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STYLIZED FACTS OF FINANCIAL TIME SERIES: A COMPREHENSIVE ANALYSIS

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***Abstract*— This article describes a set of stylized empirical facts emerging from the statistical analysis of various types of financial time series. The purpose of this article is to examine some fundamental statistical behavior of financial time series as well as nonlinearity associated with them. The knowledge of such facts could be helpful to establish better empirical models, which is nonlinear most of the times, in order to produce reliable forecasts. Also, it illustrates the importance of stationarity in financial time series analysis and the concepts behind the most used statistical tests for checking the stationarity of a series such as Augmented Dickey – Fuller Test and Phillips-Perron Test, using ten fast growing stock market indices across the globe. After observing the principal stylized facts of returns of a given asset for the empirical financial data, could be in a more comfortable situation to choose a proper model for better forecast.**

Keywords*— *financial time series; stylized facts of financial time series; stationarity; Augmented Dickey – Fuller Test; Phillips-Perron Test.

I. INTRODUCTION

Financial time series are continually brought the attention of individual, private and corporate investors, businessmen, anyone involved in international trade and the brokers and analysts. Daily news reports in newspapers, television and radio inform us for illustration of the latest stock market index values, currency exchange rates, electricity prices, and interest rates. It is often desirable to keep an eye on price behavior frequently and to try to understand the probable development of the prices in the future. Many traders deal with the risks associated with changes in prices. These risks can frequently be summarized by the variances of future returns, directly, or by their relationship with relevant co variances in a portfolio context.

Usually, a time series is defined as a sequence of values/data points/events separated/occurred by equal interval of time. A time series can be represented as a set of discrete values $\{x_1, x_2, x_3, \cdots \cdots, x_N\}$ where N is the total number of observations. A time series possess both deterministic as well as stochastic components characterized by noise interference. However financial time series data is prone to random fluctuations as compared to ordinary time series. It is characterized with high nonlinearity, non-stationary and chaotic in nature. It is often desirable to monitor price behavior frequently and to try to understand the probable development of the prices in the future.

The observations or empirical findings common across a wide range of instruments, markets, and time periods are called as stylized facts. They can be obtained by taking a common denominator among the properties observed in studies of different markets and instruments [1]. Stylized facts are usually formulated in terms of qualitative properties of asset returns and may not be precise enough to distinguish among different parametric models. The intension here is not to go the details of stylized facts, rather to study some of those for the financial time series considered, which could be helpful to establish better empirical models by the researchers in this domain. For more exhaustive study about stylized facts of financial time series, prospective readers may refer to the articles in [2-6].

The concept of stationarity has always been essential to econometric time series analysis, since most financial time series analysis necessitates that data be made stationary before any regressions can be performed. In applied econometric analysis concerning time series data, the prerequisite of stationarity is a well-known conception. A stationary time series is represented by data over time whose statistical properties remain constant regardless of a change in the time origin [7]. A financial time series can be represented by historical stock prices. Earlier research conducted on financial markets behavior suggests that financial time series follow a random

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walk [8] and the process is inherently non-stationary because of the presence of a unit root [9]. When a time series contains a unit root it is essential to difference the time series to render it stationary [10].

The objective of this study is to examine some fundamental statistical behavior of financial time series as well as nonlinearity associated with them. The knowledge of such facts could be helpful to establish better empirical models, which is nonlinear most of the times, to produce reliable forecasts. Section II discusses some of the stylized facts exhibited by the financial time series. The importance of stationarity in financial data is discussed in section III. Section IV gives the concluding remarks followed by a list of references.

II. SOME STYLIZED FACTS IN FINANCIAL TIME SERIES

In this article the daily closing prices of ten fast growing stock markets such as BSE, DJIA, NASDAQ, FTSE, TAIEX, S&P 500, ASX, LSE, SSE, and NIKKEI for period of fifteen years (01 Jan. 2000 to 31 Dec. 2014) are used. The daily closing prices are forming time series, specially called as financial time series. Figure 1 shows the daily closing indices of the stock market data used.

