CEE 362G Project 1

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1. Here, we considered four examples of know s(t) and evaluated their corresponding y(t).

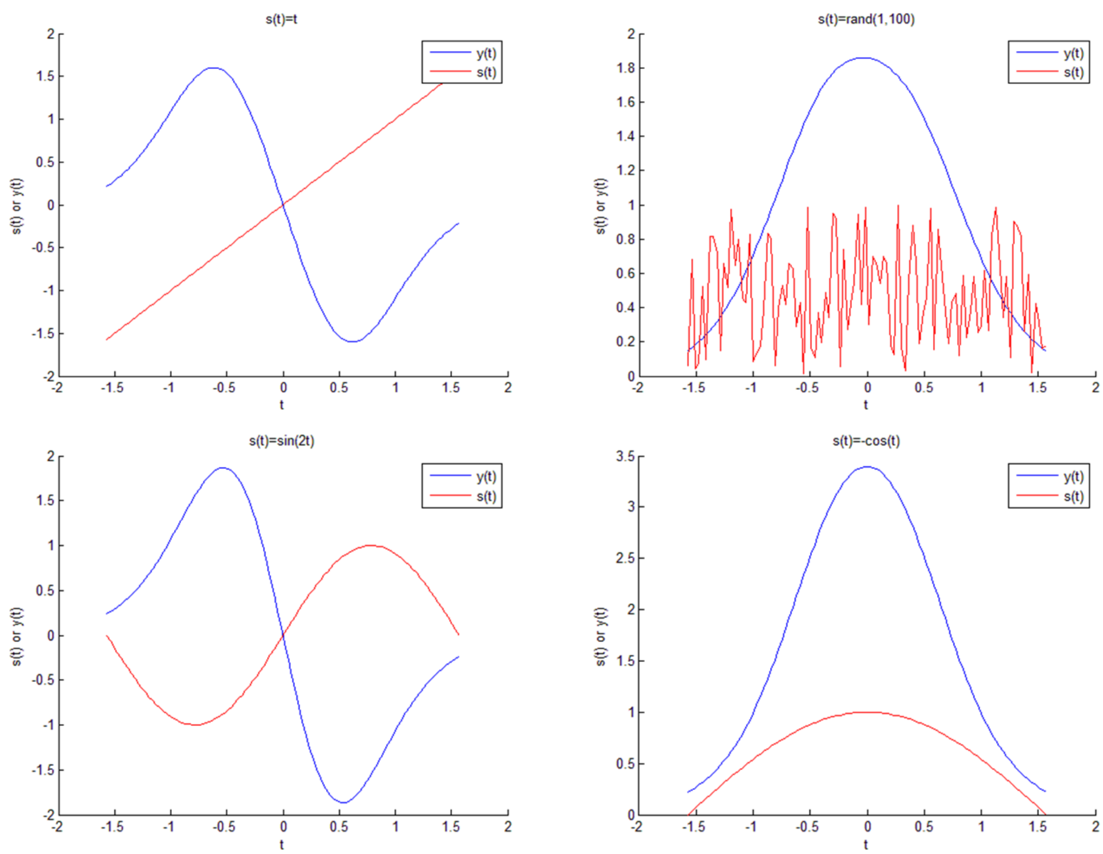

Figure 1 Examples of know s(t) and resulting y(t)

In the first example, the relationship between s and t is linear, s(t) = t. After blurring by this kernel K, the resulting y(t) becomes highly non-linear.

In the second example, s(t) is independent of t, it is random and follows a uniform distribution. Therefore, the true s(t) is very noisy. However, after blurring, the resulting y(t) is very smooth.

In the third and fourth example, s(t) are trigonometric function. In the third example, the resulting y(t) is opposite in sign with respect to s(t). In the fourth example, the resulting y(t) is amplified compared to s(t).

From the four examples considered, we can see the effects of blurring caused by this kernel are very strong and highly nonlinear. It infers that most information contained in the original image s(t) is lost. It is almost impossible to see what the original image s(t) looks like from the resulting blurred image y(t).

Another important observation is that y(t) is of similar magnitude with s(t) for all the four examples considered.

2. From the distribution of the spectrum of H, we can see the spectrum of H decays very fast. This indicates the rank of H is very low and the condition number of H is quite large. Therefore, the linear equation Hs(t)=y(t) is very ill-conditioned. In this case, a small perturbation to y(t) can cause large impacts on s(t).

