



Universitat Politècnica de Catalunya

UPC2

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template.cpp

```
Makefile 1 lines
CXXFLAGS += -g -Wall -Wextra -Wshadow -std=c++11
```

troubleshoot.txt	52 lines
------------------	----------

```
Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
```

LineContainer.h
Description: Container where you can add lines of the form $kx+m$, and query maximum values at points x . Useful for dynamic programming.
Time: $\mathcal{O}(\log N)$ 32 lines

SegmentTree.h	
Description:	Some useful functions for segment trees
Time:	$\mathcal{O}(\log N)$
	65 lines

```
const ll tot = (1 << 19); // bigger than N
ll N = 1; // ...
ll minv[tot * 2];
ll lazy[tot * 2];
```

```
// propagates lazy, msut be called before recursing
void prop(ll x, ll l, ll r) {
    if(r - l <= 1) return;
    minv[2*x] += lazy[x];
    lazy[2*x] += lazy[x];
    minv[2*x+1] += lazy[x];
    lazy[2*x+1] += lazy[x];
    lazy[x] = 0;
}
```

```
// finds the lowest value res, such that all values
// in [res, end) are >= 2 (condition can be adjusted)
ll squery(ll x, ll l, ll r, ll end) {
    if(end <= 1) return end;
    if(minv[x] >= 2) return 1; // ...
    if(r - l <= 1) {
        return r;
    }
    prop(x, l, r);
    if(minv[2*x+1] >= 2) { // ...
        return squery(2*x, l, (l+r)/2, end);
    }
    ll rend = squery(2*x+1, (l+r)/2, r, end);
    if(rend > (l+r)/2) return rend;
    return squery(2*x, l, (l+r)/2, end);
}
```

```
void example() {
    ll a = 0, b = 1;
    init(1, 0, tot);
    sadd(1, 0, tot, a, b, -2);
    ll res = squery(1, 0, tot, a);
}
```

Description: Computes sums $a[i,j]$ for all $i < I, j < J$, and increases single elements $a[i,j]$. Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h"	22 lines
struct FT2 {	

```
vector<vi> ys; vector<FT> ft;
FT2(int limx) : ys(limx) {}
void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
}
void init() {
    trav(v, ys) sort(all(v)), ft.emplace_back(sz(v));
}
int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
        ft[x].update(ind(x, y), dif);
}
ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
        sum += ft[x-1].query(ind(x-1, y));
    return sum;
}
};
```

Numerical (3)

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);

Time: $\mathcal{O}(NM \cdot \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
    }
};
```

```
swap(B[r], N[s]);
}

bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1;
        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i,0,m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}

T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        rep(i,0,m) if (B[i] == -1) {
            int s = 0;
            rep(j,1,n+1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
}
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.

Time: $\mathcal{O}(n^2 m)$

39 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = A.size(), m = x.size(), rank = 0, br, bc;
    if (n) assert(A[0].size() == m);
    // FOR(i, 0, n) FOR(j, 0, m) A[i][j] %=MOD; also b[i]...
    vi col(m); iota(col.begin(), col.end(), 0);

    FOR(i,0,n) {
        double v, bv = 0;
        FOR(r,i,n) FOR(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            FOR(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        FOR(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        FOR(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
        }
    }
}
```

```
FOR(k,i,m) A[j][k] -= fac*A[i][k];
}
rank++;
}

x.assign(m, 0);
for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    FOR(j,0,i) b[j] -= A[j][i] * b[i];
}
return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h" 7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
fail;; }
```

FFT.h

Description: Fast Fourier transform. Also includes a function for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. a and b should be of roughly equal size. For convolutions of integers, rounding the results of conv works if $(|a| + |b|) \max(a, b) < \sim 10^9$ (in theory maybe 10^6); you may want to use an NTT from the Number Theory chapter instead.

Time: $\mathcal{O}(N \log N)$

```
<valarray> 29 lines

typedef valarray<complex<double>> carray;
void fft(carray& x, carray& roots) {
    int N = sz(x);
    if (N <= 1) return;
    carray even = x[slice(0, N/2, 2)];
    carray odd = x[slice(1, N/2, 2)];
    carray rs = roots[slice(0, N/2, 2)];
    fft(even, rs);
    fft(odd, rs);
    rep(k,0,N/2) {
        auto t = roots[k] * odd[k];
        x[k] = even[k] + t;
        x[k+N/2] = even[k] - t;
    }
}

typedef vector<double> vd;
vd conv(const vd& a, const vd& b) {
    int s = sz(a) + sz(b) - 1, L = 32-__builtin_clz(s), n = 1<<L;
    if (s <= 0) return {};
    carray av(n), bv(n), roots(n);
    rep(i,0,n) roots[i] = polar(1.0, -2 * M_PI * i / n);
    copy(all(a), begin(av)); fft(av, roots);
    copy(all(b), begin(bv)); fft(bv, roots);
    roots = roots.apply(conj);
    carray cv = av * bv; fft(cv, roots);
    vd c(s); rep(i,0,s) c[i] = cv[i].real() / n;
    return c;
}
```

FFTIntegers.h

Description: NTT

Time: $\mathcal{O}(N \log N)$

39 lines

```
LL fpw(LL a, LL b, LL p){
    LL r = 1; while(b){if(b&1) r=r*a%p; a=a*a%p; b/=2;} return r;
}

const LL MOD = 2013265921; const LL ROOT = 440564289; // MOD == 15*(1<<27)+1 (prime)
vector<LL> e, er; // ROOT
has order 2^27
void FFT(vector<int> &x, LL d = 1){
    int n = x.size();
    if(n != e.size()){
        e.resize(n); er.resize(n);
        e[0] = 1; e[1] = fpw(ROOT, (1<<27)/n,MOD);
        er[0] = 1; er[1] = fpw(e[1],MOD-2,MOD);
        rep(i,2,n) e[i] = e[i-1] * e[1] % MOD;
        rep(i,2,n) er[i] = er[i-1] * er[1] % MOD;
    }
    if(d == -1) swap(e, er);
    rep(i,0,n){
        int j=0; for(int k=1; k<n; k<=&1, j<=&1) if(k&i) j++; // haxy i cheetosy
        j>=&1; if(i<j) swap(x[i], x[j]); // haxy i cheetosy
    }
    int k=0;
    while((1<<k)<n) k++;
    for(int s=1; s<n; s<=&1){
        --k;
        for(int i=0; i<n; i+=2*s) rep(j,0,s){
            LL u = x[i+j], v = x[i+j+s]*e[j<&k]%MOD;
            x[i+j] = u+v-(u+v>=MOD?MOD:0);
            x[i+j+s] = u-v+(u-v<0?MOD:0);
        }
    }
    if(d == -1) swap(e, er);
}

vector<int> convolution(vector<int> a, vector<int> b){
    int n = 1; while(n < (int)max(a.size(), b.size())) n *= 2;
    n *= 2; a.resize(n); b.resize(n);
    FFT(a); FFT(b); rep(i,0,n) a[i] = (LL)a[i]*b[i]%MOD*fpw(n,MOD -2,MOD)%MOD; FFT(a, -1);
    return a;
}
```

Number theory (4)

4.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

"euclid.h"	18 lines
------------	----------

```
const ll mod = 17; // change to something else
struct Mod {
    ll x;
    Mod(ll xx) : x(xx) {}
    Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
    Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
    Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
    Mod operator/(Mod b) { return *this * invert(b); }
    Mod invert(Mod a) {
        ll x, y, g = euclid(a.x, mod, x, y);
        assert(g == 1); return Mod((x + mod) % mod);
    }
}
```

```
Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
}
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes $\text{LIM} \leq \text{mod}$ and that mod is a prime.

