$$\exp(x) \underset{0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$\sin(x) \underset{0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)$$

$$\cos(x) \underset{0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$\ln(1+x) \underset{0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$\alpha \in \mathbb{R} \quad (1+x)^\alpha \underset{0}{=} 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x + \frac{\alpha(\alpha-1)(\alpha-2)}{6} x^2 + o(x^2)$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \underset{0}{=} 1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$\tan(x) \underset{0}{=} x + \frac{x^3}{3} + \frac{2x^5}{15} + o(x^6)$$

Ex 4

$$\cos(\arccos(x)) = x$$

$$\arccos'(x) \times (-\sin(\arccos(x))) = 1$$

$$\arccos'(x) = \frac{-1}{\sin(\arccos(x))}$$

On sait que :

$$\sin^2(\arccos(x)) + \cos^2(\arccos(x)) = 1$$

$$\sin^2(\arccos(x)) = 1 - x^2$$

$$\sin(\arccos(x)) = \sqrt{1-x^2}$$

car $\sin(x) \geq 0$ sur $[0, \pi]$

Donc $\arccos'(x) = -\frac{1}{\sqrt{1-x^2}}$

On a :

$$\arccos'(x) = \frac{-1}{\sqrt{1-x^2}} = -(1-x^2)^{-\frac{1}{2}}$$

$$(1-x^2)^{\frac{-1}{2}} = 1 + \frac{1}{2}x^2 + \frac{\frac{-1}{2}\left(\frac{-1}{2}-1\right)(x^2)^2}{2} + o(x^4)$$

$$= 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + o(x^4)$$

$$-(1-x^2)^{\frac{-1}{2}} = -1 - \frac{1}{2}x^2 - \frac{3}{8}x^4 + o(x^5)$$

$$\operatorname{Arccos}(x) = \int_0^x \operatorname{Arccos}'(t)\,dt + \frac{\pi}{2}$$

$\hookrightarrow$ Donc $\operatorname{Arccos}(x) \underset{0}{=} \frac{\pi}{2} - x - \frac{1}{6}x^3$

$$- \frac{3}{40}x^5 + o(x^5)$$

Intégrale

2) $\varphi(x) = \int_0^x e^{t^2}\,dt$

$\varphi(0) = 0$

$\varphi'(x) = e^{x^2} \underset{0}{=} 1 + x^2 + \frac{x^4}{2} + o(x^3)$

Donc $\varphi(x) \underset{0}{=} \underbrace{\varphi(0)}_{=0} + x + \frac{x^3}{3} + \frac{x^5}{10} + o(x^5)$

$\frac{d}{dx}(x^n) = n x^{n-1}$

$\hookrightarrow$ Une primitive de x^{n-1} est $\frac{1}{n}x^n$

Ex f :

$\frac{1}{6} \times \frac{3}{8} \Rightarrow \frac{1}{x^3}$

$$l_x \frac{\sqrt{x+2}}{\sqrt{x}} = \sqrt{1+\frac{2}{x}}$$

$$\underset{+\infty}{=} 1 + \frac{1}{x} - \frac{1}{4\times 2}\left(\frac{2}{x}\right)^2 + \frac{1}{6}\times\frac{3}{8}\times\left(\frac{2}{x}\right)^3 + o\left(\frac{1}{x^3}\right)$$

$$\underset{+\infty}{=} 1 + \frac{1}{x} - \frac{1}{2x^2} + \frac{1}{2x^3} + o\left(\frac{1}{x^3}\right)$$

E→f

$2 \to \quad \ln\left(x+\sqrt{1+x^2}\right)-\ln(x)$

$$= \ln\left(\frac{x+\sqrt{1+x^2}}{x}\right)$$

$$= \ln\Big(1+\underbrace{\sqrt{1+\frac{1}{x^2}}}_{\to 1}\Big) \to \ln(2)$$

$$= \ln(2) + \ln\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{1}{x^2}}\right)$$

$$\underset{+\infty}{=} \ln(2) + \ln\Big(1+\underbrace{\frac{1}{4x^2}-\frac{1}{8x^4}}_{x}+o\Big(\frac{1}{x^4}\Big)\Big)$$

$\to \frac{1}{2} \times \left(\frac{1}{8(x^4)}\right)^{2}$

$$\underset{+\infty}{=} \ln(2) + \frac{1}{4x^2} - \frac{1}{8x^4} - \frac{1}{32x^4} + o\left(\frac{1}{x^4}\right)$$

$$\underset{+\infty}{=} \ln(2) + \frac{1}{4x^2} - \frac{5}{32x^4} + o\left(\frac{1}{x^4}\right)$$

