Lemme d'Arden

$\mathcal{L} = A \times L \cup B$ admet pour solution $\mathcal{L} = A^* B$

$A = (\Sigma, Q, q_i, F, \delta)$

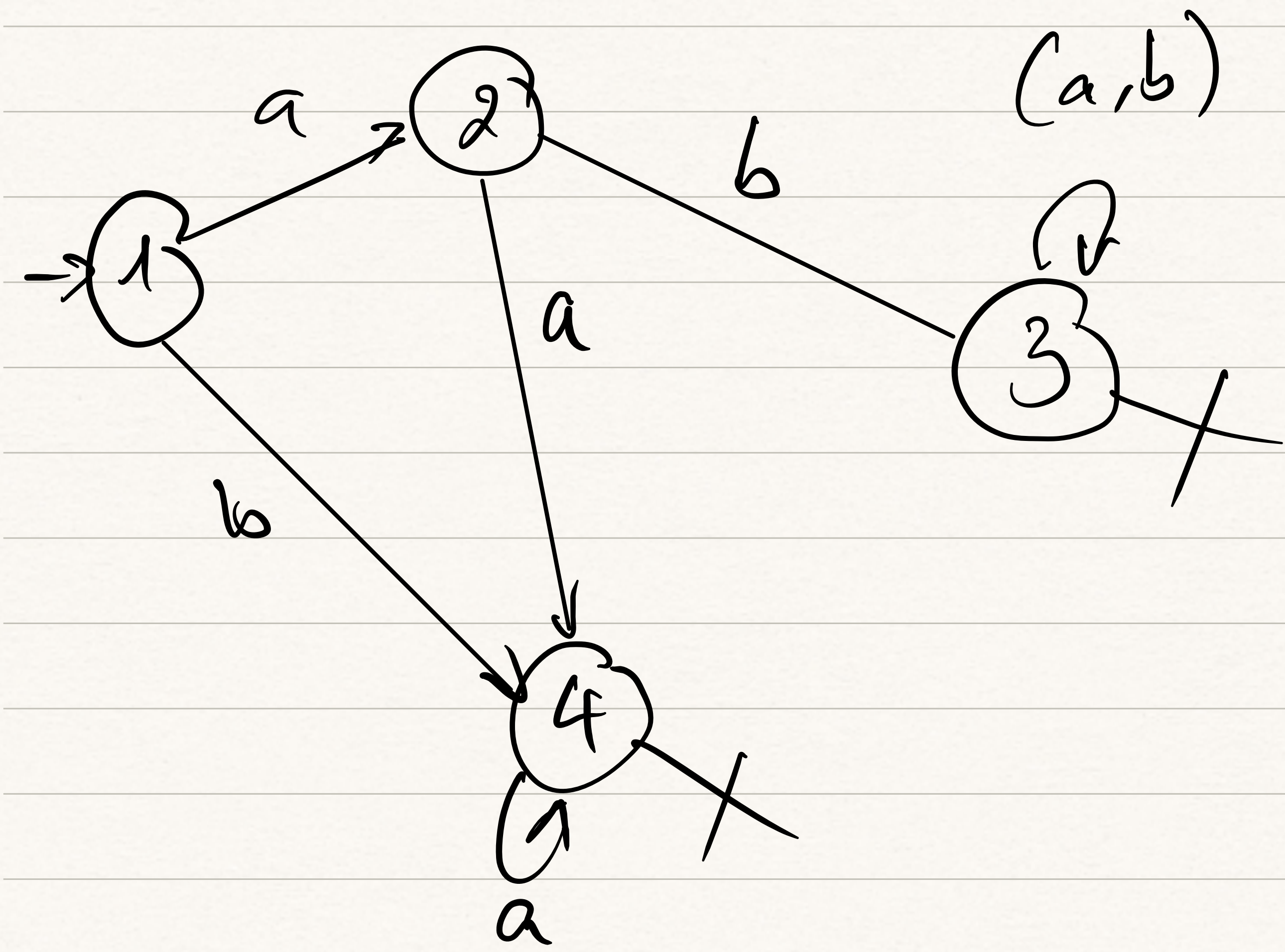
Pour chaque état $q \in Q$, on définit

Si $q \notin$ Final, $\mathcal{L}_q = \sum_{a \in \Sigma} a L_{\delta(q,a)}$

Si $q \in$ Final, $\mathcal{L}_q = \sum_{a \in \Sigma} a L_{\delta(q,a)} + \varepsilon$

Ex1 :

$$F \begin{cases} L_1 = aL_2 + bL_4 \\ L_2 = aL_4 + bL_3 \\ L_3 = (a+b)L_3 + \varepsilon \\ L_4 = aL_4 + \varepsilon \end{cases}$$



