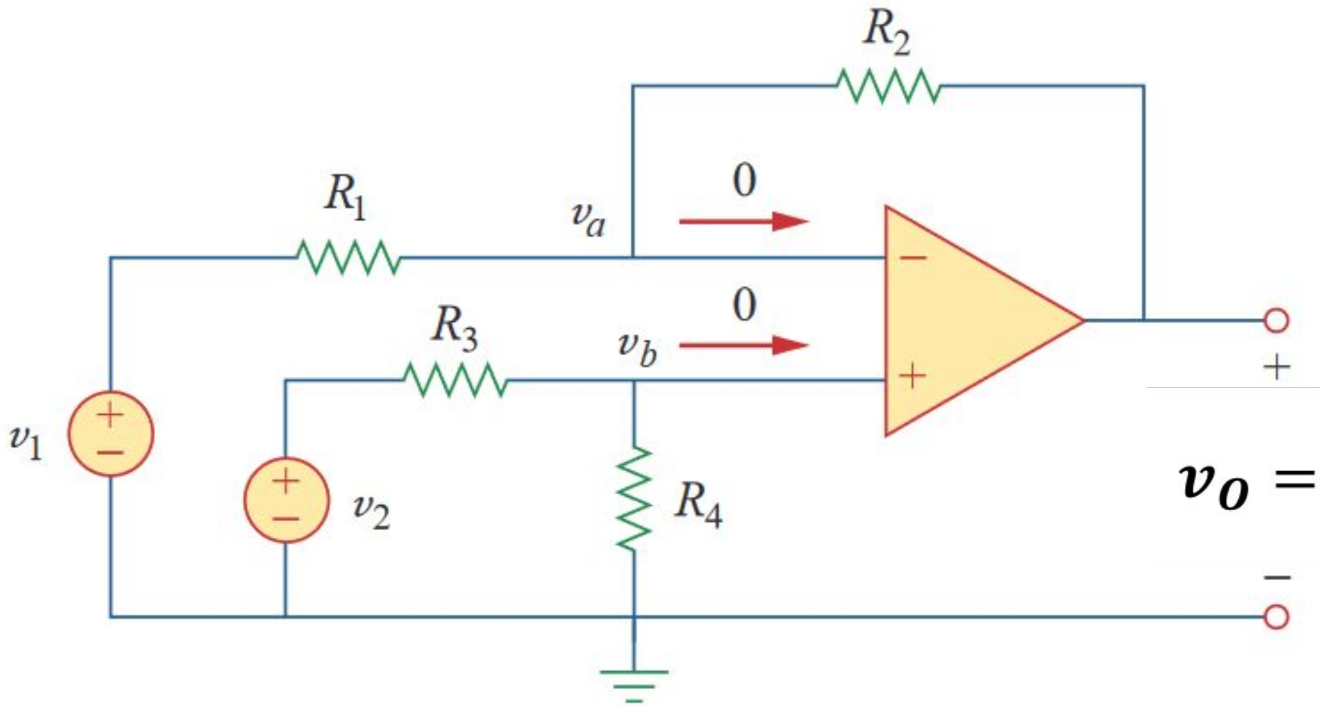
$$v_O'' = \left(1+\frac{R_2}{R_1}\right)\cdot\frac{R_4}{R_3+R_4}v_2$$

Difference Amplifier

$$v_b = v_a = \frac{R_4}{R_3 + R_4} v_2$$

$$v_O = v_O'' + v_O'$$

$$v_O = \left(1 + \frac{R_2}{R_1}\right) v_b - \frac{R_2}{R_1} \cdot v_1$$

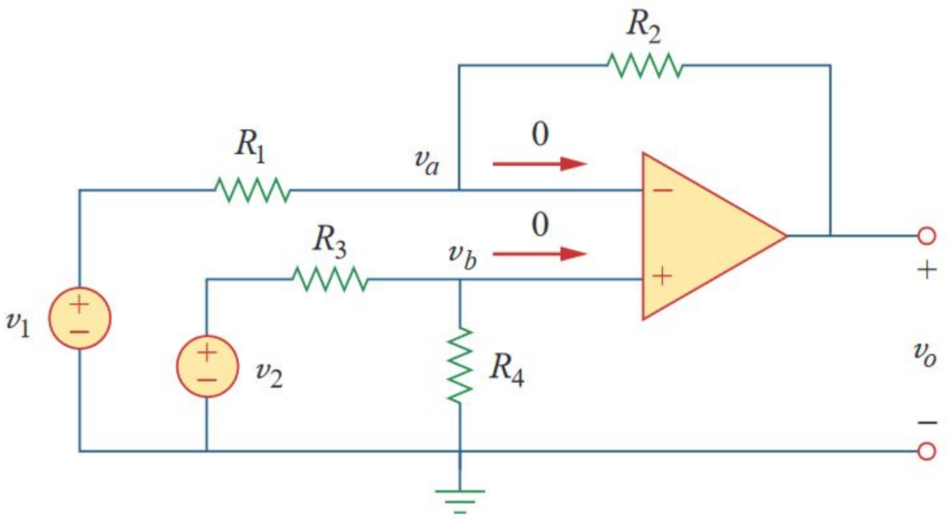


$$v_O = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} \cdot v_1$$

Difference Amplifier – Example 8

Design an op amp circuit with inputs $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ such that

$$\boldsymbol{v_o = -5v_1 + 3v_2.}$$



Solution: Method 1

$$\boldsymbol{v_O = -\frac{R_2}{R_1}\cdot v_1 + \left(1+\frac{R_2}{R_1}\right)\cdot \frac{R_4}{R_3+R_4} v_2}$$

