```
def F(n):
    if n<7: return n
    return 3 × F(n-1) + 4 × F(n-5) + F(n-7)
```

$$C(n) = \underbrace{C(n-1)}_{\geqslant C(n-7)} + \underbrace{\underbrace{C(n-5)}_{\geqslant C(n-7)} + C(n-7)}_{>0} + \underbrace{4}_{>0}$$

$$\boxed{C(n) \geqslant 3\,C(n-7)}$$

$$C(n) \geqslant 3\,C(n-7) \geqslant 3^2\,C(n-2\times 7) \ldots \geqslant 3^k\,C(n-7k) \geqslant 3^{n/7}$$

si k tq $n-7k \in [7,13]$

ie $n = 7k + i$, $i \leqslant 7$

ie $k = \frac{n}{7}$

(à peu près)

donc $C(n) \in \Omega(3^{n/7})$

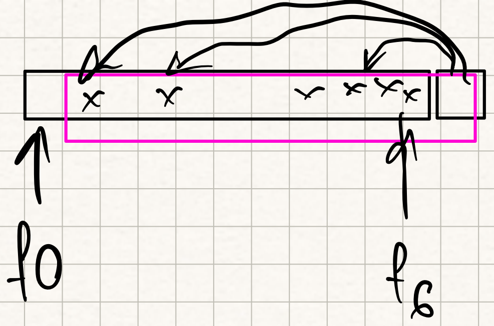
0 1 2 3 4 5 6 26

×3

×4

×1

```
def Fd(n):
    if n < 7: return n
    f0, f1, ..., f6 = 0, 1, 2, ..., 6
                                (6, n)
    for _ in range(7, n+1):
                      (0, n-6)
        | for f1 .... f6 = f1, f2, ..., f6, 3× f6
                                          + 5×f2 + f0
```



$$tmp = 3 * f_6 + 5 * f_2 + f_0$$

$$\rightsquigarrow \quad f_0 = f_1 \;\Big|\; f_5 = f_6$$
$$f_1 = f_2 \;\Big|\; f = tmp$$

$$3 * f_6 \underbrace{+}_{0,0,0} 4 * f_2 \underbrace{+}_{0} 1 * f_0$$

$$= [3, 0, 0, 0, 4, 0, 1] \cdot \begin{bmatrix} f_6 \\ f_5 \\ \vdots \\ f_0 \end{bmatrix}$$

$$\begin{bmatrix} f_6 \\ f_5 \\ \vdots \\ f_0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 & 4 & 0 & 1 \\ 1 & 0 & \cdots & & & & 0 \\ 0 & 1 & 0 & \cdots & & & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & \cdots & & & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} f_6 \\ f_5 \\ \vdots \\ f_0 \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{f_6} \\ \\ \\ \end{bmatrix} \rightarrow f_5$$

$$\begin{bmatrix} F(n) \\ F(n-1) \\ \vdots \\ F(n-6) \end{bmatrix} = M^{n-6} \begin{bmatrix} F(6) \\ \vdots \\ F(0) \end{bmatrix}$$

→ par itération de produits

$\Theta(n)$ produits (de matrices) donc $\Theta(n)$ op. arith sur des très gros entiers

→ $O(n^3)$?

$\Theta(n^2)$?

exponentiation binaire

→ $\Theta(\log n)$ produits (de matrices) donc $\Theta(\log n)$.

→ $O(n^2 \log n)$

$\Theta(n^2$ si produit naïf

→ $\Theta(n^{\log 3})$ si produit Karatsuba

$\Theta(n^{\log 3} \log n)$

Correction

$$f(n) = \begin{cases} n & \text{si } n < 4 \\ 4f(n-1) + 6f(n-2) + 2f(n-4) & \end{cases}$$

A l'étape $n-1$:

$$\underset{f(n-1)}{a}, \underset{f(n-2)}{b}, \underset{f(n-3)}{c}, \underset{f(n-4)}{d} \quad = \quad 4 \times a + 6 \times b + 2d, a, b, c$$

$$V_{n-1} = \begin{pmatrix} f(n-1) \\ f(n-2) \\ f(n-3) \\ f(n-4) \end{pmatrix}$$

$$V_n = \begin{pmatrix} f(n) \\ f(n-1) \\ f(n-2) \\ f(n-3) \end{pmatrix}$$

$$V_n = \begin{pmatrix} 4f(n-1) + 6f(n-2) + 2(n-4) \\ f(n-1) \\ f(n-2) \\ f(n-3) \end{pmatrix}$$

M tq $\quad U_n = M \; v_{n-1}$

$$\begin{pmatrix} f(n) \\ f(n-1) \\ f(n-2) \\ f(n-3) \end{pmatrix} = \begin{pmatrix} 4 & 6 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f(n-1) \\ f(n-2) \\ f(n-3) \\ f(n-4) \end{pmatrix}$$

$$V_3 = \begin{pmatrix} f(3) \\ f(2) \\ f(1) \\ f(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} V_n &= M \cdot V_{n-1} \\ &= M \cdot M v_{n-2} \\ &= M^2 \cdot U_{n-2} = M^2 \cdot M V_{n-3} \\ &\vdots \end{aligned}$$

$= M^{n-3} \cdot V_3$

$4f(n-1) + 6f(n-2) + 2(n-4)$

Inégalité :

$C(n) = C(n-1) + (n-2) + C(n-4)$

$+ 5$

$\geqslant C(n-4) + C(n-4)$

$+ C(n-4) + 0$

$= 3(n-4)$