2 Hours	12/06/2012	Hall 29	EXAMS OFFICE USE ONLY

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Course or topic No(s)	APPM 2007
Course or topic name(s) Paper Number & title	NUMERICAL TECHNIQUES
Examination/~~Test~~ to be held during month(s) of *(delete as applicable)*	JUNE 2012
Year of Study *(Art & Science leave blank)*	
Degrees/Diplomas for which This course is prescribed (BSc (Eng) should indicate which branch)	B.Sc. B.Econ Sc.
Faculty/ies presenting Candidates	SCIENCE, CLM
Internal examiners(s) And telephone extension number(s)	PROF S. ABELMAN
External examiner(s)	PROF C. M. VILLET (UJ)
Special materials required (graph/music/drawing paper) maps, diagrams, tables computer cards, etc.	NON-PROGRAMMABLE, NON-ALPHANUMERIC CALCULATORS ARE PERMITTED.

Time allowance

Course No.(s)	APPM 2007	**Hours**	2 (TWO)

Instructions to candidates (Examiners may wish to use this space to indicate, inter alia the contribution made by this examination or test towards the year mark if appropriate)

ANSWER ALL QUESTIONS COMPLETELY. SHOW ALL MAJOR CALCULATIONS AND WRITE DOWN FORMULAE USED. <u>WORK THROUGHOUT TO 4 DECIMAL PLACES UNLESS OTHERWISE STATED</u>.

150 MARKS = 100%.

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

N.B. WHERE APPROPRIATE WORK THROUGHOUT TO 4 DECIMAL PLACES

START EACH QUESTION ON A NEW PAGE

QUESTION 1: [20 marks]

The impala population $x(t)$ in the Kruger National Park in South Africa may be modelled by the equation

$$\frac{dx}{dt} = \left(r - b\,x\sin at\right)x$$

where r, b and a are constants. Write a Matlab program which inputs values for r, b and a, and initial values for x and t, and uses **Euler's method** to compute the population at monthly intervals over a period of two years.

QUESTION 2: [15 marks]

The following data are believed to follow a relationship of the form $g(x) = a + b/x$.

x	2	3	4	5	6	7	8	9
y	14.3	14.8	15.1	15.3	15.5	15.7	15.8	16.0

Determine this relationship using the **least-squares principle.**
(**Hint: Reduce to linear form**)

QUESTION 3: [38 marks]

(a) Estimate $\int_1^3 \int_0^1 y \sin x \, dx\, dy$ using the **Composite Simpson's $\frac{1}{3}$ — Rule** with $h = 0.25$ *for the x-integral* and the **Simple Trapezoidal Rule** *for the y-integral* Work to 6D and compare your answer to the true value. **[20 marks]**

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(b) Consider $I = \int_{-0.75}^{0.75} f(x)dx$. The following table gives approximations using the **Composite Simpson Rule**:

h	0.75	0.375	0.1875	0.09375
$S(h)$	1.3658440	1.3634298	1.3632869	1.3632781

Use **Richardson's extrapolation** to obtain the **best estimate** for I. State the order of your final answer. **[18 marks]**

QUESTION 4: **[37 marks]**

Given $y' = x\,y^{1/5}$; $x \in [0, 1]$, $y(0) = 1$, $h = 0.5$

Solve this initial-value problem using the scheme

$$y_{n+1} = y_n + \frac{h}{2}(y'_n + y'_{n+1}), \quad n = 0,1,2,\ldots$$

(a) Using the **modified Euler method.** **[12 marks]**

(b) At each step solving the resulting nonlinear equation by the **Newton-Raphson method.** Choose your own initial guess for the unknown at each step.

[Hint: $f(z) = 0$, $z^{(n+1)} = z^{(n)} - f(z^{(n)})/f'(z^{(n)})$] **[20 marks]**

(c) Compare the answers obtained from (a) and (b) with the **exact solution** of this differential equation. **[5 marks]**

QUESTION 5: **[40 marks]**

Consider the boundary-value problem: $y'' - 3y' + 2k^2 y = 0$; $y(0) = 0$, $y(1) = 0$.

(a) Using **central-difference approximations** with steplength h, write this differential equation as a difference equation. **[2 marks]**

(b) For $h = 0.5$, solve for k. **[3 marks]**

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(c) For $h = \frac{1}{3}$, show that the difference equation obtained in (a) reduces to an eigenvalue problem of the form $Ay = \lambda y$ where $\lambda = 4k^2$ is an eigenvalue of A, y is the corresponding eigenvector, and A is a matrix that you must determine.

[10 marks]

(d) Use **Gerschgorin's theorem** to determine regions in which the eigenvalues of A (determined in (c)) are situated. **[5 marks]**

Perform **two iterations** of the **scaled inverse power method** to obtain an estimate of the **"smallest"** eigenvalue of A (determined in (c)). Use $[1 \;\; 1]^T$ as initial eigenvector estimate. Hence estimate k. Write down the corresponding estimate of the eigenvector $\boldsymbol{y}$.

[15 marks]

<u>DO NOT DETERMINE THE INVERSE OF A.</u>

(f) Assuming errors are proportional to h^2, use the results obtained in (b) and (e) together with **Richardson extrapolation** to determine a better estimate for k.

[5 marks]

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