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UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Course or topic No(s)	APPM 2007A/APPM2020A
Course or topic name(s) Paper Number & title	METHODS A: APPLIED ORDINARY DIFFERENTIAL AND DIFFERENCE EQUATIONS
Examination/Test to be held during month(s) of *(delete as applicable)*	JUNE 2022 DEFERRED / SUPPLEMENTARY EXAMINATION
Year of Study *(Art & Science leave blank)*	SECOND
Degrees/Diplomas for which This course is prescribed (BSc (Eng) should indicate which branch)	BSc
Faculty/ies presenting Candidates	SCIENCE
Internal examiners(s) And telephone extension number(s)	DR I.S. OYELAKIN X76107
External examiner(s)	DR. SICELO GOQO
Special materials required (graph/music/drawing paper) maps, diagrams, tables computer cards, etc.	NONE

Time allowance

Course No.(s)	APPM2007A/ APPM2020A	**Hours**	2 hrs

Instructions to candidates (Examiners may wish to use this space to indicate, *inter alia* the contribution made by this examination or test towards the year mark if appropriate)

ATTEMPT ALL QUESTIONS
ONLY NON-PROGRAMMABLE CALCULATORS ARE PERMITTED
NO CELLPHONES ALLOWED
Total Marks Available= 54
100% = 50 marks

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Computer Science and Applied Mathematics

APPM2007/APPM2020A: Methods A - Applied ordinary differential and difference equations

Deferred Examination — 2022

Lecturer: Dr Ibukun Oyelakin **Total Marks: 54** **Time: 2hrs**

- **Answer all questions and show all workings.**
- **In all the questions below, the prime $'$ stands for differentiation with respect to x and overdot $\dot{x}$ stands for differentiation with respect to t.**
- **This exam has 4 questions, for a total of 54 marks but 50 marks is full marks.**

QUESTION ONE [10 MARKS]

(a) Consider the linear inhomogeneous first order ordinary differential equation

$$(b(x) - a(x)y)dx - dy = 0. \qquad (\dagger)$$

Show that $I = e^{\int a(x)dx}$ is an integrating factor of the ordinary differential equation given in $(\dagger)$. **[3 Marks]**

(b) Hence or otherwise, find the general solution to the first order linear inhomogeneous ordinary differential equation

$$\left(e^{\lambda x} + 2y\right) dx - dy = 0 \qquad (\bullet)$$

where λ is a constant, such that $\lambda \neq 2$. **[3 Marks]**

(c) Solve equation $(\bullet)$ with $\lambda = 2$ and find the particular solution if $y(1) = 2$. **[4 Marks]**

QUESTION TWO [10 MARKS]

(a) Evaluate and simplify as much as possible, the integral

$$\int \frac{x+1}{x^2-6x+8} \quad dx$$

[2 Marks]

(b) Find the differential equation $y'' = f(x, y, y')$ whose general solution is

$$y = C_1e^x + C_2e^{-x} + x.$$

[2 Marks]

(c) Consider the first order ordinary differential equation

$$\frac{dy}{dx} = \frac{x+y+3}{2x+2y+1}.$$

(i) Using an appropriate substitution, reduce the equation to separable form. **[3 Marks]**

(ii) Find the solution to the reduced ordinary differential equation in (i). **[3 Marks]**

QUESTION THREE [12 MARKS]

Given the second order ordinary differential equation

$$y'' + a(x)y' + b(x)y = 0,$$

where $a(x)$ and $b(x)$ are continuous functions of x and $y_1(x)$ is a known solution of the differential equation.

(i) Use the substitution $y_2(x) = u(x)y_1(x)$ to show that $u(x)$ satisfies the linear second order ordinary differential equation

$$u'' + \left(2\frac{y_1'}{y_1} + a(x)\right) u' = 0.$$

[5 Marks]

(ii) Hence, use reduction of order to find the general solution of the second order differential equation

$$x^2y'' - 3xy' + 4y = 0,$$

if $y_1(x) = x^2$ is a known solution of the differential equation. **[7 Marks]**

QUESTION FOUR [22 MARKS]

(a) Find the general solution to the non-homogeneous second order difference equation

$$y_{k+2} - 2y_{k+1} + 5y_k = k$$

[6 Marks]

(b) Consider the first order system of ordinary differential equation

$$\begin{aligned} \dot{x}_1 &= x_2 \qquad & x_1(0) &= -1 \\ \dot{x}_2 &= 2x_1 - x_2 \qquad & x_2(0) &= 1. \end{aligned}$$

(i) Write the system in vector-matrix form and its corresponding initial values in vector form. **[3 Marks]**

(ii) Find the exponential matrix e^{At} for the system in (i). **[7 Marks]**

(iii) Solve the resulting initial value problem in (i) using the exponential matrix obtained in (ii). **[4 Marks]**

(iv) Hence write down a solution to the original second order ordinary differential equation. **[2 Marks]**