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Assignment 01
Curvilinear Coordinate Systems

APPM2023
2023-03-24

Question 1

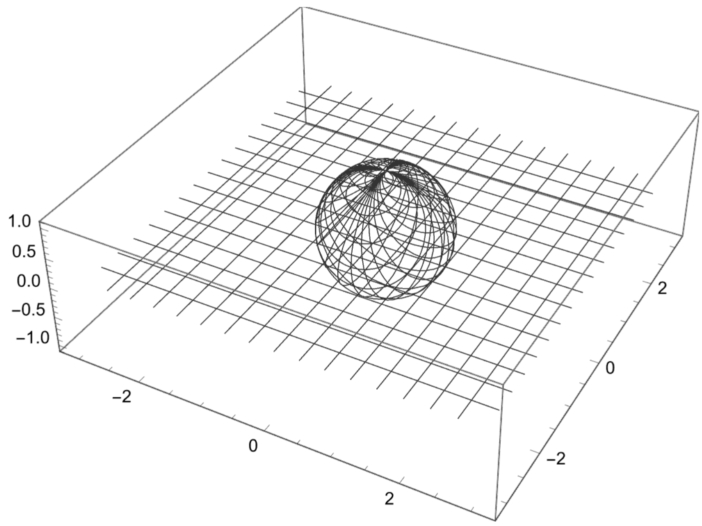

Figure 1: Plot of the portion of R bounded by $r, s \in [-3, 3]$ and the entirety of S

The relationship between R and S is that S is a transformation of R in the three-dimensional space. R is a flat 2 Dimensional plane described by $\{x, y, z\} = \{r, s, 0\}$ from this S applies a transformation to the x, y, z coordinates of R resulting in a 3 Dimensional graph. From this we can note the S is a so called sphere shaped transformation of R.

