Modeling and Control of a Novel Tilt – Roll Rotor Quadrotor UAV

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Abstract— The use of unmanned aerial vehicles (UAVs) for military, scientific, and civilian sectors are increasing drastically in recent years. The quadrotor platform has been used for many applications and research studies, as well. One of the limiting factors that prevents further implementation of the quadrotor system into applications, is the way quadrotor moves. It needs to tilt along the desired direction of motion. By doing this it can have necessary acceleration towards that direction. But tilting has the undesired effect of moving the onboard cameras' direction of view. This becomes an issue for surveillance and other vision based tasks. This study presents the design and control of a novel quadrotor system. Unlike previous study that uses regular quadrotor, this study proposes an alternative propulsion system formed by tilting rotors. This design eliminates the need of tilting the airframe, and it suggests superior performance with respect to regular quadrotor design. The mathematical model of the tiltable-rotor type quadrotor and designed control algorithms are explained. Various simulations are developed on MATLAB, in which the proposed quadrotor aerial vehicle has been successfully controlled. Comparison of the proposed system to regular quadrotor suggests better performance.

I. Introduction

The use of unmanned aerial vehicles (UAVs) for military, scientific, and civilian sectors are increasing drastically in recent years. UAVs have clear advantages such as higher maneuverability, lower cost, decreased radar signature, strength, as well as decreased risk for human life. These advantages lead the use of these vehicles in many applications from military operations to civilian tasks, and extensive research are carried out in many laboratories.

Currently, there are various commercial and experimental UAVs of various sizes available, and many more autonomous unmanned VTOL vehicles are being developed at universities, research centers, and by hobbyists [1-4]. The quadrotor UAV platform has been used for many applications and research studies, as well. The studies in quadrotor UAV modeling and control increased rapidly in recent years [5-16].

One of the limiting factors that prevents further implementation of the quadrotor system into applications, is the way quadrotor moves. It needs to tilt along the desired direction of motion. By doing this it can have necessary acceleration towards that direction. But tilting has the undesired effect of moving the onboard cameras' direction of view. This becomes an issue for surveillance and other vision based tasks.

Instead of tilting the aircraft, rotors can also be tilted. Quad Tilt Rotor (QTR) is a well-known variance of quadrotor airframe. QTR enables the transition from helicopter mode to aircraft mode by tilting the rotors at the same time along an axis perpendicular to the front direction of the vehicle (pitch angle). It also has the benefit of extra lift [17, 18]. There are two experimental full-size aircraft prototypes available. Curtiss-Wright X-19 Quad-tilt-rotor aircraft [19] developed in 1963, and Bell X-22A [20] was developed 1966. Recently, Bell Boeing Quad TiltRotor (V-44) is a proposed four-rotor derivative of the V-22 Osprey tiltrotor being under development jointly by Bell Helicopter and Boeing [21].

There are a few studies that have tiltable geometry unmanned quadrotors. Cetinsoy et al. [22] proposed design and construction of a quad tilt-wing UAV. The vehicle can switch between quadrotor to aircraft mode by tilting of all of the rotors at the same time. Ryll et al. [23] proposed quadrotor UAV with tilting propellers that enable 4 propellers to actively rotate (tilt) about the axes connecting them to the quadrotor main body. Jeong et al. [24] proposed a tilting mechanism of the Omni-Flymobile that allows converting vehicle from flying mode to driving mode or vice versa by tilting rotors 90°. Salazar-Cruz et al. [25] proposed stabilization and nonlinear control for a trirotor mini-aircraft, which has only one tiltable rotor.

In these studies, rotors tilt either all at the same time a pre-specified amount, or all tilt around one axis only. The main contribution of this paper is the introduction of the design and control of a novel quadrotor system. Unlike previous study that uses regular quadrotor, this study proposes an alternative propulsion system formed by tilting rotors. The rotors can be given pitch and roll angle with respect to airframe. Proposed quadrotor can enable tilting of each rotor independently along x and y axes, therefore adding 8 additional control inputs to the system. Although these additional control inputs make the system mechanically more complex, it brings various advantages. This design eliminates the need of tilting the airframe, and it suggests better performance with respect to regular quadrotor design.

