Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	7.0	*Inverse Functions*
Lesson Code	7.2.1	*Graphs of Inverse Functions 1*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this learning guide, the student will have been able to:

1. sketch the graph of the inverse of a function given the graph of the function

HOOK

Time Allocation: 9 minutes
Actual Time Allocation: _____ minutes

In the previous learning guides, we have learned that the inverse function of a given function f is represented by $f^{-1}(x)$, read as "*f inverse of x*". To recall how to determine the inverse of a function, let's answer the following.

Recall that to determine the inverse of a function, we change $f(x)$ to y, interchange x and y, then solve for y. Let's do these steps to determine the inverse of the functions below.

1. $f(x) = 1 + x^3$
$$y = 1 + x^3$$
$$x = 1 + y^3$$
$$x - 1 = y^3$$
$$y^3 = x - 1$$
$$y = \sqrt[3]{x-1}$$
$$\boldsymbol{f^{-1} = \sqrt[3]{x-1}}$$

2. $f(x) = 3 - 2x$
$$y = 3 - 2x$$
$$x = 3 - 2y$$
$$x - 3 = -2y$$
$$\frac{x-3}{-2} = y$$
$$\boldsymbol{f^{-1} = \frac{3-x}{2}}$$

Question: Which of the following functions have a defined inverse function?

1. $f(x) = |x - 3|$
2. $f(x) = x^3$
3. $f(x) = 9 - x^2$

Answers:
1. The function is not a one – to – one function, therefore, it has no defined inverse.
2. It has a defined inverse.
3. The function is not a one – to – one function, therefore, it has no defined inverse.

Recall from learning guide 7.1 how we can restrict the domain of functions 1 and 2 for them to have a defined inverse function. This time, we are going to graph the inverse of a function given the graph of the function.

IGNITE

Time Allocation: 20 minutes
Actual Time Allocation: _____ minutes

Let us start exploring our topic by answering this question: ***How are functions and their inverses related?***

Determine the inverse of $f(x) = \frac{x-3}{2}$

Step 1:

a) *Make Table*: Plug in 5 different values for x to get your y values using the table.

x	***f(x) = y***	**ordered pair**

b) *Graph function*: Plot the points from the table on the given coordinate plane below and draw the line that passes through them. Label the graph as f.

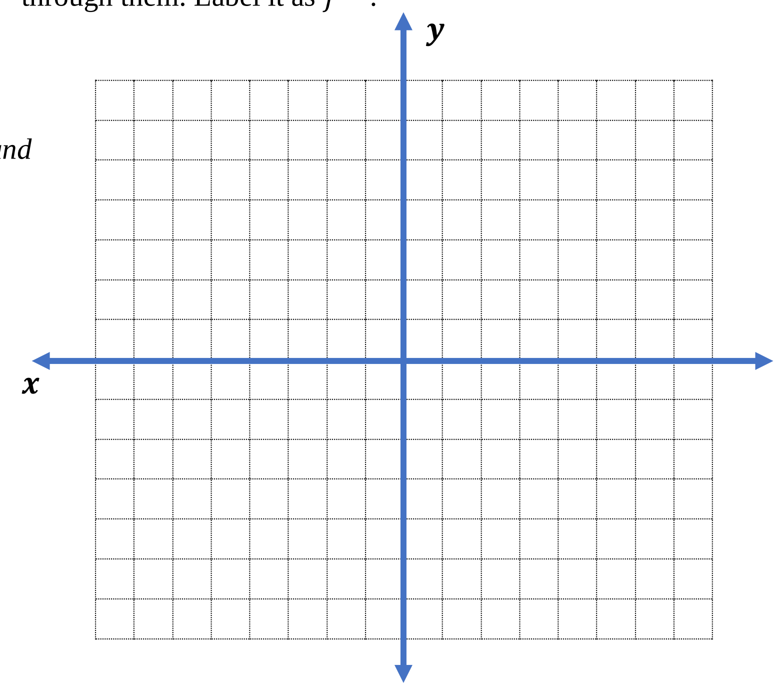
Step 2:

a) *Interchange coordinates*: Switch the x and y-coordinates of the ordered pairs found in Step 1. Example: (2, 3) → (3, 2) and write in the new table.

x	***f(x) = y***	**ordered pair**

b) *Graph function:* Plot the points on the same graph and draw the line that passes through them. Label it as f^{-1}.

