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Course Home

Advanced Logic Design

DSP Hardware Design

Methodologies

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Readings: Parhi Sec. 2 and 3

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VLSI Digital Signal Processing Systems: Design and Implementation by Prof. K. K. Parhi (UMinn)

Representing DSP Algorithms

- Example: 3-tap FIR filter

$$y(n) = ax(n) + bx(n-1) + cx(n-2)$$

- non-terminating, i.e., $n = 1 \dots \infty$

- Definitions:

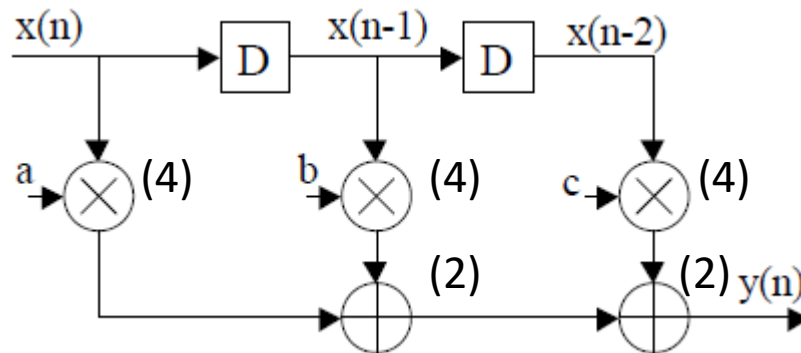
- **Iteration**: execution of all the computations in the algorithm once
- **Iteration period**: the time required for one iteration
- **Sampling rate (or throughput)**: number of samples processed per second
- **Critical path**: the longest path (computation time) between inputs and outputs (in combinational circuits), or the longest path between any two sequential elements (in sequential circuits)
- **Minimum clock period**: determined by the critical path
- **Latency**: the difference between the time an output is generated and the time at which its corresponding input was received by the system
 - Combinational circuit: latency is in terms of absolute time units
 - Sequential circuit: latency is in terms of number of clock cycles

(1) Block Diagram

- Example: 3-tap FIR filter

$$y(n) = ax(n) + bx(n-1) + cx(n-2)$$

- non-terminating, i.e., $n = 1 \dots \infty$



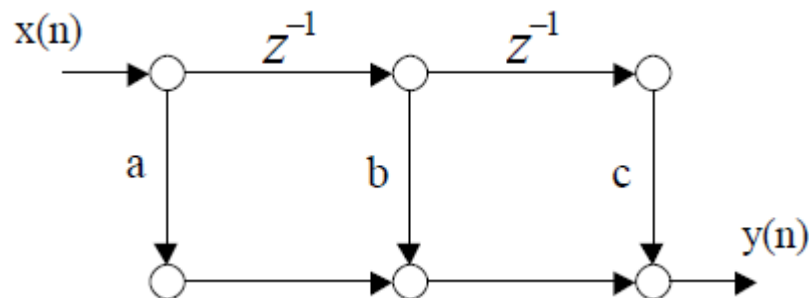
- Consists of functional blocks connected with directed edges
- Represents data flow from its input to output
- Critical path is the longest path among paths that do not contain delay elements
- 4 types of paths: input-delay, delay-output, input-output, delay-delay
- Clock period is lower bounded by the critical path computation time

(2) Signal-Flow Graph (SFG)