The paper is organized as follows. Section II describes the mathematical model of the proposed tiltable-rotor type quadrotor. The controllers are also presented in Section II. The simulation model and simulation results supporting the objectives of the paper are presented in Section III. Concluding remarks and future work are presented in Section IV

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II. MODELING AND CONTROL

A. MODELING

A regular quadrotor is an under actuated aircraft with four rotors that have fixed position with respect to the aircraft body frame. In this study, unlike in the regular definition, the rotors are able to rotate around pitch and roll angles. The required force is generated not by tilting body itself, but by tilting the required rotors. Table 1 presents the symbols and their definitions used in this section.

Rigid body representation of the quadrotor is given in Fig. 1. Body fixed frame is assumed to be at the center of gravity of the quadrotor where z axis is pointing downwards according to N, E, D (North, East, Down) geographical coordinate system. According to the Euler angle representation, angles of rotation about the aircraft's center of mass in x, y and z axes are defined respectively as roll (ϕ) , pitch (θ) and yaw (ϕ) . The earth's gravitational force mg is assumed to be constant and in downwards direction with respect to earth frame.

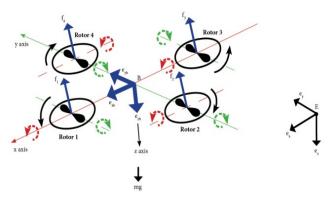


Fig. 1. Free body diagram of Tilt-Roll rotor quadrotor. North-East-Down (NED) geographical coordinate system representation is used

Consider a single rotor as shown in Fig. 2. The direction of the rotor thrust has been changed due to pitch (θ_i) and roll (ϕ_i) , where i corresponds to the rotor number (i: 1,2,3,4). This rotation can be represented by a rotation matrix with Euler angles $(\phi_i, \theta_i, \varphi_i)$ by

$$RPY(\varphi_i, \theta_i, \phi_i) = R(z, \varphi_i) \cdot R(y, \theta_i) \cdot R(x, \phi_i).$$
 (1)

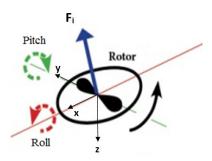


Fig. 2. Rotor angle is changed in order to obtain the necessary forces in x, y and z directions.

Table 1. Symbols and Definitions

Symbols	Definitions
e_{θ}	Error of pitch angle
θ_d	Desired pitch angle
θ_a	Actual pitch angle
e_{ϕ}	Error of roll angle
ϕ_d	Desired roll angle
	Actual roll angle
ϕ_a	Error of yaw angle
e_{φ}	Desired yaw angle
φ_d	Actual yaw angle
φ_a	Angular velocity along X axis
p	Angular velocity along Y axis Angular velocity along Y axis
q	Angular velocity along Z axis
r n	Angular acceleration along X axis
<u> </u>	Angular acceleration along Y axis
q r	Angular acceleration along Z axis
e_X	Position error along X axis
e_{Y}	Position error along Y axis
e_Z	Position error along Z axis
$e_{\dot{X}}$	Velocity error along X axis
$e_{\dot{Y}}$	Velocity error along Y axis
$e_{\dot{Z}}$	Velocity error along Z axis
$e_{\ddot{X}}$	Acceleration error along X axis
$e_{\ddot{Y}}$	Acceleration error along Y axis
e _Z	Acceleration error along Z axis
X_d	Desired position along X axis
Y_d	Desired position along Y axis
Z_d	Desired position along Z axis
X_a	Actual position along X axis
Y_a	Actual position along Y axis
Z_a	Actual position along Z axis
\dot{X}_d	Desired velocity along X axis
\dot{Y}_d	Desired velocity along Y axis
$ \vec{Z}_d $ $ \vec{X}_a $	Desired velocity along Z axis
\dot{X}_a	Actual velocity along X axis
$\dot{Y_a}$	Actual velocity along Y axis
$\dot{Z_a}$	Actual velocity along Z axis
$\ddot{X_d}$	Desired acceleration along X axis
$\ddot{Y_d}$	Desired acceleration along Y axis
$\ddot{Z_d}$	Desired acceleration along Z axis
\ddot{X}_a	Actual acceleration along X axis
Ϋ́a	Actual acceleration along Y axis
\ddot{Z}_a	Actual acceleration along Z axis
U_T	Throttle Input
F_i	Force Vector
y_{θ}	Control output for pitch angle
y_{ϕ}	Control output for roll angle
y_{φ}	Control output for yaw angle
τ_{χ}	Total torque along x axis
τ_y	Total torque along y axis
	Total torque along z axis
$\frac{\tau_z}{dT}$	Sampling Time
u I	Samping Time

