COMS3008A: Parallel Computing
Introduction to MPI II

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Blocking vs. Non-blocking

- Most of the MPI point-to-point functions can be used in either blocking or non-blocking mode.
- Blocking:
  - A blocking send function will only “return” after it is safe to modify the application buffer (your send data) for reuse.
  - A blocking send can be synchronous which means there is handshaking occurring with the receive task to confirm a safe send.
  - A blocking send can be asynchronous if a system buffer is used to hold the data for eventual delivery to the receive.
  - A blocking receive only “returns” after the data has arrived and is ready for use by the program.

Blocking vs. Non-blocking cont.

- Non-blocking:
  - Non-blocking send and receive functions behave similarly - they will return almost immediately. They do not wait for any communication events to complete.
  - Non-blocking operations simply “request” the MPI library to perform the operation when it is able.
  - It is unsafe to modify the application buffer (your variable space) until you know for a fact the requested non-blocking operation was actually performed by the library.
  - Non-blocking communications are primarily used to overlap computation with communication and exploit possible performance gains.

Outline

MPI Message Passing Function Arguments

MPI point-to-point communication functions generally have an argument list that takes one of the following formats:

Blocking sends	`MPI_Send(buffer,count,type,dest,tag,comm)`
Non-blocking sends	`MPI_Isend(buffer,count,type,dest,tag,comm,request)`
Blocking receive	`MPI_Recv(buffer,count,type,source,tag,comm,status)`
Non-blocking receive	`MPI_Irecv(buffer,count,type,source,tag,comm,request)`

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Avoiding Deadlocks

- The semantics of `MPI_Send` and `MPI_Recv` place some restrictions on how we can mix and match send and receive operations.
- Sources of deadlocks:
  - Send a large message from one process to another process
    - If the receive buffer is not large enough at the destination, the send must wait for the user to provide the memory space (through a receive)
  - Mismatched send and receive – unsafe.

- What happens with

Process 0	Process 1
Send(1)	Send(0)
Recv(0)	Recv(1)

- Order the operations.

Process 0	Process 1
Send(1)	Recv(1)
Recv(0)	Send(0)

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Avoiding Deadlocks

Example 1

Process 0 sends two messages with different tags to process 1, and process 1 receives them in reverse order.

```
int a[10], b[10], myrank;
MPI_Status status;
...
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
if (myrank == 0) {
  MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
  MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
} else if (myrank == 1) {
  MPI_Recv(b, 10, MPI_INT, 0, 2, MPI_COMM_WORLD,
      MPI_STATUS_IGNORE);
  MPI_Recv(a, 10, MPI_INT, 0, 1, MPI_COMM_WORLD,
      MPI_STATUS_IGNORE);
}
...
```

Avoiding Deadlocks

Example 2

Consider the following piece of code, in which process i sends a message to process $i+1$ (modulo the number of processes) and receives a message from process $i-1$ (modulo the number of processes). (Note that this example is different from the token ring example in Lec7.)

```
int a[10], b[10], npes, myrank;
MPI_Status status;
...
MPI_Comm_size(MPI_COMM_WORLD, &npes);
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1,
                  MPI_COMM_WORLD);
MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1,
                  MPI_COMM_WORLD, MPI_STATUS_IGNORE);
...
```

Example 2 cont.

We can break the circular wait to avoid deadlocks as follows:

```
int a[10], b[10], npes, myrank;
MPI_Status status;
...
MPI_Comm_size(MPI_COMM_WORLD, &npes);
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
if (myrank%2 == 0) {
  MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1,
      MPI_COMM_WORLD);
  MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1,
      MPI_COMM_WORLD, MPI_STATUS_IGNORE);
} else {
  MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1,
      MPI_COMM_WORLD, MPI_STATUS_IGNORE);
  MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1,
      MPI_COMM_WORLD);
}
```

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Sending and Receiving Messages Simultaneously

To exchange messages, MPI provides the following function that both sends and receives a message:

```
int MPI_Sendrecv(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, int dest, int sendtag,
        void *recvbuf, int recvcount, MPI_Datatype
            recvdatatype,
        int source, int recvtag, MPI_Comm comm,
        MPI_Status *status)
```

The arguments include arguments to the send and receive functions.

