Learning Guide Module

Subject Code Math 3	Mathematics 3
Module Code 5.0	*Other Types of Functions*
Lesson Code 5.2.3	*Piecewise-Defined Functions 3*
Time Frame	30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this learning guide, the students should be able to:

- Define floor, ceiling, and signum functions; and
- Graph floor, ceiling, and signum functions in the coordinate plane.

HOOK

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

From the previous learning guide, you have learned to graph a piecewise-defined function. Piecewise functions are functions defined by different formulas or rules that apply to different parts of the domain.

In this leaning guide, you will learn how to graph a special type of piecewise function called step functions. The graph of this type of function is a series of horizontal lines for different parts of the domain.

IGNITE

Time Allocation: 16 minutes
Actual Time Allocation: _____ minutes

Definition. The floor function $\lfloor x \rfloor$, also called the greatest integer function or greatest integer value, gives the largest integer less than or equal to x.

Example 1. Find the following values:

1. $\lfloor 0.5 \rfloor$
2. $\lfloor \sqrt{2} \rfloor$
3. $\lfloor 3.1212... \rfloor$
4. $\left\lfloor -\frac{4}{3} \right\rfloor$

Solution. From the definition, the floor of x, $\lfloor x \rfloor$, is the nearest integer less than or equal to x.

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Thus,

$\lfloor 0.5 \rfloor = 0$, since the nearest integer that is less than or equal 0.5 is 0.

$\lfloor \sqrt{2} \rfloor = 1$, since the nearest integer that is less than or equal $\sqrt{2} \approx 1.4142$ is 1.

$\lfloor 3.1212... \rfloor = 3$, since the nearest integer that is less than or equal 3.1212… is 3.

$\left\lfloor -\frac{4}{3} \right\rfloor = \lfloor -1.333... \rfloor = -2$, since the nearest integer that is less than or equal -1.333… is -2.

Example 2. Given the function $y = \lfloor x \rfloor$. Complete the table of values below.

x	*y*
0	
0.5	
0.6	
0.8	
0.99	

x	*y*
1	
1.2	
1.3	
1.8	
1.99	

x	*y*
2	
2.13	
2.5	
2.6	
2.99	

x	*y*
-1	
-0.6	
-0.7	
-0.8	
-0.99	

Note that $y = \lfloor x \rfloor$ is read as ‘ y is equal to the floor of x ’.

To complete the table of values, compute the values of y in the same manner as in Example 1. Look for the nearest integer that is less than or equal to the given value of x. Thus, we get,

x	*y*
0	0
0.5	0
0.6	0
0.8	0
0.99	0

x	*y*
1	1
1.2	1
1.3	1
1.8	1
1.99	1

x	*y*
2	2
2.13	2
2.5	2
2.6	2
2.99	2

x	*y*
-1	-1
-0.6	-1
-0.7	-1
-0.8	-1
-0.99	-1

Example 3. Graph $y = \lfloor x \rfloor$.

Using our result in Example 2, plot the points on the cartesian plane and we get the following graph,

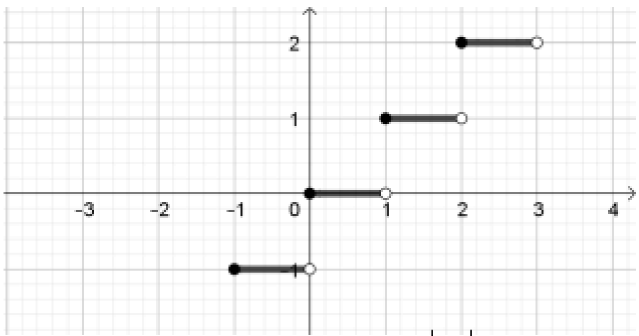


Figure 1: Graph of $y = \lfloor x \rfloor$

Note that a solid dot means "including" and an open (hollow) dot means "not including".
The floor function is sometimes called a 'step function' since it looks like an infinite staircase.

Definition. The ceiling function $\lceil x \rceil$ also called the smallest integer function or integer value, gives the smallest integer greater than or equal to x.

Example 4. Find the following values:

1. $\lceil 0.6 \rceil$
2. $\lceil \pi \rceil$
3. $\lceil -0.32 \rceil$
4. $\left\lceil -\frac{4}{3} \right\rceil$

From the definition, the ceiling of x, $\lceil x \rceil$, is the nearest integer greater than or equal to x. Thus,

$\lceil 0.6 \rceil = 1$, since the nearest integer greater than or equal to 0.6 is 1.

$\lceil \pi \rceil = 4$, since the nearest integer greater than or equal to $\pi \approx 3.1416$ is 4.

$\lceil -0.32 \rceil = 0$, since the nearest integer greater than or equal to -0.32 is 0.

