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UPC2

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Contest (1)

| template.cpp | 18 lines |
|---|----------|
| <pre>#include <bits/stdc++.h> using namespace std; typedef long long ll; typedef pair<int, int> pii; typedef vector<int> vi; const ll oo = 0x3f3f3f3f3f3f3f3fLL; #define FOR(i, a, b) for(ll i = (a); i < int(b); i++) #define FORD(i, a, b) for(ll i = (b)-1; i >= int(a); i--) #define has(c, e) ((c).find(e) != (c).end()) int main() { cin.sync_with_stdio(0); cin.tie(0); cin.exceptions(cin.failbit); return 0; }</pre> | |

| Makefile | 1 lines |
|--|---------|
| CXXFLAGS += -g -Wall -Wextra -Wshadow -std=c++11 | |

| .bashrc | 3 lines |
|--|---------|
| alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++14 \ -fsanitize=undefined,address' xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =◇ | |

| troubleshoot.txt | 52 lines |
|------------------|----------|
|------------------|----------|

Pre-submit:
Write a few simple test cases, if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all datastructures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a team mate.
Ask the team mate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a team mate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?

Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your team mates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all datastructures between test cases?

Data structures (2)

| LineContainer.h | 32 lines |
|---|----------|
| <p>Description: Container where you can add lines of the form $kx+m$, and query maximum values at points x. Useful for dynamic programming.</p> <p>Time: $\mathcal{O}(\log N)$</p> | |

```
bool Q;
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};
```

```
struct LineContainer : multiset<Line> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

| FenwickTree2d.h | 22 lines |
|--|----------|
| <p>Description: Computes sums $a[i,j]$ for all $i < I, j < J$, and increases single elements $a[i,j]$. Requires that the elements to be updated are known in advance (call <code>fakeUpdate()</code> before <code>init()</code>).</p> <p>Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)</p> | |
| <pre>"FenwickTree.h" struct FT2 { vector<vi> ys; vector<FT> ft;</pre> | |

```
FT2(int limx) : ys(limx) {}
void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
}
void init() {
    trav(v, ys) sort(all(v)), ft.emplace_back(sz(v));
}
int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
        ft[x].update(ind(x, y), dif);
}
ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
        sum += ft[x-1].query(ind(x-1, y));
    return sum;
}
};
```

Numerical (3)

| Simplex.h | 68 lines |
|---|----------|
| <p>Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns $-\text{inf}$ if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.</p> <p>Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}}}; vd b = {1,1,-4}, c = {-1,-1}, x; T val = LPSolver(A, b, c).solve(x);</p> <p>Time: $\mathcal{O}(NM * \# \text{pivots})$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.</p> | |

```
typedef double T; // long double, Rational, double + mod P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
```

```
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
```

```
struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }
};
```

```
    }

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;
    }
};
```

SolveLinear.h
Description: Solves $A \cdot x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2m)$

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        }
    }
}
```

```
        rank++;
    }

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h
Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h" 7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
fail:; }
```

FFT.h
Description: Fast Fourier transform. Also includes a function for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. a and b should be of roughly equal size. For convolutions of integers, rounding the results of conv works if $(|a| + |b|) \max(a, b) < \sim 10^9$ (in theory maybe 10^6); you may want to use an NTT from the Number Theory chapter instead.
Time: $\mathcal{O}(N \log N)$

```
<valarray> 29 lines

typedef valarray<complex<double>> carray;
void fft(carray& x, carray& roots) {
    int N = sz(x);
    if (N <= 1) return;
    carray even = x[slice(0, N/2, 2)];
    carray odd = x[slice(1, N/2, 2)];
    carray rs = roots[slice(0, N/2, 2)];
    fft(even, rs);
    fft(odd, rs);
    rep(k,0,N/2) {
        auto t = roots[k] * odd[k];
        x[k    ] = even[k] + t;
        x[k+N/2] = even[k] - t;
    }
}

typedef vector<double> vd;
vd conv(const vd& a, const vd& b) {
    int s = sz(a) + sz(b) - 1, L = 32-__builtin_clz(s), n = 1<<L;
    if (s <= 0) return {};
    carray av(n), bv(n), roots(n);
    rep(i,0,n) roots[i] = polar(1.0, -2 * M_PI * i / n);
    copy(all(a), begin(av)); fft(av, roots);
    copy(all(b), begin(bv)); fft(bv, roots);
    roots = roots.apply(conj);
    carray cv = av * bv; fft(cv, roots);
    vd c(s); rep(i,0,s) c[i] = cv[i].real() / n;
    return c;
}
```

Number theory (4)

4.1 Modular arithmetic

ModularArithmetic.h
Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

```
"euclid.h" 18 lines

const ll mod = 17; // change to something else
struct Mod {
    ll x;
    Mod(ll xx) : x(xx) {}
    Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
    Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
    Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
    Mod operator/(Mod b) { return *this * invert(b); }
    Mod invert(Mod a) {
        ll x, y, g = euclid(a.x, mod, x, y);
        assert(g == 1); return Mod((x + mod) % mod);
    }
    Mod operator^(ll e) {
        if (!e) return Mod(1);
        Mod r = *this ^ (e / 2); r = r * r;
        return e&1 ? *this * r : r;
    }
};
```

ModInverse.h
Description: Pre-computation of modular inverses. Assumes $\text{LIM} \leq \text{mod}$ and that mod is a prime.

```
3 lines

const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h
Description: Fast modular exponentiation.

ModSum.h
Description: Sums of mod'ed arithmetic progressions. $\text{modsum}(to, c, k, m) = \sum_{i=0}^{to-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

```
21 lines

typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
    ull res = k / m * sumsq(to) + c / m * to;
    k %= m; c %= m;
    if (k) {
        ull to2 = (to * k + c) / m;
        res += to * to2;
        res -= divsum(to2, m-1 - c, m, k) + to2;
    }
    return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
    c %= m;
    k %= m;
    if (c < 0) c += m;
    if (k < 0) k += m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

```
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large c .
Time: $\mathcal{O}(64/bits \cdot \log b)$, where $bits = 64 - k$, if we want to deal with k -bit numbers.