The force acting on the quadrotor body frame generated by this rotated *Rotor* i is defined as F_i . Multiplication of the equation 1 with the thrust generated by the rotor $F_i \begin{pmatrix} 0 & 0 & -1 \end{pmatrix}^T$ leads to a rotated rotor thrust force vector,

$$\vec{F}_{i} = \begin{bmatrix} F_{ix} \\ F_{iy} \\ F_{iz} \end{bmatrix} = -F_{i} \begin{bmatrix} Sin(\theta_{i}) \cdot Cos(\phi_{i}) \\ -Sin(\phi_{i}) \\ Cos(\theta_{i}) \cdot Cos(\phi_{i}) \end{bmatrix}, \qquad (2)$$

where F_{ix} , F_{iy} and F_{iz} forces are the components of the thrust force along the x, y, and z axes respectively. Thus, the thrust of rotor i is equal to $F_i = \sqrt{F_{ix}^2 + F_{iy}^2 + F_{iz}^2}$.

While, the rotation speeds of the motors are described by Ω_i , the rotor thrust force is also equal to $F_i = b\Omega_i^2$, where parameter b is a constant namely the push factor.

For a set of required F_{ix} , F_{iy} and F_{iz} forces, corresponding tilt angles of the rotor can be calculated easily. The pitch and roll angles of the rotor and its rotation speed can be calculated as

$$\theta_{i} = atan \frac{F_{ix}}{F_{iz}}$$

$$\phi_{i} = asin \frac{F_{iy}}{\sqrt{F_{ix}^{2} + F_{iy}^{2} + F_{iz}^{2}}}$$

$$\Omega_{i} = \sqrt{\frac{F_{ix}^{2} + F_{iy}^{2} + F_{iz}^{2}}{h}}$$
(3)

The idea behind this study is to fix the body of the aircraft to a user input reference point, so that the tilted motors will generate the required forces for stabilization. Thus, the axial forces must be calculated for the proposed aircraft to achieve necessary maneuvers. This is determined using a feedback control using the equations given in (4), where total thrust due to tilted four rotors are presented.

$$\sum_{i=1}^{i=4} F_{i_x} = F_x, \ \sum_{i=1}^{i=4} F_{i_y} = F_y, \ \sum_{i=1}^{i=4} F_{i_z} = F_z$$
 (4)

Note that in order to maintain the hover position, $F_z = mg$ equivalence should be determined (in ideal conditions). Choosing the earth coordinate system where Z axis is pointing upwards, the actual accelerations of the aircraft can be determined:

$$\ddot{X}_a = \sum \frac{Fx}{m}, \ \ddot{Y}_a = \sum \frac{Fy}{m}, \ \ddot{Z}_a = \sum \frac{Fz}{m} - g \tag{5}$$

Having calculated the accelerations, it is relatively easier to calculate the velocities of the aircraft simply by integrating the acceleration data in (5).

$$\dot{X}_{a} = \int_{t=0}^{t=\infty} \left(\sum \frac{Fx}{m}\right) dt , \quad \dot{Y}_{a} = \int_{t=0}^{t=\infty} \left(\sum \frac{Fy}{m}\right) dt ,$$

$$\dot{Z}_{a} = \int_{t=0}^{t=\infty} \left(\sum \frac{Fz}{m} - g\right) dt \tag{6}$$

Using the torque values in each arm of the quadrotor, the angular accelerations can be determined using;

$$\dot{p} = \sum_{l_{xx}} \frac{\tau_x}{l_{xx}}, \ \dot{q} = \sum_{l_{yy}} \frac{\tau_y}{l_{yy}}, \ \dot{r} = \sum_{l_{zz}} \frac{\tau_z}{l_{zz}}$$
 (7)

Then simply the angular velocities are found to be:

$$p = \int \dot{p} , q = \int \dot{q} , r = \int \dot{r}$$
 (8)

B. CONTROL

The control of the tilt-roll rotor quadrotor differs from the regular quadrotor. In this section, proposed controllers are presented.

