Magnetic circuit and Faradays law of Electromagnetic Induction

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MAGNETIC CIRCUITS

Introduction

A substance, which when suspended freely, points in the direction of north and south is called a MAGNET. Magnet attracts iron fillings. It is also called as permanent magnet. A current passing though a conductor also can produce magnetic effect and it is called as Electromagnet.

A permanent magnet has one north pole and one south pole. The imaginary lines which travel from north pole to south pole outside the magnet are called magnetic lines of force. They are drawn by plotting successive directions pointed out by a small compass needle in the magnetic field. Magnetic lines of forces are shown in Fig. 1 and they pass through the magnet.

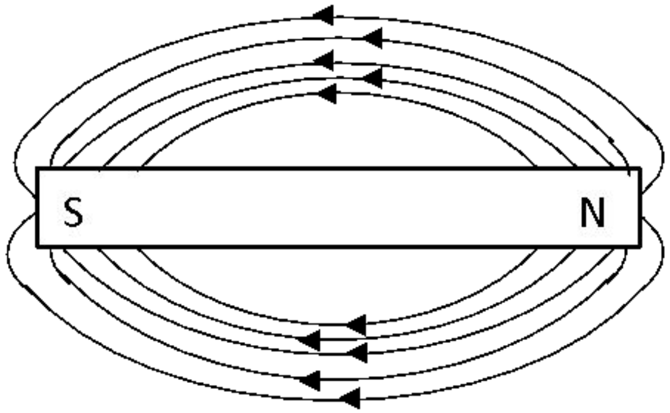


Fig. 1 Magnetic lines of forces

Flux, Magneto Motive Force and Reluctance

The magnetic lines of force in the magnetic field is called Flux. Its unit is Weber(Wb). 1 Wb = 10^8 magnetic lines. Flux is denoted by φ. Magnetic flux per unit cross sectional area is called Flux density and it is expressed in Weber / metre^2. Flux density is denoted by B.

Magneto Motive Force (mmf) is the source of producing flux in the magnetic circuit. It can be explained through Electromagnet. When a current of I ampere is passed through a coil of N turns, results in a mmf of $N I$. This $N I$ ampere turns is called the mmf and its unit is ampere turns (AT).

Reluctance is the property of magnetic circuit that opposes the setting of flux.

$$\text{Reluctance, } S = \frac{\text{mmf}}{\text{flux}}$$

Its unit is ampere turns / weber.

The following table shows the similarities between magnetic and electric circuits.

Sl. No.	Magnetic circuit	Electric circuit
1	Magnetic flux, φ webers	Electric current, I ampere
2	Magneto motive force, AT	EMF, E volts
3	Reluctance, S AT / Wb	Resistance, R ohm
4	$\varphi = \frac{\text{mmf}}{\text{reluctance}}$	$\text{Current} = \frac{\text{emf}}{\text{resistance}}$

Leakage flux and Fringing effect

The flux which do not follow the desired path in a magnetic circuit is known as *leakage flux.*

Usually we assume that all the flux lines take path of the magnetic medium. But, practically, some flux lines do not confine to the specified medium. It is because, to prevent the leakage flux, there is no perfect magnetic insulator. Some flux lines can pass through air also. All the magnetic flux which complete the desired magnetic circuit are the *useful flux.*

To account for the leakage flux, leakage coefficient is introduced. Leakage coefficient, denoted by λ is defined as follows.

$$\text{Leakage coefficient, } \lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\varphi + \varphi'}{\varphi}$$

Usually, leakage factor is greater than unity.

An air gap is often introduced in the magnetic circuit out of necessity. When crossing an air gap, the magnetic lines of force have a tendency to bulge out. This is because the magnetic lines of force repel each other when they are passing through non-magnetic material. This phenomenon is known as fringing. It is shown in Fig. 2

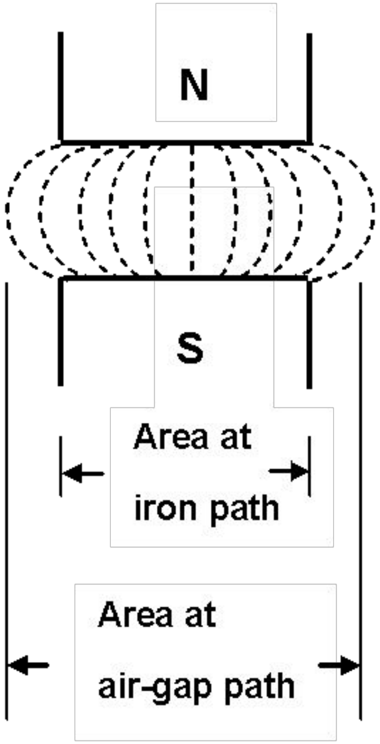


Fig. 2 Fringing effect

Fringing effect increases the effective area of cross section of the air-gap and as a result the flux density in the air-gap is reduced.