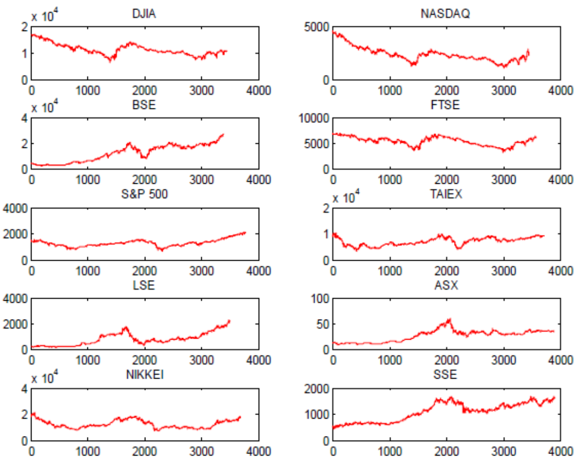

Figure 1- Daily observations of closing prices of markets (left) from top to bottom DJIA, BSE, S&P 500, LSE, NIKKEI and (right) from top to bottom NASDAQ, FTSE, TAIEX, ASX, and SSE from January 2000 to December 2014.

As can be observed from Figure 1, the financial time series generated from ten different stock markets do not seem to have anything in common. In the other hand returns exhibited more attractive statistical properties. Figure 2 plots the time series of returns for the different indexes used in this study. For common investors, returns represent a complete and scale-free summary of the investment opportunity.

Also return series are easier to handle than price series because of having more attractive statistical properties.

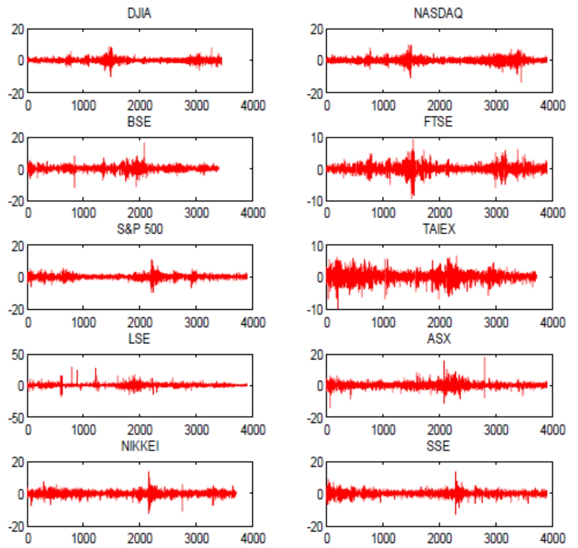

Figure 2- Return series of markets (left) from top to bottom DJIA, BSE, S&P 500, LSE, NIKKEI and (right) from top to bottom NASDAQ, FTSE, TAIEX, ASX, and SSE from January 2000 to December 2014.

The descriptive statistics of daily closing prices are summarized in Table 1. Similarly, the descriptive statistics of return series are summarized in Table 2. These statistics are used in discussion of some stylized facts exhibited by the financial time series.

The positive skewness value of the closing price as observed from Table 1 implies that all the data sets except FTSE, TAIEX, and SSE are spread out more towards right. The kurtosis analysis implies that stock price of DJIA, NASDAQ, and S&P 500 are more outlier prone where as all other financial time series are less outlier prone. Also, from the Jarque - Bera test statistics, it can be observed that all the stock price data sets are non-normal distributed.

