CSE251: Electronic Devices and Circuits

Lecture 1

Alt. Representation, CSE250 Review, IV Characteristics

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Outline

- Alternative Circuit Representation – Line diagrams
- CSE250 Review
  - KCL, KVL
  - Series, Parallel resistor network – Voltage Division, Current division
  - Examples
- IV Characteristics
  - Linear IV – Resistors, Voltage Source, Current Source, SC, OC.
  - Non-Linear IV – Piecewise Linear Model

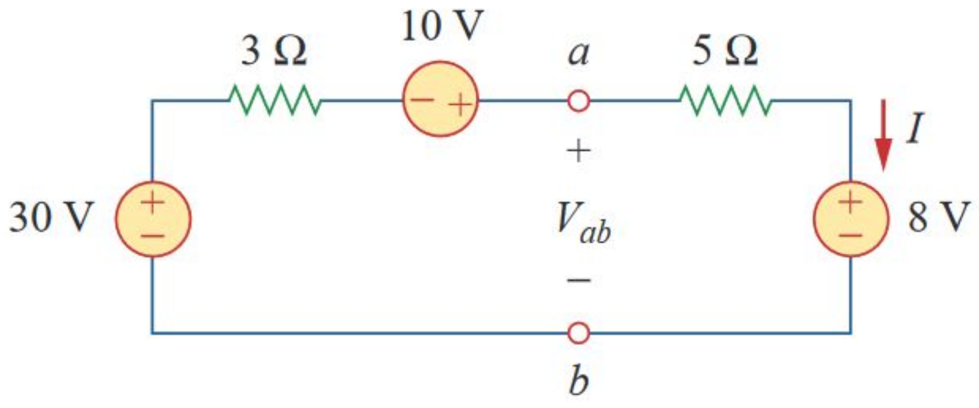
Alternative Circuit Representation: Line diagrams

Steps to decompose circuits to line diagram

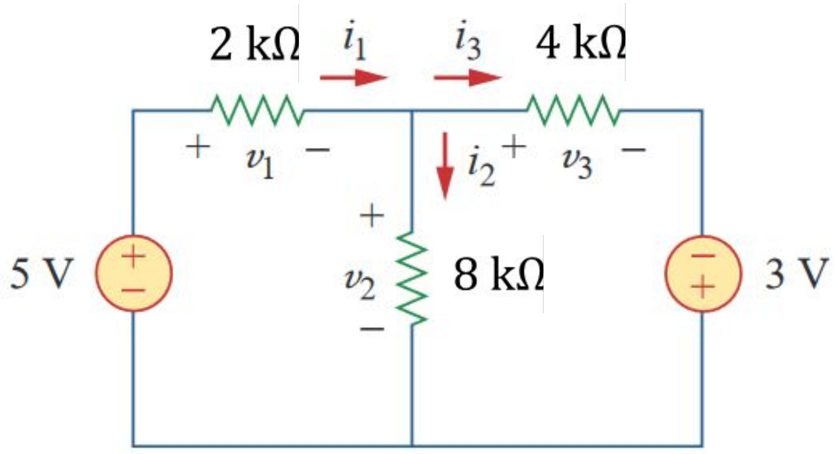
1. Set a ground so that number of **floating voltage** sources are minimized.
2. Detach the ground **from the voltage source**.
3. Convert the non-floating voltage sources (~~current sources~~) into:
   - Arrow : (→) **Fixed/Constant voltage source**
4. Keep passive elements as they are.

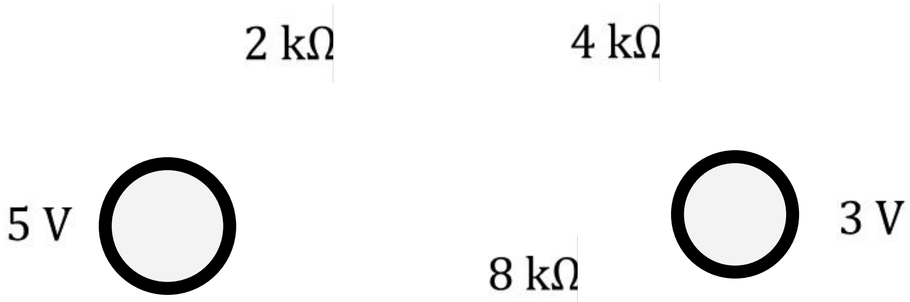
Floating voltage sources:

Voltage sources which are **not connected the ground** terminal. In the diagram, the **10 V** voltage source is floating



Line diagrams: Example 1





2 kΩ

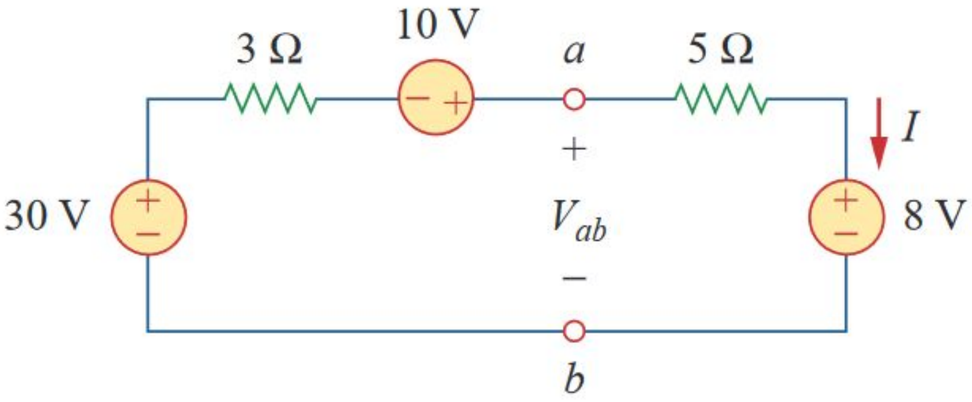
4 kΩ

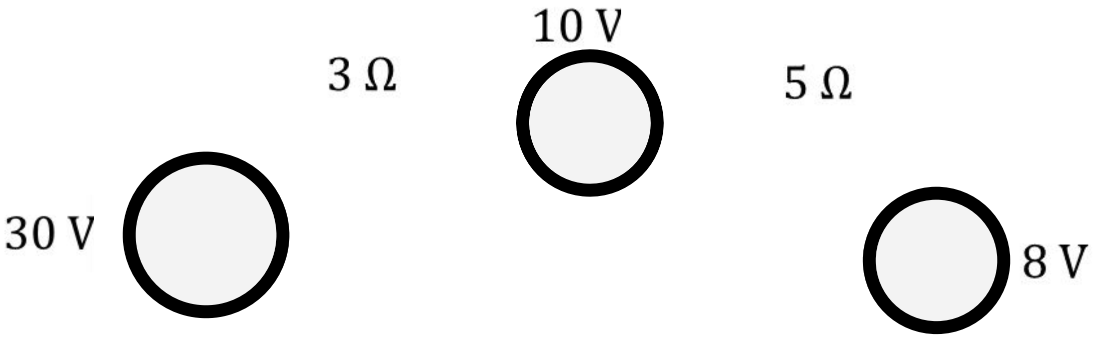
5 V

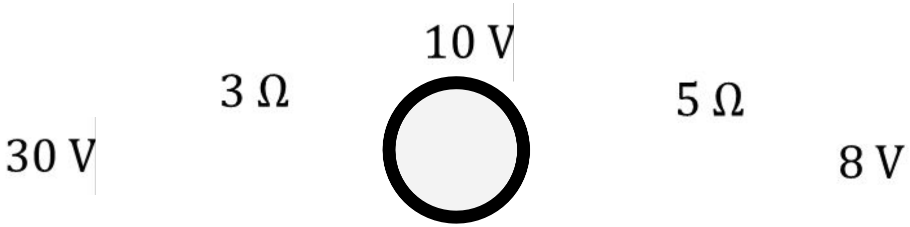
−3 V

8 kΩ

Line diagrams: Example 2

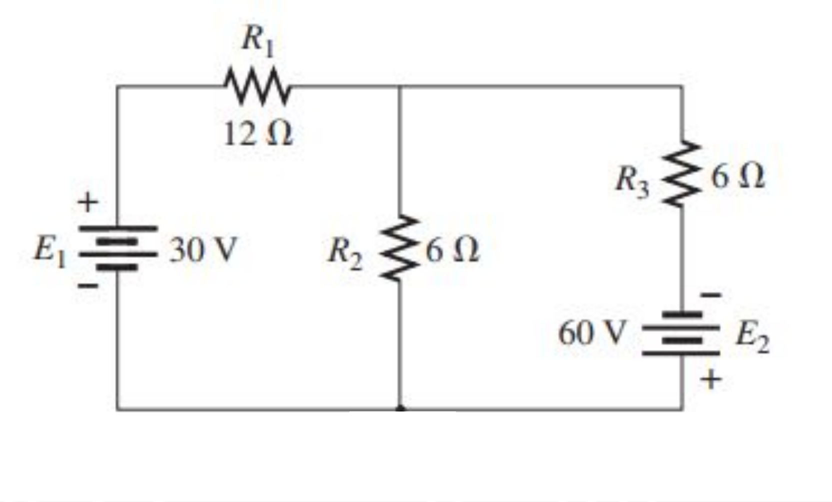






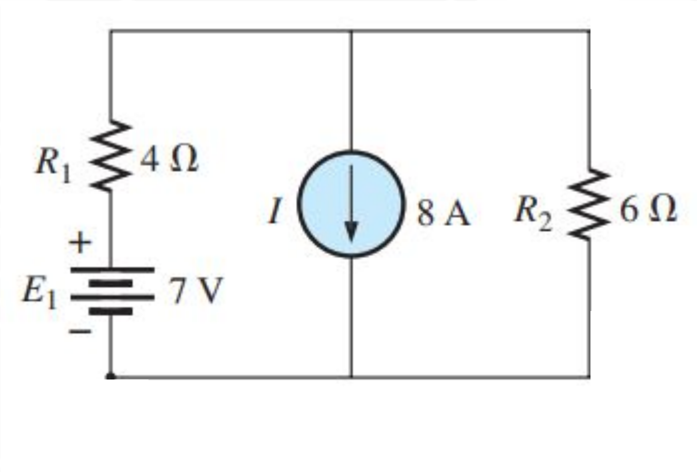
More Examples

Difficulty : 2/5



Example: 2

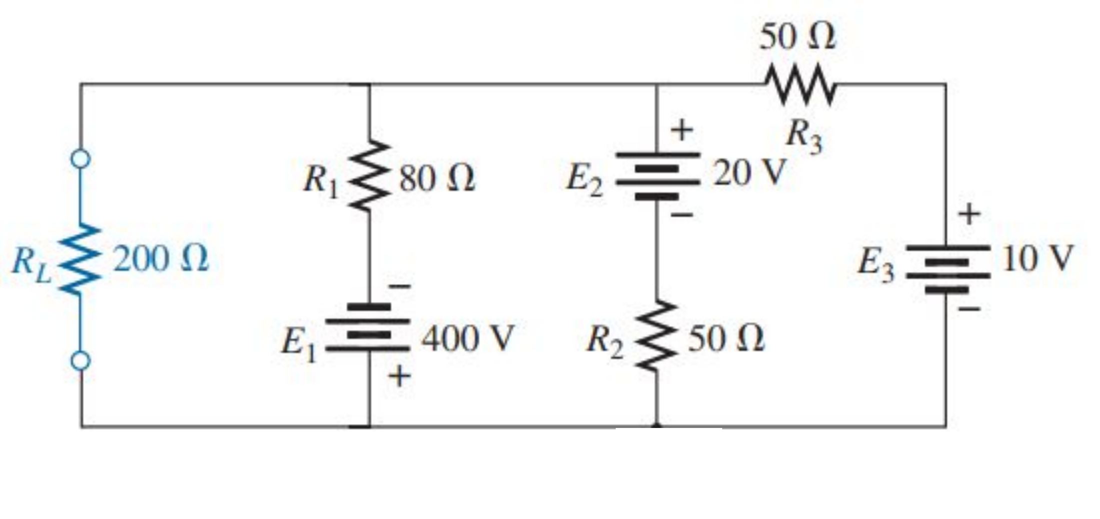
Difficulty : 3/5



Example: 3

More Examples

Difficulty : 4/5



Example: 4

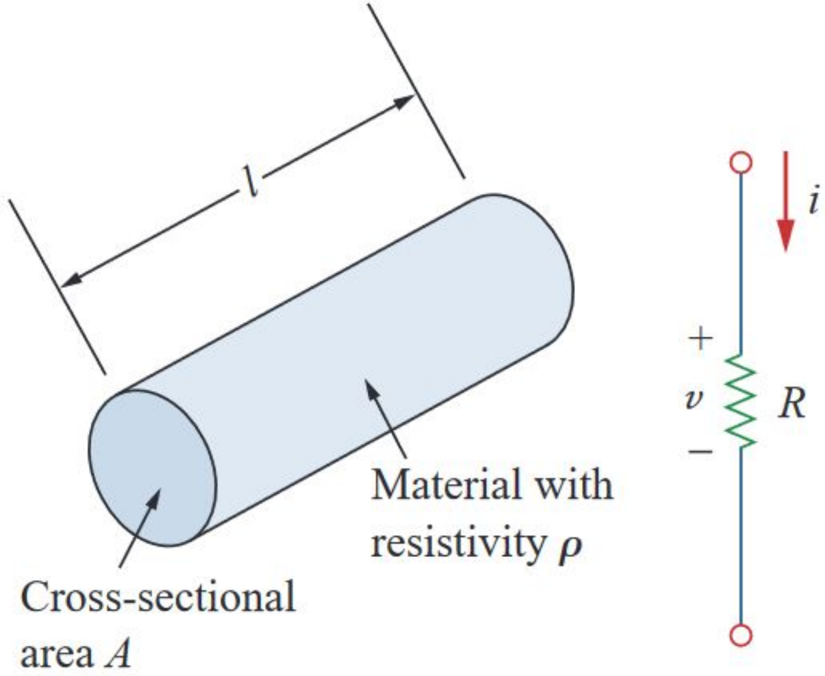
Step – (4) Make all the active elements (dc/ac type, voltage/~~current~~ sources) into single terminals (arrows/circles) using the voltages you wrote as much as you can [***THERE MIGHT BE CASES WHERE YOU CAN'T DO THAT***]

The fundamentals …

Ohm's Law –

- the voltage $\boldsymbol{v}$ across a resistor is **directly proportional** to the current $\boldsymbol{i}$ flowing through the resistor ($\boldsymbol{R}$)

$$\boldsymbol{v} \propto \boldsymbol{i}$$
$$\boldsymbol{v} = \boldsymbol{iR}$$



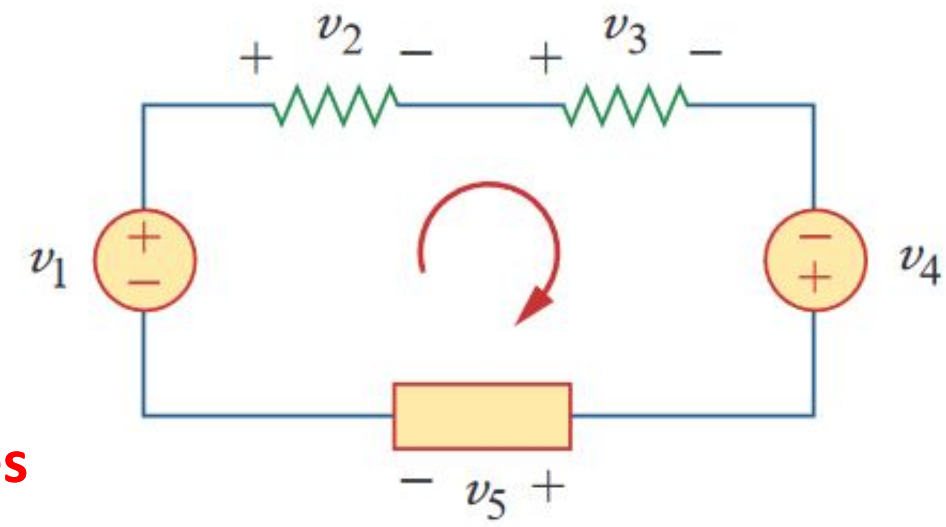
KVL: Kirchhoff's voltage law

The algebraic sum of all **voltages** around **a closed path (or loop)** is zero.