Using MPI_Sendrecv in Example 2

Example 2 can be made “safe” by using `MPI_Sendrecv`:

```
int a[10], b[10], npes, myrank;
MPI_Status status;
...
MPI_Comm_size(MPI_COMM_WORLD, &npes);
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
MPI_Sendrecv(a, 10, MPI_INT, (myrank+1)%npes, 1, b, 10,
     MPI_INT, (myrank-1+npes)%npes, 1,
     MPI_COMM_WORLD, &status);
...
```

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Overlapping Communication with Computation

- In order to overlap communication with computation, MPI provides a pair of functions for performing non-blocking send and receive operations.

```
int MPI_Isend(void *buf, int count, MPI_Datatype datatype,
        int dest, int tag, MPI_Comm comm,
        MPI_Request *request)
int MPI_Irecv(void *buf, int count, MPI_Datatype datatype,
        int source, int tag, MPI_Comm comm,
        MPI_Request *request)
```

- These operations return before the operations have been completed. Function `MPI_Test` tests whether or not the non-blocking send or receive operation identified by its `request` has finished.
- ```
  int MPI_Test(MPI_Request *request, int *flag,
          MPI_Status *status)
  ```
  ```

- `MPI_Wait` blocks until a specified non-blocking send or receive operation has completed. For multiple non-blocking operations, the programmer can specify `all` completions.

  ```
  int MPI_Wait(MPI_Request *request, MPI_Status *status)
  ```

- `MPI_Request` handle is used to determine whether an operations has completed.
  - Non-blocking wait: `MPI_Test`
  - Blocking wait: `MPI_Wait`
- Anywhere you use `MPI_Send` or `MPI_Recv`, you can use the pair of `MPI_Isend/MPI_Wait` or `MPI_Irecv/MPI_Wait`.
- It is sometimes desirable to wait on multiple requests:
  - ```
    MPI_Waitall(int count,
    MPI_Request array_of_requests[],
            MPI_Status array_of_statuses[])
    ```
  - The corresponding version of `MPI_Test`

    ```
    int MPI_Testall(int count,
    MPI_Request array_of_requests[], int *flag,
    MPI_Status array_of_statuses[])
    ```

    `flag`: true if all the requests are completed, otherwise false.

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    ```

Example 3

```
int main(int argc, char *argv[]){
    int myid, numprocs, left, right, flag=0;
    int buffer1[10], buffer2[10];
    MPI_Request request; MPI_Status status;
    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD, &numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD, &myid);
    /* initialize buffer2 */
    ......
    right = (myid + 1) % numprocs;
    left = myid - 1;
    if (left < 0)
        left = numprocs - 1;
    MPI_Irecv(buffer1, 10, MPI_INT, left, 123,
       MPI_COMM_WORLD,
        &request);
    MPI_Send(buffer2, 10, MPI_INT, right, 123,
       MPI_COMM_WORLD);
    MPI_Test(&request, &flag, &status);
    while (!flag){
        /* Do some work ... */
        MPI_Test(&request, &flag, &status);
    }
    MPI_Finalize();
}
```

Example 4

```
int main(int argc, char *argv[]){
  int numtasks, rank, next, prev, buf[2], tag1=1, tag2=2;
    MPI_Request reqs[4]; MPI_Status stats[4];
    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD, &numtasks);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);

    prev = rank-1; next = rank+1;
    if (rank == 0)  prev = numtasks - 1;
    if (rank == (numtasks - 1))  next = 0;
    MPI_Irecv(&buf[0],1,MPI_INT,prev,tag1,MPI_COMM_WORLD,
      &reqs[0]);
    MPI_Irecv(&buf[1],1,MPI_INT,next,tag2,MPI_COMM_WORLD,
      &reqs[1]);

    MPI_Isend(&rank,1,MPI_INT,prev,tag2,MPI_COMM_WORLD,
      &reqs[2]);
    MPI_Isend(&rank,1,MPI_INT,next,tag1,MPI_COMM_WORLD,
      &reqs[3]);
    MPI_Waitall(4, reqs, stats);
    MPI_Finalize();
}
```

Example 5

- We can use **the trapezoidal rule** to approximate the area between the graph of a function, $y = f(x)$, two vertical lines, and the x-axis.

