## Statistical average of R.V :

consider a R.V. X let the Possible numerical value of R.V X are $x_1, x_2, x_3$ ------ with Probability $P(x_1), P(x_2), P(x_3)$ .... If the no. of measurement N is very large, we would find that outcome $X = x_1$ would occur $NP(x_1)$ times, the outcome $X = x_2$ would occur $NP(x_2)$ times etc.





so, The arithmatic sum of all N measurement will be

$$x_1 P(x_1) N + x_2 P(x_2) N + \cdots = \cancel{\sum x_i f}$$

$$= N \sum x_i P(x_i)$$



The mean or average value of all these measurment is also called ~~mea~~ Expectation of X and is represented as $\bar{X}$ or $E(X)$ written as

$$\bar{X} = E(X) = \sum_i x_i P(x_i)$$



Now for the continuous RV.



mean $\boxed{\mu_x = m_x} = \int_{-\alpha}^{\alpha} x f_X(x)\, dx = E[X]$

2017

Week 16 / Day 100

Monday 10

It measure the variable's dispersion from the mean value.

For D.r.v

$\sum (x_i-\mu)^2 P(x_i)$

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**Variance:** It is also called 2nd moment of RV.

is denodeds $= \sigma_X^2$

$\therefore \sigma_X^2 = E\left[(X-\mu)^2\right] = \int_{-\alpha}^{\alpha} (x-\mu)^2 f_X(x)\, dx$

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The square root of variance, $\sigma_X$ is called the standard deviation of RV.

$$\therefore \sigma_x^2 = \int_{-\alpha}^{\alpha} (x-\mu)^2 f_X(x)\, dx$$

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$$= \int_{-\alpha}^{\alpha} x^2 f_X(x)\, dx - 2\mu \int_{-\alpha}^{\alpha} x f_X(x) dx + \mu^2 \int_{-\alpha}^{\alpha} f_X(x) dx$$

$$= \int_{-\alpha}^{\alpha} x^2 f_X(x) dx - 2\mu\mu + \mu^2$$

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$$= E(X^2) - \mu^2$$

$$= E[X^2] - E^2[X]$$

- Variance cannot be $-ve$ but $\mu$ may.

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## Correlation & Covariance

Consider a Pair of random variable X & Y. The Correlation is expressed as $E[XY]$ is given by,

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$$\boxed{E[XY] = \int_{-\alpha}^{\alpha}\int_{-\alpha}^{\alpha} xy\, f_{XY}(x,y)\,dx\,dy}$$

Let $\mu_x = E[X]$ and $\mu_y = E[Y]$, then the Correlation two centre random variable

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$X-\mu_x$, ie, $X-E[x]$ and $Y-\mu_y$ i.e $Y-E(Y)$ is given by.

$$COV[XY] = E[[X-E(X)][Y-E(Y)]]$$ is * called Covariance of X & Y.

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$$\boxed{COV[XY] = E[XY] - \mu_x \mu_y}$$

Now if Covariance is normalise w.r.t $\sigma_x \sigma_y$ is called Correlation Coefficient

$$\boxed{\rho_{xy} = \frac{COV[XY]}{\sigma_x \sigma_y}}$$

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- if $COV[XY] = 0$ then X & Y are uncorrelated.
- if $E[XY] = 0$ then X & Y are orthogonal.

that means, two R.V with mean is zero and if they are orthogonal then they are uncorrelated.

Notes:

| May 2017 | | | | | | |
|---|---|---|---|---|---|---|
| S | M | T | W | T | F | S |
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 28 | 29 | 30 | 31 | | | |

Prob 3 We have 3 Red balls and 5 blue balls. It select 3 red balls from it. Calculate mean, variance & standard deviation for red ball to get selected.

Sol: Possible outcome:

| | $x$ | |
|---|---|---|
| ○○○ → | 0 | → no red ball. |
| ○○⊗ → | 1 | → 1 red ball & 2 ball are blue |
| ○⊗⊗ → | 2 | → 2 " " & 1 " " |
| ⊗⊗⊗ → | 3 | → 3 " " |

here random variable $X \to$ Pick 3 ball to get red ball.

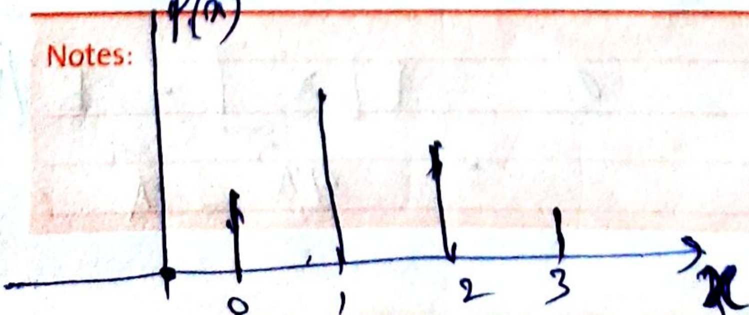
| $x_i$ | $P(x_i)$ |
|---|---|
| 0 | $\frac{{}^3C_0 \times {}^5C_3}{{}^8C_3} = \frac{10}{56}$ |
| 1 | $\frac{{}^3C_1 \times {}^5C_2}{{}^8C_3} = \frac{30}{56}$ |
| 2 | $\frac{{}^3C_2\ {}^5C_1}{{}^8C_3} = \frac{15}{56}$ |
| 3 | $\frac{{}^3C_3\ {}^5C_0}{{}^8C_3} = \frac{1}{56}$ |

$\therefore \mu = \sum x_i P(x_i) = 1.125$

$\therefore \sigma_x^2 = \sum (x_i - \mu)^2 p_i = 0.502$

$\sigma_x = \sqrt{0.502} = 0.709$

Notes:




From this, we see mean nearer to 1 as it max probability.

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Prob 4: A uniform distribution random variable X has a PDF given by

$$f_X(x) = \begin{cases} \frac{1}{2\pi} & 0 \leq x \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$$

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Determine $E[X]$, $E[X^2]$ & $E[(X-\mu)^2]$

Sol. By definition

$$\mu = E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

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$$= \int_{0}^{2\pi} x \cdot \frac{1}{2\pi}\, dx$$

$$= \frac{1}{2\pi} \frac{4\pi^2}{2} = \pi.$$

$$E[X^2] = \int_{0}^{2\pi} x^2 f_X(x)\, dx$$

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$$= \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi} = \frac{4\pi^2}{3}.$$

$$E[(X-\mu)^2] = \int_{-\infty}^{\infty} (X-\mu)^2 f_X(x)\, dx$$

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$$= \int_{0}^{2\pi} (x-\pi) \frac{1}{2\pi}\, dx.$$

| April 2017 | | | | | | |
|---|---|---|---|---|---|---|
| S | M | T | W | T | F | S |
| 30 | | | | | | 1 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 |

Notes:

$$\text{or} = E[X^2] - E^2[X]$$

$$= \frac{4\pi^2}{3} - \pi^2 = \frac{\pi^2}{3}$$