Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	6.0	*Other Types of Functions*
Lesson Code	6.3	*Transformation of Functions (Scaling)*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

By the end of this learning guide, the students will have been able to:

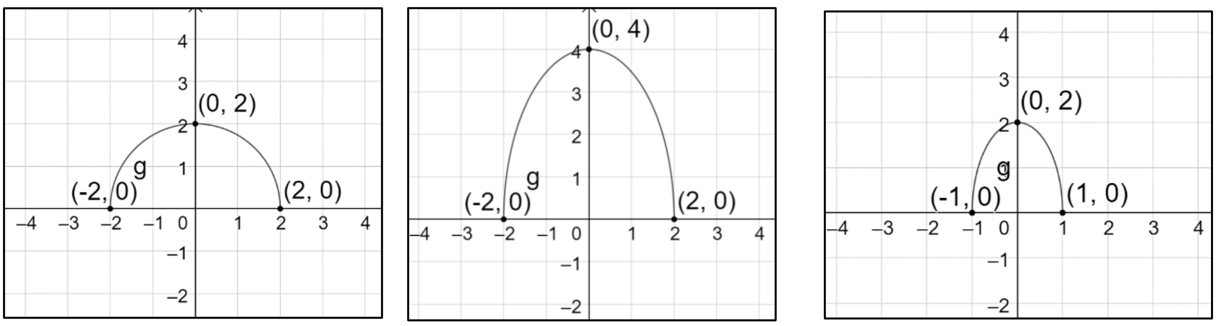
1. Understand the rule for scaling of functions.
2. Demonstrate scaling of functions using graphing tools and software.
3. Illustrate the scaling of functions on the coordinate plane.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: ______ minutes

From the previous lessons (Learning Guides 6.1 to 6.2), we learned how to translate and reflect functions. In this lesson, we will learn how to dilate functions. Recall from LG 2.5.1 that the scaling of a geometric figure changes its size. Specifically, the image is enlarged when the scale factor k is greater than one, and the image is reduced when the scale factor k is between 0 and 1. When $k = 0$, the original figure is retained.

In dilating functions, there are two types: vertical scaling and horizontal scaling. Consider the semicircle described by the function $f(x) = \sqrt{4 - x^2}$ with its graph shown below. Further, consider other semicircles as shown below.



$f(x) = \sqrt{4 - x^2}$ $\quad$ $g(x) = 2\sqrt{4 - x^2}$ $\quad$ $h(x) = \sqrt{4 - (2x)^2}$

Figure 1: Illustration of Scaling
Retrieved from: Glencoe Advanced Mathematical Concepts:
Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

Notice that the function $g(x)$ results from stretching the function $f(x)$ vertically while the function $h(x)$ results from horizontally compressing $f(x)$. We have just performed vertical and horizontal scaling. Consider the coordinate points of each graph. Did you notice anything similar to the rules of scaling from LG 2.5?

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Another kind of transformation of functions is scaling – which could either be done vertically or horizontally. Depending on the resulting function, scaling could also be classified as either a *stretch* or a *compress*. Let us explore and visualize scaling using GeoGebra.

Hands-on Activity using GeoGebra (Vertical Scaling)

1. Open your GeoGebra application or go to the link: https://www.geogebra.org/calculator
2. Graph the function $y = \sqrt{25 - x^2}$. Let us name this graph f.
3. Use the slider function of the GeoGebra and graph the function $y = a\sqrt{25 - x^2}$. Let us name this as graph g. Set its color to red. Your graph should now look like this:

