Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	5.0	*Other Types of Functions*
Lesson Code	5.2.1	*Piecewise-Defined Function 1*
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, the students should be able to:

- define piecewise-defined function;
- evaluate a piecewise-defined function; and
- determine the properties of piecewise-defined functions (domain and range; constant, increasing, decreasing on intervals).

HOOK

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

In the previous learning guide, you have learned how to evaluate the linear, polynomial, rational and square root functions and determine some of its properties.

In this learning guide, you will learn how to evaluate a function that is defined by different types of functions where each function has different domain restrictions.

IGNITE

Time Allocation: 16 minutes
Actual Time Allocation: _____ minutes

Recall that we define the absolute value function by $f(x) = |x|$ for all $x \in \Re$. Another way to describe the absolute value function is given by

$$y = f(x) = \begin{cases} x & if\, x \geq 0 \\ -x & if\, x \leq 0 \end{cases}$$

This method defines the absolute function piecewisely on $\Re$.

Definition. A piecewise function is a function in which more than one formula is used to define the output. Each formula has its own domain, and the domain of the function is the union of all these smaller domains. We notate this idea like this:

$$f(x) = \begin{cases} \text{formula 1,} & \text{if x is in domain 1} \\ \text{formula 2,} & \text{if x is in domain 2} \\ \text{formula 3,} & \text{if x is in domain 3} \end{cases}$$

Example 1. Consider the following piecewise function:

$$f(x)=\begin{cases} x+3 & if \quad x<-2 \\ |x| & if -2\le x\le 2 \\ x^2 & if \quad x>2 \end{cases}$$

Find the values of: $f(-7)$, $f(0)$, and $f(5)$.

Solution. From the given function above, it tells us which function to use depending on the value of x. Consider the following cases:

Case 1: If $x<-2$, we use $x+3$

Case 2: If $-2\le x\le 2$, we use $|x|$

Case 3: If $x>2$, we use x^2

For $f(-7)$, since $-7<-2$, we will use Case 1. Thus, $f(-7) = -7 + 3 = -4$.

For $f(0)$, since 0 is in $[-2,2]$, we will use Case 2. Thus, $f(0)=|0|=0$.

For $f(5)$, since $5>2$, we will use Case 3. Thus, $f(5)=5^2=25$.

Recall: Increasing, Decreasing, Constant

A function is increasing when the graph goes up as you travel along it from left to right.
A function is decreasing when the graph goes down as you travel along it from left to right.
A function is constant when the graph is a perfectly at horizontal line.

Example 2. Given the graph of a piecewise function below. Find the domain and range, and state the intervals where the function is increasing, decreasing, or constant.

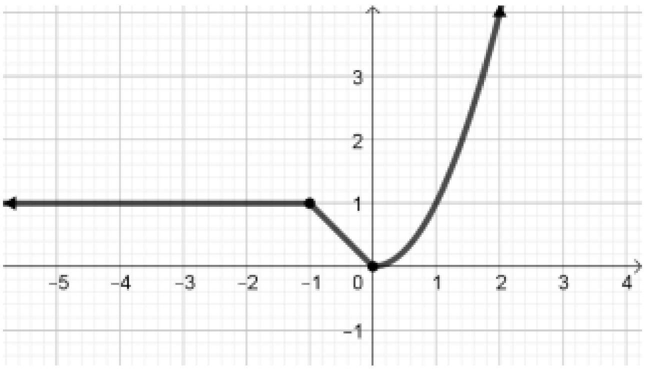


Figure 1. Graph of a Piecewise Function

Solution. Observe that the function is defined for all values of x. Thus the domain is the set of real numbers. Using set notation, we write it as $\{x \mid x \in \Re\}$ or $(-\infty,\infty)$ for the interval notation.

The output of this function (vertical direction) takes on the y-values $y\ge 0$.

Range: $[0,\infty)$

At what intervals is the function increasing, decreasing, or constant? Remember that the intervals correspond to the x-values. Thus, reading the graph from left to right, we see that the function is

- constant on the interval $(-\infty,-1]$.
- decreasing on the interval $[-1,0]$.
- increasing on the interval $[0,\infty)$.