In this work, the control algorithm is designed so that all the motors tilt with same angle values depending on the control output. This approach simplified the control work significantly, at a cost of reducing the potential benefits of the vehicle.

Calculating the θ_i and ϕ_i value of a single motor would be sufficient in terms of controllability of the system. This concept of this study lets all of the four motors to rotate at the same speed so that they produce the same amount of resultant forces $F_1 = F_2 = F_3 = F_4$. This will be different, if the motors act separately from each other with different rotational speeds and rotational angles. This concept will be considered in the future work.

For performance and applicability purposes, cascaded PID control is chosen for control and simulation of the proposed aircraft. There exists total of 4 cascaded PID loops where each of motor angles are controlled with the inputs received from the vehicle's position, velocity, altitude and total force vectors. In order to control all of these inputs, the following equations should be satisfied:

$$F_{i_{x}} = \frac{y_{\theta}}{4}, \quad F_{i_{y}} = \frac{y_{\phi}}{4}$$

$$F_{1_{z}} = F_{3_{z}} = \frac{U_{T}}{4} + \frac{y_{\phi}}{4}$$

$$F_{2_{z}} = F_{4_{z}} = \frac{U_{T}}{4} - \frac{y_{\phi}}{4}$$
(9)

where throttle, U_T , is applied as an input in order to increase/decrease motor speeds so that the aircraft can increase, decrease or maintain its altitude (hover position).

For control purposes the angle errors must be determined. The pitch angle error is calculated using the formula in (10).

$$e_{\theta} = \theta_d - \theta_a \tag{10}$$

Then control output, y_{θ} is determined using the following PID loop:

$$y_{\theta} = Kp * e_{\theta} - Kd * p + Ki * \frac{d}{dt} \sum_{t=0}^{t=\infty} e_{\theta(t)}$$
 (11)

The corresponding equations for (ϕ) and (φ) control outputs can be written as equations (10, 11) by changing the (θ) terms by (ϕ) , (ψ) and $(\dot{\theta})$ terms by $(\dot{\phi})$, $(\dot{\varphi})$.

For velocity and position control of the aircraft following equations are used:

$$e_X = X_d - X_a \tag{12}$$

$$\dot{X}_d = \frac{Kp * e_X}{dT} - Kd * \dot{X}_a \tag{13}$$

$$e_{\dot{X}} = \dot{X}_d - \dot{X}_a \tag{14}$$

$$\theta_d = Kp * e_{\dot{X}} - \frac{Kd * \ddot{X_a}}{dT} \tag{15}$$

Similarly, roll angle (ϕ) estimation of motors with respect to position and velocity output can be calculated using equations (12-15) by replacing (X), (\dot{X}) , (\ddot{X}) , (θ) with (Y), (\dot{Y}) , (\ddot{Y}) , (ϕ) , respectively.

For altitude hold control of the aircraft, equations in (16) are used:

$$e_Z = Z_d - Z_a$$

 $U_T(t) = U_T(t-1) + Kp * e_Z - Kd * \frac{(\dot{z})_a}{d^T}$ (16)

By controlling current throttle parameter in (16), one can control the altitude of the vehicle. It is useful to control the throttle value by using a feedback loop for a robust design for altitude hold purposes.

Note that the control output values in PID loops are constrained in safe limits so that the aircraft would not make too aggressive maneuvers.

III. SIMULATIONS

In order to analyze the dynamic behavior of the proposed quadrotor vehicle and the proposed controllers, a simulation model has been developed in Matlab Simulink. In this section, the simulation results will be presented and the performance of the proposed vehicle will be compared to a regular fix rotor quadrotor.