*Draw the graphs of **f** and $\boldsymbol{f^{-1}}$ on the coordinate plane at the right.*



Step 3:

Write equation: Write and equation of the line from Step 2. Write this function as

$$\boldsymbol{g(x) = mx + b}.$$

$m =$ $\qquad\qquad$ $b =$ $\qquad\qquad$ $g(x) =$

Step 4:

Compare graphs: Fold your graph paper so that the graphs of *f* and *g* coincide (lie on top of one another). How are the graphs **geometrically** related?

Your Answer: ____

GRAPH OF INVERSE FUNCTIONS

Recall that to determine the inverse of a function, we interchange x and y and then solve for y. We shall use the same concept to sketch the graph of $f^{-1}(x)$ using the given graph of *f*. If the graph of *f* contains the point *(x, f(x))*, therefore, the graph of its inverse, f^{-1} will contain the point $(f(x), x)$ and this is illustrated in Figure 1 below:

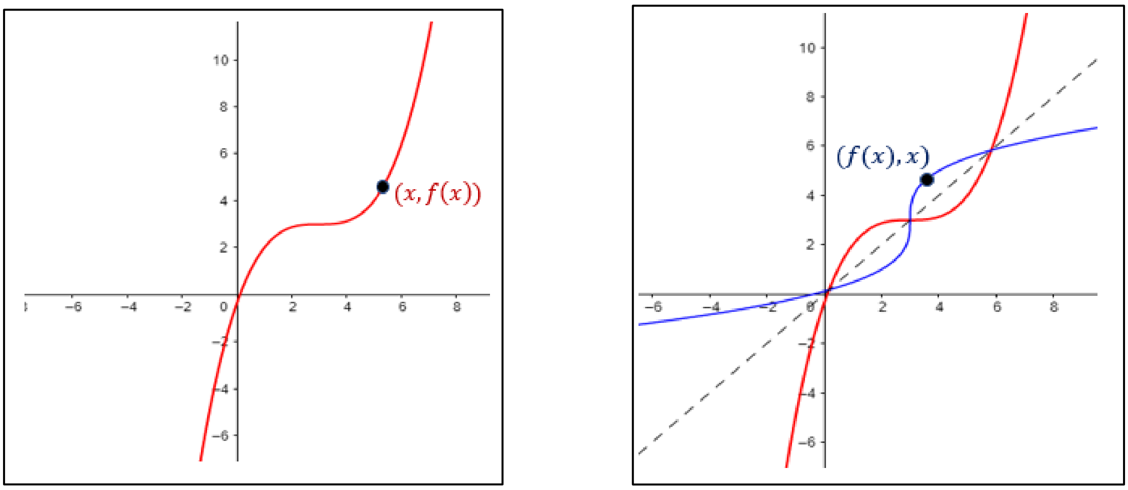


Figure 1: Graphs of a function and its inverse

Image Source: *http://www.teaching.martahidegkuti.com/shared/lnotes/4CollegeAlgebra/inverse.pdf*

We can see from our previous activity and from figure 1 that the graphs of f and f^{-1} are symmetric with respect to the line $y = x$. Recall that when we reflect a point (x, y) across the line $y = x$, the image is (y, x).

Example 1. Sketch the graph of $g^{-1}(x)$ for the given function $g(x) = \frac{1}{2}x + 3$.

Solution: Let us first graph the function g. Getting the inverse of a function literally interchanges the assignment of x and y. Thus, if $(2, 4)$ and $(-2,2)$ are points on the graph of g, then interchanging the two coordinates provides us points on the graph of its inverse. Therefore, $(4, 2)$ and $(2, -2)$ are points on the graph of the inverse. We will apply this method to obtain more points:

Points on g	$(-2, 2)$	$\left(-1, \frac{5}{2}\right)$	$(0, 3)$	$\left(1, \frac{7}{2}\right)$	$(2, 4)$	$\left(3, \frac{9}{2}\right)$	$(4,5)$
Points on g^{-1}	$(2, -2)$	$\left(\frac{5}{2}, -1\right)$	$(3, 0)$	$\left(\frac{7}{2}, 1\right)$	$(4,2)$	$\left(\frac{9}{2}, 3\right)$	$(5,4)$

Locating the points, we have the following graphs.

