09:00 06/06/19

Examinations and Graduation Office
Flower Hall

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF MATHEMATICS

MULTIVARIABLE CALCULUS MATH2007

STUDENT NO.		**Date**	
ID/PASSPORT NO.		**Venue**	
SIGNATURE		**Row & Seat**	

Internal examiner: Prof. E. G. Mphako-Banda (x76255) **External examiner:** Prof. R. Brits

Instructions to Candidates:

- Complete the information above.
- Check that this paper has a cover page and **7 pages**.
- Please do not write in red ink.
- Work done in pencil or altered will not be remarked.
- Show all working, which must be legible.
- Answer *ALL* questions.
- Approximate marks are indicated.

Markers only	
Question	**Mark**
1	/ 12
2	/ 8
3	/ 15
4	/ 25
Total	/ 60

The following identities may be useful in the examination.

$$\boxed{\begin{array}{c}\cos^2\theta+\sin^2\theta=1\\ \cos^2\theta=\frac{1}{2}(1+\cos 2\theta)\\ \sin 2\theta=2\sin\theta\cos\theta\end{array}}$$

Question 1 **[12 marks]**

(a) Let S be a hypersurface in $\mathbb{R}^n$. Define what is meant by a regular point of S. **(2)**

(b) Define what is meant by the set of tangent vectors of the hypersurface S at $\underset{\sim}{x}_0$. **(2)**

(c) Let $f(\underset{\sim}{x}) = x^2y - 6z^3$ where $\underset{\sim}{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, and $S = \{\, \underset{\sim}{x} \in \mathbb{R}^3 \mid f(\underset{\sim}{x}) = -3 \,\}$ and let $\underset{\sim}{x}_0 = \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}$.

i. Show that $\underset{\sim}{x}_0$ lies on the surface S. **(2)**

ii. Find a normal vector to S at $\underline{x}_0$. **(2)**

iii. Find $T_{\underline{x}_0}$, the set of tangent vectors to S at $\underline{x}_0$. **(3)**

iv. Find the tangent plane to S at $\underline{x}_0$. **(1)**

Question 2 **[8 marks]**

Show that if $f : \mathbb{R}^n \to \mathbb{R}$ has a local maximum at $\underline{x}_0$, then $\underline{x}_0$ is a critical point of f.

Question 3 **[15 marks]**

(a) Consider the integral $\int_{-1}^{1}\int_{|y|}^{1}(1+y^2)e^{x^3}\,dx\,dy$.

i. Sketch the region of integration, D. **(3)**

ii. Give two mathematical expressions for the region D. **(2)**

iii. Write the integral as a repeated integral in which we first integrate with respect to y and then with respect to x, stating your limits. **(2)**
Do not integrate.

(b) Evaluate $\displaystyle\iint_D 3(1 - x^2 - y^2)\, dA$, where D is the interior of the unit circle, using the transformation $x = r\cos\theta$ and $y = r\sin\theta$. **(8)**

Question 4 **[25 marks]**

(a) State Green's Theorem. **(3)**

(b) Verify Green's Theorem for $F_1(x, y) = xy$ and $F_2(x, y) = x + y$, where D is the interior of the unit circle. **(22)**

Extra space for question 4(b).