```
19 lines
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
    ull x = a * (b & (po - 1)) % c;
    while ((b >>= bits) > 0) {
        a = (a << bits) % c;
        x += (a * (b & (po - 1))) % c;
    }
    return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
    if (b == 0) return 1;
    ull res = mod_pow(a, b / 2, mod);
    res = mod_mul(res, res, mod);
    if (b & 1) return mod_mul(res, a, mod);
    return res;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$

```
30 lines
"ModPow.h"
ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1);
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1;
    int r = 0;
    while (s % 2 == 0)
        ++r, s /= 2;
    ll n = 2; // find a non-square mod p
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p);
    ll g = modpow(n, s, p);
    for (;;) {
        ll t = b;
        int m = 0;
        for (; m < r; ++m) {
            if (t == 1) break;
            t = t * t % p;
        }
        if (m == 0) return x;
        ll gs = modpow(g, 1 << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
        r = m;
    }
}
```

4.2 Number theoretic transform

NTT.h

Description: Number theoretic transform. Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For other primes/integers, use two different primes and combine with CRT. May return negative values.

```
38 lines
"ModPow.h"
const ll mod = (119 << 23) + 1, root = 3; // = 998244353
// For p < 2^30 there is also e.g. (5 << 25, 3), (7 << 26, 3),
// (479 << 21, 3) and (483 << 21, 5). The last two are > 10^9.

typedef vector<ll> vl;
void ntt(ll* x, ll* temp, ll* roots, int N, int skip) {
    if (N == 1) return;
    int n2 = N/2;
    ntt(x, temp, roots, n2, skip*2);
    ntt(x+skip, temp, roots, n2, skip*2);
    rep(i,0,N) temp[i] = x[i*skip];
    rep(i,0,n2) {
        ll s = temp[2*i], t = temp[2*i+1] * roots[skip*i];
        x[skip*i] = (s + t) % mod; x[skip*(i+n2)] = (s - t) % mod;
    }
}
void ntt(vl& x, bool inv = false) {
    ll e = modpow(root, (mod-1) / sz(x));
    if (inv) e = modpow(e, mod-2);
    vl roots(sz(x), 1), temp = roots;
    rep(i,1,sz(x)) roots[i] = roots[i-1] * e % mod;
    ntt(&x[0], &temp[0], &roots[0], sz(x), 1);
}
vl conv(vl a, vl b) {
    int s = sz(a) + sz(b) - 1; if (s <= 0) return {};
    int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
    if (s <= 200) { // (factor 10 optimization for |a|,|b| = 10)
        vl c(s);
        rep(i,0,sz(a)) rep(j,0,sz(b))
            c[i + j] = (c[i + j] + a[i] * b[j]) % mod;
        return c;
    }
    a.resize(n); ntt(a);
    b.resize(n); ntt(b);
    vl c(n); ll d = modpow(n, mod-2);
    rep(i,0,n) c[i] = a[i] * b[i] % mod * d % mod;
    ntt(c, true); c.resize(s); return c;
}
```

4.3 Primality

eratosthenes.h

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 \approx 0.8 s. Runs 30% faster if only odd indices are stored.

```
11 lines
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
    isprime.set(); isprime[0] = isprime[1] = 0;
    for (int i = 4; i < lim; i += 2) isprime[i] = 0;
    for (int i = 3; i*i < lim; i += 2) if (isprime[i])
        for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
    vi pr;
    rep(i,2,lim) if (isprime[i]) pr.push_back(i);
    return pr;
}
```

MillerRabin.h

Description: Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.

```
16 lines
"ModMulLL.h"
bool prime(ull p) {
    if (p == 2) return true;
    if (p == 1 || p % 2 == 0) return false;
    ull s = p - 1;
    while (s % 2 == 0) s /= 2;
    rep(i,0,15) {
        ull a = rand() % (p - 1) + 1, tmp = s;
        ull mod = mod_pow(a, tmp, p);
        while (tmp != p - 1 && mod != 1 && mod != p - 1) {
            mod = mod_mul(mod, mod, p);
            tmp *= 2;
        }
        if (mod != p - 1 && tmp % 2 == 0) return false;
    }
    return true;
}
```

factor.h

Description: Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run init(bits), where bits is the length of the numbers you use.
Time: Expected running time should be good enough for 50-bit numbers.

```
37 lines
"MillerRabin.h", "eratosthenes.h", "euclid.h"
vector<ull> pr;
ull f(ull a, ull n, ull &has) {
    return (mod_mul(a, a, n) + has) % n;
}
vector<ull> factor(ull d) {
    vector<ull> res;
    for (size_t i = 0; i < pr.size() && pr[i]*pr[i] <= d; i++)
        if (d % pr[i] == 0) {
            while (d % pr[i] == 0) d /= pr[i];
            res.push_back(pr[i]);
        }
    //d is now a product of at most 2 primes.
    if (d > 1) {
        if (prime(d))
            res.push_back(d);
        else while (true) {
            ull has = rand() % 2321 + 47;
            ull x = 2, y = 2, c = 1;
            for (; c==1; c = gcd((y > x ? y - x : x - y), d)) {
                x = f(x, d, has);
                y = f(f(y, d, has), d, has);
            }
            if (c != d) {
                res.push_back(c); d /= c;
                if (d != c) res.push_back(d);
                break;
            }
        }
    }
    return res;
}
void init(int bits) { //how many bits do we use?
    vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
    vector<ull> pr(p.size());
    for (size_t i=0; i<pr.size(); i++)
        pr[i] = p[i];
}
```

4.4 Divisibility

euclid.h
Description: Finds the Greatest Common Divisor to the integers a and b . Euclid also finds two integers x and y , such that $ax + by = \gcd(a, b)$. If a and b are coprime, then x is the inverse of $a \pmod{b}$.

```
7 lines
11 gcd(11 a, 11 b) { return __gcd(a, b); }
```

```
11 euclid(11 a, 11 b, 11 &x, 11 &y) {
    if (b) { 11 d = euclid(b, a % b, y, x);
        return y -= a/b * x, d; }
    return x = 1, y = 0, a;
}
```

4.4.1 Bézout’s identity

For $a \neq, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h
Description: Euler’s totient or Euler’s phi function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . The cototient is $n - \phi(n)$. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$.
 $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k, n) = 1} k = n\phi(n)/2, n > 1$
Euler’s thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat’s little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p} \forall a$.

```
10 lines
const int LIM = 5000000;
int phi[LIM];
```

```
void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for(int i = 3; i < LIM; i += 2)
        if(phi[i] == i)
            for(int j = i; j < LIM; j += i)
                (phi[j] /= i) *= i-1;
}
```