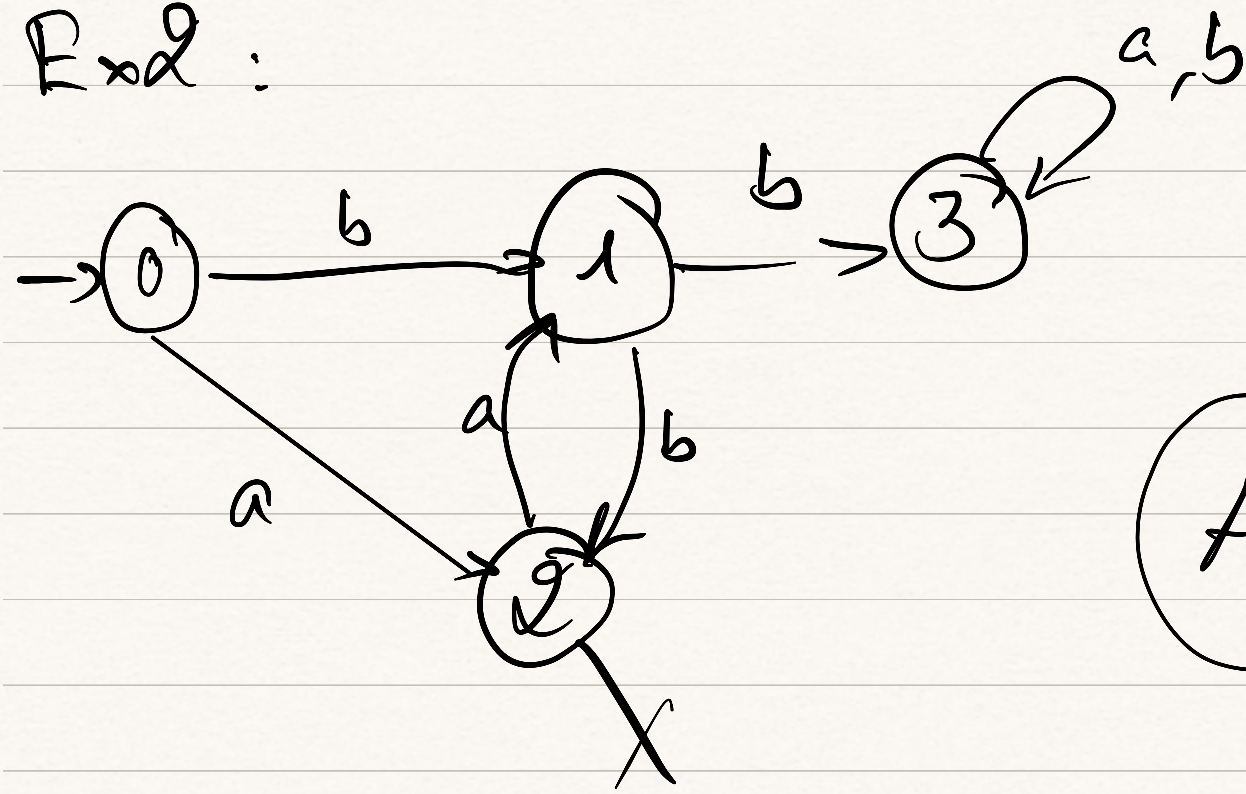
$$E \Leftrightarrow \begin{cases} L_1 = aL_2 + bL_4 \\ L_2 = aL_4 + bL_3 \\ L_3 = (a+b)^* \\ L_4 = a^* \end{cases}$$

$L = A.L + B + \varepsilon$

$L = A^* B$

$$\Leftrightarrow \begin{cases} L_1 = a\left(aa^* + b(a+b)^*\right) + ba^* \\ L_2 = aa^* + b(a+b)^* \\ L_3 = (a+b)^* \\ L_4 = a^* \end{cases}$$

Ex2 :



A_1

$$\begin{cases} L_0 = aL_2 + bL_1 \\ L_1 = \quad + bL_3 + bL_2 \\ L_2 = aL_1 + \varepsilon \\ L_3 = \cancel{(a+b)L_3 +} \emptyset \end{cases}$$

vu que L_3 est pas final peut pas aller d'ailleur

$$L_0 = aL_2 + bL_1$$

$$\begin{cases} L_1 = bL_3 + bL_2 \\ L_2 = aL_1 + \varepsilon \\ L_3 = \emptyset \end{cases}$$

concatene

$$\begin{cases} L_0 = aL_2 + bL_1 \\ L_1 = bL_2 \\ L_2 = a(bL_2) + \varepsilon \\ L_3 = \emptyset \end{cases}$$