$$\boldsymbol{\therefore \frac{R_2}{R_1} = 5}$$

$$\boldsymbol{\therefore (1+5)\cdot \frac{R_4}{R_3+R_4} = 3}$$

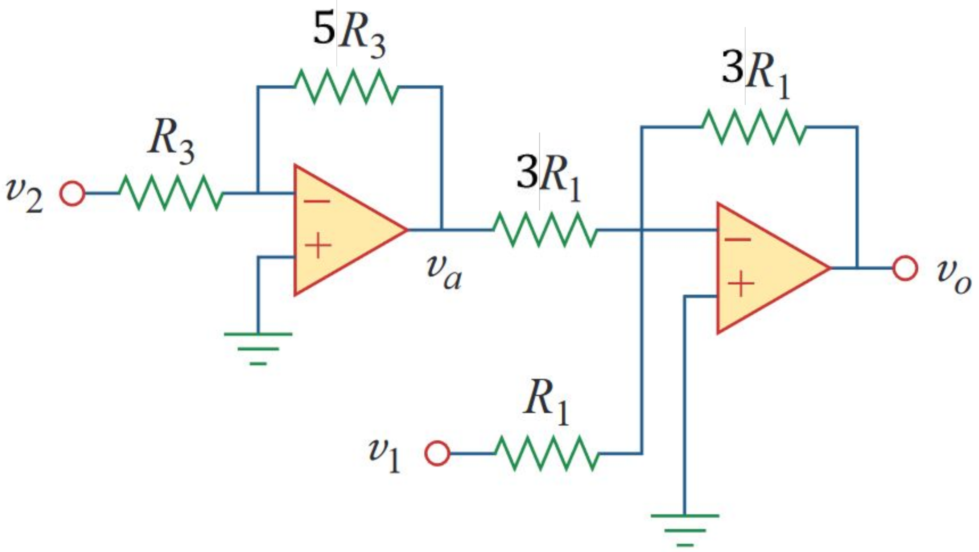
$$\boldsymbol{\Rightarrow\ R_3 = R_4}$$

* To implement functions of the form $v_O = -\boldsymbol{A}v_1 + \boldsymbol{B}v_2$, with difference amplifiers, ***B* must be less than *A* + 1**

Difference Amplifier – Example 9

Design an op amp circuit with inputs $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ such that

$$\boldsymbol{v_o = -3v_1 + 5v_2.}$$



Solution: Method 2

Two stages amplifiers must be , we can implement this function.

$\boldsymbol{-3v_1}$: Can be achieved with one stage inverting amplifier

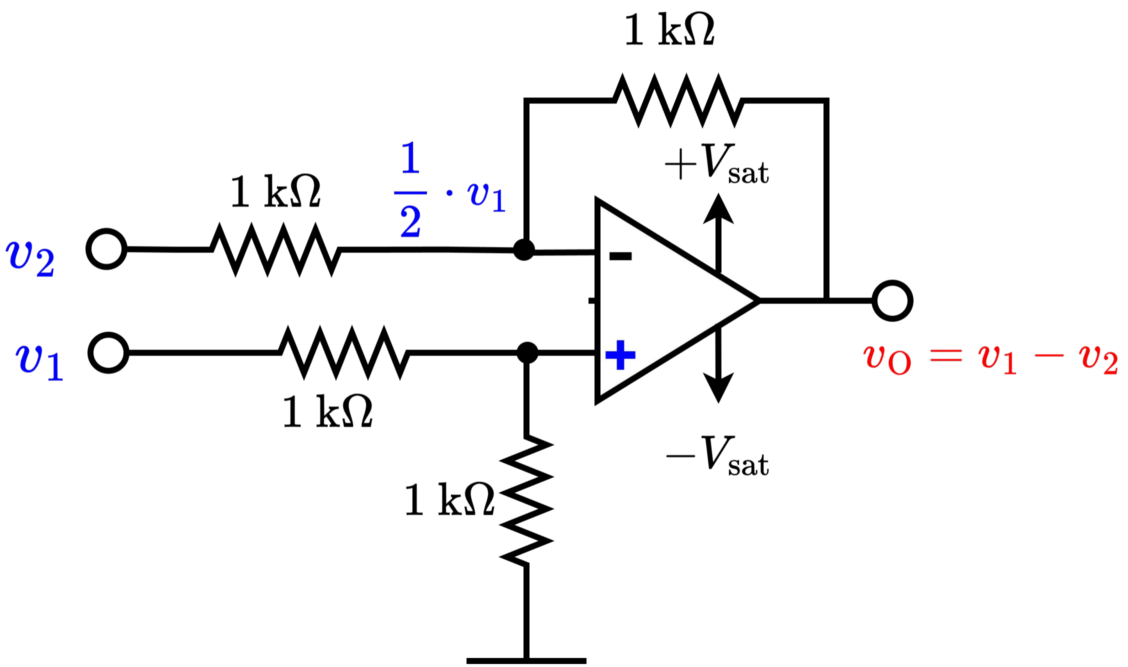
$\boldsymbol{+5v_2}$: Can be achieved by cascading two inverting amplifiers → $(- \times - = +)$

* Here, in the expression of the form $v_O = -\boldsymbol{A}v_1 + \boldsymbol{B}v_2$ given in the question $\boldsymbol{B > A + 1}$**. So, single stage difference amplifier cannot be used.**

Subtractor ($v_1 - v_2$)

Design an op amp circuit with inputs $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ such that

$$\boldsymbol{v_o = v_1 - v_2}.$$



Solution: Method 1

Inverting ratio = 1

Non-inverting ratio = 1

So, $\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}} = \mathbf{1}$ and $\left(\mathbf{1} + \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}\right) = \mathbf{2}$.

