BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

Unit-1

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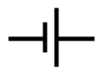
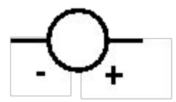
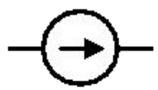
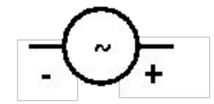
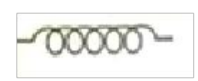
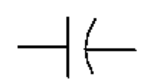
ELECTRIC CIRCUITS

Electric circuits are broadly classified as Direct Current (D.C.) circuits and Alternating Current (A.C.) circuits. The following are the various elements that form electric circuits.

D.C. Circuits

Elements	**Representation**
Voltage source	 
Current source	
Resistor	

A.C. Circuits

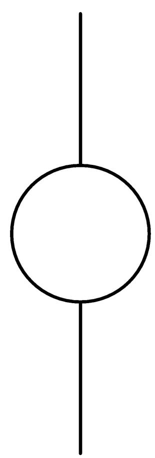
Elements	**Representation**
Voltage source	
Current source	
Resistor	
Inductor	
Capacitor	

We also will classified sources as **Independent** and **Dependent** sources

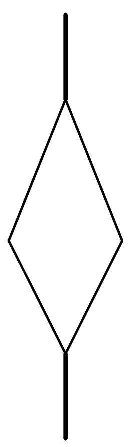
Independent source establishes a voltage or a current in a circuit without relying on a voltage or current elsewhere in the circuit

Dependent sources establishes a voltage or a current in a circuit whose value depends on the value of a voltage or a current elsewhere in the circuit

We will use circle to represent **Independent source** and diamond shape to represent **Dependent sources**



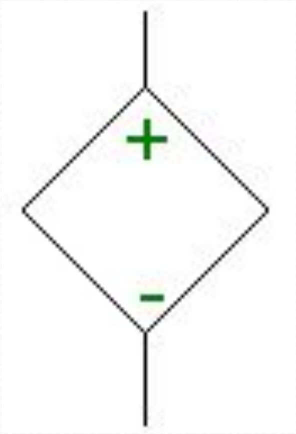
Independent source



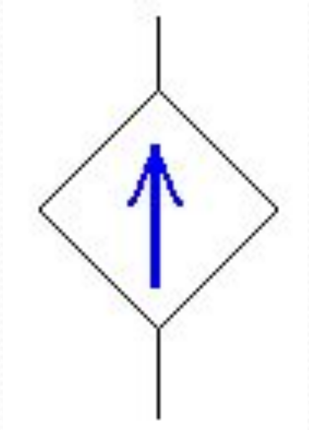
Dependent sources

Dependent Power Sources

- Voltage controlled voltage source
  - (VCVS)
- Current controlled voltage source
  - (CCVS)



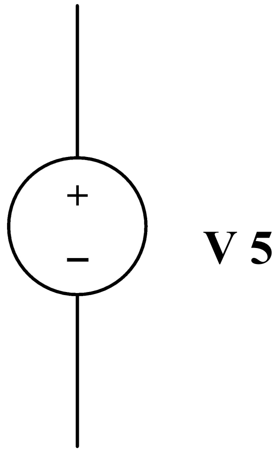
- Voltage controlled current source
  - (VCCS)
- Current controlled current source
  - (CCCS)



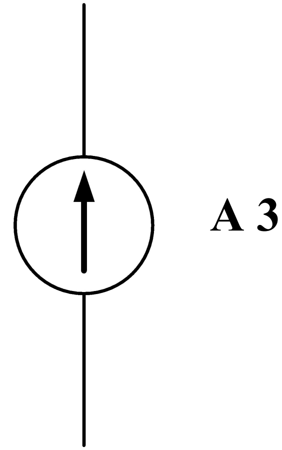
Summary

- Dependent sources are voltage or current sources whose output is a function of another parameter in the circuit.
  - Voltage controlled voltage source (VCVS)
  - Current controlled current source (CCCS)
  - Voltage controlled current source (VCCS)
  - Current controlled voltage source (CCVS)
- Dependent sources only produce a voltage or current when an independent voltage or current source is in the circuit.
- Dependent sources are treated like independent sources when using nodal or mesh analysis, but **not** with superposition.

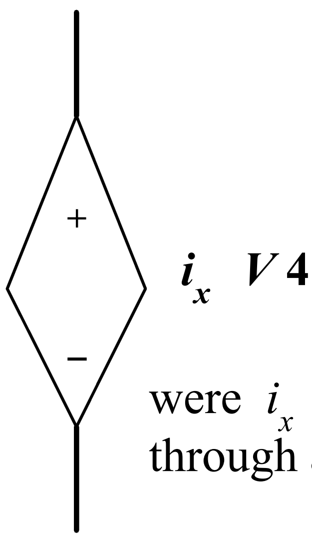
Independent and dependent voltage and current sources can be represented as



Independent voltage source

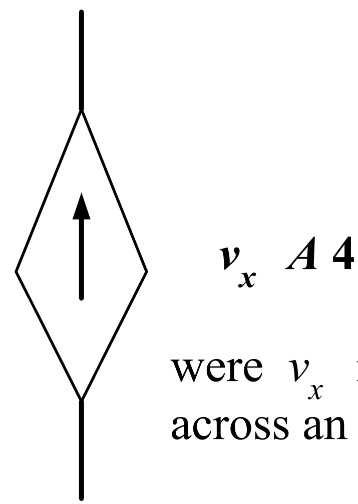


Independent current source



were i_x is some current through an element

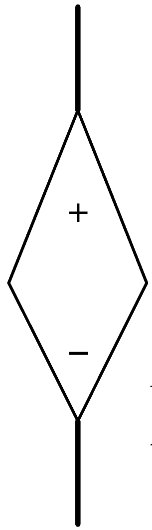
Dedependent voltage source
Voltage depend on current



were v_x is some voltage across an element

Dedependent current source
Current depend on voltage

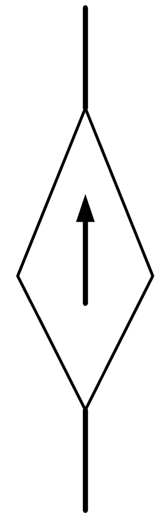
The dependent sources can be also as



v_x V **4**

were v_x is some current through an element

Dedependent voltage source
Voltage depend on voltage



i_x A **7**

were i_x is some voltage across an element

Dedependent current source
Current depend on current

First we shall discuss about the analysis of DC circuit. The voltage across an element is denoted as E or V. The current through the element is I.

Conductor is used to carry current. When a voltage is applied across a conductor, current flows through the conductor. If the applied voltage is increased, the current also increases. The voltage current relationship is shown in Fig. 1.

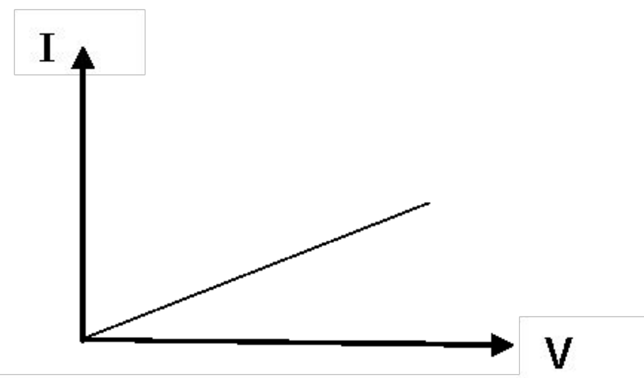


Fig. 1 Voltage – current relationship

It is seen that $I \infty V$. Thus we can write

$$I = G\,V \tag{1}$$

where G is called the conductance of the conductor.

Very often we are more interested on RESISTANCE, R of the conductor, than the conductance of the conductor. Resistance is the opposing property of the conductor and it is the reciprocal of the conductance, Thus

$$R = \frac{1}{G} \quad \text{or} \quad G = \frac{1}{R} \tag{2}$$

Therefore

$$I = \frac{V}{R} \tag{3}$$

The above relationship is known as OHM's law. Thus Ohm law can be stated as the current flows through a conductor is the ratio of the voltage across the conductor and its resistance. Ohm's law can also be written as

$$V = R\,I \tag{4}$$

$$R = \frac{V}{I} \tag{5}$$

The resistance of a conductor is directly proportional to its length, inversely proportional to its area of cross section. It also depends on the material of the conductor. Thus

$$R = \rho \frac{\ell}{A} \tag{6}$$

where ρ is called the specific resistance of the material by which the conductor is made of. The unit of the resistance is Ohm and is represented as Ω. Resistance of a conductor depends on the temperature also. The power consumed by the resistor is given by

$$P = V I \tag{7}$$

When the voltage is in volt and the current is in ampere, power will be in watt. Alternate expression for power consumed by the resistors are given below.

$$P = R I \times I = I^2 R \tag{8}$$

$$P = V \times \frac{V}{R} = \frac{V^2}{R} \tag{9}$$

KIRCHHOFF’s LAWS

There are two Kirchhoff’s laws. The first one is called Kirchhoff’s current law, KCL and the second one is Kirchhoff’s voltage law, KVL. Kirchhoff’s current law deals with the element currents meeting at a junction, which is a meeting point of two are more elements. Kirchhoff’s voltage law deals with element voltages in a closed loop also called as closed circuit.

Kirchhoff's current law

Kirchhoff's currents law states that the algebraic sum of element current meeting at a junction is zero.

Consider a junction P wherein four elements, carrying currents I_1, I_2, I_3 and I_4, are meeting as shown in Fig. 2.

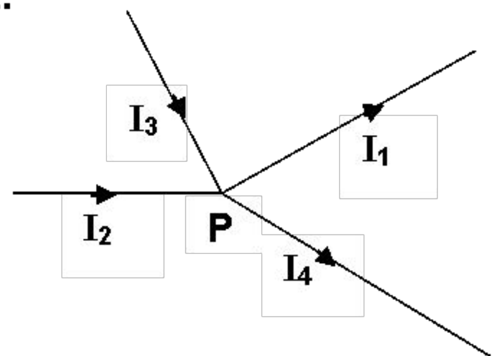


Fig. 2 Currents meeting at a junction

Note that currents I_1 and I_4 are flowing out from the junction while the currents I_2 and I_3 are flowing into the junction. According to KCL,

$$I_1 - I_2 - I_3 + I_4 = 0 \qquad (10)$$

The above equation can be rearranged as

$$I_1 + I_4 = I_2 + I_3 \qquad (11)$$

From equation (11), KCL can also stated as at a junction, the sum of element currents that flows out is equal to the sum of element currents that flows in.

Kirchhoff’s voltage law

Kirchhoff’s voltage law states that the algebraic sum of element voltages around a closed loop is zero.

Consider a closed loop in a circuit wherein four elements with voltages V_1, V_2, V_3 and V_4, are present as shown in Fig. 3.

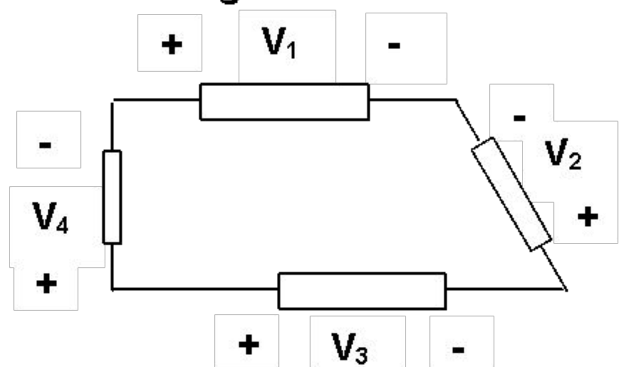


Fig. 3 Voltages in a closed loop

Assigning positive sign for voltage drop and negative sign for voltage rise, when the loop is traced in clockwise direction, according to KVL

$$V_1 - V_2 - V_3 + V_4 = 0 \quad (12)$$

The above equation can be rearranged as

$$V_1 + V_4 = V_2 + V_3 \quad (13)$$

From equation (13), KVL can also stated as, in a closed loop, the sum of voltage drops is equal to the sum of voltage rises in that loop.

Resistors connected in series

Two resistors are said to be connected in series when there is only one common point between them and no other element is connected in that common point. Resistors connected in series carry same current. Consider three resisters R_1, R_2 and R_3 connected in series as shown in Fig. 4. With the supply voltage of E, voltages across the three resistors are V_1, V_2 and V_3.

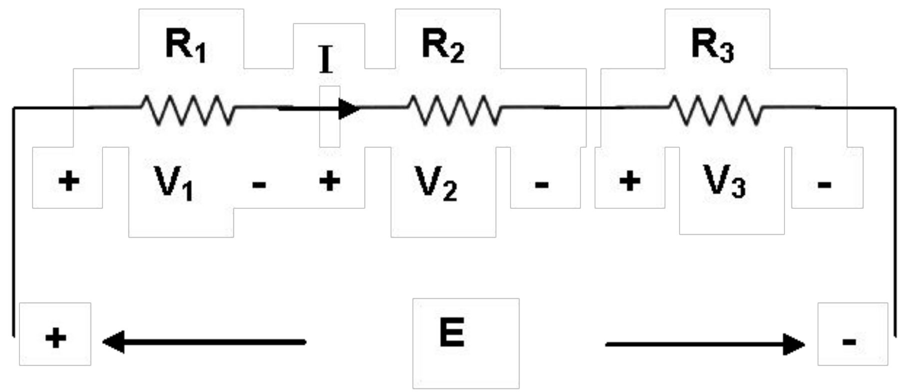


Fig. 4 Resistors connected in series

As per Ohm's law

$$\left.\begin{aligned} V_1 &= R_1\, I \\ V_2 &= R_2\, I \\ V_3 &= R_3\, I \end{aligned}\right\} \qquad (14)$$

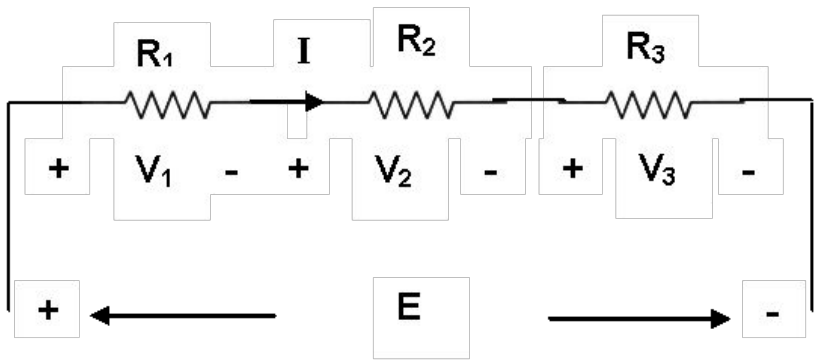


Fig. 4 Resistors connected in series

As per Ohm's law

$V_1 = R_1 I$

$V_2 = R_2 I$

$V_3 = R_3 I$

Applying KVL,

$$E = V_1 + V_2 + V_3 \tag{15}$$

$$= (R_1 + R_2 + R_3)\, I = R_{e\,q}\, I \tag{16}$$

Thus for the circuit shown in Fig. 4,

$$E = R_{e\,q}\, I \tag{17}$$

where E is the circuit voltage, I is the circuit current and $R_{e\,q}$ is the equivalent resistance. Here

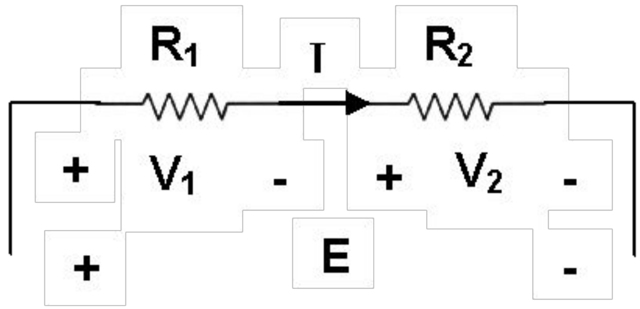
$$R_{e\,q} = R_1 + R_2 + R_3 \tag{18}$$

This is true when two are more resistors are connected in series. When n numbers of resistors are connected in series, the equivalent resistor is given by

$$R_{e\,q} = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n \tag{19}$$

Voltage division rule

Consider two resistors connected in series. Then



$V_1 = R_1\, I$

$V_2 = R_2\, I$

$E = (R_1 + R_2)\, I$ and hence $I = E / (R_1 + R_2)$

Total voltage of E is dropped in two resistors. Voltage across the resistors are given by

$$V_1 = \frac{R_1}{R_1 + R_2} E \quad \text{and} \tag{20}$$

$$V_2 = \frac{R_2}{R_1 + R_2} E \tag{21}$$

Resistors connected in parallel

Two resistors are said to be connected in parallel when both are connected across same pair of nodes. Voltages across resistors connected in parallel will be equal.

