			EXAMS OFFICE USE ONLY

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Course or topic No(s)	APPM 2007A/APPM2020A
Course or topic name(s) Paper Number & title	METHODS A: APPLIED ORDINARY DIFFERENTIAL AND DIFFERENCE EQUATIONS
Examination/Test to be held during month(s) of *(delete as applicable)*	JUNE 2022 EXAMINATION
Year of Study *(Art & Science leave blank)*	SECOND
Degrees/Diplomas for which This course is prescribed (BSc (Eng) should indicate which branch)	BSc
Faculty/ies presenting Candidates	SCIENCE
Internal examiners(s) And telephone extension number(s)	DR I.S. OYELAKIN X76107
External examiner(s)	DR. SICELO GOQO
Special materials required (graph/music/drawing paper) maps, diagrams, tables computer cards, etc.	NONE

Time allowance

Course No.(s)	APPM2007A/ APPM2020A	**Hours**	2 hrs

Instructions to candidates
(Examiners may wish to use this space to indicate, *inter alia* the contribution made by this examination or test towards the year mark if appropriate)

ATTEMPT ALL QUESTIONS
ONLY NON-PROGAMMABLE SCIENTIFIC CALCULATORS ARE PERMITTED
NO CELLPHONES ALLOWED
Total Marks Available= 57
100% = 50 marks

School of Computer Science and Applied Mathematics

APPM2007/APPM2020A: Methods A - Applied ordinary differential and difference equations

June Examination — 2022

Lecturer: Dr Ibukun Oyelakin **Total Marks: 57** **Time: 2 hours**

- **Answer all questions and show all workings.**
- **In all the questions below, the prime $'$ stands for differentiation with respect to x and overdot $\dot{x}$ stands for differentiation with respect to t.**
- **This exam has 4 questions, for a total of 57 marks but 50 marks is full marks.**

QUESTION ONE [10 MARKS]

(a) Consider the linear inhomogeneous first order ordinary differential equation

$$y' + a(x)y = b(x). \qquad (\dagger)$$

Show that if A is an arbitrary constant, then

$$y = e^{-\int a(x)dx}\left(\int b(x)e^{\int a(x)dx}dx + A\right)$$

is the solution to the ordinary differential equation given in $(\dagger)$. **[3 Marks]**

(b) Hence or otherwise, find the general solution to the first order linear inhomogeneous ordinary differential equation

$$\frac{dy}{dx} - 2y = e^{\lambda x} \qquad (\bullet)$$

where λ is a constant, such that $\lambda \neq 2$. **[3 Marks]**

(c) Solve equation $(\bullet)$ with $\lambda = 2$ and find the particular solution if $y(1) = 2$. **[4 Marks]**

QUESTION TWO [12 MARKS]

(a) Evaluate and simplify as much as possible, the integral **[2 Marks]**

$$\int \frac{x+1}{x^2-6x+8} \quad dx$$

(b) Find the differential equation $y'' = f(x, y, y')$ whose general solution is **[2 Marks]**

$$y = C_1 e^x + C_2 e^{-x} + x.$$

(c) Consider the first order ordinary differential equation

$$\frac{2x^2+3y^2-20}{xy} dy = -dx.$$

(i) Find an appropriate integrating factor to make the differential equation exact. **[3 Marks]**

(ii) Show that the equation together with its integrating factor is exact. **[2 Marks]**

(iii) Find the solution to the exact ordinary differential equation obtained in (ii). **[3 Marks]**

QUESTION THREE [15 MARKS]

(a) Given the second order Euler or Cauchy differential equation as

$$x^2 \frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$

where a and b are constants.

(i) Use the substitution $y = x^r$ to show that the auxiliary equation associated with the Euler or Cauchy equation is **[2 Marks]**

$$r^2 + (a-1)r + b = 0.$$

(ii) If the Euler or Cauchy equation have real and equal roots $r_1 = r_2 = r$, find the values of a and b with respect to r. **[2 Marks]**

(iii) Given that one of the solutions to the Euler or Cauchy equation is $y_1 = x^r$, use reduction of order and the values of a and b in question (ii) to find a second solution y_2. **[8 Marks]**

(b) Hence or otherwise, find the general solution to the Euler or Cauchy equation **[3 Marks]**

$$4x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + y = 0.$$

QUESTION FOUR [20 MARKS]

(a) Find the general solution to the non-homogeneous second order difference equation

$$y_{k+2} + 2y_{k+1} + y_k = 2(3^k)$$

[4 Marks]

(b) Find the exponential matrix e^{At} if

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

[6 Marks]

(c) Consider the first order system of ordinary differential equation

$$\begin{aligned} \dot{x}_1 &= 2x_1 + 3x_2 \qquad x_1(0) = 2 \\ \dot{x}_2 &= 2x_1 + x_2 \qquad x_2(0) = 1. \end{aligned}$$

(i) Write the system in vector-matrix form and its corresponding initial values in vector form. **[3 Marks]**

(ii) Find a general solution to the system. **[7 Marks]**