Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	6.0	*Other Types of Functions*
Lesson Code	6.2	*Transformation of Functions (Reflection)*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this learning guide, the students will have been able to:

1. Understand the rule for reflection of functions.
2. Demonstrate reflection of functions using graphing tools and software.
3. Illustrate reflection of functions on the coordinate plane.

HOOK

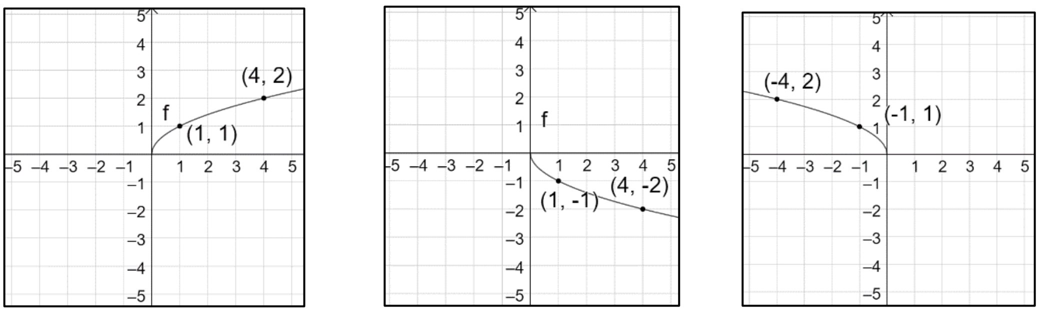
Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

From the previous lesson (Learning Guide 6.1), we learned how to vertically and horizontally translate functions. In this lesson, we will learn how to reflect functions.

Recall in your Learning Guide 2.2 that reflection is a transformation that uses a line of reflection that acts as a mirror, with an image reflected in the line. Recall further that the rules in reflecting a point (x, y) over the x-axis and y-axis are:

- Reflections over the x-axis:
  The image of the point (x, y) when reflected across the x-axis is the point $(x, -y)$.
- Reflection over the y-axis:
  The image of the point (x, y) when reflected across the y-axis is the point $(-x, y)$.

Now, consider the parent function $f(x) = \sqrt{x}$ with its graph shown below. Consider as well the square root functions g and h.



$f(x) = \sqrt{x}$ $\quad$ $g(x) = -\sqrt{x}$ $\quad$ $h(x) = \sqrt{-x}$

Figure 1: Reflection of $f(x) = \sqrt{x}$ over the x-axis and y-axis

Notice that the function $g(x)$ results from reflecting the function $f(x)$ across the x-axis. On the other hand, the function $h(x)$ results from reflecting $f(x)$ across the y-axis. We have just performed a reflection of function. Take note of the coordinate points on each graph. Did you notice anything similar to the rules of reflection from LG 2.2?

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Hands-on Activity using GeoGebra

1. Open your GeoGebra application or go to the link: https://www.geogebra.org/calculator

2. Graph the function $y = 2^x$. This function is an exponential function. You will learn more about this function in the succeeding learning guides. Let us name this graph f.

3. Graph another function $y = 2^{-x}$. Let us name this graph as g and set its color to red. Your graphs should now look like this:

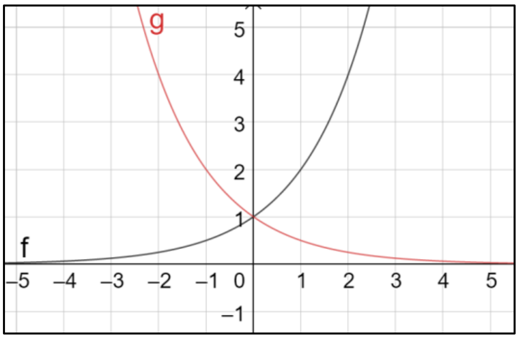


Figure 2: Functions f and g in reflection (across y-axis) hands-on activity

Notice that graph g is the result when graph f is reflected across the y-axis. What about if we want to reflect f across the x-axis?

4. Hide function g by pressing the circle on the left of the function editor. Now add a new function $y = -2^x$. Let's denote this function as h and set its color to red. Your graphs should now look like this:

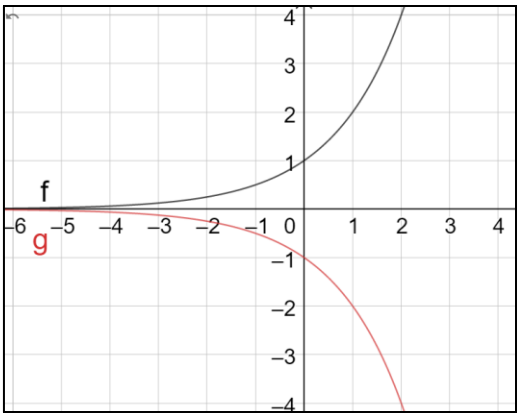


Figure 3: Functions f and g in reflection (across x-axis) hands-on activity

Notice that graph g is the result when graph f is reflected across the x-axis.

