

```

def F(n):
    if n < 7: return n
    return 3 * F(n-1) + 4 * F(n-5) + F(n-7)

```

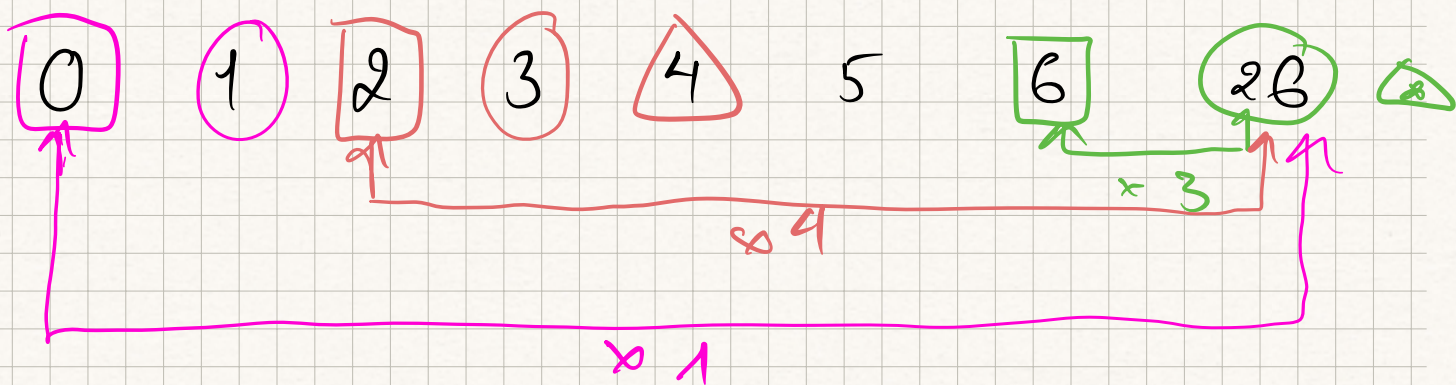
$$C(n) = \underbrace{C(n-1)}_{\geq C(n-7)} + \underbrace{C(n-5)}_{\geq C(n-7)} + \underbrace{C(n-7)}_{> 0} + \underbrace{4}_{> 0}$$

$$C(n) \geq 3 C(n-7)$$

$$C(n) \geq 3 C(n-7) \geq 3^2 C(n-2 \times 7) \dots \geq 3^k C(n-7k)$$

done $C(n) \in \Omega(3^{n/7})$

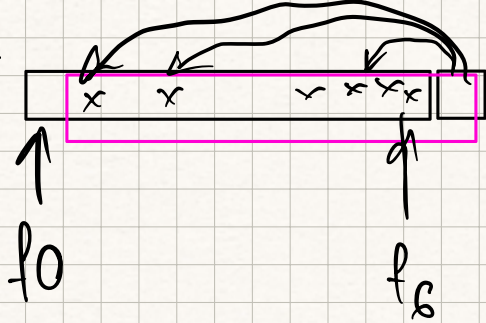
si $k + 7n - 7k \in [7, 13]$
 ie $n = k + i$
 ie $k = \frac{n}{7}$
 (\approx pour près)



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def Fd(n):
    if n < 7: return n
    f0, f1, ..., f6 = 0, 1, 2, 2, ..., 6
    for i in range(7, n+1):
        f1, f2, ..., f6, 3*f6 + 5*f5 + f0

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$$temp = 3 \times f_6 + 5 \times f_2 + f_0$$

$$\rightarrow \begin{array}{l|l} f_0 = f_1 & f_5 = f_6 \\ f_1 = f_2 & f = temp \end{array}$$

$$3 \times f_6 + 4 \times f_2 + 1 \times f_0$$

$\underbrace{\quad}_{0,0,0} \quad \underbrace{\quad}_0$

$$= [3, 0, 0, 0, 4, 0, 1] \cdot \begin{bmatrix} f_6 \\ f_5 \\ \vdots \\ f_0 \end{bmatrix}$$

$$\begin{bmatrix} f_6 \\ f_5 \\ \vdots \\ f_0 \end{bmatrix} = \begin{array}{ccccccc} 3 & 0 & 0 & 0 & 4 & 0 & 1 \\ \times & 0 & & & & & 0 \\ 0 & \times & 0 & & & & 0 \\ | & & & & & & | \\ 0 & & & & 0 & 1 & 0 \end{array}$$

$$= \begin{bmatrix} f_6 \\ \boxed{f_6} \end{bmatrix} \rightarrow f_5$$

$$\begin{bmatrix} F(n) \\ F(n-1) \\ \vdots \\ F(n-b) \end{bmatrix} = M^{n-b} \begin{bmatrix} F(b) \\ \vdots \\ F(0) \end{bmatrix}$$

→ peu d'opération de produits

$\Theta(n)$ produits (de matrices) donc $\Theta(n)$ op. arith sur des très gros entiers

→ $\Theta(n^3)$?
 $\Theta(n^2)$?

Exponentiation binaire

→ $\Theta(\log n)$ produits (de matrices) donc $\Theta(\log n)$

→ $\begin{bmatrix} O(n^2 \log n) \\ \Theta(n^2) \text{ si produit naïf} \end{bmatrix}$

→ $\begin{bmatrix} \Theta(n^{\log_3 2}) \text{ si produit Karatsuba} \\ O(n^{\log_3 2} \log n) \end{bmatrix}$

Correction

$$F(n) = \begin{cases} n & \text{si } n < 4 \\ 4F(n-1) + 6F(n-2) + 2F(n-4) \end{cases}$$

A l'étape $n-1$:

$$a, b, c, d = 4 \times a + 6 \times b + 2d, a, b, c$$

The diagram illustrates the state of variables a, b, c, d at step $n-1$. The expression $a, b, c, d = 4 \times a + 6 \times b + 2d, a, b, c$ shows the new values assigned to the variables. Arrows indicate the mapping from previous function values to the new variable values: $F(n-1)$ maps to a , $F(n-2)$ maps to b , $F(n-3)$ maps to c , and $F(n-4)$ maps to d .

$$V_{n-1} = \begin{pmatrix} F(n-1) \\ F(n-2) \\ F(n-3) \\ F(n-4) \end{pmatrix}$$

$$V_n = \begin{pmatrix} F(n) \\ F(n-1) \\ F(n-2) \\ F(n-3) \end{pmatrix}$$

$$V_n = \begin{pmatrix} 4f(n-1) + 6f(n-2) + 2(n-4) \\ f(n-1) \\ f(n-2) \\ f(n-3) \end{pmatrix}$$

M to $V_n = M V_{n-1}$

$$\begin{pmatrix} f(n) \\ f(n-1) \\ f(n-2) \\ f(n-3) \end{pmatrix} = \begin{pmatrix} 4 & 6 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f(n-1) \\ f(n-2) \\ f(n-3) \\ f(n-4) \end{pmatrix}$$

$$V_3 = \begin{pmatrix} f(3) \\ f(2) \\ f(1) \\ f(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} V_n &= M \cdot V_{n-1} \\ &= M \cdot M V_{n-2} \\ &= M^2 \cdot V_{n-2} = M^2 \cdot M V_{n-3} \\ &\vdots \end{aligned}$$

$$= n^{n-3} \cdot V_3$$

Inégalité :

$$C(n) = C(n-1) + (n-2) + C(n-4) + 5$$

$$\geq C(n-4) + C(n-4) + C(n-4) + 0$$

$$= 3C(n-4)$$