$\left\lceil -\frac{4}{3} \right\rceil = \lceil -1.33... \rceil = -1$, since the nearest integer greater than or equal to -1.33… is -1.

Example 5. Given the function $y = \lceil x \rceil$. Complete the table of values below.

x	y
0.5	
0.6	
0.8	
0.99	
1	

x	y
1.2	
1.3	
1.8	
1.99	
2	

x	y
2.13	
2.5	
2.6	
2.99	
3	

x	y
-0.6	
-0.7	
-0.8	
-0.99	
0	

Solution. To complete the table of values, compute the values of y in the same manner as in Example 4. Look for the nearest integer that is greater than or equal to the given value of x. Thus, we get,

x	y
0.5	1
0.6	1
0.8	1
0.99	1
1	1

x	y
1.2	2
1.3	2
1.8	2
1.99	2
2	2

x	y
2.13	3
2.5	3
2.6	3
2.99	3
3	3

x	y
-0.6	0
-0.7	0
-0.8	0
-0.99	0
0	0

Example 6. Graph $y = \lceil x \rceil$.

Using our result in Example 5, plot the points on the cartesian plane and we get the graph below.

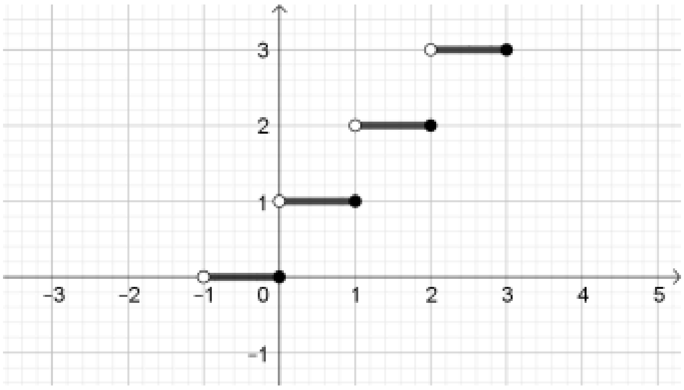


Figure 2: Graph of $y = \lceil x \rceil$

Note that a solid dot means "including" and an open dot means "not including".

Example 7. Graph $y = \left\lceil \frac{1}{3}x + 1 \right\rceil$.

Let us graph this function by identifying some ordered pairs in the table below.

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5
y	0	1	1	1	1	1	1	2	2	2	2	2	2	3

From this table, we can observe that if x is in the interval $(-3,0]$, $y = 1$. Also, if x is in the interval $(0,3]$, $y = 2$. Plotting these points on the coordinate plane, we'll get the graph below.

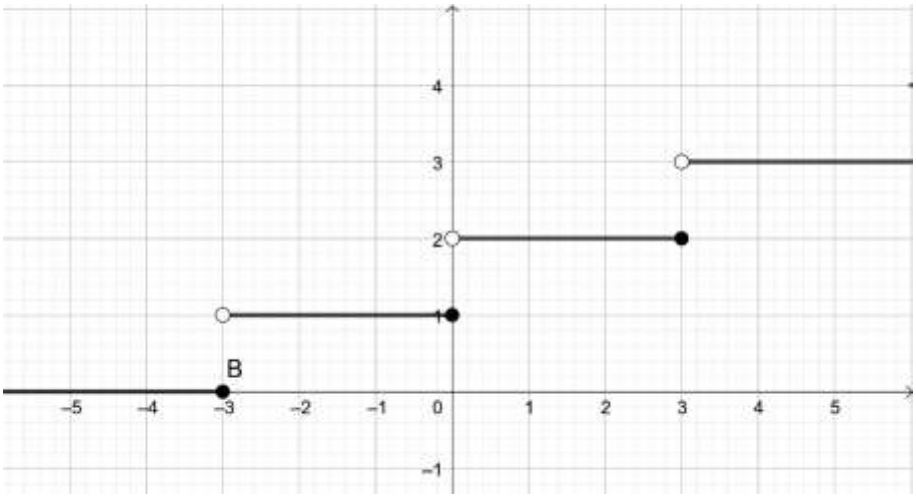


Figure 3: Graph of $y = \left\lceil \frac{1}{3}x + 1 \right\rceil$

Definition. The sign function (also called *sgn* or **signum** function), denoted by $\operatorname{sgn}(x)$, is defined as follows:

$$\operatorname{sgn}(x) = \begin{cases} -1 & if \quad x < 0 \\ 0 & if \quad x = 0 \\ 1 & if \quad x > 0 \end{cases}$$

Notice that we only have three possible output for x, i.e., -1, 0, and 1, which are all constants. If $x < 0$, the output is -1. If $x = 0$, the output is 0. If $x > 0$, the output is 1. Thus, our resulting graph is,

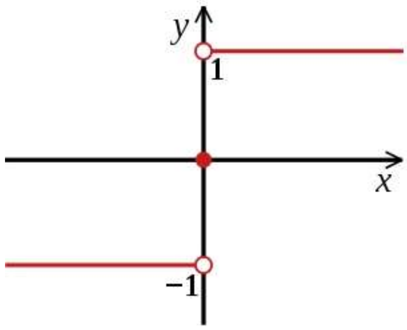


Figure 4: Graph of Signum Function
Retrieved from https://calculushowto.com/sign-function/

Example 8. Graph $y = sgn(x - 3)$.