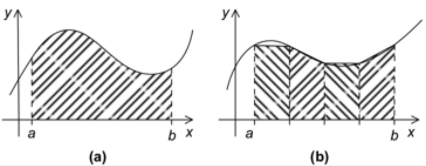

Figure: The trapezoidal rule: (a) area to be estimated, (b) estimate area using trapezoids

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Example 5 cont.

- If the endpoints of the subinterval are x_i and x_{i+1}, then the length of the subinterval is $h = x_{i+1} - x_i$. Also, if the lengths of the two vertical segments are $f(x_i)$ and $f(x_{i+1})$, then the area of the trapezoid is

$$\text{Area of one trapezoid} = \frac{h}{2}(f(x_i) + f(x_{i+1})).$$

- Since we chose the $\mathtt{N}$ subintervals, we also know that the bounds of the region are $x = a$ and $x = b$ then

$$h = \frac{b-a}{N}$$

Example 5 cont.

- The pseudo code for a serial program:

```
h = (b-a)/N;
approx = (f(a) + f(b))/2.0;
for(i=1; i<=n-1; i++){
    x_i = a + i * h;
    approx += f(x_i);
}
approx = h * approx;
```

Recall we can design a parallel program using four basic steps:

1. Partition the problem solution into tasks.
2. Identify the communication between the tasks.
3. Aggregate the tasks into composite tasks.
4. Map the composite tasks to cores.

Example 5: Parallel Algorithm for the Trapezoidal Rule

Assuming `comm_sz` evenly divides *n*, the pseudo-code for the parallel program looks like the following:

```
Get a, b, n;
h = (b - a)/n;
local_n = n/comm_sz;
local_a = a + my_rank * local_n * h;
local_b = local_a + local_n * h;
local_integral = Trap(local_a, local_b, local_n, h);
if (my_rank != 0)
  Send local integral to process 0;
else {/* my_rank == 0 */
  total_integral = local_integral;
  for (proc = 1; proc < comm_sz; proc++) {
    Receive local_integral from proc;
    total_integral += local_integral;
  }
}
if (my_rank == 0)
  print result;
```

Dealing with I/O

- In most cases, all the processes in `MPI_COMM_WORLD` have access to `stdout` and `stderr`.
- The order in which the processes' output appears is indeterministic.
- For the input, i.e., `stdin`, usually, only process 0 has access to.
- If an MPI program uses `scanf` function, then process 0 reads in the data, and sends it to the other processes.

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Collective Communications

- Communication is coordinated among a group of processes, as specified by the communicator.
- All collective operations are blocking and no message tags are used.
- All processes in the communicator must call the collective operation.
- Three classes of collective operations
    - Data movement
    - Collective computation
    - Synchronization

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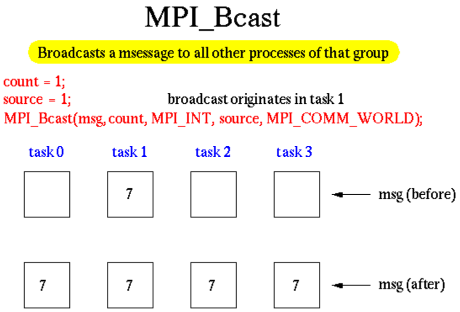
MPI_Bcast

- A collective communication in which data belonging to a single process is sent to all of the processes in the communicator is called a broadcast — `MPI_Bcast`.



- The process with rank `source` sends the contents of the memory referenced by `msg` to all the processes in the communicator `MPI_COMM_WORLD`.

Example 5 Cont.

