1 Graph Primitives

1.1 Graph

A Graph object contains a list of vertices, a list of edges, and an adjacent list. Therefore, each vertex is implicitly assigned an id from 0 to n-1, each edge is implicitly assigned an id from 0 to m-1. For each vertex, the adjacency list contains the id of its neighbors.

1.1.1 Read Graph From File: O(m+n)

```
String filename = "../../data/testgraph.txt";
Graph graph = GraphReader.Read(filename);
```

1.1.2 Insert a Vertex or an Edge: O(1)

```
Vertex vertex = new Vertex(vertexId);
graph.InsertVertex(vertex);
Edge edge = new Edge(node1Id, node2Id, edgeId);
graph.InsertEdge(edge);
```

1.1.3 Deletion of a Vertex v: O(d(v))

```
graph.DeleteVertex(vertex);
```

1.1.4 Deletion of an Edge: O(1)

```
graph.DeleteEdge(vertex1, vertex2);
```

1.1.5 Iterate Over Vertices: O(n)

```
foreach (Vertex w in G. Vertices. Values) w. Color = 0;
```

1.1.6 Iterate Over the Adjacency List: O(m+n)

```
foreach (int VertexId in graph.AdjList.Keys){
  foreach (int neighborId in graph.AdjList[VertexId]){
  }
}
```

1.2 Tree

Supports the operations as in a graph, with the added support for the parent child relationship.

1.2.1 Insert a Directed Edge: O(1)

```
Tree tree = new Tree(NumOfNodes, NumOfEdges);
//initialize nodes - each node with an unique Id...
//insertion of an edge ...
Edge e = new Edge(node1Id, node2Id, edgeId);
tree.InsertDirectedEdge(e, parentId, kidId, kidPosition)
```

1.2.2 Access Children: O(1)

For left and rightmost children, kidposition is set to 0 and Kids.Count - 1. One can access the left and right kids by invoking node.leftKid(), and node.rightKid(), respectively.

```
Node root = tree.RootNode;
Node leftKid = root.leftKid();
```

1.2.3 Iterate Over the Adjacency List: O(m+n)

```
foreach (int VertexId in graph.AdjList.Keys){
foreach (int neighborId in graph.AdjList[VertexId]) {
}
}
```

1.2.4 Check whether two Vertices are Neighbors: O(1)

```
graph.ContainsEdge(edge);
graph.AreAdjacent(vertex1, vertex2);
```

2 Graph Algorithms

2.1 BFS Ordering: O(m+n)

```
\begin{tabular}{ll} \textbf{int} & [] & bfsOrder & = & VertrexOrdering . BfsOrdering (graph , startVertex); \\ \end{tabular}
```

2.2 Connected Components: O(m+n)

```
List < Graph > components = Connected Components . get All Components (G);
```

2.3 Minimum Spanning Tree (MST): $O((m+n)\log m)$

- 3 Geometric Algorithms
- 3.1 L1 MST of a Point Set: $O(n \log n)$

4 Utilities

4.1 Nearest Neighbor

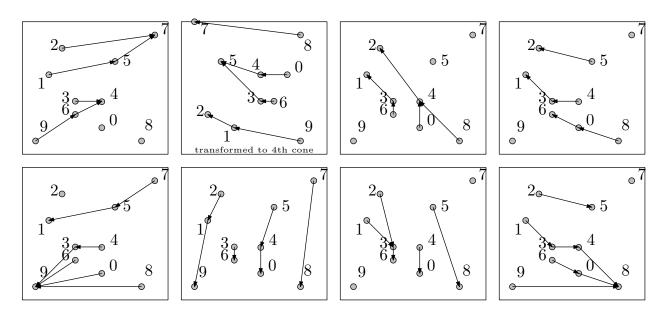


Figure 1: L1 Nearest Neighbor in a Cone

4.1.1 L1 Nearest Neighbor in a Cone: $O(n \log n)$

```
Dictionary < int , int > N = NearestNeighbor . L1NearestNeighbor InCone (P, ConeID);
```

Figure 2 visually illustrates the output of the algorithm, when ConeID = $1, 2, \dots, 8$. Uses algorithm [?].

4.2 Array Search

4.2.1 Binary Search: $O(\log n)$

```
double[] A = {.251, .5468, .84245, .84245};
int indexFound;
ArraySearch.BinarySearch(.251, A, out indexFound);
```

4.2.2 Binary Search Closest Neighbor: $O(\log n)$

```
| int l,r;
| ArraySearch.BinarySearchClosest(1.2, A, out l, out r);
```

5 Data Structures

5.1 Segment Tree

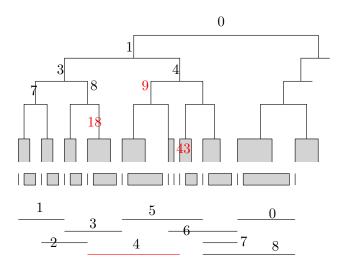


Figure 2: Segment Tree, Segment 4 is inserted in nodes 9, 18, 43

5.1.1 Construction: $O(n \log n)$

```
HInterval[] h = new HInterval[100];
//initialize h with unique ids
SegmentTree.Build(h);
```

5.1.2 Get all Intersecting Segments $O(\log n)$

```
Point q = new Point(x,y);
List<HInterval> intervals =
SegmentTree.GetAllIntersectingIntervals(q);
foreach (var interval in intervals)
Console.Write(interval.Id+",");
```

5.1.3 Get the Segment Immediately Above $O(\log^2 n)$

```
Point q = new Point(x,y);
HInterval intv =
  SegmentTree.GetImmediatelyAbove(q);
if (intv.Id > 0) Console.Write(intv.Id);
```