In this study, the body roll and pitch angles are held in zero degrees (within tolerance limits) by using the tilt-roll capabilities of the motors. However, the vehicle still can move around its z axis so that yaw angle (φ) can increase or decrease with respect to the rotational direction. During the simulations θ , φ and φ dynamically changed in order to simulate the worst case scenario.

In the Matlab simulation, user can define the starting and final points of the quadrotors and the initial values for θ , ϕ and φ . Additionally aircraft's throttle, θ , ϕ and φ angles can be disturbed during the simulation via RF simulator.

For better comparison purposes both of the quadrotors are disturbed with the same inputs and their starting-desired positions are fixed at the same points during the simulations. In this case, the starting point is set at x=1, y=1, z=0 and desired position is set at x=18, y=18, z=8 on Matlab coordinate system. The vehicles' initial φ angle is set as -10 degrees. At t=1, the vehicles φ angle is changed from -10 to +10 degrees. At t=6, their φ angles are set as 0 (zero) degrees. Moreover, θ , φ angles of the bodies are fixed to 0 (zero) degrees initially. This means that RF input is set to zero degrees but the vehicles can autonomously change their body angles with respect to their controller outputs.

In simulation 1, as presented in Fig. 3, yaw angle stabilization of the vehicle (tilt-roll rotor quadrotor) is challenged with different desired yaw angle inputs and in the meantime, the motors have dynamic values of pitch (θ) and roll (ϕ) angles depending on the control output.

In Figs. 4, 5 and 6, two types of quadrotors, the regular fixed rotor and the tilt-roll rotor, have been plotted. They are compared as they settle to some desired points in 3D coordinates.

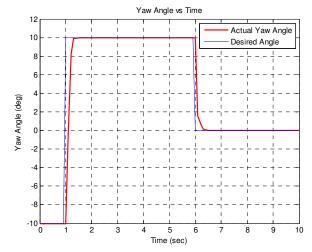


Fig. 3. Tilt-roll rotor quadrotor yaw angle (φ) vs time simulation.

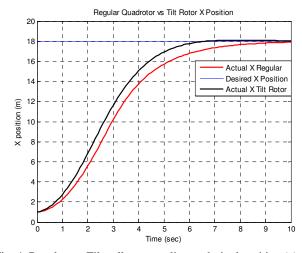


Fig. 4. Regular vs. Tilt-roll rotor settling to desired position (x)

Comparing the results in the Fig. 4, 5 and 6; one can note that the settling time of the new approach tilt-roll rotor quadrotor has better results in x and y positions while in z axis regular quadrotor's settling time is slightly less. However, the regular fixed motor quadrotor has more overshoot during the altitude (z axis) hold performance.

By examining the plots, it can be noted that in overall performance the tilt-roll rotor quadrotor has better results in terms of smaller overshoot and smaller settling time as compared to the regular fixed rotor quadrotor. However, both of the systems are controlled with relatively complex cascaded PID parameters which include at least 13 different PID coefficients. That's why additional tuning can be done by changing these coefficients so that both of the systems' performances will get better.

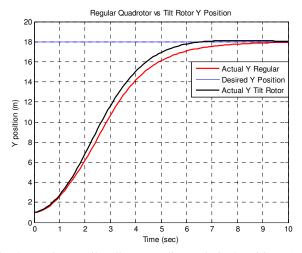


Fig. 5. Regular vs. Tilt-roll rotor settling to desired position (y)

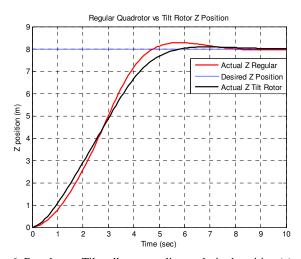


Fig. 6. Regular vs. Tilt-roll rotor settling to desired position $\left(z\right)$

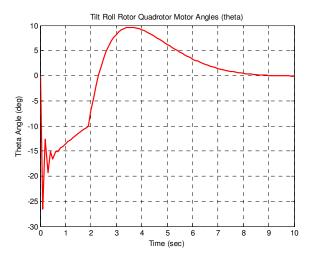


Fig. 7. Tilt-Roll Rotor quadrotor's motor angles (theta).

