Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	6.0	*Other Types of Functions*
Lesson Code	6.4	*Transformation of Functions (Sequences)*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this learning guide, the students will have been able to:

1. Understand the order of transforming functions.
2. Illustrate the order of transforming functions on the coordinate plane.
3. Apply the order of transforming functions in graphing equations from a given parent function.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

From the previous lessons (Learning Guides 6.1 to 6.3), we learned how to translate, reflect, and dilate functions. Recall the different rules in transforming of functions:

TRANSFORMATIONS

In each case, c represents a positive real number.

	Function	Draw the graph of f and:
Vertical translations	$y = f(x) + c$	Shift f upward c units.
	$y = f(x) - c$	Shift f downward c units.
Horizontal translations	$y = f(x - c)$	Shift f to the right c units.
	$y = f(x + c)$	Shift f to the left c units.
Reflections	$y = -f(x)$	Reflect f about the x-axis.
	$y = f(-x)$	Reflect f about the y-axis.
Vertical Stretching or Shrinking	$y = cf(x); c > 1$	Vertically stretch f, multiplying each of its y-coordinates by c.
	$y = cf(x); 0 < c < 1$	Vertically shrink f, multiplying each of its y-coordinates by c.
Horizontal Stretching or Shrinking	$y = f(cx); c > 1$	Horizontally shrink f, dividing each of its x-coordinates by c.
	$y = f(cx); 0 < c < 1$	Horizontally stretch f, dividing each of its x-coordinates by c.

Figure 1: Rules in Transforming Functions
Retrieved from: Glencoe Advanced Mathematical Concepts:
Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

In this lesson, we will learn how to perform a sequence of transformation.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

The sequence of transformation is the process of performing more than one transformations on a given function. Let's take a look at some examples.

Example 1: A function f is given and the indicated transformations are applied to its graph (in the given order). Denote equation for the final transformed graph as $g(x)$. Simplify your answer.

a. $f(x) = x^2$ shifted 3 units to the right then 1 unit upwards.
b. $f(x) = x^2$ shifted 1 unit upwards then 3 units to the right.
c. Compare the results from part (a) and part (b). What can you say? Do they result in the same function or not?

Answer:

a. Step 1: Shift $f(x) = x^2$ three units to the right.
$f'(x) = (x-3)^2 = x^2 - 6x + 9$
Step 2: Shift $f'(x) = (x-3)^2$ one unit upward.
$f''(x) = (x^2 - 6x + 9) + 1 = x^2 - 6x + 10$
Step 3: Denote the final transformed graph denoted as $g(x)$.
$g(x) = x^2 - 6x + 10$

b. Step 1: Shift $f(x) = x^2$ one unit upward.
$f'(x) = x^2 + 1$
Step 2: Shift $f'(x) = x^2 + 1$ three units to the right.
$f''(x) = (x-3)^2 + 1 = x^2 - 6x + 9 + 1 = x^2 - 6x + 10$
Step 3: Denote the final transformed graph denoted as $g(x)$.
$g(x) = x^2 - 6x + 10$

c. The result in (a) is equal to the result in (b). Takeaway: The order at which vertical and horizontal translations are performed in the function does not matter.

Example 2: A function f is given and the indicated transformations are applied to its graph (in the given order). Denote equation for the final transformed graph as $g(x)$. Simplify your answer.

a. $f(x) = x^2$ shifted 3 units upward then reflected across the x-axis.
b. $f(x) = x^2$ reflected across the x-axis then shifted 3 units upward.
c. Compare the results from part (a) and part (b). What can you say? Do they result in the same function or not?

Answer:

a. Step 1: Shift $f(x) = x^2$ three units upward.
$f'(x) = x^2 + 3$
Step 2: Reflect $f'(x) = x^2 + 3$ across the x-axis.
$f''(x) = -(x^2 + 3) = -x^2 - 3$
Step 3: Denote the final transformed graph denoted as $g(x)$.
$g(x) = -x^2 - 3$

b. Step 1: Reflect $f(x) = x^2$ across the x-axis.
$f'(x) = -(x^2) = -x^2$
Step 2: Shift $f'(x) = -x^2$ three units upward.
$f''(x) = -x^2 + 3$
Step 3: Denote the final transformed graph denoted as $g(x)$.
$g(x) = -x^2 + 3$

c. The result in (a) is not equal to the result in (b). Takeaway: Be mindful of the order in performing multiple types of transformations!