Similarly Table 2 summarizes the descriptive statistics of daily returns from all data sets. The positive skewness value of the return price implies that all data sets except NASDAQ, TAIEX, S&P 500, SSE, and NIKKEI are spread out more toward right. These positive skewness values suggest investment opportunities in these markets. For an example the histogram of daily returns of all the financial time series considered are presented by Figure 3. The

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peaks of the histograms are much higher than the corresponding to the normal distribution and it is slightly skewed to the right in case of BSE and slightly skewed to the left in case of NASDAQ stock data. The kurtosis analysis implies that stock price of all data sets are more outlier prone than the normal distribution. Again from the Jarque-Bera test statistics, it can be observed that all the stock price data sets are non-normal distributed.

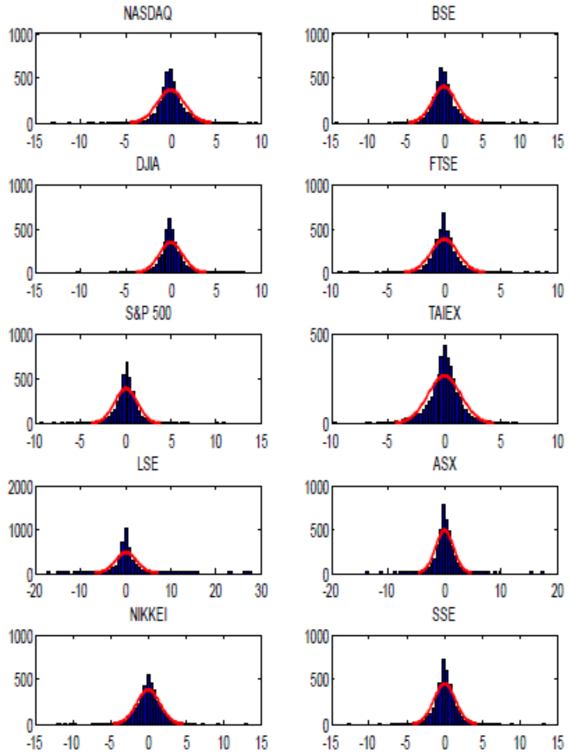

Figure 3 - Histogram of daily returns of all financial time series against the theoretical normal distribution

Table 1- Descriptive statistics of daily closing prices for ten different financial time series

Stock Index	Descriptive statistics						
	Minimum	Maximum	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera test statistics
BSE	792.1800	1.1024e+004	4.6235e+003	2.6947e+003	0.1154	1.7908	236.0430(h=1)
DJIA	6.5471e+003	1.7138e+004	1.1400e+004	2.1801e+003	0.6644	3.0512	253.8134(h=1)
NASDAQ	1.1141e+003	4.5982e+003	2.3858e+003	709.7888	1.0392	4.0027	764.3663(h=1)
FTSE	3287	6.8785e+003	5.4165e+003	836.2381	-0.2837	2.1378	158.4568(h=1)
TAIEX	3.4463e+003	1.0202e+004	6.9835e+003	1.4846e+003	-0.1776	2.0465	159.9786(h=1)
S&P 500	676.5300	2.0906e+003	1.2824e+003	269.6542	0.7109	3.5294	361.8941(h=1)
LSE	186.1040	2255	805.4437	486.4698	0.7035	2.7254	299.6108(h=1)
ASX	9.5989	59.6509	26.9877	11.2061	0.0847	2.3400	75.2181(h=1)
SSE	427.5000	1679	1.0762e+003	350.6077	-0.1679	1.5868	343.2149(h=1)
NIKKEI	7.0550e+003	2.0833e+004	1.2311e+004	3.0587e+003	0.5011	2.1468	266.5799(h=1)

Table 2- *Descriptive statistics of daily returns for ten different financial time series*