Consider two resistors R_1 and R_2 connected in parallel as shown in Fig. 5.

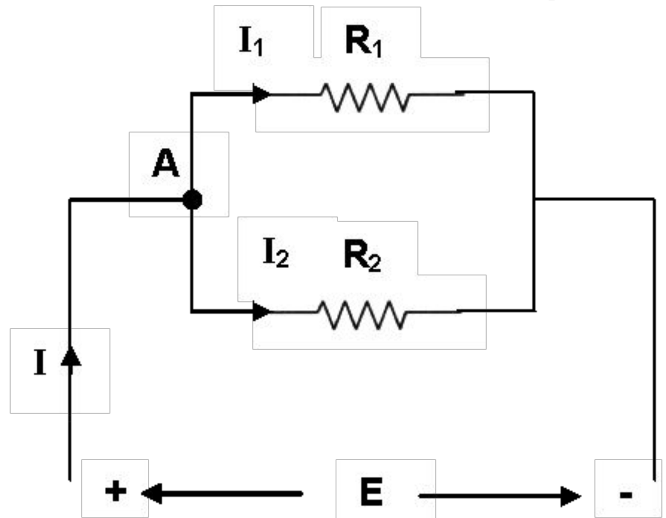
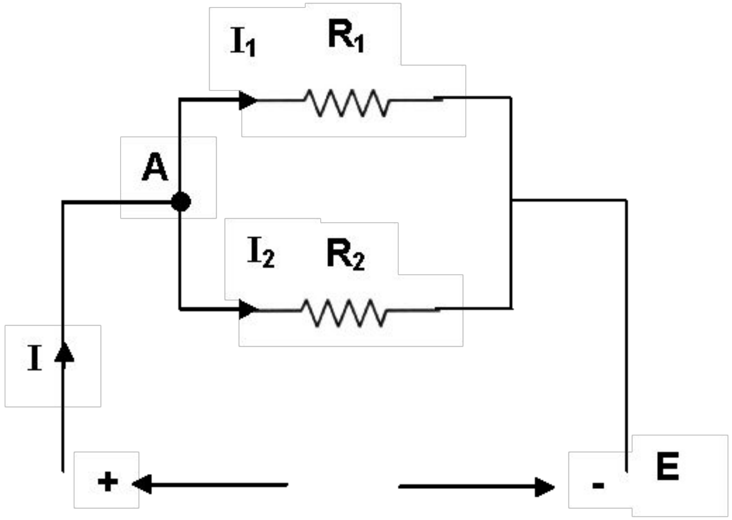


Fig. 5 Resistors connected in parallel

As per Ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \qquad (22)$$



As per Ohm's law

$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$

Applying KCL at node A

$$I = I_1 + I_2 = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_{eq}} \tag{23}$$

Thus for the circuit shown in Fig. 5

$$I = \frac{E}{R_{eq}} \tag{24}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

From the above $\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$

Thus $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ **(26)**

When n numbers of resistors are connected in parallel, generalizing eq. (25), R_{eq} can be obtained from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{R_n} \tag{27}$$

Current division rule

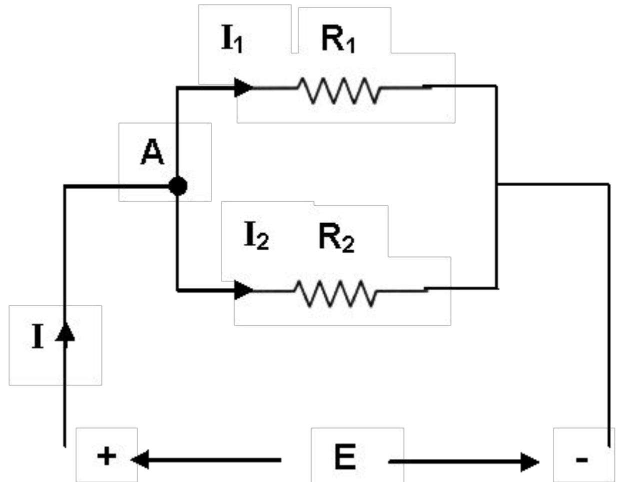


Fig. 5 Resistors connected in parallel

Referring to Fig. 5, it is noticed the total current gets divided as I_1 and I_2. The branch currents are obtained as follows.

From eq. (23)

$$E = \frac{R_1 R_2}{R_1 + R_2} I \tag{29}$$

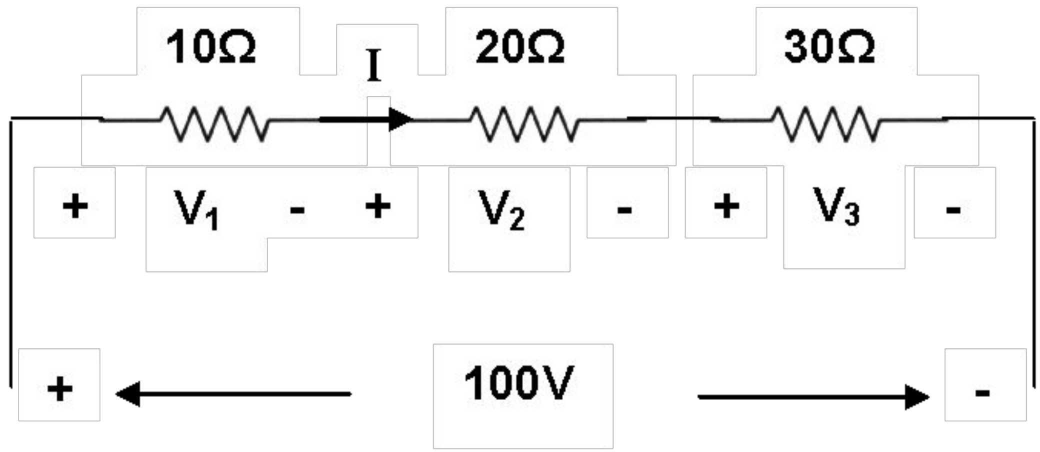
Substituting the above in eq. (22)

$$\left.\begin{aligned} I_1 &= \frac{R_2}{R_1 + R_2} I \\ I_2 &= \frac{R_1}{R_1 + R_2} I \end{aligned}\right\} \tag{30}$$

Example 1

Three resistors 10Ω, 20Ω and 30Ω are connected in series across 100 V supply. Find the voltage across each resistor.

Solution



Current $I = 100 / (10 + 20 + 30) = 1.6667$ A

Voltage across 10Ω = 10 x 1.6667 = 16.67 V

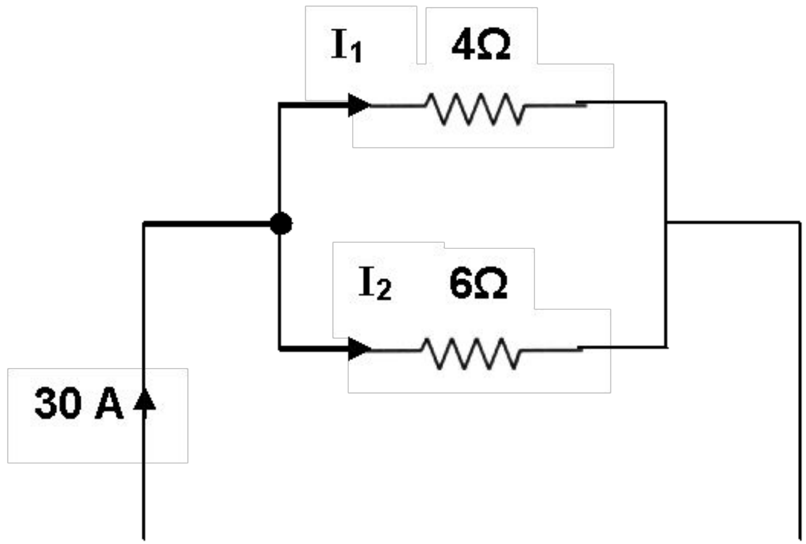
Voltage across 20Ω = 20 x 1.6667 = 33.33 V

Voltage across 30Ω = 30 x 1.6667 = 50 V

Example 2

Two resistors of 4Ω and 6Ω are connected in parallel. If the supply current is 30 A, find the current in each resistor.

Solution



Using the current division rule

$$\text{Current through } 4\Omega = \frac{6}{4+6} \times 30 = 18\,\text{A}$$

$$\text{Current through } 6\Omega = \frac{4}{4+6} \times 30 = 12\,\text{A}$$

Example 3

Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that the total power of 100 W is absorbed?

Solution

Let R_T be the total equivalent resistor. Then

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$$

Resistance $R_T = \frac{120}{154} = 0.7792\,\Omega$

Let E be the supply voltage. Then total current taken = E / 0.7792 A

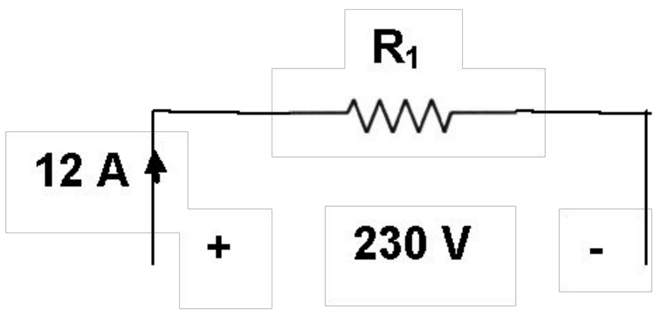
Thus $(\frac{E}{0.7792})^2 \times 0.7792 = 100$ and hence $E^2 = 100 \times 0.7792 = 77.92$

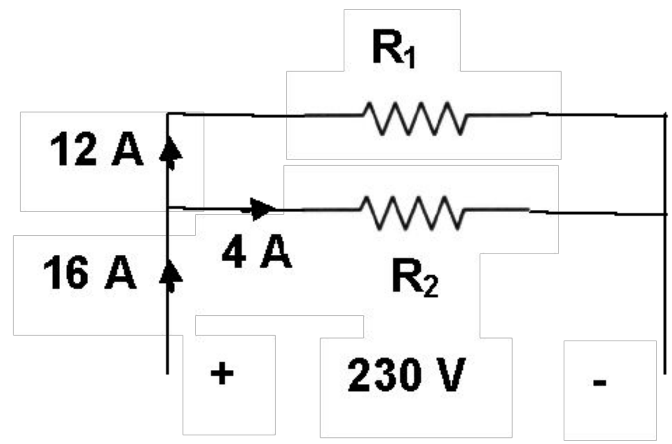
Required voltage = $\sqrt{77.92} = 8.8272$ V

Example 4

When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel, to increase the load to 16 A

Solution





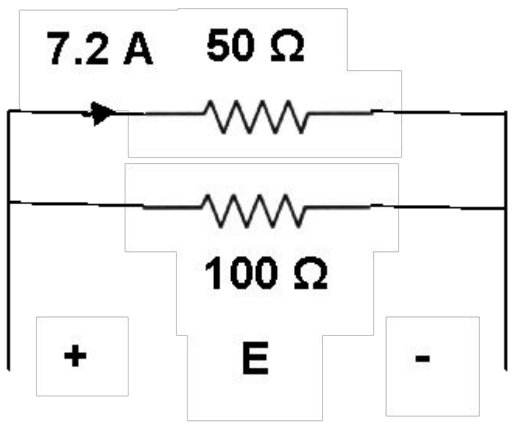
To make the load current 16 A, current through the second resistor = 16 –12 = 4 A

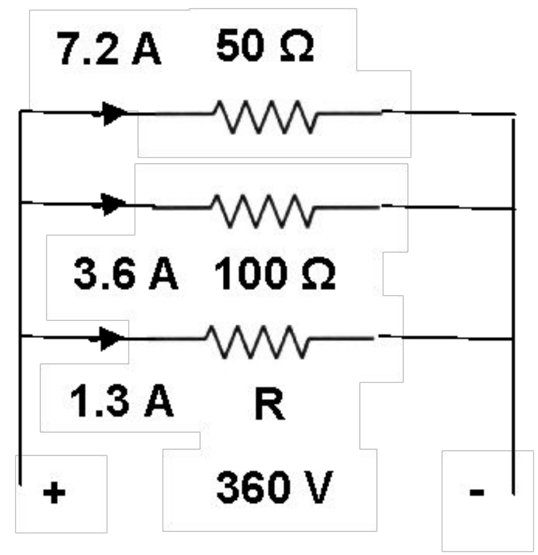
Value of second resistor R_2 = 230/4 = 57.5 Ω

Example 5

A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistor to be added in parallel to make the line current as 12.1A?

Solution





Supply voltage E = 50 x 7.2 = 360 V

Current through 100 Ω = 360/100 = 3.6 A

When the line current is 12.1 A, current through third resistor = 12.1 – (7.2 + 3.6)

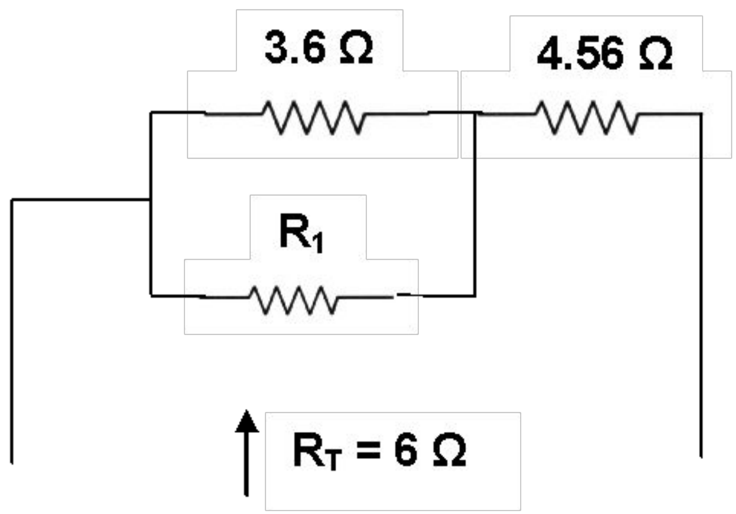
= 1.3 A

Value of third resistor = 360/1.3 = 276.9230 Ω

Example 6

A resistor of 3.6 ohms is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms, so that the total resistance of the circuit shall be 6 ohms?