- Example: 3-tap FIR filter

$$y(n) = ax(n) + bx(n-1) + cx(n-2)$$

- non-terminating, i.e., $n = 1 \dots \infty$



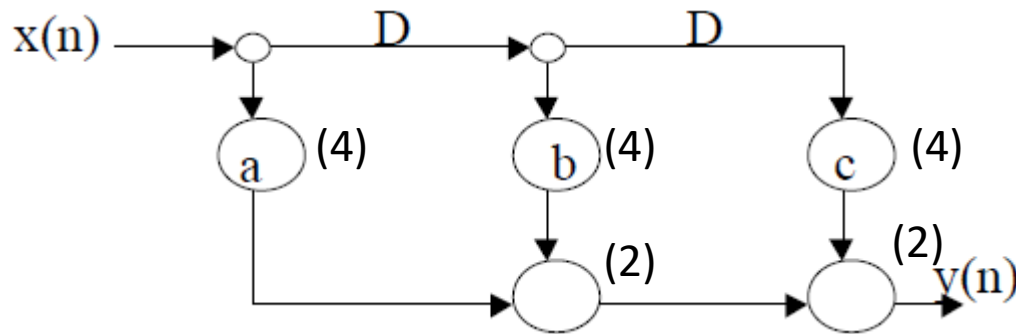
- Represent additions and multiplications
- Nodes represent variables for a computation
- A directed edge denotes multiplication or delay
- Multiple incoming edges and one outgoing edge denotes addition
- Only used for linear systems; cannot be used to describe multirate DSP systems

(3) Data-Flow Graph (DFG)

- Example: 3-tap FIR filter

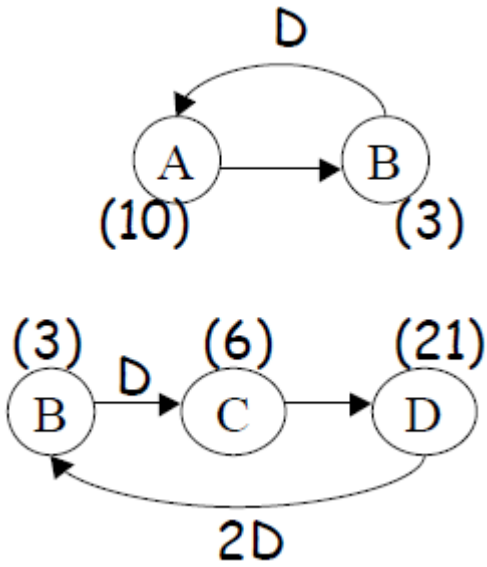
$$y(n) = ax(n) + bx(n-1) + cx(n-2)$$

- non-terminating, i.e., $n = 1 \dots \infty$



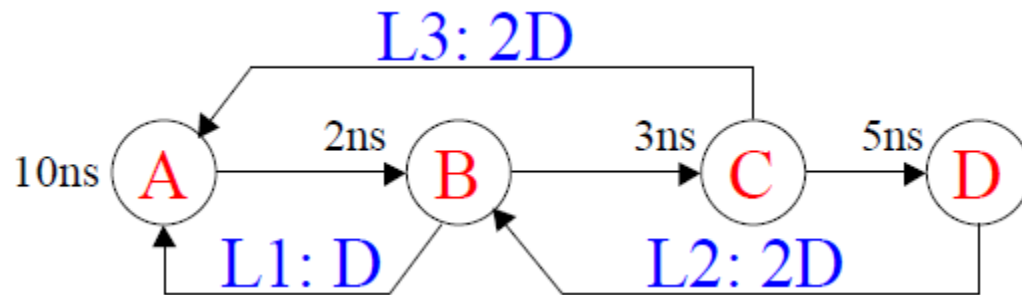
- Nodes represent computations. Each has an execution time in absolute time units. i.e., numbers in the parenthesis
- Directed edges represent data paths. It may have a delay in clock cycles.
- Any node can fire when all inputs are available, and many nodes can fire concurrently
- Intra-iteration precedence: every edge that has zero delay
- Inter-iteration precedence: every edge that has non-zero delay
- DFG is used to derive concurrent implementations onto parallel hardware

