Tutorial 3.1.1.

1. Test each of thefollowing series for convergence or divergence:

(a) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$, (b) $\displaystyle\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$, (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2-1}{n-50n^2}$.

Solution.

(a) Since $\displaystyle\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \neq 0$, it follows by the Test for Divergence that the series does not converge.

(b) From $\displaystyle\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim_{x\to 0}\frac{1}{x}\sin x = 1 \neq 0$, it follows by the Test for Divergence that the series does not converge.

(c) Here, $\displaystyle\lim_{n\to\infty}\frac{n^2-1}{n-50n^2} = -\frac{1}{50} \neq 0$ again shows that the series does not converge by the Test for Divergence.

2. Which of the following is valid? Justify your conclusions.

(a) If $a_n \to 0$ as $n\to\infty$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

Solution. This statement is false. A counter-example is the harmonic series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$, and $a_n = \dfrac{1}{n} \to 0$ but it diverges.

(b) If $a_n \not\to 0$ as $n\to\infty$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

Solution. This statement is true; it is theTest for Divergence.

(c) If $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent, then $a_n \not\to 0$ as $n\to\infty$.

Solution. This statement is the contrapositive of (a) and therefore false.