3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

6 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll a, ll e) {
    if (e == 0) return 1;
    ll x = modpow(a * a % mod, e >> 1);
    return e & 1 ? x * a % mod : x;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) = $\sum_{i=0}^{to-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

21 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
    ull res = k / m * sumsq(to) + c / m * to;
    k %= m; c %= m;
    if (k) {
        ull to2 = (to * k + c) / m;
        res += to * to2;
        res -= divsum(to2, m-1 - c, m, k) + to2;
    }
    return res;
}
```

```
ll modsum(ull to, ll c, ll k, ll m) {
    c %= m;
    k %= m;
    if (c < 0) c += m;
    if (k < 0) k += m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large c .
Time: $\mathcal{O}(64/\text{bits} \cdot \log b)$, where $\text{bits} = 64 - k$, if we want to deal with k -bit numbers.

19 lines

```
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
    ull x = a * (b & (po - 1)) % c;
    while ((b >=> bits) > 0) {
        a = (a << bits) % c;
        x += (a * (b & (po - 1))) % c;
    }
    return x % c;
}
```

```
}
ull mod_pow(ull a, ull b, ull mod) {
    if (b == 0) return 1;
    ull res = mod_pow(a, b / 2, mod);
    res = mod_mul(res, res, mod);
    if (b & 1) return mod_mul(res, a, mod);
    return res;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$

30 lines

```
ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1);
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1;
    int r = 0;
    while (s % 2 == 0)
        ++r, s /= 2;
    ll n = 2; // find a non-square mod p
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p);
    ll g = modpow(n, s, p);
    for (;;) {
        ll t = b;
        int m = 0;
        for (; m < r; ++m) {
            if (t == 1) break;
            t = t * t % p;
        }
        if (m == 0) return x;
        ll gs = modpow(g, 1 << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
        r = m;
    }
}
```

4.2 Number theoretic transform

NTT.h

Description: Number theoretic transform. Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For other primes/integers, use two different primes and combine with CRT. May return negative values.

38 lines

```
"ModPow.h"
const ll mod = (119 << 23) + 1, root = 3; // = 998244353
// For p < 2^30 there is also e.g. (5 << 25, 3), (7 << 26, 3),
// (479 << 21, 3) and (483 << 21, 5). The last two are > 10^9.

typedef vector<ll> vl;
void ntt(ll* x, ll* temp, ll* roots, int N, int skip) {
    if (N == 1) return;
    int n2 = N/2;
    ntt(x, temp, roots, n2, skip*2);
    ntt(x+skip, temp, roots, n2, skip*2);
    rep(i,0,N) temp[i] = x[i+skip];
    rep(i,0,n2) {
        ll s = temp[2*i], t = temp[2*i+1] * roots[skip*i];
        x[skip*i] = (s + t) % mod; x[skip*(i+n2)] = (s - t) % mod;
    }
}
```

```
    }
}

void ntt(vl& x, bool inv = false) {
    ll e = modpow(root, (mod-1) / sz(x));
    if (inv) e = modpow(e, mod-2);
    vl roots(sz(x), 1), tmp = roots;
    rep(i,1,sz(x)) roots[i] = roots[i-1] * e % mod;
    ntt(&x[0], &tmp[0], &roots[0], sz(x), 1);
}

vl conv(vl a, vl b) {
    int s = sz(a) + sz(b) - 1; if (s <= 0) return {};
    int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
    if (s <= 200) { // (factor 10 optimization for |a|,|b| = 10)
        vl c(s);
        rep(i,0,sz(a)) rep(j,0,sz(b))
            c[i + j] = (c[i + j] + a[i] * b[j]) % mod;
        return c;
    }
    a.resize(n); ntt(a);
    b.resize(n); ntt(b);
    vl c(n); ll d = modpow(n, mod-2);
    rep(i,0,n) c[i] = a[i] * b[i] % mod * d % mod;
    ntt(c, true); c.resize(s); return c;
}
```

4.3 Primality

eratosthenes.h
Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 ≈ 0.8 s. Runs 30% faster if only odd indices are stored.

```
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
    isprime.set(); isprime[0] = isprime[1] = 0;
    for (int i = 4; i < lim; i += 2) isprime[i] = 0;
    for (int i = 3; i*i < lim; i += 2) if (isprime[i])
        for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
    vi pr;
    rep(i,2,lim) if (isprime[i]) pr.push_back(i);
    return pr;
}
```

MillerRabin.h
Description: Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.
Time: 15 times the complexity of $a^b \bmod c$.

```
"ModMuLL.h"
bool prime(ull p) {
    if (p == 2) return true;
    if (p == 1 || p % 2 == 0) return false;
    ull s = p - 1;
    while (s % 2 == 0) s /= 2;
    rep(i,0,15) {
        ull a = rand() % (p - 1) + 1, tmp = s;
        ull mod = mod_pow(a, tmp, p);
        while (tmp != p - 1 && mod != 1 && mod != p - 1) {
            mod = mod_mul(mod, mod, p);
            tmp *= 2;
        }
        if (mod != p - 1 && tmp % 2 == 0) return false;
    }
    return true;
}
```