Loops



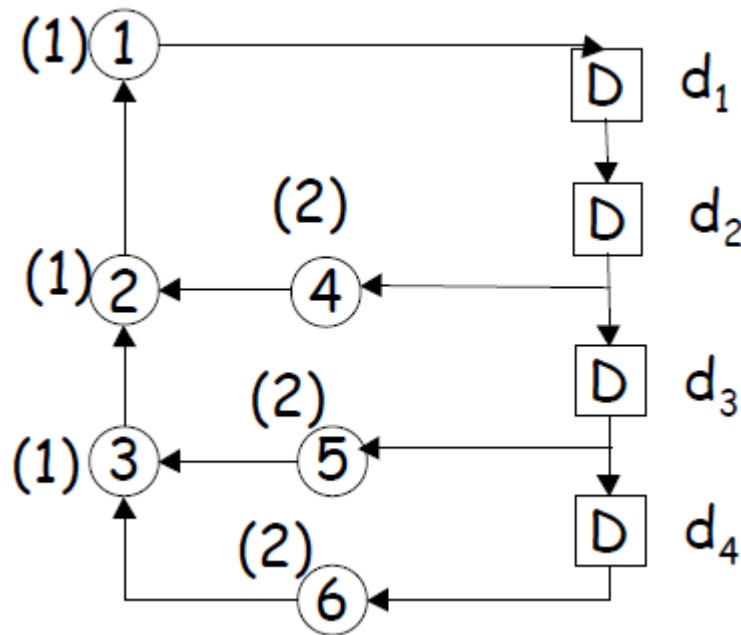
- **Loop:** directed path that begins and ends at the same node
- Loop bound: “loop computation time” divided by “loop delay”

Iteration Bound



- Three loops: L1, L2, L3.
- Loop bound of the l -th loop: t_l/w_l where t_l is the loop computation time, and w_l is the number of delays in the loop
 - Loop bound of L1: $12n/1D = 12$
 - L2: $10n/2D = 5$
 - L3: $15n/2D = 7.5$
- **Critical loop:** loop with the maximum loop bound
 - L1
- **Iteration bound:** the loop bound of the critical loop
 - $12ns (= L1)$
- Iteration bound is the *lower bound* on the iteration or sample period of the system (given speed of arithmetic operation and unlimited number of processing units). In other word, an implementation of the DSP program *can never achieve* an iteration period less than the iteration bound

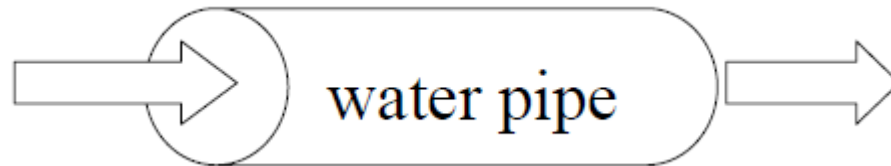
Compute Iteration Bound



- Enumerate loops and count loop bounds
 - Three loops: (1 → 4 → 2 → 1; 1 → 5 → 3 → 2 → 1; 1 → 6 → 3 → 2 → 1)
- Iteration bound is the maximum loop bound
 - $\text{Max}(4/2, 5/3, 5/4) = 2$

Pipelining and Parallel Processing

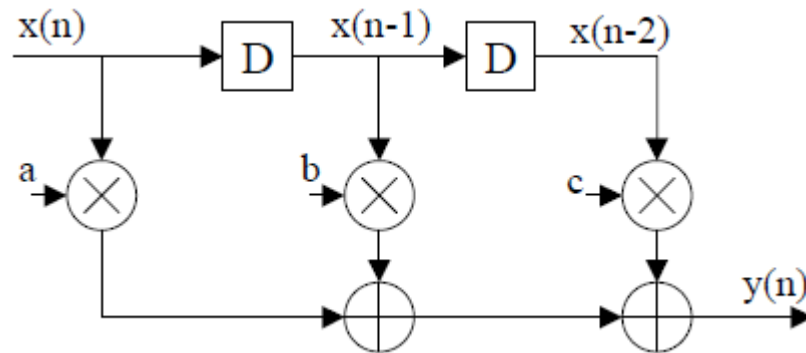
- Pipelining
 - Continue sending water without waiting for the water in the pipe to be drained