$$-\boldsymbol{v_1} + v_2 + v_3 - \boldsymbol{v_4} + v_5 = 0$$

$$v_2 + v_3 + v_5 = \boldsymbol{v_1} + \boldsymbol{v_4}$$

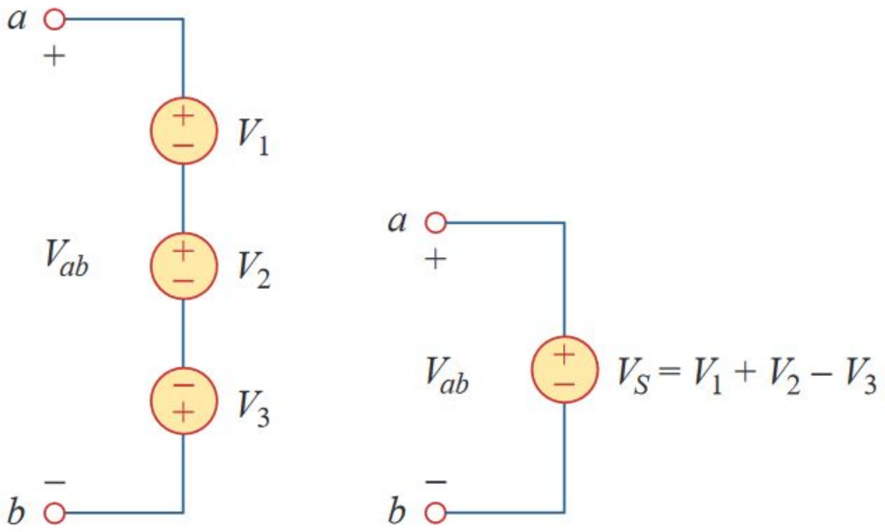
Sum of voltage drops = **Sum of voltage rises**



KVL: Kirchhoff’s voltage law

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

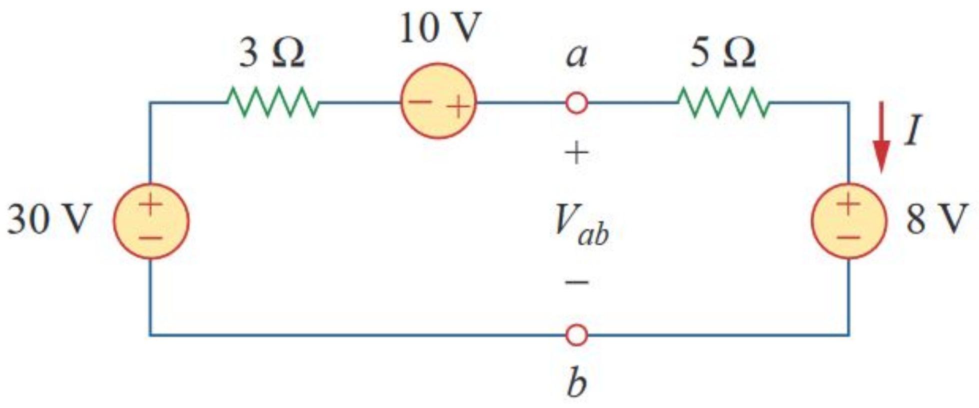
$$V_{ab} = V_1 + V_2 - V_3$$



Equivalent Circuits

KVL – Example 1

Find $\boldsymbol{I}$ and $\boldsymbol{V_{ab}}$ in the circuit



Solution:

KVL

$$-30 + 3\boldsymbol{I} - 10 + 5\boldsymbol{I} + 8 = 0$$

$$\boldsymbol{I} = \frac{32}{8}\,\mathrm{A} = 4\ \mathrm{A}$$

KVL

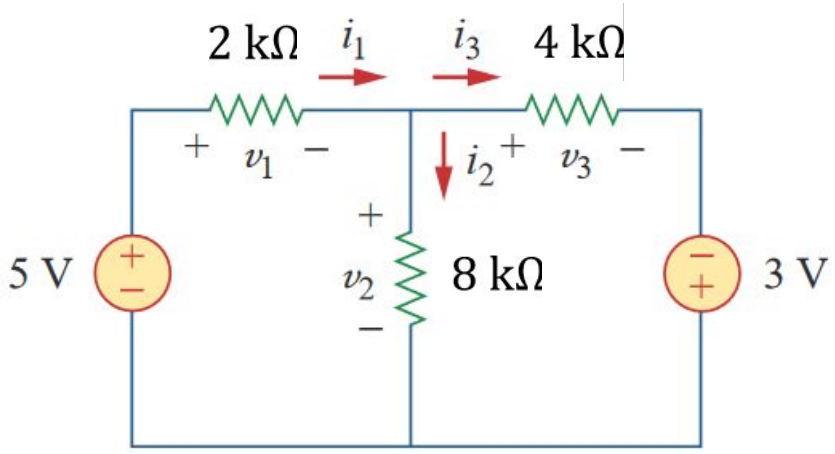
$$-V_{ab} + 5\boldsymbol{I} + 8 = 0$$

$$\boldsymbol{V_{ab}} = 28\ \mathrm{V}$$

Tip: If you find resistance values in **kΩ** instead of **Ω**, don't convert the **kΩ** values to **Ω**. Just find currents in **mA** instead of **A**.

KVL – Example 2

Find $\boldsymbol{v_1, v_2, v_3, i_1, i_2}$ and $\boldsymbol{i_3}$ in the circuit



Solution:

KVL in first loop

$$-5 + 2\boldsymbol{i_1} + 8(\boldsymbol{i_1} - \boldsymbol{i_3}) = 0$$

$$10\boldsymbol{i_1} - 8\boldsymbol{i_3} = 5$$

KVL in second loop

$$-8(\boldsymbol{i_1} - \boldsymbol{i_3}) + 4\boldsymbol{i_3} - 3 = 0$$

$$-8\boldsymbol{i_1} + 12\boldsymbol{i_3} = 3$$

Solving:

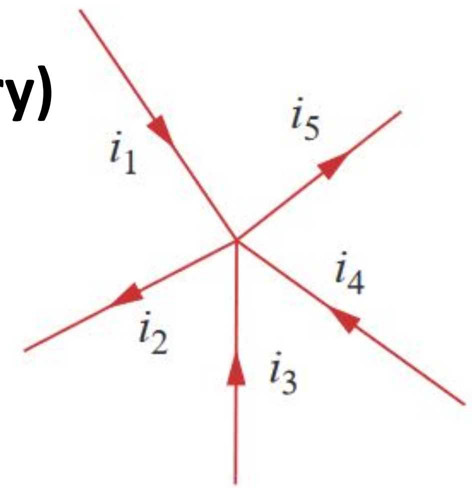
$$i_1 = 1.5 \text{ mA}$$
$$i_3 = 1.25 \text{ mA}$$
$$i_2 = i_1 - i_3 = 0.25 \text{ mA}$$

$$v_1 = 3 \text{ V}$$
$$v_2 = 2 \text{ V}$$
$$v_3 = 5 \text{ V}$$

Tip: If you find resistance values in **kΩ** instead of **Ω**, don't convert the **kΩ** values to **Ω**. Just find currents in **mA** instead of **A**.

KCL: Kirchoff's Current Law

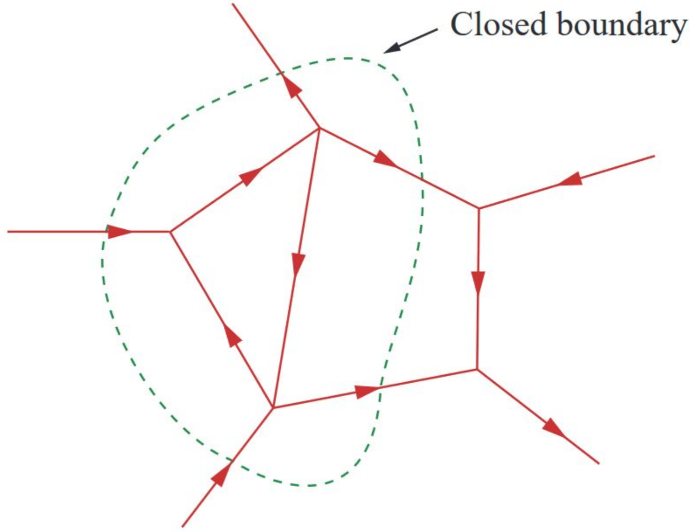
The algebraic sum of the **currents** entering a **node (closed boundary)** is equal to the sum of the currents leaving the node.



$$\boldsymbol{i_1} + (-\boldsymbol{i_2}) + \boldsymbol{i_3} + \boldsymbol{i_4} + (-\boldsymbol{i_5}) = 0$$

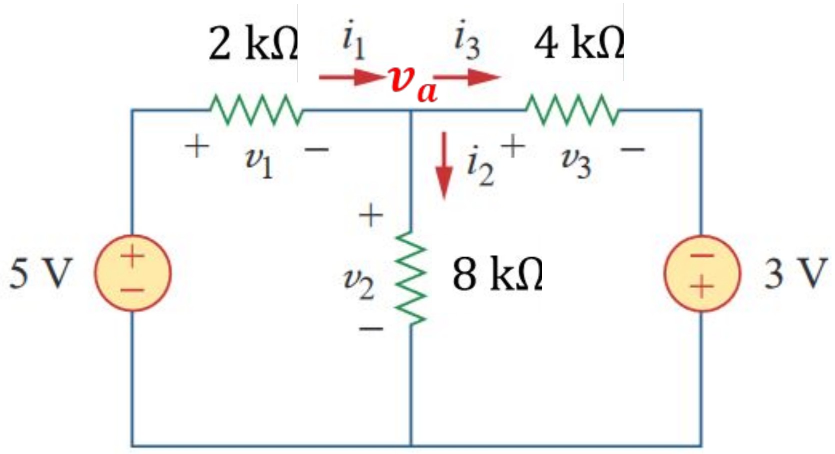
Current Entering node: **Positive**
Current Exiting node: **Negative**

Or vice versa…



KCL- Example 1

Find $\boldsymbol{v_1, v_2, v_3, i_1, i_2}$ and $\boldsymbol{i_3}$ in the circuit



Solution:

KCL in node $\boldsymbol{v_a}$. (PS: $\boldsymbol{v_a = v_2}$)

$$\frac{5 - \boldsymbol{v_2}}{2} - \frac{\boldsymbol{v_2} - (-3)}{4} - \frac{\boldsymbol{v_2} - 0}{8} = 0$$

$$v_2\left(-\frac{1}{2} - \frac{1}{4} - \frac{1}{8}\right) = -\left(\frac{5}{2} - \frac{3}{4}\right)$$

$$\boldsymbol{v_2} = \frac{7}{4} \cdot \frac{8}{7}\ \mathrm{V} = 2\ \mathrm{V}$$

$$\boldsymbol{v_1} = 5 - v_2 = 3\ \mathrm{V}$$

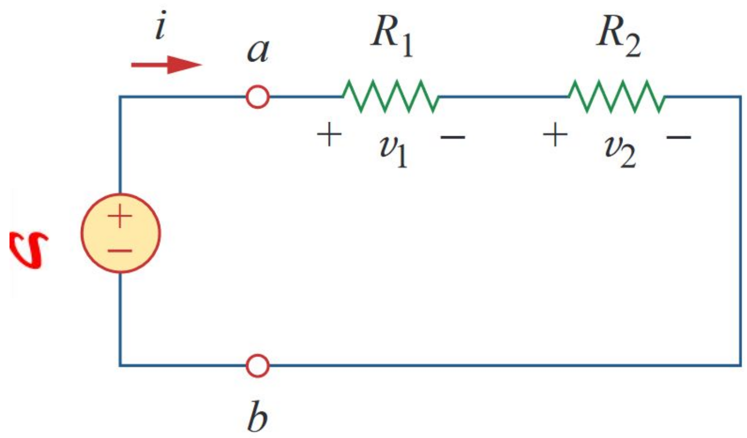
$$\boldsymbol{v_3} = v_2 - (-3) = 5\ \mathrm{V}$$

Series Resistors and Voltage Division

The **equivalent resistance** of any number of resistors connected in **series** is the sum of the individual resistances.

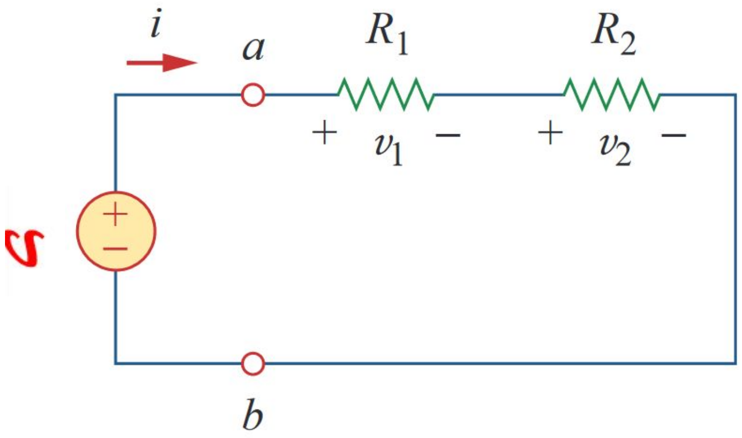
Principle of voltage division

Source voltage v - is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop.