```
factor.h
Description: Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run init(bits), where bits is the length of the numbers you use.
Time: Expected running time should be good enough for 50-bit numbers.
"MillerRabin.h", "eratosthenes.h", "euclid.h"
37 lines

vector<ull> pr;
ull f(ull a, ull n, ull &has) {
    return (mod_mul(a, a, n) + has) % n;
}

vector<ull> factor(ull d) {
    vector<ull> res;
    for (size_t i = 0; i < pr.size() && pr[i]*pr[i] <= d; i++)
        if (d % pr[i] == 0) {
            while (d % pr[i] == 0) d /= pr[i];
            res.push_back(pr[i]);
        }
    //d is now a product of at most 2 primes.
    if (d > 1) {
        if (prime(d))
            res.push_back(d);
        else while (true) {
            ull has = rand() % 2321 + 47;
            ull x = 2, y = 2, c = 1;
            for (; c==1; c = gcd((y > x ? y - x : x - y), d)) {
                x = f(x, d, has);
                y = f(f(y, d, has), d, has);
            }
            if (c != d) {
                res.push_back(c); d /= c;
                if (d != c) res.push_back(d);
                break;
            }
        }
    }
    return res;
}

void init(int bits) { //how many bits do we use?
    vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
    vector<ull> pr(p.size());
    for (size_t i=0; i<pr.size(); i++)
        pr[i] = p[i];
}
```

4.4 Divisibility

euclid.h
Description: Finds the Greatest Common Divisor to the integers a and b. Euclid also finds two integers x and y, such that $ax + by = \gcd(a,b)$. If a and b are coprime, then x is the inverse of a (mod b).

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

ll euclid(ll a, ll b, ll &x, ll &y) {
    if (b) { ll d = euclid(b, a % b, y, x);
        return y -= a/b * x, d; }
    return x = 1, y = 0, a;
}
```

4.4.1 Bézout's identity

For $a \neq 0, b \neq 0$, then $d = \gcd(a,b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x,y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h
Description: Euler's totient or Euler's phi function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . The cototient is $n - \phi(n)$. $\phi(1) = 1, p \text{ prime} \Rightarrow \phi(p^k) = (p - 1)p^{k-1}, m, n \text{ coprime} \Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$.
 $\sum_{d|n} \phi(d) = n, \sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: $a, n \text{ coprime} \Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat's little thm: $p \text{ prime} \Rightarrow a^{p-1} \equiv 1 \pmod p \forall a$.

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for(int i = 3; i < LIM; i += 2)
        if(phi[i] == i)
            for(int j = i; j < LIM; j += i)
                (phi[j] /= i) *= i-1;
}
```

4.5 Chinese remainder theorem

chinese.h
Description: Chinese Remainder Theorem. chinese(a, m, b, n) returns a number x, such that $x \equiv a \pmod m$ and $x \equiv b \pmod n$. For not coprime n,m, use chinese_common. Note that all numbers must be less than 2^{31} if you have Z = unsigned long long.
Time: $\log(m+n)$

```
"euclid.h"
template <class Z> Z chinese(Z a, Z m, Z b, Z n) {
    Z x, y; euclid(m, n, x, y);
    Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
    if (ret >= m * n) ret -= m * n;
    return ret;
}

template <class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
    Z d = gcd(m, n);
    if ((b -= a) % n < 0) b += n;
    if (b % d) return -1; // No solution
    return d * chinese(Z(0), m/d, b/d, n/d) + a;
}
```

4.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

4.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.8 Estimates

$$\sum_{d \mid n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (5)

Graph (6)

globalmincut.h
Description: Given an adjacency matrix returns the global mincut and the vertices of one of the cuts.
Time: $\mathcal{O}(V^3)$

```
84 lines
/*
 * If you dont need the cut you can eliminate every thing with
 * this coment "// *****"
 *Explanation of algorithm:
 * -getting the mincut value: it does n-1 iterations. In each
 * iteration it starts by a vertex (random) as set A.
 * then it iterates until only two vertices are left by adding
 * to set A the most tightly connected vertex to A (vertex
 * not in A).
 * it insert this vertex to A. When only two vertices are left
 * , the mincut between those two is the weight W of the
 * edges between
 * the last vertex and A. mincut = min(mincut, W)
 * We then merge the two last vertices and start again.
 *
 * -getting the cut: basically when we merge two nodes we
 * merge them with mfset. When we obtain a new best mincut
 * value, a cut
 * is represented buy the nodes in the same component as the
 * last node;
 */

// Maximum number of vertices in the graph
#define NN 256
// Maximum edge weight (MAXW * NN * NN must fit into an int)
#define MAXW 1000
// Adjacency matrix and some internal arrays
int v[NN], w[NN];
bool a[NN];

int pare[NN]; // *****
int par (int b){ // *****
    if (pare[b] == b) return b;
```

```
pare[b] = par(pare[b]);
return pare[b];
}

inline void merge (int b, int c){ // *****
    pare[par(b)] = par(c);
}

pair < int, vi > minCut(vvi& g, int n) {
    int nl = n;
    // init the remaining vertex set
    for (int i = 0; i < n; i++){
        v[i] = i;
        pare[i] = i; // *****
    }
    // run Stoer-Wagner
    int best = MAXW * n * n;
    vi cut; // *****
    while (n > 1) {
        // initialize the set A and vertex weights
        a[v[0]] = true;
        for (int i = 1; i < n; i++) {
            a[v[i]] = false;
            w[i] = g[v[0]][v[i]];
        }
        // add the other vertices
        int prev = v[0];
        for (int i = 1; i < n; i++) {
            // find the most tightly connected non-A vertex
            int zj = -1;
            for (int j = 1; j < n; j++)
                if (!a[v[j]] && (zj < 0 || w[j] > w[zj])) zj = j;
            // add it to A
            a[v[zj]] = true;
            // last vertex?
            if (i == n - 1) {
                // remember the cut weight
                if (best > w[zj]){
                    best = w[zj];
                    cut.clear(); // *****
                    for (int ko = 0; ko < nl; ko++) if (par(ko) == par(v[
                        zj])) cut.push_back(ko); // *****
                }

                // merge prev and v[zj]
                merge(prev, v[zj]); // *****
                for (int j = 0; j < n; j++)
                    g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][v[j]];
                v[zj] = v[--n];
                break;
            }
            prev = v[zj];
            // update the weights of its neighbours
            for (int j = 1; j < n; j++)
                if (!a[v[j]]) w[j] += g[v[zj]][v[j]];
        }
    }
    return {best, cut};
}
```

EulerianCycle.h
Description: returns de eulerian cycle/tour starting at u, cycle is in reverse order. If its a tour it must start at a vertex with odd degree. It is common to add edges between odd vertex to find a pseudo euler tour.
Usage: Call find cycle with a vertex where a eulerian tour/cycle is possible, when adding edges make sure that two vertex have the same edge iff it is undirected.
Time: $\mathcal{O}(E)$

```
typedef vector<int> vi;
struct edge{
    int u, v;
    bool used;
};

void Eulerdfs(int u, vi &nxt, vi &Euler, vector<edge> &E, const
    vector<vi> &adj) {
    while(nxt[u] < adj[u].size()){
        int go = adj[u][nxt[u]++];
        if(!E[go].used){
            E[go].used = 1;
            int to = (E[go].u ^ E[go].v ^ u);
            Eulerdfs(to, nxt, Euler, E, adj);
        }
    }
    Euler.push_back(u);
}

vi Eulerian(int u, vector<edge> &E, const vector<vi> &adj) {
    vi nxt (adj.size(),0);
    vi Euler;
    Eulerdfs(u, nxt, Euler, E, adj);
    reverse(Euler.begin(), Euler.end());
    return Euler;
}
```

MaxFlow.h
Description: Returns maximum flow.
Usage: To obtain a cut in the mincut problem one must bfs from the source. All the vertices reached from it using only edges with capacity > 0 are in the same cut
Time: $\mathcal{O}(VE^2)$ for general graphs and $\mathcal{O}(E * \sqrt{V})$ for the maximum matching problem (bipartite unit weighted graf). It is generally very fast.

