14h00

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| 01 hrs | 29 / 05 / 2015 | Graduation Office ~~Flower Hall~~ | Exams Office Use Only |

**University of the Witwatersrand, Johannesburg**

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|---|---|
| Course or topic No(s) | MATH2019 |
| Course or topic name(s) Paper number & title | Linear Algebra |
| Examination/Test* to be held during month(s) of (*delete as applicable) | June Examination |
| Year of study (Art & Sciences leave blank) | Second Year |
| Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch) | BSc; BCom; BA |
| Faculty/ies presenting candidates | Science, Commerce; Humanities |
| Internal examiner(s) and telephone number(s) | Prof Ye Zelenyuk Ext 76247 |
| External examiner(s) | Dr D Shkatov Ext 79999 |
| Calculator policy | |

Time allowance

| Course No's | MATH2019 | Hours | 1h00 |
|---|---|---|---|

Instruction to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

**Answer *ALL* questions.**
**Show all working**
**Total : 60**
**Duration : 1 hour**

Internal Examiners or Heads of Department are requested to sign the Declaration overleaf

## Linear Algebra Exam 2015

**Question 1** The linear operator $\mathcal{A} : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

in the standard basis. Find the matrix $B$ of $\mathcal{A}$ in the basis $\{(2, 0, 5), (-1, 1, -1), (1, 0, 3)\}$.

[10]

**Question 2** Prove that the characteristic polynomial of a linear operator does not depend on the choice of a basis.

[10]

**Question 3** Determine whether the matrix

$$A = \begin{pmatrix} -4 & 0 & 6 \\ -3 & -1 & 6 \\ -3 & 0 & 5 \end{pmatrix}$$

is diagonalizable, and if yes, find a diagonal matrix $D$ and a matrix $T$ such that $D = T^{-1}AT$.

[10]

**Question 4** Prove that for any vectors $x$, $y$ of an inner product space,

$$|(x, y)| \leq ||x|| \cdot ||y||.$$

[10]

**Question 5** Using the Gram-Schmidt process, transform the basis $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of $\mathbb{R}^3$ into an orthonormal basis.

[10]

**Question 6** Find a system of linear equations whose solution space is the subspace $\langle a_1, a_2, a_3 \rangle \subseteq \mathbb{R}^5$, where

$$a_1 = (1, 1, 0, 1, -1), a_2 = (1, 1, -1, 0, 1), a_3 = (1, 0, 1, -1, 1).$$

[10]