Universitat Politècnica de Catalunya

UPC 2

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Contest (1)

content.txt
10 lines

```
Contest (1)
Mathematics (2)
Data structures (3)
Numerical (4)
Number theory (5)
Combinatorial (6)
Graph (7)
Geometry (8)
Strings (9)
Various (10)
```

template.cpp
21 lines

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

const ll oo = 0x3f3f3f3f3f3f3f3fLL;

#define FOR(i, a, b) for(int i = (a); i < int(b); i++)
#define FORD(i, a, b) for(int i = (b)-1; i >= int(a); i--)
#define trav(i, v) for(auto &i : v)
#define has(c, e) ((c).find(e) != (c).end())
#define sz(c) int((c).size())
#define all(c) c.begin(), c.end()
#define debug(x) cerr << #x << ": " << x << endl;

int main() {
  ios::sync_with_stdio(0); cin.tie(0);
  return 0;
}
```

Makefile
11 lines

```
CXXFLAGS = -Wall -Wextra -Wconversion -Wshadow -Wfatal-errors -
    std=c++14 -O2
DEBUG = -g -fsanitize=undefined,address

%.o : %.cc
  g++ $(CXXFLAGS) $(DEBUG) $< -o $@

%.exe : %.cc
  g++ $(CXXFLAGS) $< -o $@

clean:
  rm *.o *.exe
```

troubleshoot.txt
53 lines

```
Pre-submit:
Write a few simple test cases, if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all datastructures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
(N = 0, N = 1)
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a team mate.
Ask the team mate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a team mate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your team mates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all datastructures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The extremum is given by $x=-b/2a$.

$$\begin{array}{l} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{l} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

In general, given an equation $Ax=b$, the solution to a variable x_i is given by

$$x_i=\frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n=c_1a_{n-1}+\cdots+c_ka_{n-k}$, and $r_1,\ldots,r_k$ are distinct roots of $x^k+c_1x^{k-1}+\cdots+c_k$, there are $d_1,\ldots,d_k$ s.t.

$$a_n=d_1r_1^n+\cdots+d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n=(d_1n+d_2)r^n$.

2.3 Trigonometry

$$\sin(v+w)=\sin v\cos w+\cos v\sin w$$
$$cos(v+w)=\cos v\cos w-\sin v\sin w$$

$$\tan(v+w)=\frac{\tan v+\tan w}{1-\tan v\tan w}$$
$$\sin v+\sin w=2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\cos v+\cos w=2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2=(V-W)\tan(v+w)/2$$

where V,W are lengths of sides opposite angles v,w.

$$a\cos x+b\sin x=r\cos(x-\phi)$$
$$a\sin x+b\cos x=r\sin(x+\phi)$$

where $r=\sqrt{a^2+b^2}, \phi=\operatorname{atan2}(b,a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a,b,c

Semiperimeter: $p=\dfrac{a+b+c}{2}$

Area: $A=\sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R=\dfrac{abc}{4A}$

Inradius: $r=\dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two):

$$s_a=\sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$
Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$
Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

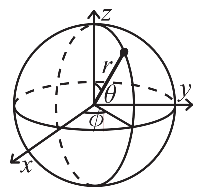
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \text{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

2.4.4 Pick's theorem

$A = i + b/2 - 1$

2.4.5 Volumes

	Sphere	Cube	Tetrahedron
Area	$4\pi r^2$	$6a^2$	$a^2\sqrt{3}$
Volume	$\frac{4}{3}\pi r^3$	a^3	$\frac{1}{12}a^3\sqrt{2}$

	Octahedron	Dodecahedron	Icosahedron
Area	$2\sqrt{3}a^2$	$3a^2\sqrt{25+10\sqrt{5}}$	$5\sqrt{3}a^2$
Volume	$\frac{1}{3}\sqrt{2}a^3$	$\frac{1}{4}(15+7\sqrt{5})a^3$	$\frac{5}{12}(3+\sqrt{5})a^3$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \quad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \quad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\text{erf}(x) \quad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

2.6.1 Faulhaber's formulae

$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$
$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$
$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n^2+n}{2}\right)^2 = \frac{n^4+2n^3+n^2}{4}$
$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{6n^5+15n^4+10n^3-n}{30}$
$1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{2n^6+6n^5+5n^4-n^2}{12}$
$1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{6n^7+21n^6+21n^5-7n^3+n}{42}$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x(x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.7.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu = np, \ \sigma^2 = np(1-p)$$

$\text{Bin}(n,p)$ is approximately $\text{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \ k = 1, 2, \ldots$$
$$\mu = \frac{1}{p}, \ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\, \sigma^2 = \lambda$$

2.7.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a<b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki} t_k$.

2.8.1 Cadenes de Markov

Absorbents Tothom pot arribar a un estat absorbent. $P = \left(\begin{smallmatrix} Q & R \\ 0 & I \end{smallmatrix}\right)$ on p_{ij} és la probabilitat d'anar de s_i a s_j $N = (I-Q)^{-1}$ on n_{ij} és el nombre de cops esperats que passarem per s_j si comencem a s_i. $t = Nc$ on t_i és el nombre esperat de passos abans de ser absorbits. $B = NR$ on b_{ij} és la probabilitat de ser absorbit a l'estat absorbent s_j si comencem a s_i.

Ergòdiques Tothom pot arribar a tothom. Sigui w l'únic vector de probabilitat tal que $w = wP$ llavors W és la matriu que té per files el vector w. $Z = (I - P + W)^{-1}$ on $m_{ij} = \frac{z_{jj} - z_{ij}}{w_j}$ és el nombre de passos esperats per arribar a s_j per primer cop des de s_i. $r_i = \frac{1}{w_i}$ on r_i és el nombre de passos esperats per anar de s_i a ell mateix.

Regulars Hi ha una P^n on tots els elements són positius. $W = limP^n$ on totes les files són iguals al vector w i w_i és la probabilitat d'acabar a l'estat s_i. w és l'únic VEP de VAP $= 1$ on els elements sumen 1.

2.9 Multiplicative Functions

$g(n) = \sum_{d|n} f(d) \implies \sum f(n) = \sum_{d|n} g(d)\mu(\frac{n}{d})$

```
for (int i = 1, la; i <= n; i = la + 1) {
  la = n / (n / i);
  //n / x yields the same value for i <= x <= la.
}
```

Data structures (3)

HashMap.h

Description: Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 (if provided).

2 lines

```
#include <bits/extc++.h>
__gnu_pbds::gp_hash_table<ll, int> h({},{},{},{}, {1 << 16});
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming.
Time: $\mathcal{O}(\log N)$

32 lines

```
bool Q;
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const {
    return Q ? p < o.p : k < o.k;
  }
};

struct LineContainer : multiset<Line> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
    return l.k * x + l.m;
  }
};
```

SegmentTreeAP.h

Description: Update adds a arithmetic progression.

72 lines

```
struct SegTreeAp {
  struct Node {
    int sum, lazy, lazyAp;
    Node() {sum = 0; lazy = 0; lazyAp = 0;}
    Node(int val) {
      sum = val; lazy = 0; lazyAp = 0;
    }
    Node operator +(const Node& n) {
      Node ans;
      ans.sum = sum + n.sum;
      ans.lazy = ans.lazyAp = 0;
      return ans;
    }
  };
  vector<Node> S;
  SegTreeAp(int n) : S(4*n) {}

  void update(int id, int l, int r) {
    if(S[id].lazy) {
      S[id].sum += (r - l) * S[id].lazy;
      if(l + 1 != r) {
        S[id<<1].lazy += S[id].lazy;
        S[id<<1|1].lazy += S[id].lazy;
      }
      S[id].lazy = 0;
    }
    if(S[id].lazyAp) {
      int m = (l + r)>>1;
      S[id].sum += ((r - l) * (r - l + 1) / 2) * S[id].lazyAp;
      if(l + 1 != r) {
        S[id<<1].lazyAp += S[id].lazyAp;
        S[id<<1|1].lazyAp += S[id].lazyAp;
        S[id<<1|1].lazy += S[id].lazyAp * (m - l);
      }
      S[id].lazyAp = 0;
    }
  }

  void build(int id, int l, int r, vi& v) {
    if(l + 1 == r) {
      S[id] = Node(v[l]);
      return;
    }
    int m = (l + r)>>1;
    build(id<<1, l, m, v);
    build(id<<1|1, m, r, v);
    S[id] = S[id<<1] + S[id<<1|1];
  }

  void update(int id, int l, int r, int x, int y, int val, int
       prog) {
    update(id, l, r);
    if(l >= y || r <= x) return;
    if(l >= x && r <= y) {
      S[id].lazy += val + (l - x - 1)*prog;
      S[id].lazyAp += prog;
      update(id, l, r);
      return;
    }
    int m = (l + r)>>1;
    update(id<<1, l, m, x, y, val, prog);
    update(id<<1|1, m, r, x, y, val, prog);
    S[id] = S[id<<1] + S[id<<1|1];
  }

  Node query(int id, int l, int r, int x, int y) {
    update(id, l, r);
    if(l >= y || r <= x) return Node();
    if(l >= x && r <= y) return S[id];
    int m = (l + r)>>1;
    return query(id<<1, l, m, x, y) + query(id<<1|1, m, r, x, y
         );
  }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$. 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j].
Time: $\mathcal{O}\left(\log^2 N\right)$. 22 lines

```
struct FT2D {
    ll R, C;
    vector<vector<ll>> tree;

  // note r+1 & c+1
    FT2D(ll r, ll c) : R(r), C(c), tree(r+1, vector<ll>(c+1)) {
          }

    void update(int row, int col, int diff) {
        for(int i = row+1;i<=R;i += (i & -i))
            for(int j = col+1;j<=C;j += (j & -j))
                tree[i][j] += diff;
    }

    int sum(int row, int col) {
        int r = 0;
        // Not i >= 0
        for(int i = row;i > 0;i -= (i & -i))
            for(int j = col;j > 0;j -= (j & -j))
                r += tree[i][j];
        return r;
    }
};
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.
Usage: x.order_of_key(key) returns how many elements are smaller than key, find_by_order(i) returns the i-th element
Time: $\mathcal{O}(\log N)$ 6 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
```

RMQ.h

Description: Range minimum query
Usage: Call function build and then use query to return the smallest value, ST stores the index to the value.
Time: $<\mathcal{O}(N\log N), \mathcal{O}(1)>$ 35 lines

```
struct SparseTable {
    vector<vector<int> > ST;
    vector<int> P;
    vector<int> v;
    int N;
    int MAXLOG = 0;

    void build(int n, const vector<int>& V){
        N = n;
        v = V;
        while ((1 << MAXLOG) <= N) ++MAXLOG;
        ST = vector<vector<int> > (N, vector<int> (MAXLOG));
        P = vector<int> (N+1);
        int LOG = 0;
        for(int i = 1; i < N + 1; ++i){
            P[i] = ((1<<LOG) > i ?  LOG-1 : ++LOG-1);
        }

        for(int i = 0; i < N; ++i) ST[i][0] = i;
        for (int j = 1; j < MAXLOG; ++j) {
            for (int i = 0; i + (1 << j) - 1 < N; ++i) {
                if (V[ST[i][j - 1]] < V[ST[i + (1 << (j - 1))][
                    j - 1]])
                    ST[i][j] = ST[i][j - 1];
                else
                    ST[i][j] = ST[i + (1 << (j - 1))][j - 1];
            }
        }
    }

  // minimum in range [l, r] (both inclusive)
    int query(int l, int r){
        int LOG = P[r-l+1];
        return min(v[ST[l][LOG]], v[ST[r - (1 << LOG) + 1][LOG
             ]]);
    }
};
```

