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Energy Efficient Multi-beacon Guard Method for Periodic Data Gathering in Time-synchronized WSN

Debanjan Sadhukhan · Seela Veerabhadreswara Rao

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Abstract In periodic data-gathering, sensors can switch on the transceiver only during packet transmission to save energy. Exact clock-synchronization is difficult to achieve because of error present in synchronization protocols. Clock-disagreement increases with time in the absence of synchronization. In this paper, we propose a multi-beacon method to decrease the energy consumption by minimizing the awake time of sender and receiver by periodically switching on and off the receivers during the guard-time. We determine the optimal number-of-times the receiver needs to wake up along with the wake-up intervals to collectively minimize the total energy consumption of the sender-receiver pair during transmission. We show the effectiveness of our approach in energy conservation and compare with existing approaches using ns2 simulation.

Keywords Synchronization · Energy consumption · Wireless sensor networks · Ad-hoc networks

1 Introduction

The sensor nodes periodically generate data in most of the wireless sensor network (WSN) applications like environmental monitoring [1], fire detection [2], battlefield surveillance [2], and so on. In these applications, energy efficient hierarchical forwarding strategies like a tree or cluster based strategies are often preferred to increase the lifetime of the WSNs [3]. Energy efficiency is an important performance metric [4, 5] in WSN. The sleep/wake (s/w) scheduling techniques are used to conserve energy in periodic data gathering where sensor

Debanjan Sadhukhan
Indian Institute of Technology Guwahati
E-mail: debanjan.sadhukhan1987@gmail.com

Seela Veerabhadreswara Rao
Indian Institute of Technology Guwahati
E-mail: svrao@iitg.ernet.in

nodes switch on their transceiver only when they intend to send a packet or expect to receive a packet [6]. A sender-receiver pair knows the exact time for communication in synchronous s/w scheduling[1].

Exact clock synchronization is required for synchronous s/w scheduling, which is very difficult to achieve because of the imperfect crystal oscillator [6]. The most governing factors for clock-disagreement are phase-offset and clock-skew. Phase-offset is the clock difference (or time difference) between two sensors at an instant of time. The clock-skew refers to the difference in actual and expected crystal oscillator frequency. Clock-skew depends on environmental factors like temperature, pressure, radiation, magnetic fields, etc. It is measured in terms of parts-per-million (ppm), where one ppm denotes the clock drift of one micro second (μs) in a second. The clock-skew rate of Mica Motes is up to 50 ppm. A typical range of clock-skew is generally between 40 and 100 ppm [6]. Hence, clock-skew plays an important role in designing any energy efficient synchronous s/w scheduling protocol for WSNs.

Time synchronization protocols mitigate the effect of clock-skew by calculating expecting phase-offset and clock-skew. The clocks may still diverge due to the estimation error involved in synchronization [6]. Frequent synchronization may reduce the clock divergence, but it is not energy efficient for application with a long periodic event interval. One way of reducing synchronization error is to exchange more synchronization messages during synchronization interval, but this strategy consumes more energy. Note that, it is difficult to completely eliminate synchronization error as a result of non-determinism present in external environmental factors like temperature, pressure, humidity, etc., that affect the clock-skew. Due to the inaccuracy and the non-determinism present in the time synchronization protocol, nodes wake up earlier than the scheduled wake up time, to circumvent message loss, known as the guard time. If the length of the guard-time is high, this approach consumes more energy. In this work, we propose a multi-beacon method to reduce the energy consumption by minimizing the awake time of sender and receiver pair by periodically switching on and off the receiver during the guard-time. It determines the optimal number-of-times the receiver needs to wake-up along with the wake-up intervals to collectively minimize the total energy consumption of the sender-receiver pair during transmission. Henceforth, it is referred as Energy efficient Multi-Beacon Guard (EMBG) method.

The next section reviews the significant contributions in the literature. Section 3 derives expression for EMBG to minimize the total collective energy consumption during a transmission between a sender and receiver pair. We analyze the effectiveness of EMBG in energy conservation and lifetime extension over a conventional strategy in Section 4 and Section 5 respectively. We show the effectiveness of our approach in energy conservation and compare with existing approaches in Section 6. The last and final section concludes the paper.

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[1] On the other hand, the packets are exchanged without synchronization in asynchronous techniques [7, 8]. In this work we are interested in synchronous s/w scheduling.

2 Related Work

A large number of techniques appeared in the literature for energy efficient data-gathering in WSNs. Generally, these data-gathering protocols are cross layer, integrating routing and MAC layer with energy management mechanism. These protocols can be divided into synchronous or asynchronous based on the wake-up mechanism.

An asynchronous preamble-sampling based protocol which exploits Low-Power-Listening (LPL), is proposed in [9]. Nodes periodically listen for a short time to decide ongoing transmission.

If an event is detected[2], the sender attaches a long preamble before transmitting data. LPL minimizes energy consumption when no event is detected, whereas consumes more energy due to long preambles. In order to minimize the energy consumption for long preamble, short and strobed preamble based strategies are proposed [10], using additional low power radio, which increases sensor cost. In any data-gathering application these protocols must be used with a data-collection protocol which supports packet routing. Collection tree protocol (CTP) [11] is one such protocol which provides packet routing. These protocols need the network to be active all the time. Hence these protocols are not applicable for a network with sleep/wake mechanism. An adaptation layer is added on top of CTP that allows duty cycling is implemented in [12].

The trade-off between energy efficiency and latency associated with waking up the nodes is shown in [13]. In these strategies, each node wakes up independently and waits for the next hop node to wake up before transmitting any packet and this increases delay. In order to minimize this delay, anycasting forwarding schemes are proposed, where each node maintains a set of candidate nodes (closer to base-station), known as the forwarding set, and forwards the packet to the first node wakes up within this forwarding set. Kim et al. proposed an anycasting forwarding technique, in which neighboring nodes are added to the forwarding set only when they collectively minimize overall expected end-to-end delay (e-delay) [7]. But packets may follow a longer route to the base station. In order to minimize path length, the same authors developed a delay optimal anycasting scheme [8], where nodes do not immediately forward the packet, instead they wait for some time and then opportunistically forward only when expected delay involves for waiting is more. But these techniques are applicable for rare event detection scenario and are not energy efficient for periodic data-gathering. In periodic data-gathering synchronous sleep/wake scheduling techniques are more applicable but time-synchronization is required for these techniques.

Different time synchronization protocols are proposed in the literature to handle the clock-skew. We only present the significant contributions in the literature, detailed survey of these protocols can be found in [14, 15]. In the traditional synchronization scheme, the sender periodically sends messages

[2] Note that, periodic events like temperature or pressure are detected by the corresponding sensing device. In this work, we assume that the communication device, which consumes more energy, is switched on only when a communication is expected.

containing its current clock value to synchronize the time with the receiver. An extension of this approach is a two-way message passing to estimate the propagation delay in order to accurately synchronize the clocks [16]. A Receiver-Broadcast Synchronization (RBS) scheme that removes the nondeterminism present in sender time, access time and propagation delay is proposed in [17]. Each node compares the arrival time of physical layer beacons by other nodes and computes the phase-offset and the clock-skew using least-square linear regression model. RBS consumes a significant amount of energy due to several rounds of message exchange. The authors of [18] proposed a timing-sync protocol for sensor networks (TPSN) in which nodes exchange two-way time-stamped synchronization messages at the medium access control (MAC) layer. It successfully determines and eliminates the synchronization error, but is not able to estimate the clock-skew. A flooding time synchronization protocol to estimate the clock-skew is proposed in [19]. It is a combination of TPSN and RBS methods proposed in [17] and [18] respectively. Ganeriwal et al. proposed an energy efficient long term synchronization scheme that transmits synchronization beacons at periodic intervals to maintain a desired bound on the synchronization error is proposed in [20]. It adapts to change in clock drift and environmental conditions to achieve application-specific precision with high probability. The authors used the least square based scheme to compute the clock-skew from a set of received beacons. Similar approaches are followed in [21, 22, 17, 20] to compute the expected phase shift and the clock-skew among the neighbors. The effect of clock-skew is also investigated in the presence of asynchronous sleep/wake scheduling in [23]. Typically, these methods utilize linear regression in synchronization protocol to determine the expected clock-skew and the expected phase-offset.