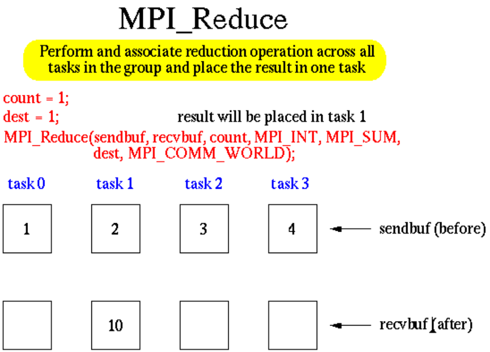
1. In our example `mpi_trapezoid_1.c`, we are using

```c
if (my_rank == 0) {
  for(dest = 1; dest < comm_sz; dest++){
    MPI_Send(a_p, 1, MPI_DOUBLE, dest, 0,
        MPI_COMM_WORLD);
    MPI_Send(b_p, 1, MPI_DOUBLE, dest, 0,
        MPI_COMM_WORLD);
    MPI_Send(n_p, 1, MPI_INT, dest, 0, MPI_COMM_WORLD
        );
  }
} else {/* my rank != 0 */
    MPI_Recv(a_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD
        , MPI_STATUS_IGNORE);
    MPI_Recv(b_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD
        , MPI_STATUS_IGNORE);
    MPI_Recv(n_p, 1, MPI_INT, 0, 0, MPI_COMM_WORLD,
        MPI_STATUS_IGNORE);
}
```

2. Instead of using point-to-point communications, you can use collective communications here. Write another function to implement this part using — `MPI_Bcast()`.

MPI_Reduce

- `MPI_Reduce` combines data from all processes in the communicator and returns it to one process.
- In many numerical algorithms, `Send/Receive` can be replaced by `Bcast/Reduce`, improving both simplicity and efficiency.



MPI_Reduce cont.

- When the `count` is greater 1, `MPI_Reduce` operates on arrays instead of scalars.

```
double local_x[N], sum[N];
...
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0, \
    MPI_COMM_WORLD);
```

Example 5 cont.

1. In our example `mpi_trapezoid_1.c`, we are using

```c
/* Add up the integrals calculated by each process */
  if (my_rank != 0) {
    MPI_Send(&local_int, 1, MPI_DOUBLE, 0, 0,
         MPI_COMM_WORLD);
  } else {
    total_int = local_int;
    for (source = 1; source < comm_sz; source++) {
      MPI_Recv(&local_int, 1, MPI_DOUBLE, source, 0,
           MPI_COMM_WORLD, MPI_STATUS_IGNORE);
      total_int += local_int;
    }
```

2. Instead of using point-to-point communications, you can also use collective communications here. Rewrite this part using appropriate collective communication.

MPI_Reduce

- Suppose that each process calls `MPI_Reduce` with operator `MPI_SUM`, and destination process 0. What happens with the following multiple calls of `MPI_Reduce`? What are the values for `b` and `d`?

Time	Process 0	Process 1	Process 2
0	`a=1; c = 2;`	`a=1; c = 2;`	`a=1; c = 2;`
1	`MPI_Reduce(&a,&b,1,...)`	`MPI_Reduce(&c,&d,1,...)`	`MPI_Reduce(&a,&b,1,...)`
2	`MPI_Reduce(&c,&d,1,...)`	`MPI_Reduce(&a,&b,1,...)`	`MPI_Reduce(&c,&d,1,...)`

- The order of the calls will determine the matching.
- What will happen with the following code?

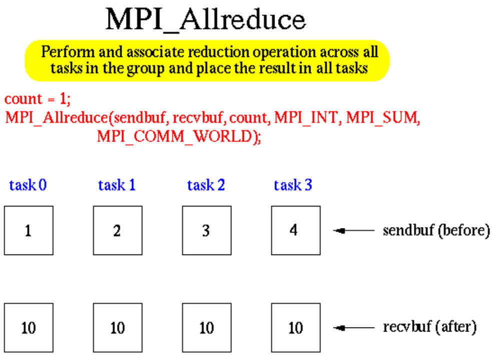
```
MPI_Reduce(&x, &x, 1, MPI_DOUBLE, MPI_SUM, 0, comm);
```

MPI_Allreduce

- If the result of the reduction operation is needed by all processes, MPI provides:

```
int MPI_Allreduce(void *sendbuf, void *recvbuf, int count,
MPI_Datatype datatype, MPI_Op op, MPI_Comm comm)
```

- This is equivalent to an `MPI_Reduce` followed by an `MPI_Bcast`.



