



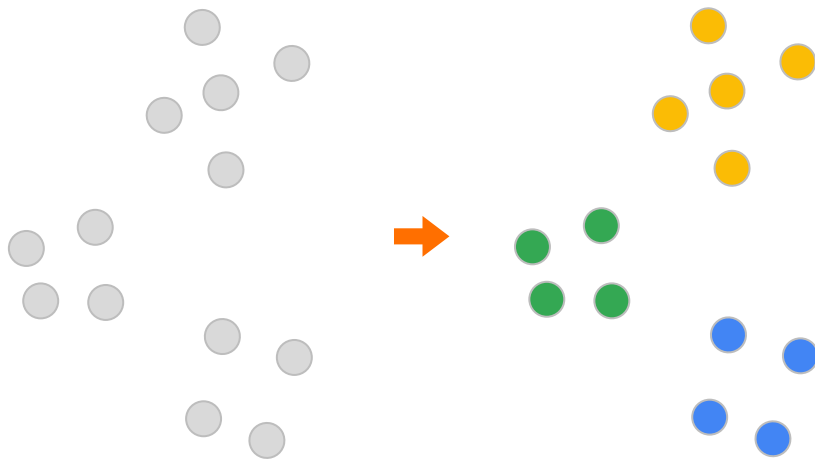
Learning to Cluster using Neural Clustering Process

