REPUBLIC OF THE PHILIPPINES
BATANGAS STATE UNIVERSITY
THE NATIONAL ENGINEERING UNIVERSITY
COLLEGE OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT

LABORATORY ACTIVITY 1

AS A REQUIREMENT FOR THE COURSE
ECE 425 CONTROL SYSTEMS ENGINEERING

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Part 1.

Solve for the Inverse Laplace Transform of the following:

1. $L\{3 - e^{-3t} + 5\sin 2t\} = F(s)$

$F(s) = 3\mathcal{L}\{1\} - \mathcal{L}\{e^{-3t}\} + 5\mathcal{L}\{\sin 2t\}$

$= \frac{3}{s} - \frac{1}{s+a} + \frac{\omega}{s^2+\omega^2}(5)$; where $a = 3$, $\omega = 2$

$= \frac{3}{s} - \frac{1}{s+3} + (5)\frac{2}{s^2+2^2}$

$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$

2. $\mathcal{L}\{3 + 12t + 42t^3 - 3e^{2t}\} = F(s)$

$F(s) = 3\mathcal{L}\{1\} + 12\mathcal{L}\{t\} + 42\mathcal{L}\{t^3\} - 3\mathcal{L}\{e^{2t}\}$

$= \frac{3}{s} + \frac{12}{s^2} + 42\left(\frac{n!}{s^{n+1}}\right) - 3\left(\frac{1}{s+a}\right)$; where: $n = 3$, $a = -2$

$= \frac{3}{s} + \frac{12}{s^2} + 42\left(\frac{3!}{s^{3+1}}\right) - 3\left(\frac{1}{s-2}\right)$

$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$

$= 12\left(\frac{1}{4s} + \frac{1}{s^2}\right) + \frac{252}{s^4} - \frac{3}{s-2}$

$\boxed{F(s) = \frac{12\left(\frac{s}{4}+1\right)}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$

3. $\mathcal{L}\{(t+1)(t+2)\} = F(s)$

$F(s) = \mathcal{L}\{t^2 + 3t + 2\}$

$= \mathcal{L}\{t^2\} + 3\mathcal{L}\{t\} + 2\mathcal{L}\{1\}$

$= \frac{n!}{s^{n+1}} + \frac{3}{s^2} + \frac{2}{s}$; where: $n = 2$

$= \frac{2!}{s^{2+1}} + \frac{3}{s^2} + \frac{2}{s}$

$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$

$= \frac{2}{s^3} + 3\left(\frac{1}{s^2} + \frac{2}{3s}\right)$

$\boxed{F(s) = \frac{2}{s^3} + 3\left(\frac{2s/3 + 1}{s^2}\right)}$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left\{\dfrac{8-3s+s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\dfrac{8}{s^3}-\dfrac{3s}{s^3}+\dfrac{s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\dfrac{8}{s^3}-\dfrac{3}{s^2}+\dfrac{1}{s}\right\} = f(t)$

$$f(t) = 8\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$= 8\left(\frac{1}{2!}\right)\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\} - 3\left(\frac{1}{1!}\right)\mathcal{L}^{-1}\left\{\frac{1!}{s^2}\right\} + 1$$

$$= \frac{8}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\} - \frac{3}{1!}\mathcal{L}^{-1}\left\{\frac{1!}{s^2}\right\} + 1$$

$$= \frac{8}{2!}t^2 - \frac{3}{1!}t + 1$$

$\boxed{f(t) = 4t^2 - 3t + 1}$

2. $\mathcal{L}^{-1}\left\{\dfrac{5}{s-2}-\dfrac{4s}{s^2+9}\right\} = f(t)$

$$f(t) = 5\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$$

$$= 5(e^{2t}) - 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+(3)^2}\right\}$$

$$= 5e^{2t} - 4\cos(3t)$$

$\boxed{f(t) = 5e^{2t} - 4\cos 3t}$

3. $\mathcal{L}^{-1}\left\{\dfrac{7}{s^2+6}\right\} = f(t)$

$$f(t) = 7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\}$$

$$= 7\left(\frac{1}{\sqrt{6}}\right)\mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+(\sqrt{6})^2}\right\}$$

$$= \left(\frac{7}{\sqrt{6}}\sin\sqrt{6}t\right)\frac{\sqrt{6}}{\sqrt{6}}$$

$\boxed{f(t) = \dfrac{7\sqrt{6}}{6}\sin\sqrt{6}\,t}$

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PART 3

1. $F(S) = \dfrac{1}{S(S^2+2S+2)}$

$$\mathcal{L}^{-}\left\{\frac{1}{S(S^2+2S+2)}\right\} = \frac{A}{S} + \frac{BS+C}{S^2+2S+2}$$

