

Principles of Data Science Project 3

Feature Encoding

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Abstract—Hello

Index Terms—something

I. INTRODUCTION

A. SIFT

Scale-invariant feature transform (SIFT) is a machine vision algorithm used to detect and describe local features in an image. It looks for extreme points in the spatial scale and extracts its position, scale, rotation invariant. This algorithm was published by David Lowe in 1999, and was summarized in 2004. [1] The description and detection of local image features can help identify objects. SIFT features are based on some local appearance points of interest on the object and are not related to the size and rotation of the image. The tolerance to light, noise, and slight changes in viewing angle is also quite high. Based on these characteristics, they are highly conspicuous and relatively easy to capture. In a huge feature database, objects are easy to identify and rarely misidentified.

The main steps of the sift algorithm are as follows:

1) *Scale-space extrema detection*: The images are convolved with Gaussian filters at different scales, and then continuous Gaussian blur is used to blur the image differences to find the key points. The key point is based on the maximum and minimum Gaussian difference (DoG) at different scales. In other words, the $D(x, y, \sigma)$ of the DoG image is caused by:

$$D(x, y, \sigma) = L(x, y, k_i \sigma) - L(x, y, k_j \sigma) \quad (1)$$

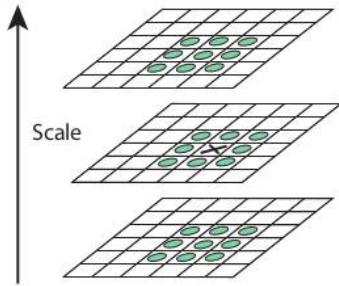


Fig. 1. Local Extrema

$L(x, y, k\sigma)$ is the convolution of the original image $I(x, y)$ and Gaussian blur $G(x, y, k\sigma)$ under the condition of the scale $k\sigma$, for example:

$$L(x, y, \sigma) = G(x, y, k\sigma) \times I(x, y) \quad (2)$$

$G(x, y, \sigma)$ is a variable-scale Gaussian function:

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \quad (3)$$

Once the DoG image is obtained, the maximum and minimum values in the DoG image can be found as key points. In order to determine the key points, each pixel in the DoG image will be made with eight pixels around the center of itself, and nine pixels in the same position of the adjacent scale magnification in the same group of DoG images, for a total of 26 points. For comparison, if this pixel is the maximum and minimum of these twenty-six pixels, this pixel is called a key point.

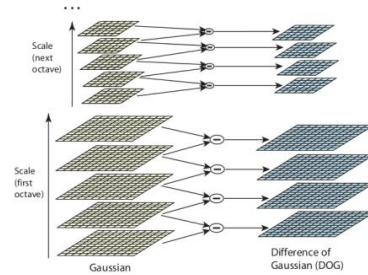


Fig. 2. DoG

2) *Keypoint localization*: There may be too many key points in different size spaces, and some key points may be relatively difficult to identify or susceptible to noise interference. The next step of the SIFT algorithm will locate each key point by the information of pixels near the key point, the size of the key point, and the main curvature of the key point, thereby eliminating the key points that are located on the side or are susceptible to noise

3) *Orientation assignment*: After the above steps, feature points that exist at different scales have been found. In order to achieve image rotation invariance, the direction of the feature points needs to be assigned. Use the gradient distribution characteristics of the pixels in the neighborhood of the feature

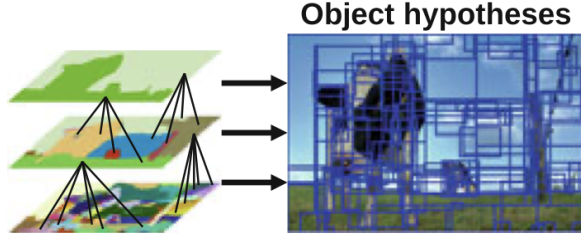


Fig. 3. Hierarchical Grouping Algorithm. [2]

point to determine its direction parameters, and then use the gradient histogram of the image to find the stable direction of the local structure of the key point.

4) *Keypoint descriptor*: Through the above steps, the location, scale and direction information of SIFT feature points have been found. Next, use a set of vectors to describe the key points, that is, to generate feature point descriptors. There are roughly three steps in generating feature descriptors:

- Correct the main direction of rotation to ensure rotation invariance.
- Generate descriptors and ultimately form a 128-dimensional feature vector
- In the normalization process, the feature vector length is normalized to further remove the influence of lighting.

