1:00 hrs	25/10/12	EXH

University of the Witwatersrand, Johannesburg

Course or topic numbers	MATH2016
Course or topic name(s) Paper Number & title	Advanced Analysis
Examination to be held during month(s) of	October 2012
Year of Study	
Degrees/Diplomas for which this course is prescribed	
Faculty/ies presenting candidates	
Internal examiner(s) and telephone numbers	Prof. Manfred Möller – Ext 76220
Moderator	Prof. Coenraad Labuschagne
Special materials required	
Time allowance	Course: MATH2016 Hours: 1
Instructions to candidates	60 marks in 60 minutes.

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

University of the Witwatersrand School of Mathematics
MATH2016–Advanced Analysis Examination 2012

Attempt all questions and write your answers in the answer book provided.

Question 1 .... [8 marks]

Let $[a, b]$ be an interval.

(a) Write down what is meant by a partition P of $[a, b]$. (2 marks)

(b) Write down what it means that f is Riemann integrable.
Explain the notation you use. (6 marks)

Question 2 .... [10 marks]

Show that an increasing function f on an interval $[a, b]$ is Riemann integrable.

Question 3 .... [8 marks]

Let f be a Riemann integrable function on an interval $[a, b]$ and define

$$f_+(x) = \begin{cases} f(x) & \text{if } f(x) \geq 0, \\ 0 & \text{if } f(x) < 0, \end{cases} \qquad x \in [a, b].$$

Show that f_+ is Riemann integrable.

Question 4 .... [12 marks]

(a) Write down the definition of a metric space. (4 marks)

(b) Show that if (X, d) is a metric space, then $d(x, y) \geq 0$ for all $x, y \in X$. (3 marks)

(c) Let d_1 and d_2 be metrics on X and let $d(x, y) = d_1(x, y) + d_2(x, y)$ for all $x, y \in X$.
Show that d is a metric on X. (5 marks)

Question 5 .... [12 marks]

Let (X, d) be a metric space and let (x_n) be a sequence in X.

(a) Define what it means that (x_n) is convergent. (2 marks)

(b) Define what it means that (x_n) is a Cauchy sequence. (2 marks)

(c) Show that the limit of (x_n) is unique if it exists. (5 marks)

(d) Show that if (x_n) converges, then (x_n) is a Cauchy sequence. (3 marks)

Question 6 .... [10 marks]

(a) Define what is meant by a generalized contraction.
Explain the notation you use. (3 marks)

(b) Let $T : C[0,1] \to C[0,1]$ be defined by

$$(Tf)(x) = 1 + \int_0^x \frac{f(t)}{4+t^2}\, dt.$$

Show that T is a (generalized) contraction. (7 marks)

(c) Find the fixed point of T. (3 bonus marks)