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Introduction of Engineering Mathematics & Linear Algebra

Best Course on Engineering Mathematics for GATE/ESE by GC Sir

8:20 AM

Gurupal Singh Chawla • Lesson 1 • Dec 13, 2024

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Welcome to

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Introduction to

Engineering Mathematics

EE/EC/IN => GATE 2026/27

GURUPALL CHAWLA

Engineering Mathematics

$\frac{13}{13}$

Engineering Mathematics is the art of applying **maths.**

Combining M**athematical** theory, practical **engineering** and scientific computing to SOLVE technological challenges.

- **Engineering Mathematics GATE/ESE**
  - Linear Algebra 2-3
  - Probability & Statistics 2-3
  - Calculus 4-5
  - Vector Analysis 2
  - Differential Equations 2
  - ESE Numerical Methods
  - COMPLEX VAR. 2-3

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Requirements for this Classes (GC sir)

0. Presence of Your Mind with Me (LIVE DAILY)
1. Big size Copy (Spiral if Possible)
2. 2 pens (Blue,Red) & 1 Highlighter

For Reference.....

1. WORK_BOOK (updated with GCPS)

2. YOUR NOTE BOOK (PLUS Classes)

3. PYQ (***)

In our Syllabus

Unit 01 Linear Algebra

- Vector Space and Basis
- **Matrix Algebra**
- **Eigen Values and Eigenvectors**
- Linear Dependence and Independence
- **Rank**
- **Solution of system of linear equations– existence and uniqueness.**

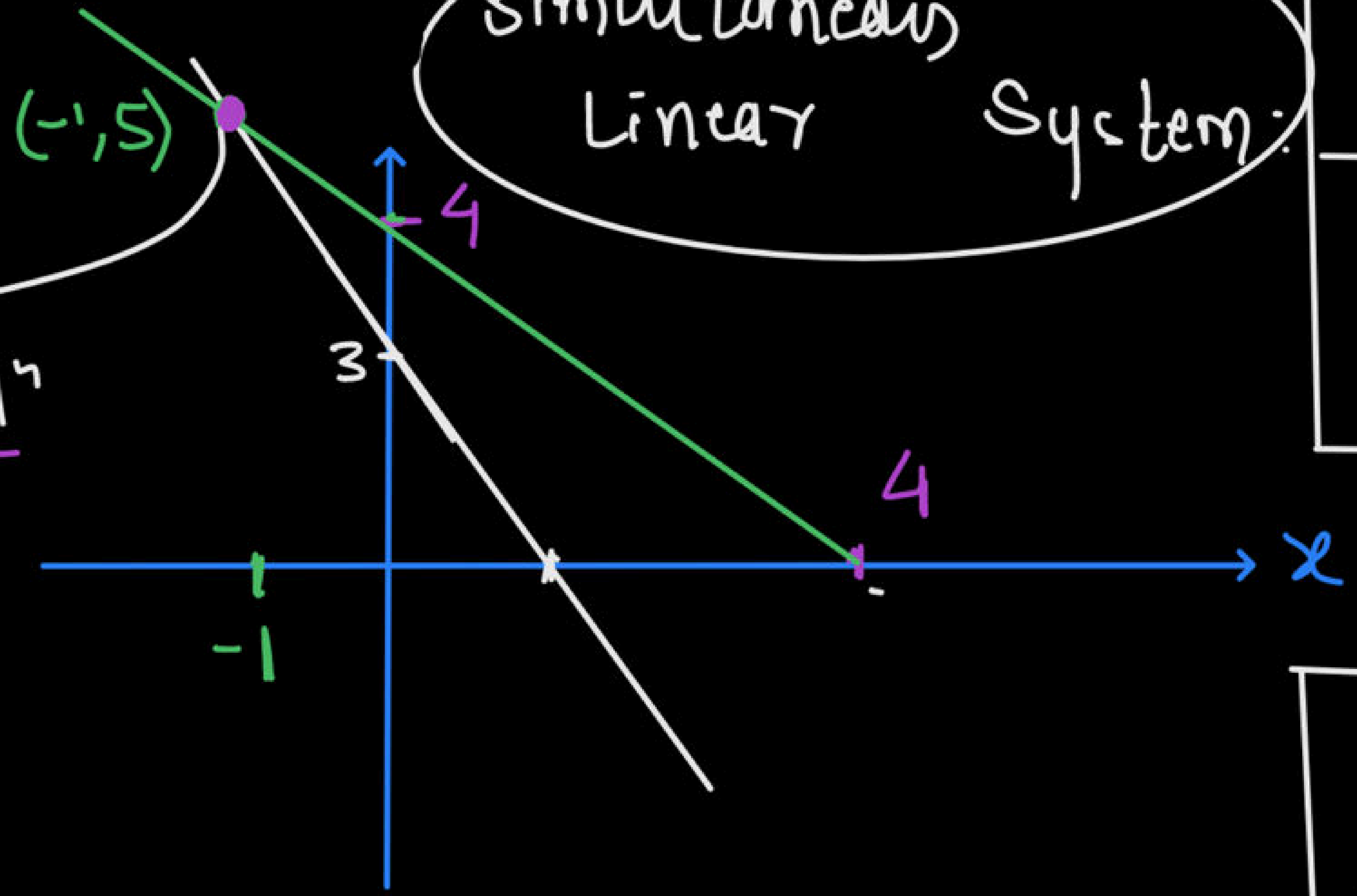
Let's Start Line_ar Algebra for CS GATE/ESE