Treap.h

Description: Treap
Time: Expected $\mathcal{O}(\log N)$ 84 lines

```
struct Tree {
  Tree *left, *right, *parent; // if parent needed.
  ll x, y, count;
  Tree (ll x) : x(x), y(rand()), count(1) {
    left = right = parent = nullptr;
  }
};

inline int card(Tree* t) {return (t ? t->count : 0);}
inline void setp(Tree *t, Tree *p) {
  if(t) t->parent = p; }

void update(Tree* t) { //Update when pointers change
  if (!t) return;
  t->count = 1 + card(t->left) + card(t->right);
  setp(t->left, t);
  setp(t->right, t);
}
```

```
Tree* merge(Tree* t1, Tree* t2) {
  if (t1 == nullptr) return t2;
  if (t2 == nullptr) return t1;
  if (t1->y >= t2->y) {
    t1->right = merge(t1->right, t2);
    update(t1);
    return t1;
  } else {
    t2->left = merge(t1, t2->left);
    update(t2);
    return t2;
  }
}

// leaves on the left all nodes less than x.
pair<Tree*, Tree*> split(Tree* t, ll x) {
  if (t == nullptr) return {nullptr, nullptr};
  if (t->x < x) {// if (card(t->left) + 1 <= x) {
  // auto p = split(t->right, x - card(t->left) - 1);
    auto p = split(t->right, x);
    t->right = p.first;
    update(t);
    setp(t, nullptr);
    return {t, p.second};
  } else {
    auto p = split(t->left, x);
    t->left = p.second;
    update(t);
    setp(t, nullptr);
    return {p.first, t};
  }
}

Tree* insert(Tree* t, Tree* n) {
  auto p = split(t, n->x);
  t = merge(p.first, n);
  t = merge(t, p.second);
  return t;
}

// Devuelve cuantos hay <= x
int count(Tree* t, int x) {
  if (!t) return 0;
  if (t->x <= x)
    return 1 + card(t->left) + count(t->right, x);
  else return count(t->left, x);
}

Tree* update_node(Tree* &root, Tree* node, ll nx) {
  setp(node->left, nullptr);
  setp(node->right, nullptr);
  auto m = merge(node->left, node->right);
  auto p = node->parent;
  node->left = node->right = node->parent = nullptr;
  node->x = nx;
  update(node);
  if (p) {
    p->left == node ? p->left = m : p->right = m;
    if (m) m->parent = p;
    while (p) { update(p); p = p->parent; }
  } else {
    root = m;
  }
  return insert(root, node);
}
```

Numerical (4)

Polynomial.h

17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = sz(a) - 1; i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    FOR(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  FOR(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      FOR(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \ldots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  FOR(k,0,n-1) FOR(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  FOR(k,0,n) FOR(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

Integrate.h

Description: Fast integration using an adaptive Simpson's rule.
Usage:
```
double z, y;
double h(double x) { return x*x + y*y + z*z <= 1; }
double g(double y) { ::y = y; return quad(h, -1, 1); }
double f(double z) { ::z = z; return quad(g, -1, 1); }
double sphereVol = quad(f, -1, 1), pi = sphereVol*3/4;
```

19 lines

```
typedef double d;
template<typename F>
d simpson(F f, d a, d b) {
  d c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}
template<typename F>
d rec(F f, d a, d b, d eps, d S) {
  d c = (a+b) / 2;
  d S1 = simpson(f, a, c);
  d S2 = simpson(f, c, b), T = S1 + S2;
  if (abs (T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2);
}
template<typename F>
d quad(F f, d a, d b, d eps = 1e-8) {
    return rec(f, a, b, eps, simpson(f, a, b));
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  FOR(i,0,n) tmp[i][i] = 1, col[i] = i;

  FOR(i,0,n) {
    int r = i, c = i;
    FOR(j,i,n) FOR(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    FOR(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    FOR(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      FOR(k,i+1,n) A[j][k] -= f*A[i][k];
      FOR(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    FOR(j,i+1,n) A[i][j] /= v;
    FOR(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) FOR(j,0,i) {
    double v = A[j][i];
    FOR(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  FOR(i,0,n) FOR(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.
68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      FOR(i,0,m) FOR(j,0,n) D[i][j] = A[i][j];
      FOR(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      FOR(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    FOR(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      FOR(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    FOR(j,0,n+2) if (j != s) D[r][j] *= inv;
    FOR(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      FOR(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      FOR(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      FOR(i,0,m) if (B[i] == -1) {
        int s = 0;
        FOR(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    FOR(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$
39 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = A.size(), m = x.size(), rank = 0, br, bc;
  if (n) assert(A[0].size() == m);
  // FOR(i, 0, n) FOR(j, 0, m) A[i][j] %= MOD; also b[i]...
  vi col(m); iota(col.begin(), col.end(), 0);

  FOR(i,0,n) {
    double v, bv = 0;
    FOR(r,i,n) FOR(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      FOR(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    FOR(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    FOR(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      FOR(k,i,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    FOR(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:
"SolveLinear.h" 7 lines

```
FOR(j,0,n) if (j != i) // instead of FOR(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
FOR(i,0,rank) {
  FOR(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}(n^2 m)$
34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  FOR(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      FOR(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    FOR(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    FOR(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    FOR(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

FFT.h

Description: Fast Fourier transform to multiply polynomials. Can be modified to multiply modulus x. Error around $\frac{max_{ans}}{1e15}$ or $\frac{max_{ans}}{2.5e18}$ for double/long double.
Time: $\mathcal{O}(N \log N)$
80 lines

```
using LD = double; //long double 2-2.5 times slower

struct C { LD x,y;
  C operator * (const C& C1) const {
    return C{x*C1.x - y*C1.y, x*C1.y + y*C1.x};
  }
  void operator += (const C& C1) {
    x += C1.x, y += C1.y;
  }
};

void FFT(vector<C> & a, int rev) {
  const int N = a.size();
  for(int i = 1, k = 0; i < N; ++i) {
    for(int bit = N / 2; (k ^= bit) < bit; bit /= 2);
    if(i < k) swap(a[i], a[k]);
  }
  for(int len = 1, who = 0; len < N; len *= 2, ++who) {
    static vector<C> t[30];
    vector<C> & om = t[who];
    if(om.empty()) {
      om.resize(len);
      const LD ang = 2 * acosl(0) / len;
      FOR(i, 0, len) om[i] = i%2 || !who ?
        C{cos(i*ang), sin(i*ang)} : t[who-1][i/2];
    }
    for(int i = 0; i < N; i += 2 * len)
      FOR(k, 0, len) {
        const C x = a[i+k], y = a[i+k+len]
          * C{om[k].x, om[k].y * rev};
        a[i+k] += y;
        a[i+k+len] = C{x.x - y.x, x.y - y.y};
      }
  }
  if(rev == -1) FOR(i, 0, N) a[i].x /= N;
}

template<typename T> //add ll mod to header
vector<T> PolyMul(const vector<T> & a, const vector<T> & b, /*
     ll mod,*/ bool split = false) {
  if(a.empty() || b.empty()) return {};
  int n = a.size() + b.size();
  vector<T> ans(n - 1);
  n = 1 << (32 - __builtin_clz(int(a.size() + b.size() - 1)));
  auto speed = [&](const vector<C> & w, int i, int k) {
    int j = i ? n - i : 0, r = k ? -1 : 1;
    return C{w[i].x + w[j].x * r, w[i].y
        - w[j].y * r} * (k ? C{0, -0.5} : C{0.5, 0});
  };
  if(!split) {
    vector<C> in(n), done(n);
    FOR(i, 0, a.size()) in[i].x = a[i];
    FOR(i, 0, b.size()) in[i].y = b[i];
    FFT(in,  1);
    FOR(i, 0, n) done[i] = speed(in, i, 0) * speed(in, i, 1);
    FFT(done, -1);
    FOR(i, 0, ans.size()) {ans[i] = is_integral<T>::value ?
        llround(done[i].x) : done[i].x;} //ans[i] %= mod;
  //FOR(i,0,ans.size())err=max(err,abs(done[i].x-ans[i]));
  }
  else { // Split big INTEGERS into pairs a1*M+a2,
    const T M = 1<<10; // where M = sqrt(max_absvalue).
    vector<C> t[2]; // This version is 2.2-2.5 times slower.
    FOR(x, 0, 2) {
      t[x].resize(n);
      auto & in = x ? b : a; // below use (in[i]-big) if
           normalized
      FOR(i, 0, in.size()) t[x][i]=C{LD(in[i]%M), LD(in[i]/M)};
      FFT(t[x], 1);
    }
    T mul = 1;
    for(int s = 0; s < 3; ++s, mul *= M /* ,mul%= mod*/) {
      vector<C> prod(n);
      FOR(x, 0, 2) FOR(y, 0, 2) if(x + y == s) FOR(i, 0, n)
        prod[i] += speed(t[0], i, x) * speed(t[1], i, y);
      FFT(prod, -1);
      FOR(i, 0, ans.size()) ans[i] += llround(prod[i].x)*mul;
                            //ans[i] += (llround(prod[i].x)%mod)*
                                 mul, ans[i] %= mod; //(
                                 substitute)
    }
  }
  return ans;
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x]\cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N\log N)$ 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) FOR(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) trav(x, a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  FOR(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$ 33 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  FOR(i,0,n) {
    int b = i;
    FOR(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    FOR(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) FOR(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}

ll det(vector<vector<ll>>& a, ll mod) {
  int n = sz(a); ll ans = 1;
  FOR(i,0,n) {
    FOR(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) FOR(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
"../number-theory/ModPow.h" 20 lines

```
vector<ll> BerlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  FOR(i,0,n) { ++m;
    ll d = s[i] % mod;
    FOR(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    FOR(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  trav(x, C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0\dots n-1]$ and $tr[0\dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}(n^2\log k)$ 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(S);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    FOR(i,0,n+1) FOR(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) FOR(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  FOR(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.
"euclid.h" 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM ≤ mod and that mod is a prime.

3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
FOR(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

6 lines

```
ll modpow(ll a, ll e, const ll mod) {
  ll cur = 1;
  for(;e;e >>= 1, a = (a*a)%mod){
    if (e&1) {cur *= e; cur %= mod;}
  } return cur;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) = $\sum_{i=0}^{\text{to}-1}(ki+c)\%m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large c.
Time: $\mathcal{O}(64/bits \cdot \log b)$, where $bits = 64 - k$, if we want to deal with k-bit numbers.

19 lines

```
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
  ull x = a * (b & (po - 1)) % c;
  while ((b >>= bits) > 0) {
    a = (a << bits) % c;
    x += (a * (b & (po - 1))) % c;
  }
  return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
  if (b == 0) return 1;
  ull res = mod_pow(a, b / 2, mod);
  res = mod_mul(res, res, mod);
  if (b & 1) return mod_mul(res, a, mod);
  return res;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$

"ModPow.h" 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1);
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

5.2 Primality

eratosthenes.h

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 ≈ 0.8 s. Runs 30% faster if only odd indices are stored.

11 lines

```
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vi pr;
  FOR(i,2,lim) if (isprime[i]) pr.push_back(i);
  return pr;
}
```

MillerRabin.h

Description: Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.
Time: 15 times the complexity of $a^b \mod c$.

"ModMulLL.h" 16 lines

```
bool prime(ull p) {
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ull s = p - 1;
  while (s % 2 == 0) s /= 2;
  FOR(i,0,15) {
    ull a = rand() % (p - 1) + 1, tmp = s;
    ull mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
}
```

factor.h

Description: Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run init(bits), where bits is the length of the numbers you use. to get factor multiple times, uncomment comments with (*)
Time: Expected running time should be good enough for 50-bit numbers.

"MillerRabin.h", "eratosthenes.h", "euclid.h" 37 lines

```
vector<ull> pr;
ull f(ull a, ull n, ull &has) {
  return (mod_mul(a, a, n) + has) % n;
}
vector<ull> factor(ull d) {
  vector<ull> res;
  for (size_t i = 0; i < pr.size() && pr[i]*pr[i] <= d; i++)
    if (d % pr[i] == 0) {
      while (d % pr[i] == 0) /*{ */ d /= pr[i];
      res.push_back(pr[i]); /*} (*)*/
    }
  //d is now a product of at most 2 primes.
  if (d > 1) {
    if (prime(d))
      res.push_back(d);
    else while (true) {
      ull has = rand() % 2321 + 47;
      ull x = 2, y = 2, c = 1;
      for (; c==1; c = gcd((y > x ? y - x : x - y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      }
      if (c != d) {
        res.push_back(c); d /= c;
        if (d != c /* || true (*)*/) res.push_back(d);
        break;
      }
    }
  }
  return res;
}
void init(int bits) {//how many bits do we use?
  vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
  pr.resize(p.size());
  for (size_t i=0; i<pr.size(); i++)
    pr[i] = p[i];
}
```

5.3 Divisibility

euclid.h

Description: Finds the Greatest Common Divisor to the integers a and b. Euclid also finds two integers x and y, such that $ax + by = \gcd(a, b)$. If a and b are coprime, then x is the inverse of $a \pmod b$.