The nodes, in periodic data-gathering, exchange data packets in periodic time intervals and every sender and receiver pair associate a slot for communication. In order to save energy, each sender and receiver pair wakes up at the same time to exchange the data packets which is widely known as the synchronous s/w scheduling technique. Nodes exchange control messages to negotiate slots for communication at periodic intervals [24, 25, 26, 27]. Due to the inaccuracy and the non-determinism present in the time synchronization protocol, the nodes wake up earlier than the scheduled wake up time, to circumvent message loss, known as the guard time. The authors of [28] used a fixed guard time to avoid the message loss. In the absence of time synchronization, the guard time tends to increase with the time which in turn increases the energy consumption. Wu *et al.* dynamically varied the guard-time to conserve more energy while satisfying a given threshold on the message capture probability in the presence of normally distributed synchronization error [6].

Bober *et al.* [29] proposed a method to conserve the energy consumption during guard-time by periodically switching on/off within the guard-time during data-collection. In BailighPulse [29], the authors proposed a scheme in which a sensor node wakes up multiple times within the guard-time and polls the channel for any activity. They also extended the mechanism for multi-hop network and proposed a multi-hop wake-up scheme that recovers network synchronization after long off periods. Specifically, the authors addressed the problem of minimizing energy consumption of the sensor network operating in multi-hop mode while taking into consideration of the clock-drift. The mean radio duty cycle, which is the average duration of the time a node is awake in the network, is minimized assuming the data-sampling and collection schedule for each node is known. The approach is extended for homogeneous, which requires that all nodes sample and send the data at the same time, and heterogeneous, allows nodes to have different sampling and collection time, network. In order to minimize the energy consumption during a wake-up, the guard-time is divided into several uniform polling intervals. During each polling intervals the receiver polls the medium for any beacon message. If the sender wants to send any data it sends beacon messages and the data packet is transfered only when the receiver acknowledges that it receives a beacon message. In order to minimize mean radio duty cycle, BailighPulse optimizes polling intervals using mechanism given in [30]. The authors showed that BailighPulse reduces energy consumption significantly compared to existing strategies using simulation and testbed results.

Motivation and Contribution: Due to the nondeterminism present in the synchronization, the guard time increases with time which in turn increases the power consumption. In a dense sensor network, wherein multiple nodes forward data to a single forwarding node (like in cluster based strategies), the forwarding node needs to apply guard time for each sender. Hence, additional energy consumption for guard time increases rapidly with the increasing number of senders associated with the forwarding node. In optimal s/w scheduling [6, 3], the receiver keeps its transceiver on during guard-time until it receives a data-packet. This in turn drains out energy quickly for longer guard-time.

In BailighPulse [29], the authors estimated the maximal drift between two nodes and use this information to estimate guard-time. The guard-time is uniformly divided into several polling intervals to conserve energy consumption. Actual guard-time follows normal distribution and this fact is not considered while estimating polling intervals. We show that instead of dividing the guard-time into uniform polling intervals if the guard-time is divided into unequal intervals, then the energy consumption reduces significantly. Moreover, non-uniform beacon interval reduces total expected waiting time within the guard time, which may reduces overall e2e delay of the data-packet. We also show that the number of expected beacons exchanged before any data-transmission is also less. We optimize the energy consumption within the guard-time by finding the beacon-exchange pattern within the guard-time. In other words, we divide the guard-time into unequal intervals for a sender-receiver pair and show that this method is energy efficient compared to existing strategies.

In this paper, we propose *Energy efficient Multi-Beacon Guard* [EMBG] method, in which the sensor nodes wake-up multiple times within the guard-time if the length of the guard-time is more than a threshold, otherwise follow the simple guard-time approach to conserve energy. The major contributions of our work are as follows:

– We derive an expression for energy consumption in multi-beacon approach during a data-packet transmission. We use this analysis to determine the optimal number of times the receiver wakes up and the wake-up intervals that minimize the expected energy consumption of a sender-receiver pair.
– We derive the expression for the expected energy consumption in guard-time approach and use this to derive a threshold for the guard-time duration beyond which multi-beacon approach is energy efficient.
– We also derive the expressions to estimate the expected energy conservation and the expected lifetime extended in EMBG.
– We show the effectiveness in energy saving and lifetime extension of EMBG.

3 Energy Efficient Multi-Beacon Guard

The sensor nodes in most of the existing approaches keep the transceiver on during the guard-time and waits for communication. The energy consumption is directly proportional to the waiting time. Therefore, the receiver in our EMBG method wakes up multiple times within the guard-time. If the length of guard-time is more than the optimal threshold, then the multi-beacon approach is followed, otherwise simple guard-time is followed. The guard-time is divided into unequal intervals in our EMBG approach. The receiver wakes up and sends a beacon message at the beginning of each interval, and waits for an average round trip time (RTT) for an acknowledgment from the sender. If the sender is awake and receives a beacon message then it starts the data transmission. The receiver, upon receiving the data packet, sends an acknowledgment to the sender. On the contrary, if the sender is not awake then the receiver goes to sleep state after a round trip time, wakes up at the beginning of the next interval, and follows the same procedure. The receiver reduces the total waiting time by going to sleep mode after each unsuccessful data transfer attempt in our EMBG approach. If the sender wakes up at the scheduled wake-up time then its transceiver remains active till it sends the message successfully. We minimize the total energy consumption by reducing the total waiting time of the sender and the receiver collectively during a packet transmission.

3.1 Wake-up Pattern of Receiver

We assume that the sensor nodes use RBS [17] for time synchronization. In the following lemma we show that the synchronization error in RBS [3] follows normal distribution.

Lemma 1 *If the sensor nodes follow RBS [17] for time synchronization, then the synchronization error between two nodes follow normal distribution.*

Proof See the Appendix.

[3] Note that for simplicity we assume that the synchronization protocol is RBS. Similar analysis can be provided for other synchronization protocols.

Let τ_p and τ_p' denotes the scheduled and actual arrival time of message p respectively. In other words, the receiver is expected to receive the message at time τ_p, but the message arrives at time τ_p'. Due to the non-determinism present in the system and measurement[4], τ_p' follows normal distribution with standard deviation σ_p. The receiver is awake during the interval $[\tau_p - T_g, \tau_p + T_g]$ to successfully receive the message, where $[-T_g, T_g]$ (or $2T_g$) is the duration of guard-time. Let X be the random variable for the event that denotes the message arrives at time X and $P(X = x)$ be the probability of the message arrives at time x. Note that the probability of the message arrives within the guard time is normally distributed. In other words, the pdf is defined as follows.

$$P(X = x) = \begin{cases} N(\tau_p, \sigma_p^2), & \text{if } \tau_p - T_g \leq x \leq \tau_p + T_g, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $N(\tau_p, \sigma_p^2)$ denotes the normal distribution with mean τ_p and standard deviation σ_p. We also assume that the probability of the message arrives within the guard-time is almost one. In other words, in order to make $\int\limits_{-\infty}^{-T_g} P(X = x)dx + \int\limits_{T_g}^{\infty} P(X = x)dx \simeq 0$, the standard deviation σ_p can be adjusted. We choose $\delta > 0$ to be very small number such that $P\left(|\tau_p - \tau_p'| \geq \sqrt{\frac{1}{\delta}}\sigma_p\right) \leq \delta$ and $T_g \geq \frac{\sqrt{\frac{1}{\delta}}\sigma_p}{2}$. Note that for simplicity we use Chebyshev's inequality [31], one can also use Chernoff's inequality for tighter bound [32]. Under this truncated definition $P(X = x) \sim N(\tau_p, \sigma_p^2)$ if $\tau_p - T_g \leq x \leq \tau_p + T_g$ otherwise $P(X = x) = 0$. For the sake of simplicity of the analysis, we assume $\tau_p = 0$ in the rest of the work[5].