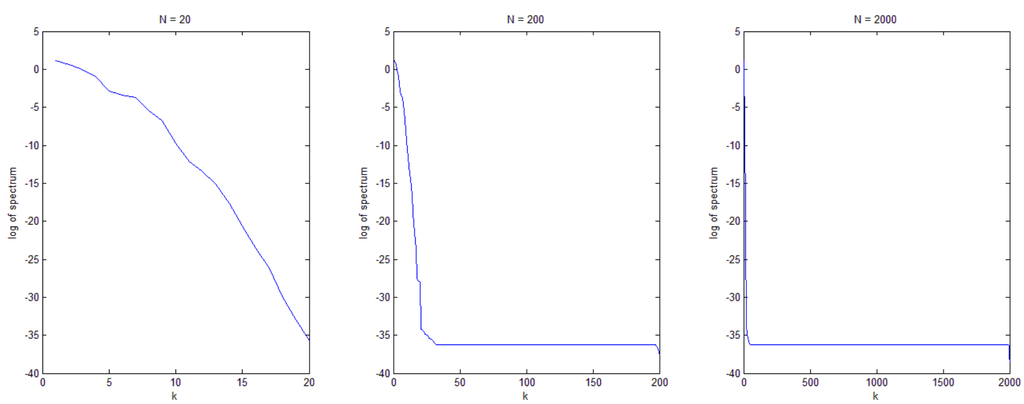

Figure 2 Distribution of spectrum of H

3. In this case, MATLAB gives rank(H)=20. We can see that the first 20 vectors in V present different periodic trends. These 20 vectors can be considered as the ‘most important’ vectors that span the rowspace of H. Other vectors are the linear combinations of these 20 vectors.

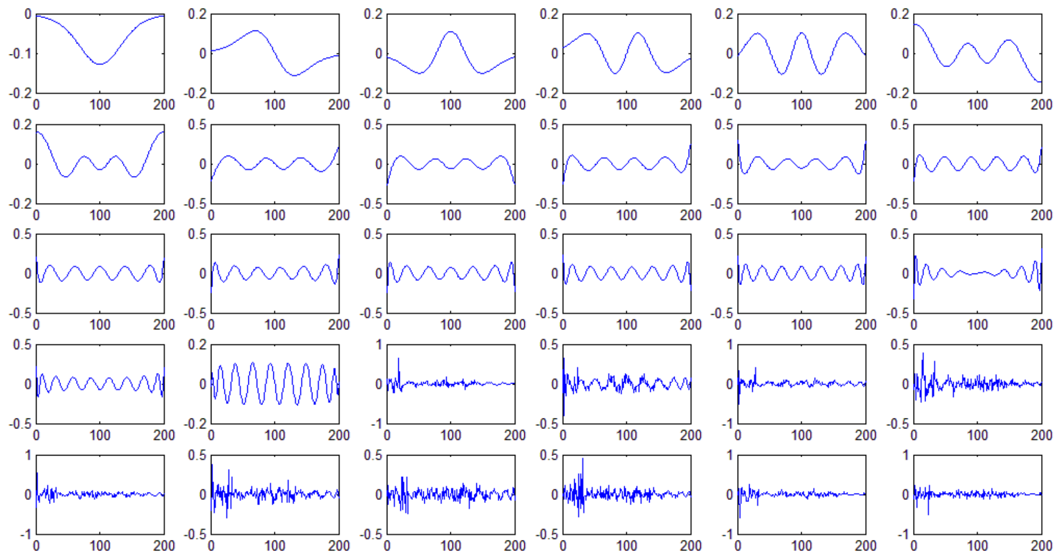

Figure 3 Column Vectors of Matrix V (the first 20 are 'most important')

4. From Figure 5, we can see for this particular s(t), using Pesudo-inverse yielded the right result. This indicates that this s(t) lies on the row space of H.