B. Selective Search

Selective Search is an object recognition algorithm which combines the strength of both an exhaustive search and segmentation. [2] We know that the problem of target detection is more complicated than image classification. An important reason is that there may be multiple objects in an image that need to be located and classified separately. Obviously, before training the classifier, you need to use some methods to divide the image into small areas. Selective search method has three main advantages: capture different scales, diversification, fast to compute. Selective search algorithm mainly includes two steps: hierarchical grouping algorithm and diversification strategies, which can be summed as follows:

1) *Hierarchical Grouping Algorithm*: Hierarchical grouping algorithm uses the method of Felzenszwalb and Huttenlocher to generate the initial region of the image, and uses the greedy algorithm to iteratively group the regions. In each grouping iteration, a larger area is formed and added to the area proposal list. Algorithm will create a regional proposal from smaller segments to larger segments in a bottom-up behavior, as shown in Fig. 3.

2) *Diversification Strategies*: In order to diversify the sampling in grouping, selective search algorithm presents three diversification strategies:

- Use various color spaces.
- Use different similarity measures.
- Changing the starting area to make sampling.

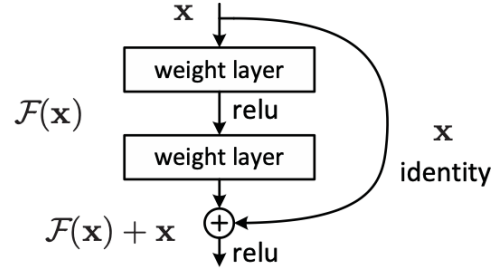


Fig. 4. Residual learning: a building block. [3]

C. ResNet

Residual Network (ResNet) [3] is presented to ease the training of networks that are substantially deeper than those used previously and have the best performance in ILSVRC & COCO 2015 competitions. We can conclude its motivation that ResNet adds a skip connection in every building block to solve the degradation problem as the learning networks become deeper and deeper. The framework of ResNet can be shown in Fig. 4. We can express its principle by the following expression:

$$x_i = x_{i-1} + F(x_{i-1})$$

where x_i is the network input and x_{i-1} is the output, $F(\cdot)$ presents the transform of network block.

As an state-of-the-art model in deep learning field, its skip connection structure can be seen as a smoother in training process [4], and the information can be maintained in forward training, which can be seen in the network loss [5].

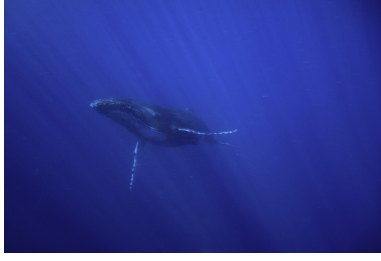
II. EXPERIMENTS

A. Sift descriptors

We used sift to extract the local descriptors of the images, and then used BOW, VLAD, FV to encode the features, and finally put them into the SVM for classification. As shown in Fig. 5, when using sift to extract local descriptors, we encountered some images that could not be extracted. We improved the image contrast to process these images, and finally got the results.

1) *BOW*: In this experiment, we used the BOW model to encode the descriptors extracted by sift, and set different clusters k in [8, 16, 32, 64, 128, 256, 512, 1024]. Besides, we normalized the feature vectors by z-score. After that, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

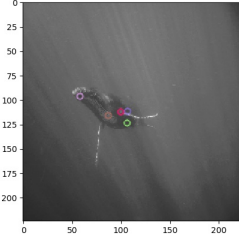
Our experiment results are shown in Tab. I. We can find that as k increases, the experimental results get better. We deem the reason is that when k is larger, the bag of words after codebook construction is larger, so the difference among the different types of images after feature encoding is also



(a) original image



(b) processed image



(c) image after sift

Fig. 5. humpback

greater. Besides, we can find that rbf kernel performs better than linear kernel. We think this is because the dimension of the feature vector obtained by the BOW model is k , and rbf kernel is more suitable for classification with fewer feature dimensions than linear kernel. From the results we can see BOW's performance is poor because its highest accuracy is only 0.2488. It is obvious because when mapping, BOW uses the bag of words to quantify the image features for constructing a word frequency histogram. And the word frequency histogram is the encoded feature vector, so much information is lost.