Solution



$3.6 \parallel R_1 = 6 - 4.56 = 1.44\ \Omega$

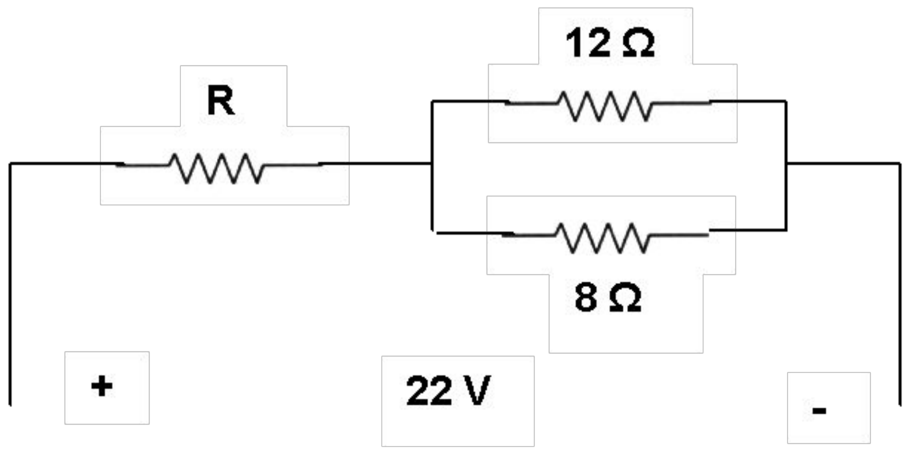
Thus $\frac{3.6 \times R_1}{3.6 + R_1} = 0.4$; Therefore $\frac{3.6 + R_1}{R_1} = \frac{1}{0.4} = 2.5$; $\frac{3.6}{R_1} = 1.5$

Required resistance $R_1 = 3.6/1.5 = 2.4\ \Omega$

Example 7

A resistance R is connected in series with a parallel circuit comprising two resistors 12 Ω and 8 Ω respectively. Total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Calculate the value of the resistor R.

Solution



Total current taken = 70 / 22 = 3.1818 A

Equivalent of 12 Ω ‖ 8 Ω = 96/20 = 4.8 Ω

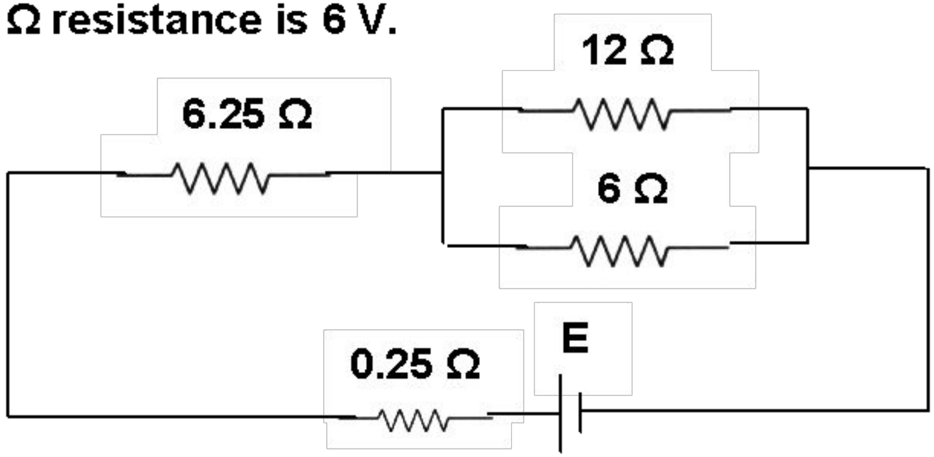
Voltage across parallel combination = 4.8 x 3.1818 = 15.2726 V

Voltage across resistor R = 22 – 15.2726 = 6.7274 V

Value of resistor R = 6.7274/3.1818 = 2.1143 Ω

Example 8

The resistors 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance and a battery which has an internal resistance of 0.25 Ω. Determine the emf of the battery if the potential difference across 6 Ω resistance is 6 V.



Voltage across 6 Ω = 6 V

Solution

Current in 6 Ω = 6/6 = 1 A

Current in 12 Ω = 6/12 = 0.5 A

Therefore current in 25 Ω = 1.0 + 0.5 = 1.5 A

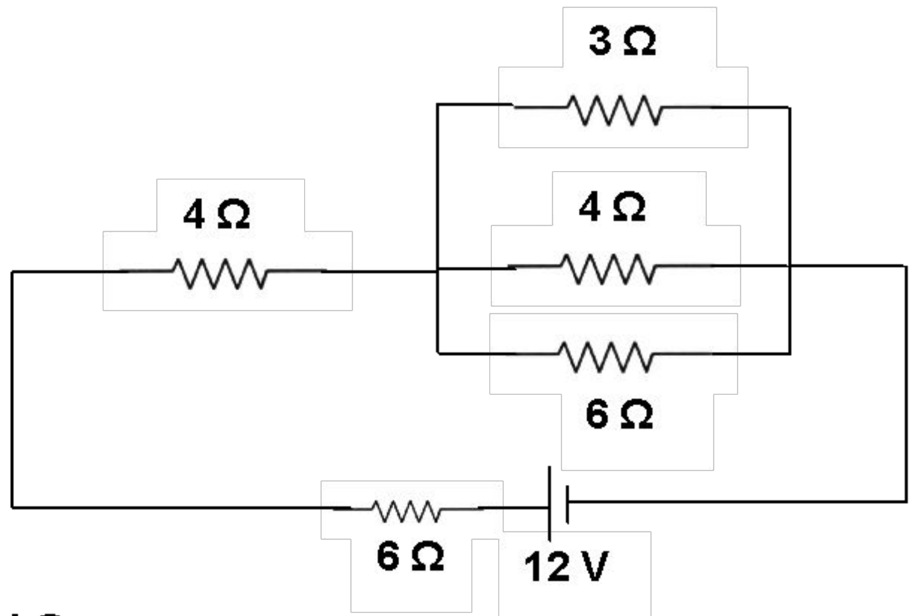
Using KVL E = (0.25 x 1.5) + (6.25 x 1.5) + 6 = 15.75 V

Therefore battery emf E = 15.75 V

Example 9

A circuit consist of three resistors 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of 12 V and an internal resistance of 6 Ω is connected across the circuit. Find the total current in the circuit and the terminal voltage across the battery.

Solution



$4\ \Omega \parallel 6\ \Omega = 24/10 = 2.4\ \Omega$

$1.4\ \Omega \parallel 3\ \Omega = 7.2/5.4 = 1.3333\ \Omega$

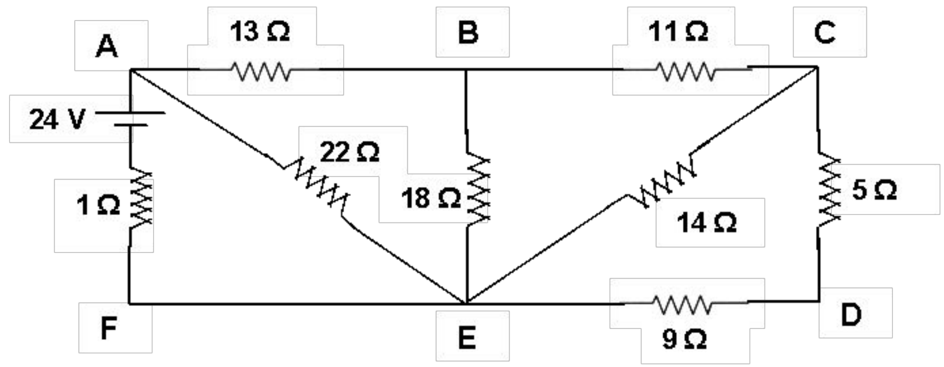
Total circuit resistance = 4 + 6 + 1.3333 = 11.3333 Ω

Circuit current = 12/11.3333 = 1.0588 A

Terminal voltage across the battery = 12 – (6 x 1.0588) = 5.6472 V

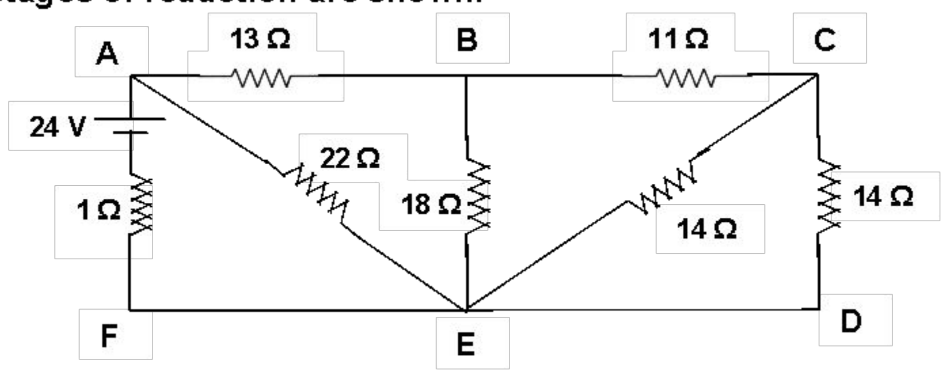
Example 10

An electrical network is arranged as shown. Find (i) the current in branch AF (ii) the power absorbed in branch BE and (iii) potential difference across the branch CD.

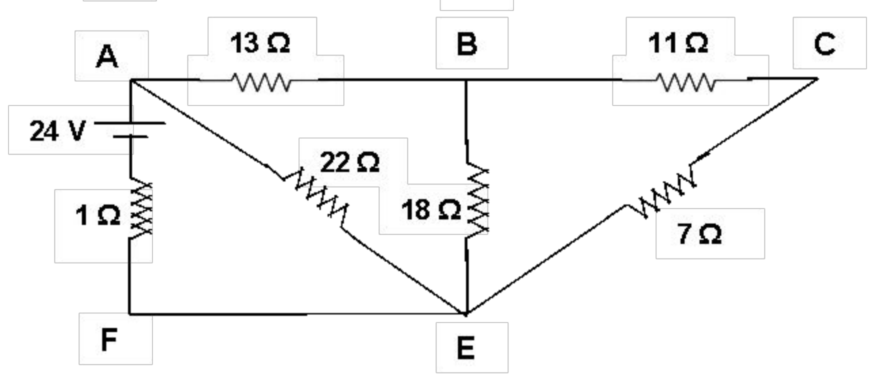


Solution

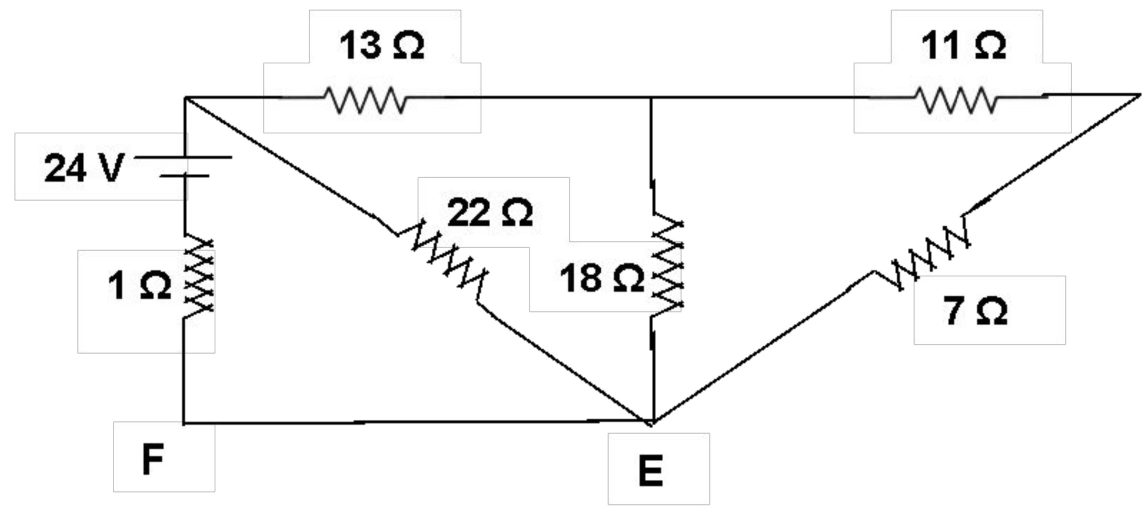
Various stages of reduction are shown.