Since the sender's wake up time follows normal distribution[6], the receiver needs to wake up more frequently at the mean of the distribution to minimize the expected energy consumption of the sender and the receiver. Hence, the sender must have equal expected waiting time between consecutive wake ups of the receiver. That is, the area under the normal curve between consecutive wake ups of the receiver is equal as shown in Fig. 1).

[4] The non-determinism occurs as a result of an imperfect crystal oscillator and governs by several environmental factors like: temperature, pressure, radiation and magnetic fields, etc. Moreover, estimation errors involve in clock-synchronization further add this non-determinism.

[5] Note that, τ_p denotes the mean of the distribution. The scheduled arrival time of the message is nonzero in a real system. If we assume $\tau_p = 0$, then we actually shift the mean of the distribution and keep the standard deviation same. The analysis derived in this paper holds for a node in a real system given that the corresponding mean needs to be adjusted by τ_p.

[6] We also assume that the delay between a sender wakes up and the message arrives at the receiver is negligible. In other words, the actual wake-up time of the sender follows the same distribution of τ_p'

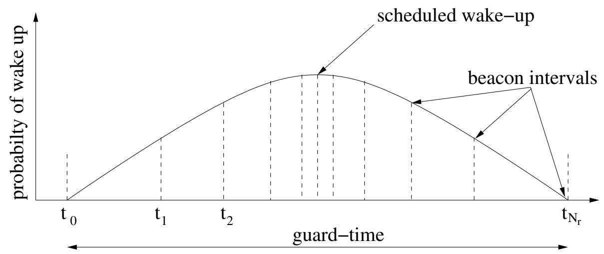


Fig. 1: Wake-up pattern of receiver within guard-time

Let t_1, t_2, t_3, ..., t_{N_r} denote the exact wake up times of the receiver where N_r denotes the number of times the receiver wakes up. Assuming that the area under the probability distribution function (PDF) is unity, the area under PDF between t_k and t_{k+1} is $\frac{1}{N_r}$, for $1 \le k \le N_r - 1$. That is,

$$\int_{-T_g}^{t_1} P(X = x)dx = \int_{t_k}^{t_{k+1}} P(X = x)dx = \frac{1}{N_r}, \quad 1 \le k \le (N_r - 1) \tag{2}$$

Therefore, t_i, for $1 \le i \le N_r$, can be found using the standard normal distribution as shown below:

$$t_i = \sum_{n=0}^{\infty} (2\pi)^{\frac{2n+2}{2}} \frac{C_{2n+1}}{(2n+1)!} \left(i \frac{1}{N_r \sigma} - \frac{1}{2} \right)^{2n+1}, \tag{3}$$

where $C_{n+1} = \sum_{j=0}^{n-1} \binom{n}{j+1} C_j C_{n-j}$, and $C_1 = 1$ [33].

3.2 Minimizing Transmission Energy

In this subsection, we use the wake-up pattern obtained in the previous subsection to analyze the effectiveness in energy saving. First, we derive the expressions for the expected number of times the receiver wakes up and the expected waiting time of the sender in Lemma 2 and Lemma 3 respectively. Then, we derive the expression for expected energy consumption in Lemma 4 using Lemma 2 and Lemma 3.

Lemma 2 *Assume that the actual wake-up time of the sender follows normal distribution and the guard-time is divided into N_r intervals. The expected number of times the receiver wakes up for a successful packet transmission is* $\frac{N_r+1}{2}$.

Proof See the Appendix.

Lemma 3 *Assume that the actual time of the sender follows the normal distribution within the interval* $[\tau_p - T_g, \tau_p + T_g]$. *The expected waiting time of a sender for a successful packet transmission is* $\frac{T_g}{N_r}$.

Proof See the Appendix.

The total expected energy consumption E_{mb} during transmission is the sum of expected energy consumption of the receiver E_{r_mb} and the sender E_{s_mb}. Every time the receiver wakes up, it transmits a beacon message and waits for RTT time to receive a data packet from the sender. After successfully receiving the data packet it sends an acknowledgment. Hence, the expected energy consumption of a receiver, using the notations given in Table 1, is as follows:

$$\begin{aligned} E_{r\_mb} = (E_{sw} + E_{txbcn} + P_{idle} \times T_{rtt}) \times \frac{N_r + 1}{2} \\ + E_{rxdata} + E_{txack}. \end{aligned} \tag{4}$$

However, the sender wakes up and waits for a beacon message from the receiver to transmit the data packet. Hence, the expected energy consumption of the sender is as follows:

$$E_{s\_mb} = E_{sw} + P_{idle} \times \frac{T_g}{N_r} + E_{rxbcn} + E_{txdata} + E_{rxack}. \tag{5}$$

Therefore, the total expected energy consumption is

$$\begin{aligned} E_{mb} = (E_{sw} + E_{txbcn} + P_{idle} T_{rtt}) \frac{N_r + 1}{2} + E_{rxdata} \\ + E_{txack} + E_{sw} + P_{idle} \frac{T_g}{N_r} + E_{rxbcn} \\ + E_{txdata} + E_{rxack}. \end{aligned} \tag{6}$$

The following lemma minimizes the expression for the expected energy consumption E_{mb}.

Lemma 4 *The expression for the expected energy consumption* [7] *in the multi-beacon approach given in Eq.6 is convex with respect to* N_r *and the optimal value occurs at*

$$N_r = \sqrt{\frac{2P_{idle} \times T_g}{E_{sw} + E_{txbcn} + P_{idle} T_{rtt}}}, \tag{7}$$

[7] Note that we ignore the effect of control packet-loss, while estimating expected energy consumption, as a result of the physical property of the wireless medium or any other factors. In our future work, we aim to account the uncertainty present in the wireless medium and provide better estimation for the expected energy consumption.

Table 1: Notations

Symbol	**Description**
σ_p	Standard deviation of actual arrival time of message
τ_p, τ_p'	Respectively denote scheduled and actual arrival time of message p
E_{sw}	Transition energy required from sleep to awake state
E_{txdata}/E_{rxdata}	Energy required to transmit/receive data packet (in μJ)
E_{txack}/E_{rxack}	Energy required to transmit/receive acknowledge packet (in μJ)
E_{txbcn}/E_{rxbcn}	Energy required to transmit/receive beacon packet (in μJ)
T_{rtt}	Average rtt time between any two neighboring node (in Sec)
P_{idle}	Power required for idle listening (in mW)
$[\tau_p - T_g, \tau_p + T_g]$	Guard time interval during which receiver is awake
N_r	Number of times receiver wakes up within guard-time
t_k	The time-instance when the receiver wakes up for k^th time
E_{mb}	Total expected energy consumption during transmission
E_{r_mb}, E_{s_mb}	Expected energy consumption of receiver and sender during transmission, respectively
Th_{mb}	Threshold for multi-beacon
E_{T_g}	Expected energy consumption in guard-time approach
T_e	Epoch duration
T_s	Synchronization interval
T_t	Sub-transmission interval
$T_{i,q}$	Time elapsed from last synchronization
M	Number of cluster member nodes
E_{cdata}	Expected energy conservation during transmission
$T_{n,m}$	n^{th} sender's time interval in m^{th} sub-transmission interval
χ_j^k	Aggregated message for a cluster-head j at level k contains m cluster member

where $N_r, P_{idle}, T_g, E_{sw},\ E_{txbcn}, T_{rtt}$ *denote the number-of-times the receiver wakes up, power required for idle listening, guard time, transition energy required from sleep to awake state, energy required to transmit a beacon and average round trip time respectively.*

3.3 Threshold for multi-beacon

When the guard time is too low, the energy required for the multi-beacon approach for control packets may be more than the guard-time approach. In this subsection, we compute the threshold for T_g beyond which the multi-beacon is more energy efficient than the guard-time approach. We denote the threshold for the multi-beacon approach by Th_{mb}.