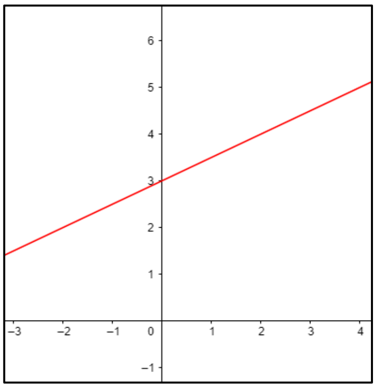


Figure 2: Graph of g

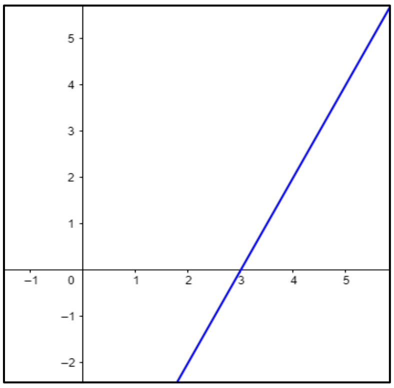


Figure 3: Graph of g^{-1}

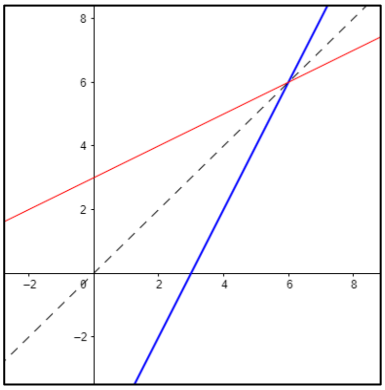


Figure 4: Graphs of g and g^{-1}

Example 2. Given $h(x) = x^2$, sketch the graph of its inverse along with the line $y = x$.

Solution: We will determine some points on the graph of h. Then, we will interchange their coordinates to obtain points on the graph of h^{-1}.

Points on h	$(-3, 9)$	$(-2,4)$	$(-1, 1)$	$(0,0)$	$(1,1)$	$(2,4)$	$(3,9)$
Points on h^{-1}	$(9, -3)$	$(4, -2)$	$(1, -1)$	$(0,0)$	$(1,1)$	$(4,2)$	$(9,3)$

Plotting and connecting the points, we have the graphs below.

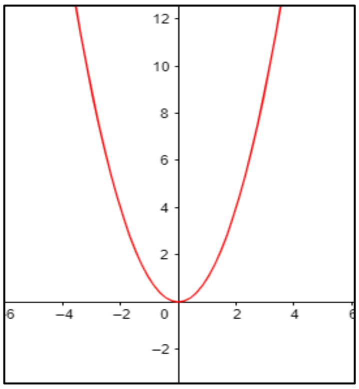


Figure 5: Graph of h

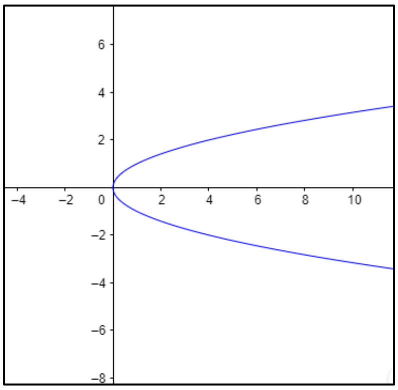


Figure 6: Graph of h^{-1}

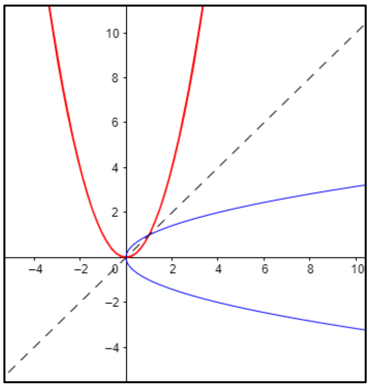


Figure 7: Graph of h and h^{-1}

As we can observe from the figures above, the graph of g^{-1} illustrates a function while the graph of h^{-1} does not. Between g and h, only g will pass the horizontal line test. Thus, g is a one-to-one function. We have also learned that the inverse of one – to – one functions is a function. Therefore, g^{-1} is a function while h^{-1} is not a function since h is not one – to – one.