Example 3. Given the graph of a piecewise function below. Find the domain and range, and state the intervals where the function is increasing, decreasing, or constant.

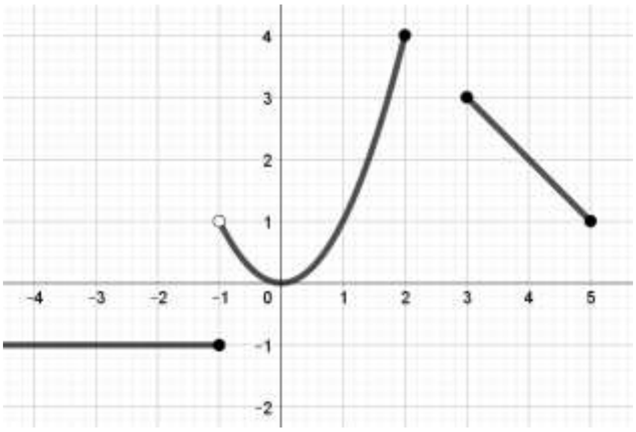


Figure 2

Solution:

In Figure 3, you can see the intervals where the points of the graph have x-values. To find the domain, we take the union of these intervals, i.e.,

$$(-\infty, -1] \cup (-1,2] \cup [3,5] = (-\infty, 2] \cup [3,5]$$

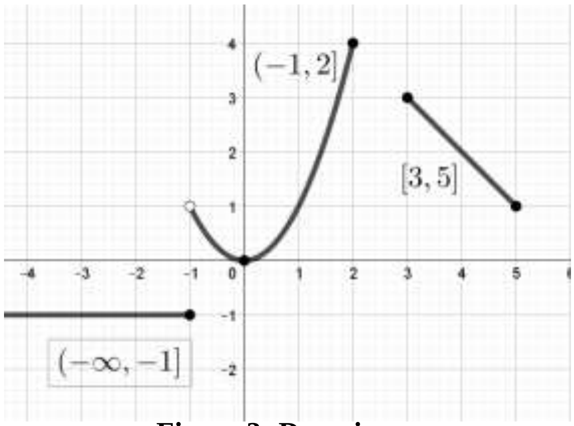


Figure 3: Domain

Therefore, the domain is

$$(-\infty, 2] \cup [3,5].$$

The range is the set of all y-values. From Figure 4, the range is

$$\{-1\} \cup [0,4].$$

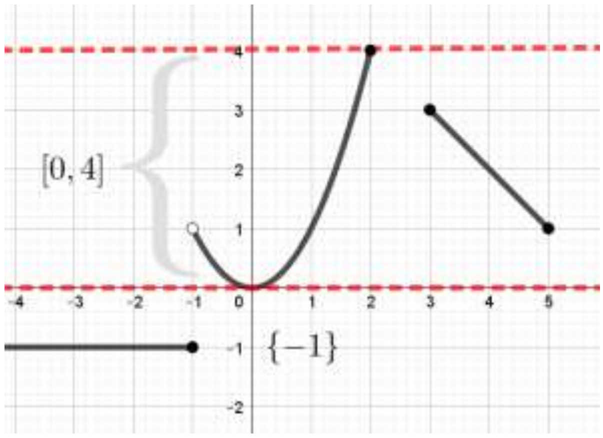


Figure 4: Range

From Figure 5, the green color indicates that the function is constant, the red color indicates that the function is decreasing, and the blue color indicates that the function is increasing.

Thus, reading the graph from left to right, we see that the function is

- constant on the interval $\left(-\infty,-1\right]$.
- decreasing on the interval $(-1,0]\cup[3,5]$.
- increasing on the interval $\left[0,2\right]$.