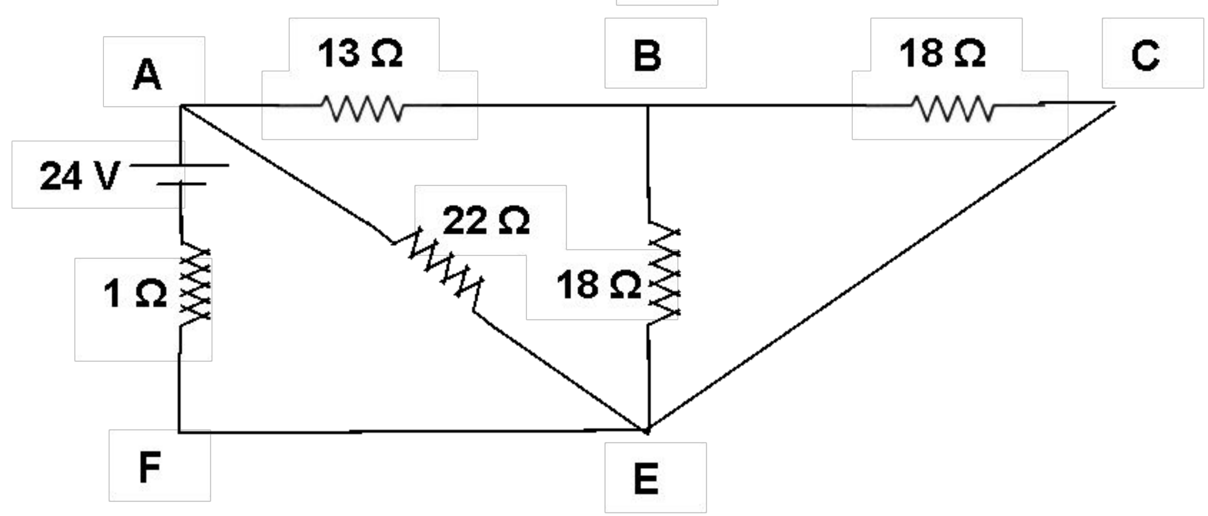
1



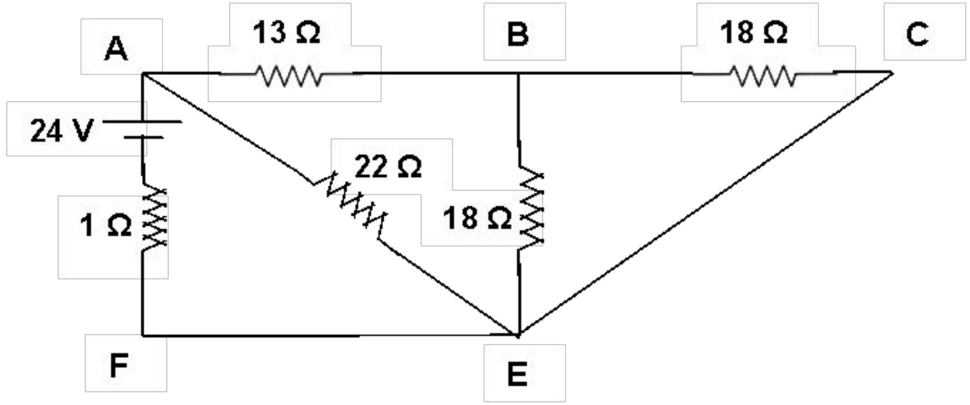
2



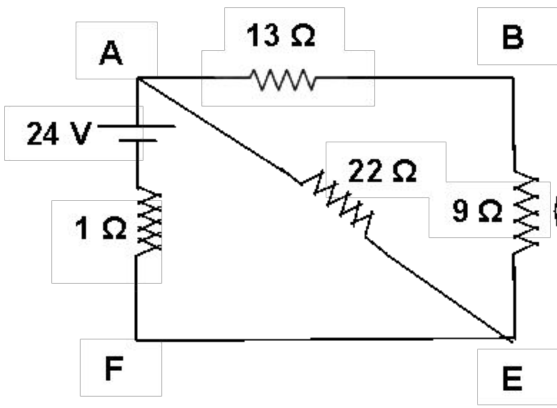
2



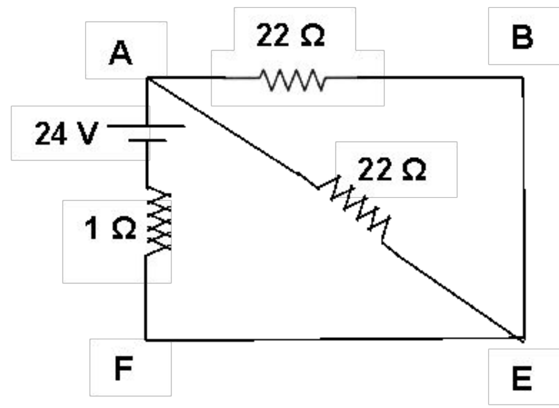
3


A
13 Ω
B
18 Ω
C
24 V
22 Ω
18 Ω
1 Ω
F
E

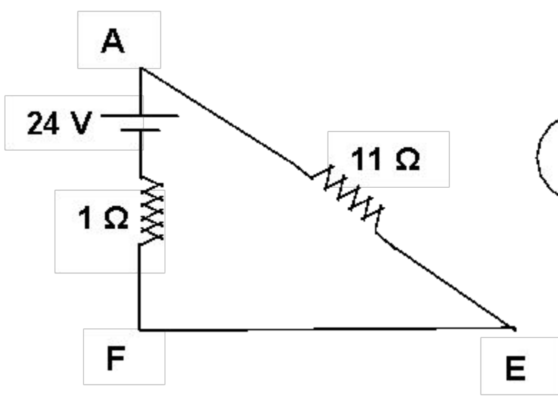
3


A
13 Ω
B
24 V
22 Ω
9 Ω
1 Ω
F
E

4


A
22 Ω
B
24 V
22 Ω
1 Ω
F
E

5


A
24 V
11 Ω
1 Ω
F
E

6

Current in branch AF = 24/12 = 2 A from F to A

Using current division rule current in 13 Ω in Fig. 4= 1 A

Referring Fig. 3, current in branch BE = 0.5 A

Power absorbed in branch BE = 0.5^2 x 18 = 4.5 W

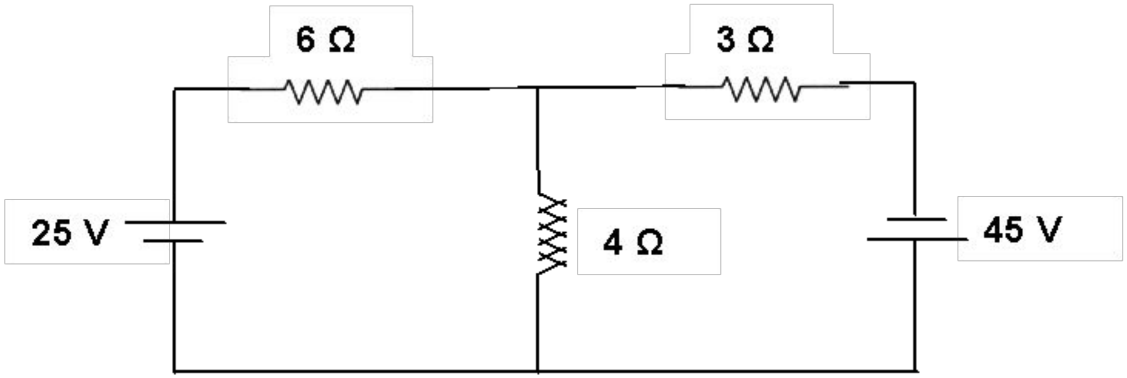
Voltage across BE = 0.5 x 18 = 9 V

Voltage across CE in Fig. 1 = $\frac{7}{18} x 9 = 3.5$ V

Referring Fig. given in the problem, using voltage division rule, voltage across in branch CD = $\frac{5}{14} x 3.5 = 1.25$ V

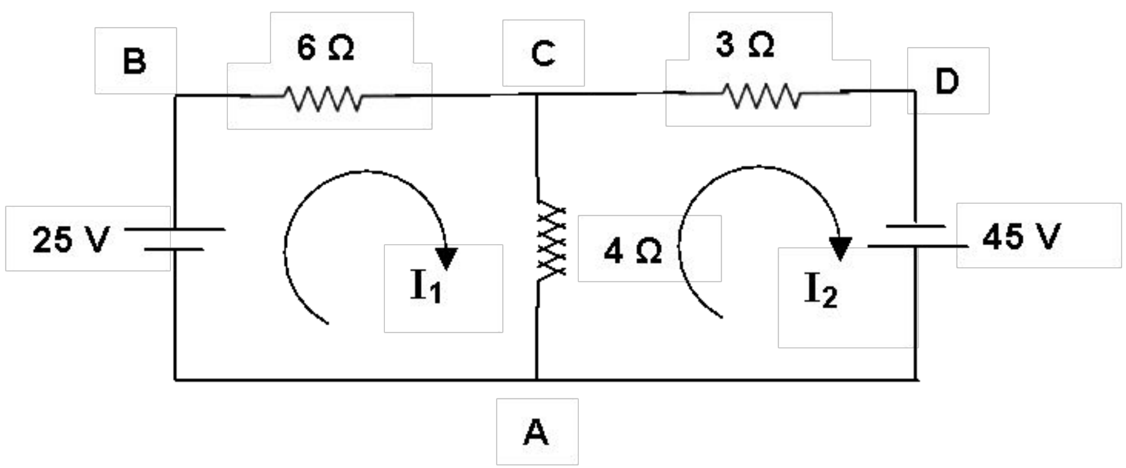
Example 11

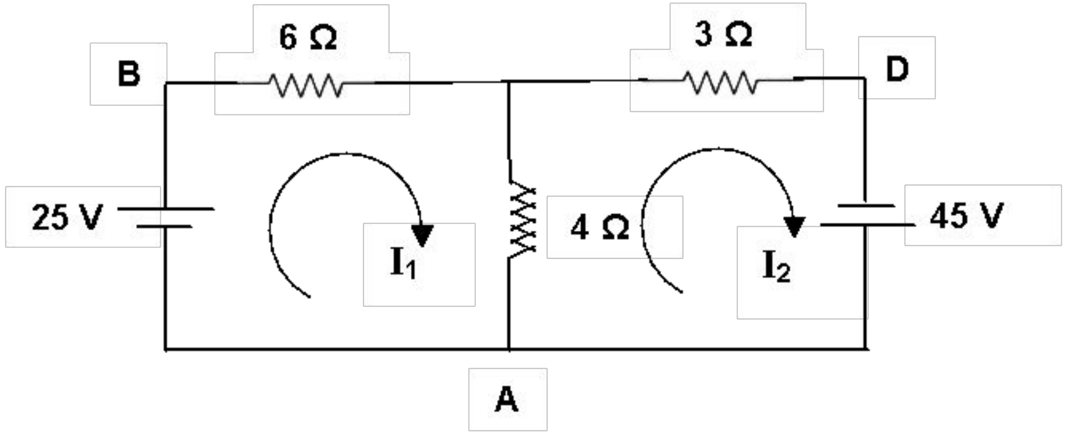
Using Kirchhoff's laws, find the current in various resistors in the circuit shown.



Solution

Let the loop current be I_1 and I_2





Considering the loop ABCA, KVL yields

$6\ I_1 + 4\ (I_1 - I_2) - 25 = 0$

For the loop CDAC, KVL yields

$3\ I_2 - 45 + 4\ (I_2 - I_1) = 0$

Thus $10\ I_1 - 4\ I_2 = 25$

$-4\ I_1 + 7\ I_2 = 45$

On solving the above $I_1 = 6.574$ A; $I_2 = 10.1852$ A

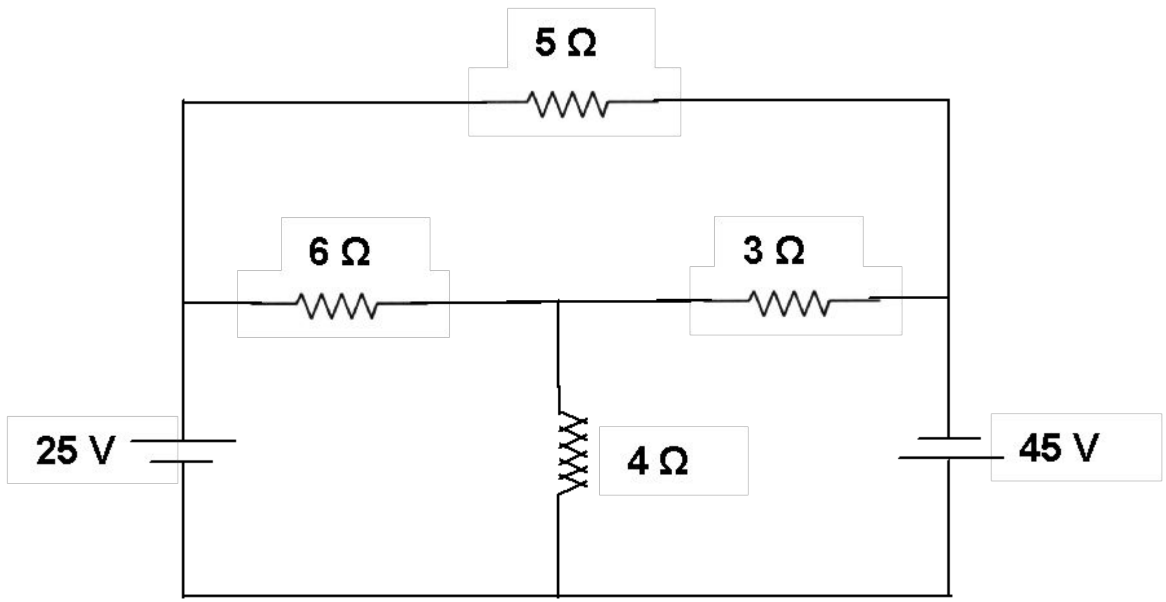
Current in 4Ω resistor = $I_1 - I_2 = 6.574 - 10.1852 = -3.6112$ A

Thus the current in 4Ω resistor is 3.6112 A from A to C

Current in 6 Ω resistor = 6.574 A; Current in 3 Ω resistor = 10.1852 A

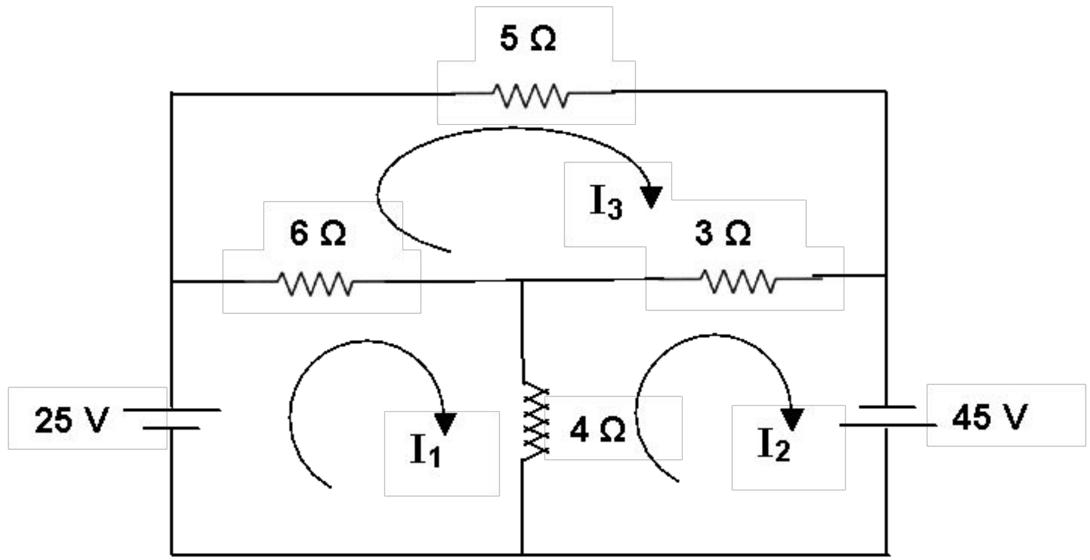
Example 12

Find the current in 5 Ω resistor in the circuit shown.



Solution

Let the loop current be I_1, I_2 and I_3.



Three loops equations are:

$6\,(I_1 - I_3) + 4\,(I_1 - I_2) - 25 = 0$

$4\,(I_2 - I_1) + 3\,(I_2 - I_3) - 45 = 0$

$5\,I_3 + 3\,(I_3 - I_2) + 6\,(I_3 - I_1) = 0$

On solving

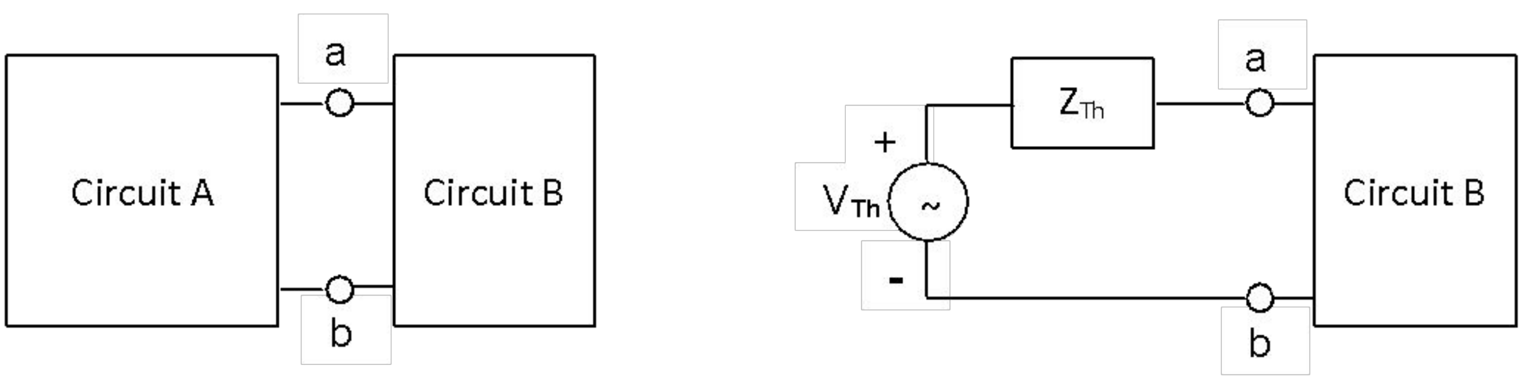
Current in 5 Ω resistor, $I_3 = 14$ A

CIRCUIT THEOREMS

THEVENIN'S THEOREM

In many practical applications, we may not be interested in getting the complete analysis of the circuit, namely finding the current through all the elements and voltages across all the elements. We may be interested to know the details of a portion of the circuit; as a special case **it may be a single element such as load impedance.** In such a situation it is very convenient to use Thevenin's theorem to get the solution.

Fig. illustrates the Thevenin's equivalent of sub-circuit A.



(a) (b)

Fig. Thevenin's equivalent.

