

Quiz on Divide And Conquer Answers with Explanation wherever possible

Q1) Let $T(n) = T(n-1) + 1/n$. Then $T(n)$ is:

- a. $\Theta(1)$
- b. $\Theta(\log n)$
- c. $\Theta(\log \log n)$
- d. $\Theta(n)$

Answer is b $O(\log N)$

Explanation: By unfolding

$$T(n) = T(n-1) + 1/n = T(n-2) + 1/n + 1/(n-1) = \dots = T(0) + \sum_{k=1}^n \frac{1}{k}$$

Now we can easily approximate the sum on the RHS using that

$$\sum_{k=1}^n \frac{1}{k} \leq 1 + \int_1^n \frac{1}{x} dx = 1 + \log n - \log 1 = 1 + \log n$$

Therefore $T(n) = O(\log n)$

Q2) Which of the following algorithms is NOT a divide & conquer algorithm by nature?

- a. Heap Sort
- b. Euclidean algorithm to compute the greatest common divisor
- c. Cooley-Tukey fast Fourier transform
- d. Quick Sort

Answer is a) Heap Sort

Q3) Consider the following function

```
find (int n)
{
    if (n < 2 ) then return;
    else
    {
        sum= 0;
        for (i= 1; i ≤ 4; i++) find (n/2);
        for (i=1; i ≤ n*n; i++) sum= sum + 1;
    }
}
```

Assume that the division operation takes constant time and “sum” is global variable. What is the time complexity of “find (n)” ?

- a. n^2
- b. n^3
- c. $n^2 \log n$
- d. None of these

Answer is c) $n^2 \log n$

Explanation:

```
Find (int n) ----- T(n)
{
  if (n < 2) then return; ----- O(1)
  else
  {
    sum = 0;
    for (i = 1; i <= 4; i++) ----- 4 times
      find (n/2); ----- T(n/2)
    for (i = 1; i <= n * n; i++)
      sum = sum + 1; ----- n^2
  }
}
```

⇒ we can write recurrence relation as :-

$$T(n) = \begin{cases} O(1) & n < 2 \\ 4T(n/2) + n^2 & n \geq 2 \end{cases}$$

By master theorem

$$T(n) = O(n^2 \log n)$$

Q4) Maximum Subarray Sum problem is to find the subarray with maximum sum. For example, given an array {12, -13, -5, 25, -20, 30, 10}, the maximum subarray sum is 45.

The naive solution for this problem is to calculate sum of all subarrays starting with every element and return the maximum of all. We can solve this using Divide and Conquer, what will be the worst case time complexity using Divide and Conquer.

- a. $O(\log N)$
- b. $O(N)$
- c. $O(N \log N)$
- d. $O(N^2)$

Answer is c $O(N \log N)$

Explanation : The recurrence relation for the same is $T(n) = 2T(n/2) + O(n)$

The above recurrence is similar to [Merge Sort](#) and can be solved either using Recurrence Tree

method or Master method. It falls in case II of Master Method and solution of the recurrence is $\Theta(n \log n)$.

Q5) The number of comparisons required to find maximum and minimum in the given array of n -element using divide and conquer:

- a. $\lceil 3n/2 \rceil$
- b. $\lceil 3n/2 \rceil + 2$
- c. $\lfloor 3n/2 \rfloor$
- d. $\lfloor 3n/2 \rfloor - 2$

Answer is d)

Explanation: It will require $(3n/2) - 2$ comparisons

Q6) If a problem can be solved by combining optimal solutions to non-overlapping problems, the strategy is called _

- A. Dynamic Programming
- B. Greedy
- C. Divide And Conquer

Answer is c) Divide and Conquer

Q7) What is the complexity of the following recurrence :

$$7T(n/2) + an^2$$

- a. $O(n^2)$
- b. $O(n^{1.51})$
- c. $O(n \log n)$
- d. $O(n^{2.81})$
- e. $O(\log n)$

Answer is d) $O(n^{2.81})$ using masters theorem

Q8) What will be the time complexity of the following recurrence relation:

$$3T(n/3) + \sqrt{n}$$

- A. $O(\sqrt{n})$
- B. $O(n)$
- C. $O(n \log n)$
- D. $O(n^2)$

Answer is b $O(n)$ using masters theorem