4.5 Chinese remainder theorem

chinese.h
Description: Chinese Remainder Theorem.
chinese(a, m, b, n) returns a number x , such that $x \equiv a \pmod{m}$ and $x \equiv b \pmod{n}$. For not coprime n, m , use chinese_common. Note that all numbers must be less than 2^{31} if you have Z = unsigned long long.
Time: $\log(m + n)$

```
13 lines
"euclid.h"
template <class Z> Z chinese(Z a, Z m, Z b, Z n) {
    Z x, y; euclid(m, n, x, y);
    Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
    if (ret >= m * n) ret -= m * n;
```

```
return ret;
}

template <class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
    Z d = gcd(m, n);
    if ((b -= a) % n < 0) b += n;
    if (b % d) return -1; // No solution
    return d * chinese(Z(0), m/d, b/d, n/d) + a;
}
```

4.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

4.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.8 Estimates

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (5)

Graph (6)

globalmincut.h
Description: Given an adjacency matrix returns the global mincut and the vertices of one of the cuts.
Time: $\mathcal{O}(V^3)$

```
84 lines
/*
 * If you dont need the cut you can eliminate every thing with
 *   this coment "// *****"
 * Explanation of algorithm:
 * -getting the mincut value: it does n-1 iterations. In each
 *   iteration it starts by a vertex (random) as set A.
 * then it iterates until only two vertices are left by adding
 *   to set A the most tightly connected vertex to A (vertex
 *   not in A).
 * it insert this vertex to A. When only two vertices are left
 *   , the mincut between those two is the weight W of the
 *   edges between
```

```
* the last vertex and A. mincut = min(mincut, W)
* We then merge the two last vertices and start again.
*
* -getting the cut: basically when we merge two nodes we
*   merge them with mfset. When we obtain a new best mincut
*   value, a cut
* is represented buy the nodes in the same component as the
*   last node;
*/
```

```
// Maximum number of vertices in the graph
#define NN 256
// Maximum edge weight (MAXW * NN * NN must fit into an int)
#define MAXW 1000
// Adjacency matrix and some internal arrays
int v[NN], w[NN];
bool a[NN];
```

```
int pare[NN]; // *****
int par (int b){ // *****
    if (pare[b] == b) return b;
    pare[b] = par(pare[b]);
    return pare[b];
}
```

```
inline void merge (int b, int c){ // *****
    pare[par(b)] = par(c);
}
```

```
pair < int, vi > minCut(vvi& g, int n) {
    int n1 = n;
    // init the remaining vertex set
    for (int i = 0; i < n; i++){
        v[i] = i;
        pare[i] = i; // *****
    }
    // run Stoer–Wagner
    int best = MAXW * n * n;
    vi cut; // *****
    while (n > 1) {
        // initialize the set A and vertex weights
        a[v[0]] = true;
        for (int i = 1; i < n; i++) {
            a[v[i]] = false;
            w[i] = g[v[0]][v[i]];
        }
        // add the other vertices
        int prev = v[0];
        for (int i = 1; i < n; i++) {
            // find the most tightly connected non-A vertex
            int zj = -1;
            for (int j = 1; j < n; j++)
                if (!a[v[j]] && (zj < 0 || w[j] > w[zj])) zj = j;
            // add it to A
            a[v[zj]] = true;
            // last vertex?
            if (i == n - 1) {
                // remember the cut weight
                if (best > w[zj]){
                    best = w[zj];
                    cut.clear(); // *****
                    for (int ko = 0; ko < n1; ko++) if (par(ko) == par(v[
                        zj])) cut.push_back(ko); // *****
                }

                // merge prev and v[zj]
                merge(prev, v[zj]); // *****
                for (int j = 0; j < n; j++)
                    g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][v[j]];
```

```
        v[zj] = v[--n];
        break;
    }
    prev = v[zj];
    // update the weights of its neighbours
    for (int j = 1; j < n; j++)
        if (!a[v[j]]) w[j] += g[v[zj]][v[j]];
    }
}
return {best, cut};
}
```

EulerianCycle.h
Description: returns de eulerian cycle/tour starting at u, cycle is in reverse order. If its a tour it must start at a vertex with odd degree
Time: $\mathcal{O}(E)$

```
//undirected graph
struct edge{
    int u, v;
    bool used;
};
```

```
vector<edge> E;
vector<vector<int>> adj; //adj stores the index in E
vector<int> nxt;
vector<int> cycle;
int p;
```

```
void find_cycle(int u){
    while(nxt[u] < adj[u].size()){
        int go = adj[u][nxt[u]++];
        if(!E[go].used){
            E[go].used = 1;
            int to = (E[go].u ^ E[go].v ^ u);
            find_cycle(to);
        }
    }
    cycle.push_back(u);
}
```

```
//directed graph
vector<int> next; //stores last visited vertex index
vector<vector<int>> adj;
vector<int> cycle;
```

```
void find_cycle(int u){
    while(next[u] < adj[u].size()){
        find_cycle(adj[u][next[u]++]);
        cycle.push_back(u);
    }
}
```

Geometry (7)

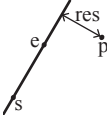
7.1 Geometric primitives

Point.h
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

```
template <class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
```

```
P operator+(P p) const { return P(x+p.x, y+p.y); }
P operator-(P p) const { return P(x-p.x, y-p.y); }
P operator*(T d) const { return P(x*d, y*d); }
P operator/(T d) const { return P(x/d, y/d); }
T dot(P p) const { return x*p.x + y*p.y; }
T cross(P p) const { return x*p.y - y*p.x; }
T cross(P a, P b) const { return (a-*this).cross(b-*this); }
T dist2() const { return x*x + y*y; }
double dist() const { return sqrt((double)dist2()); }
// angle to x-axis in interval [-pi, pi]
double angle() const { return atan2(y, x); }
P unit() const { return *this/dist(); } // makes dist()==1
P perp() const { return P(-y, x); } // rotates +90 degrees
P normal() const { return perp().unit(); }
// returns point rotated 'a' radians ccw around the origin
P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a));
};
```

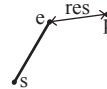
lineDistance.h
Description: Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.



```
"Point.h" 4 lines

template <class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a) / (b-a).dist();
}
```

