

I/O is so expensive. Even slight improvements in demand-paging methods yield large gains in system performance.

There are many different page-replacement algorithms. Every operating system probably has its own replacement scheme. How do we select a particular replacement algorithm? In general, we want the one with the lowest page-fault rate.

We evaluate an algorithm by running it on a particular string of memory references and computing the number of page faults. The string of memory references is called a **reference string**. We can generate reference strings artificially (by using a random-number generator, for example), or we can trace a given system and record the address of each memory reference. The latter choice produces a large number of data (on the order of 1 million addresses per second). To reduce the number of data, we use two facts.

First, for a given page size (and the page size is generally fixed by the hardware or system), we need to consider only the page number, rather than the entire address. Second, if we have a reference to a page p , then any references to page p that *immediately* follow will never cause a page fault. Page p will be in memory after the first reference, so the immediately following references will not fault.

For example, if we trace a particular process, we might record the following address sequence:

```
0100, 0432, 0101, 0612, 0102, 0103, 0104, 0101, 0611, 0102, 0103,
0104, 0101, 0610, 0102, 0103, 0104, 0101, 0609, 0102, 0105
```

At 100 bytes per page, this sequence is reduced to the following reference string:

```
1, 4, 1, 6, 1, 6, 1, 6, 1, 6, 1
```

To determine the number of page faults for a particular reference string and page-replacement algorithm, we also need to know the number of page frames available. Obviously, as the number of frames available increases, the number of page faults decreases. For the reference string considered previously, for example, if we had three or more frames, we would have only three faults—one fault for the first reference to each page. In contrast, with only one frame available, we would have a replacement with every reference, resulting in eleven faults. In general, we expect a curve such as that in Figure 10.11. As the number of frames increases, the number of page faults drops to some minimal level. Of course, adding physical memory increases the number of frames.

We next illustrate several page-replacement algorithms. In doing so, we use the reference string

```
7, 0, 1, 2, 0, 3, 0, 4, 2, 3, 0, 3, 2, 1, 2, 0, 1, 7, 0, 1
```

for a memory with three frames.

10.4.2 FIFO Page Replacement

The simplest page-replacement algorithm is a first-in, first-out (FIFO) algorithm. A FIFO replacement algorithm associates with each page the time when that page was brought into memory. When a page must be replaced, the oldest page is chosen. Notice that it is not strictly necessary to record the time when a

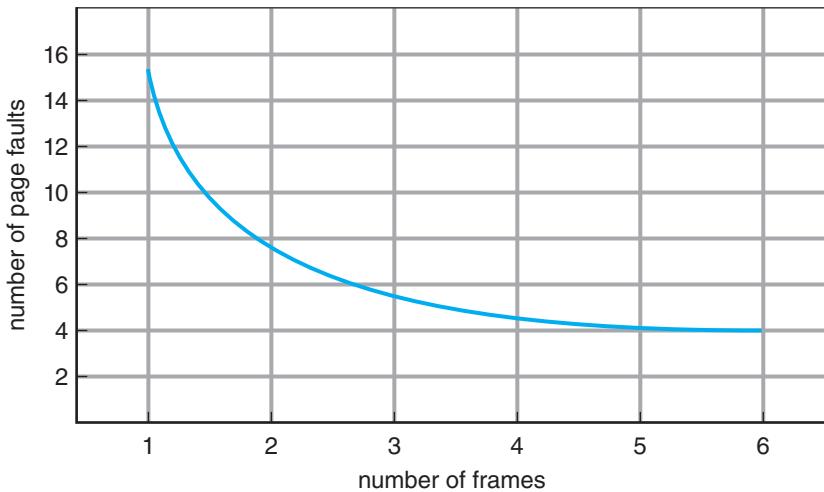


Figure 10.11 Graph of page faults versus number of frames.

page is brought in. We can create a FIFO queue to hold all pages in memory. We replace the page at the head of the queue. When a page is brought into memory, we insert it at the tail of the queue.

For our example reference string, our three frames are initially empty. The first three references (7, 0, 1) cause page faults and are brought into these empty frames. The next reference (2) replaces page 7, because page 7 was brought in first. Since 0 is the next reference and 0 is already in memory, we have no fault for this reference. The first reference to 3 results in replacement of page 0, since it is now first in line. Because of this replacement, the next reference, to 0, will fault. Page 1 is then replaced by page 0. This process continues as shown in Figure 10.12. Every time a fault occurs, we show which pages are in our three frames. There are fifteen faults altogether.