2) *VLAD*: In this experiment, we used the VLAD model to encode the descriptors extracted by sift. The dimension of VLAD feature vector is $k * d$, where k is the cluster number and d is the dimension of each descriptor which equals 128. Because the feature dimension is too high, we use LDA and PCA for feature reduction. Besides, the experiments of VLAD consume time and computing resources, we can't set many different cluster numbers and large cluster numbers. So we only set k in [4, 8, 16] for experiments. After feature encoding and reduction, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

TABLE I
ACCURACY OF STFT FEATURES BASED ON BOW(Z-SCORE) MODEL

Acc. k	M	SIFT + BOW + SVM			
		C	linear kernel	C	rbf kernel
8		0.0005	0.0699	0.5	0.1361
8		0.001	0.0854	1.0	0.1394
8		0.005	0.1138	5.0	0.1379
8		0.01	0.1213	10	0.1369
16		0.0005	0.0882	0.5	0.1744
16		0.001	0.1134	1.0	0.1786
16		0.005	0.1470	5.0	0.1823
16		0.01	0.1580	10	0.1763
32		0.0005	0.1183	0.5	0.1953
32		0.001	0.1401	1.0	0.2018
32		0.005	0.1767	5.0	0.1963
32		0.01	0.1826	10	0.1885
64		0.0005	0.1495	0.5	0.2151
64		0.001	0.1723	1.0	0.2205
64		0.005	0.2049	5.0	0.2145
64		0.01	0.2103	10	0.2059
128		0.0005	0.1774	0.5	0.2257
128		0.001	0.2037	1.0	0.2342
128		0.005	0.2210	5.0	0.2260
128		0.01	0.2218	10	0.2211
256		0.0005	0.2054	0.5	0.2291
256		0.001	0.2262	1.0	0.2434
256		0.005	0.2321	5.0	0.2351
256		0.01	0.2255	10	0.2317
512		0.0005	0.2293	0.5	0.2242
512		0.001	0.2390	1.0	0.2444
512		0.005	0.2272	5.0	0.2400
512		0.01	0.2133	10	0.2359
1024		0.0005	0.2408	0.5	0.2231
1024		0.001	0.2440	1.0	0.2511
1024		0.005	0.2164	5.0	0.2488
1024		0.01	0.2036	10	0.2473

TABLE II
ACCURACY OF STFT FEATURES BASED ON VLAD MODEL AFTER LDA

Acc. k	M	SIFT + VLAD + SVM + LDA			
		C	linear kernel	C	rbf kernel
4		0.0005	0.2202	0.5	0.2722
4		0.001	0.2524	1.0	0.2724
4		0.005	0.2669	5.0	0.2544
4		0.01	0.2670	10	0.2493
8		0.0005	0.2310	0.5	0.2645
8		0.001	0.2506	1.0	0.2649
8		0.005	0.2590	5.0	0.2519
8		0.01	0.2578	10	0.2462
16		0.0005	0.2459	0.5	0.2517
16		0.001	0.2542	1.0	0.2517
16		0.005	0.2557	5.0	0.2451
16		0.01	0.2540	10	0.2399

Our experiment results are shown in Tab. II, Tab. VII. Generally speaking, it performs better when k is smaller. We deem the reason is that the larger the cluster number, the higher the feature dimension, and the more information is lost after dimensionality reduction. From the results we can find that linear kernel and rbf kernel perform similarly after LDA dimensionality reduction, but linear kernel performance is much better than rbf kernel after PCA dimensionality reduction. We think that the data put into the SVM is linearly separable, and the classification effect of the rbf kernel is greatly affected by the parameters. The parameters we choose are not suitable

TABLE III
ACCURACY OF STFT FEATURES BASED ON VLAD MODEL AFTER PCA

Acc. k	M	SIFT + VLAD + SVM + PCA			
		C	linear kernel	C	rbf kernel
4	0.0005	0.2389	0.5	0.0915	
4	0.001	0.2476	1.0	0.1574	
4	0.005	0.2513	5.0	0.1633	
4	0.01	0.2513	10	0.1628	
8	0.0005	0.2437	0.5	0.0590	
8	0.001	0.2499	1.0	0.0809	
8	0.005	0.2491	5.0	0.0886	
8	0.01	0.2489	10	0.0888	
16	0.0005	0.2644	0.5	0.0509	
16	0.001	0.2660	1.0	0.0627	
16	0.005	0.2646	5.0	0.0683	
16	0.01	0.2625	10	0.0683	

for data classification after VLAD feature encoding and PCA dimensionality reduction. Overall, VLAD performs better than BOW because it uses the residual of each descriptor with respect to its assigned cluster while BOW only involved simply counting the number of descriptors associated with each cluster in a codebook.

