14;00 hrs

10 /06 /2015

Examinations and Graduation Office
Old Mutual Sports Hall

University of the Witwatersrand, Johannesburg

Course or topic numbers	MATH2001
Course or topic name(s) Paper Number & title	Basic Analysis
Examination to be held during month(s) of	June 2015
Year of Study	
Degrees/Diplomas for which this course is prescribed	
Faculty/ies presenting candidates	
Internal examiner(s) and telephone numbers	Prof. Manfred Möller – Ext 76220
Moderator	Prof. C. Labuschagne
Special materials required	
Time allowance	Course: MATH2001 Hours: 1
Instructions to candidates	60 marks in 60 minutes. No calculators are allowed.

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

Math2001 – Basic Analysis Examination 2015

Question 1 ..... [12 marks]

This question consists of four multiple choice parts. Answer these multiple choice questions on the first page of the answer book. There is ONLY ONE correct choice to each question.

(a) Let f be a function which is defined in a neighbourhood of $a \in \mathbb{R}$. Which of the following statements is not equivalent to 'f is continuous at a'? (3)

A. $\forall \varepsilon > 0\ \exists \delta > 0\ (0 < |x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon)$,

B. $\forall \varepsilon > 0\ \exists \delta > 0\ (|x - a| < \delta \Rightarrow 0 < |f(x) - f(a)| < \varepsilon)$,

C. $\forall \varepsilon > 0\ \exists \delta > 0\ (|x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon)$,

D. $\lim_{x \to a} f(x) = f(a)$,

E. $\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$.

(b) Let I be an interval and let f be continuous on I. Which of the following statements may be false? (3)

A. If $I = (a, b)$ with $a, b \in \mathbb{R}$ and $a < b$, then $f(I) = (c, d)$ with $-\infty \leq c < d \leq \infty$.

B. If $I = [a, b]$ with $a, b \in \mathbb{R}$ and $a < b$, then $f(I) = [c, d]$ with $-\infty < c \leq d < \infty$.

C. $f(I)$ is an interval or a singleton.

D. If $f(I)$ is a singleton, then f is constant on I.

E. If $I = [a, b]$ with $a, b \in \mathbb{R}$ and $a < b$, then $f(I)$ is bounded.

(c) Let f be a function which is differentiable at a. Which of the following statements may be false: (3)

A. f is continuous at a.

B. $\lim_{x \to a} \dfrac{f(x) - f(a) - f'(a)(x - a)}{(x - a)^2}$ exists.

C. There is a function $\tilde{f}$ which is continuous at a such that $f(x) = f(a) + \tilde{f}(x)(x - a)$.

D. $\lim_{x \to a} \dfrac{f(x) - f(a) - f'(a)(x - a)}{(x - a)} = 0$.

E. There is $\delta > 0$ such that $f(a - \delta, a + \delta)$ is bounded.

(d) Which of the following series converges conditionally? (3)

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3}}$,

B. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{(-3)^n}{n}$,

C. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3}}$ and $\sum_{n=1}^{\infty} \frac{(-3)^n}{n}$,

D. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3}}$ and $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$,

E. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ and $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$.

Question 2 ..... [14 marks]

(a) State the Intermediate Value Theorem. (3)

(b) Prove the Intermediate Value Theorem. (11)

Question 3 ..... [7 marks]

Let $a < b$ and let f be a continuous function on $[a, b]$ such that $f([a, b]) \subset [a, b]$.

(a) Show that there is $x \in [a, b]$ such that $f(x) = x$. (4)

(b) Assume additionally that f is differentiable on (a, b). Show that there is $c \in (a, b)$ such that $|f'(c)| \leq 1$. (3)

Question 4 ..... [14 marks]

(a) State the theorem on the Chain Rule for differentiable functions (Theorem 4.7 in the study guide). (4)

(b) Prove the theorem on the Chain Rule for differentiable functions. (10)
Hint. You may use a correct statement from Question 1(c).

Question 5 ..... [6 marks]

(a) State the Comparison Test for series. (3)
Note. It suffices to state one of the two (equivalent) statements.

(b) Prove the statement from part (a). (3)

Question 6 ..... [7 marks]

(a) Define the radius of convergence of a power series. (4)

(b) Find the radius of convergence of the power series (3)

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{n^n}.$$

Question 7 (Bonus question) ..... [2 marks]

Give an example of a functions f for your answer in

(a) Question 1(b) (1)

(b) Question 1(c) (1)

for which that statement is false. No proofs are required here.