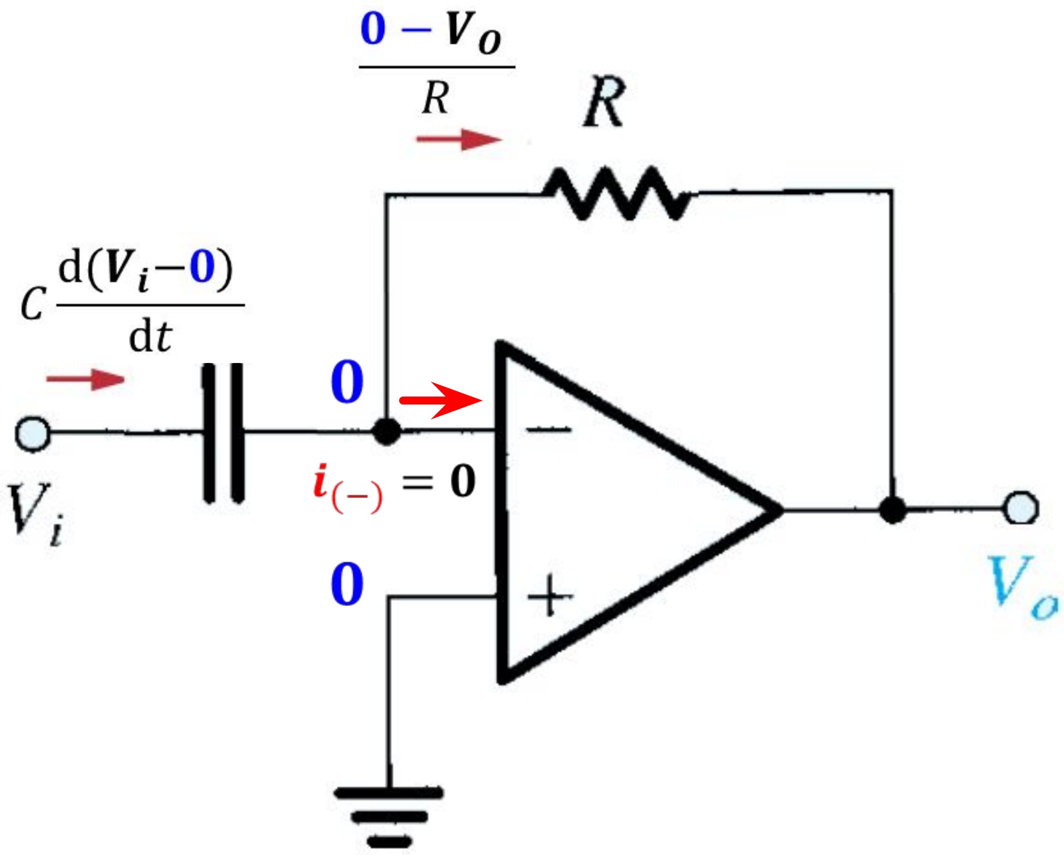
$$\therefore\ \boldsymbol{R_1 = R_2}$$

So, to get $\left(\mathbf{1} + \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}\right) \cdot \frac{\boldsymbol{R_4}}{\boldsymbol{R_3 + R_4}} = \mathbf{1}$,

