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APPM3039A
Mechanics III

Quiz 03

Total: 5 Marks　　　　　　　　　　　　　　　　　**Duration:** 15 Minutes

Instructions:

- Write your student number on this answer sheet.

- Show all workings.

Question 1　　　　　　　　　　　　　　　　　　　**[5 Marks]**

Show that for any property of the continuum C

$$\frac{D}{Dt}\int_V \rho C\, dV = \int_V \rho \frac{DC}{Dt}\, dV.$$

Assume that mass is conserved.　　　　　　　　　　　　　(5 marks)

Solution:　　LHS　　transforming　coordinates:

$$\frac{D}{Dt}\int_{V_0} \rho C J\, dV_0$$

$$= \int_{V_0} \frac{D}{Dt}(\rho C J)\, dV_0$$

$$\int_{V_0}\left(\frac{D\rho}{Dt} C J + \rho \frac{DC}{Dt} J + \rho C \frac{DJ}{Dt}\right) dV$$

we have that since mass is conserved

$$\int_{V_0}\frac{D\rho}{Dt} C J\, dV_0 = -\int \rho C \frac{DJ}{Dt}\, dV_0\,?$$

$\cdot$ Hence

$$\int_{V_0}\left(\frac{D\rho}{Dt} C \frac{DJ}{Dt} + \rho \frac{DC}{Dt}\, dV\right) + \rho C \frac{DJ}{Dt}\, dV_0$$

$$\int_{V_0} \rho \frac{DC}{Dt} J\, dV_0$$

Transform back so we have

$$\int_V \rho \frac{DC}{Dt}\, dV = LHS$$

as required

Hence $\quad \dfrac{D}{Dt}\displaystyle\int_V \rho C\, dV = \int_V \rho \frac{DC}{Dt}\, dV$