Lemma 5 *The expected energy consumption in guard-time approach is:*

$$\begin{aligned} E_{T_g} &= (E_{sw} + E_{txdata} + E_{rxack}) \\ &\quad + (E_{sw} + P_{idle} \times T_g + E_{rxdata} + E_{txack}), \end{aligned} \tag{8}$$

Proof See the Appendix.

The threshold for the multi-beacon approach can be found by solving the inequality $E_{T_g} > E_{mb}$.

$$T_g(P_{idle} - \frac{P_{idle}}{N_r}) > E_{rxbcn}$$
$$+ \{E_{sw} + E_{txbcn} + P_{idle}T_{rtt}\}\frac{N_r + 1}{2} - E_{sw}. \tag{9}$$

Substituting Eq.7 in Eq.9, gives the following inequality

$$T_g > \frac{\left(c_3 + \sqrt{c_2}\right)^2}{2P_{idle}{}^2} \tag{10}$$

where $c_2 = c_3^2 + 4P_{idle}(E_{rxbcn} + \frac{1}{2}(c_1) - E_{sw})$, $c_3 = c_1\frac{1}{2}\sqrt{\frac{p_{idle}\times 2}{c_1}} + \sqrt{\frac{P_{idle}c_1}{2}}$, and $c_1 = (E_{sw} + E_{txbcn} + P_{idle}T_{rtt})$.

Lemma 6 *Energy consumption in the multi-beacon approach is less than the guard-time approach when $T_g > Th_{mb}$, where T_g and Th_{mb} denotes half of the guard time and the threshold for the multi-beacon approach respectively.*

In order to optimize the energy consumption in EMBG, simple guard-time strategy is followed if $T_g \leq Th_{mb}$. Otherwise, multi-beacon approach is followed.

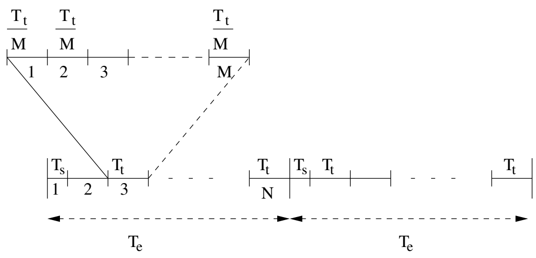


Fig. 2: Data-generation model

4 Expected Energy Conservation Estimation

In previous subsection, we derived expressions for expected energy consumption in EMBG and guard-time approaches using the wake-up pattern obtained in Subsection 3.1. Here, we extend our analysis to show how it saves energy consumption for a particular data-generation model. Although we show the effectiveness of our approach in saving energy for a particular data-generation model, our approach is applicable for any synchronized periodic data-gathering protocol.

We use the data-generation model shown in Fig. 2 that divides the time into constant duration epochs (T_e) [6, 3, 34, 35, 25]. Each epoch begins with a synchronization interval (T_s) followed by a transmission interval, which in turn consists of one or more sub-transmission intervals T_t, i.e., $T_e = T_s + NT_t, N \geq 1$. Moreover, each sub-transmission interval is split into M equal slots, one for each sender. Hence, each sender exchange data-packets during each sub-transmission interval. During each epoch, if sub-transmission interval increases, time elapsed from last synchronization increases as well.

If sender i is communicating with the receiver in q^{th} sub-transmission interval T_q, then time elapsed from last synchronization is as follows:

$$T_{i,q} = \frac{T_t}{M}(i-1) + T_t(q-1). \tag{11}$$

Using the implementation of RBS [17] given in [6], the standard deviation σ_p of the actual arrival time of the message p is as follows:

$$\sigma_p = \sqrt{\frac{{\sigma_0}^2}{{a_{ij}}^2}\left[\frac{1}{N_s} + \frac{1}{N_s}\frac{\left(\tau_p - \overline{T(j)}\right)^2}{\overline{T^2(j)} - \left(\overline{T(j)}\right)^2}\right]}, \tag{12}$$

where N_s denotes the number of pairs of time instants exchanged during synchronization between node i and j or $((t_i(k), t_j(k)), k = 1...N_s,)$, $\overline{T(j)} = \frac{\sum_{k=1}^{N_s} t_j(k)}{N_s}$, $\overline{T^2(j)} = \frac{\sum_{k=1}^{N_s} {t^2}_j(k)}{N_s}$, and σ_0 is the standard deviation of synchronization error [6].

Note that $\overline{T(j)}$ is the average of N_s time instants of the receiver. Hence, $(\tau_p - \overline{T(j)}) \geq T_{i,q}$, i.e., $(\tau_p - \overline{T(j)}) = T_{i,q} + \epsilon$, where $0 \leq \epsilon \leq T_s$. Moreover, the standard deviation σ'_p in terms of $T_{i,q}$ can be written as shown below:

$${\sigma_p}'(T_{i,q}) = \sqrt{\frac{{\sigma_0}^2}{{a_{ij}}^2}\left[\frac{1}{N_s} + \frac{1}{N_s}\frac{\left(T_{i,q}\right)^2}{\overline{T^2(j)} - \left(\overline{T(j)}\right)^2}\right]} \leq \sigma_p. \tag{13}$$

When ${\sigma_p}'(T_{i,q})$ is given, T_g can be computed as shown in Section 3.1. The expected energy conservation during a data-packet transmission E_{cdata} in multi-beacon approach can be given by,

$$
\begin{aligned}
E_{cdata} &= E_{T_g} - E_{mb}, \\
&= E_{s\_w} - (E_{sw} + E_{txbcn} + P_{idle}T_{rtt})\frac{N_r+1}{2} \\
&\quad + E_{rxbcn} + P_{idle}\left(\frac{\sqrt{\frac{1}{\delta}}\sigma_p}{2} - \frac{\sqrt{\frac{1}{\delta}}\sigma_p}{2N_r}\right),
\end{aligned}
$$

by substituting $\sigma_p'(T_{i,q})$ in place of σ_p we get

$$
\begin{aligned}
E_{cdata} \geq\; & E_{s\_w} - (E_{sw} + E_{txbcn} + P_{idle} \times T_{rtt})\frac{N_r+1}{2} \\
& + E_{rxbcn} \\
& + P_{idle}\left(\frac{\sqrt{\frac{1}{\delta}}\sigma_p{}'(T_{i,q})}{2} - \frac{\sqrt{\frac{1}{\delta}}\sigma_p{}'(T_{i,q})}{2N_r}\right).
\end{aligned}
$$