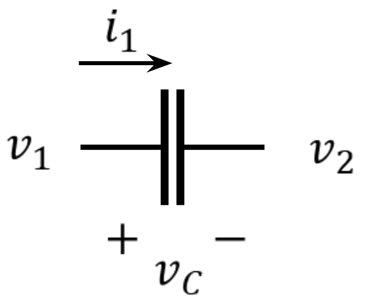
$$\therefore\ \boldsymbol{R_4 = R_3}$$

Op Amp as Inverting Differentiator

Since ideal op-amp, $\boldsymbol{i_- = i_+ = 0}$, so $\boldsymbol{i_1 = i_2}$



Review – Capacitor



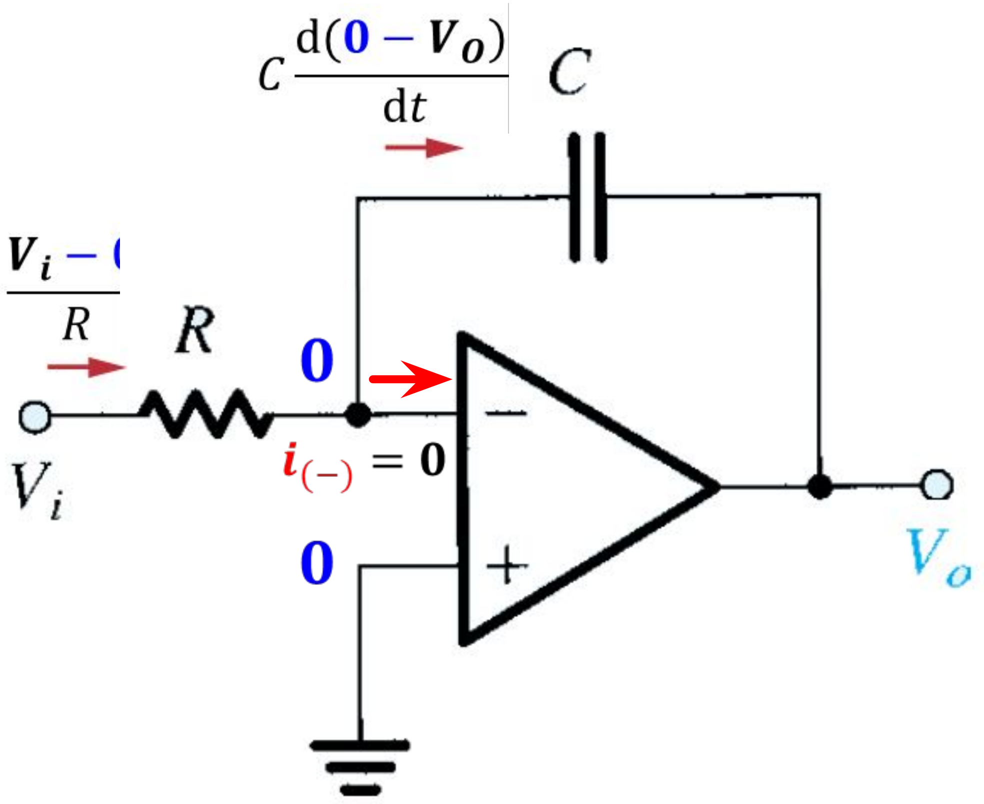
$$i_1 = C\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

$$\Rightarrow -\frac{V_o}{R} = C\frac{dV_i}{dt}$$

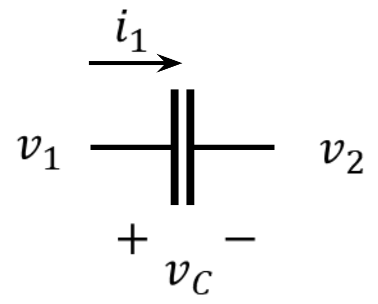
$$\boxed{\Rightarrow V_O = -RC\frac{dv_i}{dt}}$$

Op Amp as Inverting Integrator

Since ideal op-amp, $\boldsymbol{i_-} = \boldsymbol{i_+} = \mathbf{0}$, so $\boldsymbol{i_1} = \boldsymbol{i_2}$



Review – Capacitor



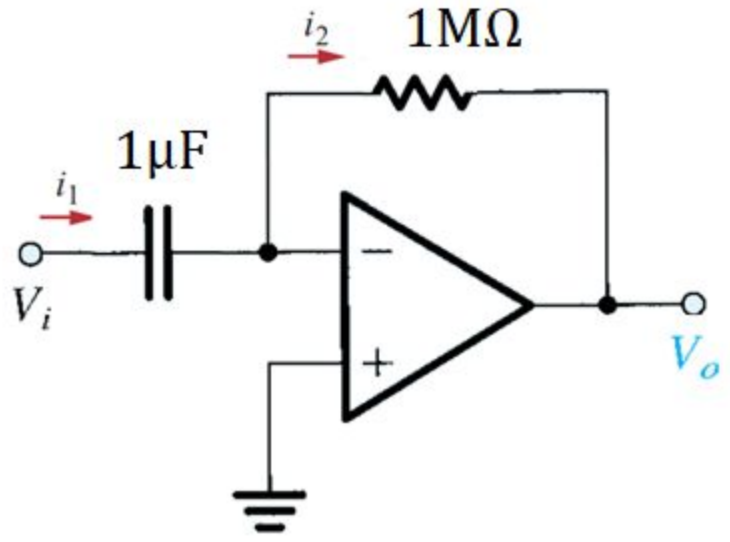
$$i_1 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

$$\Rightarrow \frac{\boldsymbol{V_i}}{R} = -C\frac{\mathrm{d}\boldsymbol{V_o}}{\mathrm{d}t}$$

$$\boxed{\Rightarrow \boldsymbol{V_o} = -\frac{1}{RC}\int \boldsymbol{V_i}(t)\mathrm{d}t}$$

Example 10

Observe the following Figure. If V_i = $5 \cdot \sin(6t)$, Find the value of $\boldsymbol{V_0}$



Solution:

This is a **differentiator**.

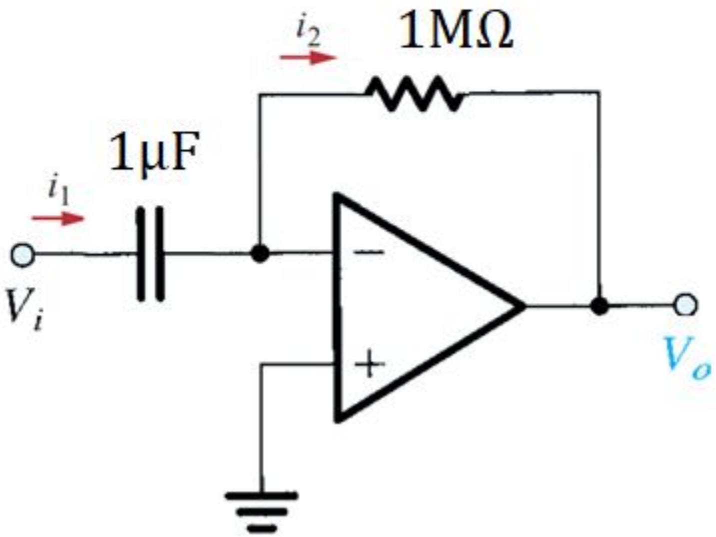
$$v_o = -RC\frac{\mathbf{d}V_i}{\mathbf{d}t}$$

$$= -(1 \times 10^6) \cdot (1 \times 10^{-6}) \times \frac{\mathbf{d}(5 \cdot \sin(6t))}{\mathbf{d}t}$$

