

$$1021 \times 1201$$

$$a_1, b_1$$

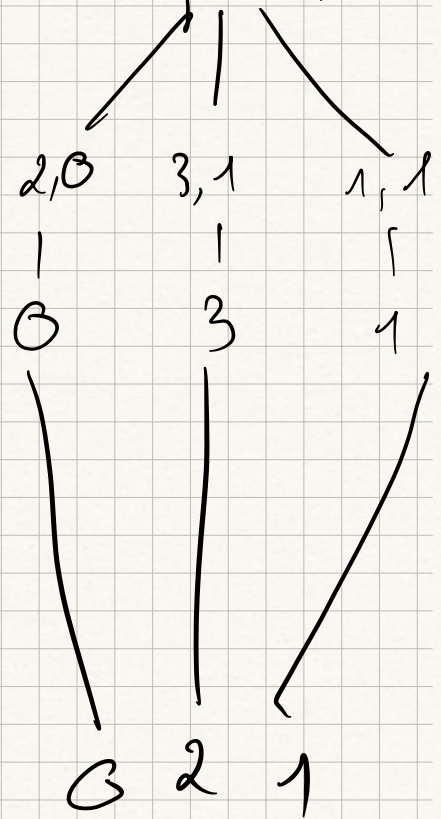
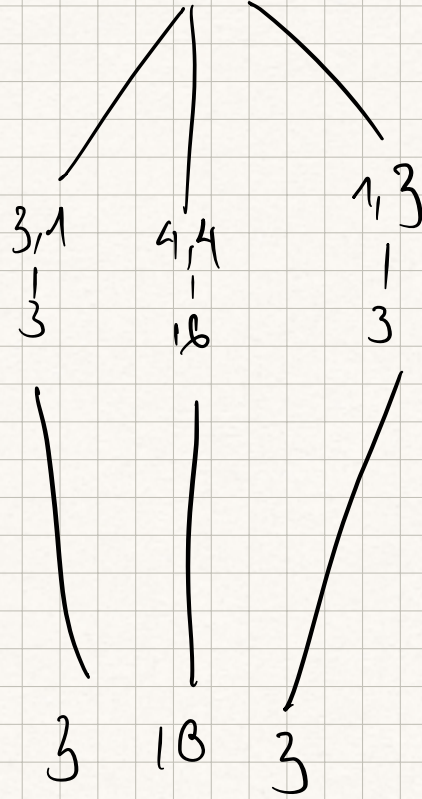
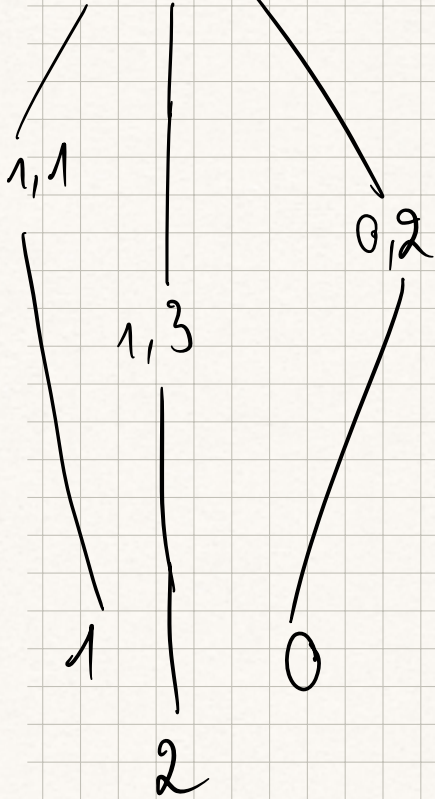
$$a_1 + a_2, b_1 + b_2$$

