Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	5.0	*Other Types of Functions*
Lesson Code	5.2.2	*Piecewise-Defined Functions 2*
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this learning guide, the students should be able to:

- graph a piecewise-defined function.

HOOK

Time Allocation: 3 minutes
Actual Time Allocation: _____ minutes

In learning guide 5.2.1, we have discussed how to evaluate a piecewise-defined function and determine some of its properties. In this learning guide, we will tackle how to graph a function that is defined by pieces of different functions over intervals and all in one graph just like in the figure shown below.

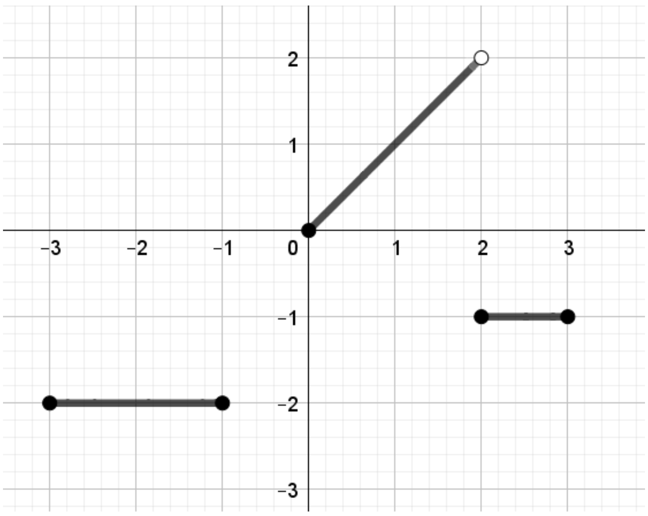


Figure 1

We use piecewise functions to describe situations in which a rule or relationship changes as the input value crosses certain “boundaries.” For example, we often encounter situations in business for which the cost per piece of a certain item is discounted once the number of pieces ordered exceeds a certain value. Tax brackets are another real-world example of piecewise functions.

IGNITE

Time Allocation: 12 minutes
Actual Time Allocation: _____ minutes

A piecewise-defined function is defined in terms of pieces of other functions. Thus, we draw the graph of each individual function and, then, for each function darken the piece corresponding to its part of the domain.

Let us discuss how to graph piecewise functions with some examples.

Example 1. Graph the piecewise function

$$f(x)=\begin{cases}-x-1 & if\ x<-2\\ x+1 & if\ x\geq -2\end{cases}$$

Solution. To graph a piecewise function, we will look into the given regions of the domain. From the given function, it would be:

$$x<-2$$
$$x\geq -2$$

From here, we can see that $x=-2$ is the point to focus on since this is where the pieces of the graph change over. On your graphing paper, draw a vertical dotted line for $x = -2$. Then draw all the functions given. Draw them very lightly with a pencil. You will have two or more functions which may cross but don't worry about that.

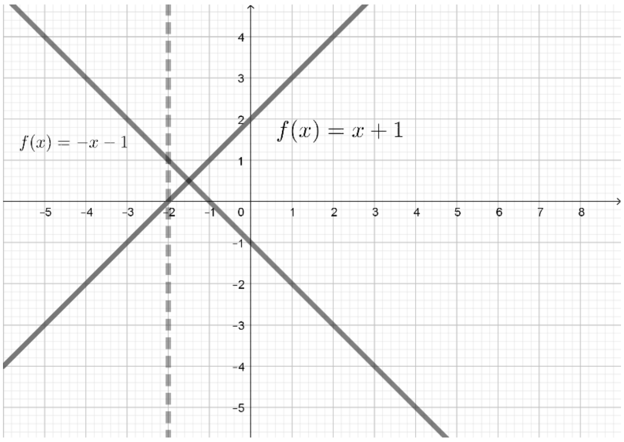


Figure 2

Now let's investigate each piece. When $x<-2$, the function is defined by $f(x)=-x-1$, so darken/color that particular function to the left of $x=-2$. Take note that there is an open hole at $x=-2$ since the discontinuity exists at this point. A *discontinuity* occurs when a gap or hole appears in the graph. The open hole indicates up to but not including $x = -2$. When $x\geq -2$, the function is defined by $f(x)=x+1$, so darken that particular function to the right of $x=-2$.

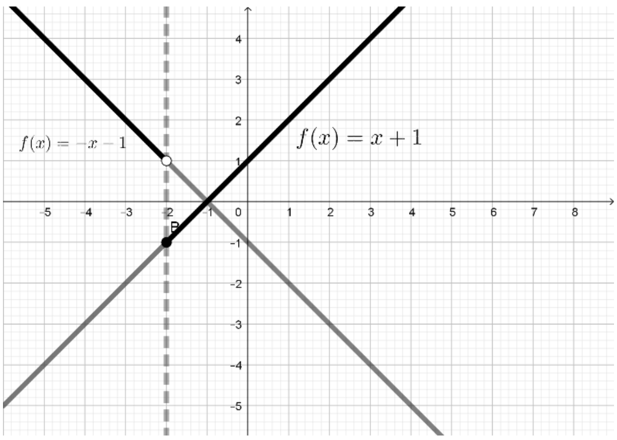


Figure 3

Below is our final graph.