1857

"Common Sense"

equation of line :

Simultaneous Linear System:



Unique Sol^n

→ Graphical Method

$$2x + y = 3 \quad \text{—} \ (\text{i})$$

$$x + y = 4 \quad \text{—} \ (\text{ii})$$

$$\boxed{\begin{aligned} x &= -1 \\ y &= 5 \end{aligned}}$$ ✓ yes!

only 1 Sol^n

(ellimination)

Cayley → Thinking

$$2x + y = 3$$
$$x + y = 4$$
$$\longrightarrow \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}_{2\times 2}}_{\text{Coefficient mat}}\overset{A}{} \underbrace{\overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}}_{\text{Un known}} = \underbrace{\overset{B}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}}}_{\text{known}}$$

E
G
Cm
S
Matrix Method

only collection of No.

$$A:B = \begin{bmatrix} 2 & 1 & \vdots & 3 \\ 1 & 1 & \vdots & 4 \end{bmatrix}_{2\times 3} \longrightarrow \text{"Matrix Method"}$$

Rows → equations

col → variables

There are two types of Linear simultaneous equations

(i) (Non-homogenous linear equations). $AX = B$ — (i)

(ii) (Homogenous linear equations). $AX = 0$ — (ii)

✓ Solving these Linear equations equations by $\boxed{\text{Matrix}}$ Method is Linear Algebra

MATRIX ?? ⇒ "Collection of Numbers in fixed No of Rows | col."

(Real | complex)

Analysis of MATRIX--

eg.

$$\begin{array}{cccccc} & \overbrace{C_1 \quad\quad C_2 \quad\quad C_3}^{\text{Columns}} & & \\ & \uparrow \quad\quad \uparrow \quad\quad \uparrow & & \\ A = & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} & \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix} & \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} \text{Rows} \end{array}$$

3×3

Matrix[Name] $\quad$ Element

$$[A] = \{a_{ij}\}_{R \times C}$$

dimension of Mat.

Position in row

Position in column

Number of rows

Number of columns

$R \times C \rightarrow$ Shape/ size/ Area/ No. of elements

Dim.

4 $\quad$ 3 $\quad$ 1 $\quad$ 2

$$A = \{a_{ij}\}_{\cancel{R \times C}}$$

Types of Matrix According to it's Dimensions (R/C)---

Case (I) $R = C$: ✓ <u>Square Matrix</u> : $A_{3\times3}$

Row = col ✓

Equat^n = Variable
Balanced System ✓
Determined System ✓

$$A_{3\times3} = \{a_{ij}\} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \rightarrow \text{order of Mat.}$$

$i<j$

$i>j$

$i=j$

- off diag

or

- secondary Diag

$\Rightarrow$ Diagonal OR

Connects 1st element to last element with $\underline{i=j}$

Main diag, Leading diag

Principle diag, on diag

* <u>Note</u>: ✓

Case II : $R \neq C$: Non Square < Rectangular>

"Unbalanced System"

(a) $R < C$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3} \rightarrow \text{Horizontal}$$

eqⁿ < variable

→ Under Determined sys.

(b) $R > C$

$$A = \begin{bmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}_{3\times2} \rightarrow \text{Vertical}$$

Eqⁿ > variable

→ overdet. System

(c) $R = 1,\ C > 1$

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Row mat / Row vector

(d) $R > 1,\ C = 1$

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Col Mat or Col vector

Formations of Matrix According to it's elements positions(i/j)

eg:

$$A = \{a_{ij}\}_{3\times4} = \begin{cases} i+j & : i<j \\ i-j & ; i>j \\ i & ; i=j \end{cases}$$

$i<j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4} = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 2 & 1 & 3 & 7 \end{bmatrix}_{3\times4}$$

$i>j$ $\qquad$ $i=j$

ie:

Que: Matrix

GC.

PYQ.

$$A_{3\times4} = \{a_{ij}\} = \begin{cases} i + j + k & ; \; i<j \\ i - j - k & ; \; i>j \\ \dfrac{i+j}{2} + k & ; \; i=j \end{cases}$$

if the sum of all elements is 100 then value of k is ____.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4} = \boxed{\begin{matrix} 1+k & 3+k & 4+k & 5+k \\ 1-k & 2+k & 5+k & 6+k \\ 2-k & 1-k & 3+k & 7+k \end{matrix}}$$

$\rightarrow$ sum of all elements $= 100$

$\Rightarrow 40 + 6k = 100$

$\Rightarrow \boxed{k = 10}$ Ans.