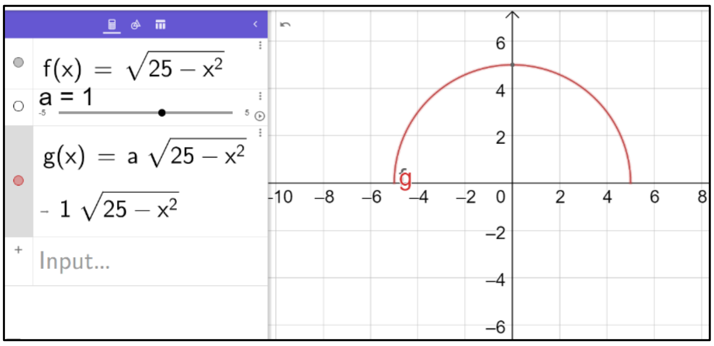


Figure 2: Functions f and g in vertical scaling hands-on activity

4. Explore the slider by dragging its point to the left or to the right. What do you notice as you change the value of a? More specifically, what do you notice when $0 < a < 1$? When $a > 1$? Or when $a = 1$? You might observe the following:
   a. When $a = 1$, the function g is equal to the function f.
   b. When $0 < a < 1$, the function g is compressed vertically.
   c. When $a > 1$, the function g is stretched vertically.
   Note: Notice that we did not consider the case when $a < 0$. This is because when a is negative, two transformations are now performed - aside from scaling the function, the graph is also reflected across the x-axis. We only consider scaling this time.
5. Let us take a closer look at vertical scaling by considering the case when $a = 2$. Drag your slider accordingly so that you have $a = 2$. Your graph should look like this:

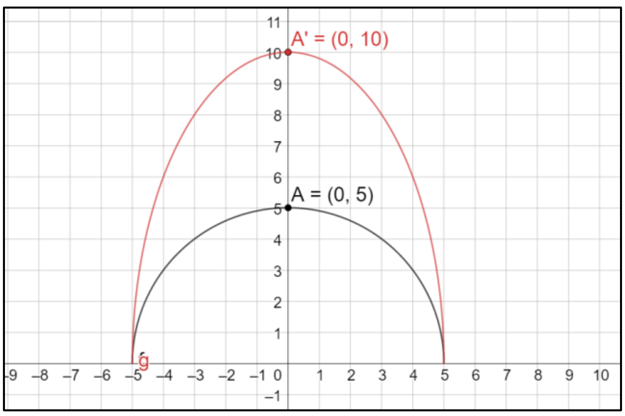


Figure 3: Close look at vertical scaling.

There are two observations that you should take note of:

1. Since $a = 2$, all function value (i.e. the y-coordinate of each point of the function) is multiplied by 2. This means that from the original point of (x, y), its corresponding point on the scaled graph is $(x, 2y)$. Notice that this was the case as shown by points $A(0,5)$ and $A'(0,10)$. In such a case, we say that $f(x)$ is vertically stretched by a factor of 2.
2. Only the y value is changed and the x value is retained. As a result, visually, only the height of the graph is changed while the width is retained.

Hands-on Activity using GeoGebra (Horizontal Scaling)

1. Open your GeoGebra application or go to the link: https://www.geogebra.org/calculator
2. Graph the function $y = \sqrt{25 - x^2}$. Let us name this graph f.
3. Use the slider function of the GeoGebra and graph the function $y = \sqrt{25 - (ax)^2}$. Let us name this graph g and set its color to red. Your graphshould now look like this:

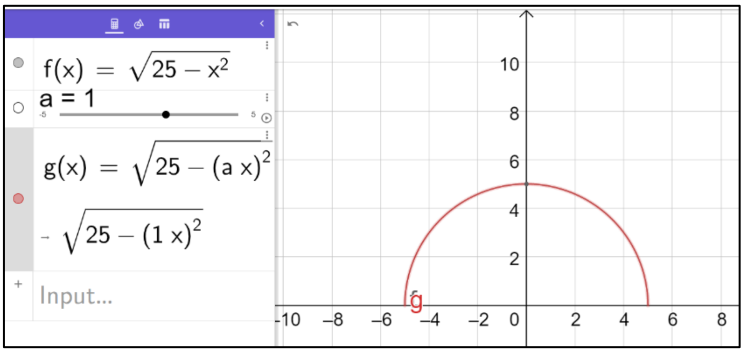


Figure 4: Functions f and g in horizontal scaling hands-on activity

4. Explore the slider by dragging its point to left or to right. What do you notice as you change the value of a? More specifically, what do you notice when $0 < a < 1$? When $a > 1$? Or when $a = 1$? You might observe the following:
   a. When $a = 1$, the function g is equal to the function f.
   b. When $0 < a < 1$, the function g is stretched horizontally.
   c. When $a > 1$, the function g is compressed horizontally.