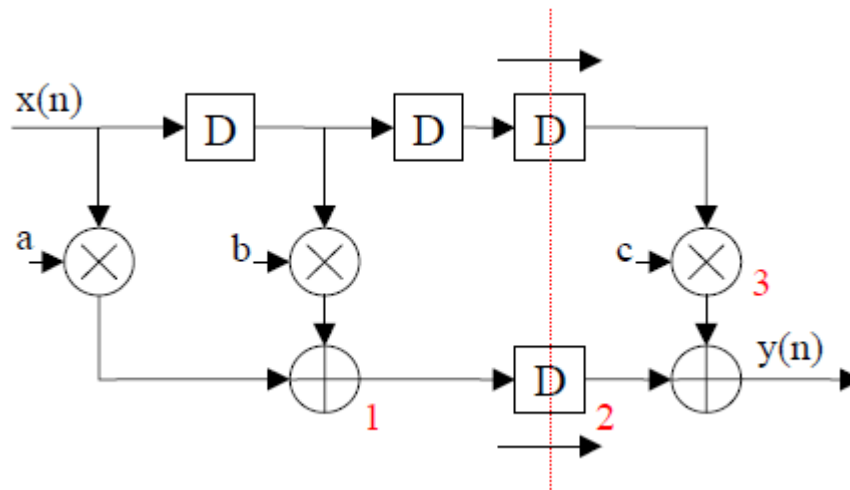
- Shortens the critical path
 - Allows an increased clock speed, sampling rate (throughput) or reduced power consumption at the same throughput
- Parallel processing
 - Multiple outputs are computed in parallel in a clock period
 - The effective sampling speed is increased by the level of parallelism
 - Allows an increased sampling rate (throughput) or reduced power consumption at the same throughput

Pipelining Digital Filters

- Direct-form 3-tap FIR filter

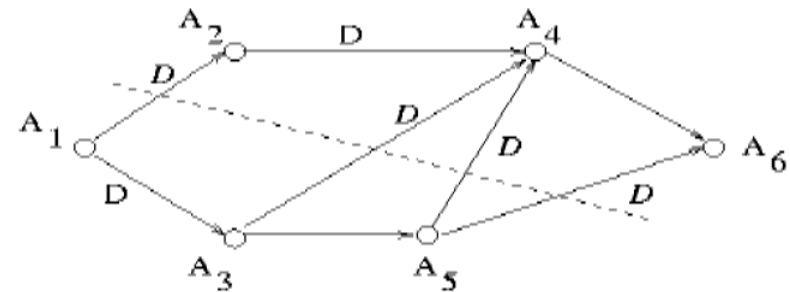
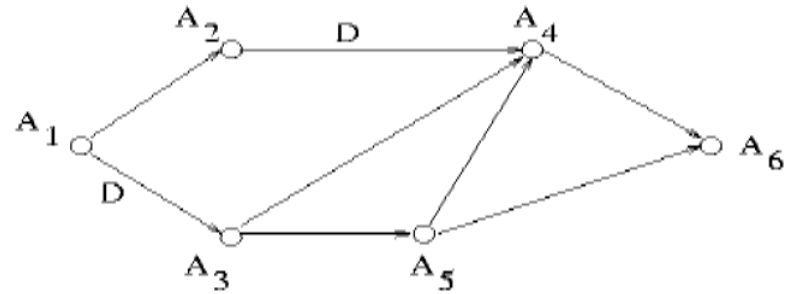