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Clustering

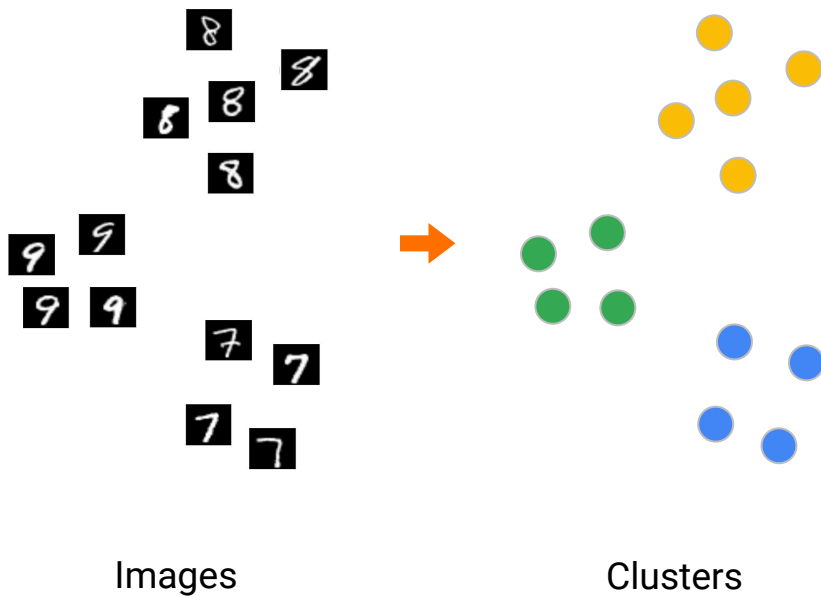


Input Data

Clusters

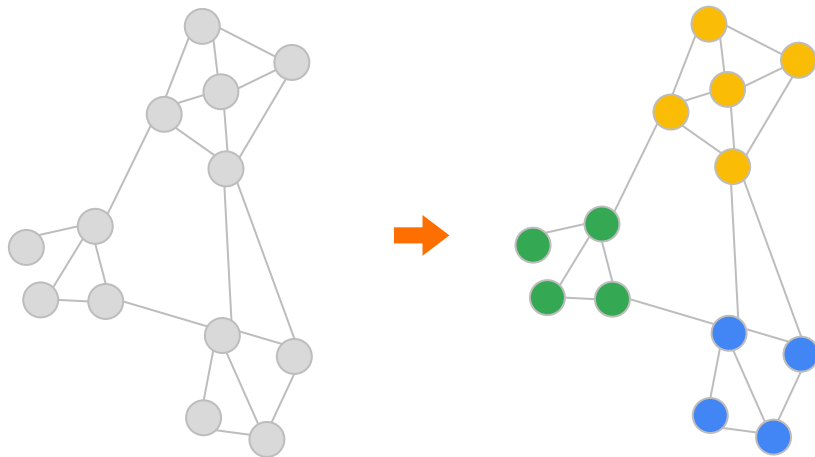


Image Clustering





Graph Clustering



Graph Nodes

Clusters



Framework for Neural Clustering

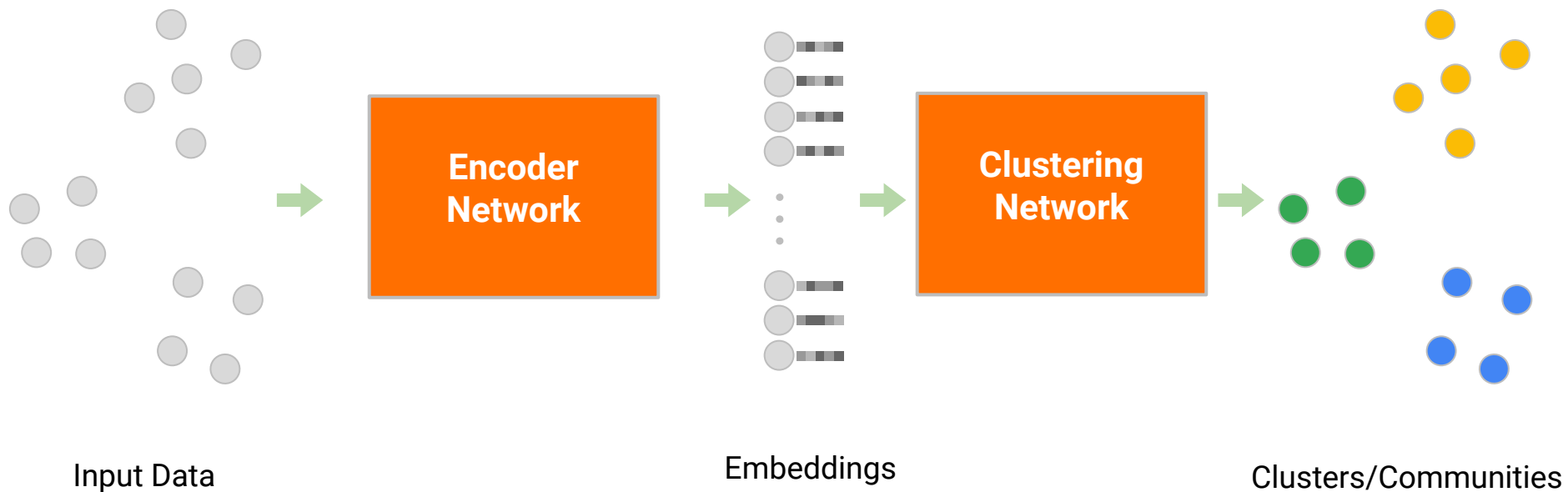
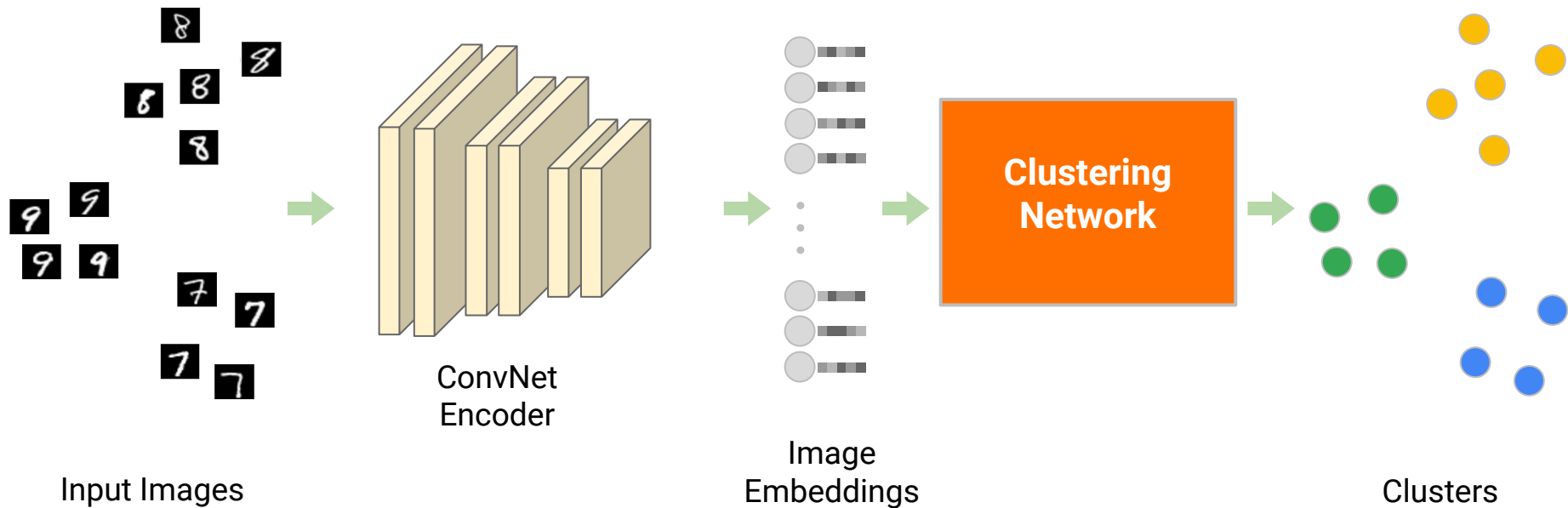