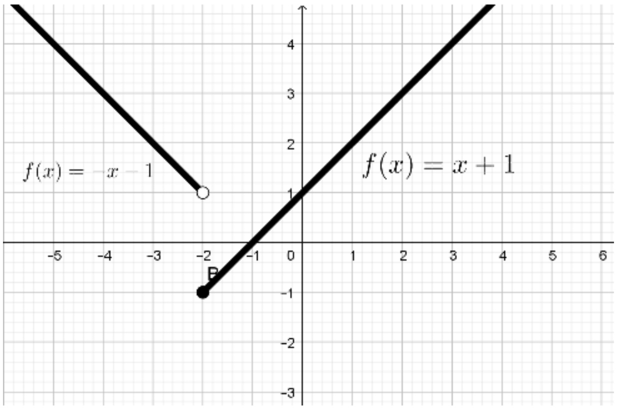


Figure 4

The domain of this function is $(-\infty, \infty)$ while the range is $[-1, \infty)$.

Example 2. Let f be the function defined by

$$f(x) = \begin{cases} -x+2 & if \quad x < -1 \\ x^2 & if\ -1 \le x \le 2 \\ \sqrt{x-2} & if\ x > 2 \end{cases}$$

Solution.
To graph $f(x)$, we will break it up into several functions that make it up. Thus, we get,

$$f(x) = -x + 2$$
$$f(x) = x^2$$
$$f(x) = \sqrt{x-2}$$

Figure 5 shows the graph of each functions.

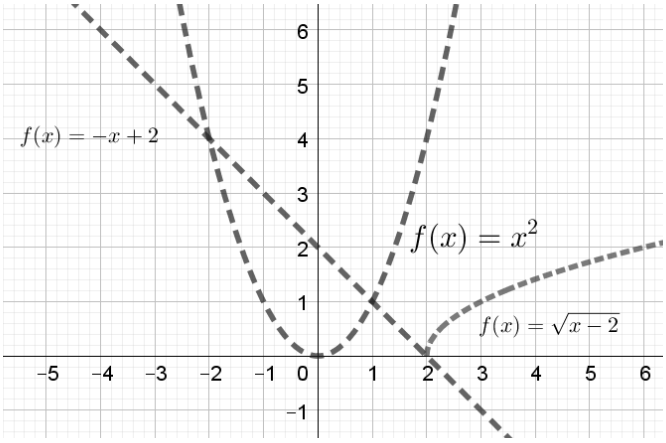


Figure 5

The points to focus on in particular are the values where the pieces change over -- that is, $x = -1$ and $x = 2$.

Now let's investigate each piece. When $x < -1$, the function is defined by $f(x) = -x + 2$, so darken that particular function to the left of $x = -1$. When $-1 \le x \le 2$, the function is defined by $f(x) = x^2$, so darken that particular function from $x = -1$ to $x = 2$. When $x > 2$, the function is defined by

$f(x)=\sqrt{x-2}$, so darken that particular function to the right of $x=2$. Note that there is an open circle at $(-1,3)$ for $f(x)=-x+2$ and at $(2,0)$ for $f(x)=\sqrt{x-2}$ since the intervals $x<-1$ and $x>2$ does not include -1 and 2. Thus, our resulting graph is

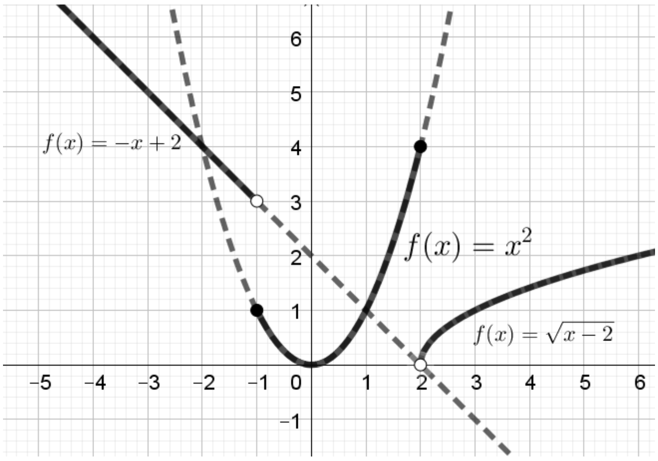


Figure 6

Figure 7 displays our final graph.