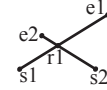
SegmentDistance.h
Description: Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;



```
"Point.h" 6 lines

typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h
Description: If a unique intersestion point between the line segments going from s1 to e1 and from s2 to e2 exists r1 is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists 2 is returned and r1 and r2 are set to the two ends of the common line. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long. Use segmentIntersectionQ to get just a true/false answer.
Usage: Point<double> intersection, dummy;
if (segmentIntersection(s1,e1,s2,e2,intersection,dummy)==1) cout << "segments intersect at " << intersection << endl;



```
"Point.h" 27 lines

template <class P>
int segmentIntersection(const P& s1, const P& e1,
    const P& s2, const P& e2, P& r1, P& r2) {
```

```
if (e1==s1) {
    if (e2==s2) {
        if (e1==e2) { r1 = e1; return 1; } //all equal
        else return 0; //different point segments
    } else return segmentIntersection(s2,e2,s1,e1,r1,r2); //swap
}
//segment directions and separation
P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
auto a = v1.cross(v2), al = v1.cross(d), a2 = v2.cross(d);
if (a == 0) { //if parallel
    auto b1=s1.dot(v1), c1=e1.dot(v1),
        b2=s2.dot(v1), c2=e2.dot(v1);

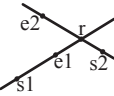
    if (al || a2 || max(b1,min(b2,c2))>min(c1,max(b2,c2)))
        return 0;
    r1 = min(b2,c2)<b1 ? s1 : (b2<c2 ? s2 : e2);
    r2 = max(b2,c2)>c1 ? e1 : (b2>c2 ? s2 : e2);
    return 2-(r1==r2);
}
if (a < 0) { a = -a; al = -al; a2 = -a2; }
if (0<al || a<-al || 0<a2 || a<-a2)
    return 0;
r1 = s1-v1*a2/a;
return 1;
}
```

SegmentIntersectionQ.h
Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

```
"Point.h" 16 lines

template <class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
    if (e1 == s1) {
        if (e2 == s2) return e1 == e2;
        swap(s1,s2); swap(e1,e2);
    }
    P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
    auto a = v1.cross(v2), al = d.cross(v1), a2 = d.cross(v2);
    if (a == 0) { // parallel
        auto b1 = s1.dot(v1), c1 = e1.dot(v1),
            b2 = s2.dot(v1), c2 = e2.dot(v1);
        return !al && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
    }
    if (a < 0) { a = -a; al = -al; a2 = -a2; }
    return (0 <= al && al <= a && 0 <= a2 && a2 <= a);
}
```

lineIntersection.h
Description: If a unique intersestion point of the lines going through s1,e1 and s2,e2 exists r is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists -1 is returned. If s1==e1 or s2==e2 -1 is returned. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.
Usage: point<double> intersection;
if (1 == LineIntersection(s1,e1,s2,e2,intersection))
cout << "intersection point at " << intersection << endl;



```
"Point.h" 9 lines
```

```
template <class P>
int lineIntersection(const P& s1, const P& e1, const P& s2,
    const P& e2, P& r) {
    if ((e1-s1).cross(e2-s2)) { //if not parallell
        r = s2-(e2-s2)*(e1-s1).cross(s2-s1)/(e1-s1).cross(e2-s2);
        return 1;
    } else
```

```
    return -((e1-s1).cross(s2-s1)==0 || s2==e2);
}

sideOf.h
Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: bool left = sideOf(p1,p2,q)==1;
"Point.h" 11 lines
```

```
template <class P>
int sideOf(const P& s, const P& e, const P& p) {
    auto a = (e-s).cross(p-s);
    return (a > 0) - (a < 0);
}
template <class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```

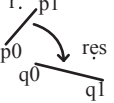
```
onSegment.h
Description: Returns true iff p lies on the line segment from s to e. Intended for use with e.g. Point<long long> where overflow is an issue. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.
"Point.h" 5 lines
```

```
template <class P>
bool onSegment(const P& s, const P& e, const P& p) {
    P ds = p-s, de = p-e;
    return ds.cross(de) == 0 && ds.dot(de) <= 0;
}
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

```
Angle.h
Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping.
Usage: vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) {
    while (v[j] < v[i].t180()) ++j;
} // sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i
"Angle.h" 37 lines
```

```
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle a) const { return {x-a.x, y-a.y, t}; }
    int quad() const {
        assert(x || y);
        if (y < 0) return (x >= 0) + 2;
        if (y > 0) return (x <= 0);
        return (x <= 0) * 2;
    }
}
```

```
Angle t90() const { return {-y, x, t + (quad() == 3)}; }
Angle t180() const { return {-x, -y, t + (quad() >= 2)}; }
Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.quad(), a.y * (1l)b.x) <
        make_tuple(b.t, b.quad(), a.x * (1l)b.y);
}
bool operator>=(Angle a, Angle b) { return !(a < b); }
bool operator>(Angle a, Angle b) { return b < a; }
bool operator<=(Angle a, Angle b) { return !(b < a); }

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180() ?
        make_pair(a, b) : make_pair(b, a.t360()));
}
```

```
Angle operator+(Angle a, Angle b) { // where b is a vector
    Angle r(a.x + b.x, a.y + b.y, a.t);
    if (r > a.t180()) r.t--;
    return r.t180() < a ? r.t360() : r;
}
```

7.2 Circles

```
CircleIntersection.h
Description: Computes a pair of points at which two circles intersect. Returns false in case of no intersection.
"Point.h" 14 lines
```

```
typedef Point<double> P;
bool circleIntersection(P a, P b, double r1, double r2,
    pair<P, P>* out) {
    P delta = b - a;
    assert(delta.x || delta.y || r1 != r2);
    if (!delta.x && !delta.y) return false;
    double r = r1 + r2, d2 = delta.dist2();
    double p = (d2 + r1*r1 - r2*r2) / (2.0 * d2);
    double h2 = r1*r1 - p*p*d2;
    if (d2 > r*r || h2 < 0) return false;
    P mid = a + delta*p, per = delta.perp() * sqrt(h2 / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