Question 2

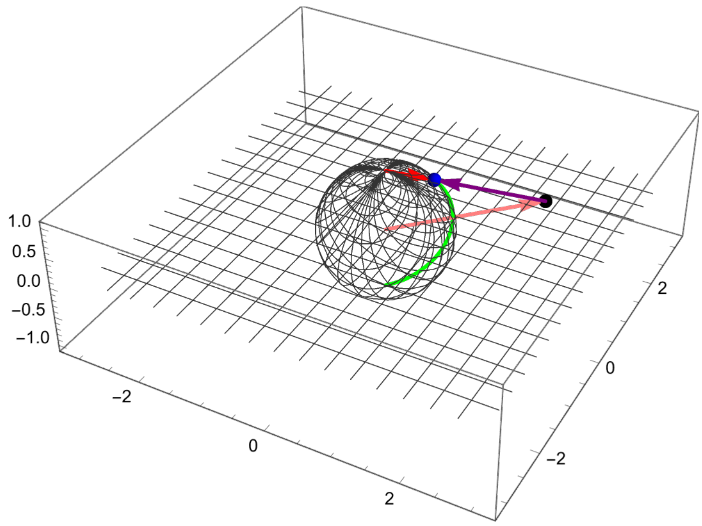

Figure 2: Plot of the different parametric mappings given $t = 0.75$

Question 3

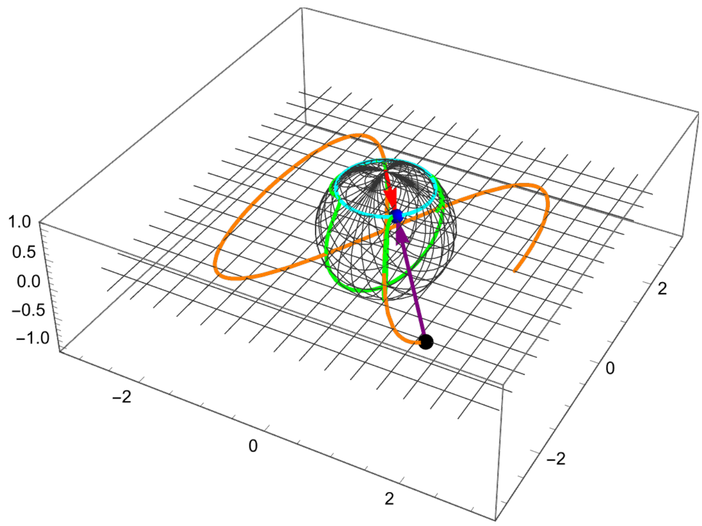

Figure 3: Plot of the different parametric mappings given $t = 0.9$

Question 4

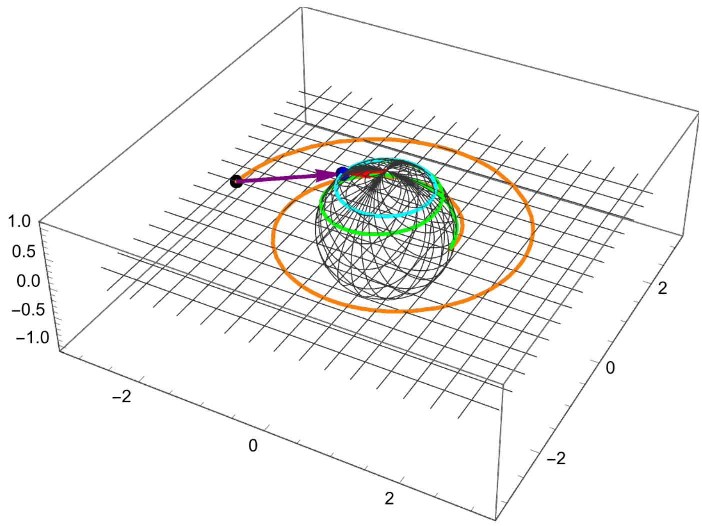

Figure 4: Plot of the different parametric mappings given $t = 0.75$

Question 5

Computed path lengths for:

$p_1(t)$ on $S = 3.14159$
$p_2(t)$ on $S = 12.5301$
$p_3(t)$ on $S = 10.159$

Where in each case $t \in [0, 1]$

Question 6

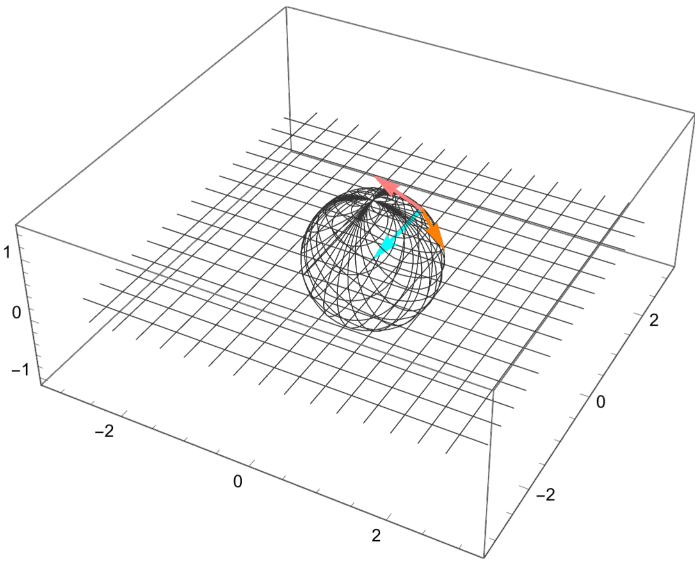

Figure 5: Graphic showing the parametric mappings $\vec{T}_1(t), \vec{N}_1(t)$ and $\vec{B}_1(t)$ at time $t = 0.75$

<u>We can see the relative orientation of each of these parametric mappings:</u>

Firstly we see that $\vec{T}_1(t), \vec{N}_1(t)$ and $\vec{B}_1(t)$ act at perpendicularly to each other which is true for all values of $t \in [0, 1]$. We see that $\vec{T}_1(t)$ (in pink) acts as the direction in which $p_1(t)$ on S moves. $\vec{N}_1(t)$ (in cyan) acts inward toward the centre of the sphere since it is the centripetal force. Lastly, $\vec{B}_1(t)$ (in orange) acts perpendicular to both $\vec{T}_1(t)$ and $\vec{N}_1(t)$ and gives us the instantaneous axis of rotation of the sphere at t time. The direction of $\vec{B}_1(t)$ gives us this axis of rotation.

