Tutorial 1.1: Questions

1a. Prove Theorem 1.3.

b. Prove Theorem 1.4, (c) – (g).

Define the following function on $\mathbb{R}$ called the *absolute value function*:

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

2. Prove the following statements for $x, y \in \mathbb{R}$ using the correct axioms, definitions and theorems:

   a. $|x| \geq 0$
   b. $|x| = 0$ if and only if $x = 0$.
   c. $|-x| = |x|$
   d. $|xy| = |x||y|$
   e. $|y| < x \Leftrightarrow -x < y < x$ whenever $x > 0$
   f. $-|x| \leq x \leq |x|$
   g. *Triangle inequality*: $\big||x| - |y|\big| \leq |x \pm y| \leq |x| + |y|$

3. Let $x, y, z \in \mathbb{R}$. Which of the following statements are **TRUE** and which are **FALSE**? Motivate your answer by referring to the correct axioms, definitions and theorems.

   a. $x \leq y \Rightarrow xz \leq yz$
   b. $0 < x \leq y \Rightarrow \frac{1}{y} \leq \frac{1}{x}$
   c. $x < y < 0 \Rightarrow \frac{1}{y} < \frac{1}{x}$
   d. $x^2 < 1 \Rightarrow x < 1$
   e. $x^2 < 1 \Rightarrow -1 < x < 1$
   f. $x^2 > 1 \Rightarrow x > 1$

4. In each of the following questions fill in the $\boxdot$ with $<$ or $>$. Motivate your answer by referring to the correct axioms, definitions and theorems.

   a. $a \geq 3 \Rightarrow \frac{a-2}{7} \boxdot \frac{a}{7}$
   b. $a \geq 1 \Rightarrow \frac{3}{a+1} \boxdot \frac{3}{a}$
   c. $a > 1 \Rightarrow \frac{9}{a} \boxdot \frac{10}{a-1}$
   d. $a > 1 \Rightarrow \frac{1}{a^2} \boxdot \frac{1}{a}$

e. $a \geq 2 \Rightarrow \frac{1}{a^2-1} \boxdot \frac{1}{a}$

f. $a > 3 \Rightarrow -\frac{3}{a} \boxdot -\frac{2}{a-1}$

5. Let $x \geq 0$ and $y \geq 0$. Show that $x < y \Leftrightarrow x^2 < y^2$.

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