Important! For consistency, the following order of transformation will be followed in transforming functions involving more than one transformation:

1. Horizontal Translation
2. Dilation
3. Reflecting
4. Vertical Translation

Example 3: Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x-3)^2 - 1$.

Answer:

1. Our graphs will be transformed in the following order:

a. Step 1: Horizontal translation. Graph $y = (x-3)^2$ by shifting the graph $of\ f(x) = x^2$ three units to the right.

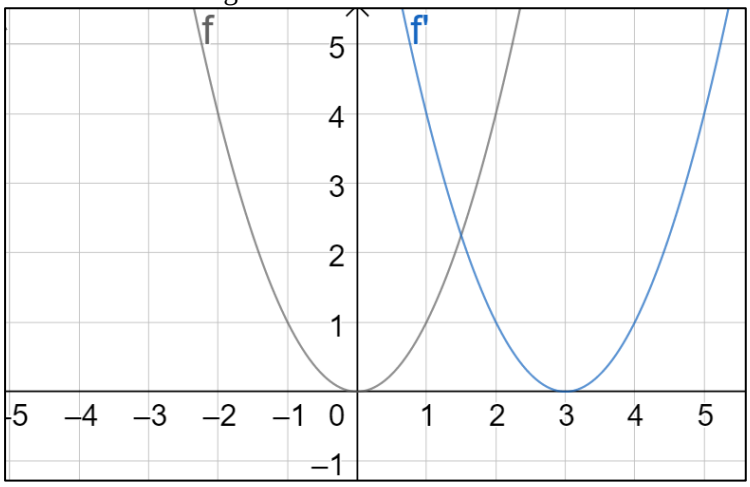

Figure 2: Step 1. Sequence of Transformations

b. Step 2: Stretching: Graph y = $2(x-3)^2$ by stretching the previous graph by a factor of 2.

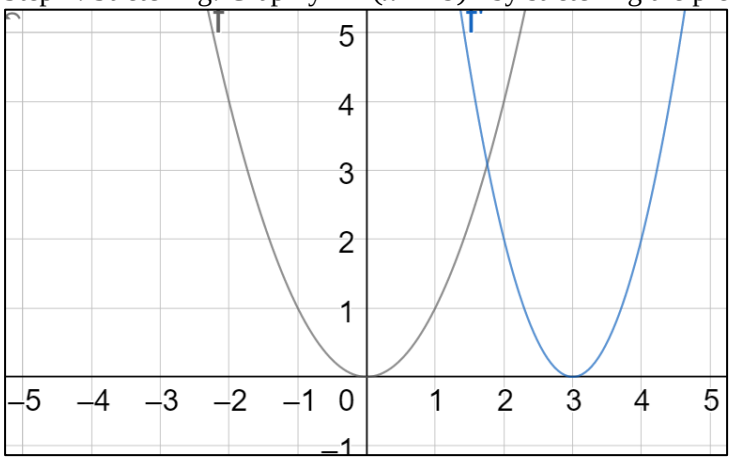

Figure 3: Step 2. Sequence of Transformations

c. Vertical translation: Graph $g(x) = 2(x-3)^2 - 1$ by shifting the previous graph down 1 unit.