<u>Next the motion of $\vec{T}_1(t), \vec{N}_1(t)$ and $\vec{B}_1(t)$ along S is this:</u>

$\vec{T}_1(t), \vec{N}_1(t)$ and $\vec{B}_1(t)$ all follow the path of $p_1(t)$ on S and thus wrap around the circumference of S as t increases. They act at their respective angles to each other at the point $p_1(t)$ on S.

Question 7

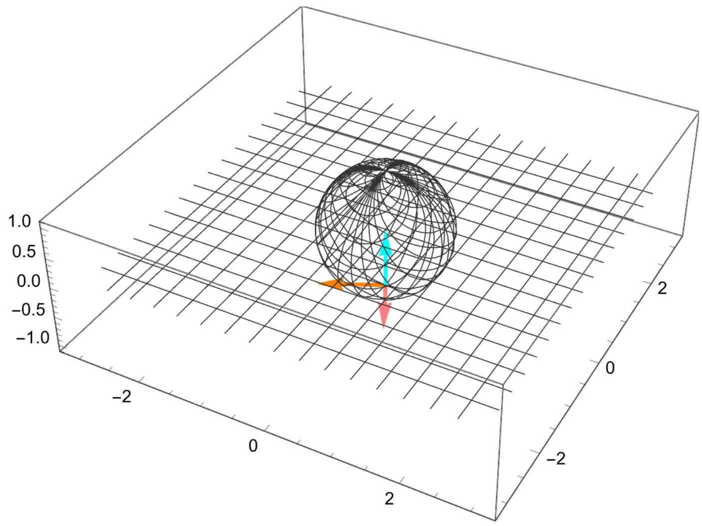

Figure 6: Graphic showing the parametric mappings $\vec{T_2}(t), \vec{N_2}(t)$ and $\vec{B_2}(t)$ at time $t = 0.75$

We can see the relative orientation of each of these parametric mappings:

Firstly we see that $\vec{T_2}(t), \vec{N_2}(t)$ and $\vec{B_2}(t)$ act at perpendicularly to each other which is true for all values of $t \in [0, 1]$. We see that $\vec{T_2}(t)$ (in pink) acts as the direction in which $p_2(t)$ on S moves. $\vec{N_2}(t)$ (in cyan) acts toward the centre when moving along circumference path on S but behaves differently when it rotates from following one circumference path on S to another. It shoots outward as it changes which circumference path on S it follows then points back inward as it begin to follow the new circumference path. Lastly, $\vec{B_2}(t)$ (in orange) acts perpendicular to both $\vec{T_2}(t)$ and $\vec{N_2}(t)$ and gives us the instantaneous axis of rotation of the sphere at t time. The direction of $\vec{B_2}(t)$ gives us this axis of rotation. This axis of rotation is constantly changes as we follow the path $p_1(t)$ on S due to the nature of following numerous circumference paths.

Next the motion of $\vec{T_2}(t), \vec{N_2}(t)$ and $\vec{B_2}(t)$ along S is this:

$\vec{T_2}(t), \vec{N_2}(t)$ and $\vec{B_2}(t)$ all follow the path of $p_2(t)$ on S as t increases and act at their respective angles to each other at $p_2(t)$ on S.

Question 8

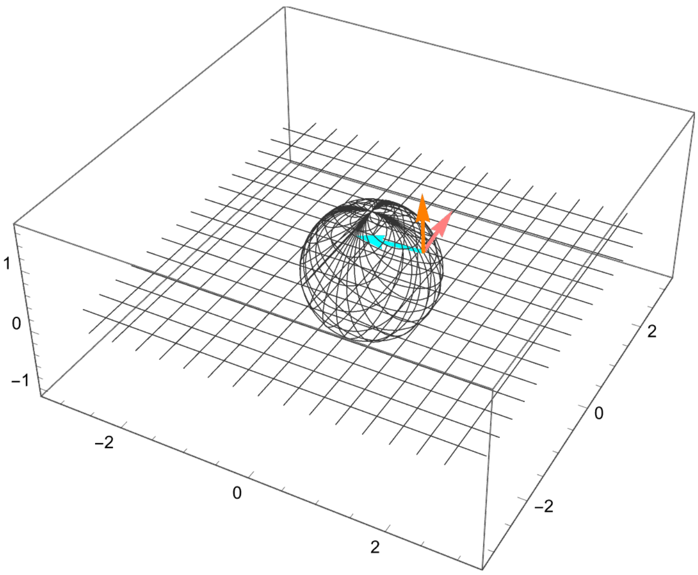

Figure 7: Graphic showing the parametric mappings $\vec{T}_3(t), \vec{N}_3(t)$ and $\vec{B}_3(t)$ at time $t = 0.5$