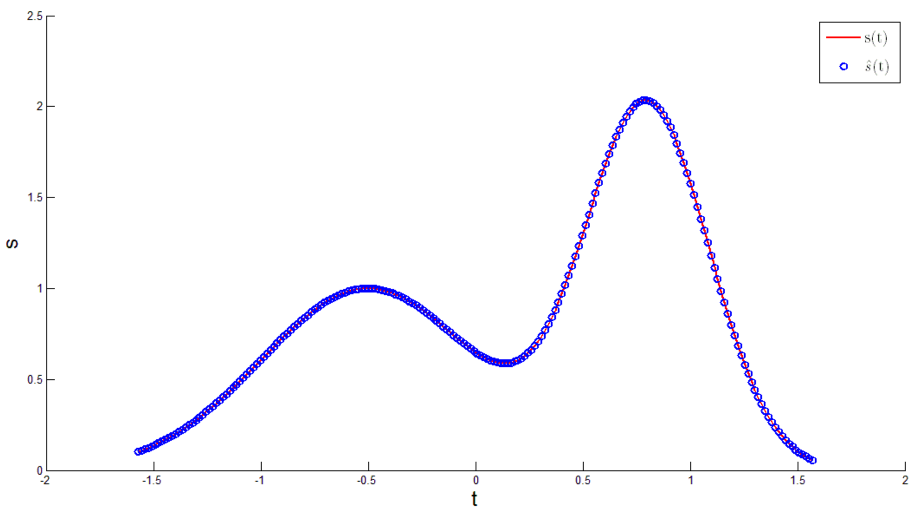

Figure 4 Comparison between the True s and the s from Pseudo-inverse (without noise)

5. After adding noise, even a very small amount, the pseudo-inverse solution deviates significantly from the true s(t). This is consistent with the fact that H has low rank and high condition number.

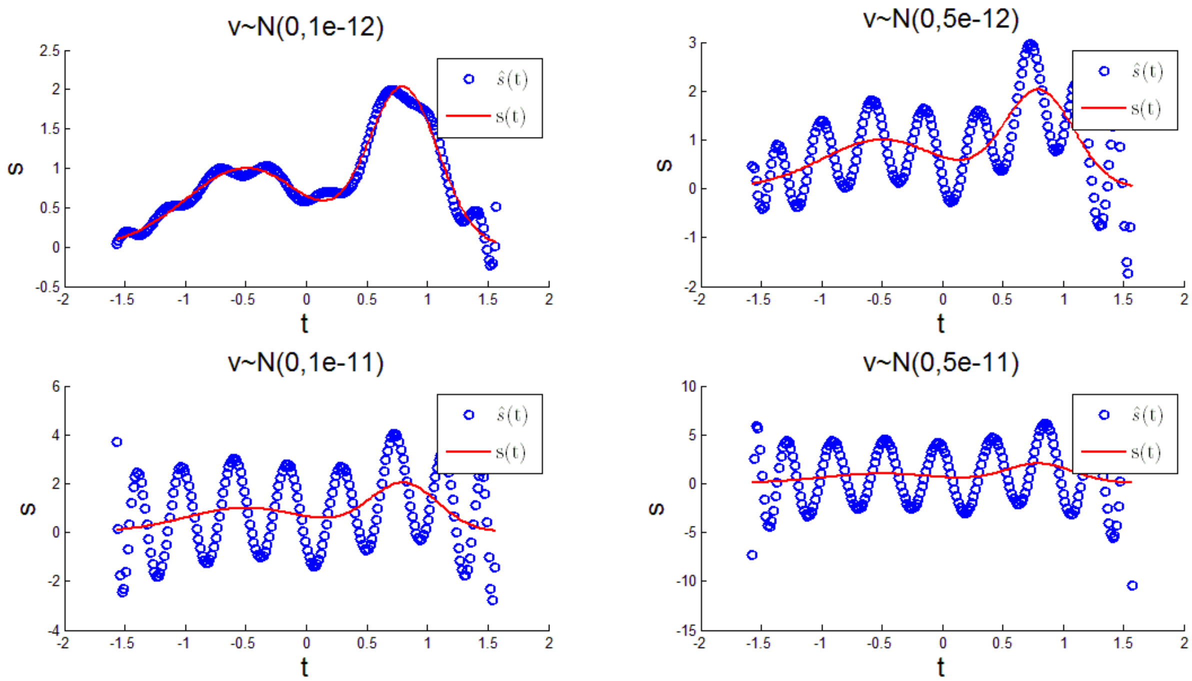

Figure 5 Comparison between the True s and the s from Pseudo-inverse (with noise)

6. In this section, we were given data coordinate *t* and data value (blurred image) *y(t)*. The linear inversion method is used to estimate true image *s(t)*. As we are dealing with an image deblurring problem. We made one additional assumptions about what the true image should look like.