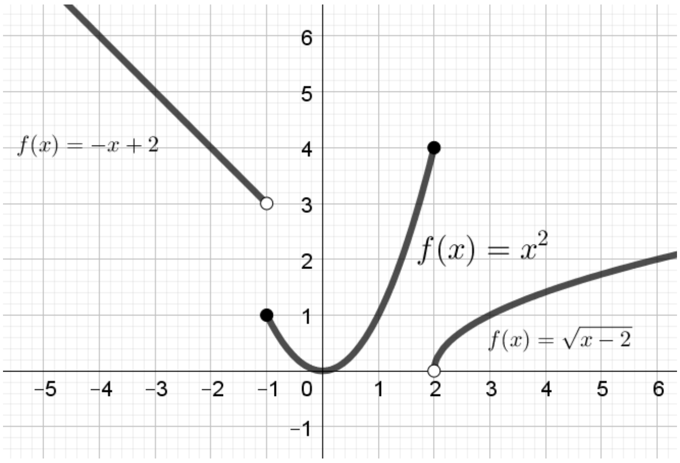


Figure 7

The domain of this piecewise function is $(-\infty,\infty)$ while the range is $[0,\infty)$.

NAVIGATE

Time Allocation: 12 minutes
Actual Time Allocation: _____ minutes

Note: Items marked with an asterisk (*) will be graded.

I. Match the piecewise function with its graph.

$$1. f(x)=\begin{cases} x^2 & \text{if } x\le 0 \\ x+2 & \text{if } x>0 \end{cases}$$

$$*2. f(x)=\begin{cases} 0 & \text{if } x<0 \\ x^2 & \text{if } x\ge 0 \end{cases}$$

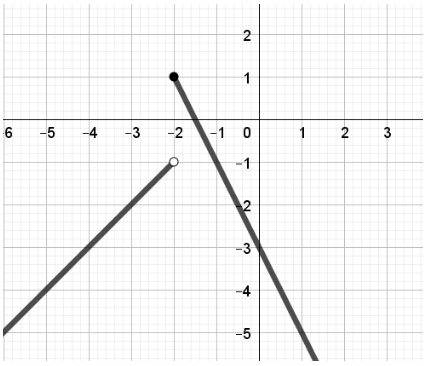
$$3. f(x)=\begin{cases} x+1 & \text{if } x<1 \\ x^3 & \text{if } x\ge 1 \end{cases}$$

$$*4. f(x)=\begin{cases} x+1 & \text{if } x<-2 \\ -2x-3 & \text{if } x\ge -2 \end{cases}$$

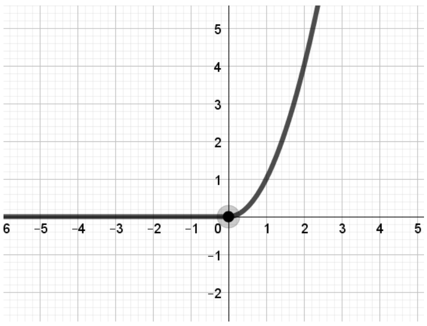
$$5. f(x)=\begin{cases} |x| & \text{if } x\le 2 \\ 1 & \text{if } 2<x<4 \\ x-2 & \text{if } 4\le x\le 6 \end{cases}$$

$$*6. f(x)=\begin{cases} 2x-1 & \text{if } x<1 \\ |x| & \text{if } 1\le x\le 2 \\ 2 & \text{if } x>2 \end{cases}$$

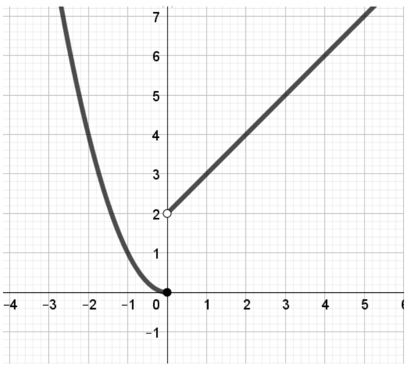
a)



b)



c)