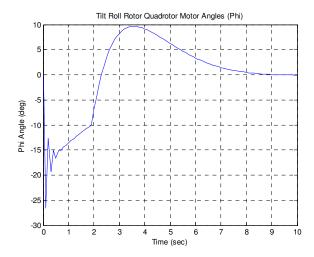


Fig. 8. Tilt-Roll Rotor quadrotor's motor angles (phi).

The biggest benefit obtained by the tilt-roll rotor quadrotor is due to how it moves. It moves by tilting the rotors, as opposed to tilting the airframe as the regular quadrotor does. The ability of tilting rotors promises a more stable flight of the airframe. The tilt (theta) and roll (phi) of rotors (motor angles) during the simulation are shown in Fig., 7 and 8. These motor angles are with respect to earth frame, and they are same for all of the rotors of the vehicle. Due to the fact that the simulation inputs (from x=1, y=1 to x=18, y=18) and starting points of θ and ϕ are the same, the two graphs of θ and ϕ exactly match each other in this simulation.

As seen on the same graphs, at t=9 both of the motor angles get to zero degrees because the desired positions of x and y are reached by the aircraft. At this point, there is only need for vertical force which can be achieved by having the motors to be in upward position with respect to the earth frame (theta=phi= 0°).

IV. CONCLUSIONS

One of the limiting factors that prevents further implementation of the quadrotor systems into various applications, is the way quadrotor moves. It needs to tilt along the desired direction of motion. This tilting enables motion of the quadrotor; on the other hand, it has an undesired effect of changing the direction of view of on board camera. This becomes an issue for surveillance and other vision based tasks. This study presented the design and control of a novel quadrotor system. Unlike previous study that uses regular quadrotor, this study proposes an alternative propulsion system formed by tilting rotors. This design eliminates the need of tilting the airframe, and it suggests superior performance with respect to regular quadrotor design. The mathematical model of the tiltable-rotor type quadrotor and designed control algorithms are explained. Various simulations are developed on Matlab, in which the proposed quadrotor aerial vehicle has been successfully controlled. Comparison of the proposed system to regular quadrotor suggests better performance.

Determining optimum control response at different conditions is a challenge, which will be investigated in our future work. We are at the process of designing the proposed vehicle. As the fabrication is complete, experimental validation of the design and control algorithms will be possible.

REFERENCES

- P. Castillo, R. Lozano, and A. E. Dzul, Modelling and Control of Mini-flying Machines, Advances in Industrial Control series, ISSN 1430-9491, Springer, 2005.
- [2] H. Y. Chao, Y. C. Cao, and Y. Q. Chen, "Autopilots for small unmanned aerial vehicles: a survey," International Journal of Control, Automation, and Systems, vol. 8, no. 1, pp. 36-44, 2010.
- [3] D. Lee, I. Kaminer, V. Dobrokhodov, and K. Jones, "Autonomous feature following for visual surveillance using a small unmanned aerial vehicle with gimbaled camera system," International Journal of Control, Automation, and Systems, vol. 8, no. 5, pp. 957-966, 2010.
- [4] D. Han, J. Kim, C. Min, S. Jo, J. Kim, and D. Lee, "Development of unmanned aerial vehicle (UAV) system with waypoint tracking and vision-based reconnaissance," International Journal of Control, Automation, and Systems, vol. 8, no. 5, pp. 1091-1099, 2010.
- [5] T. Hamel, R. Mahony, R. Lozano, and J. Ostrowski, "Dynamic modeling and configuration stabilization for an X4-flyer," Proc. of IFAC 15th Triennial World Congress, Barcelona, Spain, 2002.
- [6] E. Altuğ, J. P. Ostrowski, and C. J. Taylor, "Control of a quadrotor helicopter using dual camera visual feedback," The International Journal of Robotics Research, vol. 24, no. 5, pp. 329-341, 2005.
- [7] D. Suter, T. Hamel, and R. Mahony, "Visual servo control using homography estimation for the stabilization of an X4-flyer," Proc. of the 41st IEEE Conf. on Decision and Control, pp. 2872-2877, 2002.
- [8] A. Moktari and A. Benallegue, "Dynamic feedback controller of Euler angles and wind parameters estimation for a quadrotor unmanned aerial vehicle," Proc. of the IEEE Conf. on Rob. and Auto., pp. 2359-2366, 2004.
- [9] J. Dunfied, M. Tarbouchi, and G. Labonte, "Neural network based control of a four rotor helicopter," Proc. of IEEE Int. Conf. on Industrial Technology, pp. 1543-1548, 2004.
- [10] M. G. Earl and R. D'Andrea, "Real-time attitude estimation techniques applied to a four rotor helicopter," Proc. of IEEE Conf. on Decision and Control, pp. 3956-3961, 2004.
- [11] S. Slazar-Cruz, A. Palomino, and R. Lozano, "Trajectory tracking for a four rotor mini-aircraft," Proc. of the 44th IEEE Conf. on Decision