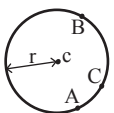
```
circleTangents.h
Description:
Returns a pair of the two points on the circle with radius r centered around c whos tangent lines intersect p. If p lies within the circle NaN-points are returned. P is intended to be Point<double>. The first point is the one to the right as seen from the p towards c.
Usage: typedef Point<double> P;
pair<P,P> p = circleTangents(P(100,2),P(0,0),2);
"Point.h" 6 lines
```

```
template <class P>
pair<P,P> circleTangents(const P &p, const P &c, double r) {
    P a = p-c;
    double x = r*r/a.dist2(), y = sqrt(x-x*x);
    return make_pair(c+a*x+a.perp()*y, c+a*x-a.perp()*y);
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist() /
        abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2() - c*b.dist2()).perp() / b.cross(c) / 2;
}
```

```
MinimumEnclosingCircle.h
Description: Computes the minimum circle that encloses a set of points. Time: expected O(n)
"circumcircle.h" 28 lines
```

```
pair<double, P> mec2(vector<P>& S, P a, P b, int n) {
    double hi = INFINITY, lo = -hi;
    rep(i,0,n) {
        auto si = (b-a).cross(S[i]-a);
        if (si == 0) continue;
        P m = ccCenter(a, b, S[i]);
        auto cr = (b-a).cross(m-a);
        if (si < 0) hi = min(hi, cr);
        else lo = max(lo, cr);
    }
    double v = (0 < lo ? lo : hi < 0 ? hi : 0);
    P c = (a + b) / 2 + (b - a).perp() * v / (b - a).dist2();
    return {(a - c).dist2(), c};
}
pair<double, P> mec(vector<P>& S, P a, int n) {
    random_shuffle(S.begin(), S.begin() + n);
    P b = S[0], c = (a + b) / 2;
    double r = (a - c).dist2();
    rep(i,1,n) if ((S[i] - c).dist2() > r * (1 + 1e-8)) {
        tie(r,c) = (n == sz(S) ?
            mec(S, S[i], i) : mec2(S, a, S[i], i));
    }
    return {r, c};
}
pair<double, P> enclosingCircle(vector<P> S) {
    assert(!S.empty()); auto r = mec(S, S[0], sz(S));
    return {sqrt(r.first), r.second};
}
```

7.3 Polygons

insidePolygon.h

Description:

Returns true if p lies within the polygon described by the points between iterators begin and end. If strict false is returned when p is on the edge of the polygon. Answer is calculated by counting the number of intersections between the polygon and a line going from p to infinity in the positive x-direction. The algorithm uses products in intermediate steps so watch out for overflow. If points within epsilon from an edge should be considered as on the edge replace the line "if (onSegment..." with the comment below it (this will cause overflow for int and long long).

Usage: typedef Point<int> pi;
vector<pi> v; v.push_back(pi(4,4));
v.push_back(pi(1,2)); v.push_back(pi(2,1));
bool in = insidePolygon(v.begin(),v.end(), pi(3,4), false);
Time: O(n)

"Point.h", "onSegment.h", "SegmentDistance.h" 14 lines

```

template <class It, class P>
bool insidePolygon(It begin, It end, const P& p,
    bool strict = true) {
    int n = 0; //number of isects with line from p to (inf,p.y)
    for (It i = begin, j = end-1; i != end; j = i++) {
        //if p is on edge of polygon
        if (onSegment(*i, *j, p)) return !strict;
        //or: if (segDist(*i, *j, p) <= epsilon) return !strict;
        //increment n if segment intersects line from p
        n += (max(i->y,j->y) > p.y && min(i->y,j->y) <= p.y &&
            ((*j-*i).cross(p-*i) > 0) == (i->y <= p.y));
    }
    return n&1; //inside if odd number of intersections
}

```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```

"Point.h"
6 lines

template <class T>
T polygonArea2(vector<Point<T>&& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}

```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

```

"Point.h"
10 lines

typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
    auto i = v.begin(), end = v.end(), j = end-1;
    Point<double> res{0,0}; double A = 0;
    for (; i != end; j=i++) {
        res = res + (*i + *j) * j->cross(*i);
        A += j->cross(*i);
    }
    return res / A / 3;
}

```

PolygonCut.h

Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

```

"Point.h", "lineIntersection.h"
15 lines

typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0;
        if (side != (s.cross(e, prev) < 0)) {
            res.emplace_back();
            lineIntersection(s, e, cur, prev, res.back());
        }
        if (side)
            res.push_back(cur);
    }
    return res;
}

```

ConvexHull.h

Description: Returns a vector of indices of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Usage: vector<P> ps, hull;
trav(i, convexHull(ps)) hull.push_back(ps[i]);
Time: $\mathcal{O}(n \log n)$

```

"Point.h"
20 lines

typedef Point<ll> P;
pair<vi, vi> ulHull(const vector<P>& S) {
    vi Q(sz(S)), U, L;
    iota(all(Q), 0);
    sort(all(Q), [&S](int a, int b){ return S[a] < S[b]; });
    trav(it, Q) {
#define ADDP(C, cmp) while (sz(C) > 1 && S[C[sz(C)-2]].cross(\
S[it], S[C.back()]) cmp 0) C.pop_back(); C.push_back(it);
        ADDP(U, <=); ADDP(L, >=);
    }
    return {U, L};
}

vi convexHull(const vector<P>& S) {
    vi u, l; tie(u, l) = ulHull(S);
    if (sz(S) <= 1) return u;
    if (S[u[0]] == S[u[1]]) return {0};
    l.insert(l.end(), u.rbegin()+1, u.rend()-1);
    return l;
}

```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

```

"ConvexHull.h"
19 lines

vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
    vector<pii> ret;
    int i = 0, j = sz(L) - 1;
    while (i < sz(U) - 1 || j > 0) {
        ret.emplace_back(U[i], L[j]);
        if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
            .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
        else --j;
    }
    return ret;
}

pii polygonDiameter(const vector<P>& S) {
    vi U, L; tie(U, L) = ulHull(S);
    pair<ll, pii> ans;
    trav(x, antipodal(S, U, L))
        ans = max(ans, {(S[x.first] - S[x.second]).dist2(), x});
    return ans.second;
}

```

PointInsideHull.h

Description: Determine whether a point t lies inside a given polygon (counter-clockwise order). The polygon must be such that every point on the circumference is visible from the first point in the vector. It returns 0 for points outside, 1 for points on the circumference, and 2 for points inside.
Time: $\mathcal{O}(\log N)$