By substituting Eq. 13 in place of $\sigma_p{}'(T_{i,q})$ in the above equation we get,

$$
\begin{aligned}
E_{cdata}(T_{i,q}) =\; & E_{s\_w} - (E_{sw} + E_{txbcn} + P_{idle}T_{rtt})\frac{N_r+1}{2} + E_{rxbcn} \\
& + \frac{P_{idle}\sqrt{\frac{1}{\delta}}\sqrt{\frac{{\sigma_0}^2}{{a_{ij}}^2}\left[\frac{1}{N_s} + \frac{1}{N_s}\frac{(T_{i,q})^2}{\overline{T^2(j)} - \left(\overline{T(j)}\right)^2}\right]}}{2} \\
& - \frac{P_{idle}\sqrt{\frac{1}{\delta}}\sqrt{\frac{{\sigma_0}^2}{{a_{ij}}^2}\left[\frac{1}{N_s} + \frac{1}{N_s}\frac{(T_{i,q})^2}{\overline{T^2(j)} - \left(\overline{T(j)}\right)^2}\right]}}{2N_r} \\
\leq\; & E_{cdata}.
\end{aligned} \tag{22}
$$

The length of guard-time and the standard deviation σ_p of actual message arrival time increases as time elapsed from last synchronization increases. The multi-beacon approach starts conserving energy when T_g reaches to Th_{mb}. Let $(T_{n,m})$ denotes n^{th} sender's time interval in m^{th} sub-transmission interval in an epoch. For a given T_t and M, if there exists $(T_{n,m})$ such that $T_g > Th_{mb}$ and $T_g \leq Th_{mb}$ for $(T_{n-1,m})$ if $n > 1$, else $T_g \leq Th_{mb}$ for $(T_{M,m-1})$ if $n = 1$, then expected energy conservation for node i in multi-beacon approach in an epoch is at least,

$$E_{epoch}(i) = E_{sync}(i) + \sum_{l=1}^{N} \left(\sum_{j=1}^{m} \left(\chi_j^{k+1} E_{rx} + E_{txack} \right) + \chi_i^k E_{tx} + E_{rxack} \right). \quad (14)$$

$$\begin{aligned} \mathrm{E}_{rx}^{EMBG,\mathrm{gt(T(i,l))}} &= (E_{sw} + E_{txbcn} + P_{idle} T_{rtt}) \frac{N_r + 1}{2}, && \text{if } gt(T(i,l)) > TH_{mb}, \\ &= E_{sw} + P_{idle} gt(T(i,l)), && \text{otherwise.} \end{aligned} \quad (15)$$

$$\begin{aligned} \mathrm{E}_{tx}^{EMBG,\mathrm{gt(T(i,l))}} &= E_{sw} + P_{idle} \frac{gt(T(i,l))}{n_r} + E_{rxbcn}, && \text{if } gt(T(i,l)) > TH_{mb}, \\ &= E_{sw}, && \text{otherwise.} \end{aligned} \quad (16)$$

$$\begin{aligned} \mathrm{E}_{epoch}^{EMBG}(i) &= E_{sync}(i) \\ &+ \sum_{l=1}^{N} (\sum_{j=1}^{m_i} \left(\chi_j^{k+1} E_{rx} + \mathrm{E}_{rx}^{EMBG,\mathrm{gt(T(j,l))}} + E_{txack} \right) + \chi_i^k E_{tx} + \mathrm{E}_{tx}^{EMBG,\mathrm{gt(T(i,l))}} \\ &+ E_{rxack}) \end{aligned} \quad (17)$$

$$\begin{aligned} \mathrm{E}_{epoch}^{GT}(i) &= E_{sync}(i) \\ &+ \sum_{l=1}^{N} (\sum_{j=1}^{m_i} \left(\chi_j^{k+1} E_{rx} + \mathrm{E}_{rx}^{GT,\mathrm{gt(T(j,l))}} + E_{txack} \right) + \chi_i^k E_{tx} + \mathrm{E}_{tx}^{GT,\mathrm{gt(T(i,l))}} \\ &+ E_{rxack}) \end{aligned} \quad (18)$$

$$\begin{aligned} \mathrm{E}_{epoch,CH}^{EMBG,dy}(i) &= E_{sync,CH}^{dy} \\ &+ \sum_{l=1}^{N} (\sum_{j=1}^{m_i} \left(\chi_j^{2} E_{rx} + \mathrm{E}_{rx}^{EMBG,\mathrm{gt(T(j,l))}} + E_{txack} \right) + \chi_i^1 E_{tx} + \mathrm{E}_{tx}^{EMBG,\mathrm{gt(T(i,l))}} \\ &+ E_{rxack}) \end{aligned} \quad (19)$$

$$\begin{aligned} \mathrm{E}_{epoch,NCH}^{EMBG,dy}(i) &= E_{sync,NCH}^{dy} \\ &+ \sum_{l=1}^{N} \left(\chi_j^{2} E_{rx} + \mathrm{E}_{rx}^{EMBG,\mathrm{gt(T(j,l))}} + E_{txack} \right) + \chi_i^1 E_{tx} + \mathrm{E}_{tx}^{EMBG,\mathrm{gt(T(i,l))}} \\ &+ E_{rxack}. \end{aligned} \quad (20)$$

$$E_{dy}^{EMBG}(i) = \left(r_e E_{epoch,CH}^{EMBG,dy}(i) + e_{con}^{CH}(i) \right) + (r-1) \left(r_e E_{epoch,NCH}^{EMBG,dy}(i) + e_{con}^{NCH}(i) \right) \quad (21)$$

$$\begin{aligned} E_{cepoch}(i) &= \sum_{s=n}^{M} E_{cdata}(T_{s,m}) \\ &+ \sum_{l=m+1}^{N} \sum_{k=1}^{M} E_{cdata}(T_{k,l}), && \text{if } n > 1, \\ &= \sum_{l=m}^{N} \sum_{k=1}^{M} E_{cdata}(T_{k,l}), && \text{if } n = 1. \end{aligned} \quad (23)$$

5 Maximizing Network Lifetime

Hierarchical clustering strategies are known to be energy efficient in periodic data-gathering. Hence, we analyze the effect of EMBG on the lifetime of a hierarchical clustering network in this section. We assumed that the base-station is located at level 0 (the highest level), and each cluster contains a single cluster head (CH) and multiple cluster members [36, 37]. Neighboring clusters use orthogonal frequency channel to avoid collisions. We used the data-aggregation model described in [3]. If a cluster head j at level k contains m cluster members (labeled as $1...m$) then the aggregated message is given as below:

$$\chi_j^k = r^k \left(\left(\sum_{i=1}^{m} \chi_i^{k+1} \right) + ml_j \right) + c^k, \tag{24}$$

where ml_j is the message length of the node j, $r^k \leq 1$, and c^k corresponds to the overhead of aggregation.

We need to measure the lifetime of a node to maximize the lifetime of the network,. The lifetime of node i can be given as: $L(i) = \frac{Q_i - E_{con}(i)}{E_{epoch}(i)} epoch_{dur}$, where $Q_i, E_{con}(i), E_{epoch}(i)$, and $epoch_{dur}$ denotes the initial energy, energy required during configuration phase, energy consumption per epoch, and epoch duration of node i respectively.

In each sub-transmission interval of an epoch, a sensor node receives packets from lower level nodes and sends the aggregated packets to the higher level node. Energy consumption per epoch is the sum of energy consumption during synchronization, data-reception, and transmission. If E_{rx} and E_{tx} respectively denotes receiving and transmitting energy for a unit length packet, then total energy consumption during an epoch of a node i is given in Eq. 14, where $E_{sync}(i)$ is the amount of energy consumption for synchronization and N is the number of sub-transmission intervals. As time lapses from synchronization increases, the guard-time gt increases. Hence, cluster head awake-time before scheduled transmission also increases. Therefore, we need to consider additional energy consumption during this gt to find total energy consumption. If node i is sending data in l^{th} sub-transmission interval, then the guard-time $[-gt(T(i,l)), gt(T(i,l))]$ can be calculated using the Chebyshev inequality discussed in Section 3.1. Note that, expressions for expected energy consumption in multi-beacon and guard-time, for a sender and receiver pair is given in Eq. 6 and 32 respectively. Hence, while excluding energy consumption for packet transmission and reception, expected energy consumption of a receiver and sender in EMBG during $[-gt(T(i,l)), gt(T(i,l))]$ is presented in Eq. 15 and 16 respectively. Total energy consumption of node i in level k of a hierarchical clustering network is presented in Eq. 17, which is obtained by substituting Eq. 15 and 16 in Eq. 14. Similarly, total energy consumption of node i at level k of the hierarchical clustering network with guard-time approach is given in Eq. 18.