   True image s(t) should be of same magnitude with the blurred image y(t).

   This assumption is consistent with our observation in problem 1 and problem 4. Based on this assumption, we determine whether the solution from the linear inversion method is reasonable or not.

   We tried different Q and R to see their effects on the solution. The data error *v* is assumed to be independently identical distributed (IID), namely its covariance matrix $R=\sigma(v)I$, where I is the identity matrix. For the parameter covariance matrix Q, we tried different covariance types, including linear, exponential, Gaussian and nugget. We also varied the standard deviation σ(s) and correlation length *l*.

In case 1, we set the standard deviation of *s(t)* to be a very small value, $\sigma(s)=10^{-3}$ to mimic the scenario when we significantly underestimate the variance of parameters, the correlation length *l* is set to be $\frac{\pi}{10}$. The standard deviation of error *σ(v)* is set to be 0.1. Result is shown in Figure 6. We can see that all solutions are very flat compared with *y(t)*. This is not consistent with our additional assumption in terms that solution *s(t)* is of much smaller magnitude compared with *y(t)*. Therefore, this solution is not reasonable.

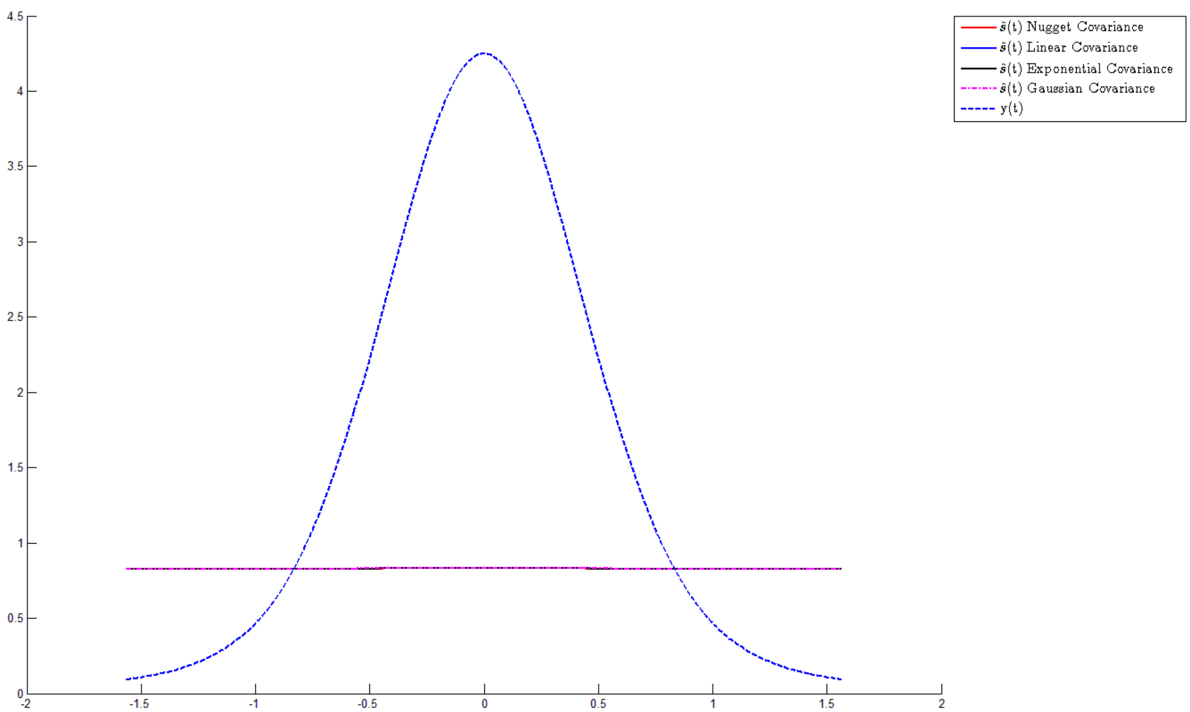

Figure 6 Solution s(t) in case 1 as compared with y(t)

In case 2, we set the standard deviation of *v* to be a very small value, $\sigma(v)=10^{-12}$ to mimic the scenario when we significantly underestimate the variance of measurement error, the correlation length *l* is set to be $\frac{\pi}{10}$. The standard deviation of parameters *σ(s)* is set to be 2. Result is shown in Figure 7. We can see that solutions are of much bigger magnitude compared with *y(t)*. In addition, these solutions resemble the pseudo-inverse solution with noise in problem 5. Therefore, we believe they are not reasonable neither.