Stock Index	Descriptive statistics						
	Minimum	Maximum	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera test statistics
BSE	-14.6179	12.4398	-0.0460	1.5724	0.5305	9.7199	7.2066e+003(h=1)
DJIA	-10.5083	8.2005	-0.0016	1.2983	0.2038	9.8306	7.2907e+003(h=1)
NASDAQ	-13.2546	9.5877	-0.0092	1.5094	-0.0871	9.0069	5.8730e+003(h=1)
FTSE	-9.3842	9.2646	-0.0010	1.1949	0.1516	10.1496	8.3278e+003(h=1)
TAIEX	-9.9360	6.5246	-0.0037	1.4603	-0.2257	5.9269	1.3657e+003(h=1)
S&P 500	-9.4695	10.9572	0.0089	1.2703	-0.1811	11.1322	1.0776e+004(h=1)
LSE	-17.4640	27.7143	0.0593	2.3315	1.0482	22.5719	6.3010e+004(h=1)
ASX	-14.0695	17.7496	0.0293	1.5867	0.3217	14.2521	2.0657e+004(h=1)
SSE	-12.9754	13.4709	0.0304	1.4215	-0.1572	10.0935	8.1989e+003(h=1)
NIKKEI	-12.1110	13.2346	-0.0071	1.5555	-0.3997	9.1350	5.9600e+003(h=1)

- **Gain/Loss Asymmetry**

This is a stylized fact in financial time series where one observes large draw downs in stock index values but not equally large upward movements. The skewness of a financial time series is a measure of the asymmetry of the distribution of the series. It may be noted that all symmetric distributions including the normal distribution posses skewnees value equal to zero. As observed from the return statistics presented in Table 2, BSE, DJIA, FTSE, LSE, and ASX have positive skewness values which might point to possible investment opportunities in these emerging markets. Positive skewness implies that the right tail of the distribution is fatter than the left tail which indicates that positive returns tend to occur more often than large negative returns. Interested readers may refer [1, 5].

- **Fat tails**

The fact that the distribution of stock returns is fat-tailed has important implications in financial time series analysis. Since the probability of observing extreme values is higher for fat-tail distributions compared to normal distributions, it leads to a gross underestimation of risk. A random variable is said to possessing fat tails if it exhibits more extreme outcomes than a normally distributed random variable with the same mean and variance [2]. This indicates that the stock market has more relatively large and small outcomes than one would expect under the normal distribution.

The degree of peakedness of a distribution relative to its tails is measured by its kurtosis value. The normal distribution has kurtosis value 3. Higher kurtosis value (leptokurtosis) is a signal of fat tails, means that most of the variance is due to infrequent extreme deviations than predicted by the normal distribution. As observed from Table 2, all the stock data sets have excess kurtosis which establishes the fact of fat tails and evidence against normality.

The commonly used graphical method for analyzing the tails of a distribution is the Quantile - Quantile (QQ) plot. For an example, the QQ plots for NASDAQ, BSE, DJIA, and FTSE are presented by Figure 4. From this it can be observed that returns have fatter tails to fit the normal distribution.

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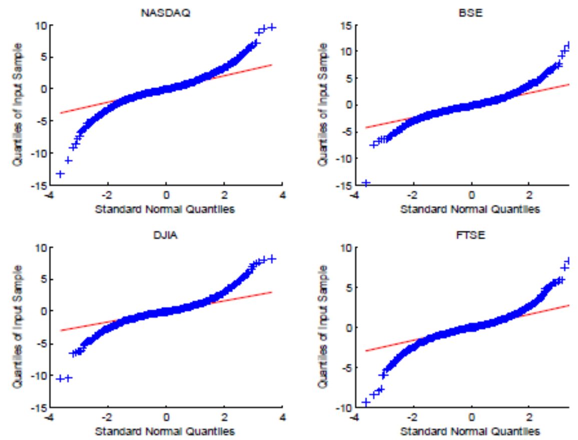

Figure 4- QQ plot of NASDAQ, BSE, DJIA, and FTSE financial time series data

- **Slow decay of autocorrelation in returns**

This fact tells that the autocorrelation function of absolute returns decays slowly as a function of the time lag. It is a well-known fact that price movements in markets do not exhibit any significant linear autocorrelation

[1]. The autocorrelation function measures how returns on a given day are correlated with returns on previous days. If such correlations are statistically significant, there is strong evidence for predictability. From Figure 5, it can be clearly seen that the autocorrelation function for BSE rapidly decays to zero after a lag.

