

# Applications of Algorithms

## Tutorial 2

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### 1 Trees

#### 1.1 Properties of trees

**Theorem 1.1.** *Let  $T$  be a graph with  $n$  vertices. Then the following statements are equivalent.*

1.  $T$  is a tree;
2.  $T$  is connected, and has  $n - 1$  edges;
3.  $T$  contains no cycles and has  $n - 1$  edges;
4.  $T$  is connected, and every edge is a bridge;
5. Any two vertices of  $T$  are connected by exactly one path;
6.  $T$  contains no circuits, but the addition of any new edge creates exactly one circuit.

1. Prove (5)  $\Rightarrow$  (4)

2. Prove (4)  $\Rightarrow$  (2)

#### 1.2 Minimum-weighted spanning tree

Consider the graph shown in Figure 1.

1. Work through the algorithm done in class in detail to find a minimum-weighted spanning tree rooted at 0.
  2. In general there can be a number of MWSTs rooted at the same vertex. Give an example of a graph which has multiple MWSTs and in this case verify that the MWSTs have the same weight.
  3. Show that there are other MWST rooted at other vertices, but that they have the same weight.
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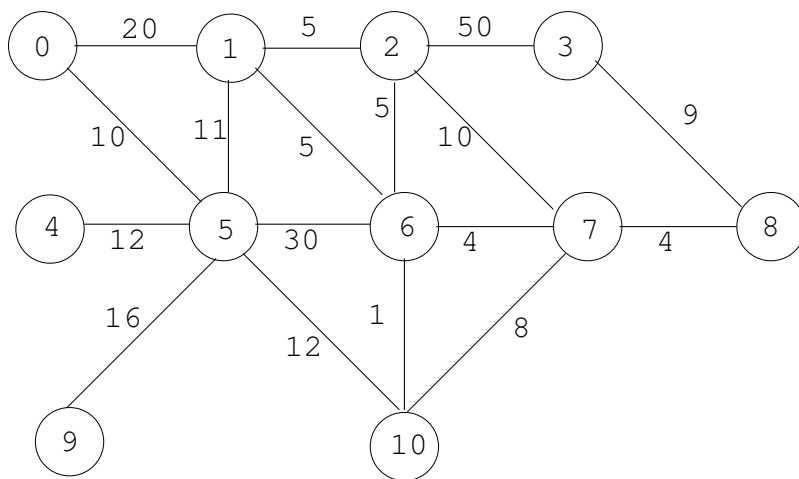


Figure 1: Graph for Search Trees