Solutions for Selected Problems from Aluffi's *Algebra: Chapter 0*

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Contents

Chapter 1

Chapter 1: Fundamentals of Algebra

1.1 Introduction to Sets and Numbers

Problem 1.1.1: Basic Set Operations

Consider the sets $A = \{x \in \mathbb{Z} \mid 0 < x \leq 5\}$ and $B = \{x \in \mathbb{N} \mid x \text{ is even and } x < 10\}$. Determine $A \cup B$ and $A \cap B$.

Solution: First, let's list the elements of each set: $A = \{1, 2, 3, 4, 5\}$ (integers strictly greater than 0 and less than or equal to 5) $B = \{2, 4, 6, 8\}$ (natural numbers that are even and less than 10)

Now, we find the union: $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

And the intersection: $A \cap B = \{2, 4\}$ □

Problem 1.1.2: Number Systems - Classification

Classify the following numbers as rational or irrational: some test change a) $\sqrt{9}$ b) π c) $0.333\ldots$ d) $\frac{1}{7}$

Solution: a) $\sqrt{9} = 3$. Since 3 can be written as $\frac{3}{1}$, it is a **rational number**. b) π is a non-repeating, non-terminating decimal, so it is an **irrational number**. c) $0.333\cdots = \frac{1}{3}$. Since it can be expressed as a fraction of two integers, it is a **rational number**. d) $\frac{1}{7}$ is already in the form of a fraction of two integers, so it is a **rational number**. □

Chapter 2

Chapter 2: Functions and Graphs