- Pipelined filter: reduces critical path, increases latency and register use



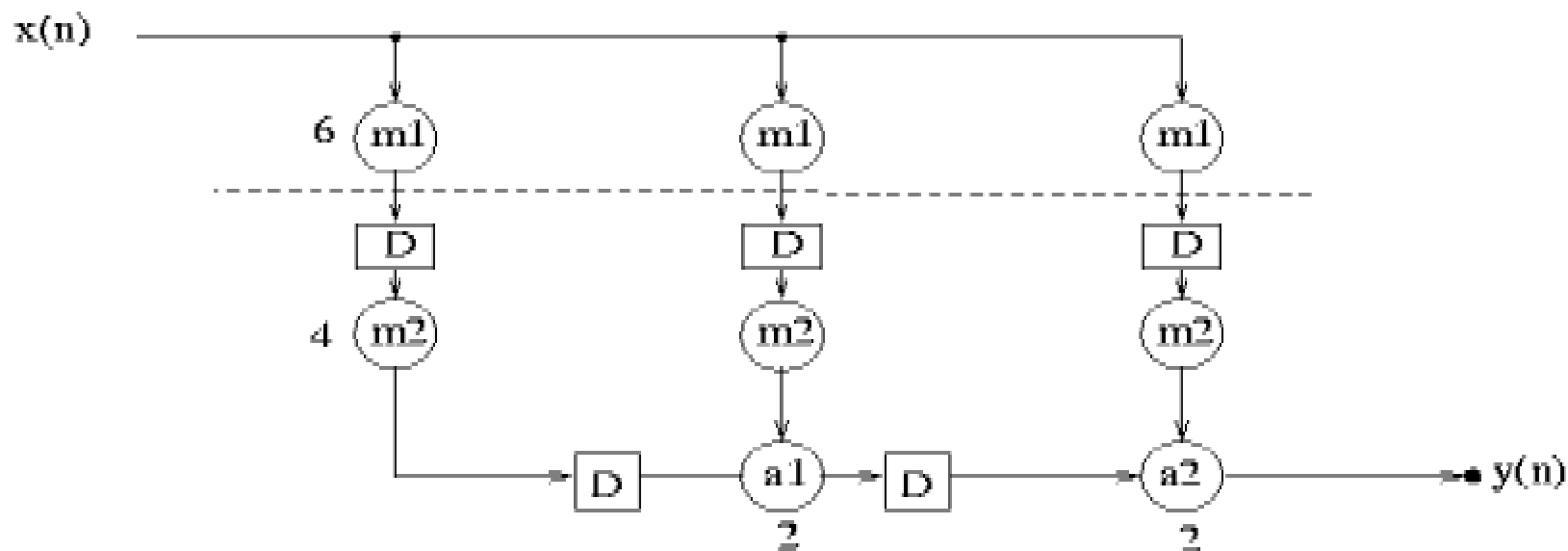
Pipeline Register Placement

- Start with DFG
- Cut: separate the vertices of a graph into two disjoint subsets
- Cutset: the *set of edges* whose end points are in different subsets
- Feed-forward cutset: data move in the forward direction on all the edges of the cutset
- Place registers across any feed-forward cutset of the graph



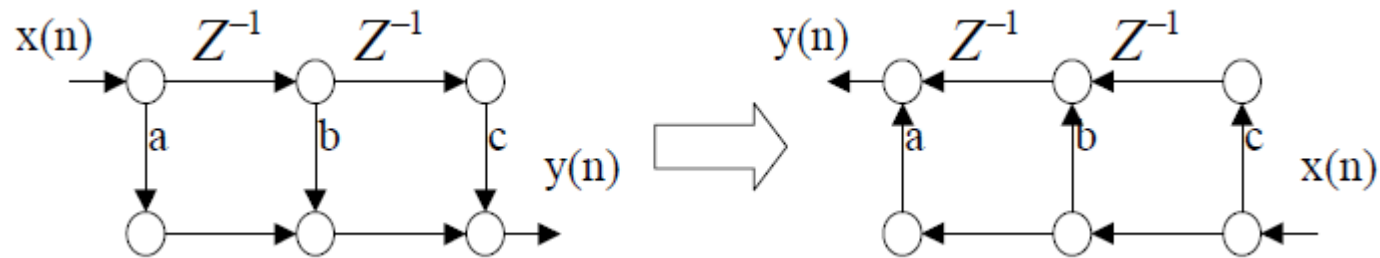
Fine-Grain Pipelining

- Break functional units for a better balance and shorten critical path

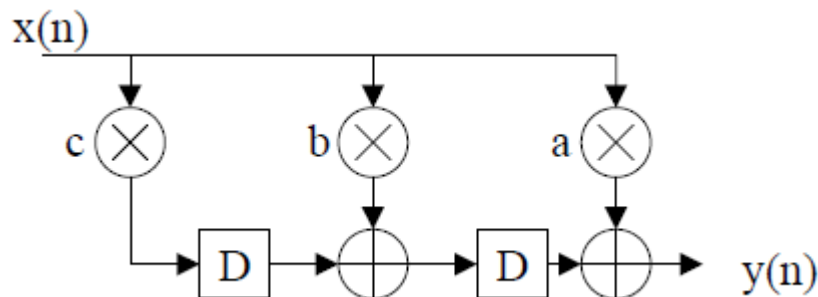


Data-Broadcast Structure

- Transposed SFG
 - Reverse the direction of all the edges in the SFG and interchange the input and output preserves the functionality of the system



