Ex 3

2) $\exp(\sin(x))$ à l'ordre 4 en 0

$\sin(x) \underset{x \to 0}{\sim} 0$

$\sin(x) = x - \frac{x^3}{6} + o(x^4)$

$\exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$

$\exp(\sin(x))$

$= 1 + x - \frac{x^3}{6} + \left(x - \frac{x^3}{6}\right)^2 + \frac{x^3}{6}$

$+ \frac{x^4}{24} + o(x^4)$

$= 1 + x - \frac{x^3}{6} + x^2 - \frac{x^4}{3} + \frac{x^9}{36} + \frac{x^3}{6}$

$$+\frac{x^4}{24}+o(x^4)\Big)$$

$$=1+x-\frac{x^3}{6}+\frac{x^2}{2}-\frac{x^4}{6}+\frac{x^3}{6}$$

$$+\frac{x^4}{24}+o(x^4)\Big)$$

$$=1+x+\frac{x^2}{2}-\frac{3x^4}{24}+o(x^4)\Big)$$

3) $e^{\cos(x)}$ à l'ordre 5 en 0

$\exp(\cos(x))$

$$=\exp(1)\times\exp\left(1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)$$

$\cos(x)$

$$=1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)$$

$$\frac{}{0} \qquad \frac{}{2} \qquad \frac{}{24}$$

$$\exp(x) \underset{0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$\exp(\cos(x))$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \frac{1}{2}\left(\frac{-x^2}{2} + \frac{x^4}{24}\right)^2$$

$$+ \frac{1}{6}\left(\frac{-x^2}{2} + \frac{x^4}{24}\right)^3 + o(5)$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^4}{8} + o(5)$$

$$\text{Dy}\left(\cos(x)^{\sin(x)}\right) \text{ à l'ordre } 5$$

$$\sin(x) \underset{0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)$$

$$\cos(x) \underset{0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)$$