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Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

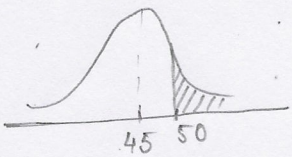
$\mu = 45 \quad \sigma = 8$

let, X is the time required to service.

Since, he starts 10 mins late, we need to find the probability of $X > 50$.

$P(X > 50) = 1 - P(X < 50)$

$Z\ Score_{50} = \frac{50 - 45}{8} = 0.625$



$\therefore P(X < 50) = 0.73$ [From the Z score table.]

$\therefore P(X > 50) = 1 - P(X < 50)$
$= 1 - 0.73 = 0.26$

$\therefore$ There is 26% chance the service manager cannot meet his commitment.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

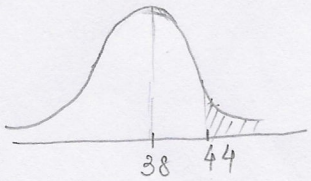
   A. More employees at the processing center are older than 44 than between 38 and 44. False

$\mu = 38 \quad \sigma = 6$

let, X be the age of an employee.

$P(X > 44) = 1 - P(X < 44)$

$Z\ Score_{44} = \frac{44 - 38}{6} = 1$



$\therefore P(X < 44) = 0.841$ [From table]

$\therefore P(X > 44) = 1 - 0.841 = 0.158$

No. of people with age $> 44 = 0.158 \times 400$
$= 63.2 \simeq 63$

Now, $P(X > 38) = 1 - P(X < 38)$

$Z\ score_{38} = \frac{38 - 38}{6} = 0$

$P(X < 38) = 0.5$ [From table]

$\therefore P(X > 38) = 0.5$

Now, $P(38 < X < 44) = 0.841 - 0.5 = 0.341$

No. of people with age between 38 and 44 $= 0.341 \times 400$
$= 136$

$\therefore$ The above statement is false.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. True

$P(X < 30)$. Z Score $= \frac{30 - 38}{6} = -1.33$

$\therefore P(X < 30) = 0.09$ [From table]

No. of people of age $< 30 = -1.33 \times 400$

$= 36$

$\therefore$ The above statement is true.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

$2X_1 \sim N(2\mu, 4\sigma^2)$

$X_1 + X_2 \sim N(2\mu, \sigma^2 + \sigma^2)$

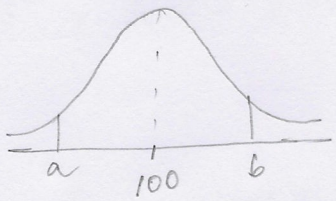
or, $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$.

The variance of $2X_1$ is double the variance of $X_1 + X_2$.

Here, μ is the mean and σ^2 is the variance.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
✓D. 48.5, 151.5
E. 90.1, 109.9



There is 0.01 probability that any random number(x) will not fall between a and b.

$\therefore P(X < a) = 0.005 \quad Z_a = -2.58$

$P(X > b) = 0.005 \quad Z_b = 2.58$

$P(a < X < b) = 0.99$

$Z_a = \frac{X - \mu}{\sigma} \Rightarrow -2.58 = \frac{X_a - 100}{20} \Rightarrow X_a = -2.58 \times 20 + 100$

$\Rightarrow a = 48.4$

Similarly,

$b = 2.58 \times 20 + 100 = 151.5$

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

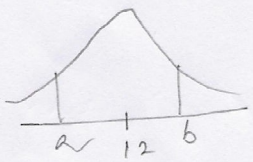
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

A. let, X_1 = Profit$_1$ $\sim N(5, 3^2)$

X_2 = Profit$_2$ $\sim N(7, 4^2)$

$X_1 + X_2 \sim N(12, 3^2 + 4^2)$

or, $X_1 + X_2 \sim N(12, 5^2)$



$P(X < a) = \frac{0.05}{2} = 0.025$

$Z = -0.67$

$\therefore a = -0.67 \times 5 + 12 = 8.65$

$b = 0.67 \times 5 + 12 = 15.38$

$\therefore$ The range is (8.65 to 15.38).

c) For, $X_1 < 0$,

$Z = \frac{0-5}{3} = -\frac{5}{3} = -1.6$

$\therefore P(X_1 < 0) = 0.0548 = 5.48\%$.

For, $X_2 < 0$,

$Z = \frac{0-7}{4} = -\frac{7}{4} = -1.75$

$P(X_2 < 0) = 0.040 = 4\%$.

$\therefore P_1 \sim N(5, 3^2)$ has more chance of getting loss.