5.1 Dynamic Clustering Network

In dynamic clustering, the CH's change over time to increase the overall network lifetime. For simplicity, we assume that the nodes use the fixed power-transmission model and the cluster members and cluster-heads communicate directly to the cluster head and base-station respectively [36]. Maximum value of k is 2 because it is a three level hierarchy. The time is divided into several rounds and a cluster-head is selected randomly in each round, to collect data and forward to the base-station such that in each r rounds, each node becomes CH once and remains non-CH for $r-1$ rounds. Let $e_{con}^{CH}(i)$ and $e_{con}^{NCH}(i)$ denote the amount of energy consumption by the node i when it is configured as CH and non-CH respectively. Hence, energy consumption for r rounds is given in Eq. 21, where $E_{epoch,CH}^{EMBG,dy}(i)$ and $E_{epoch,NCH}^{EMBG,dy}(i)$ denote the energy consumption of node i as a CH and non-CH, as given in Eq. 19 and Eq. 20 respectively (both obtained from Eq. 17), and r_e denotes number of epochs in each round.

The expected lifetime of a node i is given by

$$L_{dy}^{EMBG}(i) = \frac{Q_i}{E_{dy}^{EMBG}(i)} r_{dur}, \tag{25}$$

where r_{dur} is the duration of r rounds. Hence, the expected lifetime of the network is given by

$$L_{dynamic}^{EMBG} = \min \left\{ L_{dy}^{EMBG}(i) \right\}, \quad \forall i. \tag{26}$$

Similarly, we can determine the expected lifetime of the network for guard-time (L_{dy}^{GT}). Hence, the expected lifetime increased using EMBG is denoted as

$$L^{*} = L_{dy}^{EMBG} - L_{dy}^{GT}. \tag{27}$$

6 Simulation

We deployed 100 nodes in 500×500 m^2 area using a random uniform distribution and simulated EMBG using the network simulator 2.34 (ns2). We use Mica 2 motes [38] parameters along with the power requirements shown in [39],[6]. Other parameters used in the simulation are given in Table 2. We define the lifetime of the network as the lifetime of the first node that depletes its energy completely.

Clustering approaches are used to conserve the overall energy consumption in the network. In LEACH [36], CHs are selected randomly across the network. Random selection may increase energy consumption especially if the CHs are located at very far from the base station or adjacent to each other. Hence, in this paper we use deterministic LEACH [40] where the remaining energy is

Table 2: Simulation Parameters

Name	**Value**
Area under simulation	$500 \times 500\ m^2$
Communication range	$250\,m$
Data rate	19.2 kbps
Initial energy	1000 J
Receiving/Idle Power	13.0 mW
Transmission Power	19.5 mW
Energy required from sleep to awake	22.0 μJ
Data packet length	44 Byte
Beacon/Ack packet length	16 Byte
Synchronization protocol	RBS
Wireless media	802.11

also considered while selecting a CH. We set the number of clusters in each round as 4% of the total nodes. The cluster-head collects data-packets from cluster-members, aggregate the packets ($r = 0.5$) and forwards the resultant packets to the base-station. To avoid collision, TDMA and CDMA based approaches are used for intra and inter cluster communication respectively. For synchronization, we used the RBS [17] implementation described in [6].

6.1 Comparing simulation results for different approaches

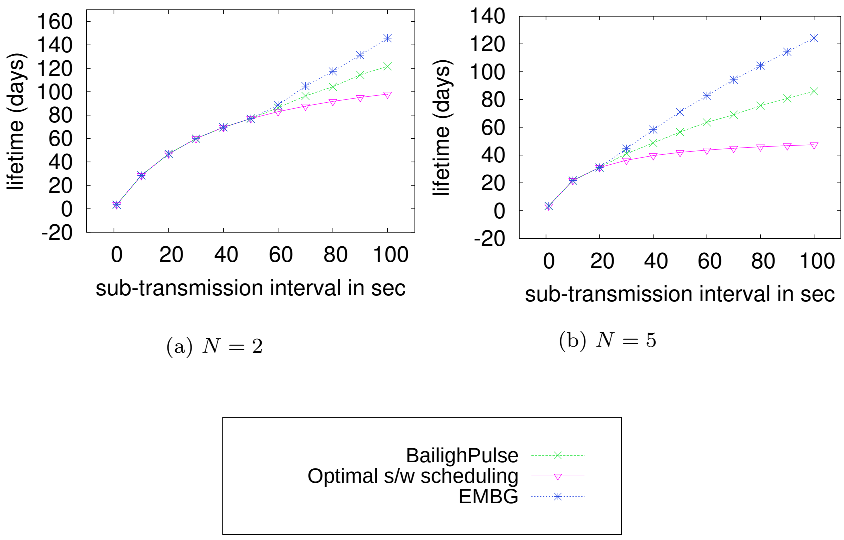
In this sub-section, we compare the performance of EMBG with BailighPulse [29] and optimal s/w scheduling [6]. Optimal s/w scheduling uses the simple guard-time approach whereas BailighPulse uses equal intervals multi beacon approach.

6.1.1 Lifetime

Fig. 3 gives the effectiveness of EMBG over optimal s/w scheduling[6] and BailighPulse [29] in increasing network lifetime for various data-generation intervals. In optimal s/w scheduling[6], the guard-time increases with the data-generation interval, which in turn increases the energy consumption during a transmission. Hence, the lifetime of EMBG increases rapidly with the data-generation interval when compared to the optimal s/w scheduling[6]. In addition, the lifetime of EMBG increases when compared to the BailighPulse because of a less number of wake-up intervals in EMBG. Moreover, we can also observe that EMBG is effective in extending the network lifetime for higher values of N.

6.1.2 Packet delivery ratio

Fig. 4 show that the percentage of the packet delivery ratio of EMBG and BailighPulse decreases marginally as density increases. The amount of beacons exchanged during an epoch increases with the density in EMBG and



(a) $N = 2$

(b) $N = 5$

(c) legend

Fig. 3: Comparing simulation results for different approaches: (a-b): lifetime

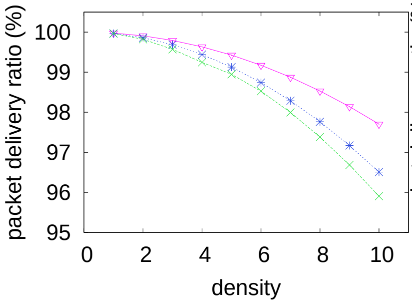
BailighPulse approaches, which in turn increases the probability of collision with data-packets. Since the number of beacon packets is more in BailighPulse [29] than EMBG, BailighPulse [29] suffers from higher packet loss as guard-time increases. This phenomenon is more visible for the higher value of N because the number of control packets exchanged increases with the value of N.

6.1.3 Delay

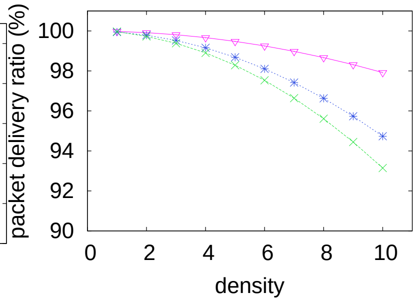
There is no deviation in the end-to-end delay for EMBG, BailighPulse and optimal s/w scheduling because all these protocols follow TDMA based data-packet transmission.