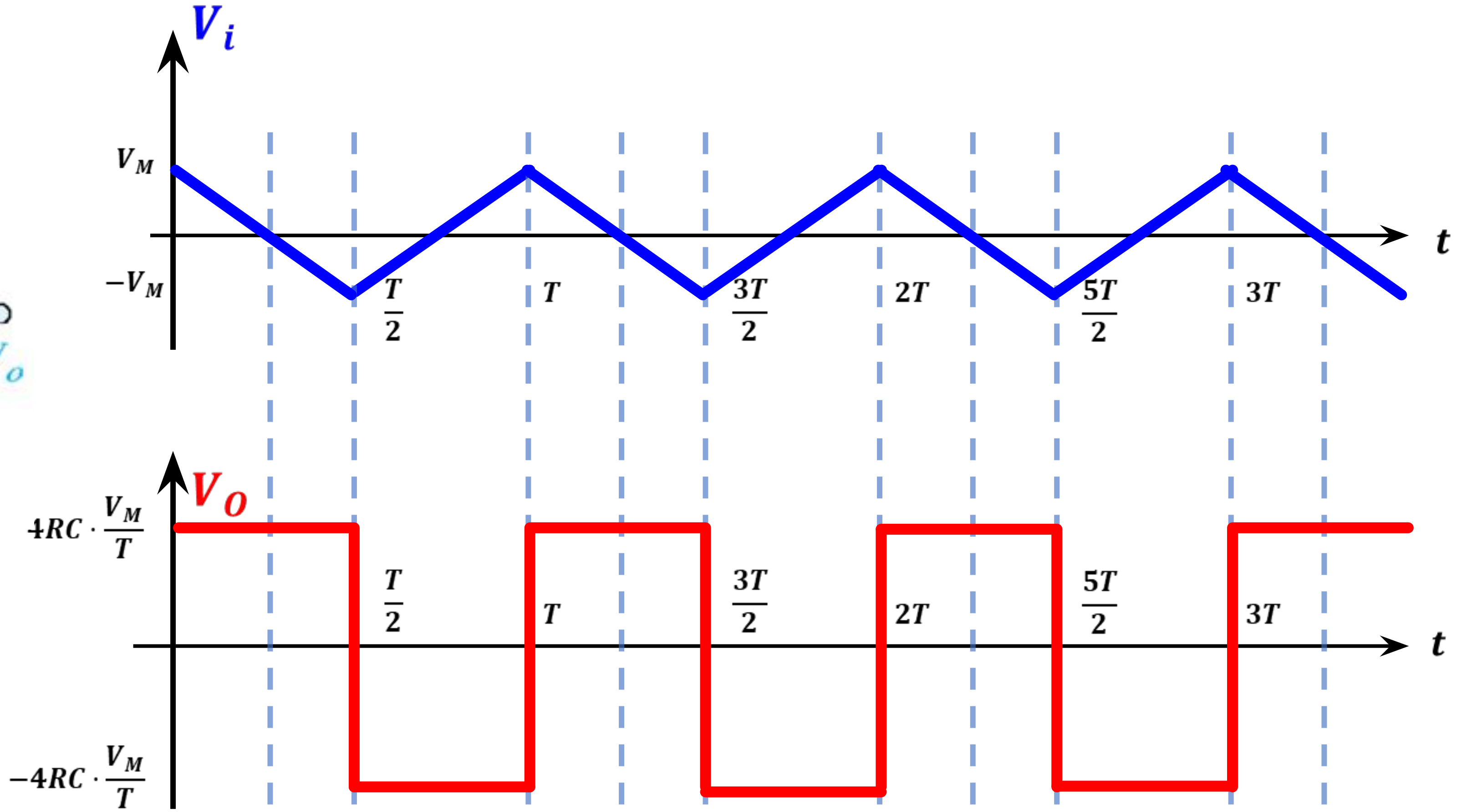
$$= -1 \times (5 \times 6\cos(6t))$$

$$= \mathbf{-30\cos(6}\boldsymbol{t}\mathbf{)}$$

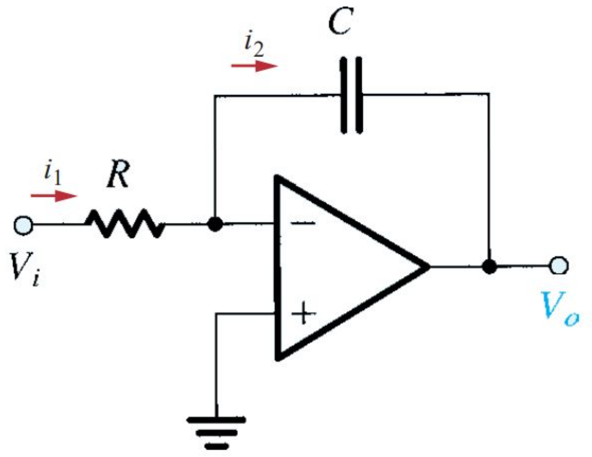