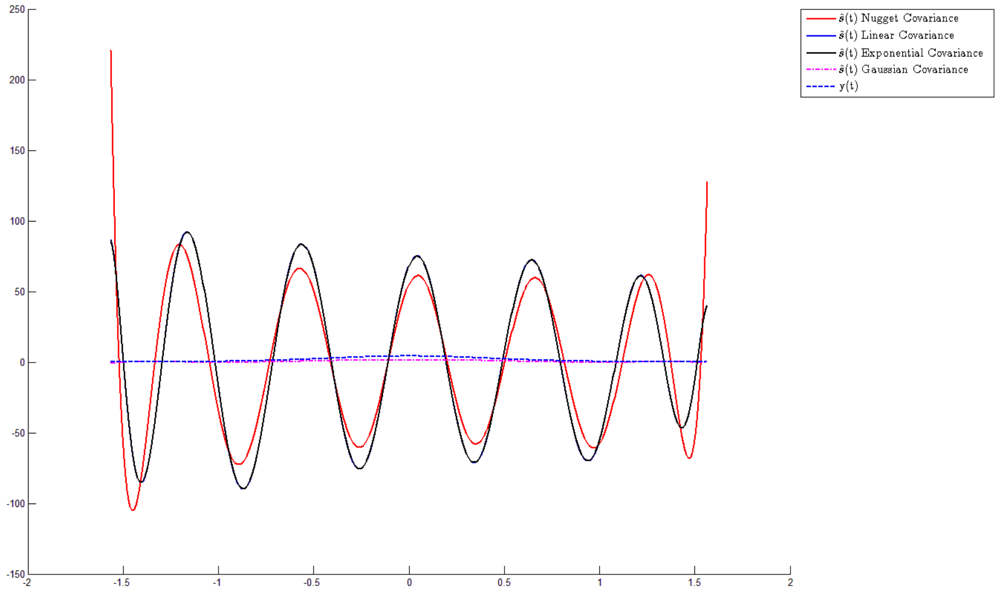

Figure 7 Solution s(t) in case 2 as compared with y(t)

In case 3, we chose σ(v)=0.1 and *σ(s)=var(y)=2*. The correlation length *l* is kept to be $\frac{\pi}{10}$. Result is shown in Figure 8. In this case, the magnitude of *s(t)* and *y(t)* is similar. In addition, solutions with different covariance types are similar to each other. We believe these solutions are reasonable.