```

"Point.h", "sideOf.h", "onSegment.h"
22 lines

typedef Point<ll> P;
int insideHull2(const vector<P>& H, int L, int R, const P& p) {
    int len = R - L;
    if (len == 2) {
        int sa = sideOf(H[0], H[L], p);
        int sb = sideOf(H[L], H[L+1], p);
        int sc = sideOf(H[L+1], H[0], p);
        if (sa < 0 || sb < 0 || sc < 0) return 0;

```



```

    if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R == sz(H)))
        return 1;
    return 2;
}

int mid = L + len / 2;
if (sideOf(H[0], H[mid], p) >= 0)
    return insideHull2(H, mid, R, p);
return insideHull2(H, L, mid+1, p);
}

```

```

int insideHull(const vector<P>& hull, const P& p) {
    if (sz(hull) < 3) return onSegment(hull[0], hull.back(), p);
    else return insideHull2(hull, 1, sz(hull), p);
}

```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. isct(a, b) returns a pair describing the intersection of a line with the polygon: $\bullet (-1, -1)$ if no collision, $\bullet (i, -1)$ if touching the corner i , $\bullet (i, i)$ if along side $(i, i + 1)$, $\bullet (i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon.
Time: $\mathcal{O}(N + Q \log n)$

```

"Point.h"
63 lines

ll sgn(ll a) { return (a > 0) - (a < 0); }
typedef Point<ll> P;
struct HullIntersection {
    int N;
    vector<P> p;
    vector<pair<P, int>>> a;

    HullIntersection(const vector<P>& ps) : N(sz(ps)), p(ps) {
        p.insert(p.end(), all(ps));
        int b = 0;
        rep(i,1,N) if (P{p[i].y,p[i].x} < P{p[b].y, p[b].x}) b = i;
        rep(i,0,N) {
            int f = (i + b) % N;
            a.emplace_back(p[f+1] - p[f], f);
        }
    }

    int qd(P p) {
        return (p.y < 0) ? (p.x >= 0) + 2
            : (p.x <= 0) * (1 + (p.y <= 0));
    }

    int bs(P dir) {
        int lo = -1, hi = N;
        while (hi - lo > 1) {
            int mid = (lo + hi) / 2;
            if (make_pair(qd(dir), dir.y * a[mid].first.x) <
                make_pair(qd(a[mid].first), dir.x * a[mid].first.y))
                hi = mid;
            else lo = mid;
        }
        return a[hi%N].second;
    }
}

```

```

bool isign(P a, P b, int x, int y, int s) {
    return sgn(a.cross(p[x], b)) * sgn(a.cross(p[y], b)) == s;
}

int bs2(int lo, int hi, P a, P b) {
    int L = lo;
    if (hi < lo) hi += N;
    while (hi - lo > 1) {
        int mid = (lo + hi) / 2;

```



```
        if (isign(a, b, mid, L, -1)) hi = mid;
        else lo = mid;
    }
    return lo;
}

pii isct(P a, P b) {
    int f = bs(a - b), j = bs(b - a);
    if (isign(a, b, f, j, 1)) return {-1, -1};
    int x = bs2(f, j, a, b)%N,
        y = bs2(j, f, a, b)%N;
    if (a.cross(p[x], b) == 0 &&
        a.cross(p[x+1], b) == 0) return {x, x};
    if (a.cross(p[y], b) == 0 &&
        a.cross(p[y+1], b) == 0) return {y, y};
    if (a.cross(p[f], b) == 0) return {f, -1};
    if (a.cross(p[j], b) == 0) return {j, -1};
    return {x, y};
}
};
```

7.4 Misc. Point Set Problems

closestPair.h
Description: i1, i2 are the indices to the closest pair of points in the point vector p after the call. The distance is returned.
Time: $\mathcal{O}(n \log n)$

```
"Point.h" 58 lines

template <class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template <class It>
bool y_it_less(const It& i, const It& j) {return i->y < j->y;}

template<class It, class IIt> /* IIt = vector<It>::iterator */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
    typedef typename iterator_traits<It>::value_type P;
    int n = yaend-ya, split = n/2;
    if(n <= 3) { // base case
        double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
        if(n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).dist()
            ;
        if(a <= b) { i1 = xa[1];
            if(a <= c) return i2 = xa[0], a;
            else return i2 = xa[2], c;
        } else { i1 = xa[2];
            if(b <= c) return i2 = xa[0], b;
            else return i2 = xa[1], c;
        }
    }
    vector<It> ly, ry, stripy;
    P splitp = *xa[split];
    double splitx = splitp.x;
    for(IIt i = ya; i != yaend; ++i) { // Divide
        if(*i != xa[split] && (*i-splitp).dist2() < 1e-12)
            return i1 = *i, i2 = xa[split], 0;// nasty special case!
        if (*i < splitp) ly.push_back(*i);
        else ry.push_back(*i);
    } // assert((signed)lefty.size() == split)
    It j1, j2; // Conquer
    double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
    double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2);
    if(b < a) a = b, i1 = j1, i2 = j2;
    double a2 = a*a;
    for(IIt i = ya; i != yaend; ++i) { // Create strip (y-sorted)
        double x = (*i)->x;
        if(x >= splitx-a && x <= splitx+a) stripy.push_back(*i);
    }
    for(IIt i = stripy.begin(); i != stripy.end(); ++i) {
```

```
        const P &p1 = **i;
        for(IIt j = i+1; j != stripy.end(); ++j) {
            const P &p2 = **j;
            if(p2.y-p1.y > a) break;
            double d2 = (p2-p1).dist2();
            if(d2 < a2) i1 = *i, i2 = *j, a2 = d2;
        }
        return sqrt(a2);
    }
}

template<class It> // It is random access iterators of point<T>
double closestpair(It begin, It end, It &i1, It &i2 ) {
    vector<It> xa, ya;
    assert(end-begin >= 2);
    for (It i = begin; i != end; ++i)
        xa.push_back(i), ya.push_back(i);
    sort(xa.begin(), xa.end(), it_less<It>);
    sort(ya.begin(), ya.end(), y_it_less<It>);
    return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
}
```

```
kdTree.h
Description: KD-tree (2d, can be extended to 3d)
"Point.h" 63 lines

typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if the box is wider than high (not best heuristic...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
            // divide by taking half the array for each child (not best performance with many duplicates in the middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()});
        }
    }
};

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
        }
```

```
        return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
        best = min(best, search(s, p));
    return best;
}