7 lines

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's totient* or *Euler's phi* function is defined as $\phi(n) :=$ # of positive integers $\le n$ that are coprime with n. The *cototient* is $n - \phi(n)$. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  FOR(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for(int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for(int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Chinese remainder theorem

chinese.h

Description: Chinese Remainder Theorem.
chinese(a, m, b, n) returns a number x, such that $x \equiv a \pmod m$ and $x \equiv b \pmod n$. For not coprime n, m, use chinese_common. Note that all numbers must be less than 2^{31} if you have Z = unsigned long long.
Time: $\log(m + n)$

"euclid.h" 13 lines

```
template <class Z> Z chinese(Z a, Z m, Z b, Z n) {
  Z x, y; euclid(m, n, x, y);
  Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
  if (ret >= m * n) ret -= m * n;
  return ret;
}

template <class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
  Z d = gcd(m, n);
  if (((b -= a) %= n) < 0) b += n;
  if (b % d) return -1; // No solution
  return d * chinese(Z(0), m/d, b/d, n/d) + a;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.8 Number Theoretic Transform

NTT.h

Description: Number theoretic transform. Can be used for convolutions modulo specific nice primes of the form $2^a \times b + 1$, where the convolution result has size at most 2^a. For other primes/integers, use two different primes and combine with CRT. May return negative values. Consider Bruteforcing for small N, 10times slower than FFT
Time: $\mathcal{O}(N \log N)$

"ModPow.h" 43 lines

```
const ll mod[] = {(119<<23)+1, (5<<25)+1, 7<<26|1, 479<<21|1,
     483<<21|1};
const ll root[] = {3,3,3,3,5};

template<typename T>
void ntt(T* x, T* temp, T* roots, int N, int skip, ll mod) {
  if (N == 1) return;
  int n2 = N>>1;
  ntt(x     , temp, roots, n2, skip<<1, mod);
  ntt(x+skip, temp, roots, n2, skip<<1, mod);
  FOR(i,0,N) temp[i] = x[i*skip];
  FOR(i,0,n2) {
    ll s = temp[i<<1], t = temp[i<<1|1] * roots[skip*i];
    x[skip*i] = (s + t) % mod; x[skip*(i+n2)] = ((s - t) % mod
         + mod) %mod;
  }
}

template<typename T>
void ntt(vector<T>& x, ll mod, ll root,  bool inv = false) {
  ll e = modpow(root, (mod-1) / sz(x), mod);
  if (inv) e = modpow(e, mod-2, mod);
  vector<T> roots(sz(x), 1), temp = roots;
  FOR(i,1,sz(x)) roots[i] = roots[i-1] * e % mod;
  ntt(&x[0], &temp[0], &roots[0], sz(x), 1, mod);
}

template<typename T>
vector<T> conv(vector<T> a, vector<T> b, ll mod, ll root) {
  int s = sz(a) + sz(b) - 1; if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
  a.resize(n); ntt(a, mod, root);
  b.resize(n); ntt(b, mod, root);
  vector<T> c(n); ll d =  modpow(n, mod-2, mod);
  FOR(i,0,n) c[i] = a[i] * b[i] % mod * d % mod;
  ntt(c, mod, root, true); c.resize(s); return c;
}

template<typename T>
vector<T> PolyMul(const vector<T>&  a,const vector<T>& b) {
  vector<T> ans1 = conv(a, b, mod[1], root[1]);
  vector<T> ans2 = conv(a, b, mod[2], root[2]);
  for(int i = 0; i < sz(ans1); ++i) ans1[i] = (chinese(ans1[i],
        mod[1], ans2[i], mod[2]));
  return ans1;
}
```

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)
Time: $\mathcal{O}(n)$

6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x, v) r = r * ++i + __builtin_popcount(use & -(1 << x)),
    use |= 1 << x;                          // (note: minus, not ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g $(g.x = x)$.

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Binomials

binomialModPrime.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_kp^k + ... + n_1p + n_0$ and $m = m_kp^k + ... + m_1p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
Time: $\mathcal{O}(\log_p n)$ 10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] % p;
    n /= p; m /= p;
  }
  return c;
}
```

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$. 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  FOR(i,1,sz(v)) FOR(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.2 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.3.3 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.3.4 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

6.3.5 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_iC_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

ArticulationPointAndBridges.h

Description: Finds all articulation points and bridges of a undirected graph.
Time: $\mathcal{O}(E+V)$ 33 lines

```
void ArticulationPointandBridges(vector<vi>& adj, vector<Edge>
    E, vi& articulation, vi& bridges) {
  int n = sz(adj), m = sz(E);
  articulation = vi(n, -1);
  bridges = vi(m, -1);
  vi dfs_low(n, -1), dfs_num(n ,-1), parent(n, -1);
  articulation = vi(n, 0);
  bridges = vi(m, 0);
  int dfsroot, rootc, cnt = 0;
  function<void(int)> ABdfs =
    [&](int u) {
```

```
    dfs_num[u] = dfs_low[u] = cnt++;
    for(int t : adj[u]){
      int v = E[t].u ^ E[t].v  ^ u;
      if(dfs_num[v] == -1){
        if(u == dfsroot) rootc++;
        parent[v] = t;
        ABdfs(v);
        dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        if(dfs_low[v] >= dfs_num[u])articulation[u] = 1;
        if(dfs_low[v] > dfs_num[u]) bridges[t] = 1;
      }
      else if (t != parent[u]){
        dfs_low[u] = min(dfs_low[u], dfs_num[v]);
      }
    }
  };
  FOR(i, 0, n) {
    if(dfs_num[i] == -1) {
      rootc = 0, dfsroot = i;
      ABdfs(i);
    }
  }
}
```

EulerianCycle.h

Description: returns de eulerian cycle/tour starting at u. If its a tour it must start at a vertex with odd degree. It is common to add edges between odd vertex to find a pseudo euler tour.

Usage: adj should contain index of edge in the vector<edge>, if undirected add index to both rows of adj list. If directed make sure if it needs to be connected, difference between in/out edges. If it is a tour then u must be a vertex with odd degree, else it can be any edge.

Time: $\mathcal{O}(E)$

23 lines

```
struct edge{
  int u, v;
  bool used;
};

void Eulerdfs(int u, vi &nxt, vi &Euler, vector<edge> &E, const
      vector<vi> &adj) {
  while(nxt[u] < adj[u].size()){
    int go = adj[u][nxt[u]++];
    if(!E[go].used){
      E[go].used = 1;
      int to = (E[go].u ^ E[go].v ^ u);
      Eulerdfs(to, nxt, Euler, E, adj);
    }
  }
  Euler.push_back(u);
}

vi Eulerian(int u, vector<edge> &E, const vector<vi> &adj) {
  vi nxt (adj.size(), 0); vi Euler;
  Eulerdfs(u, nxt, Euler, E, adj);
  reverse(Euler.begin(), Euler.end());
  return Euler;
}
```

HLD.h

Description: Creates a Heavy Light Decomposition which also allows subtree updates. Subtree of v is [in[v], out[v]). [in[nxt[v]],in[v]] is the heavy path of v

Time: $\mathcal{O}(V+E)$

22 lines

```
struct HLD {
  vi sz, in, out, rin, nxt;
  int t;
  HLD(int n) : sz(n), in(n,0), out(n), rin(n), nxt(n), t(0){}

  void dfs_sz(int v = 0, vector<vi>& adj) {
    sz[v] = 1;
    for(auto &u: adj[v]) {
      dfs_sz(u); sz[v] += sz[u];
      if(sz[u] > sz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }

  void dfs_hld(int v = 0, vector<vi>& adj) {
    in[v] = t++; rin[in[v]] = v;
    for(auto u: adj[v]) {
      nxt[u] = (u == adj[v][0] ? nxt[v] : u);
      dfs_hld(u);
    }
    out[v] = t;
  }
};
```

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: Use addedge to add edges in a directed graph, after calling Kosaraju comp will save the component number of each vertex ordered by topological order.

Time: $\mathcal{O}(E+V)$

28 lines

```
struct SCC {
  vi comp, m;
  vector<vi> adj[2];
  stack<int> st;
  int c;
  int n;
  SCC(int n) : comp(n), m(n,0), c(0), n(n) {
    adj[0] = adj[1] = vector<vi>(n);
  }

  void addEdge(int u, int v) {
    adj[0][u].push_back(v);
    adj[1][v].push_back(u);
  }

  void dfs(int u, int t) {
    m[u] = 1;
    for(int v : adj[t][u]) if(!m[v]) dfs(v,t);
    if(t) comp[u] = c;
    else st.push(u);
  }

  void kosaraju() {
    for(int i = 0; i < n; ++i) if(!m[i]) dfs(i,0);
    m = vi(n,0);
    for(;st.size();st.pop()) if(!m[st.top()]) dfs(st.top(),1),
        c++;
  }
};
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&$... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
a xor b is (a or b) and (a or b)

Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

57 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f^1].push_back(j);
    gr[j^1].push_back(f);
  }
  void set_value(int x) { either(x, x); }

  void at_most_one(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    trav(e, gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    ++time;
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = time;
      if (values[x>>1] == -1)
        values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    FOR(i,0,2*N) if (!comp[i]) dfs(i);
    FOR(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph.
Time: $\mathcal{O}(E+V)$ 35 lines

```
struct Edge {
  int u, v; //Fill this
  int bicomp; //Index of the bic. comp. (don't fill)
};

vi bic_vertex; // Vertex repr for each bic. comp.
vi num, low; //Initialize to -1
int act, bicos; //Put this to zero for every testcase
stack<int> S;

void dfs(int u, vector<vi>& adj, vector<Edge>& E, int p=-1){
  num[u] = low[u] = act++;
  for (int i = 0; i < adj[u].size(); ++i) {
    int v = E[adj[u][i]].v;
    if (v == u) v = E[adj[u][i]].u;
    if (num[v] == -1) {
      int size = S.size();
      S.push(adj[u][i]);
      dfs(v, adj, E, u);
      low[u] = min(low[u], low[v]);
      if (low[v] >= num[u]) {
        while (S.size() != size) {
          E[S.top()].bicomp = bicos;
          S.pop();
        }
        bic_vertex.push_back(u);
        ++bicos;
      }
    }
    else if (v != p and num[v] < num[u]) {
      S.push(adj[u][i]);
      low[u] = min(low[u], num[v]);
    }
  }
}
```

MinimalArborescence.h

Description: Computes a minimal spanning tree in a directed graph from a root under the condition, that it exists. Set the values marked with TODO. Algo changes N and from!
Time: $\mathcal{O}(n \log n)$ 65 lines

```
int N, R; // TODO number of nodes and number of root
vector<pii> from[2*MAXN]; // TODO for each node list of (
    predecessor, cost)
int pred[2*MAXN], label[2*MAXN];
bool del[2*MAXN];

bool cycle(int n, int p) {
  if (label[n] == 2) return false;
  if (label[n] == 1) {
    label[n] = -1;
    return true;
  }
  label[n] = 1;
  if (cycle(pred[n], n)) {
    if (label[n] == 1) {
      label[n] = -1;
    } else if (p != -1) {
      label[p] = 2;
    }
    return true;
  } else {
    label[n] = 2;
    return false;
  }
}
int rek() {
  int res = 0;
  pred[R] = -1;
  FOR(i, 0, N) {
    if (i == R || del[i]) continue;
    int m = oo;
    FOR(j, 0, sz(from[i])) m = min(m, from[i][j].second);
    res += m;
    FOR(j, 0, sz(from[i])) {
      from[i][j].second -= m;
      if (from[i][j].second == 0) pred[i] = from[i][j].first;
    }
  }
  FOR(i, 0, N) label[i] = 0;
  label[R] = 2;
  FOR(i, 0, N) {
    if (del[i] || label[i] == 2) continue;
    if (cycle(i, -1)) {
      FOR(j, 0, N) {
        if (del[j]) continue;
        if (label[j] == -1) {
          FOR(k, 0, sz(from[j])) if (label[from[j][k].first] !=
              -1)
            from[N].pb(from[j][k]);
          from[j].clear();
          del[j] = true;
        } else {
          FOR(k, 0, sz(from[j])) if (label[from[j][k].first] ==
              -1)
            from[j][k].first = N;
        }
      }
      del[N++] = false;
      return res + rek();
    }
  }
  return res;
}

int min_arbor() {
  FOR(i, 0, N) del[i] = false;
  return rek();
}
```

BinaryJumps.h

Description: Computes lca with binaryjumps. Insert nodes in dfs preorder. NODES MUST BE 1 INDEXED! (or add 1 to the parameters in each function here)
Time: $\mathcal{O}(n \log n)$ 29 lines