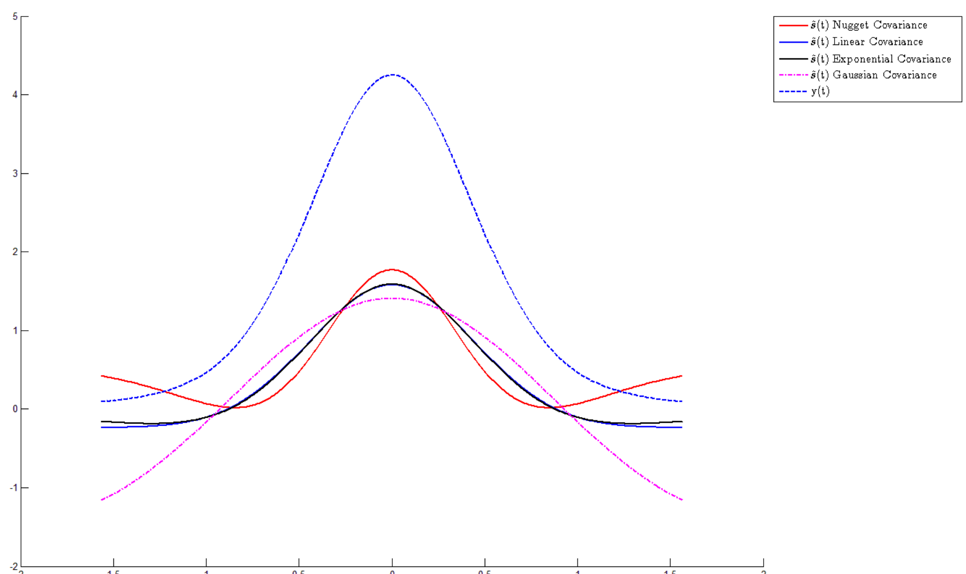

Figure 8 Solution *s(t)* in case 3 as compared with *y(t)*

The posterior standard deviation of s(t) is plotted in Figure 9. It can be seen, in this case, Nugget covariance gives highest posterior standard deviation, while Gaussian prior covariance gives the lowest. In addition, linear covariance and exponential covariance results are very close to each other. Another observation is that s(t) has less variance when t is closer to 0.

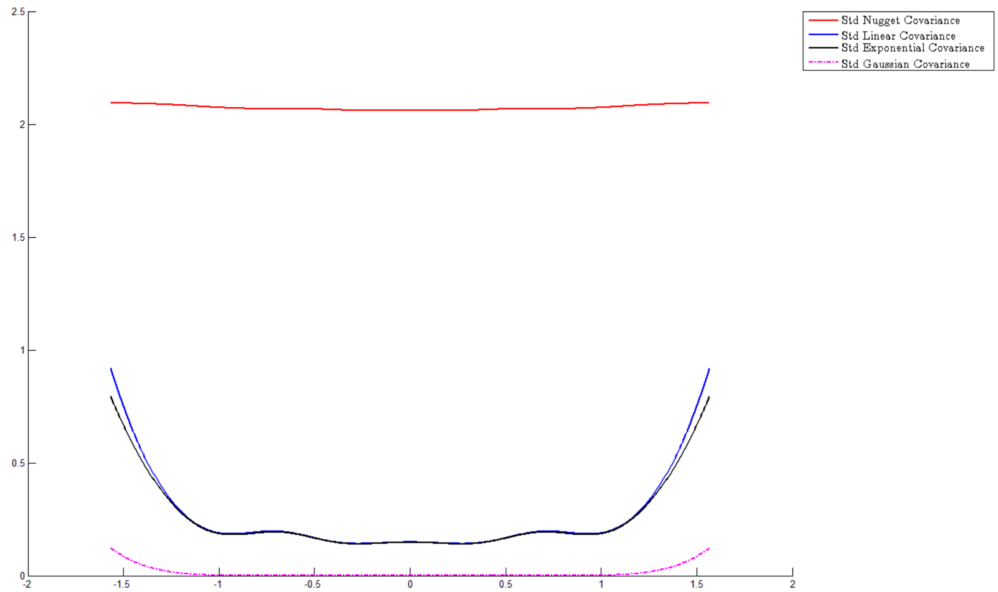

Figure 9 Posterior standard deviation of s(t) in case 3

We also plotted $\hat{y}(t) = H\hat{s}(t)$ as compared to *y(t)*. From Figure 10, we can see all $\hat{y}(t)$ match *y(t)* very well.

