

Linear Algebra Exam 2021

Question 1

Question 1 8 pts

A linear operator $\mathcal{A} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is given by the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & -2 \\ 3 & -2 & 0 \end{pmatrix}$ in the standard basis. Find the matrix B of \mathcal{A} in the basis $\{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$.

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Question 1 8 pts

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Question 2

Question 2 8 pts

Prove that the characteristic polynomial of a linear operator does not depend on the choice of a basis.

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Question 2 8 pts

Prove that the characteristic polynomial of a linear operator does not depend on the choice of a basis.

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Question 3

Question 3 9 pts

Determine whether the matrix $A = \begin{pmatrix} -1 & 0 & 1 \\ 2 & 1 & -1 \\ 2 & 0 & 0 \end{pmatrix}$ is diagonalizable, and

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Question 3 9 pts

and if yes, find a diagonal matrix D and a matrix T such that $D = T^{-1}AT$.

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Question 4

Question 4 8 pts

From the Cauchy-Bunyakowski inequality deduce that for any vectors x, y

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Question 4 8 pts

e that for any vectors x, y of an inner product space, $\|x+y\| \leq \|x\| + \|y\|$.

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Question 5

Question 5 9 pts

Using the Gram-Schmidt process, transform the basis $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of \mathbb{R}^3 into an orthonormal basis.

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Question 5 9 pts

Transform the basis $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of \mathbb{R}^3 into an orthonormal basis.

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Question 6

Question 6 8 pts

Find a system of linear equations whose solution space is the subspace $\langle a_1, a_2 \rangle$ of \mathbb{R}^5 .

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Question 6 8 pts

$a_1, a_2, a_3, a_4 \subseteq \mathbb{R}^5$, where $a_1 = (2, 1, -1, 0, 1)$, $a_2 = (-1, 1, -2, -1, 0)$, $a_3 = (2, 0, 1, 0, -1)$, $a_4 = (3, 2, -2, -1, 0)$.

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Question 6 8 pts

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