Assignment#2

Knowledge Representation and Processing

April 16, 2021

Note: Assignment#2, due on 14:00 May 4, contributes to 10% of the total mark of the course.

Q1. Recall the syntax of the Description Logics $\mathcal{EL}$, DL-Lite and $\mathcal{ALC}$. Suppose $\mathsf{Teacher}$ and Course are concept names and $\mathsf{teaches}$ is a role name.
Let $\mathcal{E}$ be any of the following expressions:

(a) $\top \sqsubseteq \bot$

(b) $\exists \mathsf{teaches}.\mathsf{Course} \sqsubseteq \mathsf{Teacher}$

(c) $\forall \mathsf{teaches}.\mathsf{Course}$

(d) $\exists \mathsf{Course}.\mathsf{teaches}$

(e) $\exists \mathsf{teaches}^{-}.\top \sqsubseteq \mathsf{Teacher} \sqcup \mathsf{School}$

(f) $\mathsf{Teacher} \sqsubseteq \exists \mathsf{teaches}.\top$

(g) $\mathsf{Teacher} \sqsubseteq \exists \mathsf{teaches}.\bot$

(h) $\geq 3\ \mathsf{teaches}.\top \sqsubseteq \mathsf{Teacher}$

(i) $\geq 4\ \mathsf{teaches}.\mathsf{Course} \sqsubseteq \mathsf{Teacher}$

(j) $\forall \mathsf{teaches}.\top \sqsubseteq \exists \mathsf{teaches}.\mathsf{Course}$

(k) $\exists \mathsf{teaches}.\top \sqsubseteq \geq 2\ \mathsf{teaches}.\top$

(l) $\geq 2\ \mathsf{teaches}.\top \sqsubseteq \exists \mathsf{teaches}.\top$

- Translate $\mathcal{E}$ into natural language;
- State whether it is;
    - an $\mathcal{EL}$ concept;
    - an $\mathcal{EL}$ concept inclusion;
    - a DL-Lite concept;
    - a DL-Lite concept inclusion;

- an $\mathcal{ALC}$ concept;
- an $\mathcal{ALC}$ concept inclusion;
- none of the above.

- If $\mathcal{E}$ is a concept inclusion, check whether $\mathcal{E}$ follows from the empty TBox (i.e., $\emptyset \models \mathcal{E}$). If this is not the case, define an interpretation $\mathcal{I}$ such that $\mathcal{I} \not\models \mathcal{E}$.
- If $\mathcal{E}$ is a concept, check whether $\mathcal{E}$ is satisfiable. If this is the case, define an interpretation $\mathcal{I}$ such that $\mathcal{E}^{\mathcal{I}} \neq \emptyset$.

Q2. Create an $\mathcal{EL}$ TBox $\mathcal{T}$ that models the following facts:

(a) Mammals are animals.

(b) Lions are mammals that are carnivores.

(c) Giraffe are mammals that are herbivores.

(d) Carnivores eat meat.

(e) A vertebrate is any animal that has, amongst other things, a backbone.

Is the following $\mathcal{EL}$-TBox an $\mathcal{EL}$-terminology? Explain your answer. Express each concept inclusion in natural language:

(a) $\mathsf{Lion} \sqsubseteq \mathsf{Animal} \sqcap \exists \mathsf{lives}.\mathsf{Savannah}$

(b) $\exists \mathsf{eat}.\mathsf{Meat} \sqsubseteq \mathsf{Carnivore}$

(c) $\mathsf{Bird} \equiv \mathsf{Vertebrate} \sqcap \exists \mathsf{has\_part}.\mathsf{Wing} \sqcap \exists \mathsf{has\_part}.\mathsf{Leg} \sqcap \exists \mathsf{lays}.\mathsf{Egg}$

(d) $\mathsf{Reptile} \sqsubseteq \mathsf{Vertebrate} \sqcap \exists \mathsf{lays}.\mathsf{Egg}$

Q3. Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation, where

$$\Delta^{\mathcal{I}} = \{1, 2, 3, 4, 5, 6\}$$

$$A^{\mathcal{I}} = \{1, 2\}$$

$$B^{\mathcal{I}} = \{3, 4, 5, 6\}$$

$$r^{\mathcal{I}} = \{(1, 3), (1, 5), (2, 6)\}$$

Determine the extension of $C^{\mathcal{I}}$ of the following $\mathcal{EL}$-concepts C under $\mathcal{I}$:

- $A \sqcap B$
- $\exists r.B$
- $\exists r.(A \sqcap B)$
- $\top$

- $A \sqcap \exists r.B$

Which of the following are true?

- $\mathcal{I} \models A \equiv \exists r.B$
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$
- $\mathcal{I} \models \top \sqsubseteq B$
- $\mathcal{I} \models B \sqsubseteq \exists r.A$

Q4. Let $\mathcal{T} = \{\mathsf{Parent} \sqsubseteq \exists \mathsf{hasChild.Person}, \mathsf{Mother} \sqsubseteq \mathsf{Parent}\}$. Show that $\mathcal{T} \not\models \mathsf{Parent} \sqsubseteq \mathsf{Mother}$ by giving an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \not\models \mathsf{Parent} \sqsubseteq \mathsf{Mother}$.

Q5. Let $\mathcal{T}$ be an $\mathcal{EL}$-TBox containing the following (primitive) concept definitions:

$$\mathsf{Bird} \equiv \mathsf{Vertebrate} \sqcap \exists \mathsf{has\_part.Wing}$$

$$\mathsf{Reptile} \sqsubseteq \mathsf{Vertebrate} \sqcap \exists \mathsf{lays.Egg}$$

(a) Is $\mathcal{T}$ in normal form? Explain.

(b) Given $\mathcal{T}$, compute an $\mathcal{EL}$-TBox $\mathcal{T}'$ in normal form using the pre-processing algorithm from the lecture.

(c) Apply the algorithm from the lecture slides deciding whether $A \sqsubseteq_{\mathcal{T}'} B$ (equivalently $\mathcal{T}' \models A \sqsubseteq B$), where A, B are concept names. Using the normalized TBox $\mathcal{T}'$ as input and explain step-by-step which rules are applied.

(d) Using the output of the algorithm, decide whether

- $\mathsf{Reptile} \sqsubseteq_{\mathcal{T}'} \mathsf{Vertebrate}$
- $\mathsf{Vertebrate} \sqsubseteq_{\mathcal{T}'} \mathsf{Bird}$

Q6. Let $\mathcal{T}$ be an $\mathcal{EL}$-TBox containing the following concept inclusions:

$$A \sqsubseteq X$$

$$A \sqsubseteq Y$$

$$B \sqsubseteq B'$$

$$X \sqcap Y \sqsubseteq \exists r.B$$

$$\exists r.B' \sqsubseteq A'$$

(a) Is $\mathcal{T}$ in normal form?

(b) Using the output of the algorithm, decide whether

- $A \sqsubseteq_{\mathcal{T}} Z$
- $B \sqsubseteq_{\mathcal{T}} Z$
- $X \sqsubseteq_{\mathcal{T}} Y$
- $A \sqsubseteq_{\mathcal{T}} A'$
- $B \sqsubseteq_{\mathcal{T}} B'$

Q7. Show that every $\mathcal{EL}$-TBox is satisfiable (consistent). That is, show that for every $\mathcal{EL}$-TBox $\mathcal{T}$ there exists an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$.