- and Control and the European Control Conference, pp. 2505-2510, 2005.
- [12] J. Escareno, S. Salazar-Cruz, and R. Lozano, "Embedded control of a four-rotor UAV," Proc. of the American Control Conference, pp. 189-204, 2006.
- [13] S. Bouabdallah and R. Siegwart, "Backstepping and sliding-mode techniques applied to an indoor micro quadrotor," Proc. of the IEEE Conf. on Robotics and Automation, pp. 2247-2252, 2005.
- [14] L. Beji, A. Abichou, and K. M. Zemalache, "Smooth control of an X4 bidirectional rotors flying robot," 5th Int. Workshop on Robot Motion and Control, pp. 181-186, 2005.
- [15] P. Castillo, A. E. Dzul, and R. Lozano, "Real-time stabilization and tracking of a four-rotor mini rotorcraft," IEEE Trans. on control systems technology, vol. 12, no. 4, pp. 510-516, 2004.
- [16] A. Tayebi and S. McGilvray, "Attitude stabilization of a VTOL quadrotor aircraft," IEEE Trans. on Control Systems Technology, vol. 14, no. 3, pp. 562-571, 2006.
- [17] T. Gaffey, Large Cargo Rotorcraft bell Helicopter's Perspectives, AHS Forum 56, May 4, 2000.
- [18] H. Yeo, W. Johnson, "Performance and Design Investigation of Heavy Lift Tilt-Rotor with Aerodynamic Interference Effects", Journal of Aircraft, Vol. 46, No. 4, pp. 1231- 1239, July–August 2009.
- [19] H. V. Borst, "Design and Development Considerations of the X-19 VTOL Aircraft", Annals of the New York Academy of Sciences, vol 107, pp. 1749-6632, 1963.
- [20] M. J. Hirschberg, "An Overview of the History of Vertical and/or Short Take-Off and Landing (V/STOL) Aircraft", Proceedings www.vstol.org, 2006.
- [21] M. Sklar, "Diversity in design", Boeing Frontiers Magazine, pp. 44-45, December 2006 - January 2007 issue, 2006.
- [22] E. Cetinsoy, S. Dikyar, C. Hancer, K.T. Oner, E. Sirimoglu, M. Unel, M.F. Aksit, "Design and construction of a novel quad tilt-wing UAV", Mechatronics 22, pp. 723–745, 2012
- [23] M. Ryll, H. H. Bülthoff, and P. R. Giordano, "Modeling and Control of a Quadrotor UAV with Tilting Propellers", 2012 IEEE International Conference on Robotics and Automation River Centre, Saint Paul, Minnesota, USA, pp. 4606- 4613, 2012.
- [24] S. H. Jeong and S. Jung," Novel Design and Position Control of an Omni-directional Flying Automobile (Omni-Flymobile)", International Conference on Control, Automation and Systems 2010, in KINTEX, Gyeonggi-do, Korea, pp. 2480 – 2484, 2010.
- [25] S. Salazar-Cruz, R. Lozano, J. Escaren, "Stabilization and nonlinear control for a novel trirotor mini-aircraft", Control Engineering Practice 17, pp. 886–894, 2009.