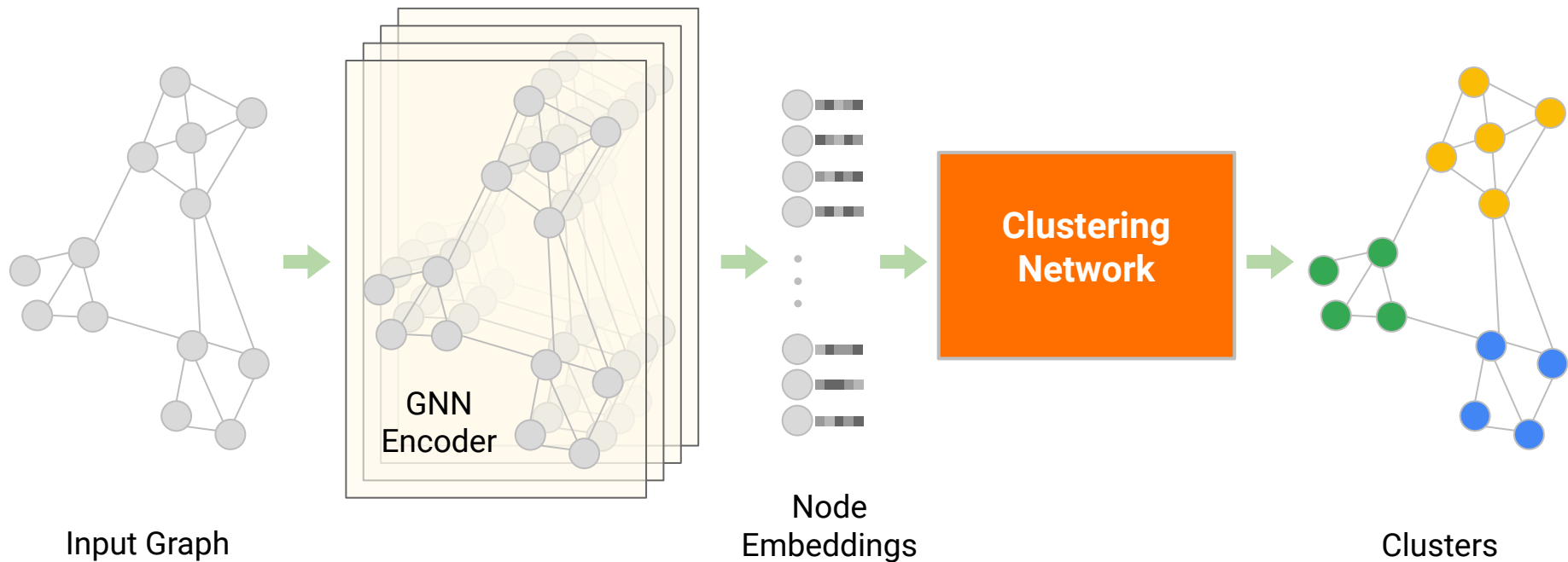


Image Clustering





Graph Clustering

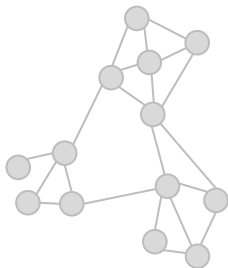




How to Cluster the Embeddings

No labeled data

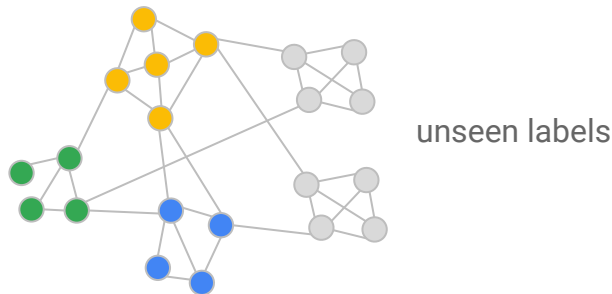
Unsupervised clustering



Not always agree with human labels

Have labeled data

Train a classifier?