5. Observe that function g was obtained by changing every x in the function f with $-x$. On the other hand, function h was obtained by multiplying the function with -1. Certainly enough, these are the rules for reflection. Let's generalize these observations by formally defining the reflection of functions.

In **Reflecting Graphs**:

- To graph $y = -f(x)$, reflect the graph of $y = f(x)$ across the $x-$ axis.
- To graph $y = f(-x)$, reflect the graph of $y = f(x)$ across the $y-$ axis.

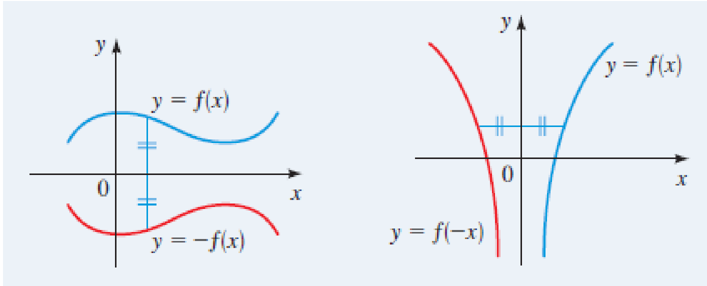


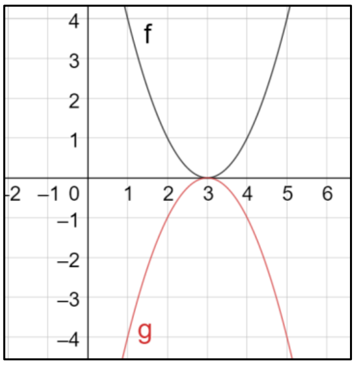
Figure 4: Reflection
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

Example 1: Reflecting a Function. Use the function $f(x) = (x-3)^2$ to sketch the graph of each function below.

a. $g(x) = -(x-3)^2$
b. $g(x) = (-x-3)^2$

Answer: Note that from the previous learning guide, the function $f(x) = (x-3)^2$ is obtained by shifting the graph of the parent function $y = x^2$ three units to the right. The function in (a) is the graph of $f(x)$ reflected across the x-axis. On the other hand, the function in (b) is the graph of $f(x)$ reflected across the y-axis.

a.



b.

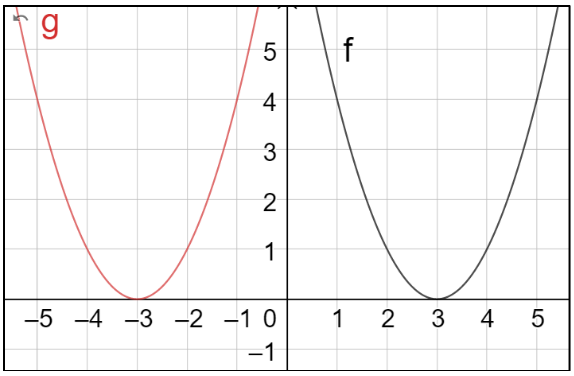


Figure 5: Functions $y = -(x-3)^2$ and $y = (-x-3)^2$

Example 2: Changes in Domain and Range. Consider the function $f(x) = \sqrt{-x^2 + 2x + 8}$. Consider as well the semicircles $g(x) = -\sqrt{-x^2 + 2x + 8}$ and $h(x) = \sqrt{-x^2 - 2x + 8}$. Do the following:

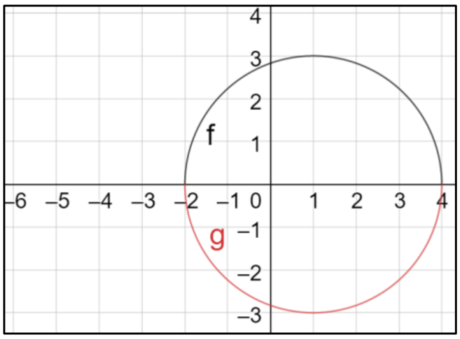
a. Describe in words how the functions $g(x)$ and $h(x)$ are related to the function $f(x)$.
b. Graph functions $g(x)$ and $f(x)$ in one coordinate plane and functions $h(x)$ and $f(x)$ in another coordinate plane
c. Using the graphs in (b), complete the table below.
d. What do you observe on the changes in domain and range after the reflection across the x-axis and y-axis?