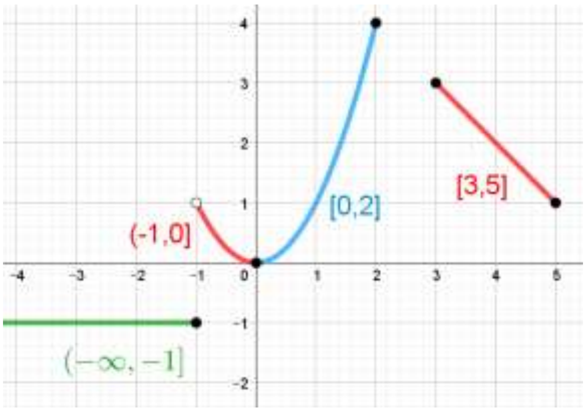


Figure 5: Increasing, Decreasing, Constant

NAVIGATE

Time Allocation: 10 minutes
Actual Time Allocation: _____ minutes

Note: Items marked with an asterisk (*) will be graded.

I. Evaluate the function for the given value of x.

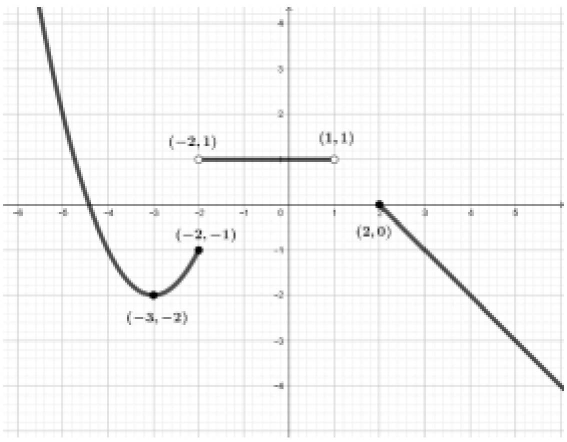
$$f(x)=\begin{cases} 4 & if \quad x<-1 \\ x+1 & if \ -1\le x\le 1 \\ (x-4)^2+1 & if \quad x>1 \end{cases}$$

$$g(x)=\begin{cases} \frac{1}{2}x & if \quad x<0 \\ 3-x & if \ 0\le x\le 3 \\ \sqrt{x} & if \quad x>3 \end{cases}$$

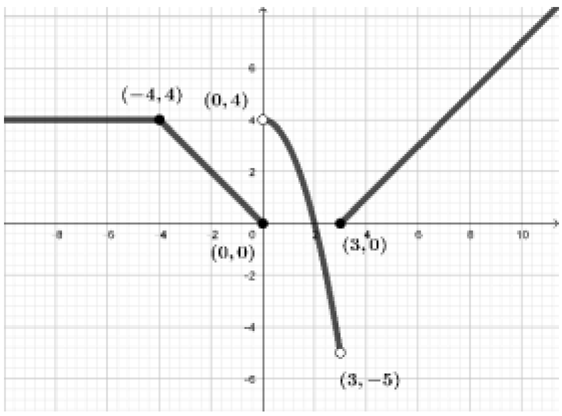
1. $f(-2)$
2. $f(-1)$ *
3. $f(0)$
4. $g(3)$ *
5. $g(8)$
6. $g(-1)$ *

II. Find the intervals where each function is increasing, decreasing and constant. Then, find the domain and range.

(1)



(2) *



(1)

Increasing: ____________________

Decreasing: ____________________

Constant: ______________________

Domain: _______________________

Range: ________________________

(2)

Increasing: ____________________

Decreasing: ____________________

Constant: ______________________

Domain: _______________________

Range: ________________________

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

In summary, here are the things you need to remember about Piecewise-Defined Functions.

- A piecewise-defined function is a function where a different formula or rule applies for different parts of the domain.
- The domain is the set of all x-values where the function is defined.
- The range is the set of all y-values that the graph of the function corresponds to.
- The function is increasing if the graph rises from left to right, decreasing if the graph falls from left to right, and constant if it is neither increasing nor decreasing.

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Quezon City: Rex Bookstore Inc
2. Young, C.(2010). *Algebra and Trigonometry*. USA:John Wiley & Sons, Inc.

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Answer Key:

I.
1. 4
3. 1
5. $2\sqrt{2}$

II.
1. Increasing: $(-3,-2)$
Decreasing: $(-\infty,-3)\cup(2,\infty)$
Constant: $(-2,1)$
Domain: $(-\infty,1)\cup[2,\infty)$
Range: $(-\infty,\infty)$