To create a table of ordered pairs, let us identify three values of x. Let us think of an $x -$ value that will make the expression inside the parentheses equal to zero. Then, get two more numbers greater than and less than this value.

x	2	3	4
y	-1	0	1

The value of x that will make $x - 3$ equal to zero is 3. For x values greater than 3, the sign of $x - 3$ is positive, therefore, $y = 1$ for these x values. For x values less than 3, the sign of $x - 3$ is negative, hence, $y = -1$ for these x values. Graphing the function, we'll have

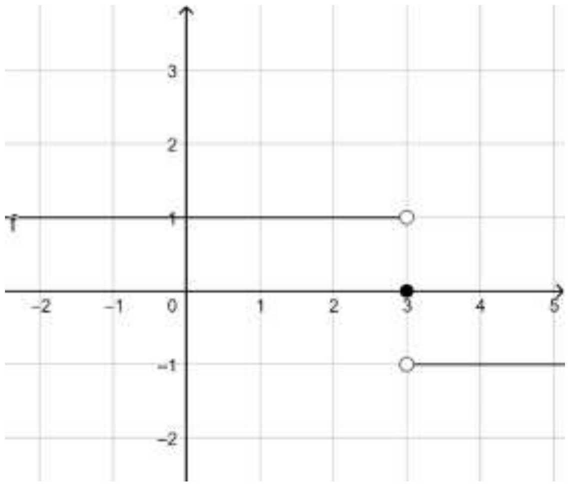


Figure 5: Graph of $y = sgn(x - 3)$

NAVIGATE

Time Allocation: 10 minutes
Actual Time Allocation: _____ minutes

Note: Items marked with an asterisk (*) will be graded.

I. Find the following values:

1. $\lfloor -3.2 \rfloor$
2. $\lfloor \pi \rfloor$ *
3. $\lfloor 1.999 \rfloor$
4. $\lceil -1.87 \rceil$ *
5. $\lceil e \rceil$
6. $\lceil 4.45 \rceil$ *

II. Graph the following functions:

7. $f(x) = \lfloor x - 3 \rfloor$
8. $g(x) = \left\lceil \frac{1}{2} x \right\rceil$ *
9. $h(x) = sgn(4x + 2)$ *

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

In summary, here are the things we need to remember about floor, ceiling and signum Functions:

- The floor function will round any number down to the nearest integer and the ceiling function will round any number up to the nearest integer.
- The sign function (or signum function) is a special function which gives us three possible outputs: -1 if $x < 0$, 1 if $x > 0$, and 0 if $x = 0$.
- It is a *real-valued step function* that tells us, numerically, whether a particular value of x is positive, negative, or zero.

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Quezon City: Rex Bookstore Inc
2. Pierce, Rod. (20 Jan 2018). "Floor and Ceiling Functions". Math Is Fun. Retrieved 20 Sep 2020 from http://www.mathsisfun.com/sets/function-floor-ceiling.html
3. Retrieved September 20, 2020 from https://calculushowto.com/sign-function/

Prepared by: Methos Kristy V. Donesa
Position: Special Science Teacher (SST) III
Campus: PSHS-SRC

Reviewed by: Arvin C. Fajardo
Position: Special Science Teacher (SST) III
Campus: PSHS-CLC

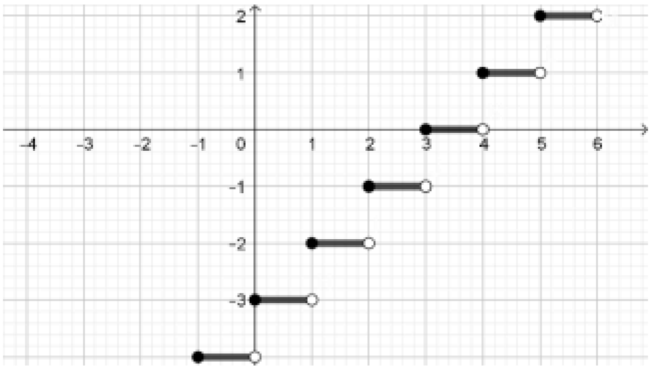
Answer Key:

I.

1. -4
3. 1
5. 3

II.

7)



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