hrs	/ /20		Exams Office Use Only

University of the Witwatersrand, Johannesburg

Course or topic No(s)	MATH2001
Course or topic name(s) Paper Number & title	BASIC ANALYSIS
Examination/Test* to be held during month(s) of (*delete as applicable)	June 2009
Year of Study (Art & Sciences leave blank)	
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	BSc, BCom, BA
Faculty/ies presenting candidates	Science, Commerce, Humanities
Internal examiners and telephone number(s)	Mr. A. Blecher - Ext. 76202 Dr. M. Hockman - Ext. 76234
External examiner(s)	Prof. I. Naidoo (UNISA)
Calculator policy	Calculators allowed but there should be no need for one.
Time allowance	Course Nos: MATH2001; Hours: 1.5 Hours
Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)	Answer section A on the computer card provided and Section B in the exam book provided. Total : 90

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

Section B.

Answer this section in the answer book provided.

Question 1

(a) Define what is meant by saying that :

(i) $\lim_{n\to\infty} a_n = -\infty$; and (2)

(ii) $\lim_{n\to\infty} a_n = \infty$. (2)

(b) Show that if $\lim_{n\to\infty} a_n = -\infty$ and k is a constant such that $k < 0$, then :

(i) $\lim_{n\to\infty} (a_n + k) = -\infty$.

(ii) $\lim_{n\to\infty} ka_n = \infty$. (6)

(c) Use your definitions in (a) to show that :

(i) $\lim_{n\to\infty} (3n - 2n^2) = -\infty$.

(ii) $\lim_{n\to\infty} (2n^2 - \sin n) = \infty$. (7)

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Question 2

(a) Provide the definitions for:

(i) $f(x) \to -\infty$ as $x \to c_-$; and

(ii) $f(x) \to \infty$ as $x \to c$. (2)

(b) State the definition of $f(x) \to \ell$ as $x \to a$. (2)

(c) Use the appropriate definition to show that:
$\lim_{x\to 2_-} \dfrac{1}{(x-2)} = -\infty$. (2)

(d) Use the appropriate definition to show that $\lim_{x\to -\infty} \dfrac{x \sin x}{x^2+1} = 0$. (5)

(e) Let $f(x) \to L$ and $g(x) \to M$ as $x \to a$.
Use the precise definition of the limit to prove that $f(x) + g(x) \to L + M$ as $x \to a$. (6)

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Question 3

(a) State the Test for Divergence for the series $\sum a_n$. (1)

(b) State both parts of the Comparison Test for two given series $\sum a_n$ and $\sum b_n$ whose terms are positive. (3)

(c) Suppose that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = r$. Show that if $r < 1$ then $\sum a_n$ is absolutely convergent. (6)

(d) State whether each of the following series converges or diverges. Prove your answer, giving full reasoning in each case:

(i) $\sum \dfrac{n^2}{7n^3 - n - 4}$. (2)

(ii) $\sum \dfrac{n^3}{7n^3 - n - 4}$ (1)

(iii) $\sum \dfrac{n}{7n^3 - n - 4}$. (2)

(iv) $\sum (-1)^n \dfrac{1}{n}$. (2)

(v) $\sum \dfrac{n!}{2^n}$ (2)

(vi) $\sum \dfrac{n^2}{2^n}$. (2)

(e) Find the values of x for which the series $\sum \dfrac{(x-2)^n}{3n+1}$ is convergent. Show all working. (5)

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Section B Total marks: [60]

Total marks: [90]