   Note: Notice again that we did not consider the case when $a < 0$. This is because when a is negative, two transformations are now performed - aside from scaling the function, the graph is also reflected with respect to the x-axis. We only consider scaling this time.
5. Let us take a closer look at horizontal scaling by considering the case when $a = 2$. Drag your slider accordingly so that you have $a = 2$. Your graph should look like this:

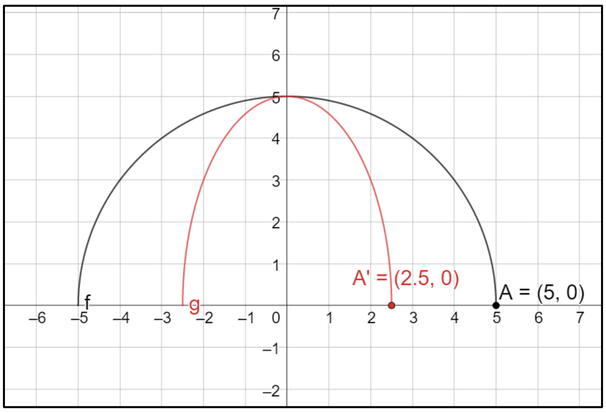


Figure 5: Close look at horizontal scaling.

There are two observations that you should take note of:

1. Since $a = 2$, all x-coordinate of each point of the function is multiplied by $\frac{1}{2}$. So, from the original point of (x, y) its corresponding point on the scaled graph is $\left(\frac{1}{2}x, y\right)$. Notice that this was the case as shown by points $A(5,0)$ and $A'(2.5,0)$. In such a case, we say that $f(x)$ is horizontally compressed by a factor of $\frac{1}{2}$.
2. Only the x value is changed and y value is retained. As a result, visually, only the width of the graph is changed while the height is retained.

Let us generalize all our observations by formally defining the vertical and horizontal compression and stretching.

In **Vertical Compression and Stretching**:
To graph $y = cf(x)$
- If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c.
- If $0 < c < 1$, compress the graph of $y = f(x)$ by a factor of c.

In **Horizontal Compression and Stretching**:
To graph $y = f(cx)$
- If $c > 1$, compress the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.
- If $0 < c < 1$, stretch the graph of $y = f(x)$ by a factor of $\frac{1}{c}$.

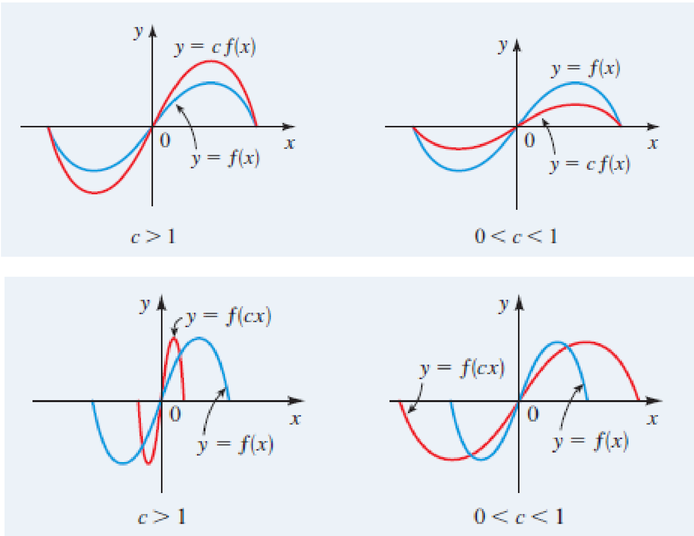


Figure 6: Vertical and Horizontal Scaling
Retrieved from: Glencoe Advanced Mathematical Concepts: Precalculus with applications by Woods, Holliday. Mcgraw-Hill Education 2003.

