Laplace transform of differential equations: MX3202

Wednesday, 7 February 2024 11:13 am

$$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$$

$$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$$

$$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0-)$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$\mathcal{L}[y'(t)] = sY(s) - y(0)$$

$$\mathcal{L}[y''(t)] = s^2Y(s) - sy(0) - y'(0)$$

$$\mathcal{L}[y'''(t)] = s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$$

$$\mathcal{L}[y''''(t)] = s^4Y(s) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0)$$

$$\mathcal{L}\left[\frac{d^5y}{dt^5}\right] = \mathcal{L}[y'''''] = s^5Y(s) - s^4y(0) - s^3y'(0) - s^2y''(0) - sy'''(0) - y''''(0)$$

$$\mathcal{L}\left[\frac{d^5f}{dt^5}\right] = s^5F(s)$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)\right\} \rightarrow \left[s^2Y(s) - \cancel{sy(0)}^{0} - \cancel{y'(0)}^{0}\right] + 12\left[sY(s) - \cancel{y(0)}^{0}\right] + 32\left[Y(s)\right] = \frac{32}{s}$$

initial conditions are zero

$$s^2Y(s) + 12sY(s) + 32Y(s) = \frac{32}{s}$$

$$Y(s)\left[s^2 + 12s + 32\right] = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2+12s+32)}$$

$$\boxed{Y(s) = \frac{32}{s(s+8)(s+4)}}$$

$$\mathcal{L}\left\{\frac{d^2x}{dt^2}+4\frac{dx}{dt}+5x=1\right\} \rightarrow \left[s^2X(s)-s\overset{1}{x(0)}-\overset{-1}{\dot{x}(0)}\right]+4\left[sX(s)-\overset{1}{x(0)}\right]$$

initial conditions $x(0) = 1$, $\dot{x}(0) = -1$.

$$+5X(s) = \frac{1}{s}$$

$$\rightarrow s^2X(s) + 4sX(s) - 4 + 5X(s) = \frac{1}{s}$$

$$4 + X(s)\left[s^2+4s+5\right] - 4 = \frac{1}{s} + 4$$

$$X(s)\left[s^2+4s+5\right] = \frac{4s+1}{s}$$

$$\boxed{X(s) = \frac{4s+1}{s(s^2+4s+5)}}$$