Note:

Consider *f* as a function and its inverse relation denoted by f^{-1}, the inverse of f is a function if and only if the function *f* is one-to-one.

Take the next example when a function is not one-to-one but we would still like to have an inverse that is a function (and not just a relation). We could have a one – to – one function by restricting its domain.

Example 3.

Let us use $h(x)$ in Example 2, applying the horizontal line test, we can say that h is not one – to – one function but we can make it so by restricting its domain. We will just choose a part of its domain to make it one – to – one, either $(-\infty, 0]$ or $[0, +\infty)$.

Using the restricted domain, $[0, +\infty)$, the graphs are shown below.

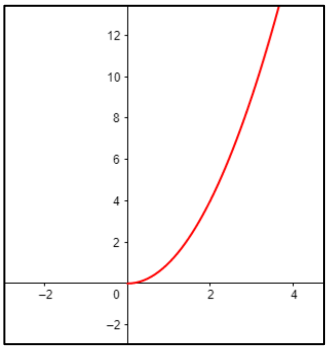


Figure 8: Graph of h

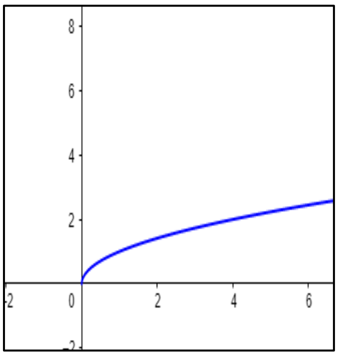


Figure 9: Graph of h^{-1}

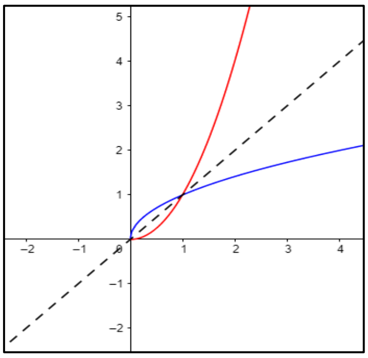


Figure 10: Graph of h and h^{-1}

Using $(-\infty, 0]$, the graphs are shown below.

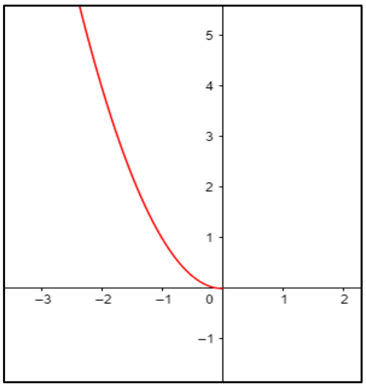


Figure 11: Graph of h

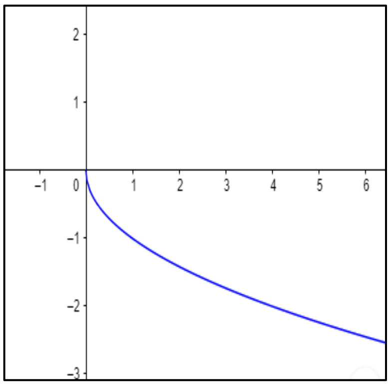


Figure 12: Graph of h^{-1}

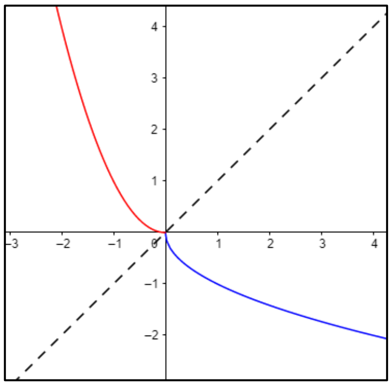


Figure 13: Graph of h and h^{-1}

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