

# COMS 3003A

## Tutorial 2: Modelling data and computational problems

DMITRY SHKATOV

22 February, 2024

### Reading:

**Boaz Barak. Introduction to Theoretical Computer Science. Sections 2.1, 2.2, 2.3, and 2.6.**

1. Which one of the following types of object can be represented by a binary string?
    - (1) integers;
    - (2) rational numbers;
    - (3) matrices;
    - (4) finite directed graphs;
    - (5) finite undirected graphs;
    - (6) files containing Python source code;
    - (7) all of the above.
  2. Write a Python program that converts English words given as input into binary strings. You will need to devise your own scheme for converting letters of the Latin alphabet into binary strings.
  3. We're representing rational numbers in the alphabet  $\Sigma = \{0, 1, \#\}$  using the following scheme:
    - (1) to represent an integer, we use the first bit to indicate a sign—0 means ‘positive’, 1 means ‘negative’—and the rest of the bits to represent a natural number;
    - (2) if  $n$  and  $m$  are integers, we represent the rational number  $\frac{n}{m}$  as  $\langle n \rangle \# \langle m \rangle$ , where  $\langle x \rangle$  is the representation of the integer  $x$  according to (1);
    - (3) if a string does not represent any rational number according to (1) and (2), we take it to be a representation of 0.
- (a) Which rational numbers are represented by the following strings of  $\Sigma$ ?
- (i) 0101#01000;
  - (ii) 0011#11010;
  - (iii) 0111#11000;
  - (iv) 1101#1011.

- (b) For each string  $s$  from Question (a), determine its length  $|s|$ .
  - (c) Give at least 5 different representations of 0.
  - (a) Write a Python program that converts a representation of a rational number given as input in the form  $\pm n / \pm m$ , where  $n$  and  $m$  are decimal representations of natural numbers, into the representation defined in (1) through (3).
4. (a) If we represent natural numbers in binary, what is the length of the representation of the number  $n$ ?
- (b) If we represent natural numbers in  $k$ -ary, with  $k > 1$ , what is the length of the representation of the number  $n$ ?
- (c) What is the ratio between representations of natural numbers in binary and in  $k$ -ary if  $k > 1$ ?
- (d) What is the ratio between representations of natural numbers in binary and in unary?
- (e) What is the ratio between representations of natural numbers in  $k$ -ary, with  $k > 1$ , and in unary?
5. Consider the following algorithm, written here in Python-style pseudo-code (an input is a string  $\langle n \rangle$  representing a natural number  $n$ ):

```

if n < 2:
    return False

for factor in range(2,n):
    if n % factor == 0:
        return False

return True

```

- (a) Determine the running time of this algorithm under the assumption that input is represented in unary.
  - (b) Determine the running time of this algorithm under the assumption that input is represented in binary.
  - (c) Determine the running time of this algorithm under the assumption that input is represented in  $k$ -ary, with  $k > 1$ .
6. Use questions 4. and 5. to work out why using unary representation of numbers might be undesirable in theoretical computer science.

7. Consider the following algorithm, written here in Python (input is given as a binary string  $s$  representing a natural number; 0 is odd):

```
def isEven( s ):
    if s[-1] == '0':
        return True

    else
        return False
```

Does this algorithm solve the problem of testing a natural number for evenness?

8. Consider the following algorithm, written here in Python (input is given as a binary string  $s$  representing a natural number):

```
def isProductOfPrimes( s ):
    if s[0] == '0':
        return False

    elif s == '1':
        return False

    else
        return True
```

Does this algorithm solve the problem of finding out if a natural number can be written as a product of prime numbers?