```
const ll maxN = 201005; // TODO
const ll MAXLOG = 20; // TODO: (1 << MAXLOG) > maxN
struct BinaryLifting {
  ll BL[maxN][MAXLOG];
  ll D[maxN];

  void init(ll root) { D[root] = 1; }
  void insert(ll n, ll par) {
    D[n] = D[par] + 1;
    BL[n][0] = par;
    FOR(j, 1, MAXLOG) BL[n][j] = BL[BL[n][j-1]][j-1];
  }
  ll lift(ll node, ll h) {
    ll cnt = 0;
    while(h) {
      if(h&1) node = BL[node][cnt];
      cnt++; h >>= 1;
    }
    return node;
  }
  int lca(int a, int b) {
    if(D[a] > D[b]) swap(a, b);
    FORD(i, 0, MAXLOG) if(D[BL[b][i]] >= D[a]) b = BL[b][i];
    if(a == b) return a;
    FORD(i, 0, MAXLOG) if(BL[a][i] != BL[b][i])
      a = BL[a][i], b = BL[b][i];
    return BL[a][0];
  }
};
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$. 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void push_flip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (push_flip(); p; ) {
      if (p->p) p->p->push_flip();
      p->push_flip(); push_flip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
```

```
    assert(!connected(u, v));
    make_root(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    make_root(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void make_root(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DominatorTree.h

Description: Computes dominator tree of a directed graph from a root. Vertex A dominates B, if all possible paths from root to B go through A. Dominator tree contrains exactly one path root -> A -> B. initialize adjancy lists, call dominator_tree(r) with root r. Afterwards dom[i] is the parent of i in the dominator tree.
Time: $\mathcal{O}((E+V)\cdot\log V)$ 48 lines

```
vi dominators(const vector<vi> &adj, int r) {
  int n = adj.size();
  vi ord, rank(n, n), prev(n, n), anc(n, n), idom(n, n), semi(n
      ), low(n);
  vector<vi> adj_rev(n), dom(n);
  function<void(int)> domdfs =
    [&](int u) {
      rank[u] = sz(ord);
      ord.emplace_back(u);
      for (int v : adj[u]){
        adj_rev[v].emplace_back(u);
        if (rank[v] < n) continue;
        prev[v] = u;
        domdfs(v);
      }
    };
  domdfs(r);
  for (int i = 0; i < n; ++i) semi[i] = low[i] = i;
  function<int(int)> eval =
    [&](int v) {
      if (anc[v] < n && anc[anc[v]] < n)
        {
          int x = eval(anc[v]);
          if (rank[semi[low[v]]] > rank[semi[x]]) low[v] = x;
          anc[v] = anc[anc[v]];
        }
      return low[v];
    };
  FORD(i, 1, ord.size()) {
    int w = ord[i];
    for (int v : adj_rev[w]) {
      int u = eval(v);
      if (rank[semi[w]] > rank[semi[u]]) semi[w] = semi[u];
    }
    dom[semi[w]].emplace_back(w);
    anc[w] = prev[w];
    for (int v : dom[prev[w]]) {
      int u = eval(v);
      idom[v] = (rank[prev[w]] > rank[semi[u]] ? u : prev[w]);
    }
    dom[prev[w]].clear();
  }
  FOR(i, 1, ord.size()) {
    int w = ord[i];
    if (idom[w] != semi[w]) idom[w] = idom[idom[w]];
  }
  FOR(u, 0, n) if (idom[u] >= n) idom[u] = -1;
  return idom;
}
```

FastMaxFlow.h

Description: Returns maximum flow.
Usage: F should be log(f). Set rc to c if it is undirected. To obtain a cut in the mincut problem one must bfs from the source. All the vertices reached from it using only edges with CAP > 0 are in the same cut. Do not reset edges when using min cap, do reset when reusing graph
If maxflow is called without the last parameter it will run a normal dinics in O(V^2*E)
Time: $\mathcal{O}(V*E*log(f))$ where f is the maxflow for general graphs. It is generally very fast. 58 lines

```
#define VEI(w,e) (E[e].u ^ E[e].v ^ w)
#define CAP(w,e) ((E[e].u == w) ? E[e].cap[0] - E[e].flow : E[e
    ].cap[1] + E[e].flow)
#define ADD(w,e,f) E[e].flow += ((E[e].u == w) ? (f) : (-(f)))

struct Edge{ int u, v; ll cap[2], flow; };
vi d, act;

bool bfs(int s, int t, vector<vi>& adj, vector<Edge>& E, ll lim
    ) {
  queue<int> Q;
  d = vi(adj.size(), -1);
  d[t] = 0; Q.push(t);
  for (Q.push(t); Q.size() and d[s] == -1; Q.pop()) {
    int u = Q.front();
    FOR(i, 0, adj[u].size()) {
      int e = adj[u][i], v = VEI(u, e);
      if (CAP(v, e) >= lim and d[v] == -1) {
        d[v] = d[u] + 1;
        Q.push(v);
      }
    }
  }
  return d[s] >= 0;
}

ll dfs(int u, int t, ll bot, vector<vi>& adj,vector<Edge>& E) {
  if (bot == 0) return 0;
  if (u == t) return bot;
  for (; act[u] < int(adj[u].size()); ++act[u]) {
    int e = adj[u][act[u]];
    if(d[u] == d[VEI(u, e)] + 1) {
      ll inc=dfs(VEI(u,e),t,min(bot,CAP(u,e)),adj,E);
      if (inc) {
        ADD(u, e, inc);
        return inc;
      }
    }
  }
  return 0;
}

ll maxflow(int s, int t, vector<vi>& adj, vector<Edge>& E, int
    F = 0) {
  //trav(e, E) e.flow = 0; reset
  ll flow = 0, bot;
  for(int lim = (1<<F); lim >= 1;lim>>=1) {
    while(bfs(s,t,adj,E,lim)) {
      act = vi(adj.size(), 0);
      while (bot = dfs(s, t, oo, adj, E)) flow += bot;
    }
  }
  return flow;
}

inline void addEdge(int x, int y, vector<vi>& adj, vector <Edge
    > & E, ll c, ll rc = 0){
  Edge e; e.u = x; e.v = y; e.flow = 0;
  e.cap[0] = c; e.cap[1] = rc;
  adj[x].push_back(sz(E)); adj[y].push_back(sz(E));
  E.push_back(e);
}
```

GomoryHu.h

Description: builds the gomory-hu maxflow tree, remember to reset edges at each maxflow call
Time: $\mathcal{O}(V)$ * $\mathcal{O}(MAXFLOW)$ 12 lines

```
void gomory_hu(vector<vi>& adj, vector<Edge>& E, vi& parent,
    vector<vector<ll>>& ans) {
  int n = sz(adj);
  parent.assign(n,0);
  ans.assign(n, vector<ll>(n,oo));
  FOR(i,1,n) {
    ll f = maxflow(i, parent[i], adj, E);
    bfs(i, parent[i], adj, E);
    FOR(j, i+1, n) if (d[j] == -1 and parent[j] == parent[i])
        parent[j] = i;
    ans[i][parent[i]] = ans[parent[i]][i] = f;
    FOR(j,0,i)  ans[i][j] = ans[j][i] = min(f, ans[parent[i]][j
        ]);
  }
}
```

PushRelabel.h

Description: Calculates de maxflow using push relabel, reset ec cur hs and H to reuse.
Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ 60 lines

```
typedef ll Flow;
struct Edge {
  int dest, back;
  Flow f, c;
};
```

```
struct PushRelabel {
  vector<vector<Edge>> g;
  vector<Flow> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}

  void addEdge(int s, int t, Flow cap, Flow rcap=0) {
    if (s == t) return;
    Edge a = {t, sz(g[t]), 0, cap};
    Edge b = {s, sz(g[s]), 0, rcap};
    g[s].push_back(a);
    g[t].push_back(b);
  }

  void addFlow(Edge& e, Flow f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  void reset() {
    int v = sz(g);
    FOR(i, 0, v) FOR(j, 0, sz(g[i])) g[i][j].c += g[i][j].f, g[
        i][j].f = 0;
    ec = vector<Flow>(v,0);
    H = vi(v,0);
    hs = vector<vi>(2*v);
  }
  Flow maxflow(int s, int t) {
    //reset();
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    FOR(i,0,v) cur[i] = g[i].data();
    for(auto& e : g[s]) add_flow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0) {
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          for(auto& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            FOR(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
          add_flow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
      }
    }
  }
};
```

MaxflowMinCap.h

Description: Normal maxflow but with minimum capacity in each edge.
Usage: Need to copy maxflow, pass source sink adj matrix, Edge matrix and choose scaling if needed
Time: $\mathcal{O}\left(V^2 * E\right)$ for general graphs. For unit capacities $\mathcal{O}\left(min(V^(2/3), E^(1/2)) * E\right)$

"FastMaxFlow.h" 49 lines

```
struct Edge { int u, v; ll cap[2], mincap, flow; };

void addEdge(int x, int y, ll c, ll m, vector<vi>& adj, vector<
    Edge>& E) {
  Edge e; e.u = x; e.v = y;
  e.cap[0] = c - m; e.cap[1] = 0; e.mincap = m;
  adj[x].push_back(E.size()); adj[y].push_back(E.size());
  E.push_back(e);
}

ll mincap(int s, int t, vector<vi>& adj, vector<Edge>& E, int F
    = 0) {
  int n = adj.size();
  int m = E.size();
  vector<ll> C(n, 0);
  for (int i = 0; i < m; ++i) {
      C[E[i].u] -= E[i].mincap;
      C[E[i].v] += E[i].mincap;
  }
  adj.push_back(vi(0));
  adj.push_back(vi(0));
  ll flowsat = 0;
  for (int i = 0; i < n; ++i) {
      if (C[i] > 0) {
          addEdge(n, i, C[i], 0, adj, E);
          flowsat += C[i];
      }
      else if (C[i] < 0) addEdge(i, n + 1, -C[i], 0, adj, E);
  }
  addEdge(t, s, oo, 0, adj, E);
  for (int i = 0; i < (int)E.size(); ++i) E[i].flow = 0;
  if (flowsat != maxflow(n, n + 1, adj, E, F)) return -1;
  maxflow(s, t, adj, E, F);
  while ((int)E.size() > m) E.pop_back();
  adj.pop_back();
  adj.pop_back();
  for (int i = 0; i < n; ++i) {
      int j = (int)adj[i].size() - 1;
      while (j >= 0 and adj[i][j] >= m) {
          --j;
          adj[i].pop_back();
      }
  }
  ll flow = 0;
  for (int i = 0; i < m; ++i) {
      E[i].flow += E[i].mincap;
      if (E[i].u == s) flow += E[i].flow;
      else if (E[i].v == s) flow -= E[i].flow;
  }
  return flow;
}
```

MaxflowMinCost.h

Description: Computes mincost maxflow using spfa algorithm, can change to BF with negative edges or dijkstra if only positive. Init takes the number of vertes, addedge ads a directed edge. MincostMaxflow returns a pair (flow,cost)
Time: $\mathcal{O}\left(|V||E|F\right)$ where F is the flow

