Review of Laplace Transform: ME-4205

Wednesday, 7 February 2024 6:19 pm

$$\mathcal{L}\{f(t)\} = F(s)$$

Time Domain → Frequency Domain

DE $\longrightarrow$ Algebra

DE $\longleftarrow$ Algebra

$$\mathcal{L}^{-}\{F(s)\} = f(t)$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

p.g 36 Table 2.1 Laplace transform table

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$ or 1	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

1) $\mathcal{L}\{f(t)\} = \mathcal{L}\{t^4\}$

$$\frac{n!}{s^{n+1}} \; ; \; n = 4$$

$$F(s) = \frac{4!}{s^{4+1}}$$

$$\boxed{F(s) = \frac{24}{s^5}}$$

2) $\mathcal{L}\{f(t)\} = \mathcal{L}\{\sin 2t\}$

$$; \; \frac{\omega}{s^2+\omega^2} \, , \; \omega = 2$$

$$\boxed{F(s) = \frac{2}{s^2+4}}$$

3) $\mathcal{L}\{f(t)\} = 5e^{2t} - t^3 + 7$

① $5\,\mathcal{L}\{e^{2t}\} = \dfrac{5}{s-2}$

② $\mathcal{L}\{t^3\} = \dfrac{6}{s^4}$

③ $7\,\mathcal{L}\{1\} = \dfrac{7}{s}$

$$\boxed{F(s) = \frac{5}{s-2} - \frac{6}{s^4} + \frac{7}{s}}$$

Inverse Laplace

1) $\mathcal{L}^{-}\{F(s)\} = \mathcal{L}^{-}\left\{\dfrac{1}{s-2}\right\}$

$a = -2$

$$\boxed{f(t) = e^{2t}u(t)}$$

2) $\mathcal{L}^{-}\{F(s)\} = \mathcal{L}^{-}\left\{\dfrac{1}{2s-1}\right\}; a = -\frac{1}{2}$

$1 = \frac{2}{2}$

$= \dfrac{1}{2}\mathcal{L}^{-}\left\{\dfrac{1}{s-\frac{1}{2}}\right\}$

$$\boxed{f(t) = \frac{1}{2}e^{\frac{1}{2}t}u(t)}$$

3) $\mathcal{L}^{-}\{F(s)\} = \mathcal{L}^{-}\left\{\dfrac{1}{s^2+3}\right\}; \omega = \sqrt{3}$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s^2+(\sqrt{3})^2} \right\}; 1 = \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \mathcal{L}^{-1}\left\{ \frac{\sqrt{3}}{s^2+(\sqrt{3})^2} \right\}$$

$$= \left[\frac{1}{\sqrt{3}} \sin \sqrt{3} t \, u(t) \right] \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{f(t) = \frac{\sqrt{3}}{3} \sin \sqrt{3} t \, u(t)}$$