Cannot generalize to unseen labels/clusters

Goal: A model that learns the clustering objective from labeled data.



Learning to Cluster using Neural Networks

- Amortized/Supervised Clustering
 - Fast inference on many statistically similar datasets after training
- Encoding prior knowledge into a trained model
 - Specify complex clustering objectives by labeled training data
- End-To-End model to cluster many types of data
 - ConvNet encoder for images, GNN encoder for graphs, etc

Neural Clustering Process (NCP)

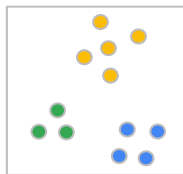
- Handles arbitrary numbers of clusters at test time
- Probabilistic clustering with well-defined posterior



Challenges in Neural Clustering

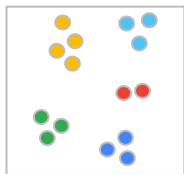
Challenges:

Unknown number of clusters at test time



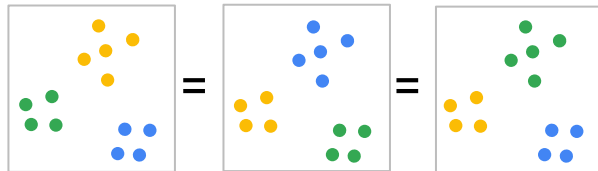
3 clusters

vs.



5 clusters

Permutation symmetry between cluster labels



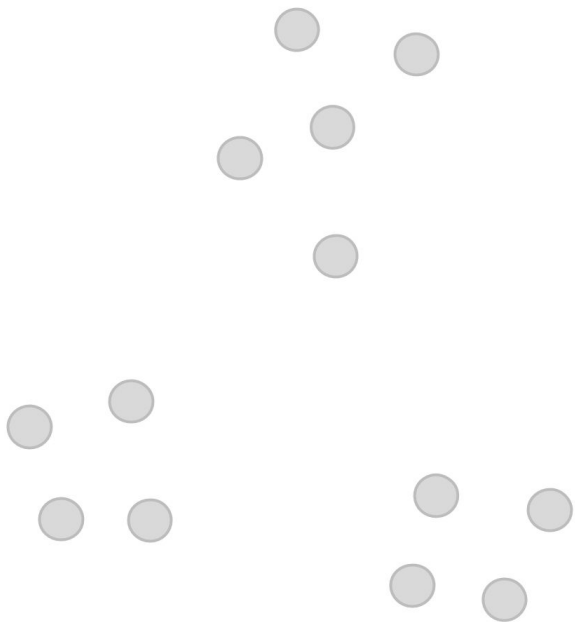
NCP:

Sequential assignment of cluster labels

Permutation-invariant neural representation



Neural Clustering Process (NCP)



- Pick any unlabeled point
- Predict
 - which existing cluster to join
 - or create a new cluster
- Iterate



Neural Clustering Process (NCP)

Pointwise expansion of the posterior

$$p(c_{1:N}|\mathbf{x}) = p(c_1|\mathbf{x}) p(c_2|c_1, \mathbf{x}) \dots p(c_N|c_{1:N-1}, \mathbf{x})$$

Each factor $p(c_i|c_{1:i-1}, \mathbf{x}) = \frac{p(c_i, c_{1:i-1}, \mathbf{x})}{\sum_{c'_i=1}^{K+1} p(c'_i, c_{1:i-1}, \mathbf{x})}$ is learned using a neural network.