63 lines

```
struct MCMF
{
  typedef int Fval;
  const int INF = 0x7f7f7f7f;
  struct edge {
    int from,to;
    Fval cap,flow,cost;
    edge(int u,int v, Fval c, Fval f, Fval co):from(u),to(v),
        cap(c),flow(f),cost(co){}
  };
  int n,m;
  vector<edge>edges;
  vector<vi> g;
  vi inq, d, p, a;
  void init(int n){
    this->n=n;
    g = vector<vi>(n);
    a = p = vi(n);
    edges.clear();
  }
  void addEdge(int from,int to,int cap,int cost){
    g[from].push_back(sz(edges));
    edges.push_back(edge(from,to,cap,0,cost));
    g[to].push_back(sz(edges));
    edges.push_back(edge(to,from,0,0,-cost));
  }
  bool spfa(int s,int t,int &flow,int &cost){
    d = vi(n,INF);
    inq = vi(n,0);
    d[s]=0;inq[s]=1;p[s]=0;a[s]=INF;
    deque<int> q; //can use queue instead of deque
    q.push_front(s);
    while(!q.empty()){
      int u=q.front();q.pop_front();
      inq[u]=0;
      for(int i=0;i<g[u].size();i++){
        edge &e=edges[g[u][i]];
        if(e.cap>e.flow&&d[e.to]>d[u]+e.cost){
          d[e.to]=d[u]+e.cost;
          p[e.to]=g[u][i];
          a[e.to]=min(a[u],e.cap-e.flow);
          if(!inq[e.to]){
            if(q.size() && d[e.to] < d[q.front()]) q.push_front
                (e.to);
            else q.push_back(e.to);
          }
          inq[e.to]=1;
        }
      }
    }
    if(d[t]==INF)return 0;
    flow+=a[t];
    cost+=d[t]*a[t];
    for(int u=t;u!=s;u=edges[p[u]].from){
      edges[p[u]].flow+=a[t];
      edges[p[u]^1].flow-=a[t];
    }
    return 1;
  }
  pair<Fval,Fval> MincostMaxflow(int s,int t){
    Fval flow=0, cost=0;
    while(spfa(s,t,flow,cost));
    return {flow,cost};
  }
};
```

StableMarriage.h

Description: findMatch finds a stable matching where the first groups is prefered given the preference lists of two groups of persons
Usage: pr1[i] = preference list of person i (from 1. list) (format A), pr2[i][j] = position in the preference list of person j (from 1. list), in preference list of i (from 2. list) (format B)
Time: $\mathcal{O}\left(N^2\right)$

28 lines

```
const int MAXN = 4 * 1024;

int pr1[MAXN][MAXN],pr2[MAXN][MAXN];
int match[MAXN]; // match[i]=partner of person i out of group 2
int res[MAXN];   // res[i]=partner of person i out of group 1
int id[MAXN];
int N;
void findMatch() {
  fill_n(match, N, -1);
  fill_n(id, N, 0);
  FOR(m,0,N) {
    int cur = m;
    while (true) {
      int ot = pr1[cur][id[cur]];
      if (match[ot] == -1) {
        match[ot] = cur;
        break;
      }
      if (pr2[ot][cur] < pr2[ot][match[ot]]) swap(match[ot],cur
          );
      id[cur]++;
    }
  }
  FOR(i,0,N) res[match[i]] = i;
}
// convert preference list of format A to format B and in
     reverse
int ranks[MAXN][MAXN]; // values to convert
int reranks[MAXN][MAXN]; // converted values
void convertRanklist() { FOR(i,0,N) FOR(j,0,N) reranks[i][ranks
     [i][j]] = j; }
```

DFSMatching.h

Description: This is a simple matching algorithm but should be just fine in most cases. Graph g should be a list of neighbours of the left partition. n is the size of the left partition and m is the size of the right partition. If you want to get the matched pairs, $match[i]$ contains match for vertex i on the right side or -1 if it's not matched.
Time: $\mathcal{O}(EV)$ where E is the number of edges and V is the number of vertices.

24 lines

```
vi match;
vector<bool> seen;
bool find(int j, const vector<vi>& g) {
  if (match[j] == -1) return 1;
  seen[j] = 1; int di = match[j];
  trav(e, g[di])
    if (!seen[e] && find(e, g)) {
      match[e] = di;
      return 1;
    }
  return 0;
}
int dfs_matching(const vector<vi>& g, int n, int m) {
  match.assign(m, -1);
  FOR(i,0,n) {
    seen.assign(m, 0);
    trav(j,g[i])
      if (find(j, g)) {
        match[j] = i;
        break;
      }
  }
  return m - (int)count(all(match), -1);
}
```

Blossom.h

Description: Computes a MaxMatching on a graph
Usage: Pass x, y the number of vertices and edges, mat[i] provides the matching or is 0 if unmatched, out prints in nice format.

<bits/stdc++.h> 49 lines

```
struct Blossom{
  int n,m,h,t,W,tot;
  vector<int> fir,la,ne,F,mat,pre,tp,q,vs;
  Blossom (int x,int y){
    n = x; m = y; W = tot = 0;
    fir = F = mat = pre = tp = q = vs = vector<int> (x+1,0);
    la = ne = vector<int> (2*y + 10,0);
  }
  void add(int x, int y){
    ins(x,y), ins(y,x);
  }
  void ins(int x,int y){ la[++tot] = y; ne[tot] = fir[x]; fir[x
      ] = tot;}
  int fd(int x){ return F[x] ? F[x] = fd(F[x]) : x;}
  int lca(int u,int v){
    for(++W;; u = pre[mat[u]], swap(u,v))
      if(vs[u = fd(u)] == W)return u;
      else vs[u] = u ? W : 0;
  }
  void aug(int u,int v){
    for (int w; u; v = pre[u=w]) w = mat[v], mat[mat[u]=v] = u;
  }
  void blo(int u,int v,int f){
    for (int w;fd(u)^f;u = pre[v=w]) {
      pre[u]=v;
      if(!(F[u])) F[u] = f;
      if(!(F[w=mat[u]])) F[w] = f;
      if(!(tp[w]^1)) tp[q[++t]=w] = -1;
    }
  }
  int bfs(int u){
    int x,E,i;
    FOR(i, 1, n+1) tp[i] = F[i] = 0;
    for(--tp[q[h=t=1]=u]; h <= t ;u = q[++h])
    for(i = fir[u]; i; i = ne[i]) if(!tp[E = la[i]]){
      if(!mat[E]) return aug(E,u),1;
      pre[E] = u, ++tp[E], --tp[q[++t]=mat[E]];
      } else if ((tp[E]^1)&&(fd(u)^fd(E))) blo(u, E, x = lca(u,
          E)), blo(E,u,x);
    return 0;
  }
  int solve(){
    int i, an=0;
    FOR(i, 1, n+1) if(!mat[i]) an += bfs(i);
    return an;
  }
  void out(){
    FOR(i, 1, n+1) cout << mat[i] << ' ';
    cout << endl;
  }
};
```

MinCostBipartite.h

Description: Given a cost matrix of a bipartite matching, finds the minimum cost matching all left elements.
Usage: Feed a n*m cost matrix, n <= m, where there are n left elements and m right elements
Time: $\mathcal{O}(n^3)$, solves 2000*2000 problems in less than 1 sec.

66 lines

```
typedef int value_type;
value_type MinAssign(const vector<vector<value_type>> &c) {
    const int n = c.size(), m = c[0].size(); // assert(n <= m);
    vector<value_type> v(m), dist(m);        // v: potential
    vector<int> matchL(n,-1), matchR(m,-1);  // matching pairs
    vector<int> index(m), prev(m);
    iota(all(index), 0);
    auto residue = [&](int i, int j) { return c[i][j] - v[j];
        };
    for (int f = 0; f < n; ++f) {
        for (int j = 0; j < m; ++j) {
            dist[j] = residue(f, j);
            prev[j] = f;
        }
        value_type w;
        int j, l;
        bool end = 0;
        for (int s = 0, t = 0;;) {
            if (s == t) {
                l = s; w = dist[index[t++]];
                for (int k = t; k < m; ++k) {
                    j = index[k];
                    value_type h = dist[j];
                    if (h <= w) {
                        if (h < w) { t = s; w = h; }
                        index[k] = index[t]; index[t++] = j;
                    }
                }
                for (int k = s; k < t; ++k) {
                    j = index[k];
                    if (matchR[j] < 0) {
                        end = 1;
                        break;
                    }
                }
                if(end) break;
            }
            int q = index[s++], i = matchR[q];
            for (int k = t; k < m; ++k) {
                j = index[k];
                value_type h = residue(i,j) - residue(i,q) + w;
                if (h < dist[j]) {
                    dist[j] = h; prev[j] = i;
                    if (h == w) {
                        if (matchR[j] < 0) {
                            end = 1;
                            break;
                        }
                        index[k] = index[t]; index[t++] = j;
                    }
                }
            }
            if(end) break;
        }
        for(int k = 0; k < l; ++k)
            v[index[k]] += dist[index[k]] - w;
        int i;
        do {
            matchR[j] = i = prev[j];
            swap(j, matchL[i]);
        } while (i != f);
    }
    value_type opt = 0;
    for (int i = 0; i < n; ++i)
        opt += c[i][matchL[i]]; // (i, matchL[i]) is a solution
    return opt;
}
```

globalmincut.h

Description: Given an nxn matrix, returns the global min cut S-T, and returns S
Time: $\mathcal{O}(V^3)$

46 lines

```
struct mfset {
  vi par;
  mfset(int n) : par(n,-1) {}
  int find(int x) {
    return par[x] == -1? x : par[x] = find(par[x]);
  }
  void merge(int x, int y) {
    par[find(x)] = find(y);
  }
};
```

```
pair<ll, vi> StoerWagner(vector<vi>& g, int n) {
  int n1 = n;
  mfset P(n);
  vi v(n), a(n), w(n,0);
  iota(v.begin(), v.end(), 0);
  ll best = 10000000;
  vi cut;
  while (n > 1) {
    a[v[0]] = 1;
    FOR(i, 1, n) {
      a[v[i]] = 0;
      w[i] = g[v[0]][v[i]];
    }
    int prev = v[0];
    FOR(i, 1, n) {
      int zj = -1;
      FOR(j, 1, n) if (!a[v[j]] && (zj < 0 || w[j] > w[zj])) zj
          = j;
      a[v[zj]] = 1;
      if (i == n - 1) {
        if (best > w[zj]){
          best = w[zj];
          cut.clear();
          FOR(k, 0, n1) if (P.find(k) == P.find(v[zj])) cut.
               push_back(k);
        }
        P.merge(prev, v[zj]);
        FOR(j, 0, n) g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][
             v[j]];
        v[zj] = v[--n];
        break;
      }
      prev = v[zj];
      FOR(j, 1, n) if (!a[v[j]]) w[j] += g[v[zj]][v[j]];
    }
  }
  return {best, cut};
}
```

7.1 Karp's Minimum Mean Cycle

1. Initialize: $d_0(r) = 0$ and $\forall v \in V - \{r\}$,$d_0(v) = \infty$.
2. For $i = 1$ to n:
— For every $v \in V$ calculate:
—– $d_i(v) = \min_{u:(u,v)\in E} = \{d_{i-1}(u) + c((u,v))\}$
Optim $= \min_{v\in V} \max_{0\le k\le n-1} \left\{\frac{d_n(v)-d_k(v)}{n-k}\right\}$

7.2 Planar Graphs

Formula Euler: v-e+f = 2
Theorem 1: If $v >= 3$ then $e <= 3v - 6$.
Theorem 2: If $v > 3$ and there are no cycles of length 3, then $e <= 2v - 4$.

7.3 Bipartite Graphs

let G = (V,E) be a Bipartite graph, we will call each part L and R respectively.
Maximum Matching is equal to **Minimum Vertex Cover**, to get the nodes in the MVC we choose the unmatched vertices of L and run a BFS/DFS in the graph with edges in the matching directed from R to L, and edges not in the matching directed from L to R. Let's all this Z, now MVC = (L\Z) ∪ (R ∩ Z).
The **Minimum Edge Cover** is obtained by doing a Maximum Matching, Then run a BFS/DFS from unmatched vertices of L, the MEC is the unreachable vertices of A and reachable vertices of B.
The **Maximum Independent Set** is the complementary of the **Minimum Vertex Cover**

7.3.1 DAG

The **Minimum Path Cover** is given by the edges in the MM of the bipartite graph after doubling the vertices.

7.4 ASSP : Johnson Algorithm

We have a sparse graph with, possibly, negative edges. We want to compute the all-pairs shortest path. Floyd Warshall may be too slow.
Algorithm:
1) A new node q is added to the graph, connected by zero-weight edges to each of the other nodes.
2) Bellman–Ford algorithm is used, starting from the new vertex q, to find for each vertex v the minimum weight h(v) of a path from q to v.
If this step detects a negative cycle, the algorithm is terminated as the shortest-path is undefined.
3) edges of the original graph are reweighted using the values computed by the Bellman–Ford algorithm:
an edge from u to v, having length w(u,v), is given by the new length w(u,v) + h(u) - h(v).
4) q is removed, and Dijkstra's algorithm is used to find the shortest paths from each node s to every other vertex in the reweighted graph.
5) Dijkstra from every node. The shortest path in this new graph is the same as in the initial graph although it's weight changes.
The idea is to do dijkstra by storing "weight in new graph" and "weight in old graph" sorting by the first one.

