Name: Andres Felipe Cadena Bolaños | **Student ID:** 139901

Report: Vehicle Routing Problem (VRP) Solution using Simulated Annealing

1. Metaheuristic: Simulated Annealing (SA)

Simulated Annealing (SA) is a well-known metaheuristic inspired by the physical process of annealing in metallurgy. It searches for near-optimal solutions by allowing both improving and, occasionally, worsening moves to escape local optima.

Here, the temperature is what controls the level of possible degradation of the solution, and with the cooling schedule, it decreases along the whole solution process. The main point in the optimization process is to focus on diversification at the beginning, and on intensification towards the end, but still allow both mechanisms all along.

- **Algorithm steps:**
    - **Initialization:** Start with an initial feasible solution and an initial relatively high temperature.
    - **Neighborhood Search:** Generate neighboring solutions by applying random modifications (neighborhood operators).
    - **Acceptance Criterion:** Accept better solutions directly. Worse solutions are accepted with a probability decreasing with temperature.
        - Probability $P = \exp(-\Delta E / T)$, where ΔE is the cost increase and T is the current temperature.
    - **Cooling Schedule:** Gradually reduce the temperature according to a cooling rate until a minimal threshold is reached.
    - **Termination:** Stop when temperature reaches a minimum or max iterations.

2. Modifications & Design Choices

At the beginning the drafted algorithm was based on:

- Getting a greedy initial solution using a basic heuristic as the Nearest Neighbor (NN).
- Using only two neighborhood operators:
    - Swap Customers: Exchanges customers between routes.

    - 2-Opt Intra-Route: Reverses customer segments within a single route to improve route efficiency.
- Completely random decision of the solution and exploring the entire neighborhood.
- Checking feasibility after generating the neighboring solutions.

To enhance the standard SA, several customizations were introduced for solving the VRP:

- **Initial Solution Construction:** Instead of using a basic heuristic, two more sophisticated construction of initial solutions were implemented, Sequential Insertion Heuristic and Clarke & Wright heuristic; the latter becoming the best starting solution. This significantly improved convergence.
- **Neighborhood Operators:** Another neighborhood operator was used, so at the end the algorithm worked with the following ones:
    - **Swap Customers:** Encourages global changes by exchanging customers between routes, promoting diversification.
    - **2-Opt Intra-Route:** Focuses on route improvement by reducing distance locally without changing the customer assignment.
    - **Relocate Customer:** Balances load distribution and fine-tunes the solution by moving one customer between routes.
- **Feasibility Enforcement:** Every operator includes capacity feasibility checks after moves, ensuring valid solutions already throughout the search, which saved some time.
- **Tournament Selection:** Instead of picking a random neighbor, the algorithm samples multiple neighbors and selects the best from a tournament, balancing exploration and intensification.

3. Experiments & Parameter Tuning

Several experiments were conducted to fine-tune the SA parameters:

Parameter	**Tested Values**	**Final Choice**
Initial Temperature	500, 1000, 1500, 2500	1500
Cooling Rate	0.98, 0.995, 0.999	0.995
Tournament Size	5, 10, 20	10

Minimum Temperature	0.001, 0.0001	0.001
Max Iterations/Temp	100, 150, 300	150
Total Max Iterations	100.000, 150.000, 200.000	200.000

Results with Tested Parameters:

Experiment	Initial Temp.	Cooling Rate	Tournament Size	Best Cost	Observation
Exp1	500	0.99	10	535.19	Fast cooling leads to risk of local optima. Initial temperature too low for sufficient exploration.
Exp2	2500	0.995	10	524.61	High initial temperature allows more exploration, but convergence is slower.
Exp3	1500	0.995	20	570.81	Larger tournament size results in slower iterations due to more candidate evaluations.
Exp4	1500	0.995	10	524.61	Good balance between exploration and exploitation.

Additional tests were conducted beyond the results presented here, including variations of total iteration limits, minimum temperature thresholds, and cooling schedules. Ultimately 200,000 total iterations were chosen to ensure the algorithm had sufficient time to explore the solution space and observe meaningful solution development over time.

This setup allowed to balance solution quality, computational time, and algorithm stability compared to the benchmark provided by the solution file.

To ensure the reproducibility of the results presented in this report, a fixed random seed was implemented during the final runs. This guarantees that the reported outcomes can be independently verified. However, in order to validate the robustness of the parameter choices, the algorithm was also tested without a fixed seed. These additional runs demonstrated that the selected parameter configuration generally performs well across multiple stochastic executions, indicating a reliable and stable behavior of the Simulated Annealing implementation even when accounting for its inherent randomness.

Testing with stochastic variability was crucial in assessing the robustness of the parameters. For this reason, although Experiment 2 achieved the known optimal solution, its parameter set did not outperform Experiment 4 considering convergence time under the influence of randomness.

Key Observations:

- Using Clarke & Wright as the initial solution reduced total cost significantly compared to other initializations. This became the one of the main factors to achieve the optimal solution more often.
- Setting the initial temperature to 1500 gave the best balance between exploration and convergence speed. Also, it confirmed that using a too high temperature led to long exploration phases to stabilize.
- The cooling rate of 0.995 provided a good trade-off between exploration and speed.
- The tournament selection consistently improved solution quality compared to random acceptance. Comparing 10 neighbors was the best approach here as it did not take a lot of time but still could compare diverse solutions.
- Minimum temperature of 0.001 combined with both 150 max iterations/temp and total max iterations of 200.000 provided a good balance between time and benchmark solution achieved on average after several runs.

4. Final Parameter Settings

Parameter	Value
Initial Temperature	1500
Cooling Rate	0.995
Tournament Size	10
Minimum Temperature	0.001
Max Iterations/Temp	150

Total Max Iterations	200.000

5. Results on VRP Instance

Instance: CMT1.vrp

Method	Total Cost
Initial Solution (Nearest Neighbor)	711.49
Initial Solution (Sequential Insert)	639.77
Initial Solution (Clarke & Wright)	584.63
Best Solution Found by SA	524.61

Route Overview (SA Best Solution):

- Number of Vehicles: 5 vehicles/routes
- Routes:
  - Route 1: [1, 33, 2, 23, 21, 36, 37, 4, 29, 32, 27, 9, 1] - Load: 149/160.0
  - Route 2: [1, 39, 10, 31, 35, 51, 17, 22, 30, 3, 12, 1] - Load: 159/160.0
  - Route 3: [1, 28, 49, 24, 8, 44, 25, 26, 15, 7, 1] - Load: 152/160.0
  - Route 4: [1, 47, 6, 50, 11, 40, 34, 46, 16, 45, 38, 13, 1] - Load: 160/160.0
  - Route 5: [1, 48, 5, 18, 43, 20, 41, 42, 14, 19, 1] - Load: 157/160.0

6. Conclusion:

The implemented Simulated Annealing with Clarke & Wright initialization and tournament-based neighborhood search achieved often high-quality solutions for the VRP. The enhancements and constant experiments made throughout the process were critical to achieve these high-quality solutions combined with convergence speed.

While the SA performed well for the CMT1 instance, further tests on larger VRP datasets could validate its scalability. The provided code is structured to allow testing with different instance files, making it a suitable for these future tests.