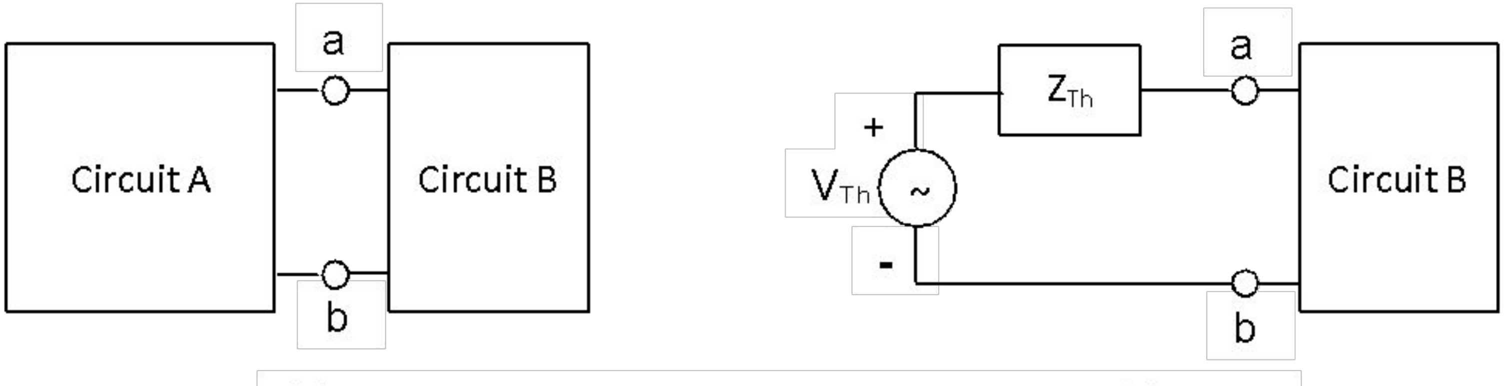


Fig. Thevenin's equivalent.

In Fig. (a) a circuit partitioned into two parts, namely circuit A and circuit B, is shown. They are connected by a single pair of terminals. In Fig.(b) circuit A is replaced by Thevenin's equivalent circuit, which consists of a voltage source V_{Th} in series with an impedance Z_{Th}.

To obtain the Thevenin's equivalent circuit, we need to find Thevenin's voltage V_{th} and Thevenin's impedance Z_{Th}. Unique procedure is available to find the Thevenin's voltage V_{Th}. When we need the Thevenin's voltage of circuit A, measure or calculate the **OPEN CIRCUIT VOLTAGE of circuit A.** This will be the Thevenin's voltage.

Thevenin's impedance can be calculated in three different ways **depending on the nature of voltage and current sources** in the circuit of our interest.

The circuit for which Thevenin's impedance is to be calculated consists of impedances and **one or more independent sources.** That is, **the circuit does not contain any dependent source**. To determine Thevenin's impedance, circuit shown in Fig. (b) is to be used.

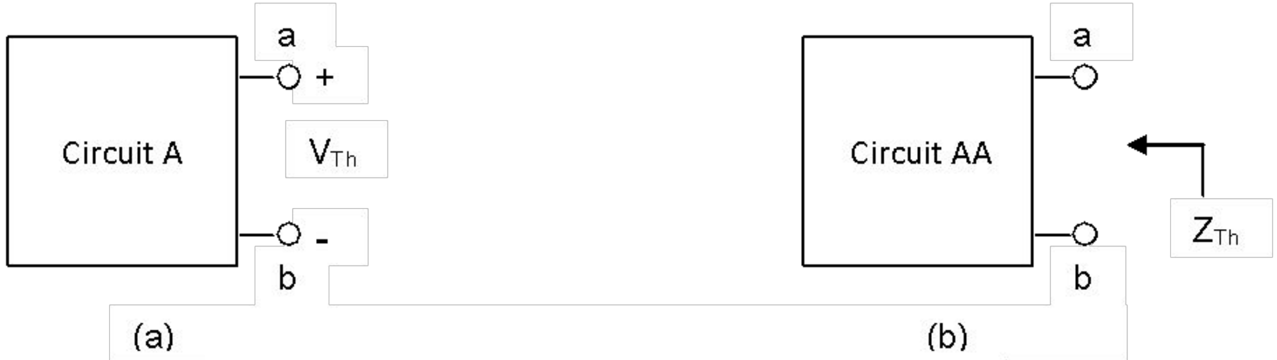


Fig. Determining Thevenin's equivalents.

The circuit AA in Fig. (b) is obtained from circuit A by replacing all the independent voltage sources by short circuits and replacing all independent current sources by open circuits. Thus in circuit AA, all the independent sources are set to zero. Then, Thevenin's impedance is the equivalent circuit impedance of circuit AA which can be obtained using reduction techniques.

The methods of finding the Thevenin's impedance depend on the nature of the circuit for which the Thevenin's equivalent is sought for. These methods are summarized below:

Circuit with independent sources only - ANY ONE OF THE FOLLOWING

1. **Make independent sources zeros and use reduction techniques to find Z_{Th}.**

2. Short circuit terminals a and b and find the short circuit current I_{sc} flowing from *a* to *b*. Then $Z_{Th} = V_{Th} / I_{sc}$

3. Set all independent sources to zero. Apply 1 V across the open circuited terminals *a*-*b* and determine the source current I_s entering the circuit through *a*. Then $Z_{Th} = 1 / I_s$. Alternatively introduce a current source of 1 A from *b* to *a* and determine the voltage V_{ab}. Then, Thevenin's impedance $Z_{Th} = V_{ab}$.

Example 1

Find the Thevenin's voltage with respect to the load resistor R_L in circuit shown in Fig.

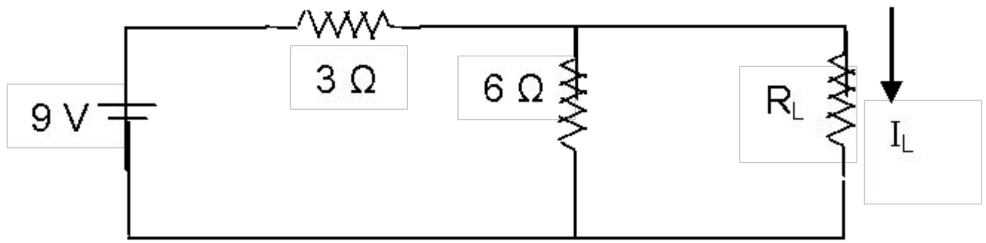
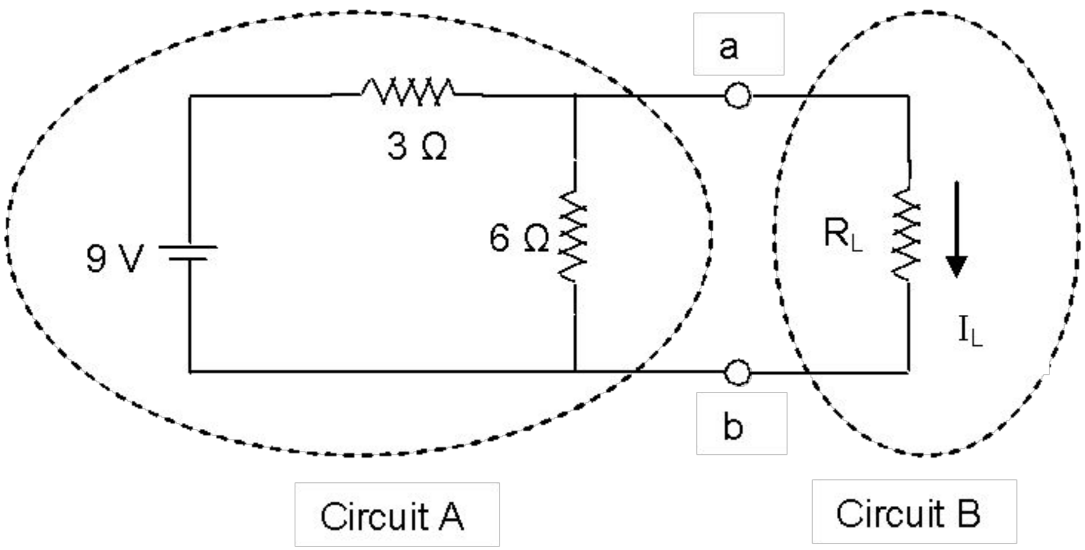
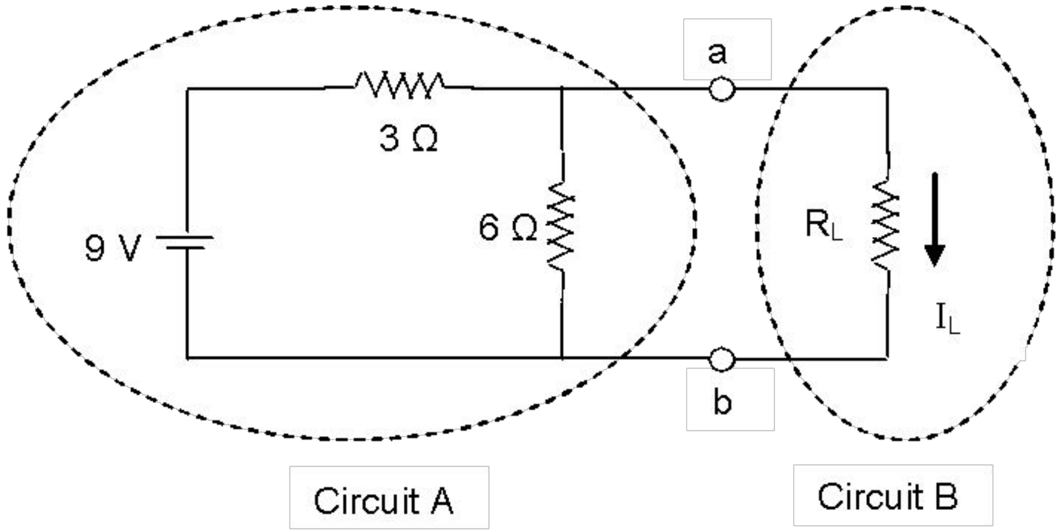


Fig. Circuit for Example1

Solution

The given circuit can be divided into two circuits as shown in Fig.





Thevenin's voltage of circuit A can be obtained from the circuit shown in Fig.

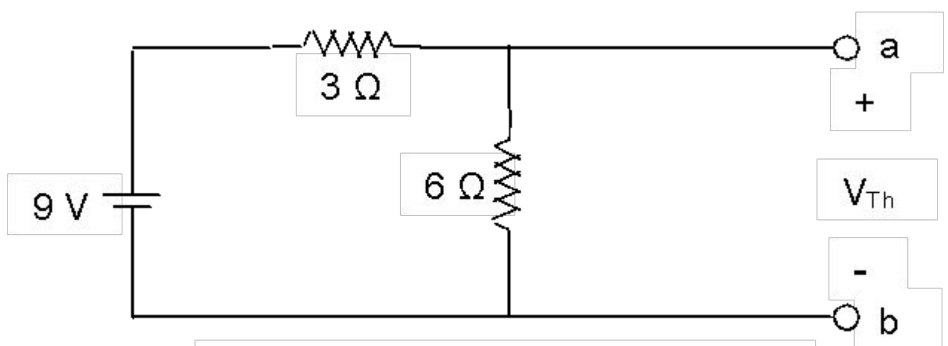


Fig. Circuit.

Using voltage division rule $V_{Th} = V_{6\Omega} = \frac{6}{9} \times 9 = 6\,V$

Example 2

Obtain the Thevenin's equivalent for the circuit shown in Fig.

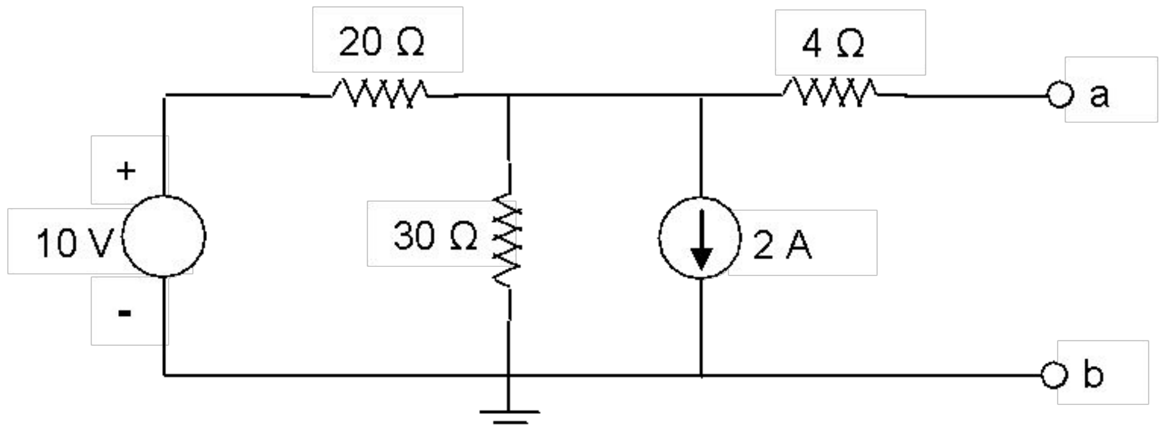


Fig. Circuit for Example 2.

Solution:

Open circuit voltage V_{ab} is the Thevenin's voltage V_{Th}.

To find Thevenin's voltage:

Note that there is no current flow in resistor of 4 Ω. Therefore, voltage V_{Th} is same as the voltage across 30 Ω resistor. Then, the node voltage equation is

$$\frac{V_{Th} - 10}{20} + \frac{V_{Th}}{30} + 2 = 0$$ On solving this, we get V_{Th} = - 18 V

To find Thevenin's impedance: Since the circuit has only independent sources, it falls under case 1

Reducing the sources to zero, the resulting circuit is shown in Fig.

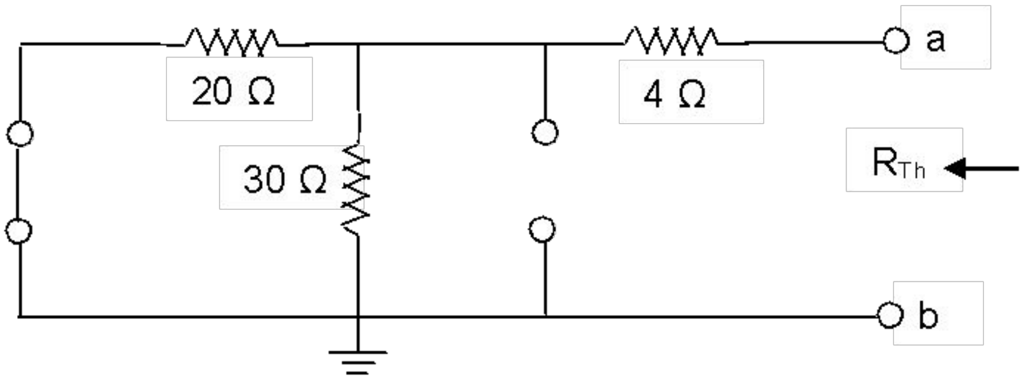


Fig. Circuit - Example 2.

Thus $R_{Th} = 4 + 20||30 = 16\ \Omega$ Thevenin's equivalent circuit is shown in Fig.

