Assignment 2

左之睿 191300087 1710670843@qq.com

Q1、

(1) Translate:

(a) There is nothing in the domain.

(b) Everybody who teaches a course is a teacher.

(c) teaches course

(d) 不存在这种表达，∃后面不能放concept name

(e) Everything that is taught is a teacher or a school

(f) Every teacher teaches something.

(g) Every teacher teaches nothing.

(h) If something teaches at least 3 things, then it is a teacher.

(i) If something teaches at least 4 courses, then it is a teacher.

(j) Everything teaches a course.

(k) Everybody teaching something at least teaches 2 things.

(l) Everybody teaching at least 2 things teaches something.

(2) State whether it is:

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在此仅列出属于的部分，若没提到则表示该expression并不属于那些类
(如(d)仅属于none of the above)
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(a) DL-Lite and $\mathcal{ALC}$ concept inclusion

(b) $\mathcal{EL}$ and $\mathcal{ALC}$ concept inclusion

(c) $\mathcal{ALC}$ concept

(d) none of the above

(e) DL-Lite with "or" concept inclusion

(f) $\mathcal{EL}$, DL-Lite and $\mathcal{ALC}$ concept inclusion

(g) $\mathcal{ALC}$ concept inclusion

(h) DL-Lite concept inclusion

(i) none of the above,但是如果对$\mathcal{ALC}$稍加扩展，则它是$\mathcal{ALC}$ concept inclusion

(j) $\mathcal{ALC}$ concpet inclusion

(k) DL_Lite concept inclusion

(l) DL-Lite concept inclusion

(3) Define an interpretation when it not follows from the empty TBox

(a) not follows from.
对任意的Interpretation $\mathcal{I}$，都有$\Delta^{\mathcal{I}} \neq \emptyset$，

(b) not follows from.
Interpretation $\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a\}$
$teaches^{\mathcal{I}} = \{(a, a)\}$
$Course^{\mathcal{I}} = \{a\}$
$Teacher^{\mathcal{I}} = \emptyset$
故$\exists(teaches.Course)^{\mathcal{I}} = \{a\} \not\subseteq Teacher^{\mathcal{I}}$

(e) not follows from.
Interpretation $\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a, b\}$
$teaches^{\mathcal{I}} = \{(a, b)\}$
$Teacher^{\mathcal{I}} = \emptyset$
$School^{\mathcal{I}} = \emptyset$
故$(\exists teaches^{-}.T)^{\mathcal{I}} = \{b\} \not\subseteq (Teacher)^{\mathcal{I}} \sqcup (Department)^{\mathcal{I}}$

(f) not follows from.
Interpretation $\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a\}$
$Teacher^{\mathcal{I}} = \{a\}$
$teaches^{\mathcal{I}} = \emptyset$
此时$Teacher^{\mathcal{I}} = \{a\} \not\subseteq (\exists teaches.T)^{\mathcal{I}}$

(g) not follows from.
Interpretation $\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a\}$
$Teacher^{\mathcal{I}} = \{a\}$
$teaches^{\mathcal{I}} = \{(a, a)\}$
此时$Teacher^{\mathcal{I}} = \{a\} \not\subseteq (\exists teaches.\bot)^{\mathcal{I}}$

(h) not follows from.
Interpretation $\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a, a_1, a_2, a_3\}$
$Teacher^{\mathcal{I}} = \emptyset$
$teaches^{\mathcal{I}} = \{(a, a_1), (a, a_1), (a, a_3)\}$
此时$(\geq 3\ teaches.T)^{\mathcal{I}} = \{a\} \not\subseteq \emptyset$

(i) not follows from.

Interpretation $\mathcal{I}$:

$\Delta^{\mathcal{I}} = \{a, b_1, b_2, b_3, b_4\}$

$Teacher^{\mathcal{I}} = \emptyset$

$Course^{\mathcal{I}} = \{b_1, b_2, b_3, b_4\}$

$teaches^{\mathcal{I}} = \{(a, b_1), (a, b_2), (a, b_3), (a, b_4)\}$

此时$(\geq 4 teaches.Course)^{\mathcal{I}} = \{a\} \not\subseteq \emptyset$

(j) not follows from.

Interpretation $\mathcal{I}$:

$\Delta^{\mathcal{I}} = \{a\}$

$teaches^{\mathcal{I}} = \emptyset$

$Course^{\mathcal{I}} = \emptyset$

此时$\forall(teaches.T)^{\mathcal{I}} = \{a\} \not\subseteq (\exists teaches.Course)^{\mathcal{I}}$

(k) not follows from.

Interpretation $\mathcal{I}$:

$\Delta^{\mathcal{I}} = \{a, b\}$

$teaches^{\mathcal{I}} = \{(a, b)\}$

我们有$a \in (\exists teaches.T)^{\mathcal{I}}$但$a \notin (\geq 2\ teaches.T)^{\mathcal{I}}$

(l) It follows from the empty TBox.