$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Line diagram: Example 3



$$\frac{\boldsymbol{R_1}}{\boldsymbol{R_1} + R_2}\boldsymbol{v}$$

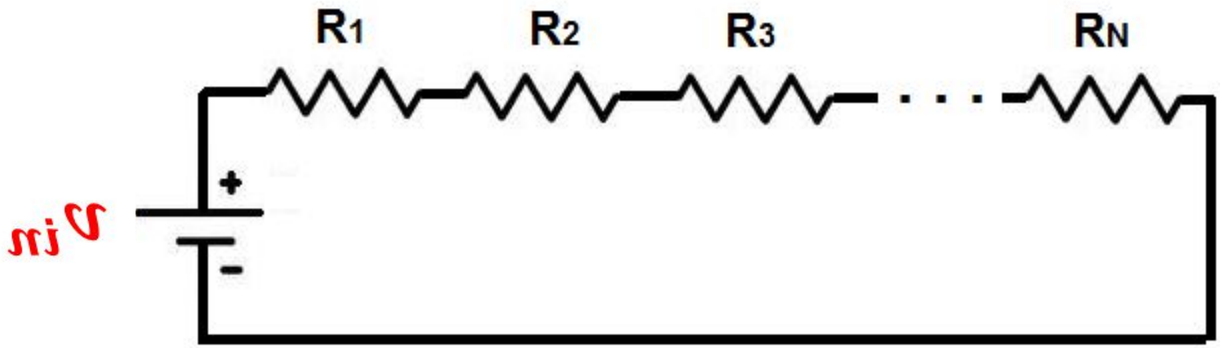
$$v_2 = \frac{\boldsymbol{R_2}}{R_1 + \boldsymbol{R_2}}\boldsymbol{v}$$

Series Resistors and Voltage Division

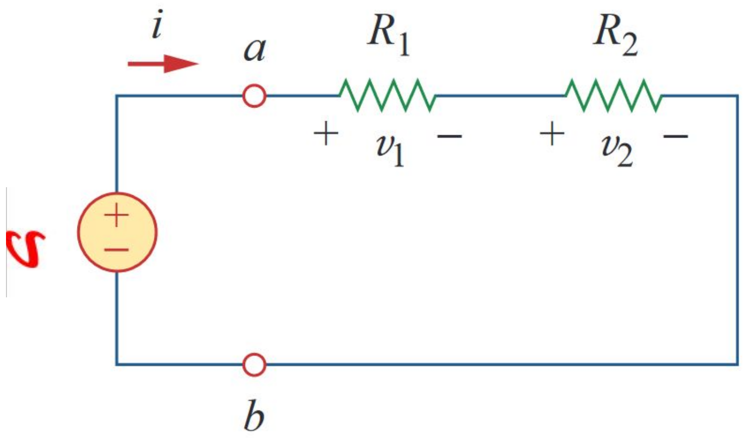
- If there are $\boldsymbol{N}$ resistors in series, the voltage across the $\boldsymbol{i}$ –th resistor is given by,

As $\boldsymbol{V} \propto \boldsymbol{R}$

$$v_i = \frac{\boldsymbol{R_i}}{\sum_i R_i} \boldsymbol{v_{in}}$$



Line diagram: Example 3



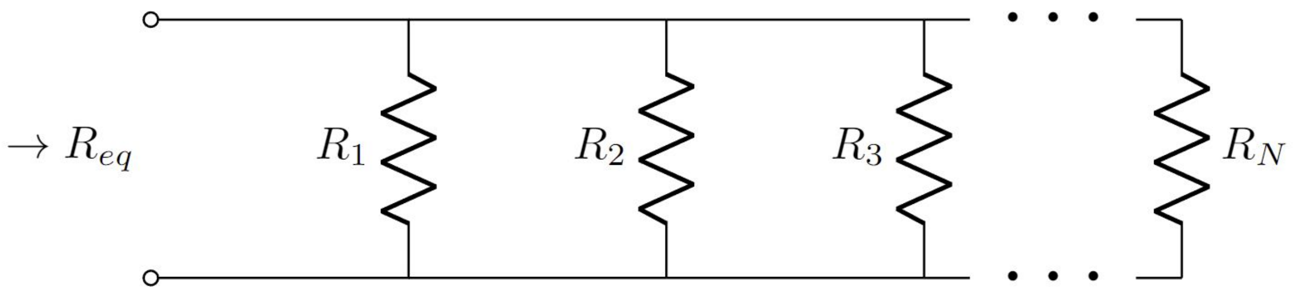
KVL (acts along a line instead of a loop)

$$v - iR_1 - iR_2 = 0$$

Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors connected in **parallel** is the inverse of the sum of the individual **conductances**.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}$$



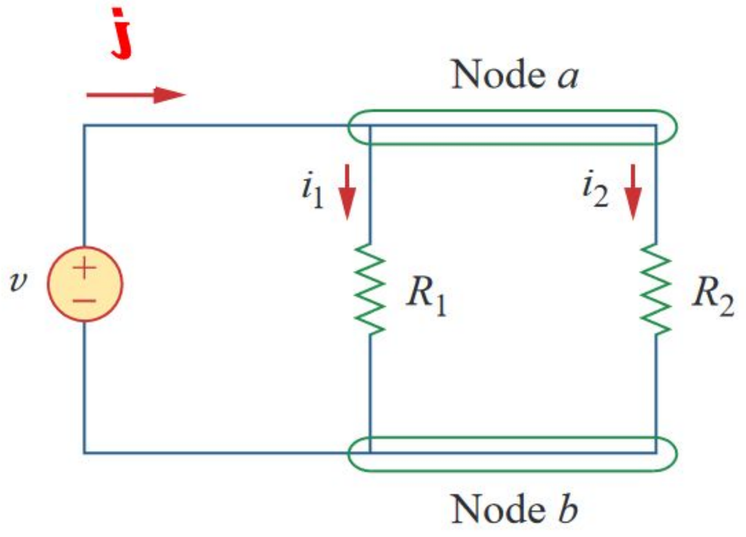
Simplification for the case when $R_1 = R_2 = R_3 \cdots = R_N$

$$R_{eq} = \frac{R_1}{N}$$

Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors connected in **parallel** is the inverse of the sum of the individual **conductances**.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_{eq} = R_1||R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

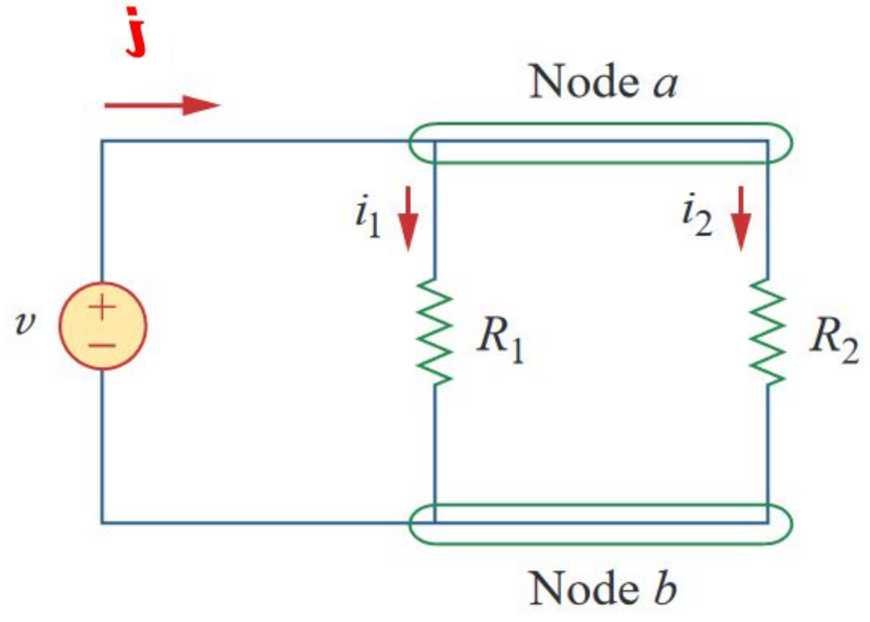


Simplification for the case when $R_1 = R_2$

$$R_{eq} = \frac{R_1}{2}$$

Parallel Resistors and Current Division

The **equivalent resistance** of any number of resistors connected in **parallel** is the inverse of the sum of the individual **conductances**.



Principle of current division

Source current i - is divided among the resistors in direct **inverse** proportion to their resistances; the larger the resistance, the larger the voltage drop.

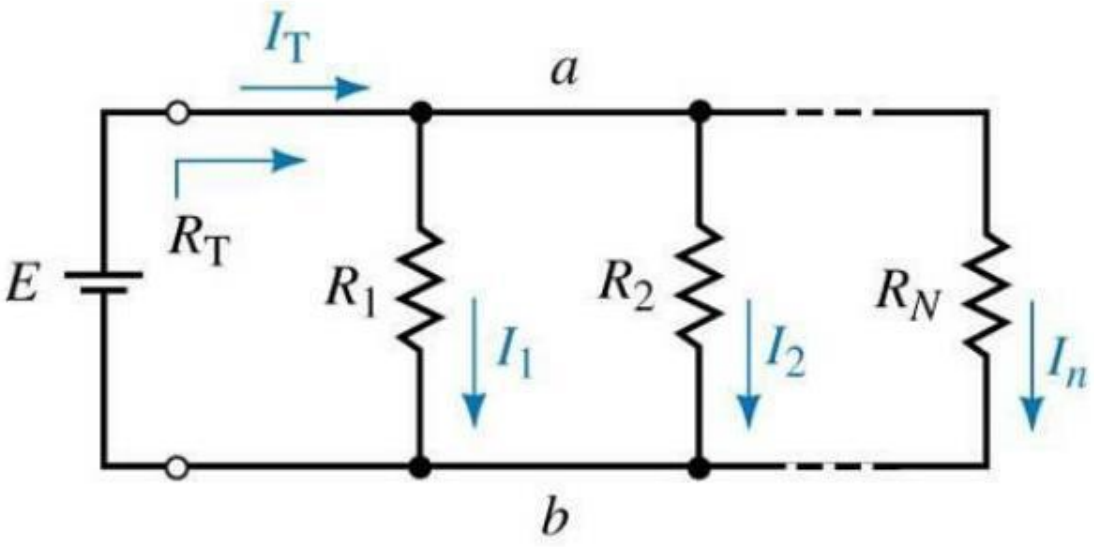
$$i_1 = \frac{1/R_1}{1/R_1 + 1/R_2} i \qquad i_2 = \frac{1/R_2}{1/R_1 + 1/R_2} i$$

Parallel Resistors and Current Division

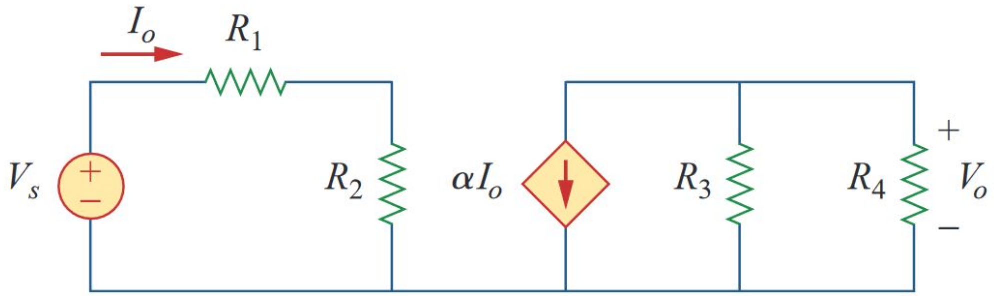
- If there are $\boldsymbol{N}$ resistors in parallel, the current through the $\boldsymbol{i}$ –th resistor is given by, $i \in \{1,2,3,\cdots N\}$

As $\boldsymbol{I} \propto \frac{\mathbf{1}}{\boldsymbol{R}}$

$$I_i = \frac{\mathbf{1}/\boldsymbol{R_i}}{\sum_i 1/R_i} \boldsymbol{I}_{\mathbf{T}}$$



Line diagrams: Example 4



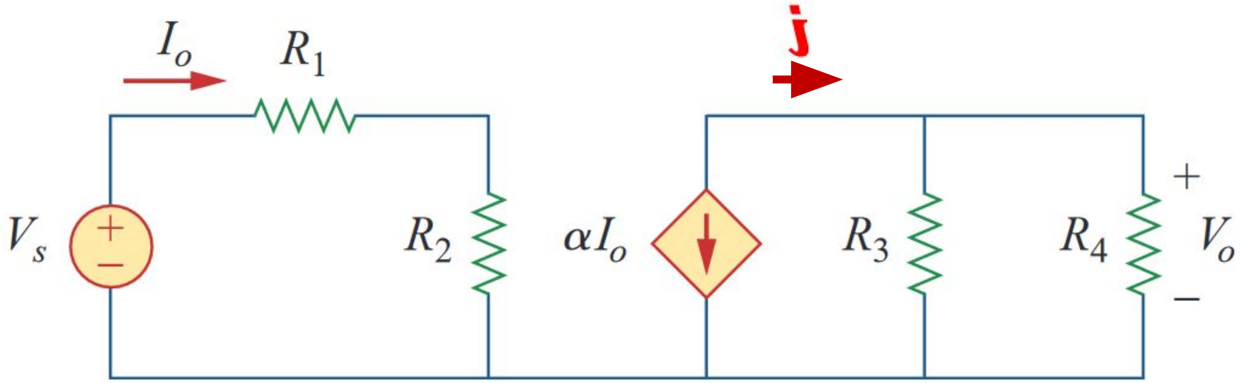
Practice Problem 1

For the circuit, find $\left|\frac{V_o}{V_s}\right|$ in terms of $\boldsymbol{\alpha}, \boldsymbol{R_1}, \boldsymbol{R_2}, \boldsymbol{R_3}$ and $\boldsymbol{R_4}$.

If $\boldsymbol{R_1} = \boldsymbol{R_2} = \boldsymbol{R_3} = \boldsymbol{R_4}$ what value of $\boldsymbol{\alpha}$ will produce $\left|\frac{V_o}{V_s}\right| = 10$?

Solution:

Ohm's Law across $\boldsymbol{R_1} + \boldsymbol{R_2}$.



$$I_O = \frac{V_S}{R_1 + R_2}$$

$$\boldsymbol{i} = -\boldsymbol{\alpha I_O}$$

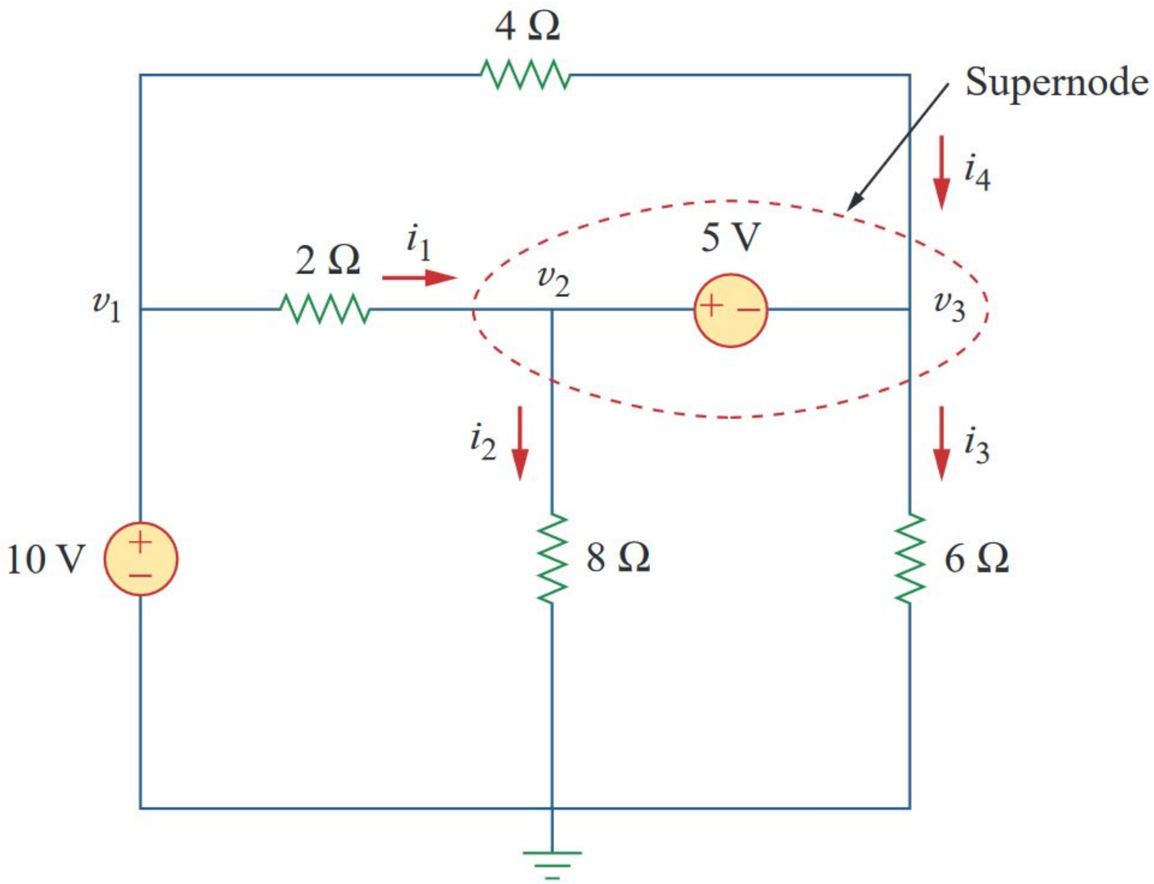
Voltage across **Parallel Resistors** $\boldsymbol{R_3}, \boldsymbol{R_4}$

