14:00 | 31|05|19 | **Examinations and Graduation Office** Flower Hall

Exams Office Use Only

**University of the Witwatersrand, Johannesburg**

| | |
|---|---|
| Course or topic No(s) | MATH2019 |
| Course or topic name(s) Paper number & title | Linear Algebra |
| Examination/Test* to be held during month(s) of (*delete as applicable) | June Exam |
| Year of study (Art & Sciences leave blank) | Second Year |
| Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch) | BSc, Bcom, Ba |
| Faculty/ies presenting candidates | Science, Commerce, Humanities |
| Internal examiner(s) and telephone number(s) | Dr R Kwashira – 76228<br>Prof Ye Zelenyuk - 76247 |
| External examiner(s) | Dr A Davison |
| Calculator policy | |

Time allowance

| Course No's | MATH2019 | Hours | 1h00 |
|---|---|---|---|

Instruction to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

**Answer all questions**
**Total : 60**
**Duration : 1h00**

## Linear Algebra Exam 2019

**Question 1** A linear operator $\mathcal{A} : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

in the standard basis. Find the matrix $B$ of $\mathcal{A}$ in the basis $\{(1, 0, -1), (0, 1, 1), (0, 2, 1)\}$.

**[10]**

**Question 2** Determine whether the matrix

$$A = \begin{pmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{pmatrix}$$

is diagonalizable, and if yes, find a diagonal matrix $D$ and a matrix $T$ such that $D = T^{-1}AT$.

**[10]**

**Question 3** Prove that for any vectors $x$, $y$ of an inner product space,

$$|(x, y)| \leq ||x|| \cdot ||y||.$$

**[10]**

**Question 4** Prove that an orthogonal system of nonzero vectors is linearly independent.

**[10]**

**Question 5** Using the Gram-Schmidt process, transform the basis $\{(0, 1, -1), (1, 0, -1), (1, 1, 0)\}$ of $\mathbb{R}^3$ into an orthonormal basis.

**[10]**

**Question 6** Find a system of linear equations whose solution space is the subspace $\langle a_1, a_2, a_3 \rangle \subseteq \mathbb{R}^5$, where

$$a_1 = (2, -2, 0, 2, -2), a_2 = (2, -2, -2, 0, -4), a_3 = (2, 0, 2, 4, 2).$$

**[10]**