K+1 possibilities:

- Join one of the K existing clusters
- Create a new cluster

Dynamically grows the number of clusters at test time



Neural Architecture

- Each cluster

$$H_k = \sum_{i:c_i=k} h(x_i)$$

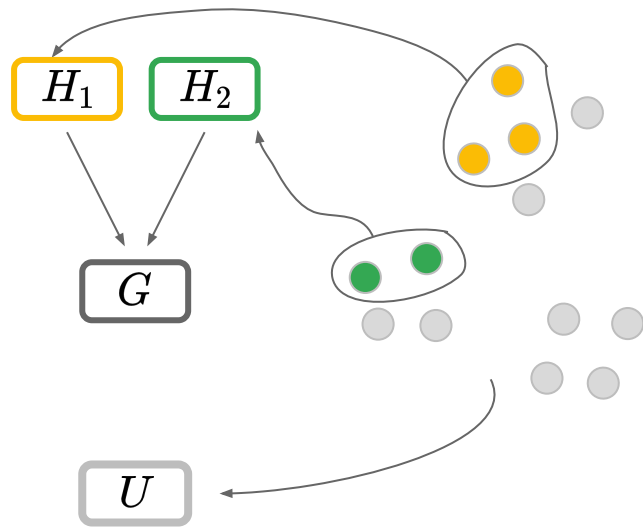
- All existing clusters

$$G = \sum_{k \in \{1 \dots K\}} g(H_k)$$

- All unassigned data

$$U = \sum_{i \in I_u} u(x_i)$$

I_u -- indices of unassigned data



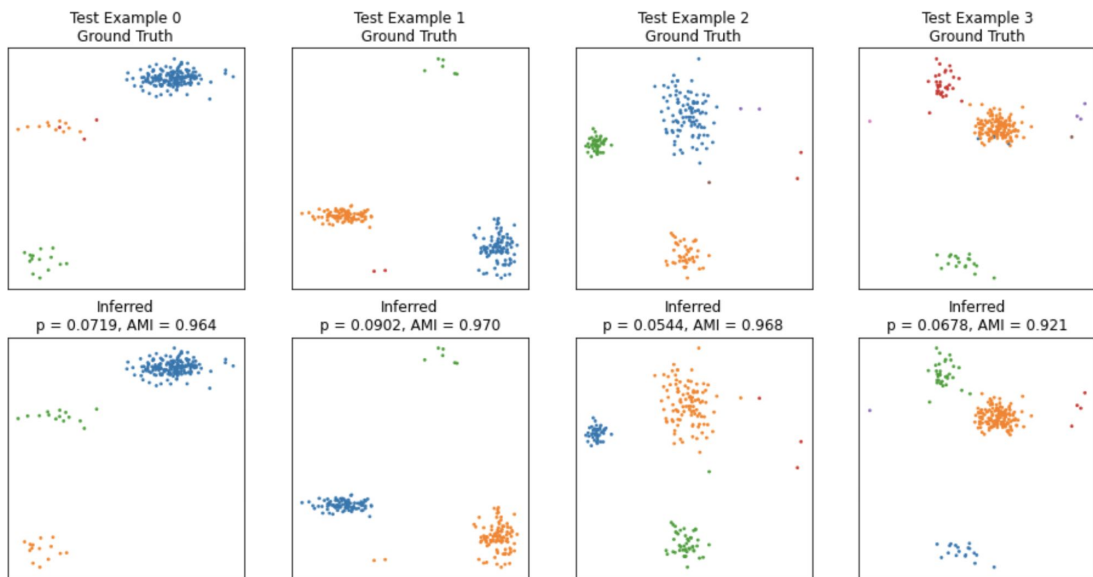
(G, U): current clustering configuration



$$p_{\theta}(c_i = k | c_{1:i-1}, \mathbf{x}) = \frac{e^{f(G_{c_i=k}, U)}}{\sum_{k'=1}^{K+1} e^{f(G_{c_i=k'}, U)}}$$



Example: Gaussian Mixtures



[Colab notebook](#)





Probabilistic Formulation for Amortized Clustering

Generative Model (e.g. Gaussian Mixtures)

$$c_1 \dots c_N \sim p(c_1 \dots c_N)$$

Cluster Labels (K distinct)

$$\mu_k \sim p(\mu) \quad k = 1 \dots K$$

Cluster Parameters

$$x_i \sim p(x_i | \mu_{c_i}) \quad i = 1 \dots N$$

Observations

Inference goal: given observations $\mathbf{x} = \{x_i\}$, produce cluster labels from $p(c_{1:N} | \mathbf{x})$

Alternative approaches: MCMC, variational inference

Amortized Inference for Probabilistic Clustering

Learn a neural network function that approximates $p(c_{1:N} | \mathbf{x})$