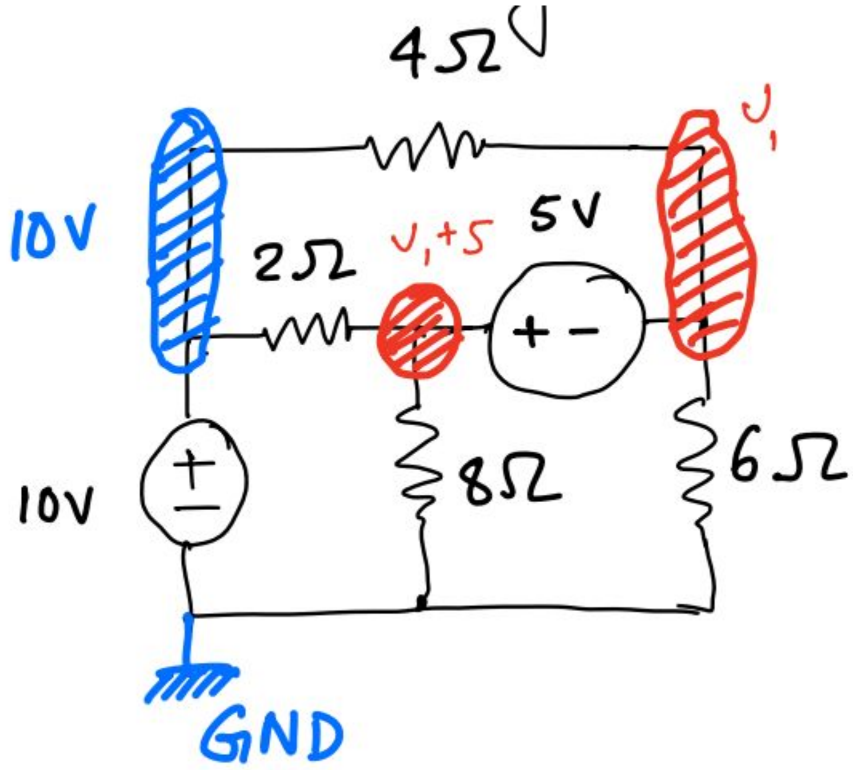
$$V_O = i(\boldsymbol{R_3}||\boldsymbol{R_4}) = -\frac{\alpha V_S}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

$$\left|\frac{V_O}{V_S}\right| = \frac{\alpha}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

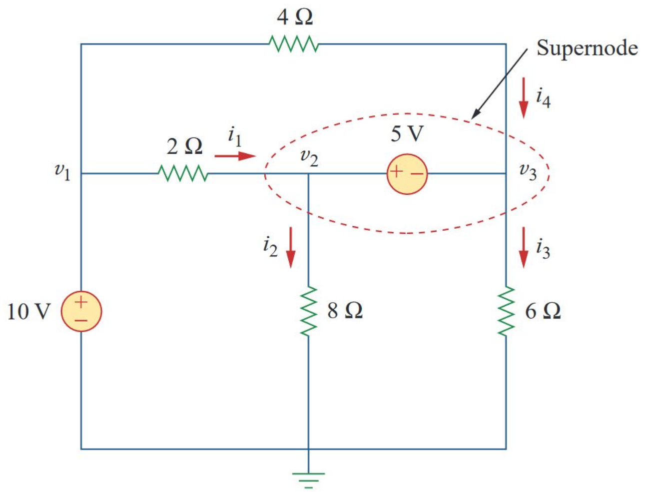
$\alpha = 40$

Example 1- Nodal Analysis





Example 1- Nodal Analysis



10V Node: $10\left(\frac{1}{4}+\frac{1}{2}\right)$

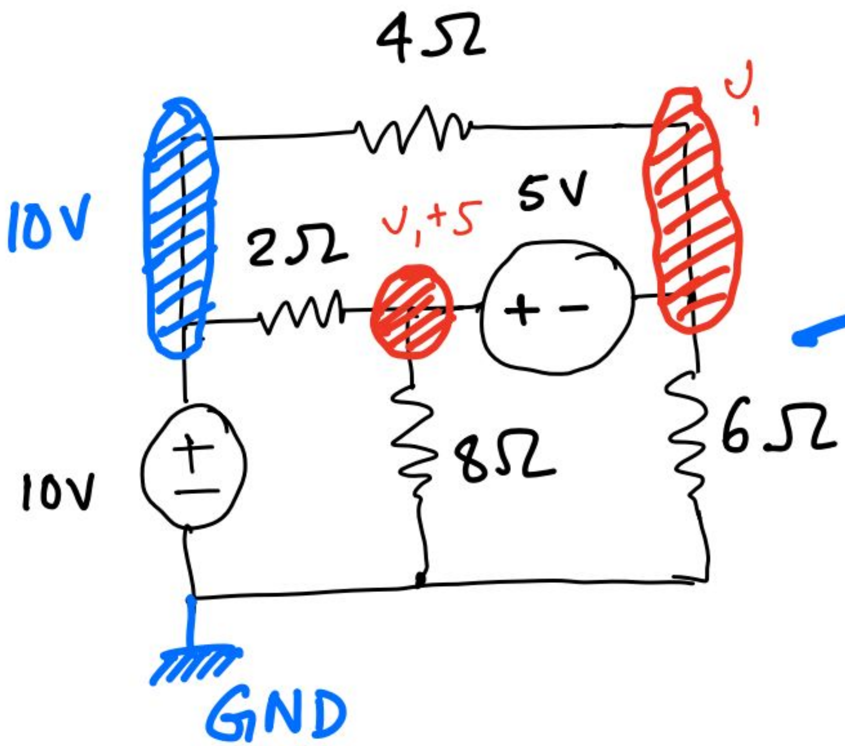
V_1 Node: $V_1\left(\frac{1}{4}+\frac{1}{6}\right)$

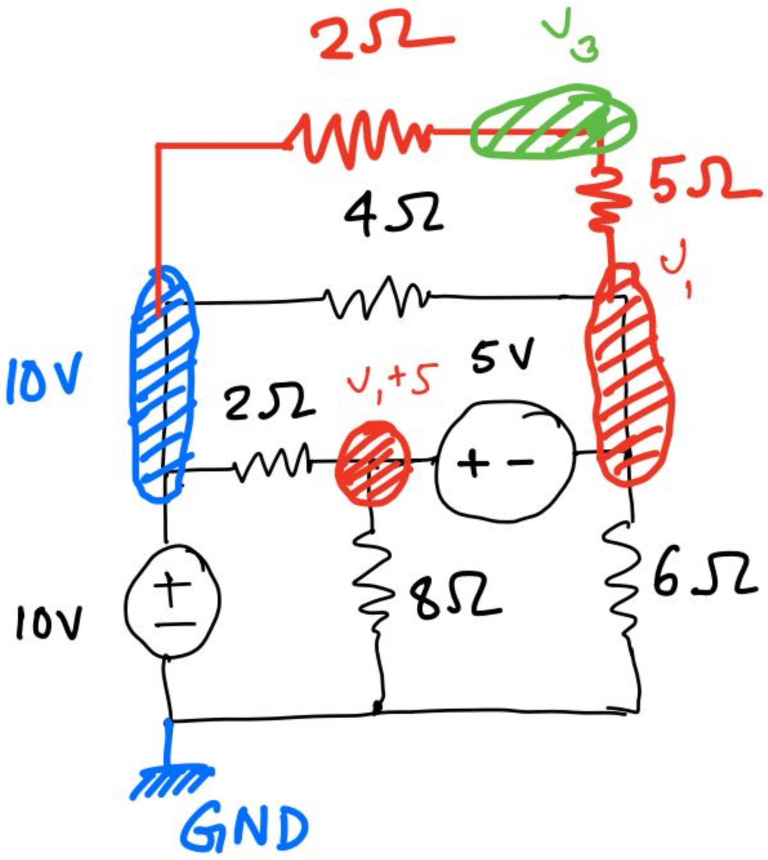
V_1+5 Node: $(V_1+5)\left(\frac{1}{2}+\frac{1}{8}\right)$

Node equation for node $\boldsymbol{v_1}$

$$v_1\left(\frac{1}{4}+\frac{1}{6}\right)+(v_1+5)\left(\frac{1}{2}+\frac{1}{8}\right)-10\left(\frac{1}{2}+\frac{1}{4}\right)=0$$

Example 2- Nodal Analysis – **Home Task 1**





Find the two node $\boldsymbol{v_1}$ and $\boldsymbol{v_3}$ equations!

Example 3

KCL at node $\boldsymbol{v_o}$

$$\frac{0.6 - 0.5}{5} = \frac{(0.5 - 0.7) - (-0.5)}{10} + \boldsymbol{I_1}$$

$$\boldsymbol{I_1} = -\mathbf{0.01}\ \mathrm{mA}$$

Example 4

- KCL at node $\boldsymbol{v_i}$

$$\frac{2 - v_i}{1} = \frac{v_i - v_o}{2} + \frac{v_i}{200}$$

$$\frac{301}{200} v_i - \frac{1}{2} v_o = 2$$

KCL at node $\boldsymbol{v_o}$

$$\frac{v_i - v_o}{2} + \frac{A v_i - v_o}{0.1} = 0$$

$$(2 \times 10^6 + 0.5) v_i - 10.5 v_o = 0$$

Example 5

- KCL at node $\boldsymbol{v_i}$

$$\frac{0 - v_i}{5} = \frac{v_i - v_o}{40} + \frac{v_i - 1}{200}$$

$$\frac{23}{100} v_i - \frac{1}{40} v_o = \frac{1}{200}$$

KCL at node $\boldsymbol{v_o}$

$$\frac{v_i - v_o}{40} + \frac{A v_i - v_o}{0.1} = \frac{v_o}{20}$$

$$(2 \times 10^6 + 0.025) v_i - 10.075 v_o = 0$$

Example 6 – Home Task 2

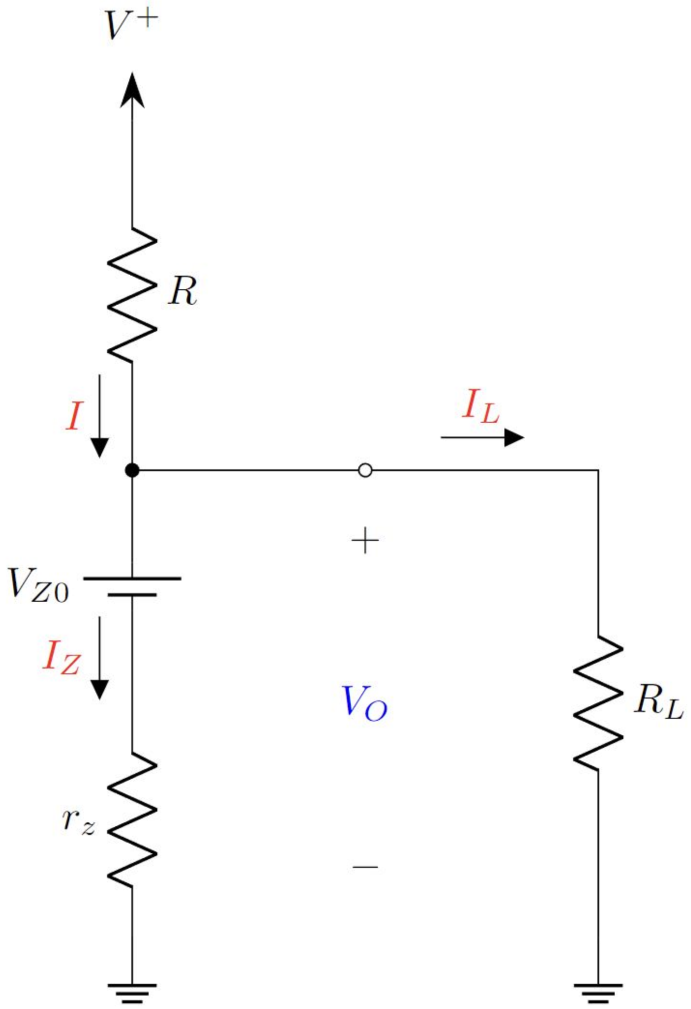
For $\boldsymbol{R} = 100\ \Omega$, $\boldsymbol{R_L} = 10\ \mathrm{k}\Omega$, $\boldsymbol{r_z} = 20\ \Omega$, $\boldsymbol{V_{ZO}} = 3\ \mathrm{V}$, and $\boldsymbol{I_Z} = 1\ \mathrm{mA}$.

a. Find $\boldsymbol{V_O}$

b. Find $\boldsymbol{I_L}$

c. Find $\boldsymbol{I}$

d. Find $\boldsymbol{V^+}$



Current-Voltage (I-V) Characteristics

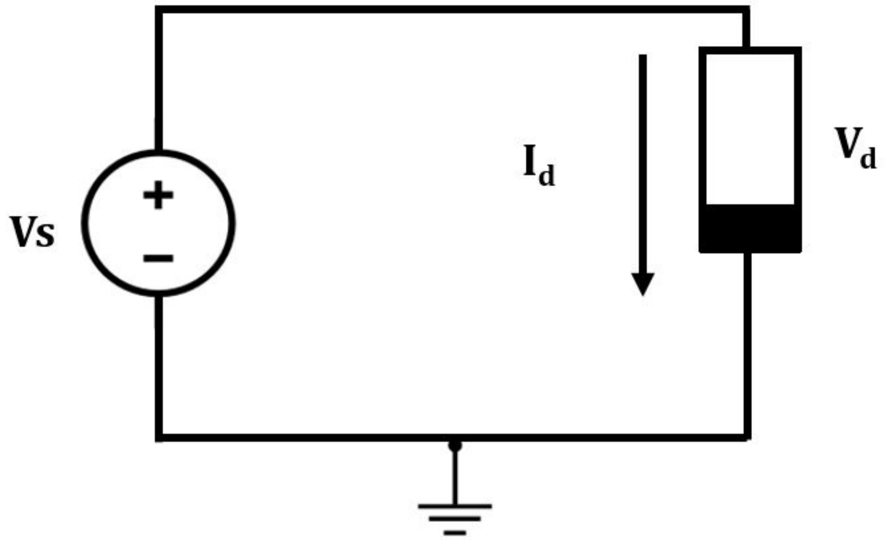
- I-V characteristic defines the relationship between the **current flow (through), I** and **voltage (across), V** an electronic device or element.

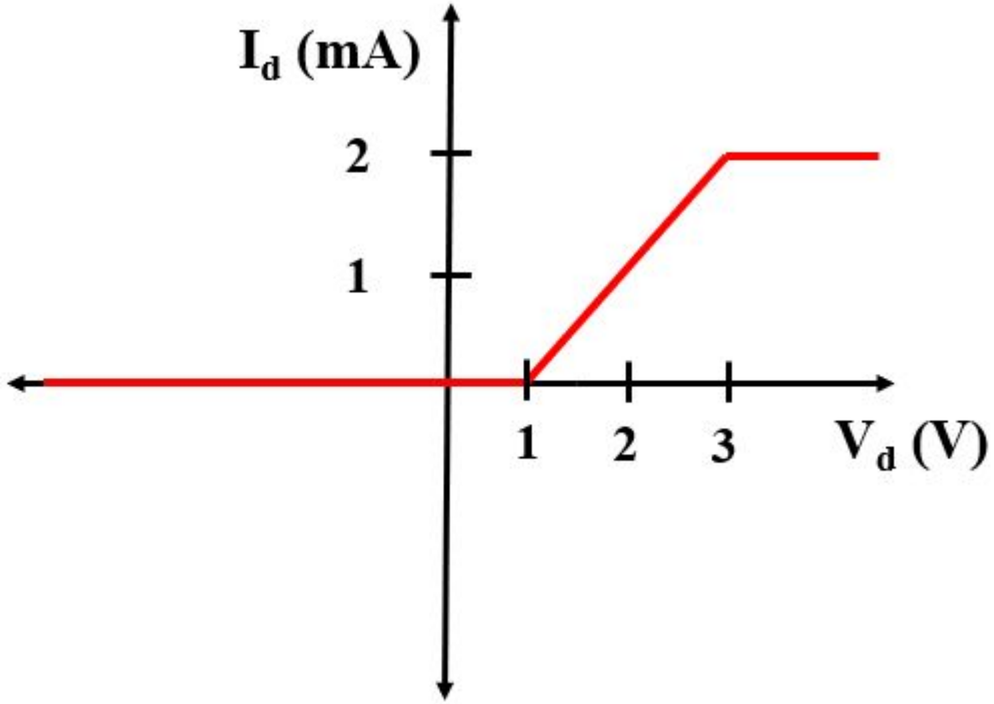
- A tool for understanding the operation of the circuit element.