$$a_2, b_2$$

$$K(10, 12)$$

$$K(31, 13)$$

$$K(21, 01)$$



$$1 \times 100 + 2 \times 10 + 0 \times 1 = 120$$

$$3 \times 100 + 10 \times 10 + 3 \times 1 = 403$$

$$0 \times 100 + 2 \times 10 + 1 \times 1 = 21$$

$$120 \cdot 10000 + (403 - 120 - 21) \cdot 100 + 21$$

$$= 120 \cdot 10000 + 262 \cdot 100 + 21$$

$$= 1226221$$

Ex 2

1 $f(n) = 3(n^2 - 1)^2$	$\in \Theta(n^4)$	$g(n) = 4n^3 + 5n$	$\in \Theta(n^3)$
2 $f(n) = \log(3(n^2 - 1)^2)$	$\in \Theta(\log n)$	$g(n) = \log(4n^3 + 5n)$	$\in \Theta(\log n)$
3 $f(n) = \log(4n^3)$	$\in \Theta(\log n)$	$g(n) = 4(\log n)^3$	$\in \Theta((\log n)^3)$
4 $f(n) = 4n^3$	polynôme	$g(n) = 3^{(4n)}$	exponentielle
5 $f(n) = 3^{(n+4)}$	$= 81 \cdot 3^n$	$g(n) = 3^{(4n)}$	$= 81^n$
6 $f(n) = 3^{(n+4)}$	$= 81 \cdot 3^n$	$g(n) = 3^n$	
7 $f(n) = \log(n^4)$	$= 4 \log n$	$g(n) = \sqrt[3]{n}$	$= n^{1/3}$
8 $f(n) = \log(4^n)$	$= 2n$	$g(n) = \sqrt[3]{n}$	$= n^{1/3}$

1 $f \in \overset{>}{\Omega}(g)$

✓

2 ✓

3 ✗

4 ✗

5 ✗

6 ✓

7 ✗

8 ✓

$f \in \overset{<}{O}(g)$

✗

✓

✓

✓

✓

✓

✓

✗

$f \in \overset{=}{\Theta}(g)$

✗

✓

✗

✗

✗

✓

✗

✗

Ex 3

def foo-1(T):

return 0 if len(T) == 0 else foo-1(T[3:]) + 3

$$A_1(n) = \begin{cases} 0 & \text{si } n = 0 \\ 1 & \text{si } n = 1 \text{ ou } 2 \\ A_2(n-3) + 1 & \text{si } n > 2 \\ + 1_{n \geq 3} \end{cases}$$

$$\Rightarrow A_1(n) = a A\left(\frac{n}{b}\right) + O(n^c)$$

$$a = 1$$

$$b = 1$$

$$c = 0$$

$$\log_3(1) = 0$$

$$\Leftrightarrow (3)^0 = 1$$

Master Théorème :

$$A(n) = a A\left(\frac{n}{b}\right) + O(n^c)$$

$$A(n) = \Theta(n^c) \text{ si } c > \log_b a$$

$$\Theta(n^{\log_b(a)} \log n) \text{ si } c = \log_b a$$

$$\Theta(n^{\log_b(a)}) \text{ si } c < \log_b(a)$$

$$A_1(n) = \left[\frac{n}{3} \right] \in \Theta(n) ?$$

def foo 2 (T)

return 0 if $\ln(T) = \varepsilon$ else

sum(e for e in T) + foo 2 (T[2:])

$$A_2(n) = \begin{cases} 0 & \text{si } n = 0 \\ 1 & \text{si } n = 1 \\ A_2(n-2) + n & \text{si } n > 0 \end{cases}$$

$$A(n) = aA\left(\frac{n}{b}\right) + \Theta(n^c)$$

$$a = 1$$

$$b = 2$$

$$c = 1$$

$$\log_{(a)}(1) = 0$$

$$A_1(n) \in \Theta(n^2)$$

def foo3(T)

return 0 if len(T) == 0 else

foo3(T[:len(T)//2]) + 1

$$A_3(n) = \begin{cases} 0 & \text{if } n = 0 \\ A_3\left(\frac{n}{2}\right) + 1 & \text{? 2} \end{cases}$$

$$a = 1$$

$$b = 2$$

$$c = 0$$

$$\log_2(1) = 0$$

$$\Theta(n^{\log_b(a)} \log n)$$

$$A_3 \in \Theta(\log n)$$

$$\Theta(n^{\log_2(1)} \log(n))$$

$$\begin{aligned} \Theta &= (n^0 \log(n)) \\ &= \log(n) \end{aligned}$$

Ex 4

def F(n):

return 1 if $n < 4$ else $2 * F(n-1) + F(n-4)$

$$C(n) = \begin{cases} 0 & \text{if } n < 4 \\ C(n-1) + C(n-4) + 2 \end{cases}$$