

**APPM2007  
Lagrangian Mechanics****Tutorial 1**

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**Question 1**

(10 Points)

Methane is a chemical compound with the chemical formula  $CH_4$  (one atom of carbon and four atoms of hydrogen). Methane is a tetrahedral molecule with four equivalent  $C - H$  bonds. Show that the angle subtended at the central Carbon atom by any two Hydrogen atoms in a methane molecule is  $109.5^\circ$ . (Hint: use the symmetry of the tetrahedron structure of the  $CH_4$  molecule to aid your computations.)

**Question 2**

(5 Points)

Consider the space  $\mathbb{R}^2$ . Compare the area element of the Rectilinear and Polar Co-ordinate systems on  $\mathbb{R}^2$ . Do the Rectilinear and Polar Co-ordinate area elements co-incide?

**Question 3**

(10 Points)

Consider the plane  $\mathbb{R}^2$  and the unit Sphere  $S^2$  with north pole marked  $p$ . Construct the following co-ordinate mappings

1.  $f : U \rightarrow S^2 \setminus \{p\}$  with  $U \subset \mathbb{R}^2$  is finite. (Hint: use Riemann Normal Co-ordinates)
2.  $f : \mathbb{R}^2 \rightarrow S^2 \setminus \{p\}$ . (Hint: use a co-ordinate projection where points in  $c = \{z \in \mathbb{R}^2 : \|z\| \leq 1\}$  are mapped to the southern hemisphere of  $S^2$  and points outside  $c$  are mapped to the northern hemisphere of  $S^2$ .)

Compare the qualitative differences and similarities of these two co-ordinatisation maps.

**Question 4**

(10 Points)

Consider a marked point in  $p \in \mathbb{R}^3$ . Let  $\alpha$ ,  $\beta$  and  $\gamma$  denote the angles subtended at the origin by the vector  $\vec{p}$  and each of the co-ordinate axes  $\hat{x}$ ,  $\hat{y}$  and  $\hat{z}$ . Show that

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1.$$