```
67 lines
typedef long long ll;
typedef vector<int> VI;
typedef vector<VI> VVI;
const ll INF = 1000000000000000000LL;

#define VEI(w,e) ((E[e].u == w) ? E[e].v : E[e].u)
#define CAP(w,e) ((E[e].u == w) ? E[e].cap[0] - E[e].flow : E[e
    ].cap[1] + E[e].flow)
#define ADD(w,e,f) E[e].flow += ((E[e].u == w) ? (f) : (-f))

struct Edge { int u, v; ll cap[2], flow; };

VI d, act;

bool bfs(int s, int t, VVI& adj, vector<Edge>& E) {
    queue<int> Q;
    d = VI(adj.size(), -1);
    d[t] = 0;
    Q.push(t);
    while (not Q.empty()) {
        int u = Q.front(); Q.pop();
        for (int i = 0; i < int(adj[u].size()); ++i) {
            int e = adj[u][i], v = VEI(u, e);
            if (CAP(v, e) > 0 and d[v] == -1) {
                d[v] = d[u] + 1;
                Q.push(v);
            }
        }
    }
    return d[s] >= 0;
}

ll dfs(int u,int t,ll bot,VVI& adj,vector<Edge>& E) {
    if (u == t) return bot;
```

```
for (; act[u] < int(adj[u].size()); ++act[u]) {
    int e = adj[u][act[u]];
    if (CAP(u, e) > 0 and d[u] == d[VEI(u, e)] + 1) {
        ll inc=dfs(VEI(u,e),t,min(bot,CAP(u,e)),adj,E);
        if (inc) {
            ADD(u, e, inc);
            return inc;
        }
    }
}
return 0;
}

ll maxflow(int s, int t, VVI& adj, vector<Edge>& E) {
    for (int i=0; i<int(E.size()); ++i) E[i].flow = 0;
    ll flow = 0, bot;
    while (bfs(s, t, adj, E)) {
        act = VI(adj.size(), 0);
        while ((bot = dfs(s,t,INF, adj, E)) flow += bot;
    }
    return flow;
}

void addEdge(int u, int v, VVI& adj, vector<Edge>& E, ll cap){
    Edge e;
    e.u = u;
    e.v = v;
    e.cap[0] = cap;
    e.cap[1] = 0;
    e.flow = 0;
    adj[u].push_back(E.size());
    adj[v].push_back(E.size());
    E.push_back(e);
}

SCC.h
Description: Finds strongly connected components in a directed graph. If vertices u,v belong to the same component, we can reach u from v and vice versa.
Usage: Use addedge to addedges in a directed graph(will also add reverse edges), after calling Kosaraju comp will save the component number of each vertex ordered by topological order.
Time: O(E+V)
27 lines

const int MAXN = 100010;
stack<int> st;
int m[MAXN], comp[MAXN];
vector<int> adj[2][MAXN];
int c = 0;

void addedge(vector<vector<int>> &adj, int u, int v) {
    adj[0][u].push_back(v);
    adj[1][v].push_back(u);
}

void dfs(int u, int t, vector<int>& m) {
    m[u] = 1;
    for(int v : adj[t][u]) if(!m[v]) dfs(v,t);
    if(t) comp[u] = c;
    else st.push(u);
}

void kosaraju(int n) {
    vector<int> m(n,0);
    for(int i = 0; i < n; ++i) if(!m[i]) dfs(i,0,m);
    m = vector<int>(n,0);
    for(;st.size();st.pop()) {
        int u = st.top();
        if(!m[u]) dfs(u,1), c++;
    }
}
```

```

}
}

Geometry (7)

7.1 Geometric primitives

Point.h
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)
25 lines

template <class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()==1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
};

lineDistance.h
Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.
4 lines

template <class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a)/(b-a).dist();
}

SegmentDistance.h
Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
6 lines

typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:

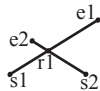
If a unique intersestion point between the line segments going from s1 to e1 and from s2 to e2 exists r1 is set to this point and l is returned. If no intersection point exists 0 is returned and if infinitely many exists 2 is returned and r1 and r2 are set to the two ends of the common line. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long. Use segmentIntersectionQ to get just a true/false answer.

Usage: Point<double> intersection, dummy;

if (segmentIntersection(s1,e1,s2,e2,intersection,dummy)==1) cout << "segments intersect at " << intersection << endl;

Point.h 27 lines

```
template <class P>
int segmentIntersection(const P& s1, const P& e1,
    const P& s2, const P& e2, P& r1, P& r2) {
    if (e1==s1) {
        if (e2==s2) {
            if (e1==e2) { r1 = e1; return 1; } //all equal
            else return 0; //different point segments
        } else return segmentIntersection(s2,e2,s1,e1,r1,r2); //swap
    }
    //segment directions and separation
    P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
    auto a = v1.cross(v2), al = v1.cross(d), a2 = v2.cross(d);
    if (a == 0) { //if parallel
        auto b1=s1.dot(v1), c1=e1.dot(v1),
            b2=s2.dot(v1), c2=e2.dot(v1);
        if (al || a2 || max(b1,min(b2,c2))>min(c1,max(b2,c2)))
            return 0;
        r1 = min(b2,c2)<b1 ? s1 : (b2<c2 ? s2 : e2);
        r2 = max(b2,c2)>c1 ? e1 : (b2>c2 ? s2 : e2);
        return 2-(r1==r2);
    }
    if (a < 0) { a = -a; al = -al; a2 = -a2; }
    if (0<al || a<-al || 0<a2 || a<-a2)
        return 0;
    r1 = s1-v1*a2/a;
    return 1;
}
```



SegmentIntersectionQ.h

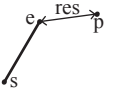
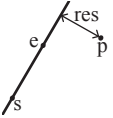
Description:

Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Point.h 16 lines

```
template <class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
    if (e1 == s1) {
        if (e2 == s2) return e1 == e2;
        swap(s1,s2); swap(e1,e2);
    }
    P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
    auto a = v1.cross(v2), al = d.cross(v1), a2 = d.cross(v2);
    if (a == 0) { // parallel
        auto b1 = s1.dot(v1), c1 = e1.dot(v1),
            b2 = s2.dot(v1), c2 = e2.dot(v1);
        return !al && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
    }
    if (a < 0) { a = -a; al = -al; a2 = -a2; }
    return (0 <= al && al <= a && 0 <= a2 && a2 <= a);
}
```

lineIntersection.h



```

pair<double, P> mec2(vector<P>& S, P a, P b, int n) {
    double hi = INFINITY, lo = -hi;
    rep(i,0,n) {
        auto si = (b-a).cross(S[i]-a);
        if (si == 0) continue;
        P m = ccCenter(a, b, S[i]);
        auto cr = (b-a).cross(m-a);
        if (si < 0) hi = min(hi, cr);
        else lo = max(lo, cr);
    }
    double v = (0 < lo ? lo : hi < 0 ? hi : 0);
    P c = (a + b) / 2 + (b - a).perp() * v / (b - a).dist2();
    return {(a - c).dist2(), c};
}

pair<double, P> mec(vector<P>& S, P a, int n) {
    random_shuffle(S.begin(), S.begin() + n);
    P b = S[0], c = (a + b) / 2;
    double r = (a - c).dist2();
    rep(i,1,n) if ((S[i] - c).dist2() > r * (1 + 1e-8)) {
        tie(r,c) = (n == sz(S) ?
            mec(S, S[i], i) : mec2(S, a, S[i], i));
    }
    return {r, c};
}

pair<double, P> enclosingCircle(vector<P> S) {
    assert(!S.empty()); auto r = mec(S, S[0], sz(S));
    return {sqrt(r.first), r.second};
}

```


7.3 Polygons

insidePolygon.h

Description: Returns true if p lies within the polygon described by the points between iterators begin and end. If strict false is returned when p is on the edge of the polygon. Answer is calculated by counting the number of intersections between the polygon and a line going from p to infinity in the positive x-direction. The algorithm uses products in intermediate steps so watch out for overflow. If points within epsilon from an edge should be considered as on the edge replace the line "if (onSegment..." with the comment bellow it (this will cause overflow for int and long long).

Usage: typedef Point<int> pi; vector<pi> v; v.push_back(pi(4,4)); v.push_back(pi(1,2)); v.push_back(pi(2,1)); bool in = insidePolygon(v.begin(),v.end(), pi(3,4), false); **Time:** $\mathcal{O}(n)$

```
"Point.h", "onSegment.h", "SegmentDistance.h" 14 lines

template <class It, class P>
bool insidePolygon(It begin, It end, const P& p,
    bool strict = true) {
    int n = 0; //number of isects with line from p to (inf,p.y)
    for (It i = begin, j = end-1; i != end; j = i++) {
        //if p is on edge of polygon
        if (onSegment(*i, *j, p)) return !strict;
        //or: if (segDist(*i, *j, p) <= epsilon) return !strict;
        //increment n if segment intersects line from p
        n += (max(i->y,j->y) > p.y && min(i->y,j->y) <= p.y &&
            ((*j-*i).cross(p-*i) > 0) == (i->y <= p.y));
    }
    return n&1; //inside if odd number of intersections
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```
"Point.h" 6 lines

template <class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

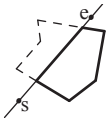
```
"Point.h" 10 lines

typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
    auto i = v.begin(), end = v.end(), j = end-1;
    Point<double> res(0,0); double A = 0;
    for (; i != end; j=i++) {
        res = res + (*i + *j) * j->cross(*i);
        A += j->cross(*i);
    }
    return res / A / 3;
}
```

PolygonCut.h

Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: vector<P> p = ...; p = polygonCut(p, P(0,0), P(1,0));



```
vector<P> res;
rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0)) {
        res.emplace_back();
        lineIntersection(s, e, cur, prev, res.back());
    }
    if (side)
        res.push_back(cur);
}
return res;
}
```

ConvexHull.h

Description: Returns a vector of indices of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Usage: vector<P> ps, hull; trav(i, convexHull(ps)) hull.push_back(ps[i]); **Time:** $\mathcal{O}(n \log n)$



```
"Point.h" 20 lines

typedef Point<ll> P;
pair<vi, vi> ulHull(const vector<P>& S) {
    vi Q(sz(S)), U, L;
    iota(all(Q), 0);
    sort(all(Q), [&S](int a, int b){ return S[a] < S[b]; });
    trav(it, Q) {
#define ADDP(C, cmp) while (sz(C) > 1 && S[C[sz(C)-2]].cross(\
        S[it], S[C.back()]) cmp 0) C.pop_back(); C.push_back(it);
        ADDP(U, <=); ADDP(L, >=);
    }
    return {U, L};
}
```

```
vi convexHull(const vector<P>& S) {
    vi u, l; tie(u, l) = ulHull(S);
    if (sz(S) <= 1) return u;
    if (S[u[0]] == S[u[l]]) return {0};
    l.insert(l.end(), u.rbegin()+1, u.rend()-1);
    return l;
}
```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

```
"ConvexHull.h" 19 lines

vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
    vector<pii> ret;
    int i = 0, j = sz(L) - 1;
    while (i < sz(U) - 1 || j > 0) {
        ret.emplace_back(U[i], L[j]);
        if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
            .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
        else --j;
    }
    return ret;
}
```

```
pii polygonDiameter(const vector<P>& S) {
    vi U, L; tie(U, L) = ulHull(S);
    pair<ll, pii> ans;
    trav(x, antipodal(S, U, L))
        ans = max(ans, {S[x.first] - S[x.second].dist2(), x});
    return ans.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a given polygon (counter-clockwise order). The polygon must be such that every point on the circumference is visible from the first point in the vector. It returns 0 for points outside, 1 for points on the circumference, and 2 for points inside.

