$$\exp(x) \underset{0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + o(x^3)$$

$$\sin(x) \underset{0}{=} x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$\ln(1+x) \underset{0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$(1+x)^\alpha \underset{0}{=} 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} + \frac{\alpha(\alpha-1)(\alpha-2)}{6} x^2 + o(x^2)$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \underset{0}{=} 1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$$\tan = \frac{\sin}{\cos} = x + \frac{x^3}{3} + \frac{2x^5}{15} + o(x^6)$$

$$\sinh \doteq x + \frac{x^3}{3!} + \frac{x^5}{5!} + O(x^5)$$

$$1y \lim_{x \to +\infty} x\left(\sqrt{x^2+x+2} - \sqrt{x^2+x+2}\right)$$

$$\sqrt{x^2+x+2} - \sqrt{x^2+x-2}$$

$$= \sqrt{x^2+x+2}\left(1 - \sqrt{\frac{x^2+x-2}{x^2+x+2}}\right)$$

$$= \sqrt{x^2+x+2}\left(1 - \sqrt{\frac{x^2+x+2-4}{x^2+x+2}}\right)$$

$$= \sqrt{x^2+x+2}\left(1 - \sqrt{1 - \frac{4}{x^2+x+2}}\right)$$

$$1 - \sqrt{1 - \frac{4}{x^2+x+2}}$$

$$= 1 - \left(1 - \frac{4}{x^2+x+2}\right)^{\frac{1}{2}}$$

On a :

$$\frac{4}{x^2+x+2} \xrightarrow{+\infty} 0$$

Or $1-(1-y)^{\frac{1}{2}} \underset{0}{\sim} \frac{1}{2}y$

donc $1-\left(1-\dfrac{4}{x^2+x+2}\right)^{\frac{1}{2}}$

$$\underset{+\infty}{\sim} \frac{2}{x}$$

En tenant compte de

$$\overbrace{x}^{x}\,\overbrace{\sqrt{x^2+x+2}}^{x} \underset{+\infty}{\sim} x^2 \ (= x.x)$$

Donc $f(x) \underset{+\infty}{\sim} x^2 \times \dfrac{2}{x} = 2x$

2) $\lim\limits_{x \to -1} \dfrac{x^3 - 3x - 2}{x^2 - 2x - 3}$

$x - 2$

$(x + 1)$

$$
\begin{array}{r|l}
x^3 - 3x - 2 & x + 1 \\ \hline
x^2 + x^2 & x^2 - x - 2 \\ \hline
-x^2 - 3x - 2 & \\
-x^2 - x & \\ \hline
-2x - 2 & \\
-2x - 2 & \\ \hline
0 &
\end{array}
\qquad
\begin{array}{r|l}
x^2 - 2x - 3 & x + 1 \\ \hline
x^2 + x & x - 3 \\ \hline
-3x - 3 & \\
-3x - 3 & \\ \hline
0 &
\end{array}
$$

$$x^3 - 3x - 2 = (x+1)(x^2 - 2x - 3)$$

$$x^2 - 2x - 3 = (x+1)(x-3)$$

$$f(x) = \frac{x^3 - 3x - 2}{x^2 - 2x - 3} = \frac{(x+1)(x^2 - 2x - 3)}{(x+1)(x-3)}$$

$$= \frac{x^2 - 2x - 3}{x - 3}$$

$$= \frac{(x+1)(x-2)}{(x-3)}$$

$$\underset{-1}{\sim} \frac{3(x+1)}{4}$$

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} (x+1)\frac{3}{4} = 0$$

3) $$\lim_{x\to 1} \frac{x^2+6}{x^2-4}$$

$$\lim_{x\to 1} \frac{x^2+6}{x^2-4} = \frac{7}{-3} \underset{1}{\sim} \frac{x^2+6}{x^2-4}$$

4) $$\lim_{x\to +\infty} \left(x+2^x\right)^{\frac{1}{2}}$$

On a $x+2^x \underset{+\infty}{\sim} 2^x$

car $\lim_{x\to+\infty} \frac{x}{2^x} = 0$

$$\left(x+2^x\right)^{\frac{1}{2}} = \sqrt{x+2^x}$$

$$= \sqrt{2^x}\sqrt{\underbrace{\frac{x}{2^x}}_{0}+1}$$

$$\lim_{x\to+\infty} \sqrt{\frac{x}{2^x}+1} = \sqrt{1} = 1$$

car $\lim_{x\to+\infty} \frac{x}{2^x} = 0$

$\frac{x}{2}\ln 2$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \sqrt{2^{x}} \cdot 1 = \lim\limits_{x\to+\infty} 2^{\frac{x}{2}} = e^{\dots}$ $\to \infty$

