Math2001–Basic Analysis April 2010 Midterm Test

SECTION 1

The questions in this section are multiple choice questions and must be answered on the computer card provided. Please ensure that your student number is entered on the card, by pencilling in the requisite digit for each block. There is ONLY ONE correct answer to each question.

Question 1 .... [2 points]

Let (a_n) be a decreasing sequence. Then

$a_{n+1} \leq a_n \ \forall n \in \mathbb{N}$

(a) $a_n \to -\infty$ as $n \to \infty$. X

(b) $a_n \to$ finite limit as $n \to \infty$. X

(c) $\lim_{n\to\infty} a_n = \sup\{a_n : n \in \mathbb{N}\}$ if (a_n) is bounded above. X

(d) $\lim_{n\to\infty} a_n = \infty$ if (a_n) is not bounded above. X

(e) Either (a) or (b) is true.

Question 2 .... [2 points]

The sequence $n^p \to \infty$ as $n \to \infty$ for

(a) $p \in \mathbb{R}$ and $p > 0$. X $p > 1$

(b) $p \in \mathbb{N}$ only. ✓

(c) $p \in \mathbb{Z}$. X 1^{-1} 2^{-1} 1^1 2^1 3^1

(d) $p \in \mathbb{R}$ and $p < 0$. X 1 $1/2$

(e) None of the above

Question 3 .... [3 points]

$\lim_{n\to\infty} \sqrt[n]{n} =$ $\lim_{n\to\infty} n^{1/n} =$

(a) ∞. 1^1 $2^{1/2}$ $3^{1/3}$ $4^{1/4}$

(b) 0. 1 $\sqrt{2}$ $\sqrt[3]{3}$ $\sqrt[4]{4} = \sqrt{2.2}$

(c) 1. 1.41 1.441

(d) e.

(e) None of the above.

2. 1.

2 26   5 25

Question 4 .... [3 points]

Let the sequence (a_n) be defined by

$$a_1 = 1,\ a_n = \sqrt{1 + a_{n-1}}\ (n \geq 2).$$

$1 = 1$ ✓

Then $a_n \to$

(a) $\sqrt{2}$.

(b) 1.

(c) 0. ✗

(d) ∞. ✗

(e) None of the above.

1.61

$\sqrt{1+1} = \sqrt{2} = 1.41$

$\sqrt{1+\sqrt{2}} = 1.55$

$\sqrt{1+\sqrt{1+\sqrt{2}}} = 1.59$

$\sqrt{1+1.59} = 1.61$

$= 1.61$

SECTION 2

Question 5 .... [10 points] 4.53

It may be assumed that the set of integers is closed under addition $+$ and multiplication $\cdot$.

(a) Define rational numbers. (2 points)

(b) Prove that the sum of two rational numbers is rational. (4 points)

(c) Prove that the sum of a rational number and an irrational number is irrational. (4 points)

Question 6 .... [20 points] 5.10

Let S be a nonempty subset of $\mathbb{R}$.

(a) Write down the definition of $\sup S$. (2 points)

(b) Show that the following are equivalent: (1) $M = \sup S$; (2) M is an upper bound of S and for each $\epsilon > 0$ there exists $s \in S$ such that $M - \epsilon < s$. (18 points)

Question 7 .... [4 points] 5.12

Let (a_n) be a sequence of real numbers. Write down the definitions for

(a) $a_n \to L$ as $n \to \infty$; (2 points)

(b) $a_n \to -\infty$ as $n \to \infty$. (2 points)

Question 8 .... [16 points]

Use the definitions given in Question ~~6~~ 7 to show that

(a) $\lim\limits_{n\to\infty} \dfrac{n-3}{n^2+1} = 0$. (8 points)

(b) $\lim\limits_{n\to\infty} (\cos n - n^2) = -\infty$. (8 points)