Que: For a Square Matrix: $A_{n\times n} = \{a_{ij}\}_{n\times n}$ find Sum of all elements of following mat,

(i) $\{a_{ij}\} = \begin{cases} 1 & : i = j \\ 0 & : i \neq j \end{cases}$

(ii) $\{a_{ij}\} = \begin{cases} i & : i = j \\ 0 & : i \neq j \end{cases}$

(iii) $\{a_{ij}\} = \begin{cases} i+j & ; i = j \\ 0 & : i \neq j \end{cases}$

(iv) $\{a_{ij}\} = \begin{cases} i \times j & : i = j \\ 0 & : \text{otherwise} \end{cases}$

(v) $\{a_{ij}\} = \begin{cases} i \times j^2 & : i = j \\ 0 & : \text{elsewhere} \end{cases}$

(vi) $\{a_{ij}\} = \{ i^2 - j^2 \ \forall\ i, j \}$

(vii) $\{a_{ij}\} = \{ i + j \ \forall\ i, j \}$

$$A_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

(i) $$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & & \vdots \\ 0 & & 1 & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix}$$

$$\text{Sum} = \sum a_{ij} = 1+1+1+ \text{ --- } n \text{ times}$$

$$= n$$

(ii) $$\begin{bmatrix} 1 & & \text{zero} \\ & 2 & \\ & & 3 & \\ \text{zero} & & & \ddots & \\ & & & & n \end{bmatrix}$$

$$\sum a_{ij} = 1+2+3+\cdots+n$$

$$= \frac{n(n+1)}{2}, \quad S_n = \frac{n}{2}(a+l$$

(iii) $$\begin{bmatrix} 2 & & & \\ & 4 & & \\ & & 6 & \\ & & & \ddots & \\ & & & & 2n \end{bmatrix}$$

$$\sum a_{ij} = 2 \times \frac{n}{2}(n+1)$$

$$= n(n+1)$$

(iv) $$\begin{bmatrix} 1^2 & & \text{zero} \\ & 2^2 & \\ & & \ddots & \\ \text{zero} & & & n^2 \end{bmatrix}$$

$$\sum a_{ij} = 1^2 + 2^2 + \cdots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

(V)

$$\begin{bmatrix} 1^3 & & \text{zero} & \\ & 2^3 & & \\ & & 3^3 & \\ \text{zero} & & & \ddots \\ & & & & n^3 \end{bmatrix}$$

$$\sum a_{ij} = 1^3 + 2^3 + 3^3 + \cdots + n^3$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

(VI)

$$\begin{bmatrix} 0 & & -ve \\ & 0 & \\ +ve & & 0 \end{bmatrix}_{n \times n}$$

$$\boxed{\sum a_{ij} = 0}$$

Check for 3X3 →

(VII) (a) $A = \{a_{ij}\} = \{1 \ \forall \ i, j$

(for every)

$$A = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

VII (a) $A = \{a_{ij}\} = \{1 \; \forall \; i, j$

(for every) $\forall$

$$A = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \end{array} \begin{array}{l} \to R_1 \\ \to R_2 \\ \vdots \\ \to R_n \end{array}$$

$\sum R_1 = 1 + 1 + \cdots 1 = n$

$\sum R_2 = n$

$\sum R_n = n$

Total Sum

$\sum R_1 + \sum R_2 + \cdots \sum R_n = n + n + \cdots + n =$

$n \times n = n^2$

$$A = \begin{bmatrix} 2 & 3 & 4 & \cdots & (1+n) \\ 3 & 4 & 5 & \cdots & (2+n) \\ \vdots & & \ddots & & \vdots \\ (n+1) & (n+2) & \cdots & & (n+n) \end{bmatrix}_{n\times n}$$

$\rightarrow \Sigma R_1 = \frac{n}{2}[2+1+n] = \frac{n}{2}[3+n]$

$\rightarrow \Sigma R_2 = \frac{n}{2}(5+n)$

$= \frac{n}{2}[7+n]$

$\cdots\cdots$

$\rightarrow \Sigma R_n = \frac{n}{2}((2n+1)+n]$

$$\text{Total Sum} = \Sigma R_1 + \Sigma R_2 + \cdots + \Sigma R_n$$

$$= \frac{n}{2}\left[(3+n)+(5+n)+\cdots+((2n+1)+n]\right] \rightarrow \underline{\underline{AP}}$$

$$= \frac{n}{2}\left[\frac{n}{2}[3+n+2n+1+n]\right] = \frac{n^2}{4}(4n+4)$$

$$= n^2(n+1)$$

THANK YOU