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MPI_Allreduce cont.

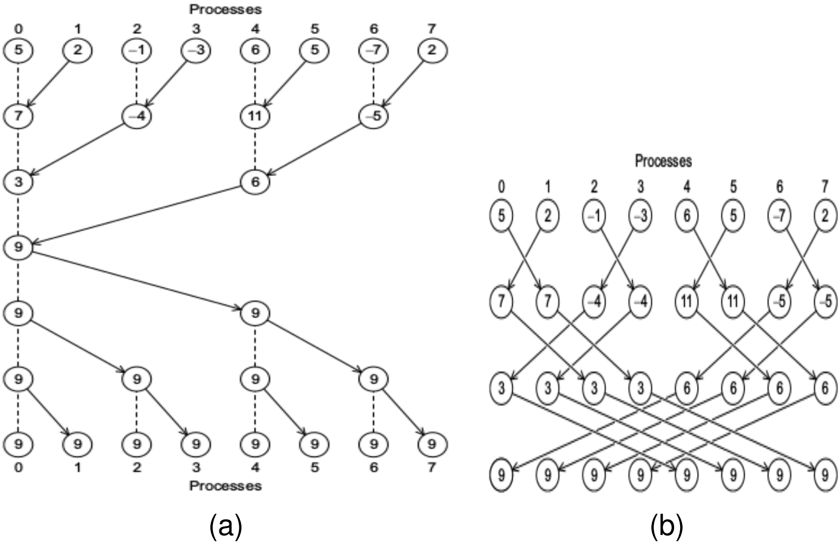


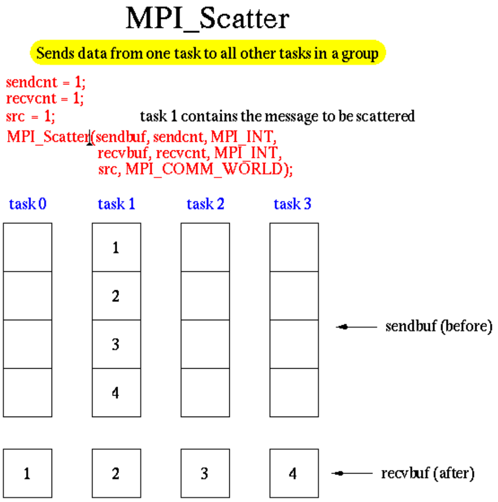
Figure: (a) A global sum followed by a broadcasting; (2) A butterfly structured global sum.

MPI_Scatter

- The scatter operation is to distribute distinct messages from a single source task (or process) to each task in the group.

```
int MPI_Scatter(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, void *recvbuf,
        int recvcount, MPI_Datatype recvdatatype,
        int source, MPI_Comm comm)
```

MPI_Scatter cont.



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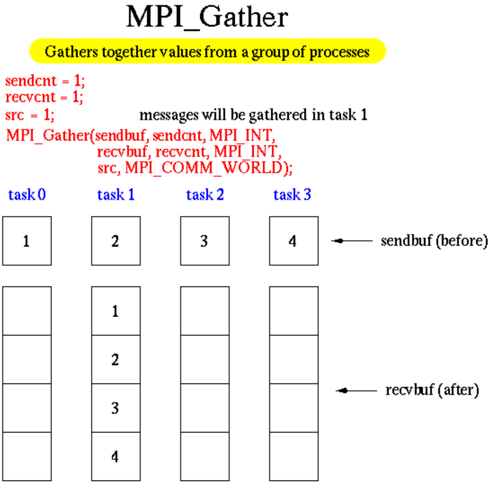
MPI_Gather

- The gather operation is performed in MPI using `MPI_Gather`.
  - Gathers distinct messages from each task in the group to a single destination task.
  - Reverse operation of `MPI_Scatter`.

```
int MPI_Gather(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, void *recvbuf,
        int recvcount, MPI_Datatype recvdatatype,
        int target, MPI_Comm comm)
```

MPI_Gather cont.