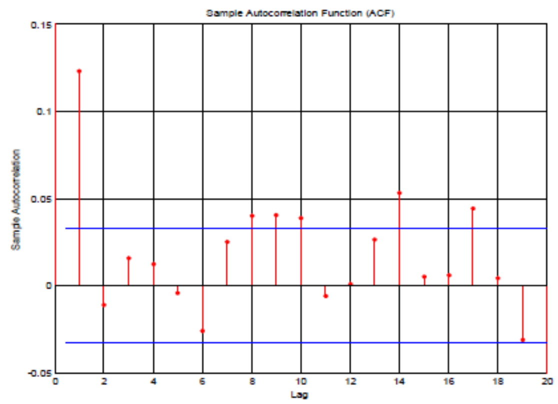

Figure 5 - Autocorrelation plot of BSE returns, along with a 95% confidence interval, for the first 20 lags

- **Volatility clustering**

This is a well-known stylized fact where different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time [1]. This implies that large price variations are more likely to be followed by large price variations; hence returns are not random walk.

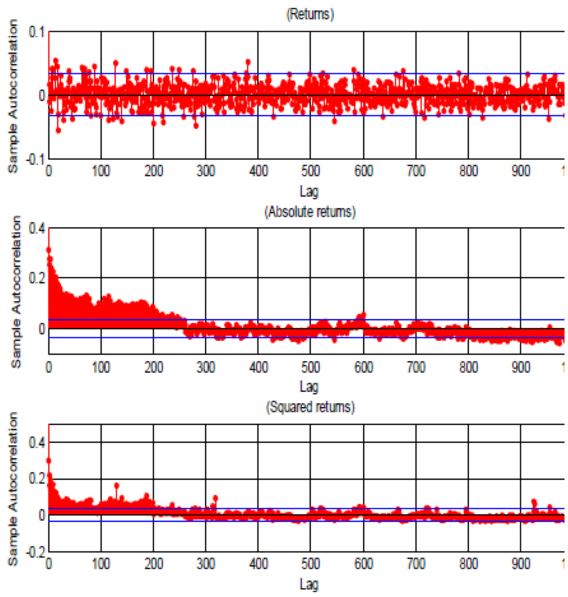

Figure 6 - Autocorrelation plots of daily BSE returns (top), absolute returns (middle) and squared Returns (bottom) for first 1000 lags.

Figure 6 shows the autocorrelation function for BSE return series (top), absolute return (middle), and Squared returns (bottom). In the top panel most of the autocorrelations lie within the interval. However in case of absolute return and squared return the autocorrelation function is significant even at long lags which provide the evidence for the predictability.

The lag plots of returns of a day rt against returns of previous day rt-1is another possible way to characterize the stylized fact of volatility/return cluster. For an example, the lag plots corresponding to returns of DJIA and NASDAQ are shown in the Figure 7.

A stylized fact that can be observed from such lag plots is that large returns tend to occur in clusters, i.e., it appears that relatively volatile periods characterized by large returns alternate with more stable periods in which returns remain small.

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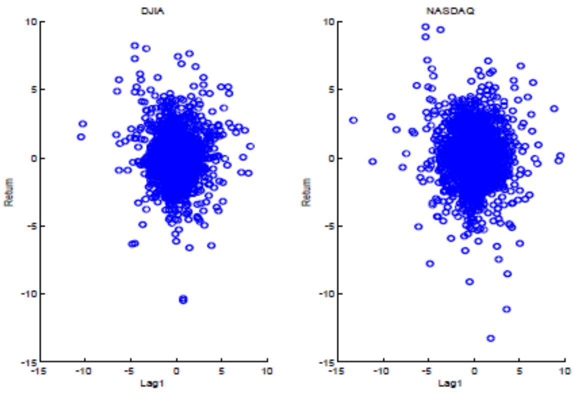

Figure 7 - Lag plots of the returns on the DJIA (left) and NASDAQ (right), on day t, against the return on day t-1

We studied some stylized facts exhibited by ten different financial time series in the above sub sections. The following observations can be drawn from the above analysis.