The FIFO page-replacement algorithm is easy to understand and program. However, its performance is not always good. On the one hand, the page replaced may be an initialization module that was used a long time ago and is no longer needed. On the other hand, it could contain a heavily used variable that was initialized early and is in constant use.

Notice that, even if we select for replacement a page that is in active use, everything still works correctly. After we replace an active page with a new

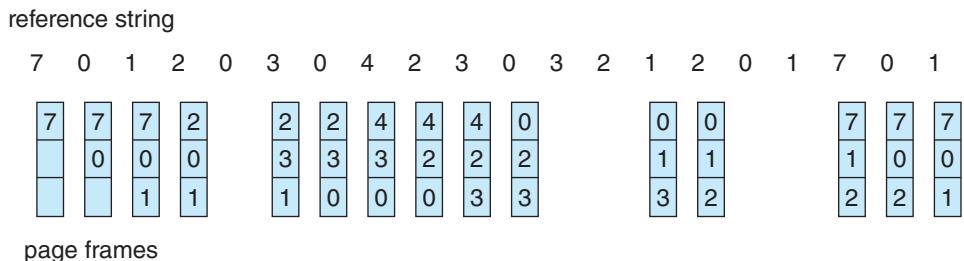


Figure 10.12 FIFO page-replacement algorithm.

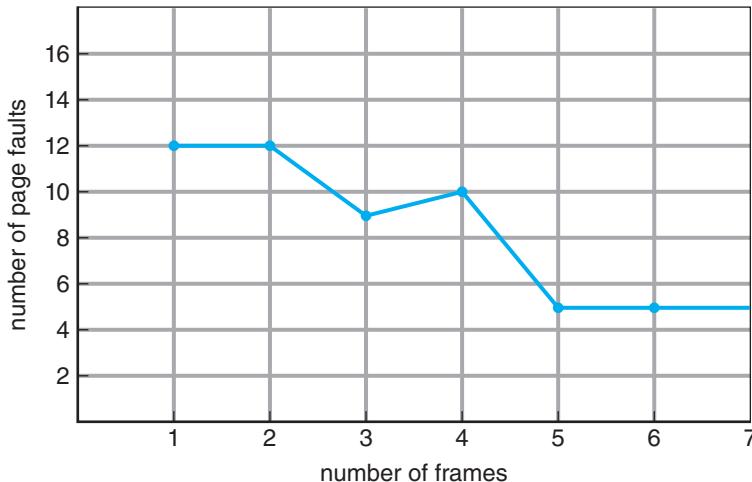


Figure 10.13 Page-fault curve for FIFO replacement on a reference string.

one, a fault occurs almost immediately to retrieve the active page. Some other page must be replaced to bring the active page back into memory. Thus, a bad replacement choice increases the page-fault rate and slows process execution. It does not, however, cause incorrect execution.

To illustrate the problems that are possible with a FIFO page-replacement algorithm, consider the following reference string:

1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Figure 10.13 shows the curve of page faults for this reference string versus the number of available frames. Notice that the number of faults for four frames (ten) is *greater* than the number of faults for three frames (nine)! This most unexpected result is known as **Belady's anomaly**: for some page-replacement algorithms, the page-fault rate may *increase* as the number of allocated frames increases. We would expect that giving more memory to a process would improve its performance. In some early research, investigators noticed that this assumption was not always true. Belady's anomaly was discovered as a result.

10.4.3 Optimal Page Replacement

One result of the discovery of Belady's anomaly was the search for an **optimal page-replacement algorithm**—the algorithm that has the lowest page-fault rate of all algorithms and will never suffer from Belady's anomaly. Such an algorithm does exist and has been called OPT or MIN. It is simply this:

Replace the page that will not be used for the longest period of time.

Use of this page-replacement algorithm guarantees the lowest possible page-fault rate for a fixed number of frames.