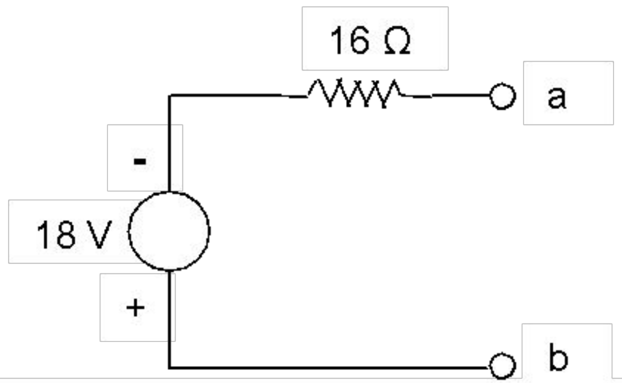
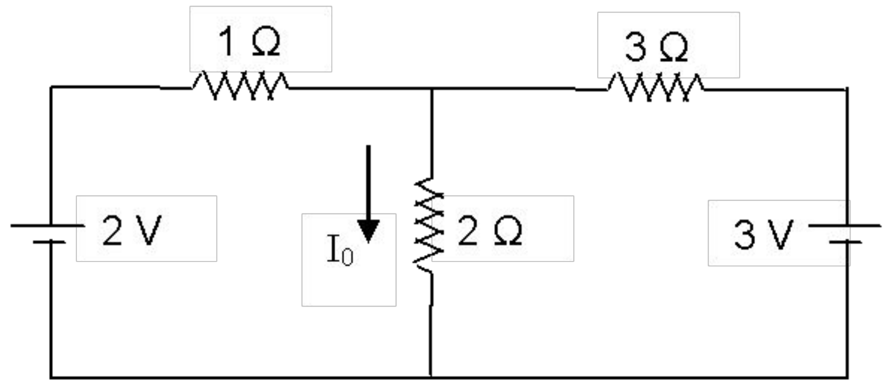


Fig. Thevenin's equivalent circuit – Example 2.

R_{Th} can be obtained by two other methods also

Example 3 Using Thevenin's equivalent circuit, calculate the current I_0 through the 2 Ω resistor in the circuit shown below.



Solution: Circuit by which V_{Th} and R_{Th} can be calculated are shown in Fig.

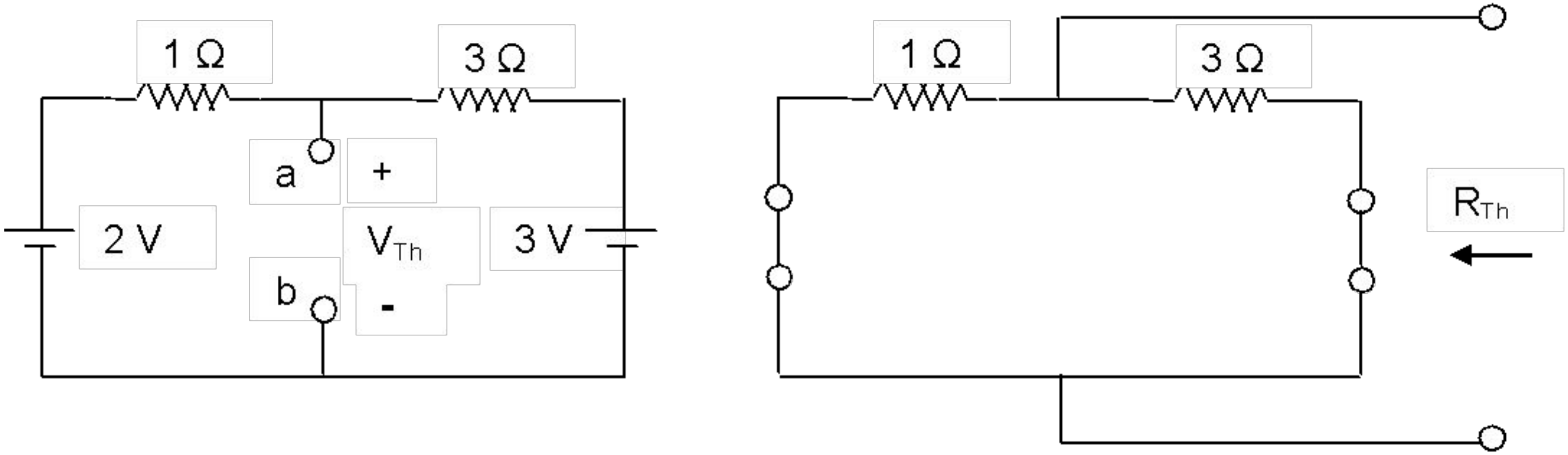
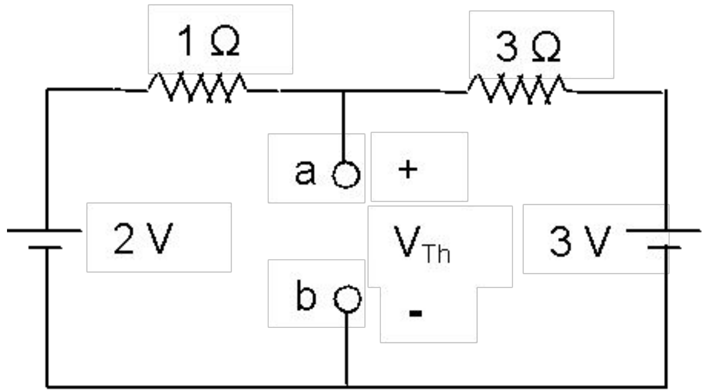
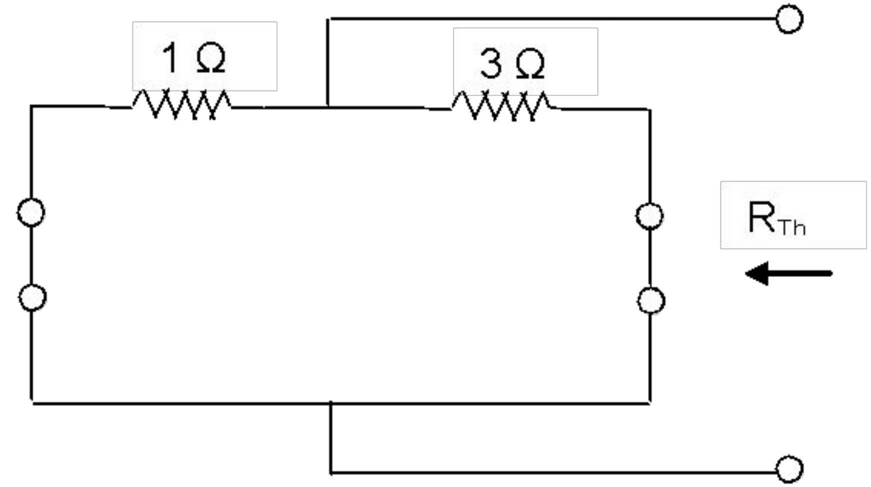


Fig. Circuits for V_{Th} and R_{Th} - Example 3.

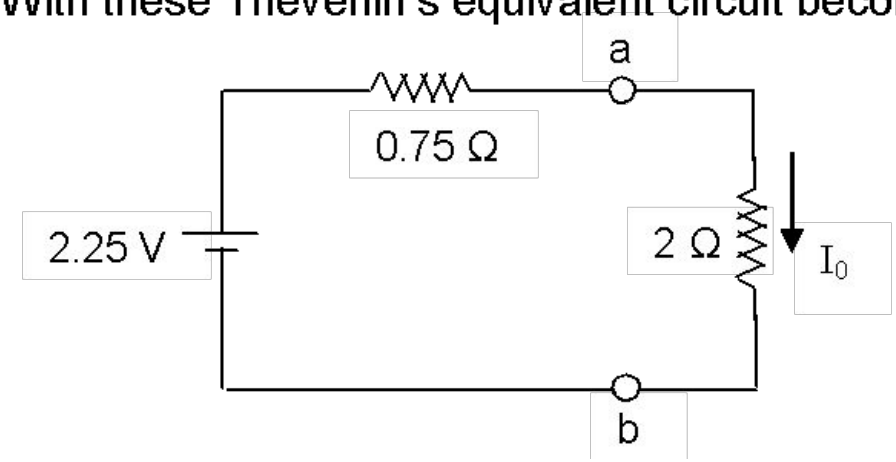




Knowing the anticlockwise current as 0.25 A

$- 2 - (1 \times 0.25) + V_{Th} = 0$. i.e. $V_{Th} = 2.25$ V; Also $R_{Th} = 1 || 3 = 0.75\ \Omega$

With these Thevenin's equivalent circuit becomes



Current $I_0 = 2.25 / 2.75 = 0.8182$ A

NORTON'S THEOREM

Much similar to Thevenin's theorem, Norton's theorem is also used to obtain the equivalent of two terminal sub-circuit.

Fig. illustrates the Norton's equivalent of sub-circuit A.

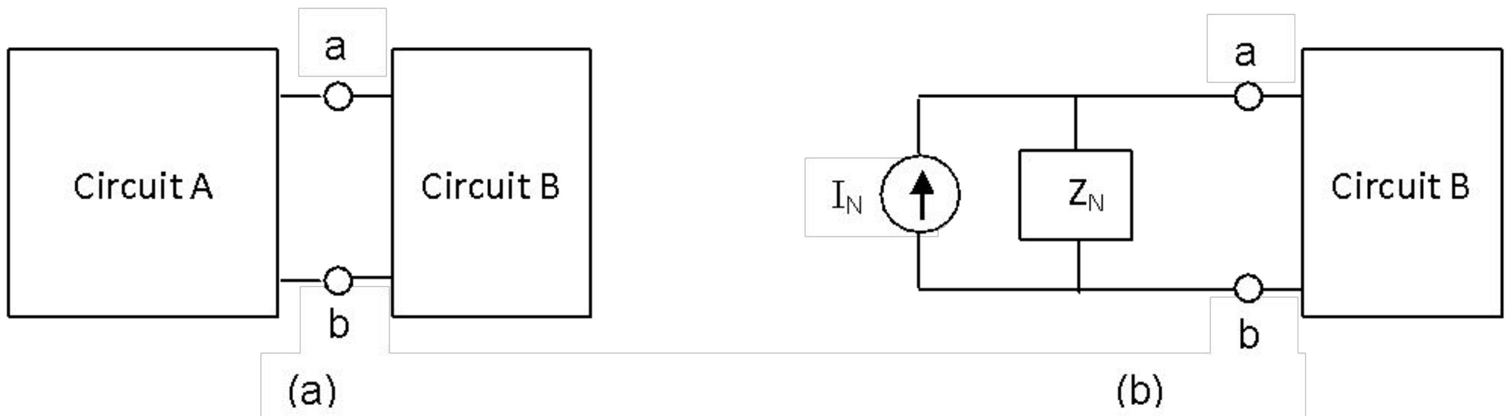


Fig. Norton's equivalent.

In Fig. (a) a circuit partitioned into two parts, namely circuit A and circuit B, is shown. They are connected by a single pair of terminals. In Fig. (b), circuit A is replaced by Norton's equivalent circuit, which consists of a current source I_N in parallel with an impedance Z_N.

Looking at the Thevenin's and Norton's equivalents shown in Fig. (a) and (b), it is clear that one can be obtained from the other through source transformation.

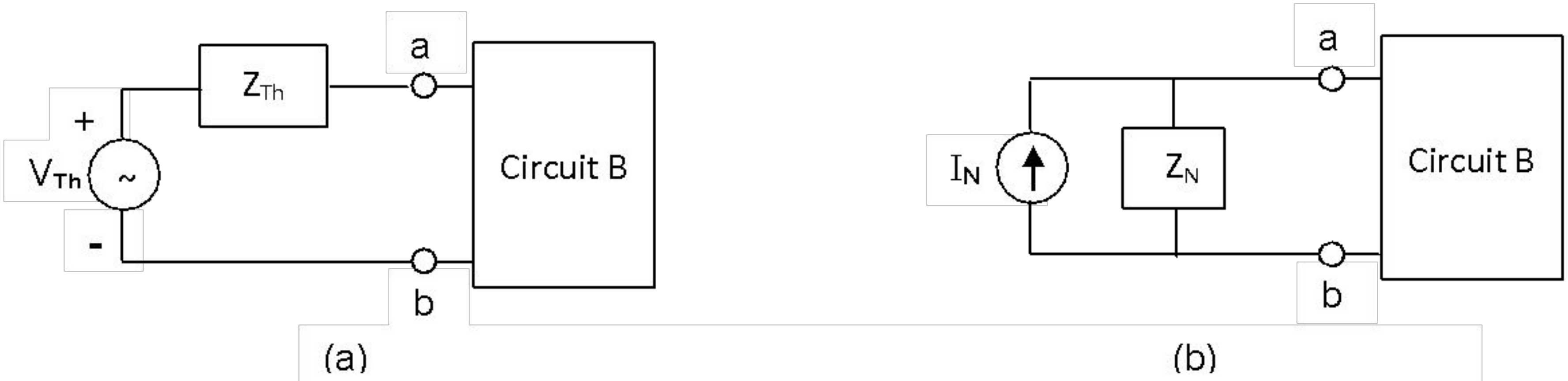


Fig. Thevenin's and Norton's equivalents.

It is to be noted that

$$Z_N = Z_{Th}$$

$$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{V_{Th}}{Z_N}$$

To obtain Norton's equivalent circuit, we need to find current I_N and the impedance Z_N. They can be obtained from Thevenin's voltage and impedance.

Otherwise Norton's current can be obtained by finding the short circuit current as indicated in Fig.

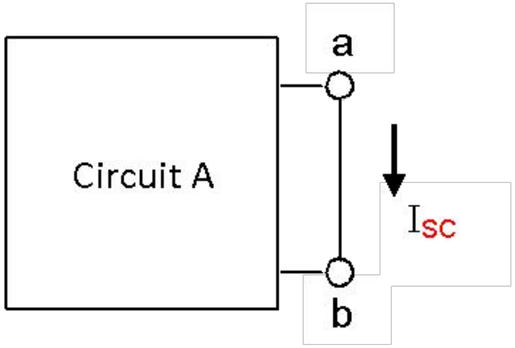


Fig. Getting short circuit current.

It is to be noted that the short circuit current is from terminal *a* to terminal *b* while Norton's current is from terminal *b* to terminal *a*.

The impedance Z_N can be got exactly same way we got Z_{Th} as discussed in previous section except that the method indicated under Case 2 is not applicable as it requires the value of V_{Th}.

Example 1

Using Norton's theorem, determine the current through the resistor R_L when R_L = 0.7, 1.2 and 1.6 Ω in the circuit shown in Fig.

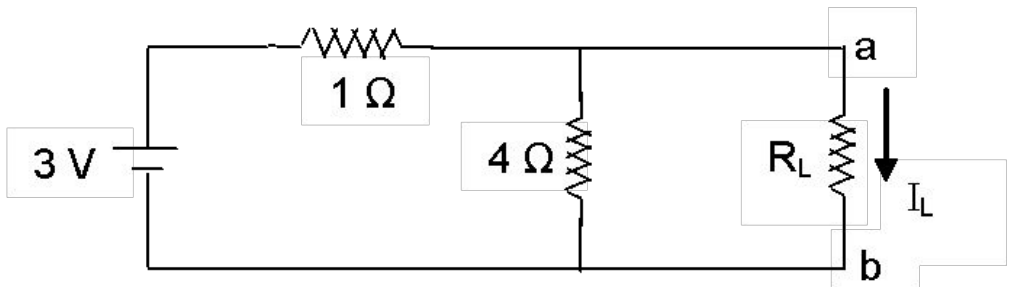


Fig. Circuit for Example 1.

Solution:

Circuits to determine I_{SC} and R_N are shown in Fig. (a) and (b).