6.1.4 Ratio of control and data packets

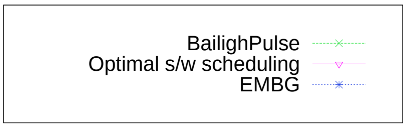
The amount of beacons exchanged in EMBG and BailighPulse increases significantly with the guard-time because of multi-beacon approach. The number of control packets generated by for a data-packet, while varying the data-generation interval is presented in Fig. 5. The ratio of control and data-packets in both EMBG and BailighPulse increases with the data-generation interval because of the multi-beacon approach.



(a) $N = 2$



(b) $N = 5$



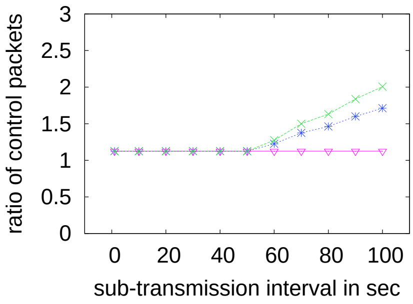
(c) legend

Fig. 4: Comparing simulation results for different approaches: (a-b): packet delivery ratio

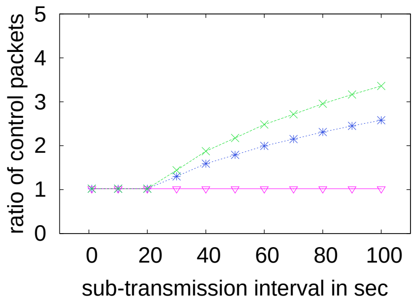
7 Conclusion and Future Work

In this paper, we proposed an *energy efficient multi-beacon guard* method where the sensor nodes follow the multi-beacon approach with unequal intervals if the length of guard-time is more, otherwise follow the simple guard-time approach to conserve energy. We studied the problem of energy efficient transmission between a sender and receiver pair and extended this analysis to maximize the lifetime of a hierarchical WSN. Further, simulation results confirms the effectiveness of our approach.

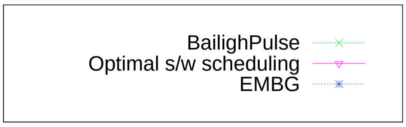
In our method we assume that the receiver switches on its transceiver for the duration of the guard-time. If the receiver switches on its transceiver for less than the duration of the guard-time, then it saves more energy during transmissions but it may loose packets and vice versa. One can control the guard-time to find the desired message capture probability and optimize lifetime. This can be pointed as a future work direction for this work. Our method can be applicable for a multi-hop data-gathering approach. As part of future work, one can extend our analysis to optimize the overall network energy consumption in a multi-hop data-gathering setup. We also believe that it would be interesting to extend our analysis for mobile ad-hoc WSN.



(a) $N = 2$



(b) $N = 5$



(c) legend

Fig. 5: Comparing simulation results for different approaches: (a-b): ratio of control and data packets

8 Appendix

Lemma 1 *If the sensor nodes follow RBS [17] for time synchronization then the synchronization error between two nodes follow normal distribution.*

Proof We collected the data from the experiment carried out in [17] and plotted the results to show that the synchronization error follows normal distribution. In order to show that the synchronization error follows normal distribution, we show that the instantaneous pairwise phase-offset between two nodes follows normal distribution. The phase-offset is the clock difference at an instant of time.

A small network consists of six nodes where one node acts as the sender and rest of the nodes act as receivers was set up. The sender node broadcast 160 packets over 3 minutes with random inter-packet delays from 200 ms to 2 seconds. For each broadcast packet 10 different phase-offsets are calculated between 5 nodes and a total of 1478 pairings are calculated.

In order to show the distribution of the receivers phase-offset (with the sender) follows normal distribution, the pairwise difference between packet reception time in X-axis and probability of pairwise difference in Y-axis is plotted in Fig. 6. The maximum phase-offset in any trial is 53 μsecs. In order to show the distribution follows normal distribution, we conduct a chi-square test which indicates 99% confidence with parameter $\mu = 0$ and $\sigma = 11.1$ μsec (refer Fig. 6).

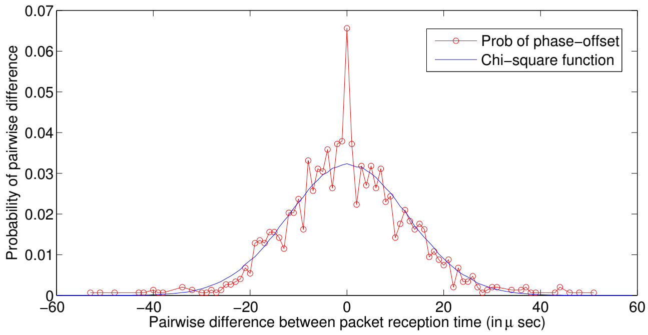


Fig. 6: Probability of pairwise difference between packet reception time

Lemma 2 *Assume the actual wake-up time of sender follows normal distribution and the guard-time is divided into N_r intervals. The expected number of time the receiver wakes up for a successful packet transmission is $\frac{N_r+1}{2}$.*

Proof The receiver wakes up, sends the beacon message, and waits for round trip time (RTT) to receive data from the sender, at times $t_i, 1 \leq i \leq N_r$. The receiver has to wake up $i+1$ times if the sender wakes up in the interval $(t_i, t_{i+1}]$, assuming propagation delay is negligible. The expected number-of-times the receiver wakes up is,

$$\begin{aligned} & P(X = x : x \leq t_1) \times 1 \\ & \quad + \sum_{i=1}^{N_r-1} P(X = x : t_i > x \leq t_{i+1}) \times (i+1) \\ & = \frac{N_r+1}{2}. \end{aligned}$$

Lemma 3 *Assume the actual time of the sender follows normal the distribution within the interval $[\tau_p - T_g, \tau_p + T_g]$, where τ_p, T_g respectively denotes the scheduled wake-up time of the sender and the half of the guard-time. The expected waiting time of a sender for a successful packet transmission is $\frac{T_g}{N_r}$.*

Proof If the area under the normal curve is equally divided into N_r regions, then the expected waiting time of the sender is,

$$W_s = \int_{-T_g}^{t_1} P(X = x)(t_1 - x)dx + \sum_{i=1}^{N_r-1} \int_{t_i}^{t_{i+1}} P(X = x)(t_{i+1} - x)dx.$$

If N_r is odd, we can rewrite the equation as,

$$
\begin{aligned}
W_s = &\int_{-T_g}^{t_1} P(X=x)(t_1-x)dx + \int_{t_{N_{r-1}}}^{t_{N_r}} P(X=x)(t_1-x)dx \\
&+ \sum_{i=1}^{\lfloor \frac{N_r}{2} \rfloor -1} \int_{t_i}^{t_{i+1}} P(X=x)(t_{i+1}-x)dx + \sum_{i=1}^{\lfloor \frac{N_r}{2} \rfloor -1} \int_{t_{N_r-(i+1)}}^{t_{N_r-i}} P(X=x)(t_{N_r-(i)}-x)dx \\
&+ \int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{t_{\lfloor \frac{N_r}{2} \rfloor +1}} P(X=x)(t_{\lfloor N_{r/2} \rfloor +1} - x)dx. \qquad (28)
\end{aligned}
$$