- Data-broadcast structure
 - Reduced critical path delay

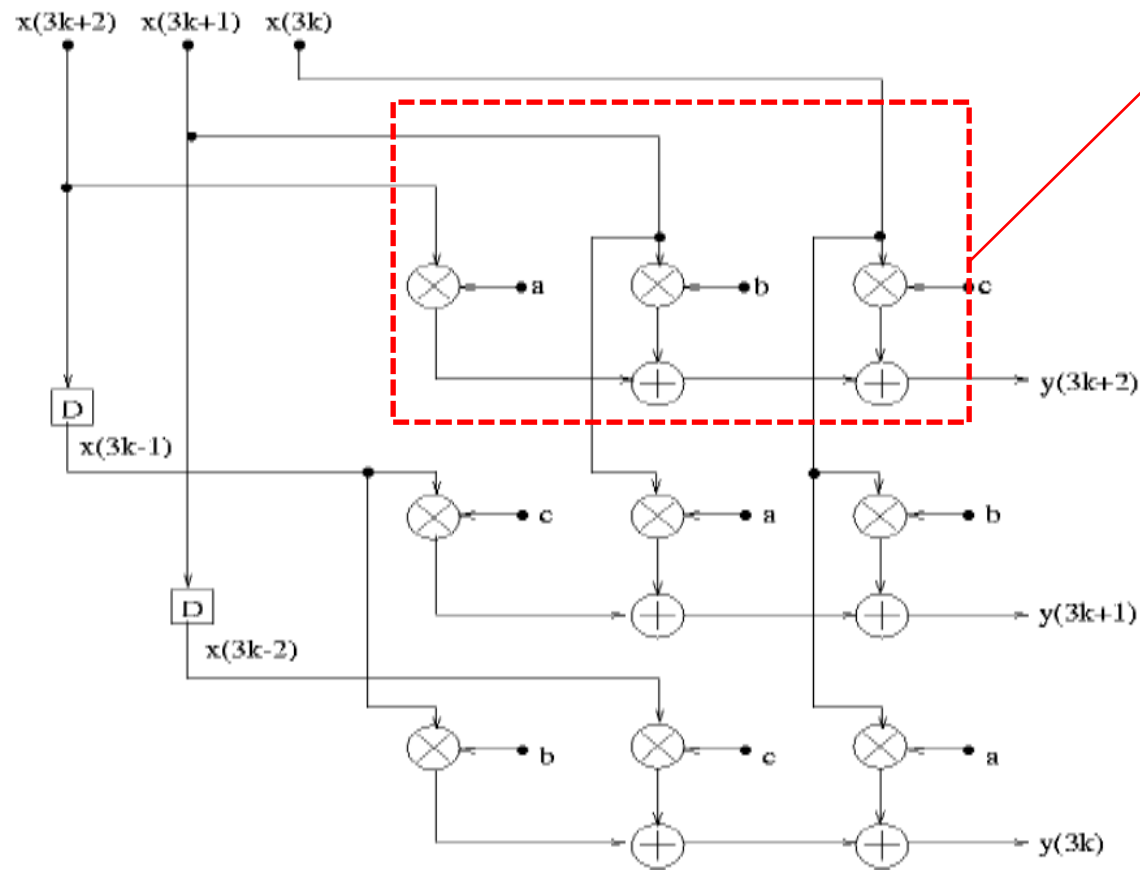


Parallel Processing

- Parallel processing is the dual of pipelining
 - Exploit concurrency in different ways:
 - Pipelining: break computation hardware into sets to improve utilization by concurrently operating on different samples
 - Parallel processing: duplicate computation hardware to operate on different samples
 - Example: $y(n) = ax(n) + bx(n-1) + cx(n-2)$
 - 3× parallel processing: k denotes clock cycle
$$y(3k) = ax(3k) + bx(3k-1) + cx(3k-2)$$
$$y(3k+1) = ax(3k+1) + bx(3k) + cx(3k-1)$$
$$y(3k+2) = ax(3k+2) + bx(3k+1) + cx(3k)$$
 - 3× sampling rate and 3× throughput at the same clock frequency
 - To maintain the same sampling rate, clock frequency can be reduced by 3×
 - Clock rate \neq sampling rate