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-2,4]$		
Range	$[0,3]$		

Answer:

a. The function $g(x)$ is the graph of $f(x)$ reflected across the x-axis. The graph $h(x)$ is the graph of $f(x)$ reflected across the y-axis.

b. The graphs are shown below.



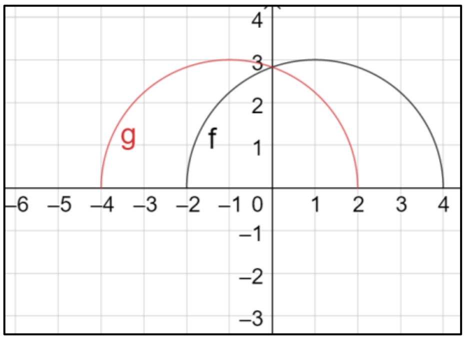


Figure 8: Functions $g(x) = -\sqrt{-x^2 + 2x + 8}$ and $h(x) = \sqrt{-x^2 - 2x + 8}$

c. The completed table is shown below.

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-2,4]$	$[-2,4]$	$[-4,2]$
Range	$[0,3]$	$[-3,0]$	$[0,3]$

d. For function $g(x)$, the domain is unchanged while the range is changed. The resulting range is negated (and arranged so that the smaller number comes first in the interval notation) as illustrated below:

Range of $g(x)$: $[0,3] \rightarrow [0(-1), 3(-1)] \rightarrow [0,-3]$, correcting it by ordering the endpoints properly will yield the interval $[-3,0]$.

For function $h(x)$, the domain is changed while the range is unchanged. The resulting domain is negated (and arranged so that the smaller number comes first in the interval notation) as illustrated below:

Domain of $h(x)$: $[-2,4] \rightarrow [-2(-1), 4(-1)] \rightarrow [2,-4]$, correcting it by ordering the endpoints properly will yield the interval $[-4,2]$.

Note: Reflection across the y-axis will affect the domain and leave the range unchanged. On the other hand, reflection across the x-axis will affect the range and leave the domain unchanged.

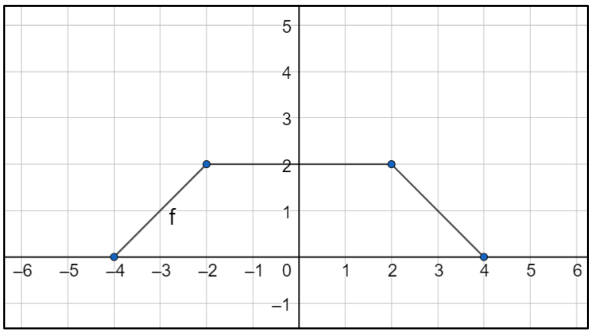
NAVIGATE

Time Allocation: 9 minutes
Actual Time Allocation: ______ minutes

Answer the following questions. Items marked with an asterisk (*) will be graded.

1. Describe how the graph of g is obtained from the graph of f.
   a. $f(x) = x^2 + 2x + 1,$ $g(x) = x^2 - 2x + 1$
   b. $*f(x) = |x| + 3x + 5,$ $g(x) = -|x| - 3x - 5$

2. The graph of $f(x)$ is given below. Consider the function $g(x) = -f(x)$ and do the following:



a. Describe in words how the $g(x)$ is related to the function $f(x)$.
b. Graph $g(x)$ in the same coordinate plane as $f(x)$.
c. Complete the table below.

	$\boldsymbol{f(x)}$	$\boldsymbol{g(x)}$
Interval at which the function is strictly increasing		
Interval at which the function is strictly decreasing		
Interval at which the function is constant		

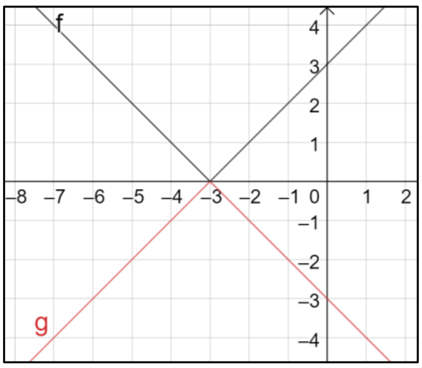
d. What are your observations?

3. *Using the same function $f(x)$ in item (3), consider the function $g(x) = f(-x)$ and repeat steps a to d.