$1 = A(S^2+2S+2) + S(BS+C)$

↳ if $S=0$

$1 = 2A$

$A = 1/2$

$$1 = \frac{S^2+2S+2}{2} + BS^2 + CS$$

$$1 = \frac{S^2+2S+2+2BS^2+2CS}{2}$$

$2 = S^2+2S+2+2BS^2+2CS$

$B = -1/2$

$C = -1$

$$\mathcal{L}^{-}\left\{\frac{1/2}{S} - \frac{1/2\,S-1}{S^2+2S+2}\right\}$$

$$\frac{1}{2} - \frac{1}{2}\mathcal{L}^{-}\left\{\frac{S+2}{S^2+2S+2}\right\} = \frac{(S+1)+1}{(S^2+2S+1)+1}$$

$$= \frac{(S+1)+1}{(S+1)^2+1} \quad ; \; w=1, \; a=1$$

$$= -1/2\, e^{-t} \cos t + \sin t$$

$= 1/2 - 1/2\, e^{-t}(\cos t + \sin t)$

$$\boxed{f(t) = \tfrac{1}{2}[1 - e^{-t}(\cos t + \sin t)]}$$

2. $F(S) = \dfrac{5(S+2)}{S^2(S+1)(S+3)}$

$$5\mathcal{L}^{-}\left\{\frac{S+2}{S^2(S+1)(S+3)}\right\} = \frac{A}{S^2} + \frac{B}{S} + \frac{C}{S+1} + \frac{D}{S+3}$$

$S+2 = [A(S+1)(S+3)] + [B(S)(S+1)(S+3)] + [C(S^2)(S+3)] + [D(S^2)(S+1)]$

↳ if $S=0$

$2 = 3A$

$A = 2/3$

if $S=-1$

$1 = 2C$

$C = 1/2$

if $S=-3$

$-1 = 18D$

$D = -1/18$

if $S=1$

$3 = 16/3 + 8B + 2 + 1/9$

$-40/9 = 8B$

$B = -5/9$

$$5\mathcal{L}^{-}\left\{\frac{2/3}{S^2} - \frac{5/9}{S} + \frac{1/2}{S+1} + \frac{1/18}{S+3}\right\}$$

$$\frac{2}{3}\mathcal{L}^{-}\left\{\frac{1}{S^2}\right\} - \frac{5}{9}\mathcal{L}^{-}\left\{\frac{1}{S}\right\} + \frac{1}{2}\mathcal{L}^{-}\left\{\frac{1}{S+1}\right\} - \frac{1}{18}\mathcal{L}^{-}\left\{\frac{1}{S+3}\right\}$$

$$5\left[\frac{2}{3}t - \frac{5}{9} + \frac{1}{2}e^{-t} - \frac{1}{18}e^{-3t}\right]$$

$$\boxed{f(t) = \frac{10}{3}t - \frac{25}{9} + \frac{5}{2}e^{-t} - \frac{5}{18}e^{-3t}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$= \mathcal{L}^{-}\left\{ \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s^2 + s} \right\}$$

$$\begin{array}{r l}
 & s^2 + s + 2 \\
s^2 + s \,\big)\!\! & \overline{s^4 + 2s^3 + 3s^2 + 4s + 5} \\
 & \underline{-s^4 + s^3} \\
 & s^3 + 3s^2 \\
 & \underline{-s^3 + s^2} \\
 & 2s^2 + 4s \\
 & \underline{-2s^2 + 2s} \\
 & 2s + 5
\end{array}$$

$$= \mathcal{L}^{-}\left\{ s^2 + s + 2 + \frac{2s+5}{s^2+s} \right\}$$

$$f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta + \frac{2s+5}{s^2+s}$$

$$\mathcal{L}^{-}\left\{ \frac{2s+5}{s(s+1)} \right\}$$

$$\mathcal{L}^{-}\left\{ \frac{(2s+2)+3}{s(s+1)} \right\}$$

$$2\mathcal{L}^{-}\left\{ \frac{s+1}{s(s+1)} \right\} + 3\mathcal{L}^{-}\left\{ \frac{1}{s(s+1)} \right\}$$

$$2\mathcal{L}^{-}\left\{ \frac{1}{s} \right\} + 3\mathcal{L}^{-}\left\{ \frac{1}{s(s+1)} \right\} = \frac{A}{s} + \frac{B}{s+1}$$

$1 = A(s+1) + Bs$

$\hookrightarrow$ if $s = -1$

$1 = -B$

$B = -1$

if $s = 0$

$A = 1$

$$3\mathcal{L}^{-}\left\{ \frac{1}{s} - \frac{1}{s+1} \right\}$$

$2 \quad + 3[1 - e^{-t}]$

$5 - 3e^{-t}$

$$\boxed{f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta + 5 - 3e^{-t}}$$