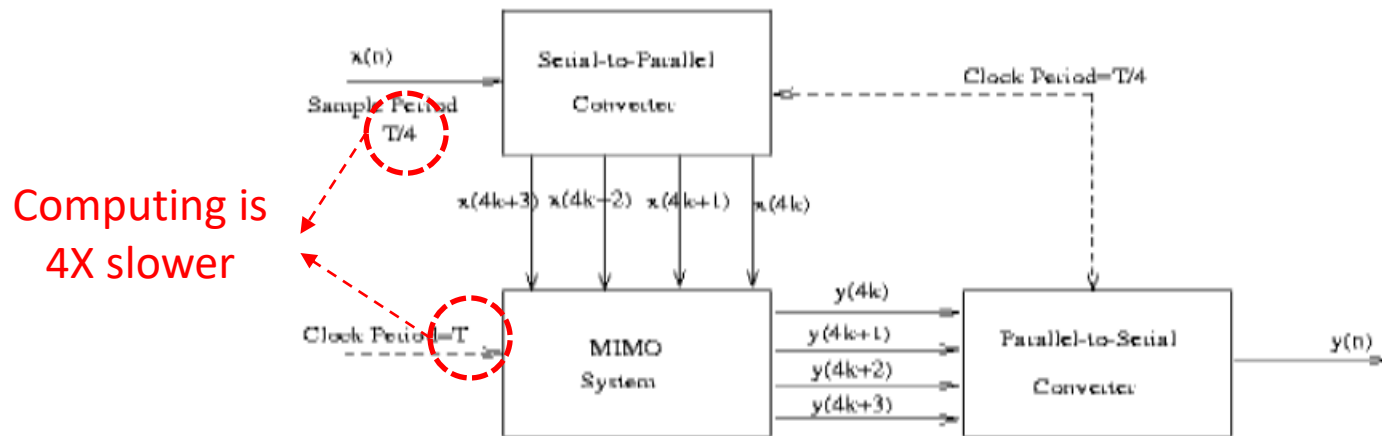
Parallel Processing

- 3× parallel direct-form FIR

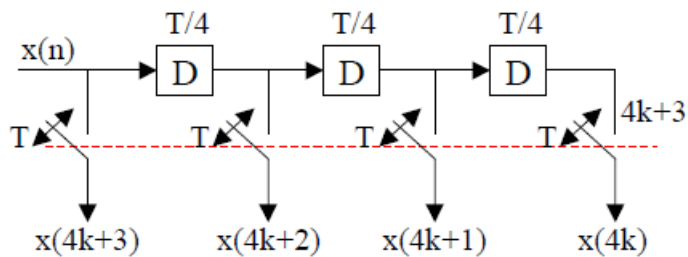


The computing
core design is
unchanged

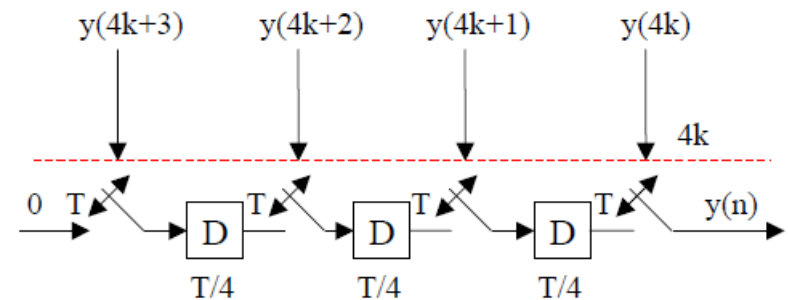
Complete Parallel System



– Serial to parallel



Parallel to serial



Communication Bound

- Why only pipelining is used?
- Pipelining can be used only to the extent, until the critical path computation time is limited by the communication or I/O
 - Pipelining can no longer increase the speed once the bound is reached
 - Use parallel processing to further increase the speed