4. Write a formula for the function g that results when the graph of $f(x)$ is transformed as described (simplify your answer as necessary). Without solving, what is the domain and range of g?
a. The graph of $f(x) = \sqrt{x+3}$ reflected across the x-axis.
b. *The graph of $f(x) = \sqrt{x+3}$ reflected across the y-axis.

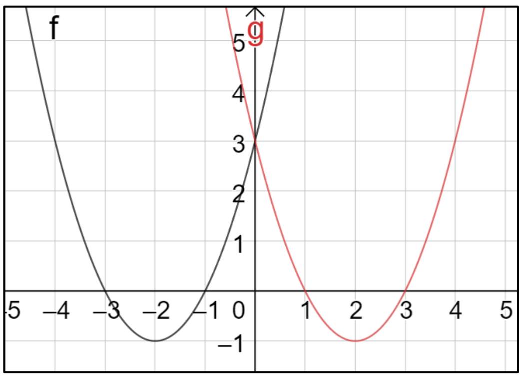
5. For the following exercises, the graphs of f and g are given. Find a formula for the function g from the function of f. Simplify your answer. Note that the graph in black is the function of f. You may verify your answer by graphing it using GeoGebra.

a.



$f(x) = |x+3|$

*b.



$f(x) = x^2 + 4x + 3$

KNOT

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

- In **Reflecting Graphs**:
  - To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the $x -$ axis.
  - To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the $y -$ axis.

- The table below summarizes the properties of the functions affected by reflection across the x-xis and y-axis.

	Reflection across x-axis	Reflection across y-axis
Domain	Not affected	Affected**
Range	Affected*	Not affected
Interval at which the function is strictly increasing	Affected*	Affected**
Interval at which the function is strictly decreasing	Affected*	Affected**
Interval at which the function is constant	Not affected	Affected**

*Can be unaffected if the function is symmetric across the x-axis since the resulting reflection will just be the original function.
**Can be unaffected if the function is symmetric across the y-axis since the resulting reflection will just be the original function.

References:

Albarico, J.M. (2013). THINK Framework. Based on *Science LINKS* by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Carter, J., Cuevas, G., Day, R., and Malloy, C., (2012). *Glencoe Geometry*. USA: The McGraw-Hill Companies, Inc.

International Geogebra Institute. (2020). *GeoGebra*. www.geogebra.org

Stewart, J., Redlin, L., Watson, S (2012). *Precalculus: Mathematics for Calculus*. Brooks/Cole, Cengage Learning.

Prepared by: Imari Joy C. Borda
Position: Special Science Teacher (SST) I
Campus: PSHS – SMC

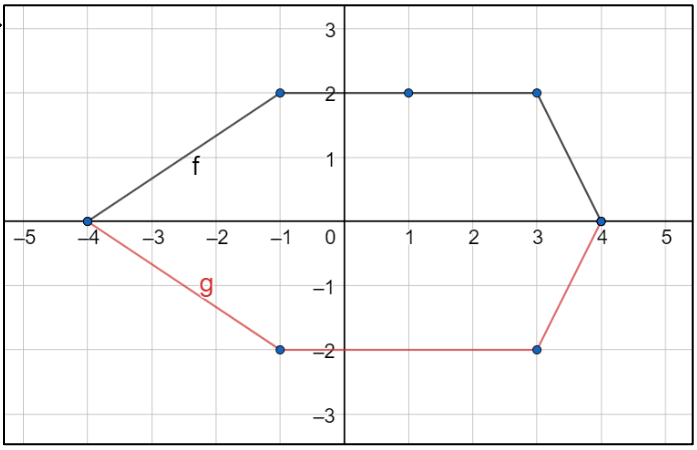
Reviewed by: Arvin Fajardo
Position: Special Science Teacher (SST) III
Campus: PSHS - CLC

Answer Key:

Navigate

1. a. The graph of $g(x)$ is the graph of $f(x)$ reflected across the y-axis.

2. a. The function $g(x)$ is the graph of $f(x)$ reflected across the x-axis.
   b.



   c.

	$f(x)$	$g(x)$
Interval at which the function is strictly increasing	$(-4-1)$	$(3,4)$
Interval at which the function is strictly decreasing	$(3,4)$	$(-4,-1)$
Interval at which the function is constant	$(-1,3)$	$(-1,3)$

   d. The intervals at which $g(x)$ is constant remains the same. On the other hand, the interval at which it is strictly increasing and the interval which it is strictly decreasing are swapped.

4. a. $g(x) = -\sqrt{x+3}$
   domain: $[-3, \infty)$
   range: $(-\infty, 0]$

5. a. $f(x) = -|x+3|$