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MPI_Allgather

- MPI also provides the `MPI_Allgather` function in which the data are gathered at all the processes.

```
int MPI_Allgather(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, void *recvbuf,
        int recvcount, MPI_Datatype recvdatatype,
        MPI_Comm comm)
```

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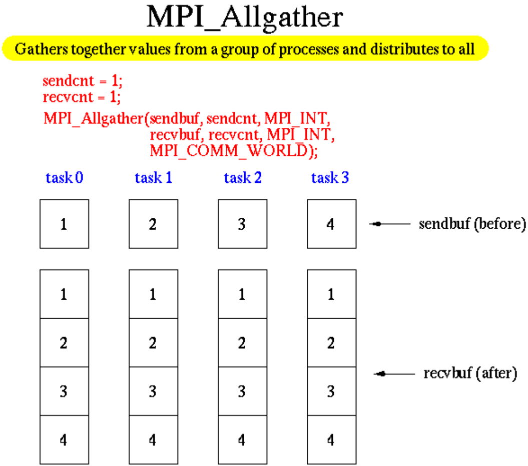
MPI_Allgather cont.



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MPI_Alltoall

- The all-to-all communication operation is performed by:

```
int MPI_Alltoall(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, void *recvbuf,
        int recvcount, MPI_Datatype recvdatatype,
        MPI_Comm comm)
```

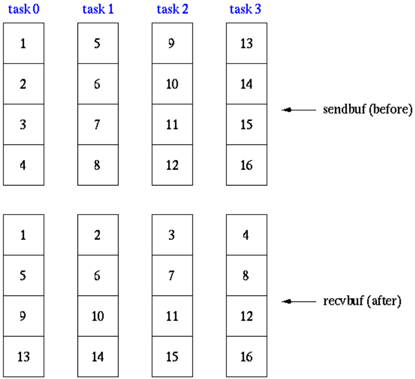
- Each task in a group performs a scatter operation, sending a distinct message to all the tasks in the group in order by index.

MPI_Alltoall cont.

MPI_Alltoall

Sends data from all to all processes. Each process performs a scatter operation.

```
sendcnt = 1;
recvcnt = 1;
MPI_Alltoall(sendbuf, sendcnt, MPI_INT,
             recvbuf, recvcnt, MPI_INT,
             MPI_COMM_WORLD);
```



Example 6 (Matrix vector multiplication)

If $A = (a_{ij})$ is an $m \times n$ matrix and $\mathbf{x}$ is a vector with n components, then $\mathbf{y} = A\mathbf{x}$ is a vector with m components. Furthermore,

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{i,n-1}x_{n-1}.$$

A serial code can be as simple as

```
    for (i = 0; i < m; i++) {
      y[i] = 0.0;
      for (j = 0; j < n; j++)
        y[i] += A[i*n+j]*x[j];
    }
```

Example 6 cont.

Process 0 reads in the matrix and distributes row blocks to all the processes in communicator `comm`.

```
    if (my_rank == 0) {
      A = malloc(m*n*sizeof(double));
      if (A == NULL) local_ok = 0;
      Check_for_error(local_ok, "Random_matrix", "Can't allocate temporary matrix", comm);
      srand(2021);
      for (i = 0; i < m; i++)
        for (j = 0; j < n; j++)
          A[i*n+j] = (double)rand( ) / RAND_MAX;
      MPI_Scatter(A, local_m*n, MPI_DOUBLE, local_A, local_m*n, MPI_DOUBLE, 0, comm);
      free(A);
    } else {
      Check_for_error(local_ok, "Random_matrix", "Can't allocate temporary matrix", comm);
      MPI_Scatter(A, local_m*n, MPI_DOUBLE, local_A, local_m*n, MPI_DOUBLE, 0, comm);
    }
```

Example 6 cont.

Each process gathers the entire vector, then proceeds to compute its share of sub-matrix and vector multiplication.