Time: $\mathcal{O}(\log N)$

```
"Point.h", "sideOf.h", "onSegment.h" 22 lines

typedef Point<ll> P;
int insideHull2(const vector<P>& H, int L, int R, const P& p) {
    int len = R - L;
    if (len == 2) {
        int sa = sideOf(H[0], H[L], p);
        int sb = sideOf(H[L], H[L+1], p);
        int sc = sideOf(H[L+1], H[0], p);
        if (sa < 0 || sb < 0 || sc < 0) return 0;
        if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R == sz(H)))
            return 1;
        return 2;
    }
    int mid = L + len / 2;
    if (sideOf(H[0], H[mid], p) >= 0)
        return insideHull2(H, mid, R, p);
    return insideHull2(H, L, mid+1, p);
}
```

```
int insideHull(const vector<P>& hull, const P& p) {
    if (sz(hull) < 3) return onSegment(hull[0], hull.back(), p);
    else return insideHull2(hull, 1, sz(hull), p);
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. isct(a, b) returns a pair describing the intersection of a line with the polygon: $\bullet (-1, -1)$ if no collision, $\bullet (i, -1)$ if touching the corner i , $\bullet (i, i)$ if along side $(i, i + 1)$, $\bullet (i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon.

Time: $\mathcal{O}(N + Q \log n)$

```
"Point.h" 63 lines

ll sgn(ll a) { return (a > 0) - (a < 0); }
typedef Point<ll> P;
struct HullIntersection {
    int N;
    vector<P> p;
    vector<pair<P, int>> a;

    HullIntersection(const vector<P>& ps) : N(sz(ps)), p(ps) {
        p.insert(p.end(), all(ps));
        int b = 0;
        rep(i,1,N) if (P{p[i].y,p[i].x} < P{p[b].y, p[b].x}) b = i;
        rep(i,0,N) {
            int f = (i + b) % N;
            a.emplace_back(p[f+1] - p[f], f);
        }
    }

    int qd(P p) {
        return (p.y < 0) ? (p.x >= 0) + 2
            : (p.x <= 0) * (1 + (p.y <= 0));
    }
}
```

```
int bs(P dir) {
    int lo = -1, hi = N;
    while (hi - lo > 1) {
        int mid = (lo + hi) / 2;
        if (make_pair(qd(dir), dir.y * a[mid].first.x) <
            make_pair(qd(a[mid].first), dir.x * a[mid].first.y))
```

```
        hi = mid;
    } else lo = mid;
}
return a[hi%N].second;
}

bool isign(P a, P b, int x, int y, int s) {
    return sgn(a.cross(p[x], b)) * sgn(a.cross(p[y], b)) == s;
}

int bs2(int lo, int hi, P a, P b) {
    int L = lo;
    if (hi < lo) hi += N;
    while (hi - lo > 1) {
        int mid = (lo + hi) / 2;
        if (isign(a, b, mid, L, -1)) hi = mid;
        else lo = mid;
    }
    return lo;
}

pii isct(P a, P b) {
    int f = bs(a - b), j = bs(b - a);
    if (isign(a, b, f, j, 1)) return {-1, -1};
    int x = bs2(f, j, a, b)%N,
        y = bs2(j, f, a, b)%N;
    if (a.cross(p[x], b) == 0 &&
        a.cross(p[x+1], b) == 0) return {x, x};
    if (a.cross(p[y], b) == 0 &&
        a.cross(p[y+1], b) == 0) return {y, y};
    if (a.cross(p[f], b) == 0) return {f, -1};
    if (a.cross(p[j], b) == 0) return {j, -1};
    return {x, y};
}
};
```

7.4 Misc. Point Set Problems

closestPair.h
Description: $i1, i2$ are the indices to the closest pair of points in the point vector p after the call. The distance is returned.
Time: $\mathcal{O}(n \log n)$

```
"Point.h" 58 lines

template<class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template<class It>
bool y_it_less(const It& i, const It& j) {return i->y < j->y;}

template<class It, class IIt> /* IIt = vector<It>::iterator */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
    typedef typename iterator_traits<It>::value_type P;
    int n = yaend-ya, split = n/2;
    if(n <= 3) { // base case
        double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
        if(n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).dist()
            ;
        if(a <= b) { i1 = xa[1];
            if(a <= c) return i2 = xa[0], a;
            else return i2 = xa[2], c;
        } else { i1 = xa[2];
            if(b <= c) return i2 = xa[0], b;
            else return i2 = xa[1], c;
        }
    }
    vector<It> ly, ry, strip;
    P splitp = *xa[split];
    double splitx = splitp.x;
    for(IIt i = ya; i != yaend; ++i) { // Divide
```

```
    if(*i != xa[split] && (**i-splitp).dist2() < 1e-12)
        return i1 = *i, i2 = xa[split], 0; // nasty special case!
    if (**i < splitp) ly.push_back(*i);
    else ry.push_back(*i);
} // assert((signed)lefty.size() == split)
It j1, j2; // Conquer
double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2);
if(b < a) a = b, i1 = j1, i2 = j2;
double a2 = a*a;
for(IIt i = ya; i != yaend; ++i) { // Create strip (y-sorted)
    double x = (*i)->x;
    if(x >= splitx-a && x <= splitx+a) strip.push_back(*i);
}
for(IIt i = strip.begin(); i != strip.end(); ++i) {
    const P &p1 = **i;
    for(IIt j = i+1; j != strip.end(); ++j) {
        const P &p2 = **j;
        if(p2.y-p1.y > a) break;
        double d2 = (p2-p1).dist2();
        if(d2 < a2) i1 = *i, i2 = *j, a2 = d2;
    }
}
return sqrt(a2);
}

template<class It> // It is random access iterators of point<T>
double closestpair(It begin, It end, It &i1, It &i2 ) {
    vector<It> xa, ya;
    assert(end-begin >= 2);
    for (It i = begin; i != end; ++i)
        xa.push_back(i), ya.push_back(i);
    sort(xa.begin(), xa.end(), it_less<It>);
    sort(ya.begin(), ya.end(), y_it_less<It>);
    return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
}
```

```
kdTree.h
Description: KD-tree (2d, can be extended to 3d)

"Point.h" 63 lines

typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if the box is wider than high (not best heuristic...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
            // divide by taking half the array for each child (not best performance with many duplicates in the middle)
```

```
        int half = sz(vp)/2;
        first = new Node({vp.begin(), vp.begin() + half});
        second = new Node({vp.begin() + half, vp.end()});
    }
}

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }

        Node *f = node->first, *s = node->second;
        T bfirst = f->distance(p), bsec = s->distance(p);
        if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

        // search closest side first, other side if needed
        auto best = search(f, p);
        if (bsec < best.first)
            best = min(best, search(s, p));
        return best;
    }

    // find nearest point to a point, and its squared distance
    // (requires an arbitrary operator< for Point)
    pair<T, P> nearest(const P& p) {
        return search(root, p);
    }
};
```

DelaunayTriangulation.h
Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are colinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}(n^2)$

```
"Point.h", "3dHull.h" 10 lines

template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
    if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
        trifun(0,1+d,2-d); }
    vector<P> p3;
    trav(p, ps) p3.emplace_back(p.x, p.y, p.dist2());
    if (sz(ps) > 3) trav(t, hull3d(p3)) if ((p3[t.b]-p3[t.a]).
        cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
        trifun(t.a, t.c, t.b);
}
```

7.5 3D

PolyhedronVolume.h
Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilst) {
    double v = 0;
    trav(i, trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

```
template <class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y),z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set.
No four points must be coplanar, or else random results will be returned.
All faces will point outwards.