- The Current-Voltage (I-V) characteristics are found by evaluating the **response** of a device/element under different **excitation** conditions. The behavior of a device depends on the **applied excitation** and can change if the excitation changes.

Current-Voltage (I-V) Characteristics

Example:



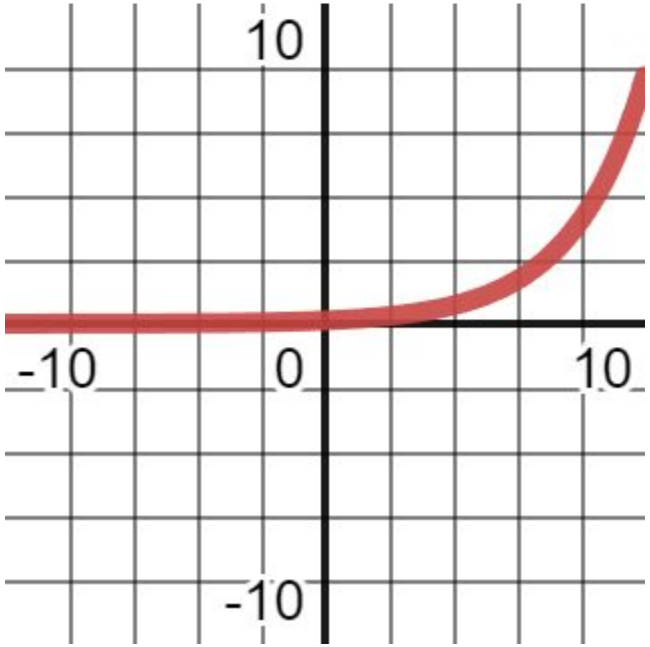


Current-Voltage (I-V) Characteristics

•

$$I = kV \qquad I = kV^2 \qquad I = \mathrm{A} \cdot \exp\left(\frac{V}{b}\right)$$

$$y = mx \qquad y = ax^2 \qquad y = \mathrm{A} \cdot \exp\left(\frac{x}{b}\right)$$



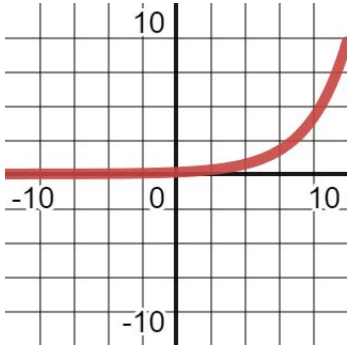
Type of (I-V) Characteristics

1. Linear Devices/Elements: The Current-Voltage relationship is linear i.e. the current through the element is a linear function of the applied voltage across it. The relationship can be characterized by:

$$I = kV$$

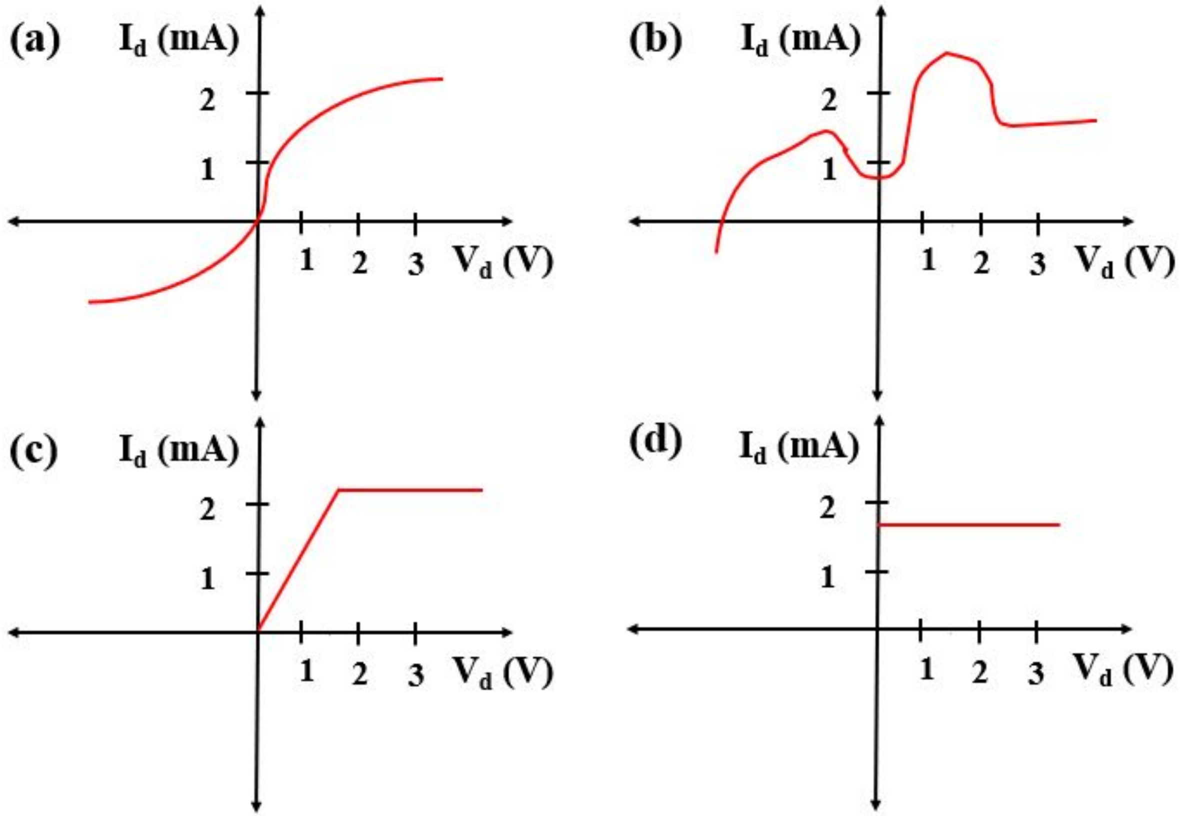
2. Non-Linear Devices/Elements: The Current-Voltage relationship is Non-linear i.e., the current through the element is a nonlinear function of the applied voltage across it.

$$I = k\sqrt{V}$$
$$I = kV^2$$
$$I = kV^3$$



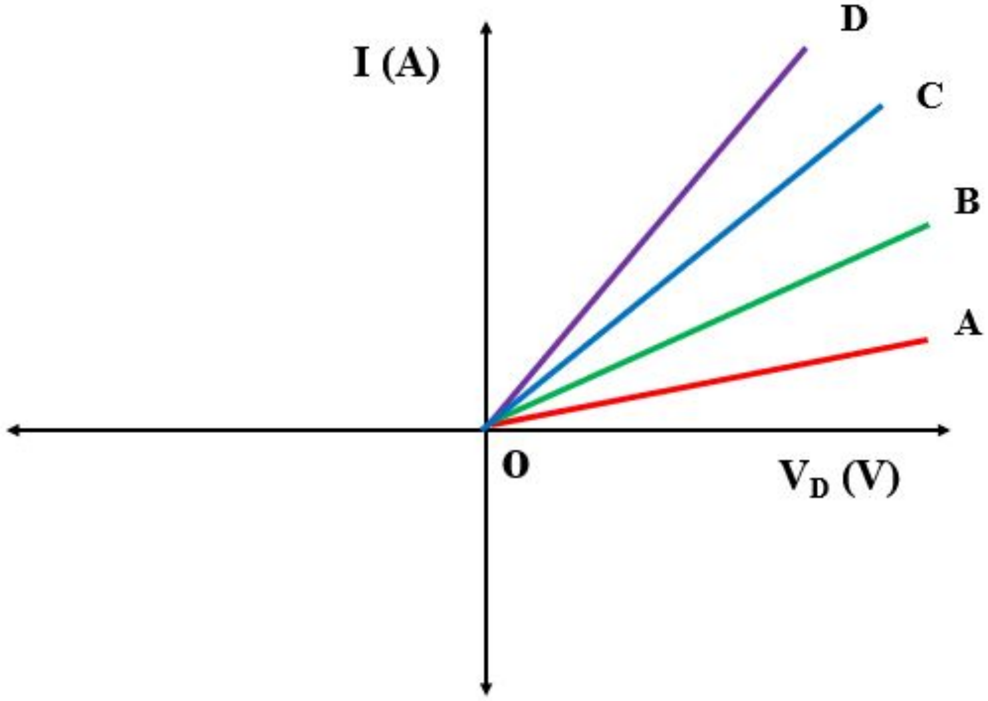
Type of (I-V) Characteristics

- Identify which of these I-V curves are Linear and which are Nonlinear



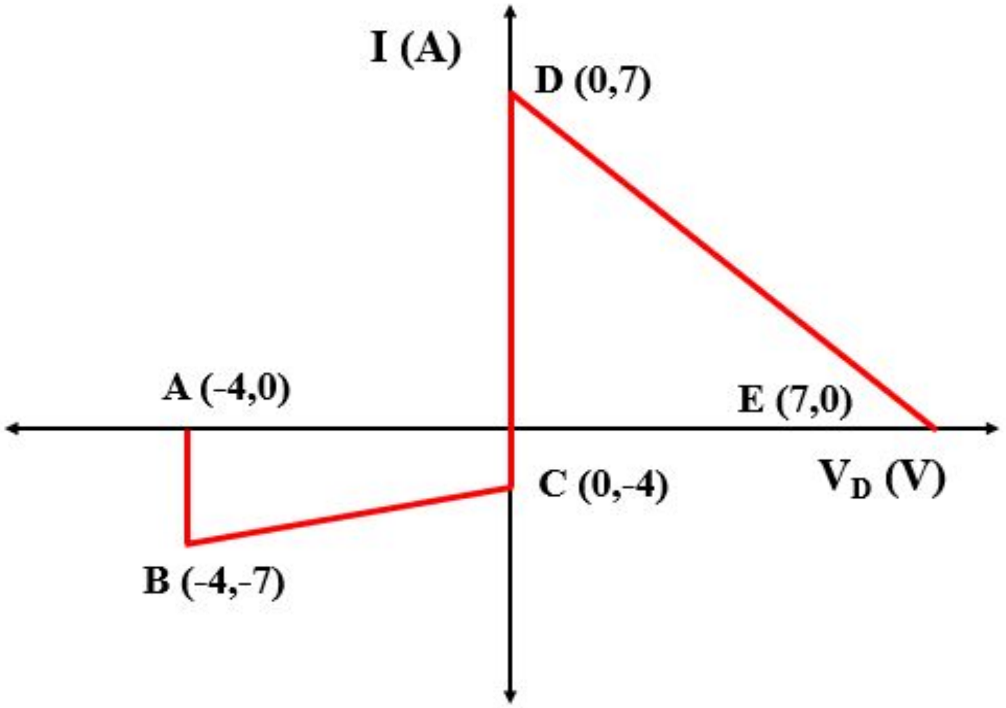
Linear Devices/Elements

- Write down the slopes of these following regions in ascending order (you do not need to calculate the slopes)



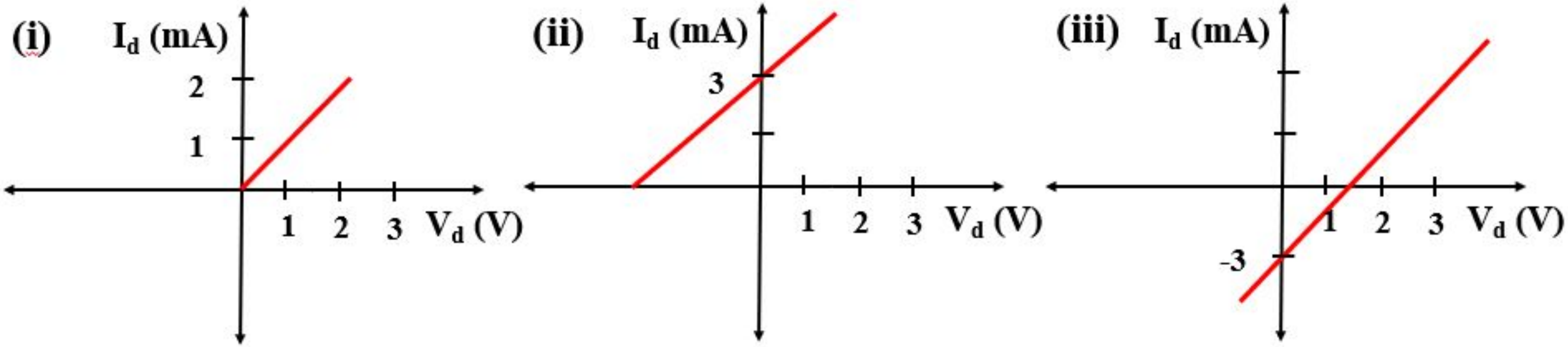
Linear Devices/Elements

- Find out the slope of the following curves



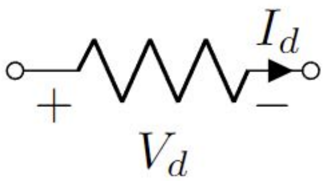
Linear Devices/Elements

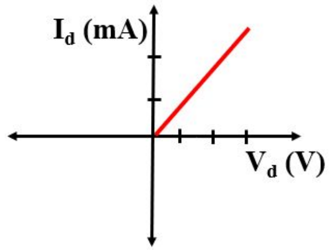
- For the lines represented by $\boldsymbol{y = mx + c}$ what is the value of $\boldsymbol{c}$ in the following figures [Figure (i), (ii) and (iii)]



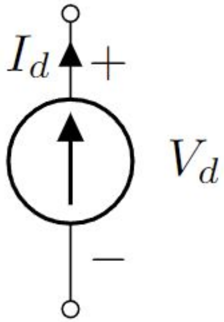
Linear Devices/Elements:

