Ex 1 :

$|a^3cbbca|_a = 4$

$|a^3cbbca|_b = 2$

$|Aiti|_{a,b} = 0$

$|aabjjdd|_a = 2$

$|aabjjdd|_b = 1$

2) $uv = abaac$

$L = \{ \varepsilon, (a, baac), (ab, aac), (aba, ac), (abaa, c),$
$(\varepsilon, abaac), (abaac, \varepsilon) \}$

3) $v = |ababab|_{aba} = 2$ ← nombre d'occurrences du facteur aba

$w = aba$

$|ababab|_{|w|} = 4$

aba

aba

ab a

aba

Ex2

1) $\alpha = \{a, ab, bb\}$ et $M = \{\varepsilon, b, a^2\}$

$\alpha M = \{a, ab, a^3, ab, abb, a^3b, bb, bbb, a^2bb\}$

2) $\alpha = \emptyset$ et $M = \{a, ba, bb\}$

$\alpha M = \emptyset$

3) $\alpha = \{\varepsilon\}$ et $M = \{a, ba, bb\}$

$\alpha M = \{a, ba, bb\} = M$

4) $\alpha = \{aa, ab, ba\}$ et $M = \{a, b\}^*$ ← infini

$\alpha M = \{aa(a+b)^*, ab(a+b)^*, ba(a+b)^*\}$

2.2)

$\alpha(M \cup N) = (\alpha M) \cup (\alpha N)$

$w \in \alpha(M \cup N)$

$w = u.v$ $\quad u \in \alpha \quad v \in M \cup N$

2e cas possible : On sait que :

$u \in L \qquad v \in M \cup N$

$\Rightarrow u \in M$

$\Rightarrow w = \underset{\to M}{u} \times \underset{\to L}{v}$

$\Rightarrow w = LM$

2) $u \in N \qquad w \in LM$

$\Rightarrow w \in (LM) \cup (LN)$

exemple :

$L = \{a\} \qquad M = \{b\} \qquad N = \{c, \varepsilon\}$

$w = ab \qquad w = ac \qquad w = a$

2ème partie :

$L(M \cap N) \neq LM \cap LN$

exemple

$L = \{a, \varepsilon\} \qquad LM = \{aa, a\}$

$M = \{a\} \qquad LN = \{aaa, aa\}$

$N = \{aa\}$

$M \cap N = \emptyset$

3)

a) $M^* = M^* . M^*$ ✓

b) $M^* = (M.M)^*$ ✗

$M = \{a\}$ $\quad M^* = \{\varepsilon, a, aa, aaa, \dots\}$ $\quad a \in \{M\}$

$a \notin \{M^*\}$

$MM = \{aa\}$

$(MM)^* = \{\varepsilon, aa, aaa, aaaa, \dots\}$

c) $M^* = M.M^*$ ✗

$M = \{a\}$ $\quad M^* = \{\varepsilon, a, aa, \dots\}$

$M.M^* = \{a, aa, aaa, \dots\}$

$a \in M^*$

$a \notin M.M^*$

d) $M^* = (M^*)^*$ ✓

e) $M.(N.M)^* = (M.N)^* . N$

$M = \{a\}$ $\quad N.M = \{ba\}$

$N = \{b\}$ $\quad NM^* = \{\varepsilon, ba, baba, \dots\}$

$N.(NM)^* = \{bba, bbaba, bbababa, \dots\}$

$M.N = ab$

$(MN)^* = \{\varepsilon, ab, abab, \ldots\}$

$(MN)^*N = \{b, abb, ababb, \ldots\}$

c)

$M^* = M^* . M^*$ ✗

$M = \{a\}$

$M^* = \{\varepsilon, a, aa, aaa, \ldots\}$

$M^* M^* = \{\varepsilon, a, aa, aaa, \ldots$

f) $(M \cup N)^* = M^* \cup N^*$ ✓ $\quad M^* = \{\varepsilon, a, aa, \ldots\}$

$M = \{a\} \qquad M \cup N = \{a, b\} \qquad N^* = \{\varepsilon, b, bb, \ldots\}$

$N = \{b\} \qquad (M \cup N)^* = \{\varepsilon, a, b, aa, ab, bb, \ldots\}$

$M^* \cup N^* = \{\varepsilon, ab, a, b, \ldots\}$

d) $M^* = (M^*)^*$ ✓

$M = \{a\} \qquad (M^*)^* = \{\varepsilon, a, aaa, \ldots\}^*$

$M^* = \{\varepsilon, a, aa, \ldots\} \qquad = \{\varepsilon, a, aa, aaa, \ldots\}$

gx $(M \cap N)^* = M^* \cap N^*$

$M = \{a\}$ $N = \{aa\}$ $M^* = \{\varepsilon, a, aa, \dots\}$

$N^* = \{\varepsilon, aa, aaa \dots\}$

$M \cap N = \emptyset$

$(M \cap N)^* = \emptyset$ $M^* \cap N^* = \{\varepsilon, aa, aaa, \dots\}$

hx $(M \cup N)^* = (M^* \cdot N^*)$

$N^* = \{\varepsilon, b, bbb, \dots\}$

$M = \{a\}$ $M \cup N = \{a, b\}$

$N = \{b\}$ $M^* = \{\varepsilon, a, aa, \dots\}$

$(M \cup N)^* = \{\varepsilon, a, b, aa, bb, ab\}$

$M^* \cdot N^* = \{\varepsilon, a, b, ab, aa, bb \dots\}$

$(M^* \cdot N^*)^* = \{\varepsilon, a, b, aba, aaa, baa, \dots\}$

i) $(M \cup N)^* = (M^* . N)^* . M^*$

$M = \{a\}$ $\quad M \cup N = \{a, b\}$ $\quad M^* = \{\varepsilon, a, aa, \dots\}$

$N = \{b\}$ $\quad (M \cup N)^* = \{\varepsilon, a, b, aa, bb, ab, abb, \dots\}$

$M^* . N = \{b, a, aab, aaab, \dots\}$

$(M^* . N)^* = \{\varepsilon, b, a, aab, aaab, aaaabb, \dots\}$

$(M^* . N)^* . M^* = \{\varepsilon, ba, baa, baaa, abba \dots\}$

Faux

Ex 3

1) $\mathcal{L}\{b^* a b^*\}$

2) $\mathcal{L}\{b^* a b^* a b^*\} = \{babab, baa, \dots\}$

3) $\mathcal{L}\{(a+b)^* a . (a+b)^* a . (a+b)^*\}$

4) $\mathcal{L}\{[(a+b)^* a . (a+b)^* a . (a+b)^*] +$
$[(a+b)^* b . (a+b)^* b . (a+b)^*]\}$

5> $\mathcal{L}\ \{b^* a b^* a b^*\}^*$

6> $\mathcal{L}\ \{(a+b)^* aa (b+a)^*\}$

7> $\mathcal{L}\ \{(b^* a b)^* + (b a b^*)^*\}$

8> $\mathcal{L}\ \{b^* a^*\}$

9> $\mathcal{L}\ \{b^* a^* b^*\}$