Example 1: Dilating a Parent Function. Use the parent function $f(x) = |x|$ to sketch the graph of each function.

a. $g(x) = 5|x|$
b. $g(x) = |0.5x|$

Answer: The function in (a) is the graph of $f(x) = |x|$ stretched vertically by a factor of 5. On the other hand, the function in (b) is the graph of $f(x) = |x|$ stretched horizontally by a factor of 2.

a. b.

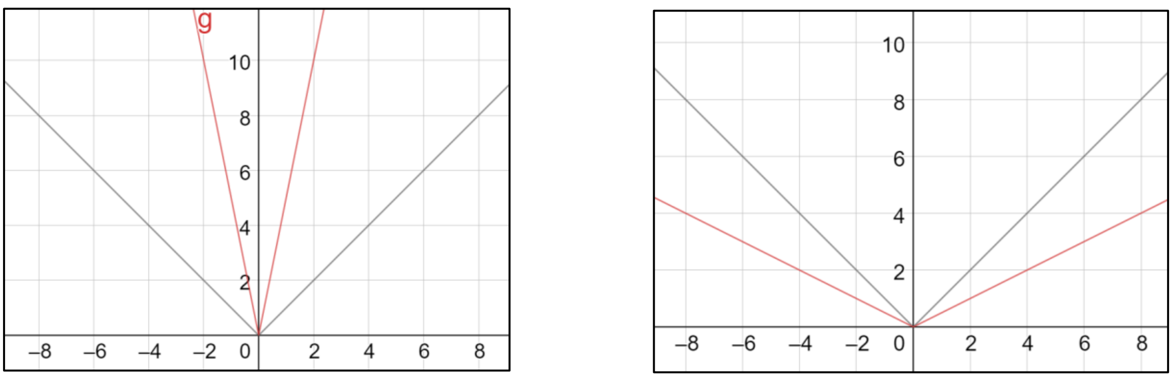


Figure 7: Functions $y = 5|x|$ and $y = |0.5x|$

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Example 2: Changes in the domain and range. Consider the function $f(x) = \sqrt{9 - x^2}$. Consider as well the semicircles $g(x) = 2\sqrt{9 - x^2}$ and $h(x) = \sqrt{9 - (2x)^2}$. Do the following:

a. Describe in words how the functions $g(x)$ and $h(x)$ are related to the function $f(x)$.
b. Graph functions $g(x)$ and $f(x)$ in one coordinate plane and functions $h(x)$ and $f(x)$ in another coordinate plane
c. Using the graphs in (b), complete the table below.
d. What do you observe on the changes in domain and range after vertical and horizontal scaling?

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-3,3]$		
Range	$[0,3]$		

Answer:

a. The function $g(x)$ is the graph of $f(x)$ stretched vertically at a factor of 2. The function $h(x)$ is the graph of $f(x)$ compressed horizontally at a factor of $\frac{1}{2}$.
b. The graphs are shown below.

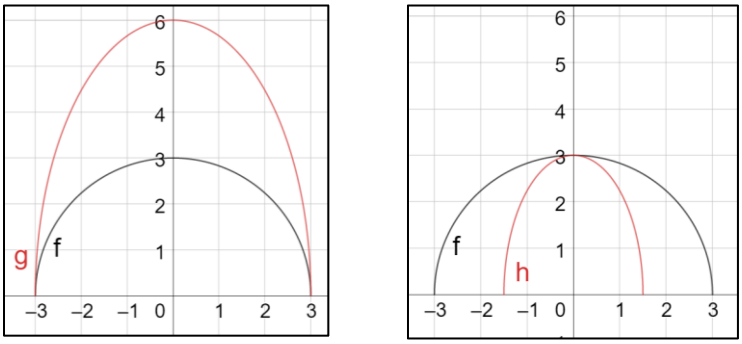


Figure 8: Functions $g(x) = 2\sqrt{9 - x^2}$ and $h(x) = \sqrt{9 - (2x)^2}$

c. The completed table is shown below.