3) *Fisher Vector*: In this experiment, we used the FV model to encode the descriptors extracted by sift. The dimension of FV feature vector is $2*k*d$, where k is the cluster number and d is the dimension of each descriptor which equals 128. Because the feature dimension is too high, we use LDA and PCA for feature reduction like the VLAD experiment,. Besides, the experiments of FV also consume time and computing resources, we can't set many different cluster numbers and large cluster numbers. So we only set k in [4, 8, 16] for experiments. After feature encoding and reduction, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

TABLE IV
ACCURACY OF STFT FEATURES BASED ON FV MODEL AFTER LDA

Acc. k	M	SIFT + FV + SVM + LDA			
		C	linear kernel	C	rbf kernel
4	0.0005	0.2504	0.5	0.2696	
4	0.001	0.2661	1.0	0.2703	
4	0.005	0.2688	5.0	0.2589	
4	0.01	0.2689	10	0.2543	
8	0.0005	0.2494	0.5	0.2578	
8	0.001	0.2594	1.0	0.2561	
8	0.005	0.2607	5.0	0.2468	
8	0.01	0.2560	10	0.2448	
16	0.0005	0.2312	0.5	0.2334	
16	0.001	0.2351	1.0	0.2280	
16	0.005	0.2336	5.0	0.2198	
16	0.01	0.2315	10	0.2190	

Our experiment results are shown in Tab. IV, V. From the results we can find that FV performs better than BOW and VLAD. This is because Fisher Vector encodes a vector with richer image information which contains 1-order information and 2-order information. Besides in general, as k increases, the experimental results get worse. We deem the reason is that when k is larger, feature vectors resulting from feature encoding contain more redundant information. As for why

TABLE V
ACCURACY OF STFT FEATURES BASED ON FV MODEL AFTER PCA

Acc. k	M	SIFT + FV + SVM + PCA			
		C	linear kernel	C	rbf kernel
4	0.0005	0.2420	0.5	0.0476	
4	0.001	0.2489	1.0	0.0829	
4	0.005	0.2530	5.0	0.0988	
4	0.01	0.2525	10	0.0988	
8	0.0005	0.2556	0.5	0.0445	
8	0.001	0.2606	1.0	0.0447	
8	0.005	0.2597	5.0	0.0453	
8	0.01	0.2587	10	0.0453	
16	0.0005	0.2630	0.5	0.0436	
16	0.001	0.2655	1.0	0.0436	
16	0.005	0.2655	5.0	0.0436	
16	0.01	0.2658	10	0.0436	

when using PCA to reduce dimensionality, rbf kernel performs better than linear kernel, we think it is the same as the reason for this phenomenon in VLAD experiment.

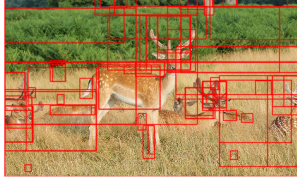
B. Selective Search + ResNet descriptors

We used selective search to extract the local proposal of the images and use pretrained ResNet to extract the local descriptor, then used BOW, VLAD, FV to encode the features, and finally put them into the SVM for classification. As shown in Fig. 6, when using selective search to extract image proposals in (a), we can limit the size of aim proposals and get the proposal-dropped one in (b), which can help us save the resources of computing.

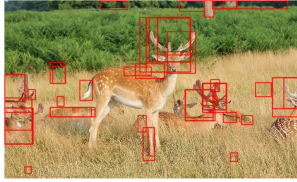
1) *BOW*: In this experiment, we used the BOW model to encode the descriptors extracted by selective search + pretrained ResNet, and set different clusters k in [8, 16, 32, 64, 128, 256, 512, 1024]. Besides, we normalized the feature vectors by z-score. After that, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

TABLE VI
ACCURACY OF SELECTIVE SEARCH + PRETRAINED RSENET FEATURES BASED ON BOW(Z-SCORE) MODEL

Acc. k	M	Selective Search + pretrained RseNet + BOW + SVM			
		C	linear kernel	C	rbf kernel
8	0.001	0.0817	5	0.1080	
8	0.005	0.1032	10	0.1036	
16	0.001	0.1105	5.0	0.1386	
16	0.005	0.1358	10	0.1299	
32	0.001	0.1695	5.0	0.1847	
32	0.005	0.1984	10	0.1718	
64	0.001	0.2108	5.0	0.2117	
64	0.005	0.2332	10	0.1977	
128	0.001	0.2481	5.0	0.2424	
128	0.005	0.2629	10	0.2322	
256	0.001	0.2626	5.0	0.2574	
256	0.005	0.2734	10	0.2482	
512	0.001	0.2586	5.0	0.2468	
512	0.005	0.2642	10	0.2393	
1024	0.001	0.2548	5.0	0.2376	
1024	0.005	0.2561	10	0.2326	