Geometry (8)

8.1 Geometric primitives

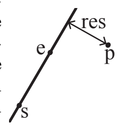
Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)
25 lines

```
template <class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T a=0, T b=0) : x(a), y(b) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.

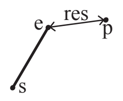


"Point.h" 4 lines

```
template <class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.

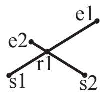


Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
"Point.h" 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersetion point between the line segments going from s1 to e1 and from s2 to e2 exists r1 is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists 2 is returned and r1 and r2 are set to the two ends of the common line. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long. Use segmentIntersectionQ to get just a true/false answer.



Usage: `Point<double> intersection, dummy;`
```
if (segmentIntersection(s1,e1,s2,e2,intersection,dummy)==1)
cout << "segments intersect at " << intersection << endl;
```
"Point.h" 27 lines
```
template <class P>
int segmentIntersection(const P& s1, const P& e1,
    const P& s2, const P& e2, P& r1, P& r2) {
  if (e1==s1) {
    if (e2==s2) {
      if (e1==e2) { r1 = e1; return 1; } //all equal
      else return 0; //different point segments
    } else return segmentIntersection(s2,e2,s1,e1,r1,r2);//swap
  }
  //segment directions and separation
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = v1.cross(d), a2 = v2.cross(d);
  if (a == 0) { //if parallel
    auto b1=s1.dot(v1), c1=e1.dot(v1),
         b2=s2.dot(v1), c2=e2.dot(v1);
    if (a1 || a2 || max(b1,min(b2,c2))>min(c1,max(b2,c2)))
      return 0;
    r1 = min(b2,c2)<b1 ? s1 : (b2<c2 ? s2 : e2);
    r2 = max(b2,c2)>c1 ? e1 : (b2>c2 ? s2 : e2);
    return 2-(r1==r2);
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
  if (0<a1 || a<-a1 || 0<a2 || a<-a2)
    return 0;
  r1 = s1-v1*a2/a;
  return 1;
}
```

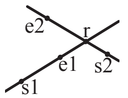
SegmentIntersectionQ.h

Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

"Point.h" 16 lines
```
template <class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
  if (e1 == s1) {
    if (e2 == s2) return e1 == e2;
    swap(s1,s2); swap(e1,e2);
  }
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = d.cross(v1), a2 = d.cross(v2);
  if (a == 0) { // parallel
    auto b1 = s1.dot(v1), c1 = e1.dot(v1),
         b2 = s2.dot(v1), c2 = e2.dot(v1);
    return !a1 && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
  return (0 <= a1 && a1 <= a && 0 <= a2 && a2 <= a);
}
```

lineIntersection.h

Description:
If a unique intersetion point of the lines going through s1,e1 and s2,e2 exists r is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists -1 is returned. If s1==e1 or s2==e2 -1 is returned. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage: `point<double> intersection;`
```
if (1 == LineIntersection(s1,e1,s2,e2,intersection))
cout << "intersection point at " << intersection << endl;
```
"Point.h" 9 lines
```
template <class P>
int lineIntersection(const P& s1, const P& e1, const P& s2,
    const P& e2, P& r) {
  if ((e1-s1).cross(e2-s2)) { //if not parallell
    r = s2-(e2-s2)*(e1-s1).cross(s2-s1)/(e1-s1).cross(e2-s2);
    return 1;
  } else
    return -((e1-s1).cross(s2-s1)==0 || s2==e2);
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 11 lines
```
template <class P>
int sideOf(const P& s, const P& e, const P& p) {
  auto a = (e-s).cross(p-s);
  return (a > 0) - (a < 0);
}
template <class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

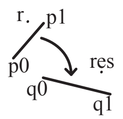
onSegment.h

Description: Returns true iff p lies on the line segment from s to e. Intended for use with e.g. Point<long long> where overflow is an issue. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h" 5 lines
```
template <class P>
bool onSegment(const P& s, const P& e, const P& p) {
  P ds = p-s, de = p-e;
  return ds.cross(de) == 0 && ds.dot(de) <= 0;
}
```

linearTransformation.h

Description:



Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 6 lines
```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
```
int j = 0; FOR(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively
oriented triangles with vertices at 0 and i
```
37 lines
```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int quad() const {
    assert(x || y);
    if (y < 0) return (x >= 0) + 2;
    if (y > 0) return (x <= 0);
    return (x <= 0) * 2;
  }
  Angle t90() const { return {-y, x, t + (quad() == 3)}; }
  Angle t180() const { return {-x, -y, t + (quad() >= 2)}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.quad(), a.y * (ll)b.x) <
         make_tuple(b.t, b.quad(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

8.2 Circles

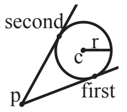
CircleIntersection.h

Description: Computes a pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 14 lines
```
typedef Point<double> P;
bool circleIntersection(P a, P b, double r1, double r2,
    pair<P, P>* out) {
  P delta = b - a;
  assert(delta.x || delta.y || r1 != r2);
  if (!delta.x && !delta.y) return false;
  double r = r1 + r2, d2 = delta.dist2();
  double p = (d2 + r1*r1 - r2*r2) / (2.0 * d2);
  double h2 = r1*r1 - p*p*d2;
  if (d2 > r*r || h2 < 0) return false;
  P mid = a + delta*p, per = delta.perp() * sqrt(h2 / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

circleTangents.h

Description:
Returns a pair of the two points on the circle with radius r centered around c whos tangent lines intersect p. If p lies within the circle NaN-points are returned. P is intended to be Point<double>. The first point is the one to the right as seen from the p towards c.



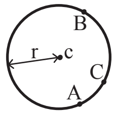
Usage: typedef Point<double> P;
pair<P,P> p = circleTangents(P(100,2),P(0,0),2);

"Point.h" 6 lines

```
template <class P>
pair<P,P> circleTangents(const P &p, const P &c, double r) {
  P a = p-c;
  double x = r*r/a.dist2(), y = sqrt(x-x*x);
  return make_pair(c+a*x+a.perp()*y, c+a*x-a.perp()*y);
}
```

circumcircle.h

Description:
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 28 lines

```
pair<double, P> mec2(vector<P>& S, P a, P b, int n) {
  double hi = INFINITY, lo = -hi;
  FOR(i,0,n) {
    auto si = (b-a).cross(S[i]-a);
    if (si == 0) continue;
    P m = ccCenter(a, b, S[i]);
    auto cr = (b-a).cross(m-a);
    if (si < 0) hi = min(hi, cr);
    else lo = max(lo, cr);
  }
  double v = (0 < lo ? lo : hi < 0 ? hi : 0);
  P c = (a + b) / 2 + (b - a).perp() * v / (b - a).dist2();
  return {(a - c).dist2(), c};
}
pair<double, P> mec(vector<P>& S, P a, int n) {
  random_shuffle(S.begin(), S.begin() + n);
  P b = S[0], c = (a + b) / 2;
  double r = (a - c).dist2();
  FOR(i,1,n) if ((S[i] - c).dist2() > r * (1 + 1e-8)) {
    tie(r,c) = (n == sz(S) ?
      mec(S, S[i], i) : mec2(S, a, S[i], i));
  }
  return {r, c};
}
pair<double, P> enclosingCircle(vector<P> S) {
  assert(!S.empty()); auto r = mec(S, S[0], sz(S));
  return {sqrt(r.first), r.second};
}
```

8.3 Polygons

insidePolygon.h

Description: Returns true if p lies within the polygon described by the points between iterators begin and end. If strict false is returned when p is on the edge of the polygon. Answer is calculated by counting the number of intersections between the polygon and a line going from p to infinity in the positive x-direction. The algorithm uses products in intermediate steps so watch out for overflow. If points within epsilon from an edge should be considered as on the edge replace the line ”if (onSegment...” with the comment bellow it (this will cause overflow for int and long long).
Usage: typedef Point<int> pi;
vector<pi> v; v.push_back(pi(4,4));
v.push_back(pi(1,2)); v.push_back(pi(2,1));
bool in = insidePolygon(v.begin(),v.end(), pi(3,4), false);
Time: $\mathcal{O}(n)$

"Point.h", "onSegment.h", "SegmentDistance.h" 14 lines

```
template <class It, class P>
bool insidePolygon(It begin, It end, const P& p,
    bool strict = true) {
  int n = 0; //number of isects with line from p to (inf,p.y)
  for (It i = begin, j = end-1; i != end; j = i++) {
    //if p is on edge of polygon
    if (onSegment(*i, *j, p)) return !strict;
    //or: if (segDist(*i, *j, p) <= epsilon) return !strict;
    //increment n if segment intersects line from p
    n += (max(i->y,j->y) > p.y && min(i->y,j->y) <= p.y &&
        ((*j-*i).cross(p-*i) > 0) == (i->y <= p.y));
  }
  return n&1; //inside if odd number of intersections
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 6 lines

```
template <class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  FOR(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

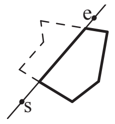
Description: Returns the center of mass for a polygon.

"Point.h" 10 lines

```
typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
  auto i = v.begin(), end = v.end(), j = end-1;
  Point<double> res{0,0}; double A = 0;
  for (; i != end; j=i++) {
    res = res + (*i + *j) * j->cross(*i);
    A += j->cross(*i);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



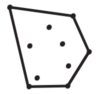
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" 15 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  FOR(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0)) {
      res.emplace_back();
      lineIntersection(s, e, cur, prev, res.back());
    }
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:
Returns a vector of indices of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.



Usage: vector<P> ps, hull;
trav(i, convexHull(ps)) hull.push_back(ps[i]);
Time: $\mathcal{O}(n \log n)$

"Point.h" 20 lines

```
typedef Point<ll> P;
pair<vi, vi> ulHull(const vector<P>& S) {
  vi Q(sz(S)), U, L;
  iota(all(Q), 0);
  sort(all(Q), [&S](int a, int b){ return S[a] < S[b]; });
  trav(it, Q) {
#define ADDP(C, cmp) while (sz(C) > 1 && S[C[sz(C)-2]].cross(\
  S[it], S[C.back()]) cmp 0) C.pop_back(); C.push_back(it);
    ADDP(U, <=); ADDP(L, >=);
  }
  return {U, L};
}

vi convexHull(const vector<P>& S) {
  vi u, l; tie(u, l) = ulHull(S);
  if (sz(S) <= 1) return u;
  if (S[u[0]] == S[u[1]]) return {0};
  l.insert(l.end(), u.rbegin()+1, u.rend()-1);
  return l;
}
```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

"ConvexHull.h" 19 lines

```
vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
  vector<pii> ret;
  int i = 0, j = sz(L) - 1;
  while (i < sz(U) - 1 || j > 0) {
    ret.emplace_back(U[i], L[j]);
    if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
          .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
    else --j;
  }
  return ret;
}

pii polygonDiameter(const vector<P>& S) {
  vi U, L; tie(U, L) = ulHull(S);
  pair<ll, pii> ans;
  trav(x, antipodal(S, U, L))
    ans = max(ans, {(S[x.first] - S[x.second]).dist2(), x});
  return ans.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a given polygon (counter-clockwise order). The polygon must be such that every point on the circumference is visible from the first point in the vector. It returns 0 for points outside, 1 for points on the circumference, and 2 for points inside.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "onSegment.h" 22 lines

```
typedef Point<ll> P;
int insideHull2(const vector<P>& H, int L, int R, const P& p) {
  int len = R - L;
  if (len == 2) {
    int sa = sideOf(H[0], H[L], p);
    int sb = sideOf(H[L], H[L+1], p);
    int sc = sideOf(H[L+1], H[0], p);
    if (sa < 0 || sb < 0 || sc < 0) return 0;
    if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R == sz(H)))
      return 1;
    return 2;
  }
  int mid = L + len / 2;
  if (sideOf(H[0], H[mid], p) >= 0)
    return insideHull2(H, mid, R, p);
  return insideHull2(H, L, mid+1, p);
}

int insideHull(const vector<P>& hull, const P& p) {
  if (sz(hull) < 3) return onSegment(hull[0], hull.back(), p);
  else return insideHull2(hull, 1, sz(hull), p);
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. isct(a, b) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon.
Time: $\mathcal{O}(N + Q\log n)$

