Project Euler Problem #9

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A Pythagorean Triple (a, b, c) is defined by two integers n and m such that $m > n$ by Euclid's Formula. Assuming:

$$a^2 + b^2 = c^2$$

$$a = 2mn,\ b = m^2 - n^2,\ c = m^2 + n^2$$

In the problem, we're given an additional condition $a + b + c = 1000$, so we have the system:

$$\begin{cases} a^2 + b^2 = c^2 \\ a + b + c = 1000 \end{cases}$$

Substituting Euclid's form into the second equation, we get the equation:

$$m(m + n) = 500$$

Which has 12 integer solutions, with some checking we find that $m = 20$ and $n = 5$ yield

$$a = 200,\ b = 375,\ c = 425$$

$$\implies a + b + c = 200 + 375 + 425 = 1000$$

$$\implies a^2 + b^2 = 40000 + 140625 = 180625 = c^2$$

Our solution is the product $abc = 31875000$.