// find nearest point to a point, and its squared distance
// (requires an arbitrary operator< for Point)
pair<T, P> nearest(const P& p) {
    return search(root, p);
}
};
```

```
DelaunayTriangulation.h
Description: Computes the Delaunay triangulation of a set of points.
Each circumcircle contains none of the input points. If any three points are
colinear or any four are on the same circle, behavior is undefined.
Time:  $\mathcal{O}(n^3)$ 
"Point.h", "3dHull.h" 10 lines

template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
    if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
        trifun(0,1+d,2-d); }
    vector<P> p3;
    trav(p, ps) p3.emplace_back(p.x, p.y, p.dist2());
    if (sz(ps) > 3) trav(t, hull3d(p3)) if ((p3[t.b]-p3[t.a]).
        cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
        trifun(t.a, t.c, t.b);
}
```

7.5 3D

```
PolyhedronVolume.h
Description: Magic formula for the volume of a polyhedron. Faces should
point outwards.
"Point.h" 6 lines

template <class V, class L>
double signed_poly_volume(const V& p, const L& trilst) {
    double v = 0;
    trav(i, trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

```
Point3D.h
Description: Class to handle points in 3D space. T can be e.g. double or
long long.
"Point.h" 32 lines

template <class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
```

```
P operator/(T d) const { return P(x/d, y/d, z/d); }
T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
}
T dist2() const { return x*x + y*y + z*z; }
double dist() const { return sqrt((double)dist2()); }
//Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
double phi() const { return atan2(y, x); }
//Zenith angle (latitude) to the z-axis in interval [0, pi]
double theta() const { return atan2(sqrt(x*x+y*y),z); }
P unit() const { return *this/(T)dist(); } //makes dist()==1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); }
//returns point rotated 'angle' radians ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
}
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

```
"Point3D.h" 49 lines
typedef Point3D<double> P3;
```

```
struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};
```

```
struct F { P3 q; int a, b, c; };
```

```
vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k};
        E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
        FS.push_back(f);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);

    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j--], FS.back());
                FS.pop_back();
            }
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
```

```
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
C(a, b, c); C(a, c, b); C(b, c, a);
}
}
trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

```
8 lines
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

Strings (8)

PalindromeTree.h

Description: Palindrome Tree for string s
Time: $\mathcal{O}(sz(s))$ for building

```
64 lines
const int maxN = 1000010; // at least sz(s) + 3
struct Node {
    int suffix;
    int len;
    map<char, int> children;

    // not needed for construction, add if needed
    char c;
    int parent;
    vector<int> suffixof;
};

int nodeid;
Node tree[maxN]; // 0: -1 root, 1: empty string
int pos2node[maxN]; // not needed for construction

int add(int parent, char c) {
    if(has(tree[parent].children, c)) {
        return tree[parent].children[c];
    }
    int newid = nodeid++;
    tree[newid].suffix = -1;
    tree[newid].len = tree[parent].len + 2;
    tree[newid].parent = parent;
    tree[newid].c = c;
    tree[parent].children[c] = newid;
    return newid;
}

void build(string& s) {
    nodeid = 2;
    tree[0].parent = -1;
    tree[0].len = -1;
    tree[1].parent = -1;
```

```
tree[0].suffixof.push_back(1);
int cur = 0;
FOR(i, 0, s.size()) {
    int newn = -1;
    while(1) {
        int curlen = tree[cur].len;
        if(i-1-curlen >= 0 && s[i-1-curlen] == s[i]) {
            newn = add(cur, s[i]);
            break;
        }
        cur = tree[cur].suffix;
    }
    pos2node[i] = newn;
    if(tree[newn].suffix != -1) {
        cur = newn;
        continue;
    }
    if(cur == 0) {
        tree[newn].suffix = 1;
    } else {
        do {
            cur = tree[cur].suffix;
        } while(i-1-tree[cur].len < 0
            || s[i-1-tree[cur].len] != s[i]);
        tree[newn].suffix = tree[cur].children[s[i]];
    }
    tree[tree[newn].suffix].suffixof.push_back(newn);
    cur = newn;
}
}
```

SuffixArray.h

Description: Builds suffix array for a string. $a[i]$ is the starting index of the suffix which is i -th in the sorted suffix array. The returned vector is of size $n+1$, and $a[0] = n$. The lcp function calculates longest common prefixes for neighbouring strings in suffix array. The returned vector is of size $n+1$, and $ret[0] = 0$.
Memory: $\mathcal{O}(N)$
Time: $\mathcal{O}(N \log^2 N)$ where N is the length of the string for creation of the SA. $\mathcal{O}(N)$ for longest common prefixes.

```
61 lines
typedef pair<ll, int> pli;
void count_sort(vector<pli> &b, int bits) { // (optional)
    //this is just 3 times faster than stl sort for N=10^6
    int mask = (1 << bits) - 1;
    FOR(it,0,2) {
        ll move = it * bits;
        vi q(1 << bits), w(q.size() + 1);
        FOR(i,0,b.size())
            q[(b[i].first >> move) & mask]++;
        partial_sum(q.begin(), q.end(), w.begin() + 1);
        vector<pli> res(b.size());
        FOR(i,0,b.size())
            res[w[(b[i].first >> move) & mask]++] = b[i];
        swap(b, res);
    }
}
struct SuffixArray {
    vi a;
    string s;
    SuffixArray(const string& _s) : s(_s + '\0') {
        int N = s.size();
        vector<pli> b(N);
        a.resize(N);
        FOR(i, 0, N) {
            b[i].first = s[i];
            b[i].second = i;
        }
        int q = 8;
```

```
while((1 << q) < N) q++;
for(int moc = 0; ; moc++) {
    count_sort(b, q); // sort(b.begin(), b.end()) can be used
                        as well
    a[b[0].second] = 0;
    FOR(i, 1, N) {
        a[b[i].second] = a[b[i-1].second]
            + (b[i-1].first != b[i].first);
    }
    if((1 << moc) >= N) break;
    FOR(i, 0, N) {
        b[i].first = (ll)a[i] << q;
        if(i + (1 << moc) < N) {
            b[i].first += a[i + (1 << moc)];
        }
        b[i].second = i;
    }
}
FOR(i, 0, a.size()) a[i] = b[i].second;
}

vi lcp() {
    int n = a.size(), h = 0;
    vi inv(n), res(n);
    FOR(i, 0, n) inv[a[i]] = i;
    FOR(i, 0, n) if (inv[i] > 0) {
        int p0 = a[inv[i] - 1];
        while(s[i+h] == s[p0+h]) h++;
        res[inv[i]] = h;
        if(h > 0) h--;
    }
    return res;
}
};
```

Hashing.h
Description: Various self-explanatory methods for string hashing. 45 lines