$b \times \emptyset$
$= \emptyset$

$L_1 = bL_2$

$L_2 = aL_1 + \varepsilon$

$a(bL_2)$

$a\,b\,a$

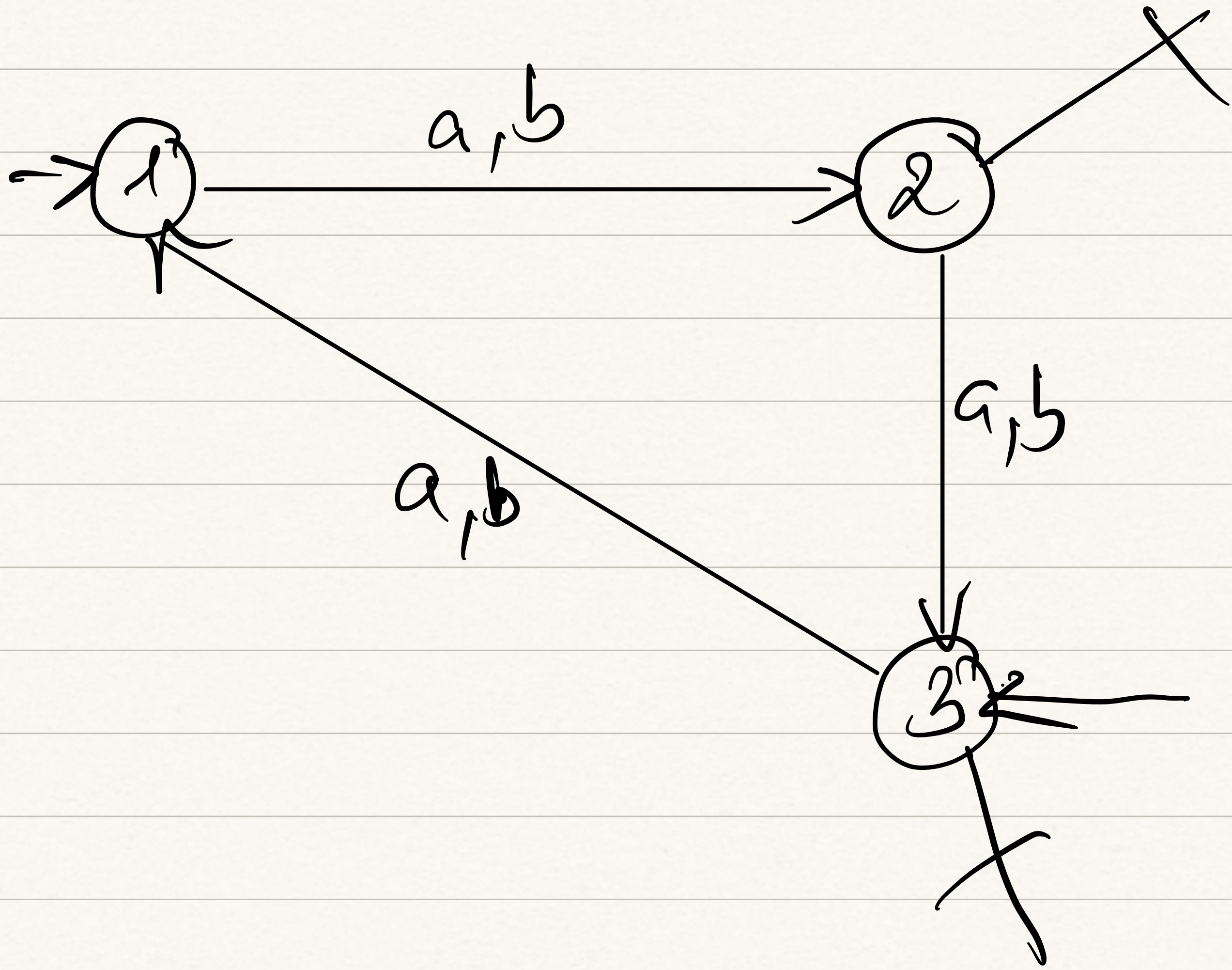
$a^* b$

$$\begin{cases} L_0 = aL_2 + bL_1 \\ L_1 = bL_2 \\ L_2 = (ab)^* \\ L_3 = \emptyset \end{cases}$$

$$\begin{cases} L_0 = aL_2 + bL_1 \\ L_1 = b(ab)^* \\ L_2 = (ab)^* \\ L_3 = \emptyset \end{cases}$$

$$\begin{cases} L_0 = a(ab)^* + bb(ab)^* \\ L_1 = b(ab)^* \\ L_2 = (ab)^* \\ L_3 = \emptyset \end{cases}$$

A_2



$$
\begin{cases}
L_1 = (a+b)L_2 \\
L_2 = (a+b)L_3 + \varepsilon \\
L_3 = (a+b)L_1 + \varepsilon
\end{cases}
$$

L_2

$$\begin{cases} L_1 = (a+b)(a+b)L_3 + (a+b) \\ L_2 = (a+b)\overbrace{(a+b)(a+b)(a+b)L_3 + (a+b)}^{L_3} + \varepsilon \\ L_3 = (a+b)\overbrace{(a+b)\left[(a+b)L_3 + \varepsilon\right]}^{L_2} + \varepsilon \end{cases}$$

$$\begin{cases} L_1 = (a+b)L_2 \\ L_2 = (a+b)\left\{\left[(a+b)^3\right]^*\left[(a+b)^2 + \varepsilon\right]\right\} \\ L_3 = (a+b)^3 L_3 + (a+b)^2 + \varepsilon \end{cases}$$

$$= \left((a+b)^3\right)^*\left[(a+b)^2 + \varepsilon\right]$$

$$L(A_2) = L_1 + L_3 = (a+b)^*$$