(4) Check satisfiable

(c) satisfiable

Interpretation $\mathcal{I}$:

$\Delta^{\mathcal{I}} = \{a\}$

$teaches^{\mathcal{I}} = \{(a, a)\}$

$Course^{\mathcal{I}} = \{a\}$

此时$(\forall teaches.Course)^{\mathcal{I}} = \{a\}$

Q2、

create:

Mammals⊑Animals

Lions⊑Mammals⊓Carnivore

Giraffe⊑Mammals⊓Herbicore

Carnivore⊑ ∃eat.meat

Vertebrate≡Animal⊓∃has.backbone

No. Because $\exists eat.Meat$ is not a concept name

(a) Lion is an animal lives in Savannah

(b) Animals which eat meat are carnivores.

(c) The vertebrate which has wing,leg and lays egg is a bird.

(d) Reptiles are vertebrates which lay egg.

Q3、

$A \sqcap B = \emptyset$
$\exists r.B = \{1,2\}$
$\exists r.(A \sqcap B) = \emptyset$
$\mathrm{T} = \{1,2,3,4,5,6\}$
$A \sqcap (\exists r.B) = \{1,2\}$

True:

$\mathcal{I} \models A \equiv \exists r.B$
$\mathcal{I} \models A \sqcap B \sqsubseteq \mathrm{T}$
$\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$

Q4、

记Parent为A，hasChild为r，Mother为B，Person为C
令$r^{\mathcal{I}} = \{(a,b),(b,c)\}$

$A^{\mathcal{I}} = \{a,b\}$

$B^{\mathcal{I}} = \{b\}$

$C^{\mathcal{I}} = \{a,b,c\}$
此时$\mathcal{I} \models \mathcal{T}$但是$\mathcal{I} \not\models Parent \sqsubseteq Mother$

Q5、

(a) No, 因为≡不能出现在normal form中

(b)
X is a fresh concept name
$Bird \sqsubseteq Vertebrate$
$Bird \sqsubseteq \exists has\_part.Wing$
$Reptile \sqsubseteq Vertebrate$
$Reptile \sqsubseteq \exists lays.Egg$

$X \sqsubseteq \exists has\_part.Wing$
$\exists has\_part.Wing \sqsubseteq X$
$Vertebrate \sqcap X \sqsubseteq Bird$
(c)首先初始化$S(A) = \{A\}, R(r) = \emptyset$ for A and r in $\mathcal{T}'$
然后使用如下四个规则进行处理
$simpleR: \ \ if \ A' \in S(A) \ and \ A' \sqsubseteq B \in \mathcal{T}' \ and \ B \notin S(A)$
$then \ S(A) := S(A) \cup \{B\}$

$conjR: \ \ if \ A_1, A_2 \in S(A) \ and \ A_1 \sqcap A_2 \sqsubseteq B \in \mathcal{T}' \ and \ B \notin S(A)$
$then \ S(A) := S(A) \cup \{B\}$

$rightR: \ \ if \ A' \in S(A) \ and \ A' \sqsubseteq \exists r.B \in \mathcal{T}' \ and \ (A, B) \notin R(r)$
$then \ R(r) := R(r) \cup (A, B)$

$leftR: \ \ if \ (A, B) \in R(r) \ and \ B' \in S(B) \ and \ \exists r.B' \sqsubseteq A' \in \mathcal{T}' \ and \ A' \notin S(A)$
$then \ S(A) := S(A) \cup \{A'\}$
最后，$\mathcal{T}' \models A \sqsubseteq B \ iff \ B \ in \ S(A)$
(d)
$Reptile \sqsubseteq_{\mathcal{T}'} Vertebrate$: Yes
$Vertebrate \sqsubseteq_{\mathcal{T}'} Bird$: No

Q6、

(a) No,因为 $X \sqcap Y \sqsubseteq \exists r.B$不是normal form
(b) 题目中并未给出Z的具体来源，在此视为对$X \sqcap Y \sqsubseteq \exists r.B$处理得到的产物，即$X \sqcap Y \sqsubseteq Z, Z \sqsubseteq \exists r.B$
$A \sqsubseteq_{\mathcal{T}} Z$: Yes
$B \sqsubseteq_{\mathcal{T}} Z$: No
$X \sqsubseteq_{\mathcal{T}} Y$: No
$A \sqsubseteq_{\mathcal{T}} A'$: Yes
$B \sqsubseteq_{\mathcal{T}} B'$: Yes

Q7、

设$\mathcal{T}$是一个$\mathcal{EL}$-TBox，定义如下的解释$\mathcal{I}$:
$\Delta^{\mathcal{I}} = \{a\}$
$A^{\mathcal{I}} = \{a\}$（对所有的concept name A都是如此）
$r^{\mathcal{I}} = \{(a, a)\}$（对所有的role name r都是如此）
此时，$for \ all \ \mathcal{EL} - concepts \ A,$有$A^{\mathcal{I}} = \{a\}$
显然$\mathcal{I} \models A \sqsubseteq B$对所有$\mathcal{EL}$-concept inclusions $A \sqsubseteq B$成立，因此$\mathcal{I}$是$\mathcal{T}$的一个model，证明完毕