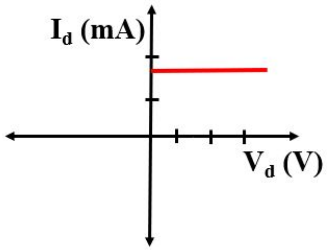
- Resistors



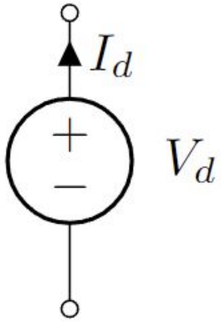


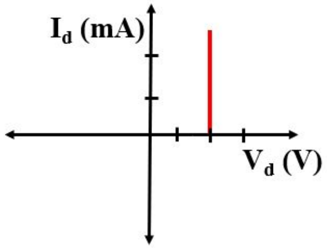
- Current Source





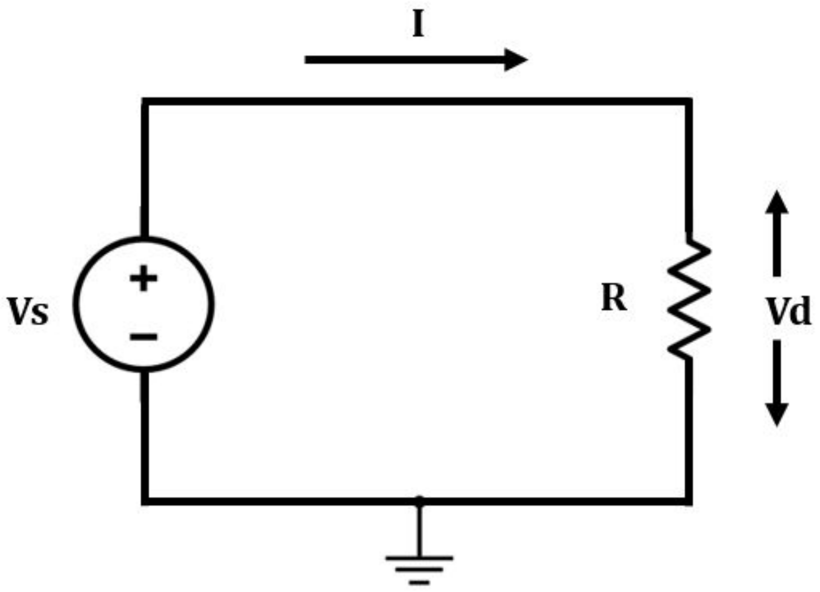
- Voltage Source





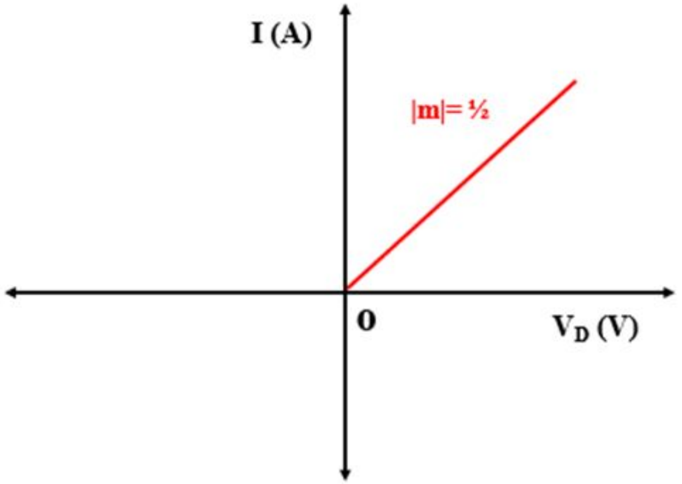
Resistor

- The relationship between current, $\boldsymbol{I}$ and voltage, $\boldsymbol{V_d}$ in a resistor of value '**R**' is defined by the "**<u>Ohm's law</u>**":



$$V_d = IR$$
$$\Rightarrow I = \frac{V_d}{R}$$
$$\Rightarrow I = \frac{1}{R} \cdot V + 0$$

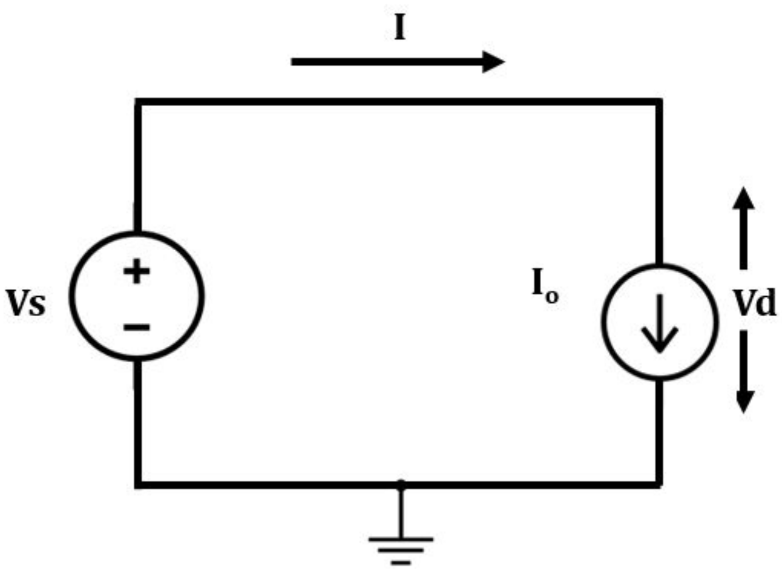
$$\boldsymbol{y = m \cdot x + c}$$



I-V curve of a 2Ω resistor

Current Source

- The value of current flow through a current source is **FIXED** and thus does not change with voltage. The equation is as follows



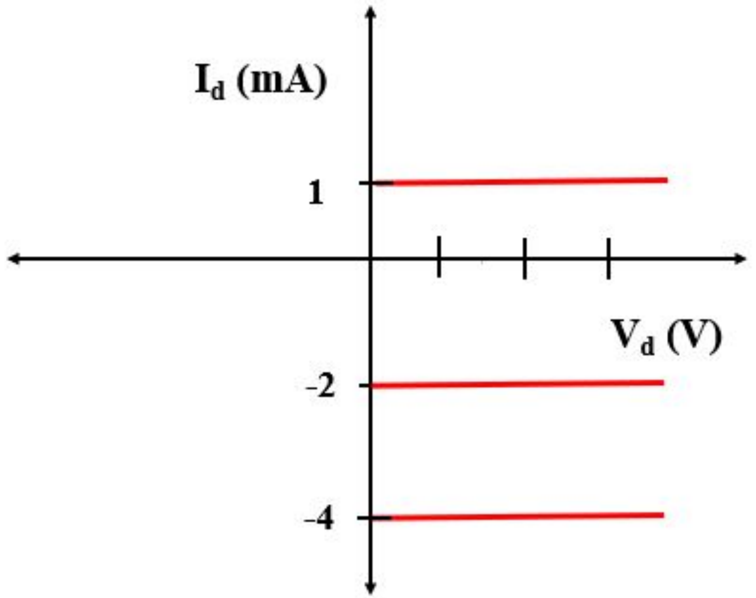
$$I = I_0$$

$$\boldsymbol{y = c}$$

$$I = 0 \cdot V_d + I_0$$

$$\boldsymbol{y = m \cdot x + c}$$

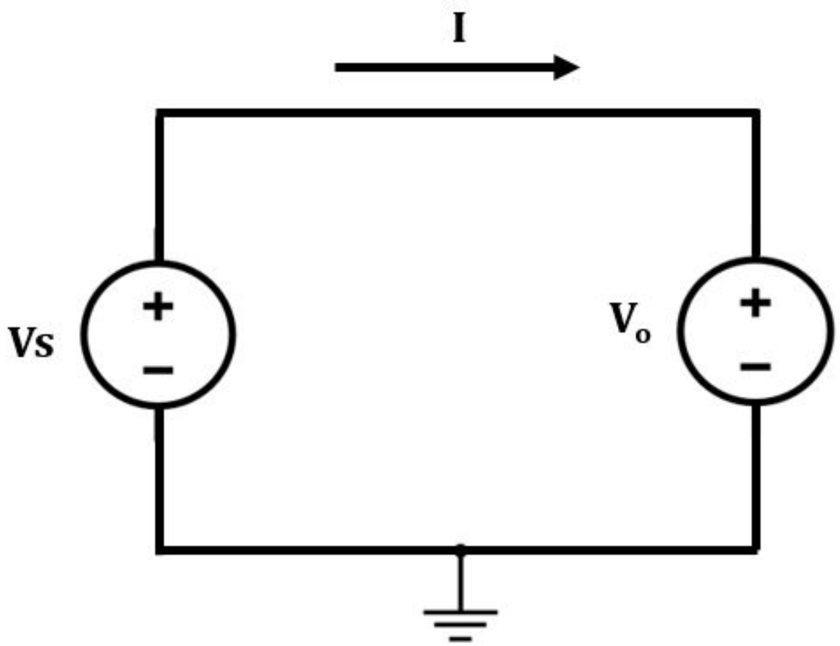
Slope: $\boldsymbol{m = 0}$



This curve intersects the y axis at $\boldsymbol{I = I_0}$.

Voltage Source

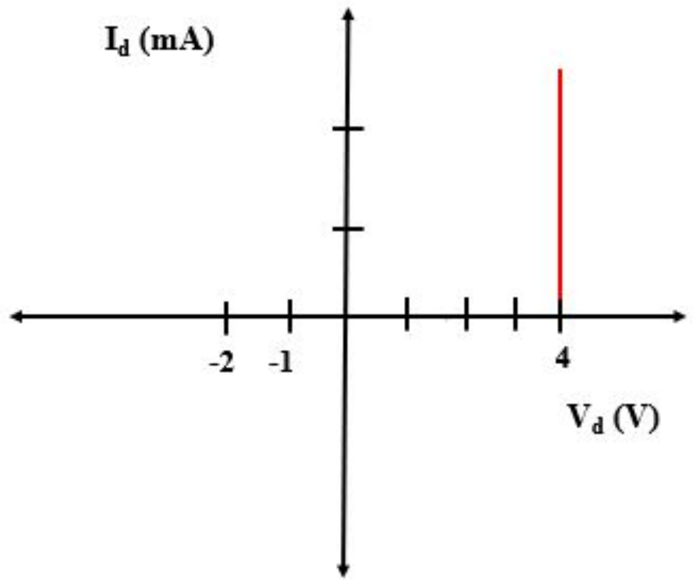
- The value of *<u>voltage across a voltage source</u>* is **FIXED** and thus does not change even if the current through the branch changes.



$$V = V_0$$

$$x = c$$

Slope: $m = \infty$

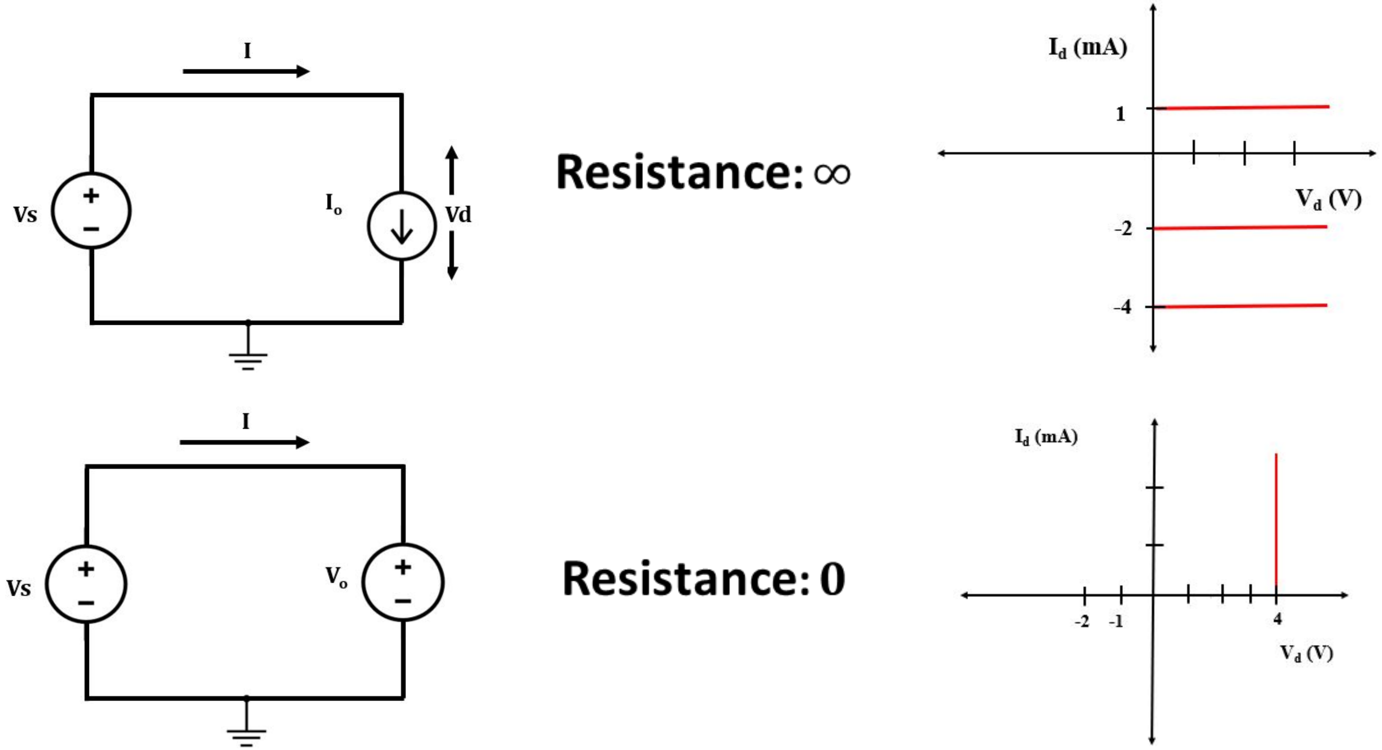


This curve intersects the x axis at $V_d = V_o$.

Electrical Sources

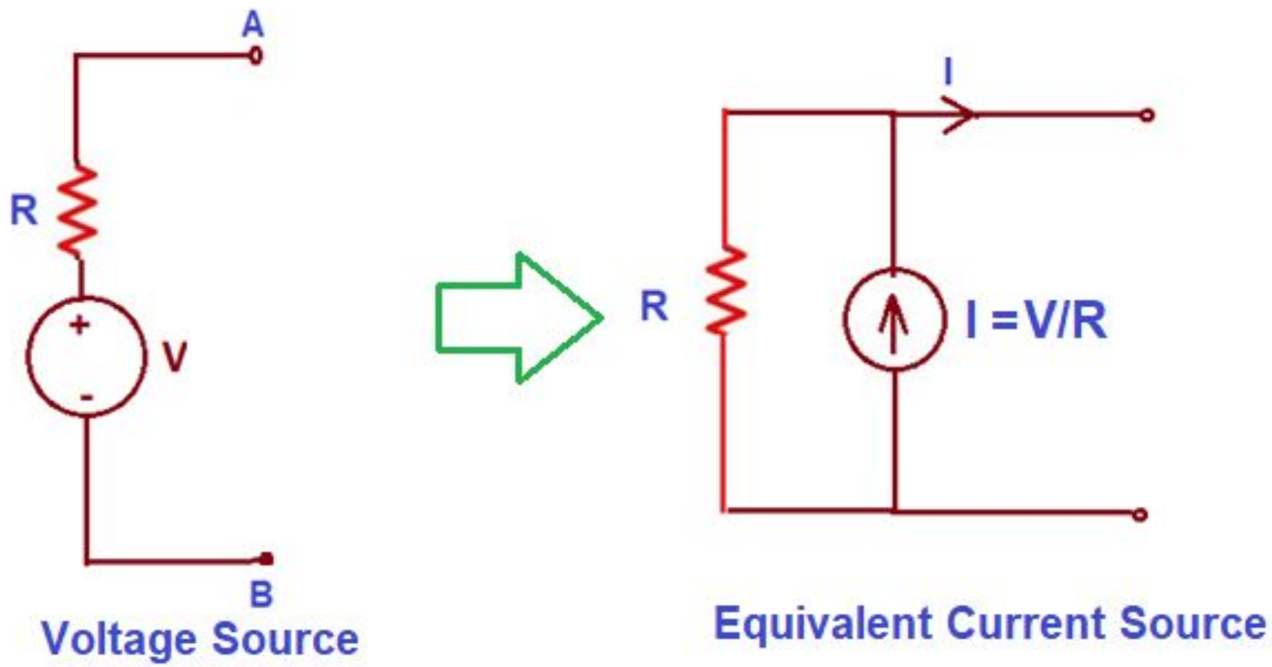
Ideally, internal resistance of a **CURRENT SOURCE** is **infinite (undefined)**

