R Notebook

Code

Market Scenario	Probability	K1 (w1 = .3)	K2 (w2 = .2)	K3 (w3 = .5
w1	.2	.3	.1	-.2
w2	.3	-.1	.3	.3
w3	.4	.2	-.1	.3
w4	.1	-.2	-.4	-.1

$$C^{-1} = \begin{bmatrix} 32 & 2.6 & 6.1 \\ 2.6 & 22 & -4.3 \\ 6.1 & -4.3 & 24 \end{bmatrix}$$

Hide

```
c.inv <- matrix(c(32,2.6,6.1,2.6,22,-4.3,6.1,-4.3,24),nrow = 3, byrow = T)
```

1. Compute the expected return of each asset.

Hide

```
Scenario <- t(c(.2,.3,.4,.1))

W <- t(c(.3,.2,.5))

k1<- t(c(.3,-.1,.2,-.2))
k2 <- t(c(.1,.3,-.1,-.4))
k3 <- t(c(-.2,.3,.3,-.1))

mu_k1 <- Scenario %*% t(k1)
mu_k2 <- Scenario %*% t(k2)
mu_k3 <- Scenario %*% t(k3)

m <- matrix(c(mu_k1 ,mu_k2,mu_k3),nrow = 1)
m
```

```
     [,1] [,2] [,3]
[1,] 0.09 0.03 0.16
```

2)Compute the convariance matrix amount assets by filling the missing entries in C.

$$c = \begin{bmatrix} C_{1\,1} & C_{1\,2} & C_{1\,3} \\ C_{2\,1} & C_{2\,2} & C_{2\,3} \\ C_{3\,1} & C_{3\,2} & C_{3\,3} \end{bmatrix}$$

$$Var(K_j) = \sum_{i=1}^{N} (K_i - \mu_{K_j})^2 \cdot p_i$$

$$Cov(J,K) = \sum_{i=1}^{N} (J_i - \mu_J)(K_i - \mu_K) \cdot p_i$$

3. compute the expected return and risk of the porfolio.

$$c = \begin{bmatrix} .033 & -.0057 & -.0094 \\ -.0057 & .048 & .01 \\ -.0094 & .01 & .046 \end{bmatrix}$$

$$\mu_v = W \cdot \mu_k$$

Hide

```
c <- matrix(c(.0329,-.0057,-.0094,-.0057,.0481,.0102,-.0094,.0102,.0464),nrow = 3 , byrow = T)
m_v <- round(m %*% t(W),2)
val_v <- round(sqrt(W %*% c%*% t(W)),2)
m_v
```

```
     [,1]
[1,] 0.11
```

Hide

```
val_v
```

```
     [,1]
[1,] 0.12
```

4. compute the weights of the MVP. Compute the expeted return and risk of the MVP

$$u = [1 \;\; 1 \;\; 1];$$
$$W_{MVP} = \frac{u \cdot C^{-1}}{u \cdot C^{-1} \cdot u^T};$$
$$\mu_{MVP} = W_{MVP} \cdot m;$$
$$\sigma_{MVP} = W_{MVP} \cdot C^{-1} \cdot (W_{MVP})^T;$$

Hide

```
u <- t(c(1,1,1))
w.mvp <- round((u%*%c.inv)/as.vector(u%*%c.inv%*%t(u)),2)
m.mvp <- round(w.mvp %*% t(m),3)
val.mvp <- round(sqrt(w.mvp %*% c %*% t(w.mvp)),2)
w.mvp
```

```
     [,1] [,2] [,3]
[1,] 0.47 0.23  0.3
```

Hide

```
m.mvp
```

```
      [,1]
[1,] 0.097
```

Hide

```
val.mvp
```

```
     [,1]
[1,] 0.11
```

5. find the weights for the MVL. It is given that

$$M^{-1} = \begin{bmatrix} 5.3 & -.51 \\ -.51 & .061 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_1 \end{bmatrix} = M^{-1} \begin{bmatrix} \mu \\ 1 \end{bmatrix}$$

$$\lambda_1 = 5.3\mu - .51 \quad \lambda_2 = .061 - .051$$

$$w_{MVP} = \lambda_1 m \cdot C^{-1} + \lambda_2 u C^{-1}$$

$$w_{MVP} = a\mu + b$$

Hide

```
M.inv <- matrix(c(5.3,-.51,-.51,.061), nrow = 2 , byrow =T)
a <- round(M.inv[1,1] * m %*% c.inv + M.inv[2,1] * u %*% c.inv,3)
b <- round(M.inv[1,2] * m %*% c.inv + M.inv[2,2] * u %*% c.inv,2)
a
```

```
      [,1]   [,2] [,3]
[1,] 0.093 -9.261 9.42
```

Hide

```
b
```

```
     [,1] [,2] [,3]
[1,] 0.48 1.13 -0.6
```

$$w_{MVL} = [0.093 \quad -9.26 \;\; 9.4]\mu + [0.48 \;\; 1.13 \quad -0.60].$$

6. Find the equation of the effcient frontier. The product between W_{MVL} and C is given

$$w_{MVL} \cdot C = [.015 - 0.038\mu \;, .046 - .35\mu \;, .34\mu - .021]$$

$$\sigma = \sqrt{w_{MVL} \cdot C \cdot w_{MVL}^T} =$$

$$\sqrt{[.015 - 0.038\mu \, , .046 - .35\mu \, , .34\mu - .021] \cdot [0.093\mu + .48 \, , -9.26\mu + 1.13 \, 9.4\mu - .60]^T} =$$

$$\sqrt{6.4\mu^2 - 1.2\mu + .071}$$

7. Given the risk-free return R = .05, compute the weights of the market portfolio.

$$w_{MVL} = \frac{(m - R * u) \cdot C^{-1}}{(m - R * u) \cdot C^{-1} \cdot u^T}$$