"Point.h" 63 lines

```
ll sgn(ll a) { return (a > 0) - (a < 0); }
typedef Point<ll> P;
struct HullIntersection {
  int N;
  vector<P> p;
  vector<pair<P, int>> a;

  HullIntersection(const vector<P>& ps) : N(sz(ps)), p(ps) {
    p.insert(p.end(), all(ps));
    int b = 0;
    FOR(i,1,N) if (P{p[i].y,p[i].x} < P{p[b].y, p[b].x}) b = i;
    FOR(i,0,N) {
      int f = (i + b) % N;
      a.emplace_back(p[f+1] - p[f], f);
    }
  }

  int qd(P p) {
    return (p.y < 0) ? (p.x >= 0) + 2
         : (p.x <= 0) * (1 + (p.y <= 0));
  }

  int bs(P dir) {
    int lo = -1, hi = N;
    while (hi - lo > 1) {
      int mid = (lo + hi) / 2;
      if (make_pair(qd(dir), dir.y * a[mid].first.x) <
        make_pair(qd(a[mid].first), dir.x * a[mid].first.y))
        hi = mid;
      else lo = mid;
    }
    return a[hi%N].second;
  }

  bool isign(P a, P b, int x, int y, int s) {
    return sgn(a.cross(p[x], b)) * sgn(a.cross(p[y], b)) == s;
  }

  int bs2(int lo, int hi, P a, P b) {
    int L = lo;
    if (hi < lo) hi += N;
    while (hi - lo > 1) {
      int mid = (lo + hi) / 2;
      if (isign(a, b, mid, L, -1)) hi = mid;
      else lo = mid;
    }
    return lo;
  }

  pii isct(P a, P b) {
    int f = bs(a - b), j = bs(b - a);
    if (isign(a, b, f, j, 1)) return {-1, -1};
    int x = bs2(f, j, a, b)%N,
        y = bs2(j, f, a, b)%N;
    if (a.cross(p[x], b) == 0 &&
        a.cross(p[x+1], b) == 0) return {x, x};
    if (a.cross(p[y], b) == 0 &&
        a.cross(p[y+1], b) == 0) return {y, y};
    if (a.cross(p[f], b) == 0) return {f, -1};
    if (a.cross(p[j], b) == 0) return {j, -1};
    return {x, y};
  }
};
```

8.4 Misc. Point Set Problems

closestPair.h

Description: $i1$, $i2$ are the indices to the closest pair of points in the point vector p after the call. The distance is returned.
Time: $\mathcal{O}(n\log n)$

"Point.h" 58 lines

```
template <class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template <class It>
bool y_it_less(const It& i,const It& j) {return i->y < j->y;}

template<class It, class IIt> /* IIt = vector<It>::iterator */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
  typedef typename iterator_traits<It>::value_type P;
  int n = yaend-ya, split = n/2;
  if(n <= 3) { // base case
    double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
    if(n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).dist()
        ;
    if(a <= b) { i1 = xa[1];
      if(a <= c) return i2 = xa[0], a;
      else return i2 = xa[2], c;
    } else { i1 = xa[2];
      if(b <= c) return i2 = xa[0], b;
      else return i2 = xa[1], c;
  } }
  vector<It> ly, ry, stripy;
  P splitp = *xa[split];
  double splitx = splitp.x;
  for(IIt i = ya; i != yaend; ++i) { // Divide
    if(*i != xa[split] && (**i-splitp).dist2() < 1e-12)
      return i1 = *i, i2 = xa[split], 0;// nasty special case!
    if (**i < splitp) ly.push_back(*i);
    else ry.push_back(*i);
  } // assert((signed)lefty.size() == split)
  It j1, j2; // Conquer
  double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
  double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2);
  if(b < a) a = b, i1 = j1, i2 = j2;
  double a2 = a*a;
  for(IIt i = ya; i != yaend; ++i) { // Create strip (y-sorted)
    double x = (*i)->x;
    if(x >= splitx-a && x <= splitx+a) stripy.push_back(*i);
  }
  for(IIt i = stripy.begin(); i != stripy.end(); ++i) {
    const P &p1 = **i;
    for(IIt j = i+1; j != stripy.end(); ++j) {
      const P &p2 = **j;
      if(p2.y-p1.y > a) break;
      double d2 = (p2-p1).dist2();
      if(d2 < a2) i1 = *i, i2 = *j, a2 = d2;
  } }
  return sqrt(a2);
}

template<class It> // It is random access iterators of point<T>
double closestpair(It begin, It end, It &i1, It &i2 ) {
  vector<It> xa, ya;
  assert(end-begin >= 2);
  for (It i = begin; i != end; ++i)
    xa.push_back(i), ya.push_back(i);
  sort(xa.begin(), xa.end(), it_less<It>);
  sort(ya.begin(), ya.end(), y_it_less<It>);
  return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if the box is wider than high (not best
          heuristic...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

DelaunayTriangulation.h

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are colinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}\left(n^2\right)$

"Point.h", "3dHull.h" 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  trav(p, ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (sz(ps) > 3) trav(t, hull3d(p3)) if ((p3[t.b]-p3[t.a]).
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

6 lines

```
template <class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
  double v = 0;
  trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

32 lines

```
template <class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  FOR(i,0,4) FOR(j,i+1,4) FOR(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  FOR(i,4,sz(A)) {
    FOR(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    FOR(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

AhoCorasick.h

Description: Builds an Ahocorasick Trie, with suffix links.
Usage: `This is an offline Algorithm, Pass the vector of patterns to Trie.init(v), find function returns the number of times each string appears`
Time: $\mathcal{O}\left(n+m+z\right)$

72 lines

```
const int MaxM = 200005;

struct Trie{
    static const int Alpha = 26;
    static const int first = 'a';
    int lst = 1;
    struct node{
        int nxt[Alpha] = {}, p = -1;
        char c;
        vi end; //if all patterns are different, can use flag
            instead
        int SuffixLink = -1;
        int cnt = 0;
    };
  vector<node> V;
  int num;
  stack<int> reversebfs;
  inline int getval(char c) {
    return c - first;
  }
  void CreateSuffixLink() {
    queue<int> q;
    for(q.push(0); q.size(); q.pop()) {
      int pos = q.front();
      reversebfs.push(pos);
      if(!pos || !V[pos].p) V[pos].SuffixLink = 0;
      else {
        int val = getval(V[pos].c);
        int j = V[V[pos].p].SuffixLink;
        V[pos].SuffixLink = V[j].nxt[val];
      }
      for(int i = 0; i < Alpha; ++i) {
        if(V[pos].nxt[i]) q.push(V[pos].nxt[i]);
        else if(!pos || !V[pos].p)  V[pos].nxt[i] = V[0].nxt[i
            ];
        else V[pos].nxt[i] = V[V[pos].SuffixLink].nxt[i];
      }
    }
  }

  void init(vector<string> &v) {
    V.resize(MaxM);
    num = v.size();
    int id = 0;
    for(auto &s : v) {
      int pos = 0;
      for(char &c : s) {
        int val = getval(c);
        if(!V[pos].nxt[val]) {
          V[lst].p = pos; V[lst].c = c; V[pos].nxt[val] = lst
              ++;
        }
        pos = V[pos].nxt[val];
      }
      V[pos].end.emplace_back(id++);
    }
    CreateSuffixLink();
  }

  vi find(string& word) {
    int pos = 0;
    vi ans(num, 0);
    for(auto &c : word) {
      int val = getval(c);
      pos = V[pos].nxt[val];
      V[pos].cnt++; //We count the times we reach each node,
          and then do a reverse propagation
    }
    for(;reversebfs.size();reversebfs.pop()) {
      int x = reversebfs.top(); //When we process x, we know we
           have been there V[x].cnt times;
      for(int i : V[x].end) ans[i] += V[x].cnt;
      if(V[x].SuffixLink != -1) V[V[x].SuffixLink].cnt += V[x].
          cnt;
    }
    return ans;
  }
};
```

PalindromeTree.h

Description: Palindrome Tree for string s
Time: $\mathcal{O}(sz(s))$ for building

64 lines

```
const int maxN = 1000010; // at least sz(s) + 3
struct Node {
  int suffix;
  int len;
  map<char, int> children;

  // not needed for construction, add if needed
  char c;
  int parent;
  vector<int> suffixof;
};

int nodeid;
Node tree[maxN]; // 0: -1 root, 1: empty string
int pos2node[maxN]; // not needed for construction

int add(int parent, char c) {
  if(has(tree[parent].children, c)) {
    return tree[parent].children[c];
  }
  int newid = nodeid++;
  tree[newid].suffix = -1;
  tree[newid].len = tree[parent].len + 2;
  tree[newid].parent = parent;
  tree[newid].c = c;
  tree[parent].children[c] = newid;
  return newid;
}

void build(string& s) {
  nodeid = 2;
  tree[0].parent = -1;
  tree[0].len = -1;
  tree[1].parent = -1;
  tree[0].suffixof.push_back(1);
  int cur = 0;
  FOR(i, 0, s.size()) {
    int newn = -1;
    while(1) {
      int curlen = tree[cur].len;
      if(i-1-curlen >= 0 && s[i-1-curlen] == s[i]) {
        newn = add(cur, s[i]);
        break;
      }
      cur = tree[cur].suffix;
    }
    pos2node[i] = newn;
    if(tree[newn].suffix != -1) {
      cur = newn;
      continue;
    }
    if(cur == 0) {
      tree[newn].suffix = 1;
    } else {
      do {
        cur = tree[cur].suffix;
      } while(i-1-tree[cur].len < 0
          || s[i-1-tree[cur].len] != s[i]);
      tree[newn].suffix = tree[cur].children[s[i]];
    }
    tree[tree[newn].suffix].suffixof.push_back(newn);
    cur = newn;
  }
}
```

Manacher.h

Description: call with String str of length n, returns: r[0..2*n-2],r[i] radius of longest palindrome with center i/2 in str
Time: $\mathcal{O}(n)$

12 lines

```
vi manacher(string s) {
  int n = sz(s); vi r(2*n - 1);
  r[0] = 0;
  int p = 0;
  FOR(i, 1, 2*n-1) {
    r[i] = (p/2 + r[p] >= (i+1)/2) ? min(r[2*p - i], p/2 + r[p]
         - i/2) : 0;
    while (i/2 + r[i] + 1 < n && (i+1)/2 - r[i] - 1 >= 0
    && str[i/2 + r[i] + 1] == str[(i+1)/2 - r[i] - 1]) r[i]++;
    if (i/2 + r[i] > p/2 + r[p]) p = i;
  }
  // FOR(i,0,2*n-1) r[i] = r[i]*2 + !(i&1); // change radius to
       diameter
}
```

SuffixArray.h

Description: lcp (x,y) = min(lcp(x,x+1),lcp(x+1,x+2)...lcp(y-1,y)) to answer queries with RMQ O(1)
Memory: Build $\mathcal{O}(N \log N)$ (Can be optimised as in B you only use the previous row to compute one row. But then you cannot do lcp
Time: Build: $\mathcal{O}(N \log N)$ where N is the length of the string for creation of the SA. LCP $\mathcal{O}(\log N)$ It is not necessary to use raddixsort if the $\mathcal{O}(nlog^2 n)$ fits the time limit, one can just use stl sort.

60 lines