That of a **VOLTAGE SOURCE** is **zero**

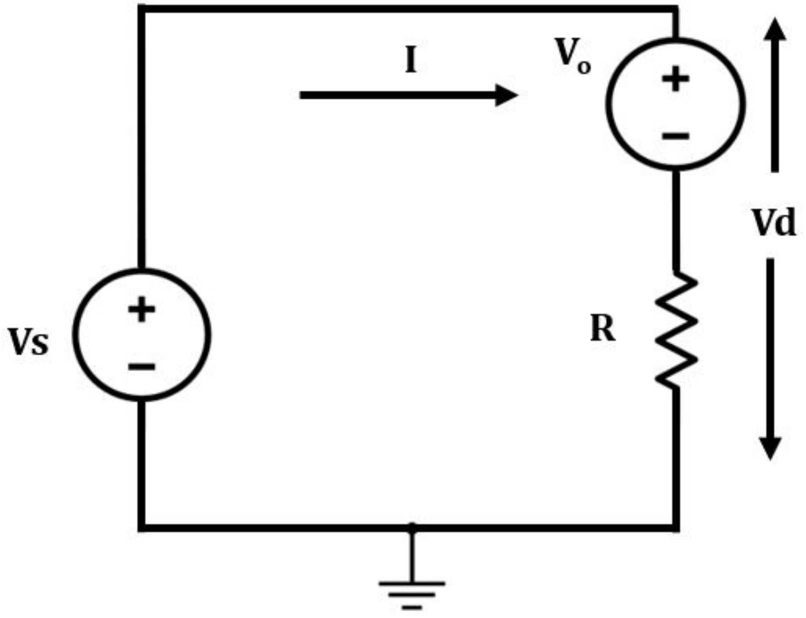


Hybrid/ Compound Linear Circuits

- Voltage Source in Series with a Resistor
- Current source in Parallel with a Resistor



Voltage Source in Series with a Resistor



$$V_d - V_o = IR$$

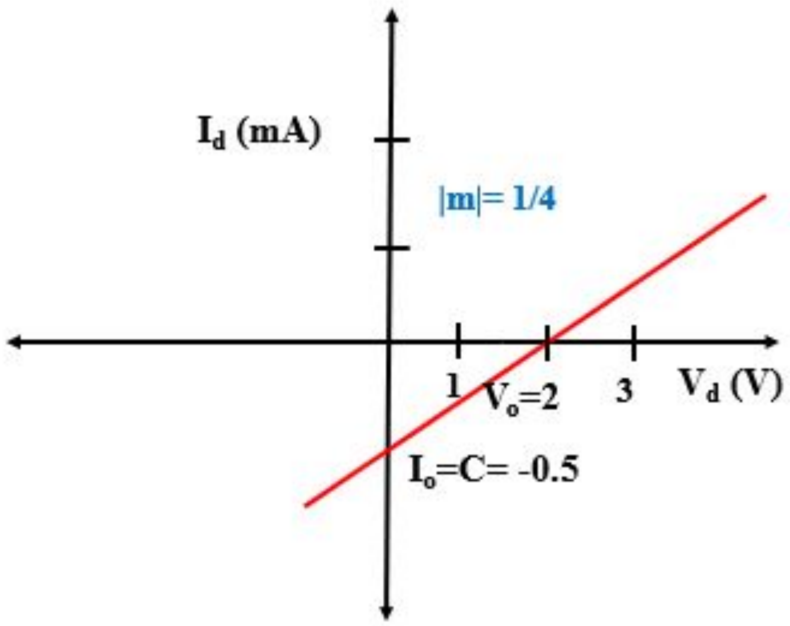
$$\Rightarrow I = \frac{V_d - V_o}{R}$$

$$\Rightarrow I = \frac{1}{R} \cdot V_d - \frac{Vo}{R}$$

$$\boldsymbol{y = mx + c}$$

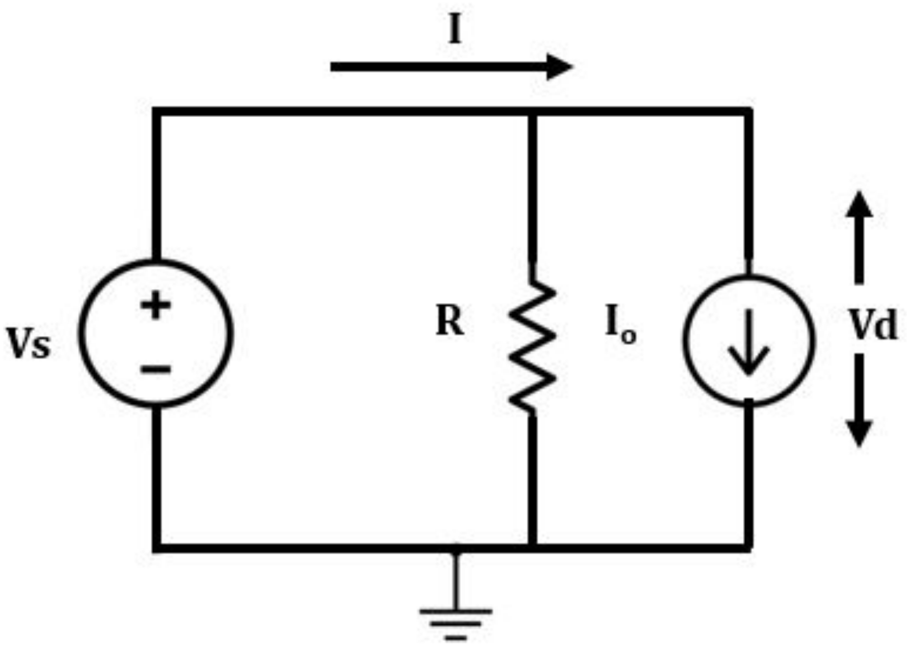
$$\boldsymbol{m = \frac{1}{R}}$$

$$\boldsymbol{c = -\frac{V_0}{R}}$$



I-V curve of a $\mathbf{4}\ \boldsymbol{k\Omega}$ resistor in series with a **2 V** voltage source

Current source in Parallel with a Resistor



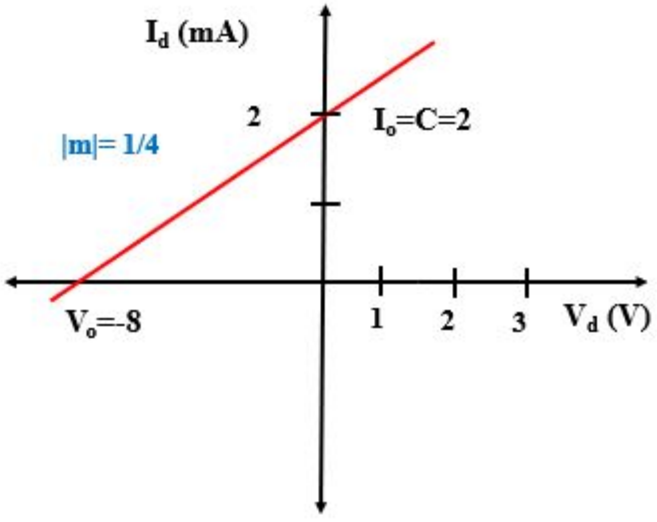
$$I = \frac{V_d}{R} + I_0$$

$$\Rightarrow I = \frac{1}{R} \cdot V_d + I_0$$

$$\boldsymbol{y = mx + c}$$

$$\boldsymbol{m = \frac{1}{R}}$$

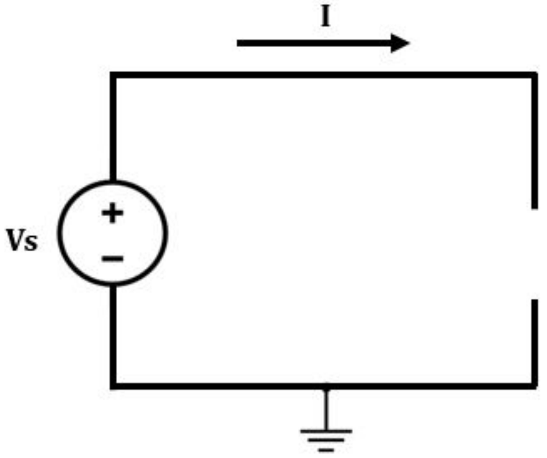
$$\boldsymbol{c = I_0}$$



The value of a resistor CAN NOT be Negative!