For example, on our sample reference string, the optimal page-replacement algorithm would yield nine page faults, as shown in Figure 10.14. The first three references cause faults that fill the three empty frames. The reference to page 2 replaces page 7, because page 7 will not be used until reference 18, whereas

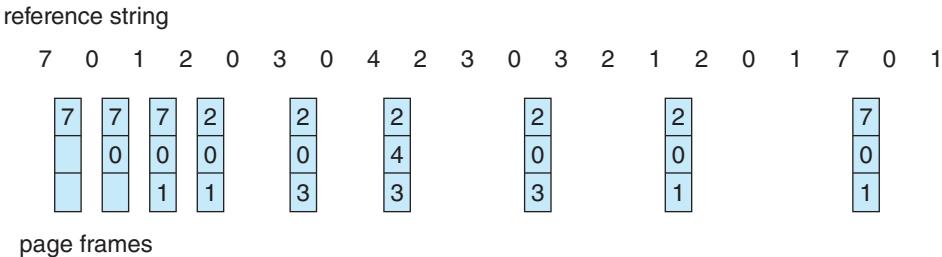


Figure 10.14 Optimal page-replacement algorithm.

page 0 will be used at 5, and page 1 at 14. The reference to page 3 replaces page 1, as page 1 will be the last of the three pages in memory to be referenced again. With only nine page faults, optimal replacement is much better than a FIFO algorithm, which results in fifteen faults. (If we ignore the first three, which all algorithms must suffer, then optimal replacement is twice as good as FIFO replacement.) In fact, no replacement algorithm can process this reference string in three frames with fewer than nine faults.

Unfortunately, the optimal page-replacement algorithm is difficult to implement, because it requires future knowledge of the reference string. (We encountered a similar situation with the SJF CPU-scheduling algorithm in Section 5.3.2.) As a result, the optimal algorithm is used mainly for comparison studies. For instance, it may be useful to know that, although a new algorithm is not optimal, it is within 12.3 percent of optimal at worst and within 4.7 percent on average.

10.4.4 LRU Page Replacement

If the optimal algorithm is not feasible, perhaps an approximation of the optimal algorithm is possible. The key distinction between the FIFO and OPT algorithms (other than looking backward versus forward in time) is that the FIFO algorithm uses the time when a page was brought into memory, whereas the OPT algorithm uses the time when a page is to be *used*. If we use the recent past as an approximation of the near future, then we can replace the page that *has not been used* for the longest period of time. This approach is the **least recently used (LRU) algorithm**.

LRU replacement associates with each page the time of that page's last use. When a page must be replaced, LRU chooses the page that has not been used for the longest period of time. We can think of this strategy as the optimal page-replacement algorithm looking backward in time, rather than forward. (Strangely, if we let S^R be the reverse of a reference string S , then the page-fault rate for the OPT algorithm on S is the same as the page-fault rate for the OPT algorithm on S^R . Similarly, the page-fault rate for the LRU algorithm on S is the same as the page-fault rate for the LRU algorithm on S^R .)

The result of applying LRU replacement to our example reference string is shown in Figure 10.15. The LRU algorithm produces twelve faults. Notice that the first five faults are the same as those for optimal replacement. When the reference to page 4 occurs, however, LRU replacement sees that, of the three frames in memory, page 2 was used least recently. Thus, the LRU algorithm replaces page 2, not knowing that page 2 is about to be used. When it then faults

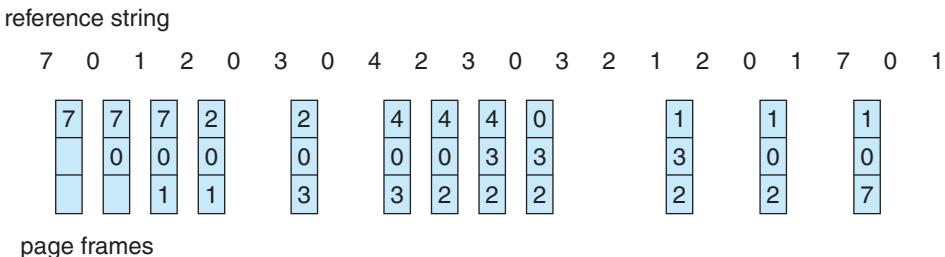


Figure 10.15 LRU page-replacement algorithm.

for page 2, the LRU algorithm replaces page 3, since it is now the least recently used of the three pages in memory. Despite these problems, LRU replacement with twelve faults is much better than FIFO replacement with fifteen.