- We showed that the probability density function of the return series of these stock indices are skewed and fat tailed.

- The positive skewness shown by BSE, DJIA, FTSE, LSE, and ASX return series implying possible investment opportunities in these markets.

- For all markets, fat tails existed with kurtosis far in excess of the corresponding to the normal distribution.

- The linear autocorrelations are insignificant after a few lags and nonlinear autocorrelations prevailed, which was evidence for the existence of volatility clusters.

These preliminary analyses of the stylized facts in financial time series could be helpful to the researchers to develop sophisticated and more accurate model for forecasting stock market behavior.

III. Stationarity of Financial Time Series

The terms non-stationary and stationary form a fundamental part of time series econometric analysis. A stationary time series refers to data whose statistical properties remain unchanged over time regardless of the change in time [7].Testing stationarity in financial time series involves testing for the order of integration in the time series. It is the test process to check whether the time series possess a unit root. There are two principal tests popular amongst econometricians for testing the null-hypothesis of a unit root to establish stationarity. These are the Augmented Dickey-Fuller (ADF) test for unit roots and the Phillips - Perron(PP) test [11]. There are also several tests for testing stationarity with stationarity as the null-hypothesis. This article discusses the outputs from ADF test and PP test conducted over the ten financial time series.

It is worth mentioning that stationarity of a financial time series is an important feature which is required to make statements about the future in order to get valid forecast. Therefore, the first thing done is to investigate the time series for the presence of a unit root to determine whether the analyzed time series is stationary or not. In case of the standard Dickey-Fuller test the assumptions to be checked can be written as follows:

H_0: $\alpha = 0$ => the series has unit root, so is non-stationary

Against the alternative hypothesis

$H_{alt.}$: $\alpha < 0$ => doesn't have unit root, so the time series is stationary, and are evaluated using t_α statistic calculated as

$t_\alpha = \frac{\hat{\alpha}}{se(\hat{\alpha})}$, where $\hat{\alpha}$ represents the estimate of α and $se(\hat{\alpha})$ represents the coefficient standard error.

The value of the statistics is calculated and then compared with the critical values of the ADF test. If the value of the statistics is less than critical value, then we reject the null hypothesis, so there is no unit root and the time series is stationary. An alternative to the ADF test is Phillips - Perron test (PP test) which is a non-parametric method and very similar to ADF test. The only difference between them is that the PP test allows residues to be auto-correlated by introducing an automatic correction in the testing procedure. The mathematical detail of these test are beyond the scope of this article. We used the ADF test and PP test available in the Matlab software for checking the time series stationarity. The outputs from these tests are summarized in Table-3.

The analysis revealed non - stationarity in levels and stationarity in first difference for all ten analyzed time series by applying the Augmented Dickey-Fuller and Phillips-Perron tests. This may be observed clearly from the Table-3 presented above. This conclusion is

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agreed with the Box –Jenkins approach for modeling time series stated that financial time series are non-stationary.

Also, for BSE index prices it can be observed very high autocorrelations for the first two lags, 1.0000 and 0.9988 respectively. This values decreasing slowly for the next lags, reaching a value of 0.9602 at the 30th lag. Also, daily DJIA index prices correlogram points out very high autocorrelations for the first two lags, 1.0000 and 0.9985 respectively, with values decreasing very slowly for the next lags, reaching a value of 0.9372 at the 30th lag.

Similar observations recorded for NASDAQ, FTSE, and TAIEX time series. This leads to the conclusion that the price series of the five stock indices are non-stationary. On the other hand, high values of Q-Stat test and zero probability to all lags confirms the presence of autocorrelation.