Problems involving simple magnetic circuits

Before doing problems involving magnetic circuits it is necessary to know some more terms associated with the magnetic circuit.

Magnetic field intensity, (also called as Magnetizing force) denoted as H, is the mmf per unit length of magnetic flux path. Thus,

$$H = \frac{N I}{\ell}$$

Flux density is proportional to magnetic field intensity. Thus $B \infty H$. The constant of proportionality is called ***permeability***, μ. Thus B = μ H or

$$\mu = B / H$$

Permeability of vacuum or free space is denoted as μ_0. Its value is

$4 \pi \times 10^{-7}$. Permeability of any other medium is given by

$$\mu = \mu_0 \mu_r$$

where μ_r is called the relative permeability of the medium.

An expression for Reluctance, S can be obtained as follows.

$$S = \frac{NI}{\varphi} = \frac{NI}{Ba} = \frac{NI}{\mu_0\ \mu_r\ H\,a}\ ; \quad \text{Since } H = \frac{NI}{\square} \quad \text{we get} \quad S = \frac{\square}{a\ \mu_0\ \mu_r}$$

Permeance, P is the reciprocal of Reluctance.

An iron core coil with a small air gap is shown in Fig. 3.

Coil has n turns.

Current through coil = I

Mean radius of magnetic path = R_m

Cross section of core is circular with diameter d

Length of air gap = ℓ_g

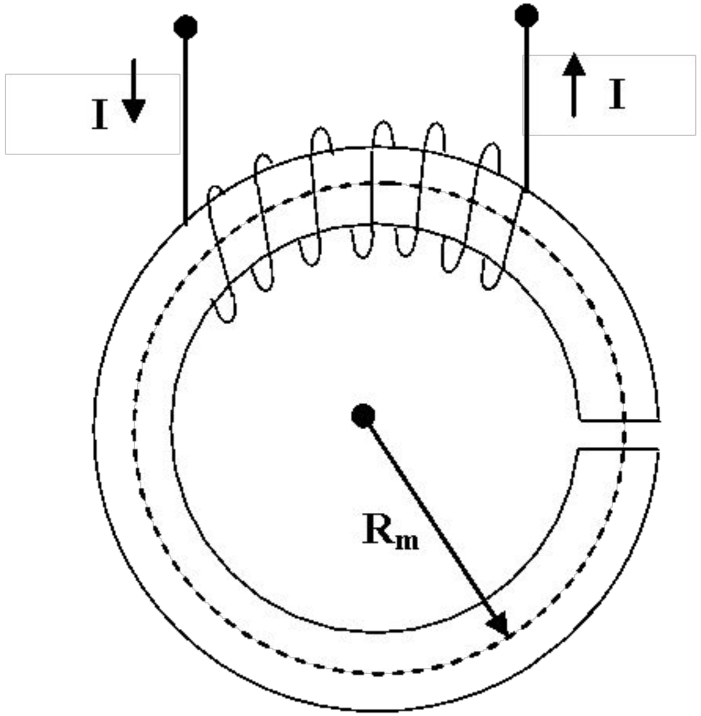


Fig. 3 Iron core coil

Example 1

A toroidal air core coil with 2000 turns has a mean radius of 25 cm. The diameter of each turn is 6 cm. If the current in the coil is 10 A, find (a) MMF (b) flux and (c) flux density.

Solution

Given N = 2000 turns; R_m = 25 cm; d = 6 cm; I = 10 A.

