Ex 3 :

2) $\exp(\sin x)$ à l'ordre 4 en 0

$$\sin(x) \underset{x\to 0}{\rightsquigarrow} 0$$

$$e^{\sin(x)} = 1 + \left(x - \frac{x^3}{3!}\right) + \frac{\left(x - \frac{x^3}{3!}\right)^2}{2}$$

$$+ \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$$

$$= 1 + x - \frac{x^3}{6} + \frac{x^2}{2} - \frac{x^4}{6} + \frac{x^3}{6}$$

$$+ \frac{x^4}{24} + o(x^4)$$

$$= 1 + x + \frac{x^2}{2} - \frac{3}{24}x^4 + o(x^4)$$

$$\left(x - \frac{x^3}{3!}\right)^2 = x^2 - \frac{x^4}{3} + \frac{x^6}{36}$$

3) $e^{\cos(x)}$ ~~en~~ l'ordre 5 en 0

$$\exp(\cos(x)) = \exp\left(1 - \frac{x^2}{2} + \frac{x^4}{25} + o(x^5)\right)$$

$\exp(1) = e^1 = e$

$$= \exp(1) \times \exp\left(\frac{-x^2}{2} + \frac{x^4}{24} + o(x^5)\right)$$

$$= e \times \left[1 - \frac{x^2}{2} + \frac{x^4}{24} + \frac{1}{2}\left(\frac{-x^2}{2}\right)^2 + o(x^5)\right)$$

$$= e \times \left[1 - \frac{x^2}{2} + \left(\frac{1}{24} + \frac{1}{8}\right)x^4 + o(x^5)\right.$$

$$= e\left[1 - \frac{x^2}{2} + \frac{x^4}{8} + o(x^5)\right]$$

$DL\left((\cos(x))^{\sin(x)}\right)$ à l'ordre 5

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)$$

$$\cos(x) \underset{0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$

$\exp(\sin(x)\ln(\cos(x)))$

$= \cos(x)$

$$\ln(1+\cos(x)-1) = -\frac{x^2}{2} + \frac{x^4}{24} - \frac{1}{2}\left(\frac{-x^2}{2}\right)^2 + o(x^4)$$

$$\ln\Big(\overbrace{1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)}^{1+x}\Big) \underset{0}{=} \frac{-x^2}{2} + \frac{x^4}{24} - \frac{x^4}{8} + o(x^4)$$

$\sin(x)\ln(\cos(x))$

$$= \left(x - \frac{x^3}{6} + o(x^3)\right)\left(\frac{-x^2}{2} - \frac{x^4}{12} + o(x^4)\right)$$

$$= \frac{-x^3}{2} - \frac{x^5}{12} + \frac{x^5}{12} + o(x^5)$$

$$= \frac{-x^3}{2} + o(x^3)$$

$$\exp\left(\frac{-x^3}{2} + o(x^3)\right) = 1 - \frac{x^3}{2} + o(x^5)$$

5) $x \cosh(x)^{\frac{1}{x}}$ en 0 ordre 4

$$\cosh(x)^{\frac{1}{x}} = \exp\left(\frac{1}{x}\ln(\cosh(x))\right)$$

$$\ln(\cosh(x)) \underset{0}{=} \ln\left(1+\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)$$

$$\underset{0}{=} \frac{x^2}{2}+\frac{x^4}{24}-\frac{1}{2}\left(\frac{x^2}{2}\right)^2+o(x^5)$$

$$= \frac{x^2}{2}-\frac{1}{12}x^4+o(x^5)$$

$$= x\left(\frac{x}{2}-\frac{1}{12}x^3+o(x^4)\right)$$

$$\exp\left(\frac{1}{x}\ln(\cosh(x))\right).$$

$$\underset{0}{=} 1 + \frac{x}{2} - \frac{1}{12}x^3 + \frac{1}{2}\left(\frac{x}{2}\right)^2 + \frac{1}{6}\left(\frac{x}{2}\right)^3$$

$$+ o\left(x^3\right) = 1 + \frac{x}{2} + \frac{x^2}{8} - \frac{1}{16}x^3 + o\left(x^3\right)$$

$$x\cosh(x)^{\frac{1}{x}} \underset{0}{=} x + \frac{x^2}{2} + \frac{x^3}{8} - \frac{x^4}{16} + o\left(x^4\right)$$

Ex 4

$\text{Arccos}(x) \underset{0}{=} ?$

$\cos(\text{Arccos}(x)) = x$

$\text{Arccos} :]-1, 1[\longrightarrow]-\pi, \pi[$

$\text{Arccos}(0) = \frac{\pi}{2}$

→ Calculer $\text{Arccos}'(x)$

→ DL de $\text{Arccos}'(x)$

→ Intégrer le DL

$$f(x) = \int_0^x e^{t^2} dt$$

$$f'(x) = e^{x^2}$$

-> DL de e^{x^2} en 0

-> Intègre le DL