Ex 1

$$f(x) = \sqrt[3]{x^3 + x^2 + 1} + \sqrt{x^2 - x + 1}$$

1) l'asymptote en $+\infty$

2) positions

$x \longrightarrow +\infty$ ssi $t = \frac{1}{x} \longrightarrow 0$

$$\sqrt[3]{x^3 + x^2 + 1} = \sqrt[3]{x^3\left(1 + \frac{1}{x} + \frac{1}{x^3}\right)}$$

$$= x\sqrt[3]{1 + \frac{1}{x} + \frac{1}{x^3}}$$

$$= x\left(1 + \frac{1}{x} + \frac{1}{x^3}\right)^{\frac{1}{3}}$$

$$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}x^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3 + \dots + o(x^n)$$

$$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + o(x^2)$$

$$= x\left(1 + \frac{1}{3}\left(\frac{1}{x} + \cancel{\frac{1}{x^3}}_{\color{red}o\left(\frac{1}{x^2}\right)}\right) - \frac{1}{9}\left(\frac{1}{x} + \cancel{\frac{1}{x^3}}\right)^2_{\color{red}o\left(\frac{1}{x^2}\right)} + o\left(\underbrace{\left(\frac{1}{x} + \frac{1}{x^3}\right)^2}\right)\right) \to \frac{1}{x^2}$$

$$\left(\frac{1}{x} + \frac{1}{x^3}\right)^2 = \frac{1}{x^2} + \frac{1}{x^6} + \frac{2}{x^4} \sim \frac{1}{x^2}$$

$$= x\left(1 + \frac{1}{3x} - \frac{1}{9}\cdot\frac{1}{x^2} + o\left(\frac{1}{x^2}\right)\right)$$

2) $f(x) = \sqrt[3]{x^3 + x^2 + 1} + \sqrt{x^2 - x + 1}$

$$\sqrt{x^2 - x + 1} = \sqrt{x^2\left(1 - \frac{1}{x} + \frac{1}{x^3}\right)}$$

$$= x\sqrt{1 \underbrace{- \frac{1}{x} + \frac{1}{x^2}}_{0}}$$

$$= x\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{2}}$$

$$= x\left(1+\frac{1}{2}\left(\frac{-1}{x}+\frac{1}{x^2}\right)-\frac{1}{3}\left(\frac{-1}{x}+\frac{1}{x^2}\right)^2\right.$$

$$\left.+\ o\left(\frac{1}{x^2}\right)\right)$$

$$(1+x)^{\frac{1}{2}} = (1+x)^{\frac{1}{2}}$$
$$= 1+\frac{1}{2}x+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}x^2+o(x^2)$$

$$= x\left(1-\frac{1}{2x}+\frac{1}{2x^2}-\frac{1}{8}\times\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$$

$$= x\left(1-\frac{1}{2x}+\frac{3}{8}\times\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)$$

$$= x-\frac{1}{2}+\frac{3}{8}\times\frac{1}{x}+o\left(\frac{1}{x}\right)$$

$$\sqrt[3]{x^3+x^2+1}\ - \quad = x+\frac{1}{3}-\frac{1}{9}\frac{1}{x}+o\left(\frac{1}{x}\right)$$

$$\sqrt[3]{x^3+x^2+1} + \sqrt{x^2-x+1}$$

$$= x + \frac{1}{3} - \frac{1}{9}\frac{1}{x} + o\left(\frac{1}{x}\right) + x - \frac{1}{2} + \frac{3}{8}\frac{1}{x}$$

$$f(x) = 2x - \frac{1}{6} + \frac{19}{72}\frac{1}{x} + o\left(\frac{1}{x}\right)$$

$y = 2x - \frac{1}{6}$ est l'asymptote de f en $+\infty$

$$f(x) - \left(2x - \frac{1}{6}\right) = \frac{19}{72} \times \frac{1}{x} + o\left(\frac{1}{x}\right)$$
$x \to +\infty$

La courbe est au dessus de l'asymptote

$$\cos(x) - 1 \underset{0}{\sim} -\frac{x^2}{2}$$

$$\lim_{0} \frac{\cos(x)-1}{x^2} = \lim_{0} \frac{\frac{-x^2}{2}}{x^2} = \frac{-1}{2}$$

$$e^x - 1 \sim x$$

$$\lim_{0} \frac{e^x - 1}{x} = 1$$

$$\lim_{0} \frac{x\cos(x) - \ln(1+x)}{x\sin(x)} \quad \text{DL2}$$

$$= \frac{x\left(1 - \frac{x^2}{2}\right) - \left(x - \frac{x^2}{2}\right) + o(x^2)}{x(x) + o(x^2)}$$

$$= \frac{x - x + \frac{x^2}{2} + o(x^2)}{x^2 + o(x^2)}$$

$$= \frac{\frac{x^2}{2} + o(x^2)}{x^2 + o(x^2)} \sim \frac{\frac{x^2}{2}}{x^2} = \frac{1}{2}$$

Rq : $f(x) = \dfrac{o(x^2)}{x^2 + o(x^2)} \sim \dfrac{o(x^2)}{x^2} \to 0$

Vecteur $\mathbb{R}^4$

$F : x + y - z - t = 0$

F sev de $\mathbb{R}^4$

Base de F : $x = -y + z + t$

$(x, y, z, t) = (-y + z + t, y, z, t)$

remplacer

$= y(-1, 1, 0, 0) + z(1, 0, 1, 0) + t(1, 0, 0, 1)$

décomposer

Base de F

$$\begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \dots \to \left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

G engendré par $\begin{cases} u_1 = (1, 0, 1, 0) \\ u_2 = (0, 1, 0, -1) \\ u_3 = (1, -1, 1, 1) \end{cases}$

Base de G ?

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{matrix} \xrightarrow{L_3 - L_1} \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{matrix} \xrightarrow{} \underset{L_3 + L_2}{} \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix}$$

(u_1, u_2) base de G