The LRU policy is often used as a page-replacement algorithm and is considered to be good. The major problem is *how* to implement LRU replacement. An LRU page-replacement algorithm may require substantial hardware assistance. The problem is to determine an order for the frames defined by the time of last use. Two implementations are feasible:

- **Counters.** In the simplest case, we associate with each page-table entry a time-of-use field and add to the CPU a logical clock or counter. The clock is incremented for every memory reference. Whenever a reference to a page is made, the contents of the clock register are copied to the time-of-use field in the page-table entry for that page. In this way, we always have the “time” of the last reference to each page. We replace the page with the smallest time value. This scheme requires a search of the page table to find the LRU page and a write to memory (to the time-of-use field in the page table) for each memory access. The times must also be maintained when page tables are changed (due to CPU scheduling). Overflow of the clock must be considered.
- **Stack.** Another approach to implementing LRU replacement is to keep a stack of page numbers. Whenever a page is referenced, it is removed from the stack and put on the top. In this way, the most recently used page is always at the top of the stack, and the least recently used page is always at the bottom (Figure 10.16). Because entries must be removed from the middle of the stack, it is best to implement this approach by using a doubly linked list with a head pointer and a tail pointer. Removing a page and putting it on the top of the stack then requires changing six pointers at worst. Each update is a little more expensive, but there is no search for a replacement; the tail pointer points to the bottom of the stack, which is the LRU page. This approach is particularly appropriate for software or microcode implementations of LRU replacement.

Like optimal replacement, LRU replacement does not suffer from Belady’s anomaly. Both belong to a class of page-replacement algorithms, called **stack algorithms**, that can never exhibit Belady’s anomaly. A stack algorithm is an algorithm for which it can be shown that the set of pages in memory for n frames is always a *subset* of the set of pages that would be in memory with n

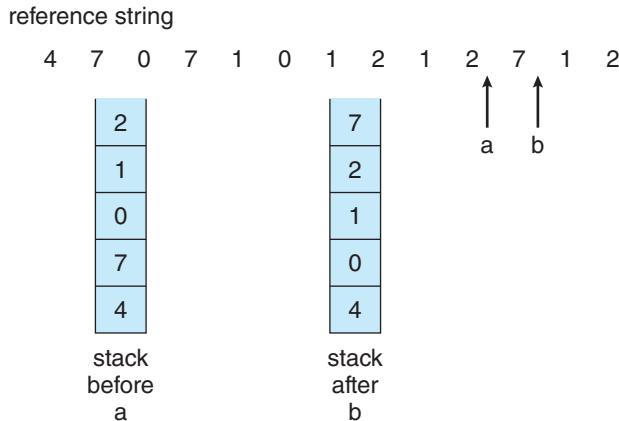


Figure 10.16 Use of a stack to record the most recent page references.

+ 1 frames. For LRU replacement, the set of pages in memory would be the n most recently referenced pages. If the number of frames is increased, these n pages will still be the most recently referenced and so will still be in memory.

Note that neither implementation of LRU would be conceivable without hardware assistance beyond the standard TLB registers. The updating of the clock fields or stack must be done for *every* memory reference. If we were to use an interrupt for every reference to allow software to update such data structures, it would slow every memory reference by a factor of at least ten, hence slowing every process by a factor of ten. Few systems could tolerate that level of overhead for memory management.

10.4.5 LRU-Approximation Page Replacement

Not many computer systems provide sufficient hardware support for true LRU page replacement. In fact, some systems provide no hardware support, and other page-replacement algorithms (such as a FIFO algorithm) must be used. Many systems provide some help, however, in the form of a **reference bit**. The reference bit for a page is set by the hardware whenever that page is referenced (either a read or a write to any byte in the page). Reference bits are associated with each entry in the page table.

Initially, all bits are cleared (to 0) by the operating system. As a process executes, the bit associated with each page referenced is set (to 1) by the hardware. After some time, we can determine which pages have been used and which have not been used by examining the reference bits, although we do not know the *order* of use. This information is the basis for many page-replacement algorithms that approximate LRU replacement.

10.4.5.1 Additional-Reference-Bits Algorithm

We can gain additional ordering information by recording the reference bits at regular intervals. We can keep an 8-bit byte for each page in a table in memory. At regular intervals (say, every 100 milliseconds), a timer interrupt transfers control to the operating system. The operating system shifts the reference bit for each page into the high-order bit of its 8-bit byte, shifting the other bits right