14:00 hrs	03/06/2005	EX. HALL	Exams Office Use Only

University of the Witwatersrand, Johannesburg

Course or topic No(s)	MATH204 / 201B
Course or topic name(s) Paper Number & title	BASIC ANALYSIS
Examination/Test* to be held during month(s) of (*delete as applicable)	June 2005
Year of Study (Art & Sciences leave blank)	
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	BSc, BCom, BA
Faculty/ies presenting candidates	
Internal examiners and telephone number(s)	Mr. A. Blecher - Ext. 76202 Dr. C.J. van Alten - Ext. 76251
External examiner(s)	Dr. F. Bullock (UNISA)
Calculator policy	Calculators allowed

Time allowance

Course Nos	MATH204 / 201B	Hours	1.5 Hours

Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)

Answer section A on the computer card provided and Section B in the exam book provided.
Total : 90

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

Question 4

Let f and g be functions from $(0, 1)$ to $\mathbb{R}$, such that $-1 \leq f(x) \leq 1$ for all $x \in (0, 1)$ and $g(x) \to -\infty$ as $x \to 1_-$. Which of the following will necessarily be true as $x \to 1_-$?

(a) $f(x)g(x) \to 0$

(b) $f(x)g(x) \to 1$

(c) $f(x)g(x) \to -1$

(d) $f(x)g(x)$ does not converge to any number or to ∞ or $-\infty$

(e) $f(x)g(x)$ could converge to any real number or diverge

Question 5

Let $f : (0, \infty) \to \mathbb{R}$ be a continuous function such that $\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to 0_+} f(x) = 0$. Which of the following is necessarily true?

(a) The range of f is $\mathbb{R}$.

(b) The equation $f(x) = -2$ has a solution in $(0, \infty)$

(c) $f(x)$ is a decreasing function.

(d) There is a number $A > 0$ such that $f(x)$ is decreasing on (A, ∞).

(e) None of the above are true.

Question 6

If, for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $c - \delta < x < c$ implies $|f(x) - K| < \varepsilon$, then

(a) $\lim_{x\to c} f(x) = K$

(b) $\lim_{x\to c_-} f(x) = \infty$

(c) $\lim_{x\to c_+} f(x) = K$

(d) $\lim_{x\to c_-} f(x) = K$

(e) $\lim_{x\to c_+} f(x) = L$

Question 11

The equation $3^x = x^2 + 6$

(a) has a solution in $(0, 1)$.

(b) has a solution in $(0, 2)$.

(c) has a solution in $(1, 2)$.

(d) has a solution in $(2, 3)$.

(e) has no solutions.

Question 12

Suppose that $\lim_{n\to\infty} a_n = 0$. Then

(a) the sequence (a_n) does not converge.

(b) the sequence (s_n) converges, where $s_n = \sum_{k=1}^{n} a_k$.

(c) the sequence (s_n) is bounded, where $s_n = \sum_{k=1}^{n} a_k$.

(d) the series $\sum_{n=1}^{\infty} a_n$ converges.

(e) the series $\sum_{n=1}^{\infty}(-1)^n a_n$ converges if $0 \le a_{n+1} \le a_n$ for all $n \in \mathbb{N}$.

Question 13

The series $\sum \dfrac{n}{n^2+2}$ can be shown to

(a) diverge, by comparison with $\sum \frac{1}{n}$.

(b) converge, by comparison with $\sum \frac{1}{n}$.

(c) converge, by the ratio test.

(d) diverge, by the ratio test.

(e) converge, by the alternating series test.

Question 14

Find the radius of convergence of the series $\sum \dfrac{n^n}{n!} x^n$

(a) 1

(b) e

(c) $\frac{1}{e}$

(d) ∞

(e) 0

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Question 6

In each of the following cases decide if the series converges or diverges. Then prove your answer.

(i) $\sum \frac{3(2^n)}{3+5^n}$

(ii) $\sum \frac{3n + n\cos^2 n}{2n^3 - 5}$

(iii) $\sum \frac{n!\, n^n}{(2n)!}$

[12]

Question 7

Consider the series $\sum_{n=2}^{\infty}(-1)^n \frac{n-2}{n^2+3}$

(a) Does the series converge absolutely? Justify your answer.

(b) Does the series converge conditionally? Justify your answer.

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Total marks: 90