(a) original selective-search image



(b) proposal-dropped image

Fig. 6. ssdemo

Our experiment results are shown in Tab. VI. We can find that as k increases, the experimental results get better. We deem the reason is that when k is larger, the bag of words after codebook construction is larger, so the difference among the different types of images after feature encoding is also greater. From the results we can see BOW's performance is poor as its highest accuracy is only 0.2642. It is obvious because when mapping, BOW uses the information of first level in codebook and information has been lost. Compared with SIFT method, we can find selective search + ResNet have poor performance, we deem the reason that We use pretrained ResNet rather than self-trained model.

2) *VLAD*: In this experiment, we used the VLAD model to encode the descriptors extracted by selective search + pretrained RseNet. The dimension of VLAD feature vector is $k*d$, where k is the cluster number and d is the dimension of each descriptor which equals 128. Because the feature dimension is too high, we use PCA for feature reduction. Besides, the experiments of VLAD consume time and computing resources, we can't set many different cluster numbers and large cluster numbers. So we only set k in $[4, 8]$ for experiments. After feature encoding and reduction, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

Our experiment results are shown in Tab. ???. We conclude

TABLE VII
ACCURACY OF SELECTIVE SEARCH + PRETRAINED RSENET FEATURES
BASED ON VLAD MODEL AFTER PCA

Acc. k	M	Selective Search + pretrained RseNet + VLAD + SVM + PCA			
		C	linear kernel	C	rbf kernel
4	0.001	0.4805	5.0	0.0719	
	0.005	0.4864	10	0.0720	
8	0.001	0.4281	5.0	0.1107	
	0.005	0.4412	10	0.1098	

rbf ones performs better than linear one because rbf depends on initial parameters sampling while model is complicated. Overall, VLAD performs better than BOW because it uses the residual of each descriptor with respect to its assigned cluster while BOW only involved simply counting the number of descriptors associated with each cluster in a codebook. And we can find in VLAD, selective search + learning-based method have better performance than SIFT one because the former maintains more complete local information of images.

3) *Fisher Vector*: In this experiment, we used the FV model to encode the descriptors extracted by selective search + pretrained RseNet. The dimension of FV feature vector is $2*k*d$, where k is the cluster number and d is the dimension of each descriptor which equals 128. Because the feature dimension is too high, we use PCA for feature reduction like the VLAD experiment. Besides, the experiments of FV also consume time and computing resources, we can't set many different cluster numbers and large cluster numbers. So we only set k in $[2, 4]$ for experiments. After feature encoding and reduction, we fed encoded feature vectors into SVM for image classification and chose two different kernels of linear and rbf with the change of C .

TABLE VIII
ACCURACY OF SELECTIVE SEARCH + PRETRAINED RSENET FEATURES
BASED ON FV MODEL AFTER PCA

Acc. k	M	Selective Search + pretrained RseNet + FV + SVM + PCA			
		C	linear kernel	C	rbf kernel
2		0.001	0.3147	5.0	0.1548
		0.005	0.3440	10	0.1519
4		0.001	0.4637	5.0	0.0531
		0.005	0.4674	10	0.0531

Our experiment results are shown in Tab. VIII. From the results we can find that FV performs better than BOW but worse than VLAD. This is because Fisher Vector encodes 2-order information but our pretrained model can't extract the exact feature from local proposals. Besides in general, as k increases, the experimental results get worse. We deem the reason is that when k is larger, feature vectors resulting from feature encoding contain more redundant information. As for why when using PCA to reduce dimensionality, linear kernel performs better than rbf kernel, we think the reason that rbf performs poor if model is too complicated to get initial parameter sampling.

III. CONCLUSION

IV. FUTHER DISCUSSION

1) *Impact of scale method on BOW experiment:* We know that each dimension of the feature vector obtained by BOW is an integer. To avoid large dimensional differences, we need to standardize the feature vector. In section II-B1, we used Z-Score. In order to get the influence of standardized methods on the experimental results, in this experiment, we used the MaxMin.