Example 11

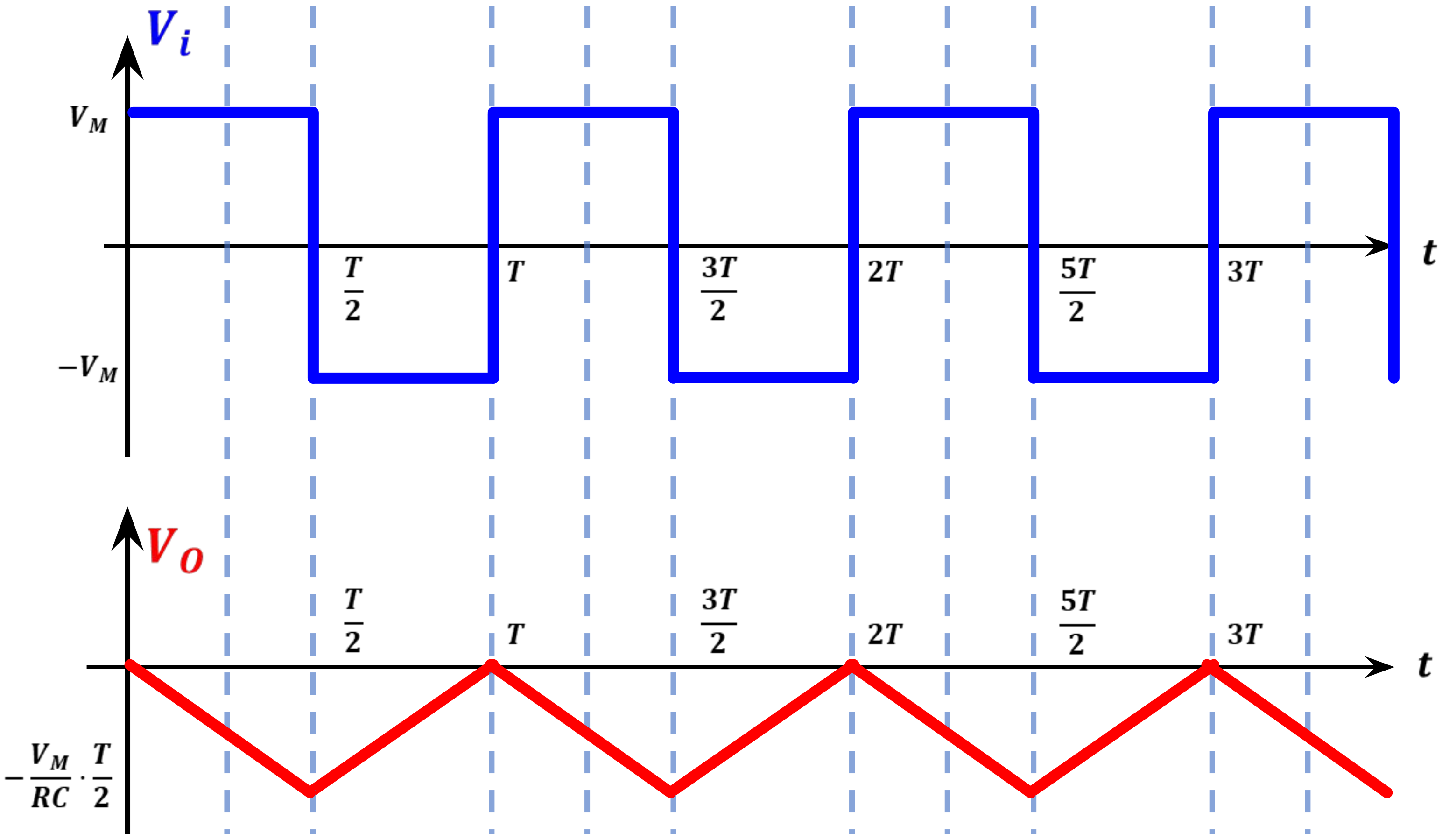


$$\text{Slope: } \left|\frac{dv}{dt}\right| = \frac{V_M-(-V_M)}{T/2} = \frac{4V_M}{T}$$