Hide

```
R <- .05
w.m <- round((((m-R*u)%*%c.inv)/as.vector((m-R*u)%*%c.inv%*%t(u))),2)
mu.m <- round(w.m %*% t(m),2)
val.m <- round(sqrt(w.m %*% c %*% t(w.m)),2)
w.m
```

```
     [,1] [,2] [,3]
[1,] 0.47 -0.2 0.73
```

Hide

```
mu.m
```

```
     [,1]
[1,] 0.15
```

Hide

```
val.m
```

```
     [,1]
[1,] 0.16
```

8. Compute the expected return and risk of the market portfolio. You can use weight in above or use the efficient frontier.

Method 1 (Use W_M).

$$\mu_M = w_{market\ portolio} \cdot m$$
$$\sigma = w_{market\ portolio} \cdot C \cdot (w_{market\ portolio})^T$$

Hide

```
mu.m <- round(w.m %*% t(m),2)
val.m <- round(sqrt(w.m %*% c %*% t(w.m)),2)
mu.m
```

```
     [,1]
[1,] 0.15
```

Hide

```
val.m
```

```
     [,1]
[1,] 0.16
```

Method 2. Use the efficient frontier. Treate σ as the y variable μ as the x variable.

$$\frac{d\sigma}{d\mu} = \frac{1}{2}\frac{2 \times 6.4 - 1.2}{\sqrt{6.4\mu^2 - 1.2\mu + .071}}$$

The CML passes through (σ_M, μ_M) and has the form

$$\sigma - \sigma_M = \frac{d\sigma}{d\mu}|_M(\mu - \mu_M)$$

The CML also passes through (R,0), leading to

$$0 - \sigma_M = \frac{d\sigma}{d\mu}|_M(\mu - \mu_M)$$

Therefore,

$$-\sqrt{6.4\mu_m^2 - 1.2\mu_M + .071} = \frac{1}{2}\frac{2 \times 6.4 - 1.2}{\sqrt{6.4\mu^2 - 1.2\mu + .071}}(R - \mu_M),$$

Which has a solution

$$\mu_M = .15$$

9. Compute the return of the market portfolio

$$K_M = []$$

10. Find the equation of the CML

$$\mu = \frac{\mu_M - R}{\sigma_M} + R = \frac{.15 - .05}{.16} + .05 = .63\sigma + .05$$

where $.63\sigma$ is called the risk premium.

11. Consider a feasible portfolio with allocation w = [.2 .4 .4]. It is given that $Cov(K_M, K_V = .01$. Compute its β_V.

Method 1.

$$\beta_V = \frac{Cov(K_M, K_V)}{\sigma_M^2}$$

Hide

```
beta.v <- .01/val.m^2
beta.v
```

```
         [,1]
[1,] 0.390625
```

Method 2.

$$\beta_V = \frac{\mu_V - R}{\mu_M - R}.$$

Hide

```
w.v <- t(c(.2,.4,.4))
mu.v <- w.v %*% t(m)
beta.v <- (mu.v-R)/(mu.m - R)
mu.v
```

```
      [,1]
[1,] 0.094
```

Hide

```
beta.v
```

```
     [,1]
[1,] 0.44
```

12. Write down the CAPM equation for the above portfolio.

$$K_V = \beta K_M + \alpha + \epsilon\ ,$$

where

$$K_V = [.02\ , .22\ , -.24]\ K_M = [-.025\ , .11\ , .33\ , -.087]$$

and

$$\hat{\beta} = \beta_V\ , \hat{\alpha} = \mu_v - \beta_V \mu_M\ , E\epsilon = 0.$$

13. Consider a portfolio on the CML with .09 of risk. Compute its β_V.

$$\beta_V = \frac{\sigma_V}{\sigma_M}$$

Hide

```
val.cml <- .09
new.beta.v <- val.cml/val.m
new.beta.v
```

```
       [,1]
[1,] 0.5625
```

14. If you want to achieve 20% of expected return using the three securities in the table, what is your allocation with the minimum variance?

Hide

```
mu.ex <- .20
w.mvl <- mu.ex * a + b
w.mvl %*% t(m)
```

```
         [,1]
[1,] 0.228648
```

[?

Hide

```
w.fv <- t(c(.48,.30,.25))
w.fv %*% t(m)
```

```
       [,1]
[1,] 0.0922
```

?]

15. How would you allocate the capital amoung risky assets and the risk-free asset if you would like to achieve 9% of expected return with the minimum variance?

$$0.05w_1 + 0.15(1 - w_1) = 0.09$$
$$- > w_1 = .6, w_2 = .4$$

16. Write down the equation of the SML.

$$\mu_V = \beta_V(\mu_M - R) + R = \beta_V = \beta_V(0.15 - .05) + 0.05 = 0.10\beta_V + 0.5$$