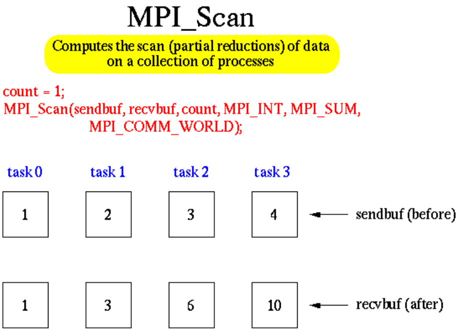
```
    MPI_Allgather(local_x, local_n, MPI_DOUBLE,
      x, local_n, MPI_DOUBLE, comm);

    for (local_i = 0; local_i < local_m; local_i++) {
      local_y[local_i] = 0.0;
      for (j = 0; j < n; j++)
        local_y[local_i] += local_A[local_i*n+j]*x[j];
    }
```

MPI_Scan

- To compute prefix-sums, MPI provides:

```
int MPI_Scan(void *sendbuf, void *recvbuf, int count,
        MPI_Datatype datatype, MPI_Op op, MPI_Comm comm)
```



- Using this core set of collective operations, MPI communications can be greatly simplified.

Outline

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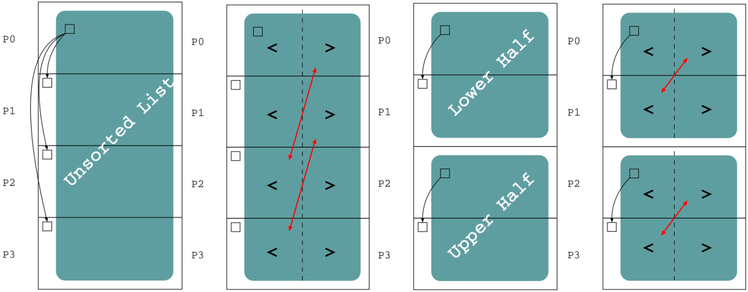
Parallel quicksort

- one process broadcast initial pivot to all processes;
- each process in the upper half swaps with a partner in the lower half
- recurse on each half
- swap among partners in each half
- each process uses quicksort on local elements

Parallel quicksort cont.



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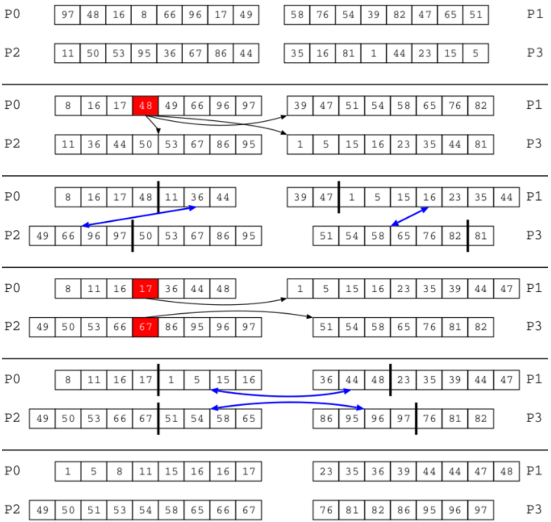
Hyperquicksort

Limitation of parallel quicksort: poor balancing of list sizes.
Hyperquicksort: sort elements before broadcasting pivot.

- sort elements in each process
- select median as pivot element and broadcast it
- each process in the upper half swaps with a partner in the lower half
- recurse on each half

Hyperquicksort cont.



Example 7

Task 0 pings task 1 and awaits return ping

```
#include "mpi.h"
#include <stdio.h>
  main(int argc, char *argv[]){
  int numtasks, rank, dest, source, rc, count, tag=1;
  char inmsg, outmsg='x';
  MPI_Status Stat;

  MPI_Init(&argc,&argv);
  MPI_Comm_size(MPI_COMM_WORLD, &numtasks);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);

  if (rank == 0){
    dest = 1;
    source = 1;
    rc = MPI_Send(&outmsg, 1, MPI_CHAR, dest, tag,
        MPI_COMM_WORLD);
    rc = MPI_Recv(&inmsg, 1, MPI_CHAR, source, tag,
        MPI_COMM_WORLD, &Stat);
  }
```

