Solutions for Selected Problems from Aluffi's *Algebra: Chapter 0*

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Chapter I

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2 Functions between sets

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3 Categories

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4 Morphisms

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5 Universal properties

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Chapter II

Groups, first encounter

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2 Examples of groups

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3 The category Grp

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4 Group homomorphisms

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Chapter III

Rings and modules

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2 The category Ring

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3 Ideals and quotient rings

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4 Ideals and quotients: Remarks and examples. Prime and maximal ideals

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5 Modules over a ring

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6 Products, coproducts, etc., in R-Mod

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7 Complexes and homology

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Chapter IV

Groups, second encounter

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2 The Sylow theorems

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3 Composition series and solvability

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4 The symmetric group

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5 Products of groups

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6 Finite abelian groups

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