09:00 hrs	01 / 06 / 2018	Examinations and Graduation Office ~~Central Block Exams Hall~~	Exams Office Use Only

University of the Witwatersrand, Johannesburg

Course or topic No(s)	MATH2019
Course or topic name(s) Paper number & title	Linear Algebra
Examination/Test* to be held during month(s) of (*delete as applicable)	June Exam
Year of study (Art & Sciences leave blank)	Second Year
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	BSc, Bcom, BA
Faculty/ies presenting candidates	Science, Commerce, Humanities
Internal examiner(s) and telephone number(s)	Prof Y Zelenyuk Ext 76247 Dr R Kwashira Ext 76228 Dr M Folly-Gbetoula Ext 76289
External examiner(s)	Dr A Davison
Calculator policy	

Time allowance

Course No's	MATH2019	Hours	1h00

Instruction to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

Answer all questions
Total : 60
Duration : 1h00

Linear Algebra Exam 2018

Question 1 The linear operator $\mathcal{A} : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

in the standard basis. Find the matrix B of $\mathcal{A}$ in the basis $\{(2,0,5), (-1,1,-1), (1,0,3)\}$.

[10]

Question 2 Prove that the characteristic polynomial of a linear operator does not depend on the choice of a basis.

[10]

Question 3 Determine whether the matrix

$$A = \begin{pmatrix} -4 & 0 & 6 \\ -3 & -1 & 6 \\ -3 & 0 & 5 \end{pmatrix}$$

is diagonalizable, and if yes, find a diagonal matrix D and a matrix T such that $D = T^{-1}AT$.

[10]

Question 4 From the Cauchy-Bunyakowski inequality deduce that for any vectors x, y of an inner product space, $||x+y|| \leq ||x|| + ||y||$.

[10]

Question 5 Using the Gram-Schmidt process, transform the basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$ into an orthonormal basis.

[10]

Question 6 Find a system of linear equations whose solution space is the subspace $\langle a_1, a_2, a_3 \rangle \subseteq \mathbb{R}^5$, where

$$a_1 = (1,1,1,1,-1), a_2 = (1,1,-1,1,1), a_3 = (1,1,1,-1,1).$$

[10]