We can see the relative orientation of each of these parametric mappings:

Firstly we see that $\vec{T}_3(t), \vec{N}_3(t)$ and $\vec{B}_3(t)$ act at perpendicularly to each other which is true for all values of $t \in [0, 1]$. We see that $\vec{T}_3(t)$ (in pink) acts as the direction in which $p_3(t)$ on S moves. $\vec{N}_3(t)$ (in cyan) acts inward toward the centre of the 2 Dimensional circle encapsulated in S at the z-axis value on S at t and a centripetal force. Lastly, $\vec{B}_3(t)$ (in orange) acts perpendicular to both $\vec{T}_3(t)$ and $\vec{N}_3(t)$ and gives us the instantaneous axis of rotation of the sphere at t time. The direction of $\vec{B}_3(t)$ gives us this axis of rotation, in this case $\vec{B}_3(t)$ always points upward as t increases.

Next the motion of $\vec{T}_3(t), \vec{N}_3(t)$ and $\vec{B}_3(t)$ along S is this:

$\vec{T}_3(t), \vec{N}_3(t)$ and $\vec{B}_3(t)$ all follow and trace the path of $p_3(t)$ on S as t increases and thus follow a spiral motion around the sphere S. Note that as it approaches the north pole of S the spiral circumference path length decreases. They act at their respective angles to each other at $p_2(t)$ on S.

Question 9

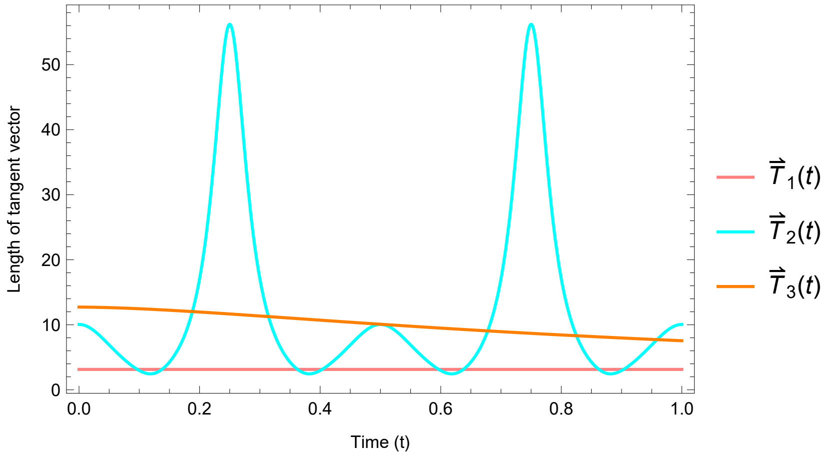

Figure 8: Graphic showing the length of tangent vectors $\vec{T_1}(t)$, $\vec{T_2}(t)$ and $\vec{T_3}(t)$ on the interval $t \in [0, 1]$

$\vec{T_1}(t)$ has a constant length as t increases. This is expected as it follows circumference of S and has a constant velocity of S for all values of t indicating that $\vec{T_1}(t)$ will remain constant.

$\vec{T_2}(t)$ has a fluctuating length as it follows the path $p_2(t)$ on S. This is due to the path turning sharply as it moves around S. Due to it twisting and turning around S it's length increases and decreases accordingly.

$\vec{T_3}(t)$ has a decreasing length as $\lim_{t\to\infty} \vec{T_3}(t)$ and will eventually be equal to 0. This is what we expect to happen as initially the point initially moves fast as it spirals around S but begins to slow down as it approaches the north pole of the sphere. Moving in smaller and smaller spirals, slower and slower as it reaches the north pole thus slowly decreasing the value of $\vec{T_3}(t)$ as t increases.