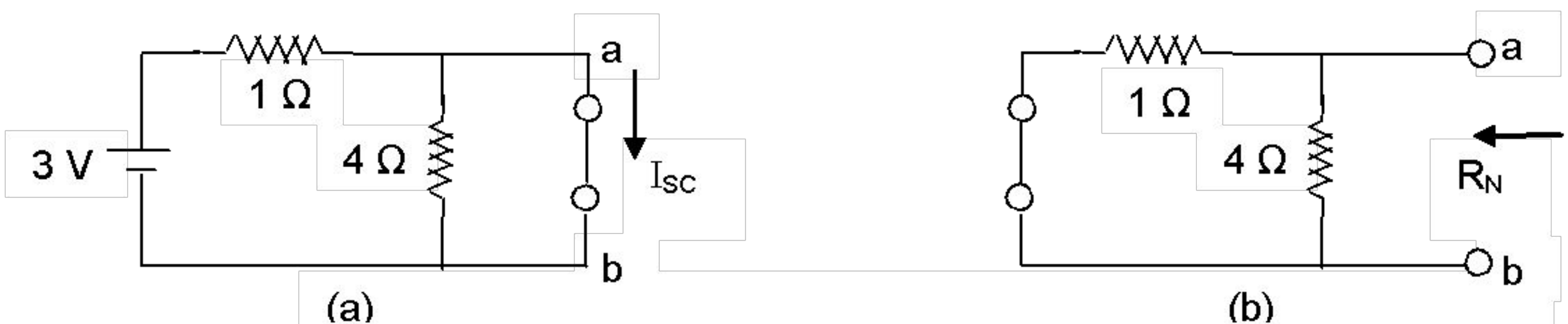
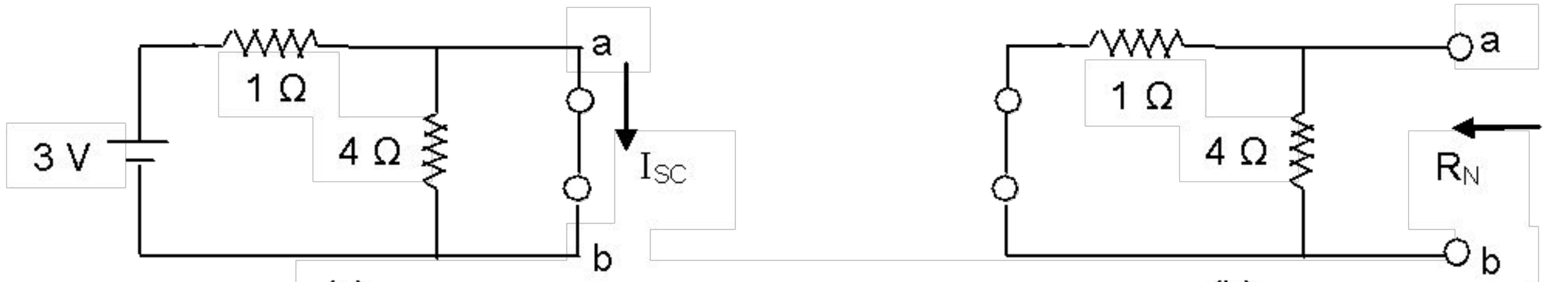


Fig. Short circuit current and Norton's resistance.



(a) (b)

It is to be noted that since there is a short circuit parallel to 4 Ω no current flows in it.

Norton's current I_N = 3 A; Norton's resistance $R_N = 1 || 4 = 0.8\ \Omega$

Norton's equivalent circuit is shown in Fig.

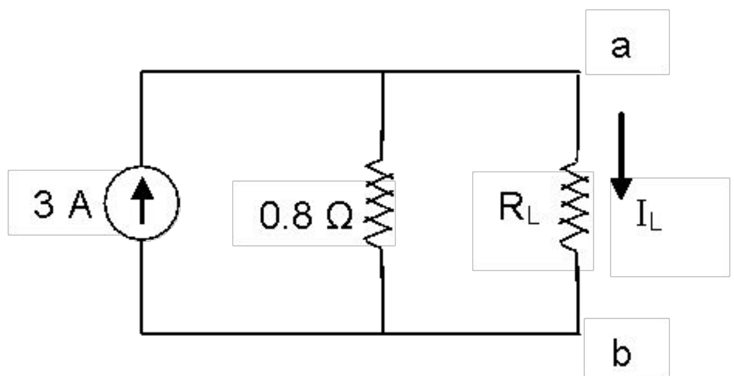


Fig. Norton's equivalent.

R_N can be obtained by another method also.

When $R_L = 0.7\ \Omega$, $I_L = (0.8 / 1.5) \times 3 = 1.6$ A; When $R_L = 1.2\ \Omega$, $I_L = (0.8 / 2) \times 3 = 1.2$ A

When $R_L = 1.6\ \Omega$, $I_L = (0.8 / 2.4) \times 3 = 1.0$ A

MAXIMUM POWER TRANSFER THEOREM

There are some applications wherein maximum power needs to be transferred to the load connected. Consider a linear ac circuit *A*, connected to a load of impedance Z_L as shown in Fig. (a). It is required to transfer maximum real power to the load. The circuit A can be replaced by its Thevenin's equivalent as shown in Fig. (b).

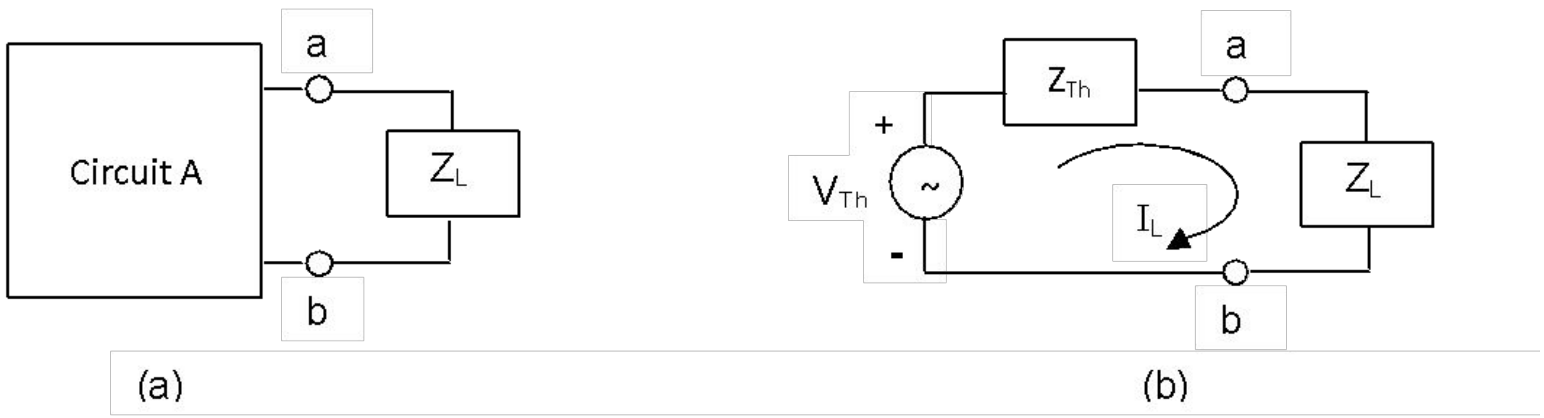


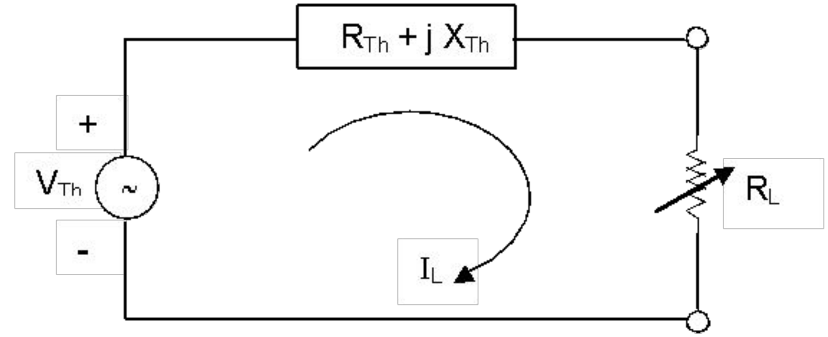
Fig. Maximum power transfer theorem - Illustration.

Let $Z_{Th} = (R_{Th} + j X_{Th})$ and $Z_L = (R_L + j X_L)$

The following maximum power transfer theorems determine the values of load impedance Z_L for which maximum real power is transferred to the load impedance.

Case 1:

Load is a variable resistance R_L



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + jX_{Th}}$ This gives $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + X_{Th}^{\ 2}}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + X_{Th}^{\ 2}}$

This can be written as $P_L = \dfrac{|V_{Th}|^2}{\dfrac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^{\ 2}}{R_L}}$

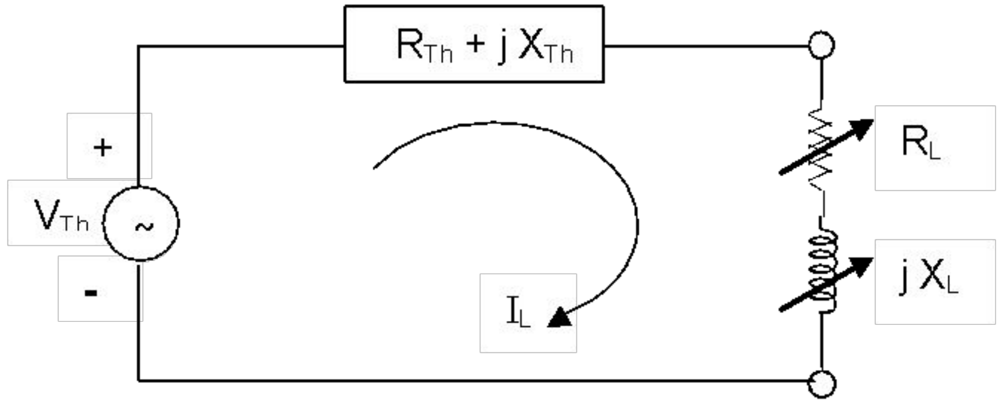
For power P_L to be maximum, $\dfrac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^{\ 2}}{R_L}$ must be minimum. Thus power P_L will be maximum when

$\dfrac{d}{dR_L}\left(\dfrac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^{\ 2}}{R_L}\right) = 0$ i.e. when $-\dfrac{R_{Th}^{\ 2}}{R_L^{\ 2}} + 1 - \dfrac{X_{Th}^{\ 2}}{R_L^{\ 2}} = 0$

i.e. when $R_L^{\ 2} = R_{Th}^{\ 2} + X_{Th}^{\ 2}$ i.e. when $R_L = \sqrt{R_{Th}^{\ 2} + X_{Th}^{\ 2}} = |Z_{Th}|$

Using this value of R_L, the current I_L and hence maximum power can be computed.

Case 2 In the load impedance, R_L and X_L are varied independently as shown.



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + jX_L)}$ Thus $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$

If R_L in Eq. is held fixed, the value of P will be maximum when $(X_{Th} + X_L)^2$ is minimum.

This will occur when $X_{Th} + X_L = 0$ i.e. when

$X_L = - X_{Th}$

Keeping $X_L = - X_{Th}$ Eq. becomes

$$P_L = \frac{|V_{Th}|^2\ R_L}{(R_{Th} + R_L)^2} = \frac{|V_{Th}|^2}{\frac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L}$$

For P_L given by Eq. to become maximum, $\frac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L$ must be minimum. This will occur when $\frac{d}{dR_L}(\frac{R_{Th}^{\ 2}}{R_L} + 2R_{Th} + R_L) = 0$ i.e. when $-\frac{R_{Th}^{\ 2}}{R_L^{\ 2}} + 1 = 0$ i.e. when

$R_L = R_{Th}$

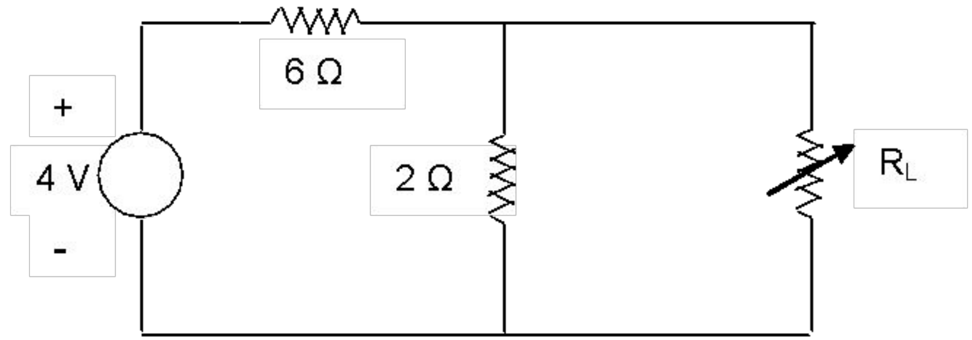
Combining Eqs. and, we can state that real power transferred to the load will be maximum when

$Z_L = R_{Th} - j\,X_{Th} = Z_{Th}^{\ *}$

Setting $R_L = R_{Th}$ and $X_L = -\,X_{Th}$ in Eq., maximum real power can be obtained as

$$P_{max} = \frac{|V_{Th}|^2}{4\,R_{Th}}$$

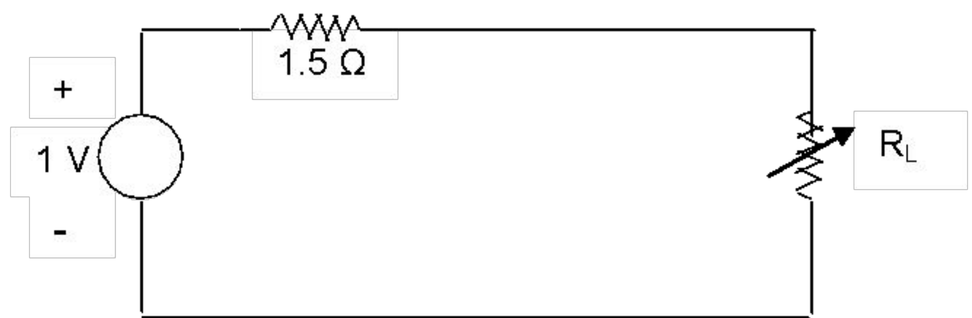
Example 1 Consider the circuit shown below. Determine the value of R_L when it is dissipating maximum power. Also find the value of maximum power dissipated.



Solution:

As a first step, Thevenin's equivalent across the load resistor is obtained.

$V_{Th} = \frac{2}{2+6} \times 4 = 1\text{ V}$; $R_{Th} = 6 || 2 = 1.5\ \Omega$ Resulting circuit is shown.



For P_L to be maximum, $R_L = 1.5\ \Omega$; Then circuit current = 1/3 = 0.3333 A

Maximum power dissipated $P_{max} = 0.3333^2 \times 1.5 = 0.16667$ W

SUPERPOSITION THEOREM

The idea of superposition rests on the linearity property. Superposition theorem is applicable to linear circuits having two or more independent sources.

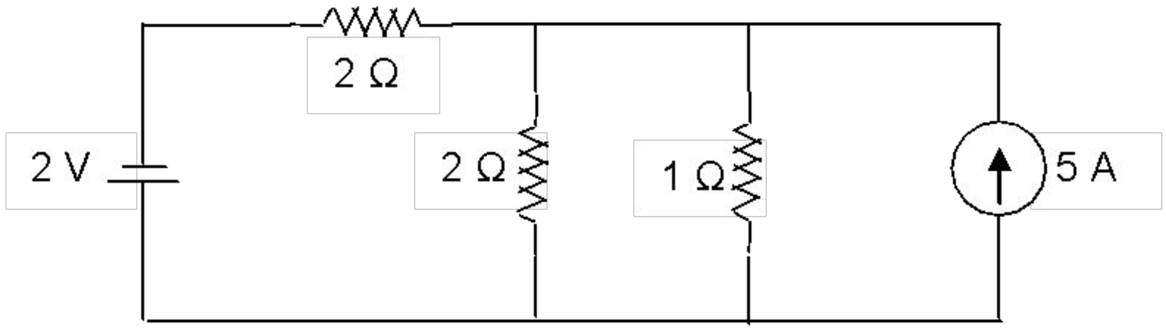
In a linear circuit having two or more independent sources, total response in an element (voltage across the element or current through the element) is equal to the algebraic sum of responses in that element due to each source applied separately while the other sources are reduced to zero.