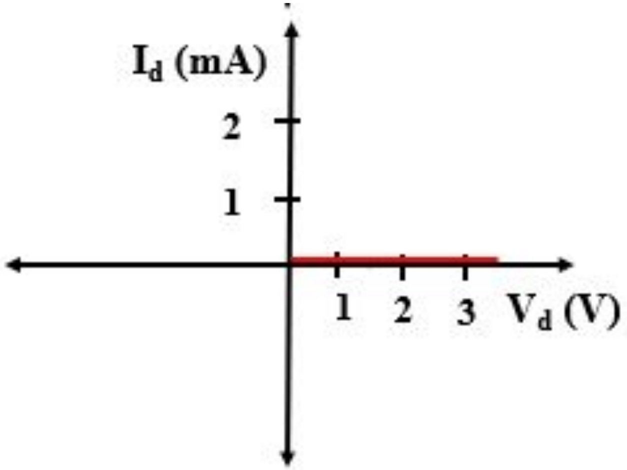
Degenerate Linear Elements

- Open Circuit

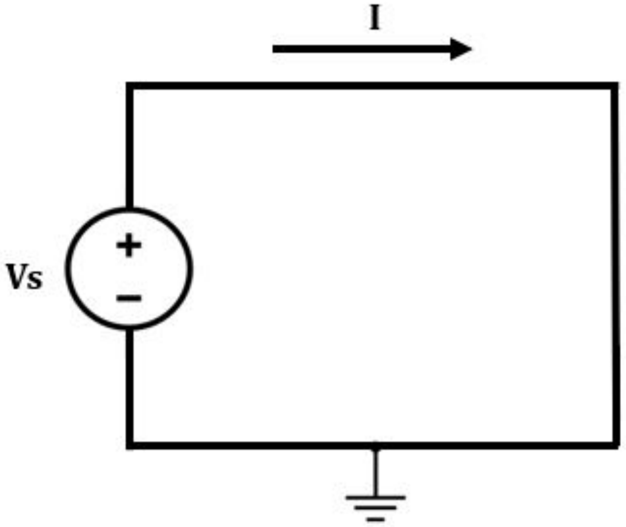


$$I_d = I_0 = 0$$

$$\boldsymbol{y = c = 0}$$

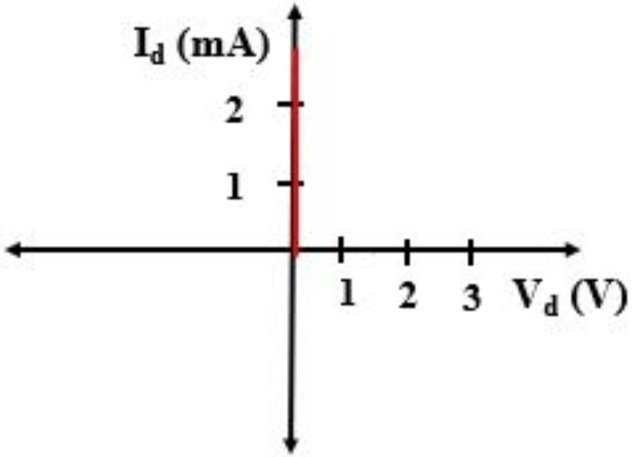


- Short Circuit



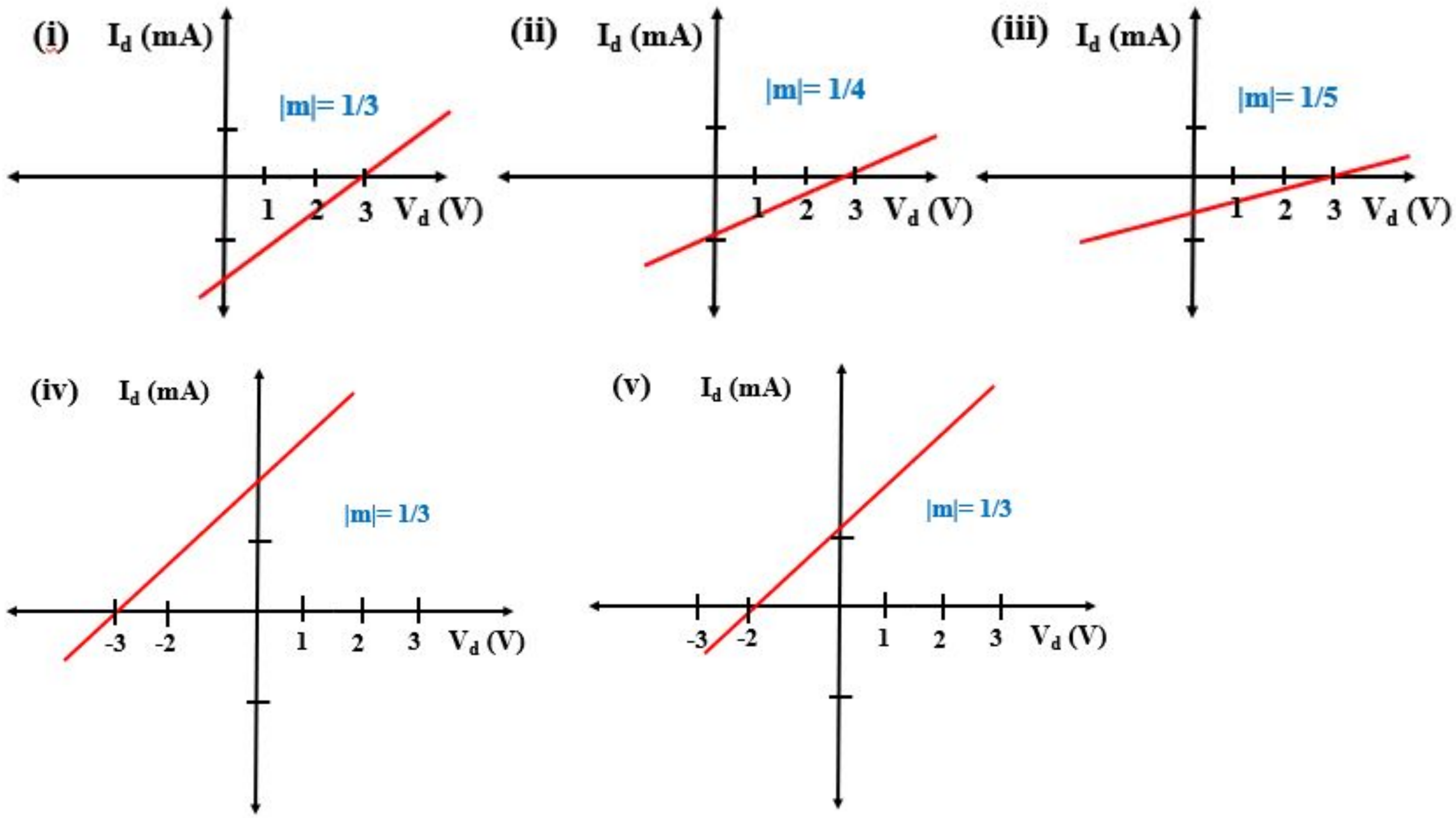
$$V = V_0 = 0$$

$$\boldsymbol{x = c = 0}$$



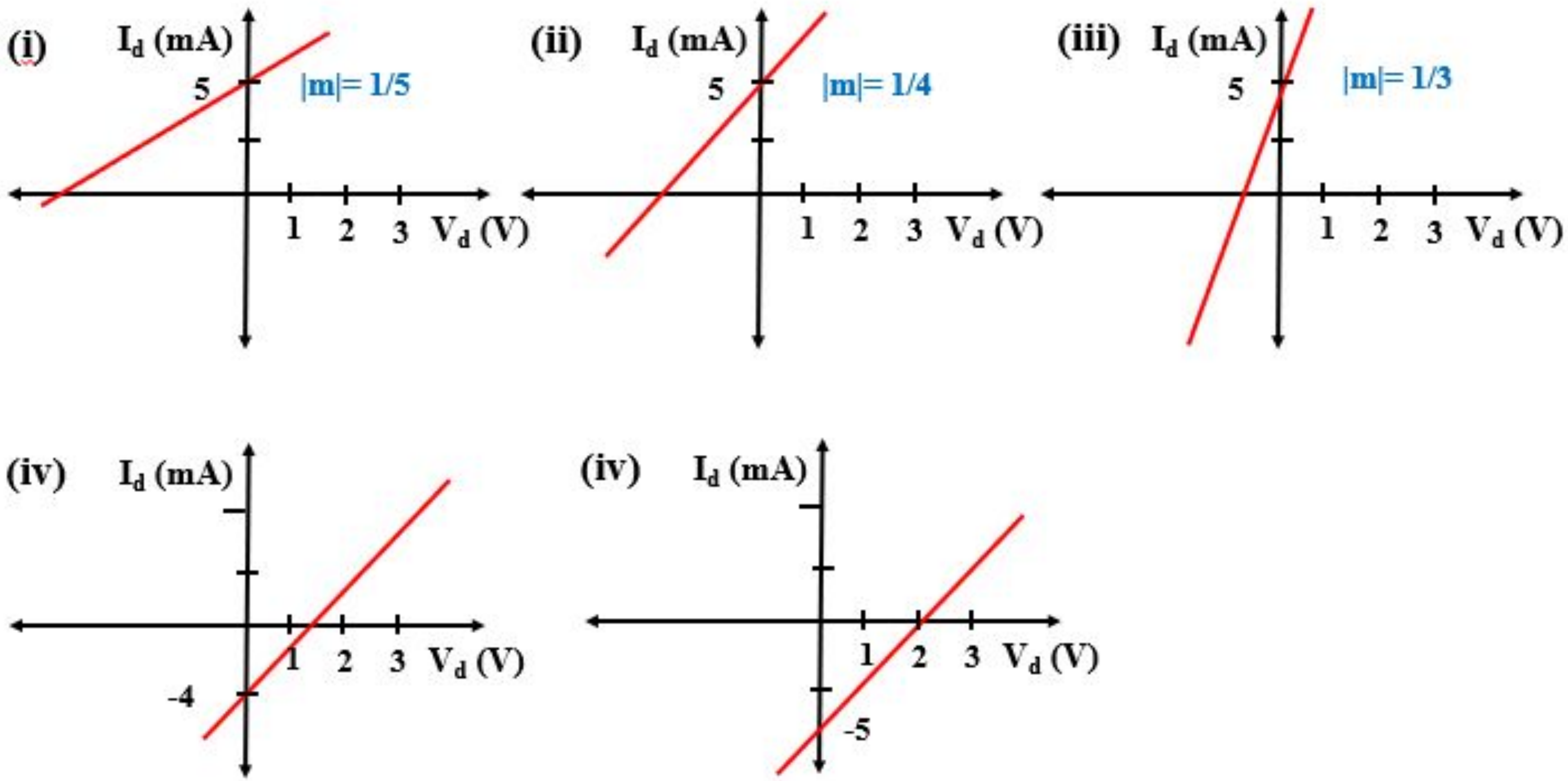
Voltage Source in Series with a Resistor

- Find the circuit



Current source in Parallel with a Resistor

Find the circuit

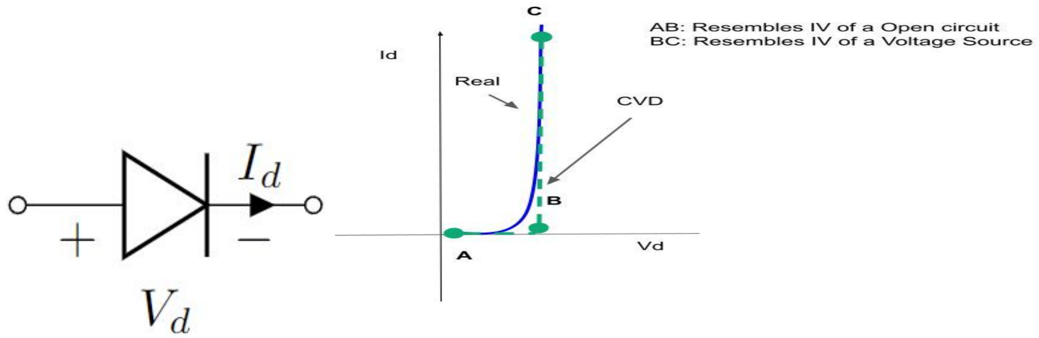


Practice Problems

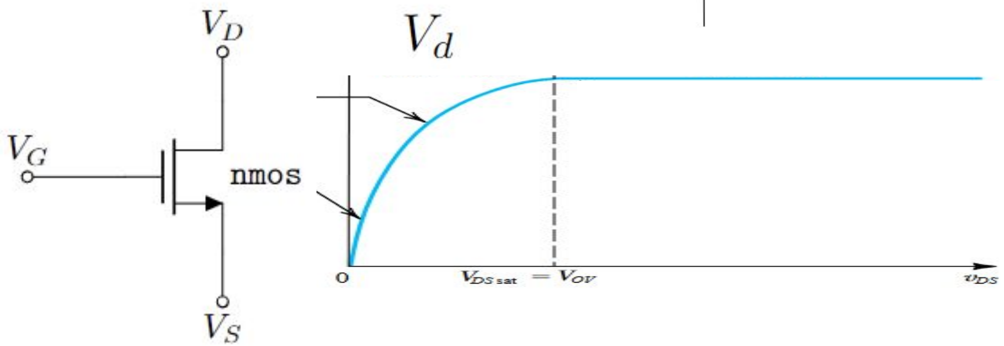
1. A Voltage Source, $V_o = -10$ V in series with a resistor of $R = 3$ kΩ.
   i. Write down the equation representing this curve
   ii. Determine the unknown parameters
   iii. Label the I-V curve
2. A Current Source, $I_o = -5$ mA in parallel with a resistor of $R = 5$ kΩ.
   i. Write down the equation representing this curve
   ii. Determine the unknown parameters
   iii. Label the I-V curve
3. A Current Source, $I_o = 5$ mA in parallel with a resistor. The slope of the curve is, $m = -5\ k\Omega^{-1}$.
   i. Write down the equation representing this curve
   ii. Determine the unknown parameters
   iii. Label the I-V curve

Non-Linear Devices/Elements

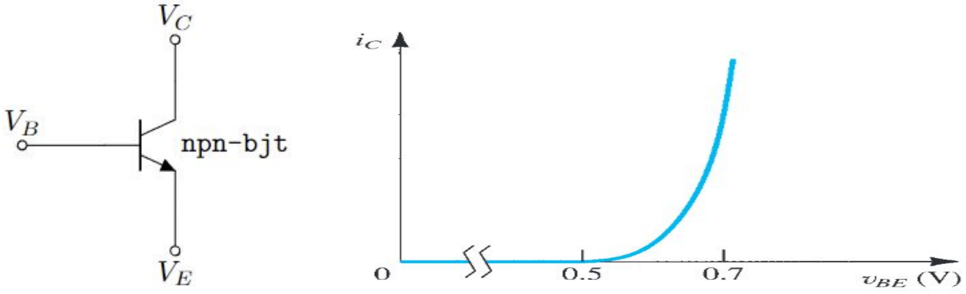
- Diode



- MOSFET

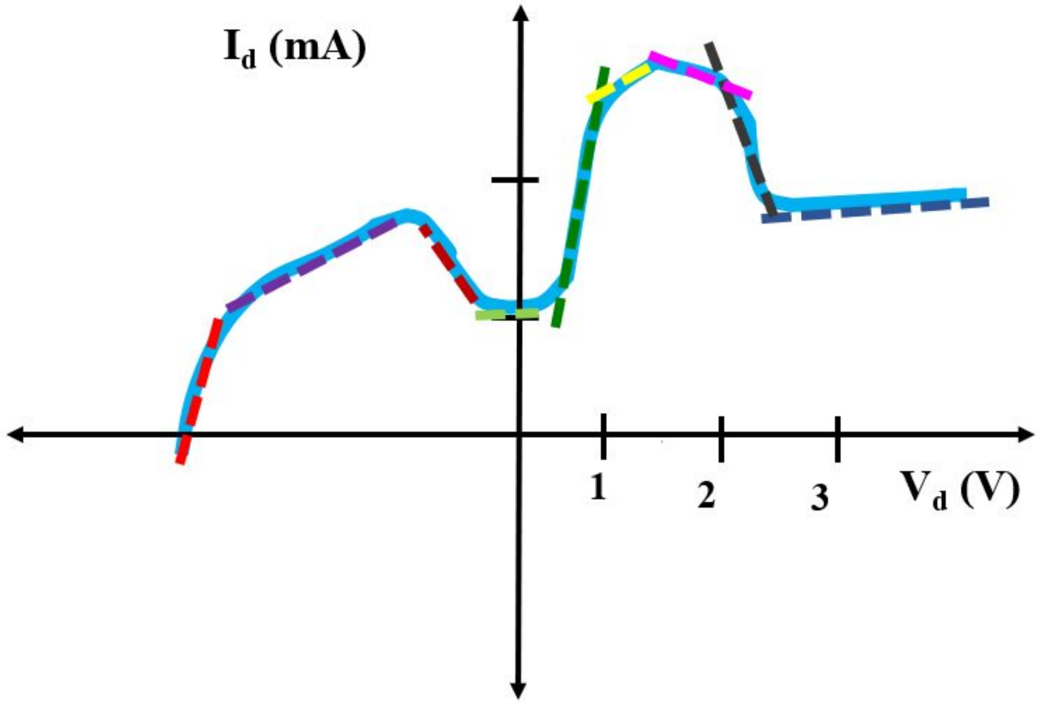


- BJT



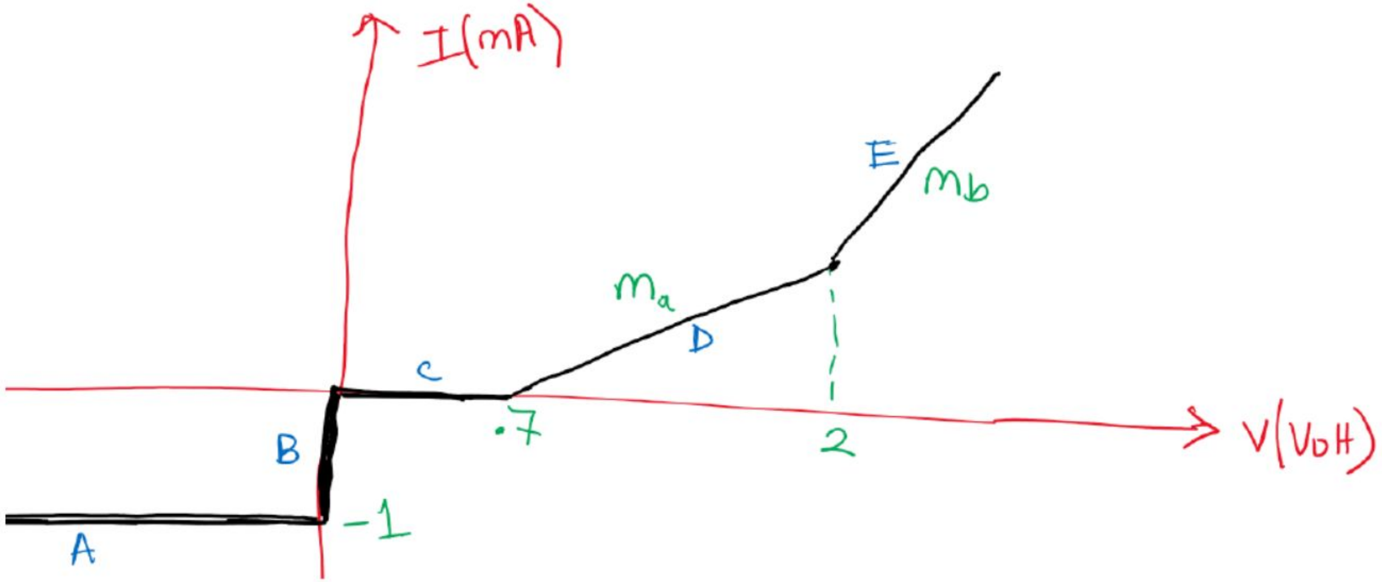
Piecewise Linear Approximation for NL devices

- Simplifying non-linear IV characteristics by piecewise linear parts.
- Non-linear functions are usually approximated by a series of linear segments that follow the tangent of the non-linear segment as can be seen from the following figure.



Piecewise Linear Approximation for NL devices

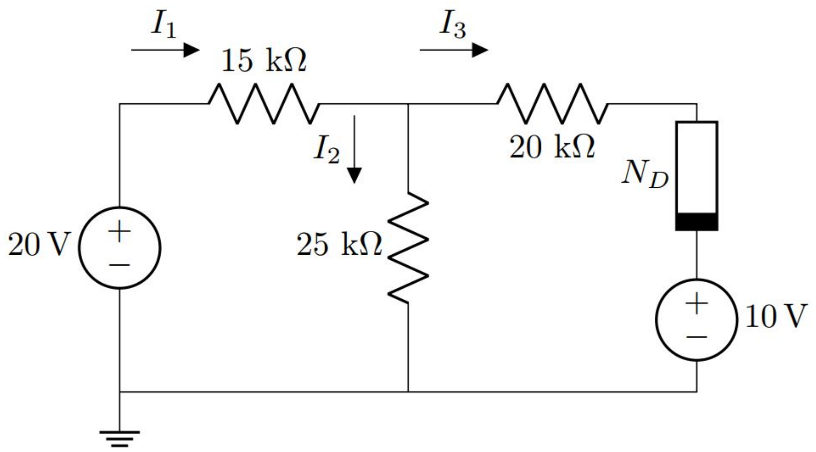
I-V curve of a hypothetical piecewise linear device is shown below.



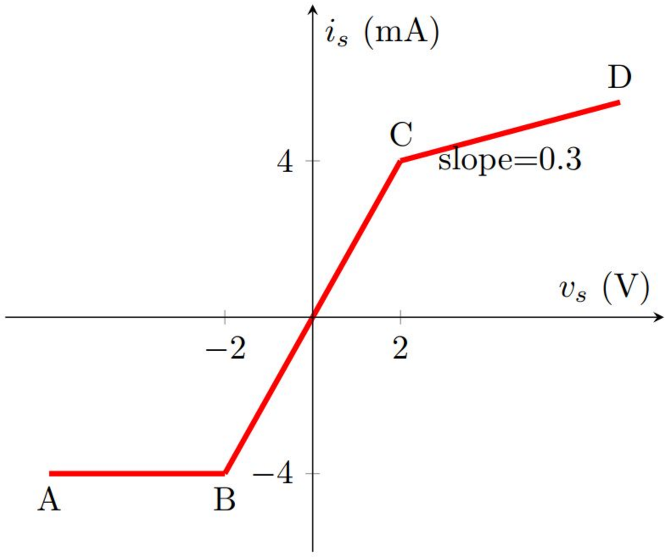
Here, P & Q that will come from your student id. For example, if the last 4 digits of your student id is 1234, then P=12, Q=34. In the graph, m_a = P and m_b = Q

What is the device model and parameter for the regions A, B, C, D, E? If the voltage across the device is 2.1v, what will be the operating region? What is the current flowing through it?

Piecewise Linear Approximation for NL dev



(a) A circuit with a non-linear device N_D



(b) IV Characteristics of the non-linear device N_D

(a) **Identify** the equivalent linear circuit models for the 3 linear regions (AB, BC, CD) shown in the IV characteristics of the non-linear device N_D (Figure (b)) and **calculate** the model parameters. [3]

(b) **Detect** the operating region for the device when $v_s = 3$ V and **calculate** the current through the device, i_s, for this voltage (hint: use Figure (b) and answers from previous part). [1+1]

(c) **Show** the alternative representation of the circuit in Figure (a). [1.5]

(d) Assume that the non-linear device N_D has been replaced with its equivalent linear device of segment BC. **Draw** the alternative representation of the circuit again by replacing N_D. [0.5]

(e) **Apply** KVL and KCL on the circuit of part (d) to calculate the values of I_1, I_2, and I_3. [3]