(a) MMF = N I = 2000 x 10 = 20000 AT

(b) Flux = MMF / Reluctance

$$\text{Reluctance, } S = \frac{\ell}{a\,\mu_0\,\mu_r}\text{; Since it is air core} = \mu_r = 1$$

$$\ell = 2\pi \times 0.25 = 1.5708 \text{ m}; \quad a = \pi r^2 = \pi \times 0.03^2 = 0.002827 \text{ m}^2$$

$$\text{Reluctance, } S = \frac{1.5708}{0.002827 \times 4\pi \times 10^{-7}} = 4.4217 \times 10^8 \text{ AT / Wb}$$

$$\text{Flux, } \varphi = \frac{20000}{4.4217 \times 10^8} = 4.5231 \times 10^{-5} \text{ Wb}$$

$$\text{(c) Flux density, } B = \frac{\varphi}{a} = \frac{4.5231 \times 10^{-5}}{0.002827} = 0.016 \text{ Wb / m}^2$$

Example 2

The flux produced in the air gap between two electro magnetic poles is 0.05 Wb. If the cross sectional area of the air gap is 0.2 m^2, find (a) flux density, (b) magnetic field intensity, (c) reluctance and (d) permeance of the air gap. Find also the mmf dropped in the air gap, given the length of air gap to be 1.2 cm.

Solution

Given Flux, $\varphi = 0.05$ Wb; $a = 0.2\ m^2$; $\ell_g = 0.012$ m

(a) Flux density, $B = \dfrac{0.05}{0.2} = 0.25\ Wb/m^2$

(b) Depending on the data H can be calculated either from $H = \dfrac{NI}{\ell}$ or

$H = \dfrac{B}{\mu}$; Magnetic field intensity, $H = \dfrac{B}{\mu} = \dfrac{0.25}{4\pi \times 10^{-7}} = 1.9894 \times 10^{5}\ AT/m$

(c) Reluctance, $S = \dfrac{\ell_g}{a\,\mu_0} = \dfrac{0.012}{0.2 \times 4\pi \times 10^{-7}} = 4.7746 \times 10^{4}\ AT/Wb$

(d) Permeance, $P = \dfrac{1}{S} = \dfrac{1}{4.7746 \times 10^{4}} = 2.0944 \times 10^{-5}\ Wb/AT$

$MMF = H \times \ell_g = 1.9894 \times 10^{5} \times 0.012 = 2.3873 \times 10^{3}\ AT$

Example 3

A ring has mean diameter of 15 cm, a cross section of 1.7 cm^2 and has a radial gap of 0.5 mm cut in it. It is uniformly wound with 1500 turns of insulated wire and a current of 1 A produces a flux of 0.1 mWb across the gap. Calculate the relative permeability of iron on the assumption that there is no magnetic leakage.

Solution

Given D_m = 15 cm; a = 1.7 cm^2; ℓ_g = 0.5 mm; N = 1500 turns; I = 1 A;

φ = 0.1 mWb

Given $D_m = 15$ cm; $a = 1.7\ cm^2$; $\ell_g = 0.5$ mm; $N = 1500$ turns; $I = 1$ A;

$\varphi = 0.1$ mWb

MMF produced = 1500 x 1 = 1500 AT

Total reluctance = MMF / Flux = 1500 / 0.0001 = 1500×10^4 AT / Wb

Total reluctance = Reluctance of air gap + Reluctance of iron path

$$\text{Reluctance of air gap} = \frac{\ell_g}{a\,\mu_0} = \frac{0.0005}{1.7 \times 10^{-4} \times 4\pi \times 10^{-7}} = 2.3405 \times 10^6 \text{ AT / Wb}$$

Reluctance of iron path = $15 \times 10^6 - 2.3405 \times 10^6 = 12.6595 \times 10^6$ AT / Wb

Length of iron path = $\pi \times 15 \times 10^{-2} - 0.05 \times 10^{-2} = 47.0739 \times 10^{-2}$ m

$$\text{Thus } 12.6595 \times 10^6 = \frac{\ell}{a\,\mu_0\,\mu_r} = \frac{47.0739 \times 10^{-2}}{1.7 \times 10^{-4} \times 4\pi \times 10^{-7} \times \mu_r} = \frac{2203.5423 \times 10^6}{\mu_r}$$

Thus $\mu_r = 2203.5423 / 12.6595 = 174.0623$

Example 4

A series magnetic circuit has an iron path of length 50 cm and an air gap of 1mm. The cross section of the iron is 6.66 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetization curve for the iron.