$x \to \infty$

$e^{\ln(x)} = \ln e^{x} = x$

5) $\lim\limits_{x\to 2} \dfrac{x^2-5x+6}{x^3-8}$

$$\begin{array}{r|l} x^2-5x+6 & x-2 \\ \hline x^2-2x & x-3 \\ \hline -3x+6 & \\ -3x+6 & \\ \hline 0 & \end{array} \qquad \begin{array}{r|l} x^3-8 & x-2 \\ \hline \cancel{x^2-2x^2} & x^2+2x+4 \\ \hline 2x^2-8 & \\ 2x^2-4x & \\ 4x-8 & \\ 4x-8 & \end{array}$$

$x^2-5x+6 = (x-2)(x-3)$

$x^3-8 = (x-2)(x^2+2x+4)$

$$\frac{x^2-5x+6}{x^3-8} = \frac{(x-2)(x-3)}{(x-2)(x^2+2x+4)} = \frac{x-3}{x^2+2x+4}$$

$\lim f(x) = \dfrac{-1}{\dots}$

$x \to 2 \quad \quad 2 \quad \quad 12$

b) $\lim\limits_{x \to 0} \frac{1}{x} \left(\sqrt{1 + x + x^2} - 1 \right)$

$$\sqrt{1 + x + x^2} - 1 \underset{0}{\sim} \frac{1}{2} \times x + x^2$$

$$\underset{0}{\sim} \frac{x + x^2}{2}$$

$$\frac{1}{x} \times \frac{x + x^2}{2} = \frac{1}{x} \times \frac{x(1 + x)}{2}$$

$$= \frac{1 + x}{2}$$

$$\underset{0}{\sim} \frac{1}{2}$$

f) $\lim\limits_{x \to +\infty} \frac{\sin(x)}{x}$

On a $|\sin(x)| \leq 1$

$\lim \frac{1}{} = 0$ donc $\lim \frac{\sin(x)}{} = 0$

$\lim\limits_{x \to +\infty} \frac{}{x}$ $\lim\limits_{x \to +\infty} \frac{}{x}$

8) $\lim\limits_{x \to +\infty} \frac{x^3 + e^x}{x^3 - 8}$

En $+\infty$ c'est e^x qui emporte sur les puissances de x donc

$$\lim_{x \to +\infty} \frac{x^3 + e^x}{3x^3 + 1} = \lim_{x \to +\infty} e^x \left(\frac{x^3}{e^x} + 1 \right) \frac{1}{3x^3 + 1}$$

$$= \lim_{x \to +\infty} \left(x^3 + e^x \right) \times \frac{1}{3x^3 + 1}$$

$$= \lim_{x \to +\infty} x^3 \left(1 + \frac{e^x}{x^3} \right) \times \frac{1}{3 + \frac{1}{x^3}}$$

Pas fini

$-1 - (-1) = 2$

$- 1 + 1 = 2$

9) $\lim_{x \to 1} \dfrac{x^3 - 2x^2 - x + 2}{x^3 - 3x^2 + 3x - 1}$

$$
\begin{array}{r|l}
x^3 - 2x^2 - x + 2 & x - 1 \\
\hline
x^3 - x^2 & x^2 - x - 2 \\
\hline
-x^2 - x + 2 & \\
-x^2 + x & \\
-2x + 2 & \\
-2x + 2 & \\
\hline
0 &
\end{array}
\qquad
\begin{array}{r|l}
x^2 - x - 2 & x + 1 \\
\hline
x^2 + x & x - 2 \\
-2x - 2 &
\end{array}
$$

$(x-1)(x+1)(x$

$$\frac{x^3 - 2x^2 - x + 2}{x^3 - 3x^2 + 3x - 1} = \frac{(x-1)(x+1)(x-2)}{(x-1)^3}$$

$$= \lim_{x \to 1} \frac{(x+1)(x-2)}{(x-1)^2} = \lim_{x \to 1} \frac{-2}{(x-1)^2} = -\infty$$

On vérifie la limite

$$\lim_{x \to 1} \frac{(x+1)(x-2)}{(x-1)^2} \cdot \frac{(x-1)^2}{-2} = \lim_{x \to 1} \frac{(x+1)(x-2)}{-2} = \frac{-2}{-2} = 1$$