Basic Analysis 2015 — Solutions of Tutorials

Section 3.2

Tutorial 3.2.1.

1. Prove from the definitions that

(a) $\lim_{x\to-\infty} \frac{1-3x}{2x-1} = -\frac{3}{2}$, (b) $\frac{1}{x-1} \to \infty$ as $x \to 1^+$, (c) $\frac{1}{x-1} \to -\infty$ as $x \to 1^-$.

Proof. (a) For $x > \frac{1}{2}$,

$$\left|\frac{1-3x}{2x-1} + \frac{3}{2}\right| = \left|\frac{2-6x+6x-3}{4x-2}\right| = \frac{1}{4x-2}.$$

Now let $\varepsilon > 0$ and let

$$K = \frac{\frac{1}{\varepsilon}+2}{4}.$$

For $x > K$ we have $x > \frac{1}{2}$ and

$$4x-2 > 4K-2 = \frac{1}{\varepsilon}.$$

Hence

$$\left|\frac{1-3x}{2x-1} + \frac{3}{2}\right| = \frac{1}{4x-2} < \varepsilon.$$

(b) Let $A > 0$ and put $\delta = \frac{1}{A} > 0$. Then, for $x \in (1, 1+\delta)$, $0 < x-1 < \delta$, and therefore

$$\frac{1}{x-1} > \frac{1}{\delta} = A.$$

(c) Let $A < 0$ and put $\delta = -\frac{1}{A} > 0$. Then, for $x \in (1-\delta, 1)$, $-\delta < x-1 < 0$, and therefore

$$\frac{1}{x-1} < -\frac{1}{\delta} = A.$$

□