	$f(x)$	$g(x)$	$h(x)$
Domain	$[-3,3]$	$[-3,3]$	$[-\frac{3}{2},\frac{3}{2}]$
Range	$[0,3]$	$[0,6]$	$[0,3]$

d. For function $g(x)$, <u>the domain is unchanged while the range is changed</u>. The resulting range is extended by twice the previous value as shown below.

Range of $g(x)$: $[0,3] \rightarrow [0(2), 3(2)] \rightarrow [0,6]$

For function $h(x)$, <u>the domain is changed while the range is unchanged</u>. The resulting domain is reduced by half of the previous value as shown below.

Domain of $h(x)$: $[-3,3] \rightarrow [-3\left(\frac{1}{2}\right), 3\left(\frac{1}{2}\right)] \rightarrow [-\frac{3}{2},\frac{3}{2}]$

Note: Any horizontal scaling will affect the domain and leave the range unchanged. On the other hand, any vertical scaling will affect the range and leave the domain unchanged.

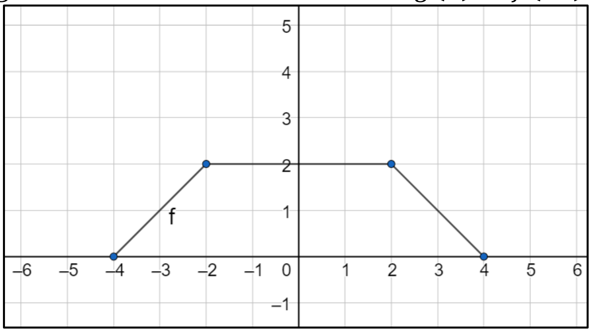
NAVIGATE

Time Allocation: 9 minutes
Actual Time Allocation: _____ minutes

Answer the following questions. Items marked with an asterisk (*) will be graded.

1. Describe how the graph of g is obtained from the graph of f.
   a. $f(x) = |x| + 1;$ $g(x) = \frac{1}{4}(|x| + 1)$
   b. $*f(x) = |x| + 1;$ $g(x) = |\sqrt{2}\,x| + 1$
   c. $f(x) = |x| + 1;$ $g(x) = |0.25x| + 1$
   d. $*f(x) = |x| + 1;$ $g(x) = 4|x| + 4$

2. The graph of $f(x)$ is given below. Consider the function $g(x) = f(2x)$ and do the following:



   a. Describe in words how the $g(x)$ is related to the function $f(x)$.
   b. Graph $g(x)$ in the same coordinate plane as $f(x)$.
   c. Complete the table below.

	$f(x)$	$g(x)$
Interval at which the function is strictly increasing		
Interval at which the function is strictly decreasing		
Interval at which the function is constant		

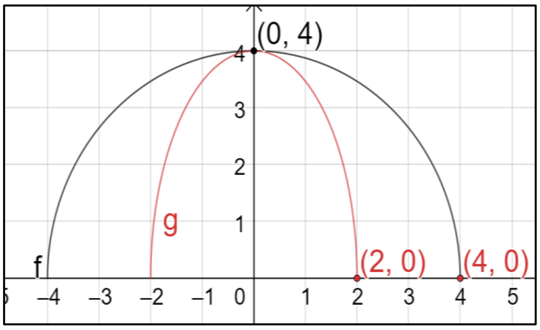
   d. What are your observations?

3. *Using the same function $f(x)$ in item (3), consider the function $g(x) = 3f(x)$ and repeat steps a to d.

4. Write a formula for the function g that results when the graph of $f(x)$ is transformed as described (simplify your answer as necessary). Without solving, what is the domain and range of g?
   a. The graph of $f(x) = \sqrt{x}$ is compressed horizontally by a factor of $\frac{1}{3}$.
   b. *The graph of $f(x) = x^2$ is stretched vertically by a factor of 3.