In order to prove this lemma, we first show that $\int_{t_i}^{t_{i+1}} P(X=x)(t_{i+1}-x)dx + \int_{t_{N_r-(i+1)}}^{t_{N_r-i}} P(X=x)(t_{N_r-i}-x)dx = \frac{1}{N_r}\left|t_i - t_{i+1}\right|$.

$$
\begin{aligned}
&\int_{t_i}^{t_{i+1}} P(X=x)(t_{i+1}-x)dx + \int_{t_{N_r-(i+1)}}^{t_{N_r-i}} P(X=x)(t_{N_r-i}-x)dx \\
&= \int\limits_{t_i}^{t_{i+1}} P(X=x)(t_{i+1})dx - \frac{-\sigma}{\sqrt{2\pi}}\left(e^{\frac{-t_{i+1}{}^2}{2\sigma^2}} - e^{\frac{-t_i{}^2}{2\sigma^2}}\right) \\
&\quad + \int\limits_{t_{N_r-(i+1)}}^{t_{N_r-i}} P(X=x)(t_{N_r-i})dx - \frac{-\sigma}{\sqrt{2\pi}}\left(e^{\frac{-t_{N_r-i}{}^2}{2\sigma^2}} - e^{\frac{-t_{N_r-(i+1)}{}^2}{2\sigma^2}}\right), \\
&\quad \text{by substituting } P(X=x) = \frac{1}{\sigma_p\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_p^2}}
\end{aligned}
$$

Since the function is symmetric on both side of mean, and $\|t_i - t_{i+1}\| = \left\|t_{N_r-(i+1)} - t_{N_r-(i)}\right\|$, $|t_{N_r-i}| = |t_i|$ for $i \leq \left\lfloor \frac{N_r}{2} \right\rfloor - 1$. The above equation can be rewritten as,

$$
\begin{aligned}
&t_{i+1}\int_{t_i}^{t_{i+1}} P(X=x)dx + t_{N_r-i}\int_{t_{N_r-(i+1)}}^{t_{N_r-i}} P(X=x)dx \\
&= t_{i+1}\frac{1}{N_r} + t_{N_r-i}\frac{1}{N_r}, \text{ by substituting } P(X=x) = \frac{1}{\sigma_p\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_p^2}} \\
&= \frac{1}{N_r}\left|t_{i+1} - t_i\right|. \qquad (29)
\end{aligned}
$$

Since the midpoint of the middle interval $[t_{\lfloor \frac{N_r}{2} \rfloor}, t_{\lfloor \frac{N_r}{2} \rfloor +1}]$ is mean of PDF, we can write $\int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{t_{\lfloor \frac{N_r}{2} \rfloor +1}} P(X=x)(t_{\lfloor \frac{N_r}{2} \rfloor +1} - x)dx$ of Eq.28 as,

$$\int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{0} P(X=x)(t_{\lfloor \frac{N_r}{2} \rfloor+1})dx - \int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{0} P(X=x)(x)dx$$
$$+ \int_{0}^{t_{\lfloor \frac{N_r}{2} \rfloor+1}} P(X=x)(t_{\lfloor \frac{N_r}{2} \rfloor+1})dx - \int_{0}^{t_{\lfloor \frac{N_r}{2} \rfloor+1}} P(X=x)(x)dx$$

$$= t_{\lfloor \frac{N_r}{2} \rfloor+1} \int\limits_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{0} P(X=x)dx - \frac{-\sigma}{\sqrt{2\pi}} \left(1 - e^{\frac{-t_{\lfloor \frac{N_r}{2} \rfloor}^{2}}{2\sigma^2}}\right)$$
$$+ t_{\lfloor \frac{N_r}{2} \rfloor+1} \int\limits_{0}^{t_{\lfloor \frac{N_r}{2} \rfloor+1}} P(X=x)dx - \frac{-\sigma}{\sqrt{2\pi}} \left(e^{\frac{-(t_{\lfloor \frac{N_r}{2} \rfloor+1})^2}{2\sigma^2}} - 1\right),$$

by substituting $P(X=x) = \frac{1}{\sigma_p\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_p^2}}$

As we know that the function is symmetric in both side of the mean, hence $\left|t_{\lfloor \frac{N_r}{2} \rfloor+1}\right| = \left|t_{\lfloor \frac{N_r}{2} \rfloor}\right|$ and the above expression becomes,

$$\int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{t_{\lfloor \frac{N_r}{2} \rfloor+1}} P(X=x)(t_{\lfloor \frac{N_r}{2} \rfloor+1} - x)dx$$
$$= t_{\lfloor \frac{N_r}{2} \rfloor+1} \int_{t_{\lfloor \frac{N_r}{2} \rfloor}}^{0} P(X=x)dx + t_{\lfloor \frac{N_r}{2} \rfloor+1} \int_{0}^{t_{\lfloor \frac{N_r}{2} \rfloor+1}} P(X=x)dx,$$
$$= t_{\lfloor \frac{N_r}{2} \rfloor+1} \times \frac{1}{N_r}. \tag{30}$$

Note that $t_{\lfloor \frac{N_r}{2} \rfloor+1}$ can be written as $\left|0 - t_{\lfloor \frac{N_r}{2} \rfloor}\right|$. Similarly, $\int_{-T_g}^{t_1} P(X = x)(t_1 - x)dx + \int_{t_{n_{r-1}}}^{t_{N_r}} P(X = x)(t_1 - x)dx = \frac{1}{N_r}|t_1 - T_g|$. Hence Eq.28 can be written as,

$$W_s = \frac{1}{N_r}|t_1 - T_g| + \frac{1}{N_r}\left(\sum_{i=1}^{\lfloor \frac{N_r}{2} - 1 \rfloor} |t_i - t_{i+1}|\right)$$
$$+ \frac{1}{N_r}\left|t_{\lfloor \frac{N_r}{2} \rfloor} - 0\right|, \text{by substituting Eq.29,30.}$$
$$= \frac{T_g}{N_r}.$$

Similarly, it can be proved for N_r is even.

Lemma 4 *The expected energy consumption in guard-time approach is*

$$\begin{aligned} E_{T_g} &= (E_{sw} + E_{txdata} + E_{rxack}) \\ &\quad + (E_{sw} + P_{idle} \times T_g + E_{rxdata} + E_{txack}). \end{aligned} \tag{31}$$

Proof In guard time approach, the receiver is awake during the whole guard-time and the sender sends the data packet as soon as it wakes up. As mentioned earlier wake up times of the sender follows the normal distribution within the guard time. If x denotes the instant when the sender wakes up, then the amount of time the receiver waits is $T_g + x$, where $-T_g \leq x \leq T_g$. Hence, the expected amount of time the receiver waits is

$$\begin{aligned} W_{r\_T_g} &= \int_{-T_g}^{T_g} P(X = x)(T_g + x)dx, \\ &= T_g \int\limits_{-T_g}^{T_g} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right) dx + \int\limits_{-T_g}^{T_g} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right) x \ dx, \\ &\quad \text{by substituting } P(X = x) = \tfrac{1}{\sigma_p\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_p^2}} \\ &= T_g. \end{aligned}$$

The receiver sends an acknowledgment after receiving the data packet from the sender and switches off the transceiver. Hence, the expected energy consumption of the receiver using the notations given in Table 1 is given as follows: $E_{r\_T_g} = E_{sw} + P_{idle} \times (W_{r-T_g}) + E_{rxdata} + E_{txack}$. The energy consumption of sender is $E_{s\_T_g} = E_{sw} + E_{txdata} + E_{rxack}$. Hence, the total expected energy consumption is

$$\begin{aligned} E_{T_g} &= (E_{sw} + E_{txdata} + E_{rxack}) \\ &\quad + (E_{sw} + P_{idle}(W_{r\_T_g}) + E_{rxdata} + E_{txack}). \end{aligned} \tag{32}$$

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