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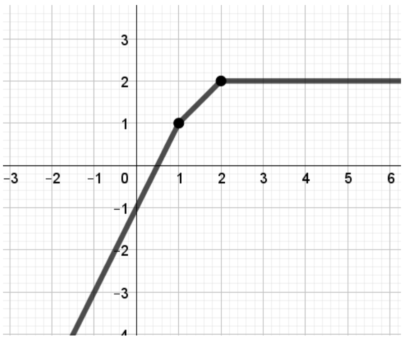
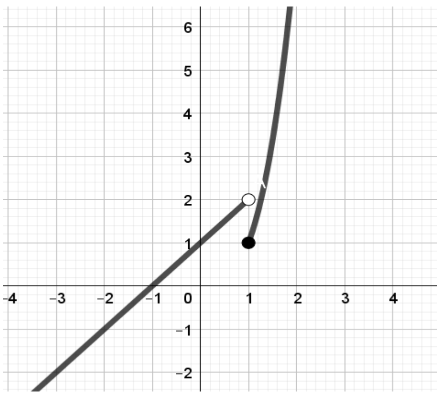
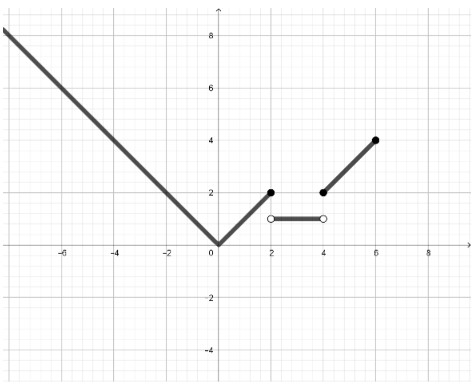


e)



f)

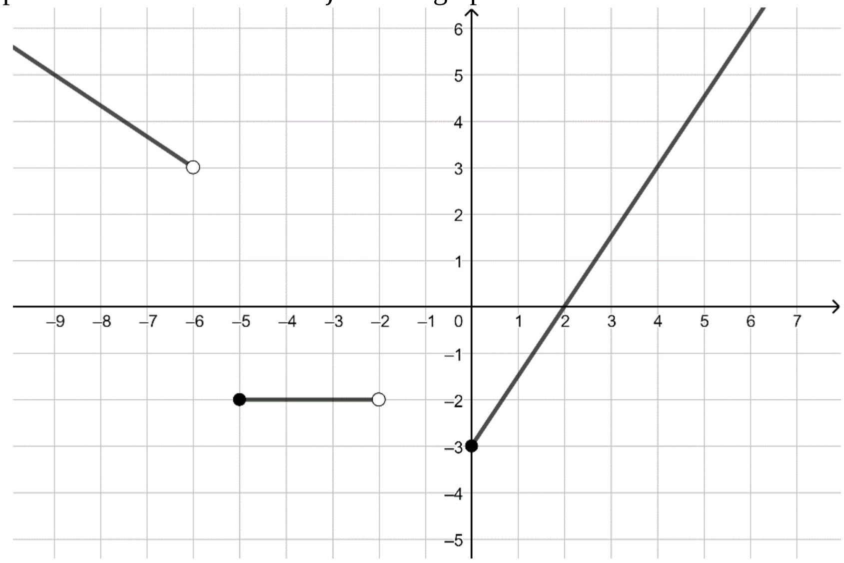


II. Graph the following piecewise functions.

$$1.\ f(x)=\begin{cases} -x & if \quad x\le -1 \\ 2 & if\ -1<x<1 \\ x & if \quad x>1 \end{cases} \qquad\qquad *\,2.\ g(x)=\begin{cases} -2x & if \quad x\le -1 \\ x^3-2x+2 & if\ -1<x<2 \\ 6 & if\ \ 2<x<5 \end{cases}$$

III. Do as indicated.

1. Find a piecewise defined function f whose graph is shown below.



* 2. Create a piecewise defined function that contains the points $(-1,2)$, $(2,4)$ and $(-3,-5)$.

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: ______ minutes

In summary, here are the things you need to remember in graphing a Piecewise-Defined Functions.

- A piecewise function is defined in terms of pieces of other functions. Thus, we draw the graph of these pieces of functions, then darken the graph of each function corresponding to its part of the domain.
- A solid circle on the left or right end of a graph indicates that the graph terminates there and the point is included in the graph.
- An open circle indicates that the graph terminates there and the point is not included in the graph.
- A graph is discontinuous when it has holes and/or gaps.

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Quezon City: Rex Bookstore Inc
2. Young, C.(2010). *Algebra and Trigonometry*. USA:John Wiley & Sons, Inc.

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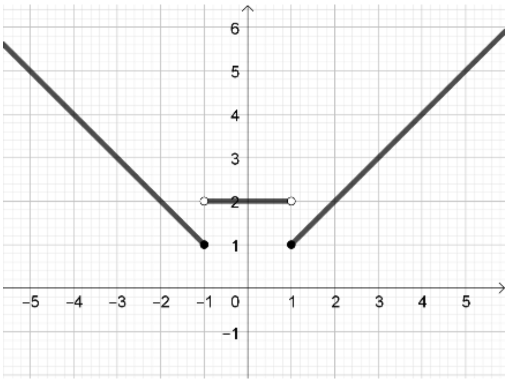
Answer Key:

I.
1. c 3. e 5. f

II.
1.



III.

1. $$y = \begin{cases} -\frac{2}{3}x - 1 & if\ x < -5 \\ -2 & if -5 \le x < -2 \\ \frac{3}{2}x - 3 & if\ x \ge 0 \end{cases}$$