```
typedef unsigned long long H;
static const H C = 123891739; // arbitrary

// Arithmetic mod 2^64-1. 5x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse).
// "typedef H K;" instead if you think test data is random.
struct K {
    typedef __uint128_t H2;
    H x; K(H x=0) : x(x) {}
    K operator+(K o){ return x + o.x + H(((H2)x + o.x)>>64); }
    K operator*(K o){ return K(x*o.x) + H(((H2)x * o.x)>>64); }
    H operator-(K o) { K a = *this + ~o.x; return a.x + !~a.x; }
};

struct HashInterval {
    vector<K> ha, pw;
    HashInterval(string& str) : ha(str.size()+1), pw(ha) {
        pw[0] = 1;
        FOR(i,0,str.size())
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
};

vector<H> getHashes(string& str, int length) {
    if (str.size() < length) return {};
    K h = 0, pw = 1;
    FOR(i,0,length)
```

```
h = h * C + str[i], pw = pw * C;
vector<H> ret = {h - 0};
FOR(i,length,str.size()) {
    ret.push_back(h * C + str[i] - pw * str[i-length]);
    h = ret.back();
}
return ret;
}

H hashString(string& s) {
    K h = 0;
    for(auto c:s) h = h * C + c;
    return h - 0;
}
```

Various (9)

Techniques (A)

| | |
|--|-----------|
| techniques.txt | 159 lines |
| Recursion | |
| Divide and conquer | |
| Finding interesting points in N log N | |
| Algorithm analysis | |
| Master theorem | |
| Amortized time complexity | |
| Greedy algorithm | |
| Scheduling | |
| Max contiguous subvector sum | |
| Invariants | |
| Huffman encoding | |
| Graph teory | |
| Dynamic graphs (extra book-keeping) | |
| Breadth first search | |
| Depth first search | |
| * Normal trees / DFS trees | |
| Dijkstra's algoritm | |
| MST: Prim's algoritm | |
| Bellman-Ford | |
| Konig's theorem and vertex cover | |
| Min-cost max flow | |
| Lovasz toggle | |
| Matrix tree theorem | |
| Maximal matching, general graphs | |
| Hopcroft-Karp | |
| Hall's marriage theorem | |
| Graphical sequences | |
| Floyd-Warshall | |
| Eulercykler | |
| Flow networks | |
| * Augumenting paths | |
| * Edmonds-Karp | |
| Bipartite matching | |
| Min. path cover | |
| Topological sorting | |
| Strongly connected components | |
| 2-SAT | |
| Cutvertices, cutedges och biconnected components | |
| Edge coloring | |
| * Trees | |
| Vertex coloring | |
| * Bipartite graphs (=> trees) | |
| * 3^n (special case of set cover) | |
| Diameter and centroid | |
| K'th shortest path | |
| Shortest cycle | |
| Dynamic programming | |
| Knapsack | |
| Coin change | |
| Longest common subsequence | |
| Longest increasing subsequence | |
| Number of paths in a dag | |
| Shortest path in a dag | |
| Dynprog over intervals | |
| Dynprog over subsets | |
| Dynprog over probabilities | |
| Dynprog over trees | |
| 3^n set cover | |
| Divide and conquer | |
| Knuth optimization | |
| Convex hull optimizations | |
| RMQ (sparse table a.k.a 2^k-jumps) | |
| Bitonic cycle | |
| Log partitioning (loop over most restricted) | |
| Combinatorics | |

| |
|--|
| Computation of binomial coefficients |
| Pigeon-hole principle |
| Inclusion/exclusion |
| Catalan number |
| Pick's theorem |
| Number theory |
| Integer parts |
| Divisibility |
| Euklidean algorithm |
| Modular arithmetic |
| * Modular multiplication |
| * Modular inverses |
| * Modular exponentiation by squaring |
| Chinese remainder theorem |
| Fermat's small theorem |
| Euler's theorem |
| Phi function |
| Frobenius number |
| Quadratic reciprocity |
| Pollard-Rho |
| Miller-Rabin |
| Hensel lifting |
| Vieta root jumping |
| Game theory |
| Combinatorial games |
| Game trees |
| Mini-max |
| Nim |
| Games on graphs |
| Games on graphs with loops |
| Grundy numbers |
| Bipartite games without repetition |
| General games without repetition |
| Alpha-beta pruning |
| Probability theory |
| Optimization |
| Binary search |
| Ternary search |
| Unimodality and convex functions |
| Binary search on derivative |
| Numerical methods |
| Numeric integration |
| Newton's method |
| Root-finding with binary/ternary search |
| Golden section search |
| Matrices |
| Gaussian elimination |
| Exponentiation by squaring |
| Sorting |
| Radix sort |
| Geometry |
| Coordinates and vectors |
| * Cross product |
| * Scalar product |
| Convex hull |
| Polygon cut |
| Closest pair |
| Coordinate-compression |
| Quadtrees |
| KD-trees |
| All segment-segment intersection |
| Sweeping |
| Discretization (convert to events and sweep) |
| Angle sweeping |
| Line sweeping |
| Discrete second derivatives |
| Strings |
| Longest common substring |
| Palindrome subsequences |

| |
|---|
| Knuth-Morris-Pratt |
| Tries |
| Rolling polynom hashes |
| Suffix array |
| Suffix tree |
| Aho-Corasick |
| Manacher's algorithm |
| Letter position lists |
| Combinatorial search |
| Meet in the middle |
| Brute-force with pruning |
| Best-first (A*) |
| Bidirectional search |
| Iterative deepening DFS / A* |
| Data structures |
| LCA (2^k-jumps in trees in general) |
| Pull/push-technique on trees |
| Heavy-light decomposition |
| Centroid decomposition |
| Lazy propagation |
| Self-balancing trees |
| Convex hull trick (wcipeg.com/wiki/Convex_hull_trick) |
| Monotone queues / monotone stacks / sliding queues |
| Sliding queue using 2 stacks |
| Persistent segment tree |