Time: $\mathcal{O}(n^2)$

```
"Point3D.h" 49 lines
```

```
typedef Point3D<double> P3;
```

```
struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};
```

```
struct F { P3 q; int a, b, c; };
```

```
vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k};
        E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
        FS.push_back(f);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);
```

```
rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
        F f = FS[j];
        if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
            E(a,b).rem(f.c);
            E(a,c).rem(f.b);
            E(b,c).rem(f.a);
            swap(FS[j--], FS.back());
            FS.pop_back();
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        }
        trav(it, FS) if ((A[it.b] - A[it.a]).cross(
            A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
        return FS;
    };
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

Strings (8)

AhoCorasick.h

Description: Builds an Ahocorasick Trie, with suffix links
Usage: This is an offline Algorithm, Pass the vector of patterns to Trie.init(), find function return the words matched in order of matching
Time: $\mathcal{O}(n + m + z)$

```
const int MaxM = 200005;

struct Trie{
    static const int Alpha = 26;
    static const int first = 'a';
    int lst = 1;
    struct node{
        int nxt[Alpha] = {}, p = -1;
        char c;
        vector<int> end;
        //bitset<MaxN> bitmask;
        int SuffixLink;
    };
    vector<node> V;

    inline int getval(char c) {
        return c - first;
```

```
};

void CreateSuffixLink() {
    queue<int> q;
    for(q.push(0); q.size(); q.pop()) {
        int pos = q.front(); if(!pos) V[pos].
            SuffixLink = -1;
        else {
            int val = getval(V[pos].c);
            int j = V[V[pos].p].SuffixLink;
            while(j > -1 && !V[j].nxt[val]) j = V[j].
                SuffixLink;
            if(j == -1) V[pos].SuffixLink = 0;
            else {
                V[pos].SuffixLink = V[j].nxt[val];
                for(auto &i : V[V[pos].SuffixLink].end) V[
                    pos].end.emplace_back(i);
                //V[pos].bitmask |= V[V[pos].SuffixLink].
                    bitmask;
            }
        }
        for(int i = 0; i < Alpha; ++i) if(V[pos].nxt[i]) q.
            push(V[pos].nxt[i]);
    }
}

void init(vector<string> &v) {
    V.resize(MaxM);
    int id = 0;
    for(auto &s : v) {
        int pos = 0;
        for(char &c : s) {
            int val = getval(c);
            if(!V[pos].nxt[val]) {
                V[lst].p = pos;
                V[lst].c = c;
                V[pos].nxt[val] = lst++;
            }
            pos = V[pos].nxt[val];
        }
        V[pos].end.emplace_back(id++);
        //V[pos].bitmask.set(id++);
    }
    CreateSuffixLink();
}

vector<int> find(string& word) {
    int pos = 0;
    vector<int> ans;
    for(auto &c : word) {
        int val = getval(c);
        while(pos > -1 && !V[pos].nxt[val]) pos = V[pos].
            SuffixLink;
        if(pos == -1) pos = 0;
        else pos = V[pos].nxt[val];
        if(V[pos].end.size()) {
            for(auto &i : V[pos].end) ans.emplace_back(i);
        }
        return ans;
    }
};
```

PalindromeTree.h

Description: Palindrome Tree for string s
Time: $\mathcal{O}(sz(s))$ for building

```
const int maxN = 1000010; // at least sz(s) + 3
```

```
struct Node {
    int suffix;
    int len;
    map<char, int> children;

    // not needed for construction, add if needed
    char c;
    int parent;
    vector<int> suffixof;
};

int nodeid;
Node tree[maxN]; // 0: -1 root, 1: empty string
int pos2node[maxN]; // not needed for construction

int add(int parent, char c) {
    if(has(tree[parent].children, c)) {
        return tree[parent].children[c];
    }
    int newid = nodeid++;
    tree[newid].suffix = -1;
    tree[newid].len = tree[parent].len + 2;
    tree[newid].parent = parent;
    tree[newid].c = c;
    tree[parent].children[c] = newid;
    return newid;
}

void build(string& s) {
    nodeid = 2;
    tree[0].parent = -1;
    tree[0].len = -1;
    tree[1].parent = -1;
    tree[0].suffixof.push_back(1);
    int cur = 0;
    FOR(i, 0, s.size()) {
        int newn = -1;
        while(1) {
            int curlen = tree[cur].len;
            if(i-1-curlen >= 0 && s[i-1-curlen] == s[i]) {
                newn = add(cur, s[i]);
                break;
            }
            cur = tree[cur].suffix;
        }
        pos2node[i] = newn;
        if(tree[newn].suffix != -1) {
            cur = newn;
            continue;
        }
        if(cur == 0) {
            tree[newn].suffix = 1;
        } else {
            do {
                cur = tree[cur].suffix;
            } while(i-1-tree[cur].len < 0
                || s[i-1-tree[cur].len] != s[i]);
            tree[newn].suffix = tree[cur].children[s[i]];
        }
        tree[tree[newn].suffix].suffixof.push_back(newn);
        cur = newn;
    }
}
```

SuffixArray.h

Description: lcp (x,y) = min(lcp(x,x+1),lcp(x+1,x+2)...lcp(y-1,y)) to answer queries with RMQ O(1)

Memory: Build $\mathcal{O}(N \log N)$ (Can be optimised as in B you only use the previous row to compute one row. But then you cannot do lcp
Time: Build: $\mathcal{O}(N \log N)$ where N is the length of the string for creation of the SA. LCP $\mathcal{O}(\log N)$ It is not necessary to use raddixsort if the $\mathcal{O}(n \log^2 n)$ fits the time limit, one can just use stl sort.

65 lines

```
struct SF {
    pair<ll, ll> ord;
    ll id;
    bool operator<(const SF& s) const { return ord < s.ord; }
};

ll lcp(ll x, ll y, vector < vector < ll > > &B, ll N, ll step)
{
    if (x == y) return N - x;
    ll res = 0;
    for (ll i = step - 1; i >= 0 and x < N and y < N; --i)
        if (B[i][x] == B[i][y]) { x += 1<<i; y += 1<<i; res += 1<<i; }
    return res;
}

void raddixSort(vector < SF > & A, vector < vector < ll > > & B,
    vector < ll > &times, vector < ll > & pos, vector < SF > &
    & L2, ll N) {
    ll k = max(N, 256LL);
    for (ll i = 0; i < k + 2; ++i) times[i] = 0;
    for (ll i = 0; i < N; ++i)
        times[A[i].ord.second + 1]++;
    pos[0] = 0;
    for (ll i = 1; i < k + 2; ++i)
        pos[i] = pos[i - 1] + times[i - 1];
    for (ll i = 0; i < N; ++i)
        L2[pos[A[i].ord.second + 1]++] = A[i];

