09:00 hrs | 18 / 11 /2019

Examinations and Graduation Office

Central Block Exams Hall

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University of the Witwatersrand, Johannesburg

Course or topic No(s)	MATH2001
Course or topic name(s) Paper number & title	BASIC ANALYSIS II
Examination/Test* to be held during month(s) of (*delete as applicable)	November Exam - 2019
Year of study (Art & Sciences leave blank)	2nd Year
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	BSc, Bcom, BA
Faculty/ies presenting candidates	Science, Commerce, Humanities
Internal examiner(s) and telephone number(s)	Prof O Olele Otafudu 76216 Dr R Maartens 76232
External examiner(s)	Dr D Ralaivaosaona – Stellenbosch University
Calculator policy	
Time allowance	Course No's: MATH2001; Hours: 1h00
Instruction to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)	**Answer all questions** **Total : 60** **Duration : 1h00**

MATH2001 Basic Analyis Exam November 2019

First Examiner: Prof. O. Olela Otafudu
Second Examiner: Mr. Ronnie Maartens
Moderator: Dr Dimbinaina Ralaivaosaona

Answer all questions. *Time allowed: 60 minutes.* *Maximum Marks 60.*

Question 1

(1.1) Let f be a real function, $a; L \in \mathbb{R}$ and assume that the domain of f contains a deleted neighbourhood of a, that is, $f(x)$ is defined for all x in a deleted neighbourhood of f. What does it mean $\lim_{x\to a} f(x) = L$? [3]

(1.2) Prove that if $\lim_{x\to a} f(x) = L$, then L is unique. [8]

(1.3) Prove that $\lim_{x\to 2}(x^2 - 3x) = -2$. [4]

[15 marks]

Question 2

(2.1) Let I and J be intervals, $g : J \to \mathbb{R}$ and $f : I \to \mathbb{R}$ with $f(I) \subseteq J$, and let $a \in I$. Assume that f is differentiable at a and g is differentiable at $f(a)$. Prove that $g \circ f$ is differentiable at a and $(g \circ f)'(a) = g'(f(a))f'(a)$. . [5]

(2.2) Show that $f(x) = \sin x$ is continuous at any $a \in \mathbb{R}$. [5]

[10 marks]

Question 3 Determine if the following series is convergent or divergent:

(3.1) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\left(\frac{4+n}{3+2n}\right)^n$. [5]

(3.2) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n+3^n}{5^n}$. [5]

(3.3) $\displaystyle\sum_{n=1}^{\infty}\cos(n\pi)$. [5]

(3.4) $\displaystyle\sum_{n=1}^{\infty}n^2e^{-n^3}$. [5]

[20 marks]

Question 4

(4.1) Let

$$f(x) = \begin{cases} \dfrac{|x| + x}{2x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Determine the right and the left continuity of $f(x)$ at $x = 0$. [5]

(4.2) State and prove the First Mean Value Theorem. [10]

[15 marks]

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