Table-3 The output of Augmented Dickey-Fuller test and Phillips - Perron test for all financial time series data

Stock Data	Indicators	ADF Test		PP Test	
		Level	1st Diff.	Level	1st Diff.
BSE	t-Statistic (Prob.)	-2.5460 (0.3223)	-41.5448 (0.0001)	-2.4068 (0.3913)	-54.0093 (0.0000)
	t-critical (5%)	-3.4138	-3.4138	-3.4138	-3.4138
DJIA	t-Statistic (Prob.)	-2.3788 (0.4051)	-44.7766 (0.0000)	-2.4392 (0.3752)	-63.6215 (0.0000)
	t-critical (5%)	-3.4139	-3.4139	-3.4139	-3.4139
NASDAQ	t-Statistic (Prob.)	-2.5920 (0.2995)	-48.0225 (0.0001)	-2.5823 (0.3043)	-64.9935 (0.0001)
	t-critical (5%)	-3.4139	-3.4138	-3.4139	-3.4138
FTSE	t-Statistic (Prob.)	-2.0381 (0.5738)	-47.2026 (0.0000)	-2.1130 (0.5367)	-65.5486 (0.0000)
	t-critical (5%)	-3.4139	-3.4138	-3.4139	-3.4138
TAIEX	t-Statistic (Prob.)	-3.2628 (0.0730)	-40.8769 (0.0000)	-3.1798 (0.0889)	-58.4660 (0.0000)
	t-critical (5%)	-3.4138	-3.4138	-3.4138	-3.4138
S&P 500	t-Statistic (Prob.)	-2.3658 (0.4076)	-42.7748 (0.0000)	-2.4397 (0.3852)	-61.6015 (0.0000)
	t-critical (5%)	-3.4235	-3.4235	-3.4235	-3.4235
LSE	t-Statistic (Prob.)	-2.1366 (0.5765)	-45.2226 (0.0000)	-3.3130 (0.5367)	-64.5482 (0.0000)
	t-critical (5%)	-3.4339	-3.4338	-3.4339	-3.4338
ASX	t-Statistic (Prob.)	-3.0460 (0.3423)	-40.5048 (0.0001)	-2.4168 (0.3913)	-52.0293 (0.0000)
	t-critical (5%)	-3.4138	-3.4138	-3.4138	-3.4138
SSE	t-Statistic (Prob.)	-2.1371 (0.5738)	-45.2026 (0.0000)	-2.1530 (0.5367)	-63.5406 (0.0000)
	t-critical (5%)	-3.4239	-3.4238	-3.4239	-3.4238
NIKKEI	t-Statistic (Prob.)	-2.3385 (0.4451)	-44.7766 (0.0000)	-2.2392 (0.3752)	-65.6265 (0.0000)
	t-critical (5%)	-3.4437	-3.4437	-3.4437	-3.4437

IV. CONCLUSIONS

This article analyzed some important stylized facts of ten different financial time series data such as BSE, JIA, NASDAQ, FTSE, TAIEX, S&P 500, ASX, LSE, SSE, and NIKKEI for a period of fifteen years. The description emphasizes properties widespread to a wide variety of emerging and developing stock markets across the globe. We observed that, it is helpful to get aware with the empirical data before looking for the suitable models. Study of such stylized facts could be the foundation for nonlinear modeling of financial assets returns. We discussed the importance of stationarity as an essential feature necessary to be achieved before analyzing a financial time series. The analysis revealed non-stationarity in levels and stationarity in first difference for all ten financial time series analyzed. This fact agreed with the Box – Jenkins approach stated that financial time series are non - stationary. The knowledge of such observations could be helpful to establish better empirical models, which is nonlinear most of the times, in order to produce reliable forecasts. The future work may concentrate on developing some sophisticated nonlinear financial forecasting models.

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