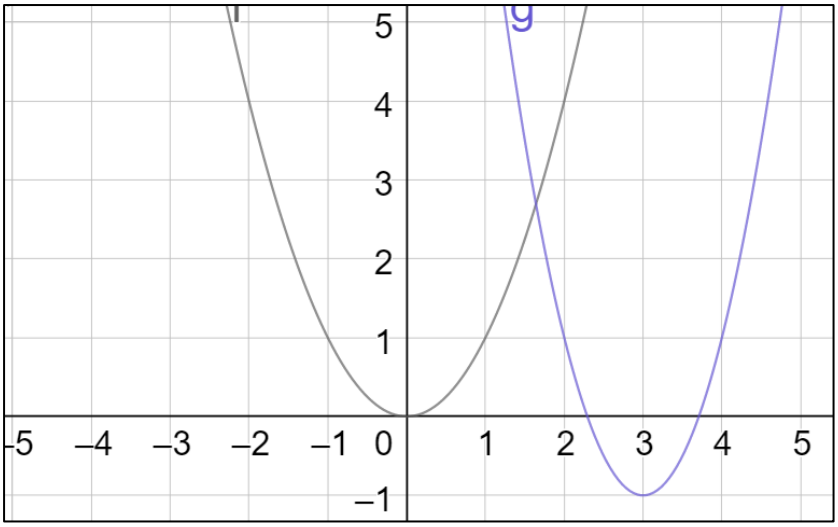

Figure 4: Step 3. Sequence of Transformations

Use geogebra to graph the function $g(x) = 2(x-3)^2 - 1$. The result should be exactly the same as the one we obtained throught transformations.

NAVIGATE

Time Allocation: 9 minutes
Actual Time Allocation: _____ minutes

Answer the following questions. Items marked with an asterisk (*) will be graded.

1. Use the graph of $f(x)$ on the right to graph each function g.
   a. $g(x) = f(x-1) - 1$
   b. $*g(x) = -f(x-1) + 1$
   c. $g(x) = 2f(\frac{1}{2}x)$
   d. $*g(x) = \frac{1}{2}f(x+1)$

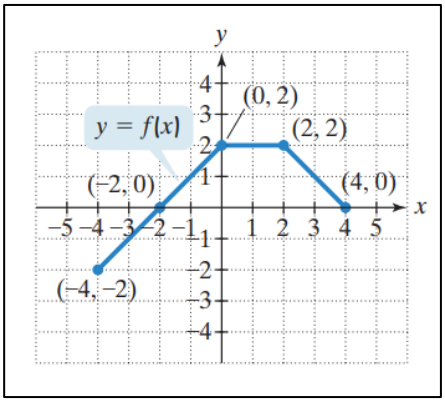

Figure 4: Function for Sequence of Transformation
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

2. Begin by graphing the parent absolute value function $f(x) = |x|$. Use the transformation of this graph to sketch the given function g.
   a. $h(x) = -|x+4|$
   b. $*h(x) = -|x+4| + 1$
   c. $h(x) = 2|x+4|$
   d. $*h(x) = -2|x+4| + 1$

3. Functions f and g are graphed in the same coordinate system. If g is obtained from f through a sequence of transformations, find an equation for g.

a.

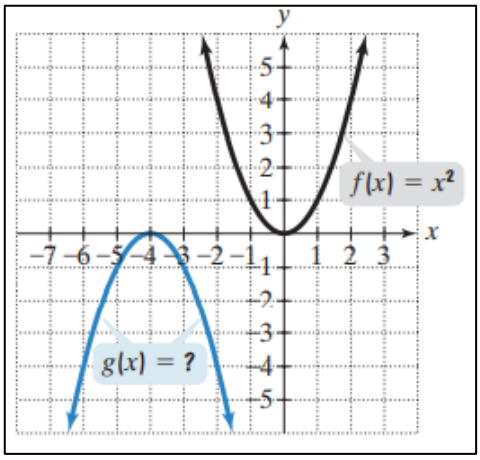

Figure 5: Absolute Value Function for Sequence of Transformation
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

b. *

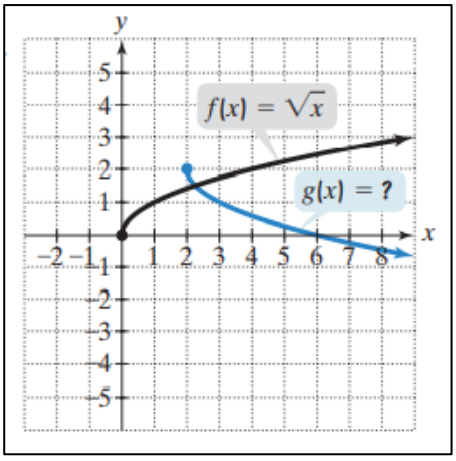

Figure 5: Semicircle Function for Sequence of Transformation
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