Ex 8

1) $\dfrac{x^2 - \sin(x^2)}{x^6} \underset{0}{\sim}$

$\sin(x^2) \underset{0}{=} x^2 - \dfrac{x^6}{6} + o(x^6)$

$x^2 - \sin(x^2) \underset{0}{=} \dfrac{x^6}{6} + o(x^6)$

Donc $\dfrac{x^2 - \sin(x^2)}{x^6} = \dfrac{1}{6} + o(1)$

2) $\dfrac{1+\ln(1+x)-e^{x}}{1-\cos(x)}$ en 0

$1-\cos(x) \underset{0}{\sim} \dfrac{x^2}{2}$

on s'arrête ou on a la diff le reste on enlève

$e^{x} \underset{0}{=} 1+x+\boxed{\dfrac{x^2}{2}}+\dfrac{x^3}{6}+o(x^3)$

$1+\ln(1+x) \underset{0}{=} 1+x\boxed{-\dfrac{x^2}{2}}+o(x^2)$

$1+\ln(1+x)-e^{x}$

$\underset{0}{=} 1+x-\dfrac{x^2}{2}-\left(1+x+\dfrac{x^2}{2}\right)+o(x^2)$

$\underset{0}{=} -x^2+o(x^2)$

$\underset{0}{\sim} -x^2 \Rightarrow \dfrac{1+\ln(1+x)-e^{x}}{1-\cos(x)} \underset{0}{\sim} \dfrac{2}{x^2}(-x^2)$

$= -2$

$$3y \quad \left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}}$$

$$= \exp\left(\frac{1}{x}\ln\left(\frac{a^x + b^x}{2}\right)\right)$$

$$\boxed{\begin{aligned} &a, b \in \mathbb{R}_+^* \\ &\text{Si } a = b \\ &a^x = b^x \\ &\left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}} = a \end{aligned}}$$

$$\frac{x\ln(a) + x\ln(b)}{2}$$

$$\ln\left(\frac{a^x + b^x}{2}\right) = \ln\left(1 + \boxed{\frac{(a^x - 1) + (b^x - 1)}{2}}\right)$$

$$\underset{0}{\sim} \frac{(a^x - 1) + (b^x - 1)}{2}$$

$$a^x - 1 = \exp(x\ln(a)) - 1$$

$$\underset{0}{=} x\ln(a) + o(x)$$

Si $a \neq \frac{1}{b}$, $\ln\left(\frac{a^x + b^x}{2}\right) = x\left(\frac{\ln a + \ln b}{2}\right) + o(x)$

Donc $\frac{1}{x}\ln\left(\frac{a^x + b^x}{2}\right) \to \frac{\ln(a) + \ln(b)}{2}$ donc

$$\left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}} \longrightarrow \exp\left(\frac{\ln(a) + \ln(b)}{2}\right)$$

$A \succ \dfrac{2x}{\ln\left(\dfrac{1+x}{1-x}\right)}$ en 0 $\underset{0}{\sim}$ 1

$$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$$

$$\ln(1+x) \underset{0}{=} \boxed{x} + o(x)$$

diff

$$\ln(1-x) \underset{0}{=} \boxed{-x} + o(x)$$

$x-(-x)$
$=2x$

$$\ln\left(\frac{1+x}{1-x}\right) \underset{0}{=} 2x + o(x)$$

Donc $\dfrac{2x}{\ln\left(\dfrac{1+x}{1-x}\right)} \underset{0}{=} \dfrac{2x}{2x} \underset{0}{=} 1$

Donc $\displaystyle\lim_{x \to 0} \frac{2x}{\ln\left(\frac{1+x}{1-x}\right)} = 1$

Ex5

$$\frac{1}{x} = \frac{1}{2} + \alpha_1 (x-2) + \alpha_2 (x-2)^2 + \alpha_3 (x-2)^3 + o((x-2)^3)$$

$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{1+\frac{(x-2)}{2}}\right)$$

$$= \frac{1}{2} - \frac{1}{2^2}(x-2) + \frac{(x-2)^2}{4}$$

$$- \frac{(x-2)^3}{8} + o((x-2)^3)$$

$$\frac{1}{x} = \frac{1}{2+(x-2)} = \frac{1}{2}\left(\frac{1}{1+\frac{x-2}{2}}\right)$$