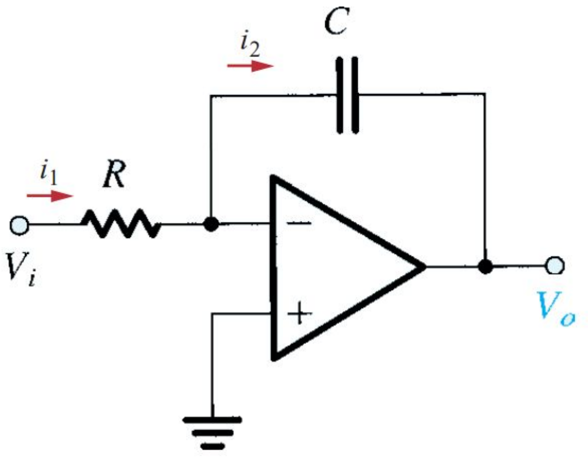
Example 12

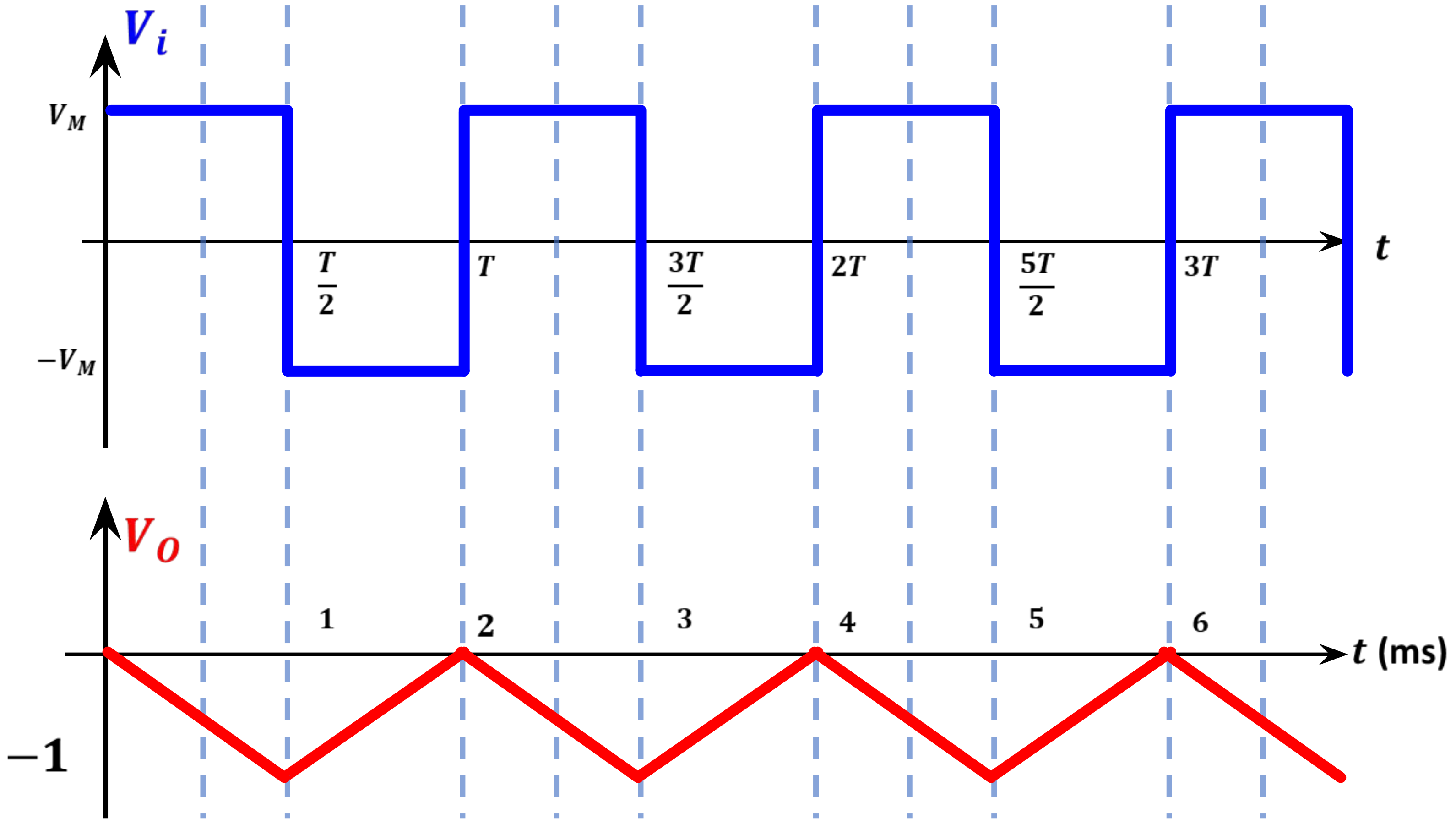




$$\int_0^t v_I\,dt = V_M \cdot t$$

Example 12





$$\int_0^t v_I \, dt = V_M \cdot t$$

APPLICATIONS:

- Implementing operational functions
- $f = -2x - 3y$
- $f = -4x + 5y$
- $f = -7x + \frac{d}{dt}y$
- $f = -\frac{1}{3}\int x \cdot dt + 2\ln y + 4z$
- $f = -3\frac{dx}{dt} + 2\exp(y) + 4z$
- $f = xy/z$

APPLICATIONS:

- Implementing operational functions
- $f = -2x - 3y$

APPLICATIONS:

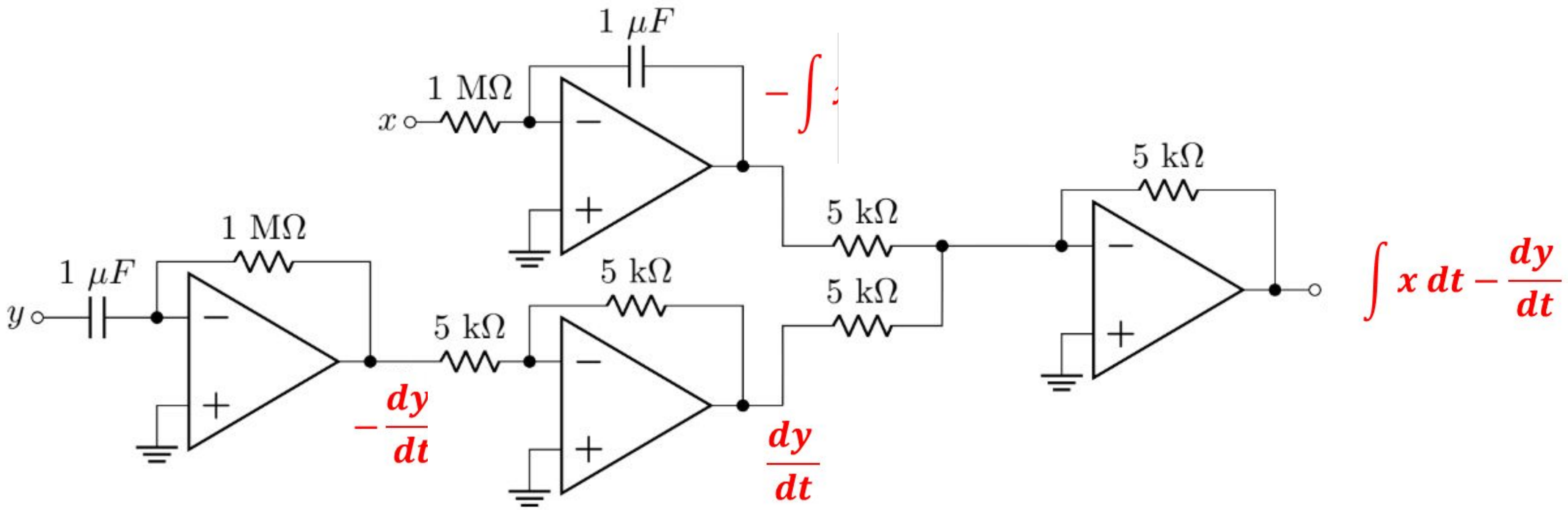
- Implementing operational functions
- $f = -4x + 5y$