TABLE IX
ACCURACY OF STFT FEATURES BASED ON BOW(MAXMIN-SCALE)
MODEL

Acc. k	M	SIFT + BOW + SVM			
		C	linear kernel	C	rbf kernel
8	0.0005	0.0430	0.5	0.0959	
8	0.001	0.0430	1.0	0.1080	
8	0.005	0.0430	5.0	0.1208	
8	0.01	0.0436	10	0.1247	
16	0.0005	0.0435	0.5	0.0965	
16	0.001	0.0435	1.0	0.1136	
16	0.005	0.0435	5.0	0.1428	
16	0.01	0.0450	10	0.1556	
32	0.0005	0.0456	0.5	0.0874	
32	0.001	0.0456	1.0	0.1154	
32	0.005	0.0464	5.0	0.1619	
32	0.01	0.0557	10	0.1773	
64	0.0005	0.0440	0.5	0.0760	
64	0.001	0.0440	1.0	0.1063	
64	0.005	0.0486	5.0	0.1711	
64	0.01	0.0608	10	0.1870	
128	0.0005	0.0442	0.5	0.0636	
128	0.001	0.0442	1.0	0.0977	
128	0.005	0.0556	5.0	0.1708	
128	0.01	0.0743	10	0.1904	
256	0.0005	0.0433	0.5	0.0587	
256	0.001	0.0433	1.0	0.0826	
256	0.005	0.0657	5.0	0.1730	
256	0.01	0.0975	10	0.2006	
512	0.0005	0.0468	0.5	0.0523	
512	0.001	0.0468	1.0	0.0756	
512	0.005	0.0916	5.0	0.1738	
512	0.01	0.1345	10	0.2071	
1024	0.0005	0.0430	0.5	0.0439	
1024	0.001	0.0442	1.0	0.0568	
1024	0.005	0.0568	5.0	0.1399	
1024	0.01	0.1418	10	0.1772	

Our experiment results are shown in Tab. IX. We can find that Z-Score performs better than MaxMin. We speculate that there may be outliers in the feature vector that affect the experimental results. Therefore Z-Score is more suitable for this project.

2) *FV containing only first-order information:* We know that FV uses gradient vectors of likelihood functions to encode pictures. In general, it contains both first-order information (expectation) and second-order information (variance). Therefore, in this section, we set the FV to include only first-order information, and conduct a comparative experiment with section II-B3.

Our experiment results are shown in Tab. X, XI. From the results we can find that FV containing only first-order information even performs slightly better than FV containing

TABLE X
ACCURACY OF STFT FEATURES BASED ON FV(1-ORDER) MODEL AFTER
LDA

Acc. k	M	SIFT + FV + SVM + LDA			
		C	linear kernel	C	rbf kernel
4	0.0005	0.2405	0.5	0.2698	
4	0.001	0.2626	1.0	0.2723	
4	0.005	0.2702	5.0	0.2576	
4	0.01	0.2678	10	0.2494	
8	0.0005	0.2446	0.5	0.2694	
8	0.001	0.2610	1.0	0.2662	
8	0.005	0.2686	5.0	0.2552	
8	0.01	0.2653	10	0.2493	
16	0.0005	0.2487	0.5	0.2555	
16	0.001	0.2561	1.0	0.2538	
16	0.005	0.2550	5.0	0.2471	
16	0.01	0.2515	10	0.2428	

TABLE XI
ACCURACY OF STFT FEATURES BASED ON FV MODEL AFTER PCA

Acc. k	M	SIFT + FV + SVM + PCA			
		C	linear kernel	C	rbf kernel
4	0.0005	0.2440	0.5	0.1214	
4	0.001	0.2496	1.0	0.1659	
4	0.005	0.2507	5.0	0.1733	
4	0.01	0.2473	10	0.1733	
8	0.0005	0.2496	0.5	0.0588	
8	0.001	0.2560	1.0	0.1022	
8	0.005	0.2568	5.0	0.1154	
8	0.01	0.2558	10	0.1154	
16	0.0005	0.2593	0.5	0.0445	
16	0.001	0.2615	1.0	0.0599	
16	0.005	0.2613	5.0	0.0702	
16	0.01	0.2604	10	0.0702	

first- and second-order information. From this we can deduce that sometimes FV can only use first-order information for feature encoding, which also reduces the requirements for computing resources. Besides, we can find that also containing only first-order information, FV performs better than VLAD. We think the reason is that GMM clustering can extract feature information better than K-Means clustering.

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