The questions in this section are multiple choice questions and must be answered on the computer card provided. Please ensure that your student number is entered by pencilling in the requisite digit for each block. There is one correct answer to each question.

Question 1 .... 2 points

Let (a_n) be a decreasing sequence. Then

(a) $a_n \to -\infty$ as $n \to \infty$.

(b) $a_n \to$ finite limit as $n \to \infty$.

(c) $\lim_{n\to\infty} a_n = \sup\{a_n : n \in \mathbb{N}\}$ if (a_n) is bounded above.

(d) $\lim_{n\to\infty} a_n = \infty$ if (a_n) is not bounded above.

(e) Either (a) or (b) is true.

Question 2 .... 2 points

The sequence $n^p \to \infty$ as $n \to \infty$ for

(a) $p \in \mathbb{R}$ and $p > 0$.

(b) $p \in \mathbb{N}$ only.

(c) $p \in \mathbb{Z}$.

(d) $p \in \mathbb{R}$ and $p < 0$.

(e) None of the above

Question 3 .... 2 points

$\lim_{n\to\infty} \sqrt[n]{n} =$

(a) ∞.

(b) 0.

(c) 1.

(d) e.

(e) None of the above.

Question 4 - - - - - - - - - - - - - - - 2 points

$a_n \to$

(e) None of the above

Question 5. BA Solutions

(a) A rational number is a number of the form p/q where p, q are integers and $q \neq 0$

(b) Let $\frac{p_1}{q_1}$ and $\frac{p_2}{q_2}$ be two rational numbers where $p_1, p_2, q_1, q_2 \in \mathbb{Z}$ and $q_1 \neq 0$ and $q_2 \neq 0$

$$\therefore \frac{p_1}{q_1} + \frac{p_2}{q_2} = \frac{p_1 q_2 + p_2 q_1}{q_1 q_2} \quad \text{--- ①} \quad (\text{def}^n \text{ of additi}$$

Now $p_1 q_2$ and $p_2 q_1 \in \mathbb{Z}$
So $p_1 q_2 + p_2 q_1 \in \mathbb{Z}$ } by given allowed assumption.

Also $q_1 q_2 \neq 0$ as $q_1 \neq 0$ and $q_2 \neq 0$

Hence ① is rational by definition ⓐ (4)

(c). Let x be rational and y be irrational
In order to get a contradiction, suppose
$x+y$ is rational.

$\therefore$ $(x+y)+(-x)$ is rational by (b)

i.e. y is rational.

But this is a contradiction (because y is given irrat.)
$\therefore$ Assumption false, i.e. $x+y$ is irrational. (4)

[10]

Question 7

(a) $\forall \varepsilon > 0$, $\exists K (>0)$ s.t. $n \geq K \Rightarrow |a_n - L| < \varepsilon$

(or $n > K \Rightarrow |a_n - L| \leq \varepsilon$)

(b) $\forall A < 0$, $\exists K (>0)$ s.that $n \geq K \Rightarrow a_n < A$.

of the form p/q where p, q are integers and $q \neq 0$

(b) Let $\frac{p_1}{q_1}$ and $\frac{p_2}{q_2}$ be two rational numbers where $p_1, p_2, q_1, q_2 \in \mathbb{Z}$ and $q_1 \neq 0$ and $q_2 \neq 0$

$$\therefore \frac{p_1}{q_1} + \frac{p_2}{q_2} = \frac{p_1 q_2 + p_2 q_1}{q_1 q_2} \quad \text{...(1)} \quad (\text{def}^n \text{ of addition})$$

Now $p_1 q_2$ and $p_2 q_1 \in \mathbb{Z}$ } given allowed by assumption.
So $p_1 q_2 + p_2 q_1 \in \mathbb{Z}$

Also $q_1 q_2 \neq 0$ as $q_1 \neq 0$ and $q_2 \neq 0$

Hence (1) is rational by definition (a) (4)

(c) Let x be rational and y be irrational
In order to get a contradiction, suppose $x+y$ is rational
$\therefore (x+y)+(-x)$ is rational by (b)
i.e y is rational.

But this is a contradiction (because y is given irrat.)
$\therefore$ Assumption false, i.e $x+y$ is irrational (4)

[10]

Question 7

(a) $\forall \varepsilon > 0, \exists K (>0)$ s.t $n > K \Rightarrow |a_n - L| < \varepsilon$
(or $n > K \Rightarrow |a_n - L| \leq \varepsilon$)

(b) $\forall A < 0, \exists K (>0)$ s.t. that $n > K \Rightarrow a_n < A$

Question 8

(a) For any $\epsilon > 0$,

$$\left|\frac{n-3}{n^2+1}\right| < \left|\frac{n}{n^2}\right| = \frac{1}{n} < \epsilon$$ ③

provided $n > K = \lceil \frac{1}{\epsilon} \rceil$ Hence ~~take~~ let

$K = \lceil \frac{1}{\epsilon} \rceil$ ③

Then for any $\epsilon > 0$, $n > K$ implies ②

$$\left|\frac{n-3}{n^2+1} - 0\right| < \epsilon$$

Hence

$$\lim_{n\to\infty} \frac{n-3}{n^2+1} = 0$$

(b) For any $A < 0$

$$\cos n - n^2 \leq 1 - n^2 \; \cancel{\leq} \; < A$$

provided

$n^2 > 1-A$ ③

① That is $n \geq \lceil \sqrt{1-A} \rceil$.

Hence let $K = \lceil \sqrt{1-A} \rceil$. Then ③

for any $A < 0$, $n > K$ implies

$\cos n - n^2 < A$

$\therefore \lim_{n\to\infty} (\cos n - n^2) = -\infty$. ②