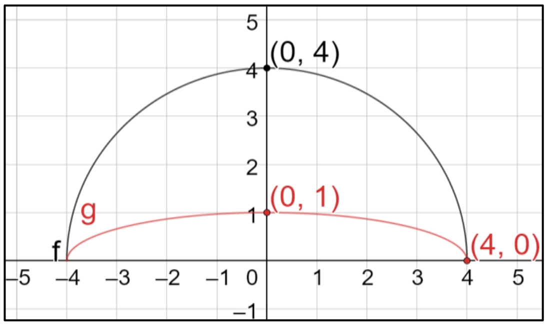
5. For the following exercises, the graphs of f and g are given. Find a formula for the function g from the function of f. Simplify your answer. Note that the graph in black is the function of f. You may verify your answer by graphing it using GeoGebra.

a.



$f(x) = \sqrt{16 - x^2}$

*b.



$f(x) = \sqrt{16 - x^2}$

KNOT

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

In **Vertical Compression and Stretching**:
To graph $y = cf(x)$

- If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c.
- If $0 < c < 1$, compress the graph of $y = f(x)$ by a factor of c.

In **Horizontal Compression and Stretching**:
To graph $y = f(cx)$

- If $c > 1$, compress the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.
- If $0 < c < 1$, stretch the graph of $y = f(x)$ by a factor of $\frac{1}{c}$.

The table below summarizes the properties of the functions affected by vertical and horizontal scaling.

	Vertical Scaling	Horizontal Scaling
Domain	Not affected	Affected
Range	Affected	Not affected
Interval at which the function is strictly increasing	Not affected	Affected
Interval at which the function is strictly decreasing	Not affected	Affected
Interval at which the function is constant	Not affected	Affected

References:

Albarico, J.M. (2013). THINK Framework. Based on *Science LINKS* by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Carter, J., Cuevas, G., Day, R., and Malloy, C., (2012). *Glencoe Geometry*. USA: The McGraw-Hill Companies, Inc.

International Geogebra Institute. (2020). *GeoGebra.* www.geogebra.org

Stewart, J., Redlin, L., Watson, S (2012). *Precalculus: Mathematics for Calculus*. Brooks/Cole, Cengage Learning.

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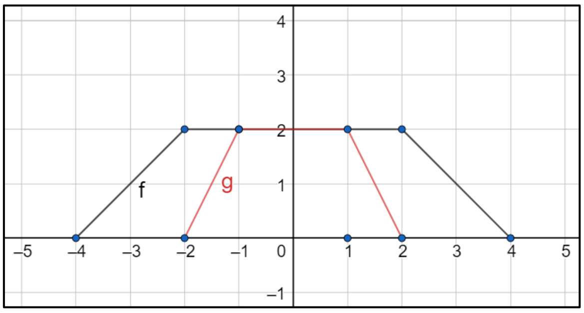
Answer Key:

Navigate

1. a. The graph of $g(x)$ is the graph of $f(x)$ compressed vertically by a factor of $\frac{1}{4}$.
   c. The graph of $g(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 4.

2. a. The function $g(x)$ is the graph of $f(x)$ compressed horizontally by a factor of $\frac{1}{2}$ units.
   b .



   c.

	$f(x)$	$g(x)$
Interval at which the function is strictly increasing	$(-4-2)$	$(-2,-1)$
Interval at which the function is strictly decreasing	$(2{,}4)$	$(1{,}2)$
Interval at which the function is constant	$(-2{,}2)$	$(-1{,}1)$

   d. The intervals at which $g(x)$ is strictly increasing, strictly decreasing, and constant are the corresponding intervals of $f(x)$ reduced by $\frac{1}{2}$.

4. a. $g(x) = \sqrt{3x}$
   domain: $[0, \infty)$
   range: $[0, \infty)$

5. a. $f(x) = \sqrt{16 - 4x^2}$

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