    for (ll i = 0; i < k + 2; ++i)
        times[i] = 0;
    for (ll i = 0; i < N; ++i)
        times[L2[i].ord.first + 1]++;
    pos[0] = 0;
    for (ll i = 1; i < k + 2; ++i)
        pos[i] = pos[i - 1] + times[i - 1];
    for (ll i = 0; i < N; ++i)
        A[pos[L2[i].ord.first + 1]++] = L2[i];

    void compute_suffix_array(vector < SF > & A, vector<vector<ll>
        > & B, ll N, string & S, ll &step) {
        ll MAXN = 3000005; //millor posar numero gran que algo en
            funcio de N pq peta
        vector < SF > L2(MAXN);
        vector < ll > pos(MAXN + 2,0), times(MAXN + 2,0);
        A.resize(N); B.resize(1); B[0].resize(N);
        for (ll i = 0; i < N; ++i) B[0][i] = S[i];
        step = 1;
        for (ll b = 0, pw = 1; b < N; ++step, pw<=&1) {
            for (ll i = 0; i < N; ++i) {
                A[i].ord.first = B[step - 1][i];
                A[i].ord.second = i + pw < N ? B[step - 1][i + pw] : -1;
                A[i].id = i;
            }
            raddixSort(A, B, times, pos, L2, N); //sort(A.begin(), A.
                end());
            B.resize(step + 1); B[step].resize(N);
            b = B[step][A[0].id] = 1;
            for (ll i = 1; i < N; ++i) {
                if (A[i - 1] < A[i]) ++b;
                B[step][A[i].id] = b;
            }
        }
    }
}
```

```
    }
}

Hashing.h
Description: Various self-explanatory methods for string hashing. 45 lines

typedef unsigned long long H;
static const H C = 123891739; // arbitrary

// Arithmetic mod 2^64-1. 5x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse).
// "typedef H K;" instead if you think test data is random.
struct K {
    typedef __uint128_t H2;
    H x; K(H x=0) : x(x) {}
    K operator+(K o){ return x + o.x + H(((H2)x + o.x)>>64); }
    K operator*(K o){ return K(x*o.x) + H(((H2)x * o.x)>>64); }
    H operator-(K o) { K a = *this + ~o.x; return a.x + !~a.x; }
};

struct HashInterval {
    vector<K> ha, pw;
    HashInterval(string& str) : ha(str.size()+1), pw(ha) {
        pw[0] = 1;
        FOR(i,0,str.size())
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
};

vector<H> getHashes(string& str, int length) {
    if (str.size() < length) return {};
    K h = 0, pw = 1;
    FOR(i,0,length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h - 0};
    FOR(i,length,str.size()) {
        ret.push_back(h * C + str[i] - pw * str[i-length]);
        h = ret.back();
    }
    return ret;
}

H hashString(string& s) {
    K h = 0;
    for(auto c:s) h = h * C + c;
    return h - 0;
}
```

Various (9)

Techniques (A)

techniques.txt	159 lines
Recursion	
Divide and conquer	
Finding interesting points in N log N	
Algorithm analysis	
Master theorem	
Amortized time complexity	
Greedy algorithm	
Scheduling	
Max contiguous subvector sum	
Invariants	
Huffman encoding	
Graph teory	
Dynamic graphs (extra book-keeping)	
Breadth first search	
Depth first search	
* Normal trees / DFS trees	
Dijkstra's algoritm	
MST: Prim's algoritm	
Bellman-Ford	
Konig's theorem and vertex cover	
Min-cost max flow	
Lovasz toggle	
Matrix tree theorem	
Maximal matching, general graphs	
Hopcroft-Karp	
Hall's marriage theorem	
Graphical sequences	
Floyd-Warshall	
Eulercykler	
Flow networks	
* Augumenting paths	
* Edmonds-Karp	
Bipartite matching	
Min. path cover	
Topological sorting	
Strongly connected components	
2-SAT	
Cutvertices, cutedges och biconnected components	
Edge coloring	
* Trees	
Vertex coloring	
* Bipartite graphs (=> trees)	
* 3^n (special case of set cover)	
Diameter and centroid	
K'th shortest path	
Shortest cycle	
Dynamic programming	
Knapsack	
Coin change	
Longest common subsequence	
Longest increasing subsequence	
Number of paths in a dag	
Shortest path in a dag	
Dynprog over intervals	
Dynprog over subsets	
Dynprog over probabilities	
Dynprog over trees	
3^n set cover	
Divide and conquer	
Knuth optimization	
Convex hull optimizations	
RMQ (sparse table a.k.a 2^k-jumps)	
Bitonic cycle	
Log partitioning (loop over most restricted)	
Combinatorics	

Computation of binomial coefficients
Pigeon-hole principle
Inclusion/exclusion
Catalan number
Pick's theorem
Number theory
Integer parts
Divisibility
Euklidean algorithm
Modular arithmetic
* Modular multiplication
* Modular inverses
* Modular exponentiation by squaring
Chinese remainder theorem
Fermat's small theorem
Euler's theorem
Phi function
Frobenius number
Quadratic reciprocity
Pollard-Rho
Miller-Rabin
Hensel lifting
Vieta root jumping
Game theory
Combinatorial games
Game trees
Mini-max
Nim
Games on graphs
Games on graphs with loops
Grundy numbers
Bipartite games without repetition
General games without repetition
Alpha-beta pruning
Probability theory
Optimization
Binary search
Ternary search
Unimodality and convex functions
Binary search on derivative
Numerical methods
Numeric integration
Newton's method
Root-finding with binary/ternary search
Golden section search
Matrices
Gaussian elimination
Exponentiation by squaring
Sorting
Radix sort
Geometry
Coordinates and vectors
* Cross product
* Scalar product
Convex hull
Polygon cut
Closest pair
Coordinate-compression
Quadtrees
KD-trees
All segment-segment intersection
Sweeping
Discretization (convert to events and sweep)
Angle sweeping
Line sweeping
Discrete second derivatives
Strings
Longest common substring
Palindrome subsequences

Knuth-Morris-Pratt
Tries
Rolling polynom hashes
Suffix array
Suffix tree
Aho-Corasick
Manacher's algorithm
Letter position lists
Combinatorial search
Meet in the middle
Brute-force with pruning
Best-first (A*)
Bidirectional search
Iterative deepening DFS / A*
Data structures
LCA (2^k-jumps in trees in general)
Pull/push-technique on trees
Heavy-light decomposition
Centroid decomposition
Lazy propagation
Self-balancing trees
Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
Monotone queues / monotone stacks / sliding queues
Sliding queue using 2 stacks
Persistent segment tree