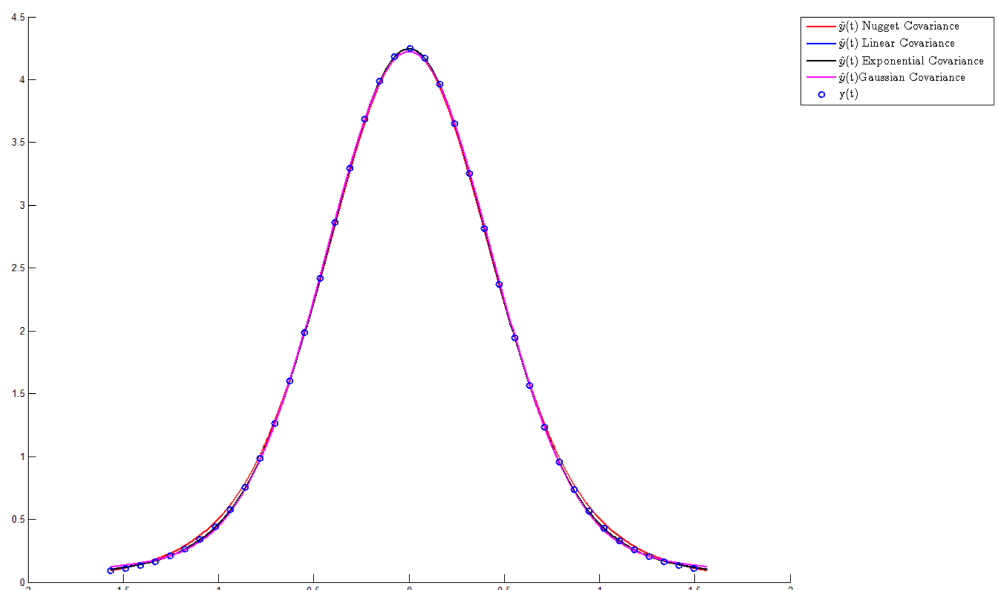

Figure 10 $\hat{y}(t)$ compared with *y(t)* in case 3

7. Ten conditional realizations are generated using the solution with exponential covariance in case 3, as shown in Figure 11. It is clear that conditional realizations are more noisy than the best estimate $\hat{s}(t)$ (red curve). In addition, in all conditional realizations, *s(t)* is less variant when t is close to 0, which is consistent with the posterior standard deviation results.

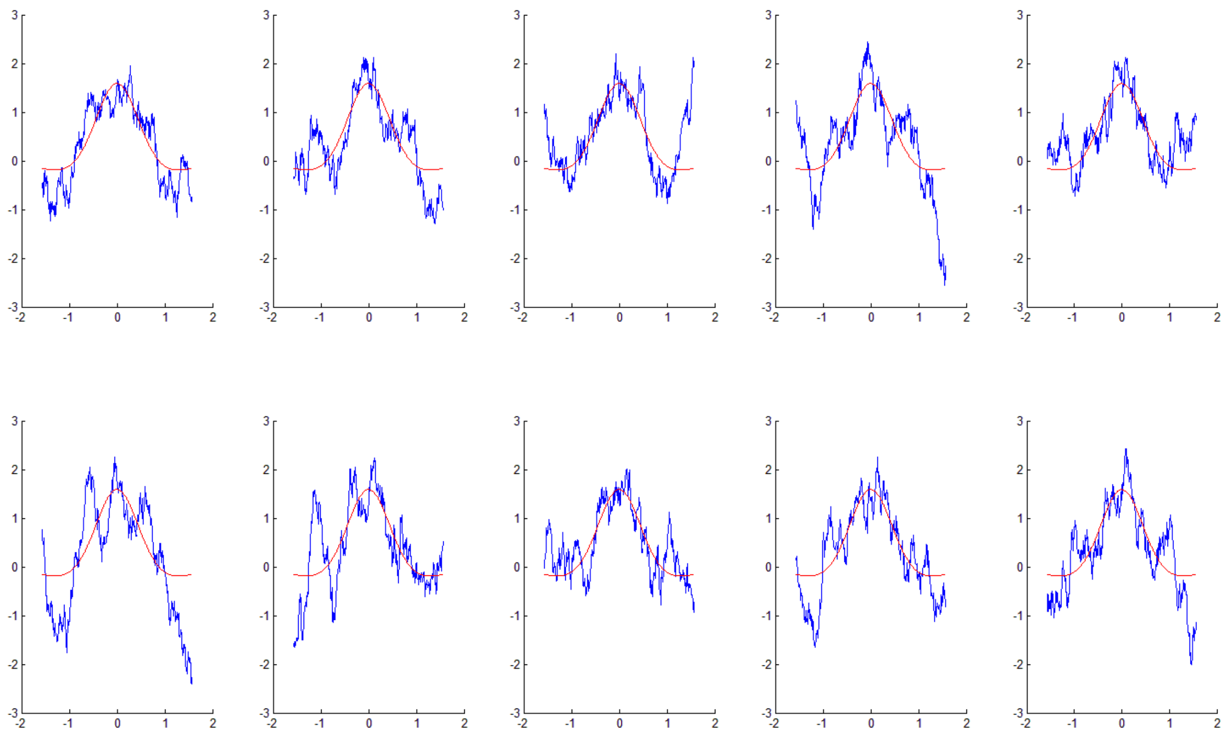

Figure 11 Ten conditional realizations of s(t) with exponential covariance