Example 7 cont.

```
  else if (rank == 1) {
    dest = 0;
    source = 0;
    rc = MPI_Recv(&inmsg, 1, MPI_CHAR, source, tag,
        MPI_COMM_WORLD, &Stat);
    rc = MPI_Send(&outmsg, 1, MPI_CHAR, dest, tag,
        MPI_COMM_WORLD);
  }

  rc = MPI_Get_count(&Stat, MPI_CHAR, &count);
  printf("Task %d: Received %d char(s) from task %d with
      tag %d \n",
         rank, count, Stat.MPI_SOURCE, Stat.MPI_TAG);


  MPI_Finalize();
}
```

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Example 8

Perform a scatter operation on the rows of an array

```
#include "mpi.h"
#include <stdio.h>
#define SIZE 4

main(int argc, char *argv[])  {
  int numtasks, rank, sendcount, recvcount, source;
  float sendbuf[SIZE][SIZE] = {
      {1.0, 2.0, 3.0, 4.0},
      {5.0, 6.0, 7.0, 8.0},
      {9.0, 10.0, 11.0, 12.0},
      {13.0, 14.0, 15.0, 16.0}  };
  float recvbuf[SIZE];

  MPI_Init(&argc,&argv);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  MPI_Comm_size(MPI_COMM_WORLD, &numtasks);
```

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Example 8 cont.

```c
 if (numtasks == SIZE) {
   source = 1;
   sendcount = SIZE;
   recvcount = SIZE;
   MPI_Scatter(sendbuf,sendcount,MPI_FLOAT,recvbuf,
      recvcount,
           MPI_FLOAT,source,MPI_COMM_WORLD);
      printf("rank= %d  Results: %f %f %f %f\n",rank,
         recvbuf[0],recvbuf[1],recvbuf[2],recvbuf[3]);
   }else
     printf("Must specify %d processors. Terminating.\n"
        ,SIZE);
   MPI_Finalize();
```

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Example 9

The Odd-Even Transposition Sort

- Sorts n elements in n phases (n is even), each of which requires $n/2$ compare-exchange operations.
- The algorithm alternates between two phases — odd and even phases.
- Let $< a_0, a_1, ..., a_{n-1} >$ be the sequence to be sorted.
  - During the odd phase, elements with odd indices are compared with their right neighbours, and if they are out of sequence they are exchanged; thus, the pairs $(a_1, a_2), (a_3, a_4), \ldots, (a_{n-3}, a_{n-2})$ are compare exchanged.
  - During the even phase, elements with even indices are compared with their right neighbours, and if they are out of sequence they are exchanged; $(a_0, a_1), (a_2, a_3), \ldots, (a_{n-2}, a_{n-1})$.
- After n phases of odd-even exchanges, the sequence is sorted. Each phase requires $n/2$ compare-exchange operations (sequential complexity $O(n^2)$).

Example 9 cont. – The serial algorithm

```
for i = 0 to n-1 do
  if i is even then
    for j = 0 to n/2 - 1 do
      compare-exchange(a(2j), a(2j+1));
  if i is odd then
    for j = 0 to n/2 - 1 do
      compare-exchange(a(2j+1), a(2j+2));
```

Example 9 cont. – The parallel algorithm

```
void oddevensort(int n)
  id = process's label;
  for i =0 to n-1 do
    if i is odd then
      if id is odd then
        compare-exchange_min(id, id + 1);//increasing
            comparator
      else
        compare-exchange_max(id, id - 1);//decreasing
            comparator
    if i is even then
      if id is even then
        compare-exchange_min(id, id + 1);
      else
        compare-exchange_max(id, id - 1);
```

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Outline

Summary

- Point-to-point communication
  - Blocking vs non-blocking
  - Safety in MPI programs
- Collective communication
  - Collective communications involve all the processes in a communicator.
  - All the processes in the communicator must call the same collective function.
  - Collective communications do not use tags, the message is matched on the order in which they are called within the communicator.
  - The meanings of *local variable* and *global variable* in MPI
  - Some important MPI collective communications we learned: `MPI_Reduce`, `MPI_Allreduce`, `MPI_Bcast`, `MPI_Gather`, `MPI_Scatter`, `MPI_Allgather`, `MPI_Alltoall`, `MPI_Scan` etc.

Bibliography

Useful references:

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