```
struct SF {
  pair<ll, ll> ord;
  ll id;
  bool operator<(const SF& s) const { return ord < s.ord; }
};

ll lcp(ll x, ll y, vector < vector < ll > > &B, ll N, ll step)
     {
  if (x == y) return N - x;
  ll res = 0;
  for (ll i = step - 1; i >= 0 and x < N and y < N; --i)
    if (B[i][x] == B[i][y]) { x += 1<<i; y += 1<<i; res += 1<<i
        ; }
  return res;
}

void raddixSort(vector < SF > & A, vector < vector <ll > > & B,
      vector < ll > &times, vector < ll > & pos, vector < SF >
      & L2, ll N) {
  ll k = max(N, 256LL);
  for (ll i = 0; i < k + 2; ++i) times[i] = 0;
  for (ll i = 0; i < N; ++i)
    times[A[i].ord.second + 1]++;
  pos[0] = 0;
  for (ll i = 1; i < k + 2; ++i)
    pos[i] = pos[i - 1] + times[i - 1];
  for (ll i = 0; i < N; ++i)
    L2[pos[A[i].ord.second + 1]++] = A[i];

  for (ll i = 0; i < k + 2; ++i)
    times[i] = 0;
  for (ll i = 0; i < N; ++i)
    times[L2[i].ord.first + 1]++;
  pos[0] = 0;
  for (ll i = 1; i < k + 2; ++i)
    pos[i] = pos[i - 1] + times[i - 1];
  for (ll i = 0; i < N; ++i)
    A[pos[L2[i].ord.first + 1]++] = L2[i];
}

void compute_suffix_array(vector < SF > & A, vector<vector<ll>
     > & B, ll N, string & S, ll &step) {
  ll MAXN = 3000005; //millor posar numero gran que algo en
       funcio de N pq peta
  vector < SF > L2(MAXN);
  vector < ll > pos(MAXN + 2,0), times(MAXN + 2,0);
  A.resize(N); B.resize(1); B[0].resize(N);
  for (ll i = 0; i < N; ++i) B[0][i] = S[i];
  step = 1;
  for (ll b = 0, pw = 1; b < N; ++step, pw<<=1) {
    for (ll i = 0; i < N; ++i) {
      A[i].ord.first = B[step - 1][i];
      A[i].ord.second = i + pw < N ? B[step - 1][i + pw] : -1;
      A[i].id = i;
    }
    raddixSort(A, B, times, pos, L2, N); //sort(A.begin(), A.
         end());
    B.resize(step + 1); B[step].resize(N);
    b = B[step][A[0].id] = 1;
    for (ll i = 1; i < N; ++i) {
      if (A[i - 1] < A[i]) ++b;
      B[step][A[i].id] = b;
    }
  }
}
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$ 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    FOR(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    FOR(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

SuffixAutomaton.h

Time: $\mathcal{O}(26N)$ 98 lines

```
const int S = 27;
struct node { int len, link, next[S]; };
struct tree_node { int t0[S], t1[S], next[S], k; };

const int N = 200000;
int sz, last;
node v[N];
tree_node tree[N];
int T[N], Tsz;

inline void init() {
  sz = 1; last = 0; v[0].len = 0; v[0].link = -1;
  memset(v[0].next, -1, sizeof(v[0].next));
}

inline void add(int c) {
  int nlast = sz++;
  v[nlast].len = v[last].len + 1;
  memset(v[nlast].next, -1, sizeof(v[nlast].next));
  int p = last;
  while (p != -1 && v[p].next[c] == -1) {
    v[p].next[c] = nlast; p = v[p].link;
  }
  if (p == -1) {
    v[nlast].link = 0;
  } else {
    int q = v[p].next[c];
    if (v[p].len + 1 == v[q].len) {
      v[nlast].link = q;
    } else {
      int clone = sz++;
      v[clone].len = v[p].len + 1;
      v[clone].link = v[q].link;
      memcpy(v[clone].next, v[q].next,
          sizeof(v[q].next));
      while (p != -1 && v[p].next[c] == q) {
        v[p].next[c] = clone; p = v[p].link;
      }
      v[nlast].link = v[q].link = clone;
    }
  }
  last = nlast;
}

int dfs_tree(int x) {
    int p[S];
    for (int i=0;i<tree[x].k;++i)p[i]=tree[x].next[i];
    memset(tree[x].next, -1, sizeof(tree[x].next));
    int minval = Tsz;
    for (int i = 0; i < tree[x].k; ++i) {
        int t1 = dfs_tree(p[i]) - 1;
        int t0 = t1 - v[p[i]].len + v[x].len + 1;
        minval = min(minval, t0);
        tree[x].next[T[t0]] = p[i];
        tree[x].t0[T[t0]] = t0;
        tree[x].t1[T[t0]] = t1;}
    return minval;}

void suffix_tree(string& a) {
    Tsz = (int)a.size();
    for (int i = 0; i < Tsz; ++i) T[i] = a[i] - 'a';
    T[Tsz++] = S-1;
    init();
    for (int i = Tsz-1; i >= 0; --i) add(T[i]);
    for (int i = 0; i < sz; ++i) tree[i].k = 0;
    for (int i = 1; i < sz; ++i)
      tree[v[i].link].next[tree[v[i].link].k++] = i;
    dfs_tree(0);}

//codi per http://codeforces.com/problemset/problem/123/D
typedef long long ll;
ll res;

int dfs(int x) {
    int conter = 0;
    for (int i = 0; i < S; ++i) {
        if (tree[x].next[i] == -1) continue;
        int next = tree[x].next[i];
        ll quants = dfs(next);
        ll dist = tree[x].t1[i] - tree[x].t0[i] + 1;
        res += quants*dist*(quants + 1)/2;
        conter += quants;
    }
    if (!conter) return 1;
    return conter;
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    string a;
    cin >> a;
    suffix_tree(a);
    res = 0;
    dfs(0);
    res -= (ll)a.size() + 1;
    cout << res << '\n';
}
```

Z.h

Description: Computes the longest prefix which ends at the i-th position 12 lines

```
vi Z(string &s) {
    int n = s.size(); int L, R;
    L = R = 0; vi Z(n, 0);
    for (int i = 1; i < n; ++i){
        if (i < R) Z[i] = min(Z[i-L], R-i);
        else Z[i] = 0;
        while (Z[i] + i < n and s[Z[i]] == s[i+Z[i]]) ++Z[i];
        if (i+Z[i] > R){
            L = i; R = i + Z[i];
        }
    }
}
```

KMP.h

Description: returns a 1 indexed array containing the length of the suffix which matches the prefix.
Time: $\mathcal{O}(|S|)$ 8 lines

```
void KMP(vi& kmp, string& s) {
  kmp.assign(sz(s) + 1, -1);
  for (int i = 0; i < sz(s); i++){
    int j = kmp[i];
    while(j >= 0 && s[i] != s[j]) j = kmp[j];
    kmp[i+1] = j + 1;
  }
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+min_rotation(v), v.end());`
Time: $\mathcal{O}(N)$

8 lines

```
int min_rotation(string s) {
  int a=0, N=sz(s); s += s;
  FOR(b,0,N) FOR(i,0,N) {
    if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
    if (s[a+i] > s[b+i]) { a = b; break; }
  }
  return a;
}
```

allSubstringLCS.h

Description: Calculates the LCS of s and t[i..j]
Time: $\mathcal{O}(|S|^2)$

24 lines

```
vector<vi> allSubstringLCS(string& s, string& t) {
  int n = sz(s), m = sz(t);
  vector<vi> M(2, vi(m));
  iota(all(M[1]), 0);
  FOR(i, 0, n) {
    int k = -1;
    FOR(j, 0, m) {
      if (s[i] == t[j]) {
        M[i&1][j] = k; k = M[(i+1)&1][j];
      } else {
        M[i&1][j] = max(M[(i+1)&1][j],k); k = min(M[(i+1)&1][j
             ],k);
      }
    }
  }
  vector<vi> ans(m, vi(m));
  FOR(i, 0, m) {
    int cur = 0;
    FOR(j, i, m) {
      if (M[(n+1)&1][j] < i) ++cur;
      ans[i][j] = cur;
    }
  }
  return ans;
}
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F f, G g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

Karatsuba.h

Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length well. See also FFT, under the Numerical chapter.
Time: $\mathcal{O}(N^{1.6})$

Josephus.h

Description: if you have n people in a circle and the first person to die is 0, and each time afterwards you kill the person k positions later, outputs the number of the last person to die

3 lines

```
cin >> n >> k; v[1]=0;
for (int i=2;i<=n;++i) v[i]=(v[i-1]+k)%i;
cout<< v[n]+1 << endl;
```

10.3 Dynamic programming

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \le k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    FOR(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

KnuthDP.h

Description: When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $dp[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$, That is $p[i][j-1] \le p[i][j] \le p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. We should calculate the values on decreasing i and increasing j. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^3) \to \mathcal{O}(N^2)$

wqsBinaryDP.h

Description: Given a problem to minimize dp[n][k] choosing k cuts/groups and dp[n][j] is convex, we can relax the second condition and perform a binary search on some integer penalization, we find the min dp[n][x] + C*x, and find the maximum C such that x <= k, then we can simply substract C*k from this answer

Time: $\mathcal{O}(N^2K) \rightarrow \mathcal{O}(NlogN)$

10.3.1 1D1D DP optimization

Let arg(x) be the position in which f is minimized. Note that even if the necessary conditions are not fulfiled, maybe we can take away some useless states so it does fulfil them.

- $f(x) = \min_{0 \le i < x}\{f(i) + w[i,x]\}$
  Necessary condition: $\forall i \le j, arg(i) \le arg(j) \iff w[i,x]$ fulfils the quadrilateral inequality
  Then we can mantain a deque of (p, startpos, endpos) where each element mantains the range where p is arg(x), we can find the starting position using a binary search, ending position is determined when a new candiate trumps our range, the deque is increasing on p.
- $f(x) = \min_{b[x] \le i < x}\{g(i)\} + w[x]\}$
  Necessary condition: $b[x] non-decreasing$
  Note if $g(x+1) \le g(x)$ we will not consider $g(x)$ to update the dp. Hence we can mantain a monotonically increasing deque/list of all the $g(x)$, and popfront until the first value fits in our range. Note that we may need to popback several elements to mantain monotonicity
- $f(x) = \min_{0 \le i < x}\{a[x] \cdot f(i) + b[x] \cdot g(i)\}$
  Necessary condition:
  We aim to minimize $P = a \cdot f(i) + b \cdot g(i)$, we can rearrange this us $g(i) = -\frac{a}{b} \cdot f(i) + \frac{P}{b}$. If we consider f(i) and g(i) as points on a plane, we are looking for the minimum $\frac{P}{b}$ or maximum if b is negative which fulfils our equation, we can calculate this with a binary search on a convex hull. If the gradient $\frac{a}{b}$ is monotone, we can use one pointer for the Upper and Lower hulls and not need a binary search.

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

10.5 Optimization tricks

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `x && !(x & (x - 1))` true, if `x` is power of 2.
- `gray_code[x] = x ^ (x >> 1)`
- `checkerboard[y][x] = (x & 1) ^ (y & 1)`
- `ffs(int x), ffs(ll x)` number of the least significant bit, `ffs(1 << i) = i+1`
- `__builtin_clz(uint x), ..._clzll(ull)` number of leading zeros, for x ¿ 0
- `__builtin_ctz(uint x), ..._ctzll(ull)` number of trailing zeros, for x ¿ 0
- `__builtin_popcount(uint x), ..._popcountll(ull)` number of 1 bits
- `#define ld_ll(X) (63-__builtin_clzll(ll(X)))` floor(log2(X))
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- ` FOR(b,0,K) FOR(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

10.5.3 Random Problem Solutions

Strings

- Distinct substrings in a word: Sum len(q) - len(suffixlink(q))
- Sum of Lengths of distincst substrings: We count how many times a letter appears in distinct substrings, as all paths in automata ara different substrings we have 1 + number of paths, we can solve this by easy dp.

n-Queens Problem $n \ge 4$ or $n = 1$

1. Divide n by 12. Remember the remainder (n is 8 for the eight queens puzzle).
2. Write a list of the even numbers from 2 to n in order. If the remainder is 3 or 9, move 2 to the end of the list.
3. Append the odd numbers from 1 to n in order, but, if the remainder is 8, switch pairs (i.e. 3, 1, 7, 5, 11, 9, ...).
4. If the remainder is 2, switch the places of 1 and 3, then move 5 to the end of the list.
5. If the remainder is 3 or 9, move 1 and 3 to the end of the list.
6. Place the first-column queen in the row with the first number in the list, place the second-column queen in the row with the second number in the list, etc.

For $n = 8$ this results in the solution shown above. A few more examples follow.

- 14 queens (remainder 2): 2, 4, 6, 8, 10, 12, 14, 3, 1, 7, 9, 11, 13, 5.
- 15 queens (remainder 3): 4, 6, 8, 10, 12, 14, 2, 5, 7, 9, 11, 13, 15, 1, 3.
- 20 queens (remainder 8): 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 3, 1, 7, 5, 11, 9, 15, 13, 19, 17.

Techniques (A)

techniques.txt

159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```