APPLICATIONS:

- Implementing operational functions
- $f = -7x + \frac{d}{dt}y$

Example

Analyze the circuit below to **find** an expression of $\boldsymbol{f}$ in terms of inputs $\boldsymbol{x}$ and $\boldsymbol{y}$.



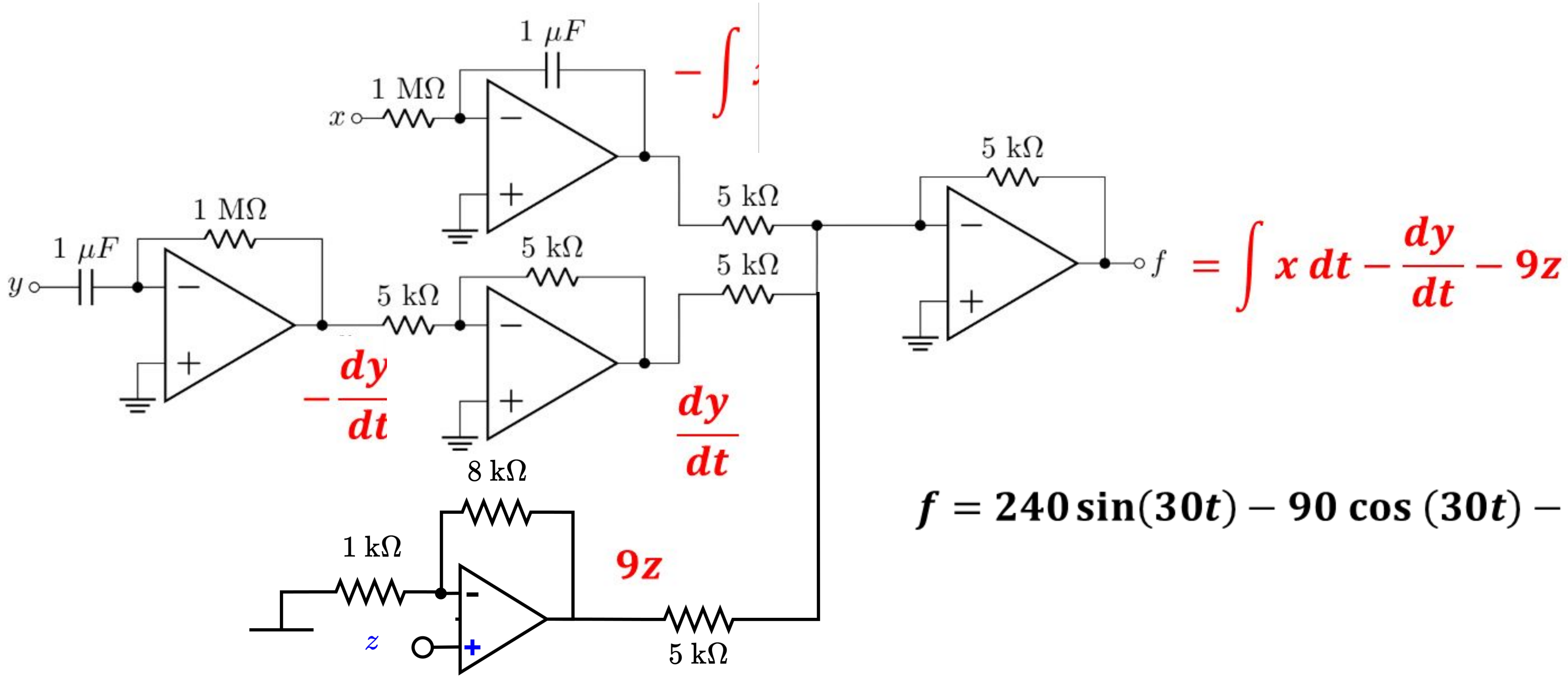
Solution:

$$v_{I_1} = -\frac{dy}{dt}\,;\, v_{I_2} = -\frac{1}{RC}\int x\,dt\,;\, v_{I_3} = -v_{I_1} = \frac{dy}{dt};\ v_o = -(v_{I_2} + v_{I_3})$$

Example

Analyze the circuit below to **find** an expression of $\boldsymbol{f}$ in terms of inputs $\boldsymbol{x}$ and $\boldsymbol{y}$.

If $\boldsymbol{x = 8\cos(30t)}$ **V** and $\boldsymbol{y = 3\sin(30t)}$ **V** and $\boldsymbol{z = 1}$ **V**



$$\boldsymbol{f = 240\sin(30t) - 90\cos(30t) - 9\ \mathrm{V}}$$

Thank You!