To make a current source to zero, it must be open circuited. Similarly, **if any voltage source is to be made zero, it must be short circuited.** When this theorem is used in circuit with initial conditions, they are to be treated as sources. Further, dependent sources if any are left intact because they are controlled by circuit variables.

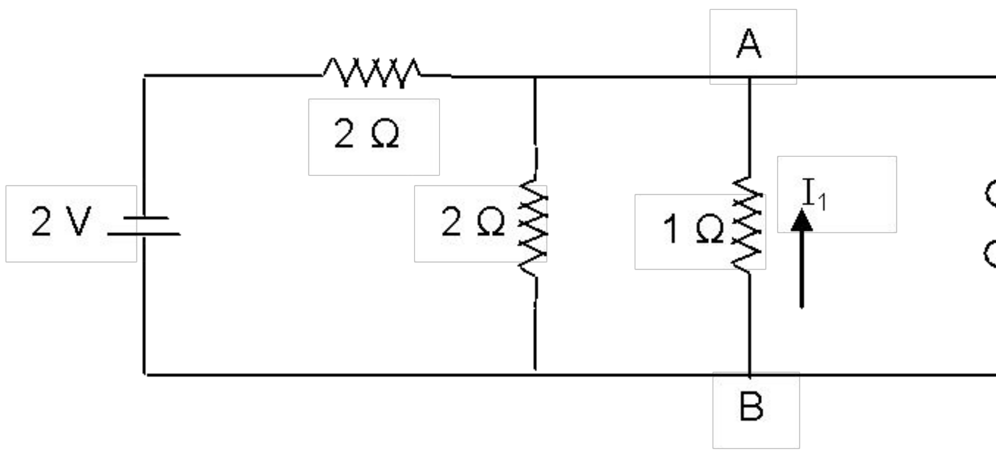
One disadvantage of analyzing a circuit using Superposition theorem is that it involves more calculations. If the circuit has three independent sources, we need to solve three simpler circuits each having only one independent source. However, when the circuit has only one independent source, several short-cut techniques can be readily applied to get the solution.

Major advantage of Superposition theorem is that it can be used to solve ac circuit having more than one source with **different frequencies.** In such case, solution in time frame is obtained corresponding to each source and added up to get the total solution.

<u>Example 1</u> Calculate the current through the 1 Ω resistor in the circuit shown below.

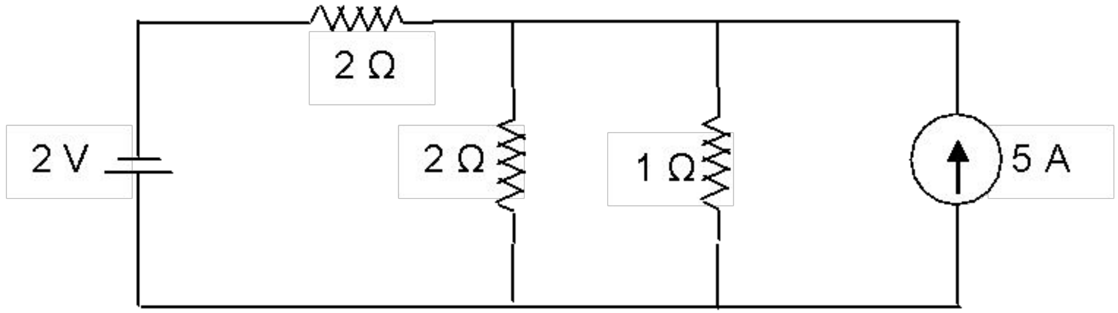


<u>Solution:</u> First calculate current I_1 due to voltage source alone. The current source is open circuited. The resulting circuit is shown below.



Total circuit resistance R_T = 2.6667 Ω. Circuit current $I_T = \dfrac{2}{2.6667} = 0.75$ A

$$\text{Current } I_1 = \frac{2}{3} \times 0.75 = 0.5 \text{ A from B to A}$$



Now calculate current I_2 due to current source alone. The voltage source is short circuited as shown in Fig.

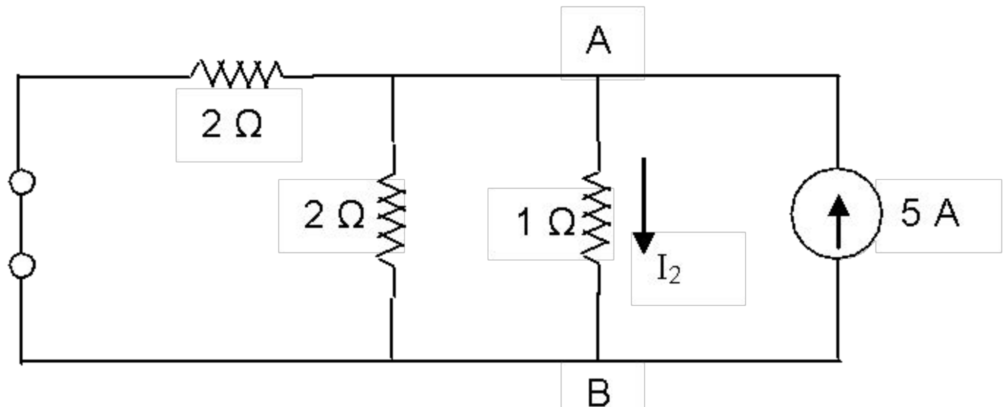


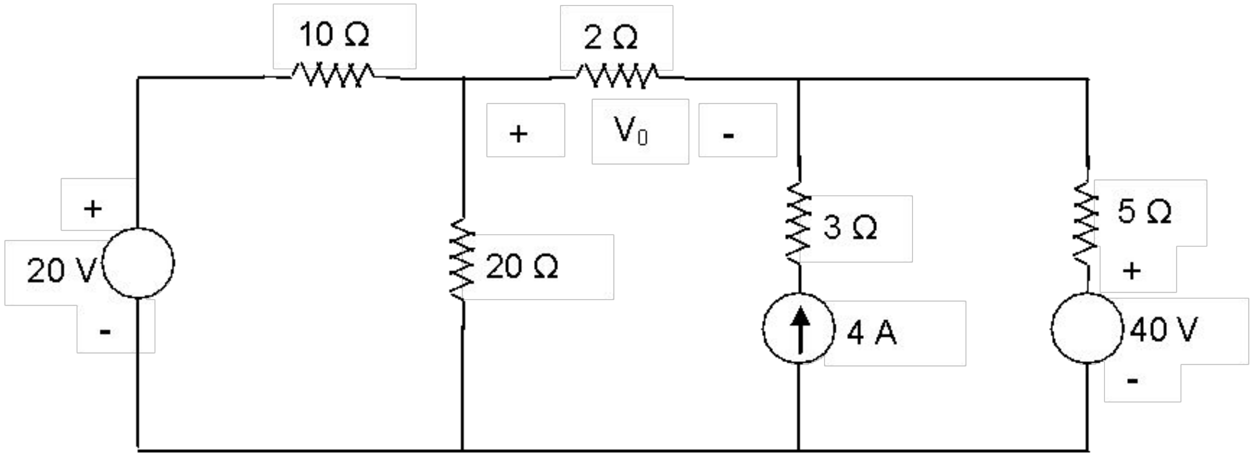
Fig. Circuit - Example 1

Noting that two 2 Ω resistors are in parallel, current I_2 = 2.5 A from A to B.

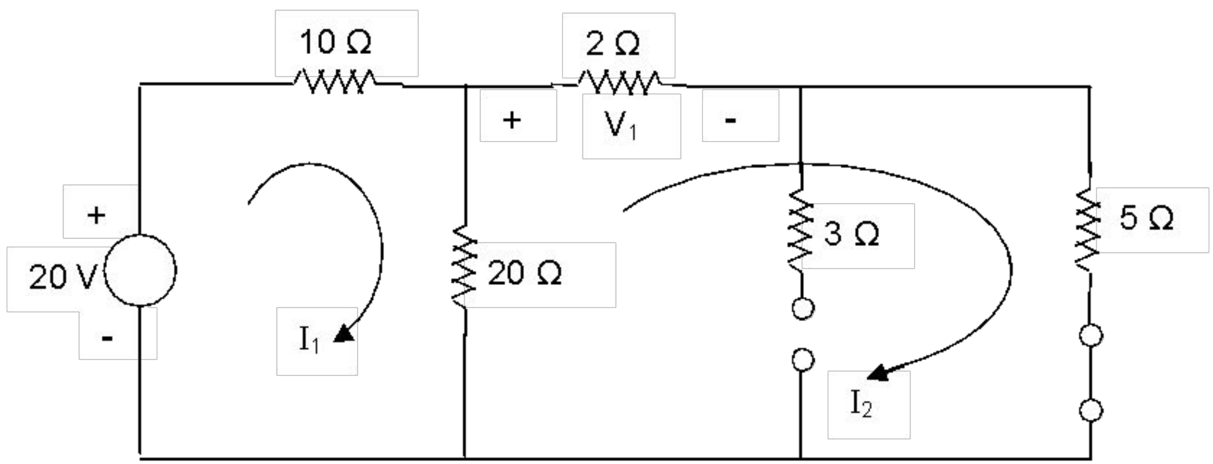
When both the sources are simultaneously present:

Current through 1 Ω resistor = 2.5 - 0.5 = 2 A from A to B.

Example 2   In the circuit shown, find the voltage drop, V_0 across the 2 Ω resistor using Superposition theorem.



Solution: 20 V source alone present: The circuit will be as shown below.

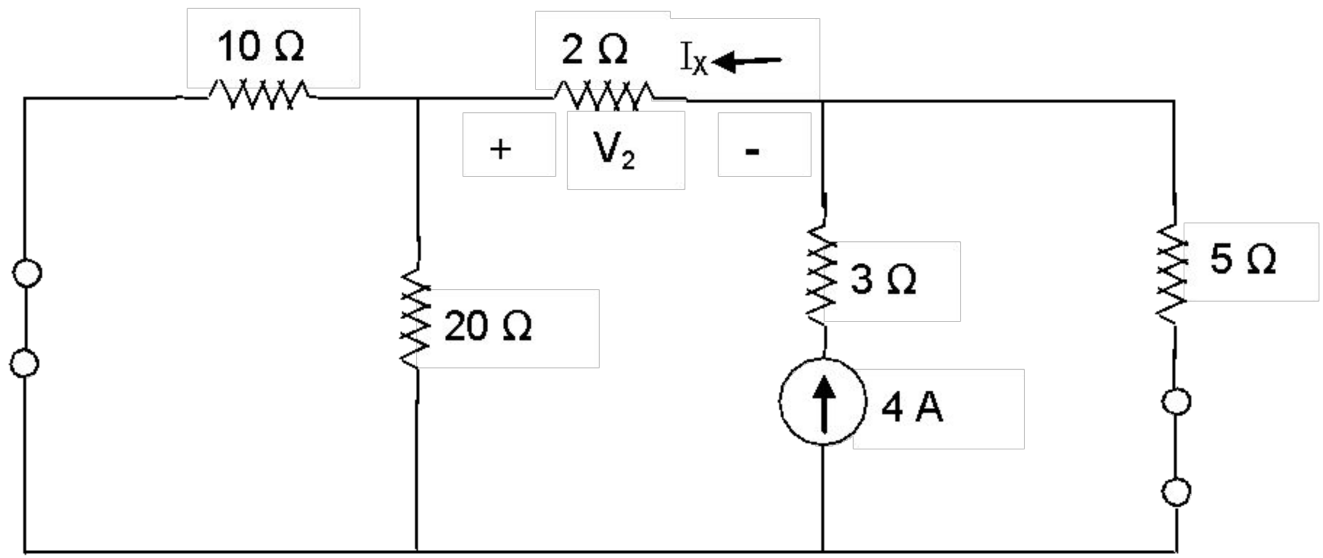


Mesh current equations: $\begin{bmatrix} 30 & -20 \\ -20 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$ On solving, $I_2 = 0.9756$ A

Thus voltage $V_1 = 2 \times 0.9756 = 1.9512$ V

4 A source alone present:

The circuit will be as shown below.



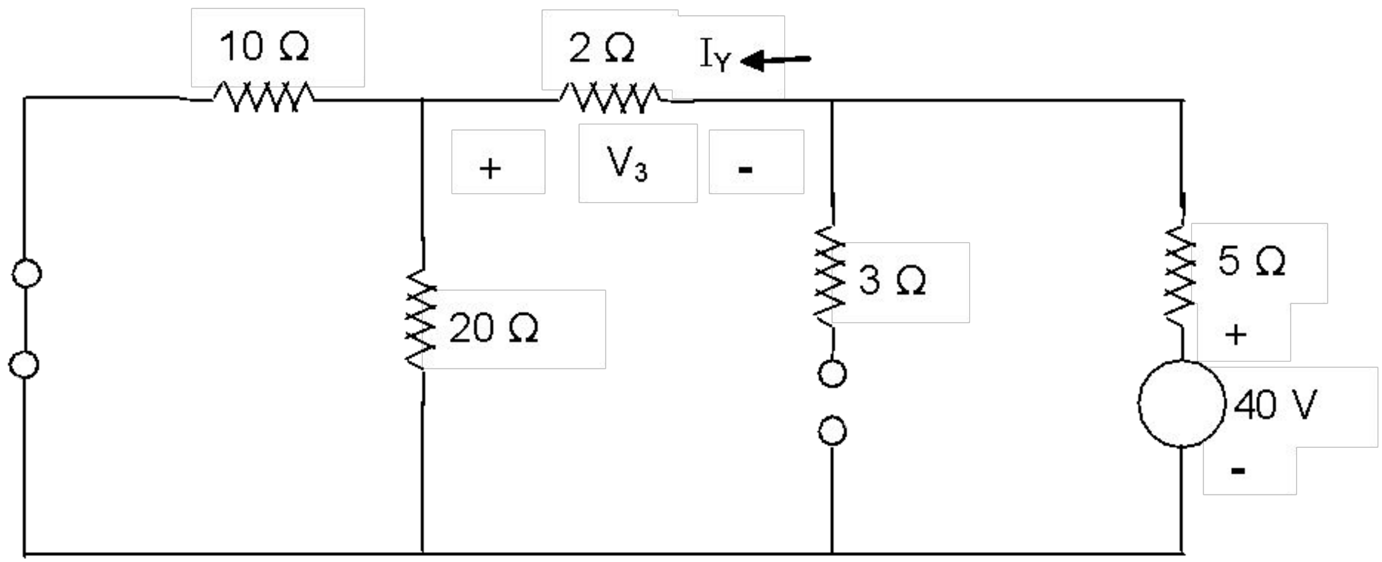
$$2 + 10 \,||\, 20 = 8.6667\ \Omega$$

$$\text{Therefore current } I_X = \frac{5}{13.6667} \times 4 = 1.4634\ \text{A}$$

$$\text{Thus voltage } V_2 = -\,2 \times 1.4634 = -\,2.9268\ \text{V}$$

40 V source alone present:

Resulting circuit is shown below.



Circuit resistance $R_T = 5 + 2 + (10||20) = 13.6667\ \Omega$

Current $I_Y = 40 / 13.6667 = 2.9268$ A; Thus voltage $V_3 = -2 \times 2.9268 = -5.8537$ V

When all the three sources are simultaneously present,

voltage across 2 Ω, i.e. $V_0 = V_1 + V_2 + V_3 = 1.9512 - 2.9268 - 5.8537 = -6.8293$ V