KNOT

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

- The sequence of transformation is the process of performing more than one transformations on a given function.
- The order of operation in performing a sequence of transformation matters. However, if the sequence only involved translation, the order at which vertical and horizontal translations are performed does not matter.
- For consistency, the following order of transformation will be followed in transforming functions involving more than one transformation:
    i. Horizontal Translation
    ii. Dilation
    iii. Reflecting
    iv. Vertical Translation

References:

Albarico, J.M. (2013). THINK Framework. Based on *Science LINKS* by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Carter, J., Cuevas, G., Day, R., and Malloy, C., (2012). *Glencoe Geometry*. USA: The McGraw-Hill Companies, Inc.

International Geogebra Institute. (2020). *GeoGebra*. www.geogebra.org

Stewart, J., Redlin, L., Watson, S (2012). *Precalculus: Mathematics for Calculus*. Brooks/Cole, Cengage Learning.

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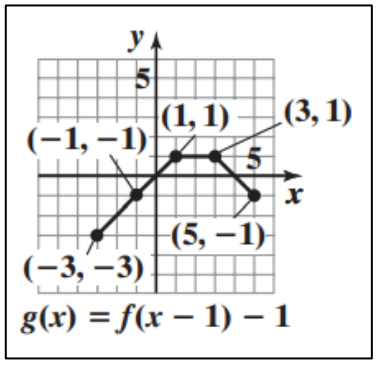
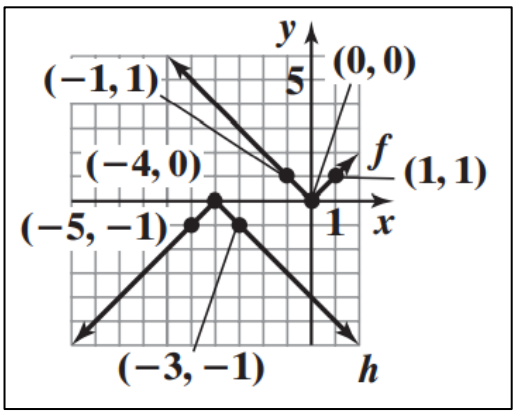
Answer Key:

Navigate

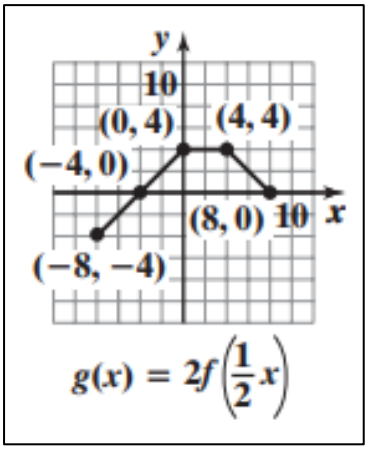
1. a.



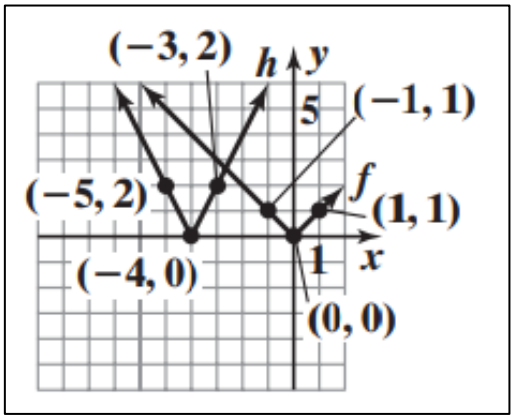
2. a.



c.



c.



3. a. $f(x) = -(x+4)^2$