Flux density (Wb / m^2):	1.2	1.35	1.45	1.55
Magnetizing force (AT / m):	500	1000	2000	4000

Solution

Given $\ell_i = 0.5$ m; $\ell_g = 1 \times 10^{-3}$ m; $a = 6.66 \times 10^{-4}$ m^2; $N = 400$; Flux = 0.9 mWb

Reluctance of air gap $= \dfrac{\ell_g}{a\,\mu_0} = \dfrac{1 \times 10^{-3}}{6.66 \times 10^{-4} \times 4\pi \times 10^{-7}} = 1.1949 \times 10^{6}$ AT / Wb

Required air gap mmf $= 0.9 \times 10^{-3} \times 1.1949 \times 10^{6} = 1075.4$ AT

Flux density in the iron path $= 0.9 \times 10^{-3} / (6.66 \times 10^{-4}) = 1.3514$ Wb / m^2

From the given data,

corresponding value of H = 1000 + (1000 x 0.0014 / 0.1) = 1014 AT / m

Required iron path mmf = 1014 x 0.5 = 507 AT

Total mmf required = 1075.4 + 507 = 1582.4 AT

Current required = 1582.4 / 400 = 3.956 A

Example 5

An iron rod of 1 cm radius is bent to a ring of mean diameter 30 cm and wound with 250 turns of wire. Assume the relative permeability of iron as 800. An air gap of 0.1 cm is cut across the bent ring. Calculate the current required to produce a useful flux of 20000 lines if (a) leakage is neglected and (b) leakage factor is 1.1.

Solution

Given $r = 1$ cm; $D_m = 0.3$ m; $N = 250$; $\mu_r = 800$; $\ell_g = 0.001$ m

$$\text{Flux } \varphi = 20000 / (10^8) = 0.2 \text{ mWb}$$

Leakage is neglected

Area of cross section, $a = \pi \times 10^{-4} = 0.0003142 \text{ m}^2$

$$\text{Reluctance of air gap} = \frac{\ell_g}{a\,\mu_0} = \frac{0.001}{0.0003142 \times 4\pi \times 10^{-7}} = 2.5327 \times 10^6 \text{ AT/Wb}$$

Required air gap mmf $= 0.0002 \times 2.5327 \times 10^6 = 506.54$ AT

Length of iron path $= (\pi \times 0.3) - 0.001 = 0.9415$ m

Reluctance of iron path =

$$\frac{\ell}{a\,\mu_0\,\mu_r} = \frac{0.9415}{0.0003142 \times 4\pi \times 10^{-7} \times 800} = 2.9807 \times 10^6 \text{ AT/Wb}$$

Required iron path mmf = $0.0002 \times 2.9807 \times 10^6 = 596.14$ AT

Total mmf required = 506.54 + 596.14 = 1102.68 AT

Current required = 1102.68 / 250 = 4.4107 A

Leakage factor is 1.1

As in previous case, required air gap mmf = $0.0002 \times 2.5327 \times 10^6 = 506.54$ AT

To maintain useful flux of 0.2 mWb in the air gap,

flux required in the iron path = 1.1 x 0.2 = 0.22 mWb

Required iron path mmf = $0.00022 \times 2.9807 \times 10^6 = 655.754$ AT

Total mmf required = 506.54 + 655.754 = 1162.294 AT

Current required = 1162.294 / 250 = 4.6492 A

Hysteresis and eddy current losses

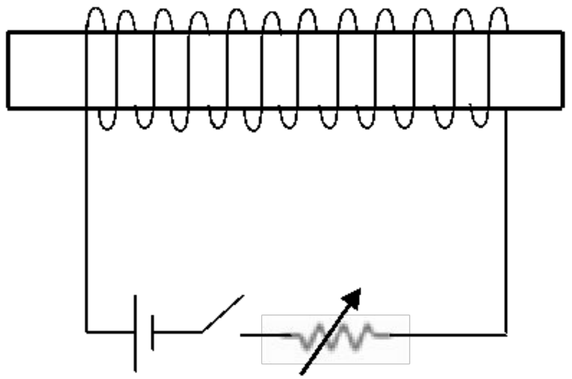


Fig. 6 – Circuit for B-H curve

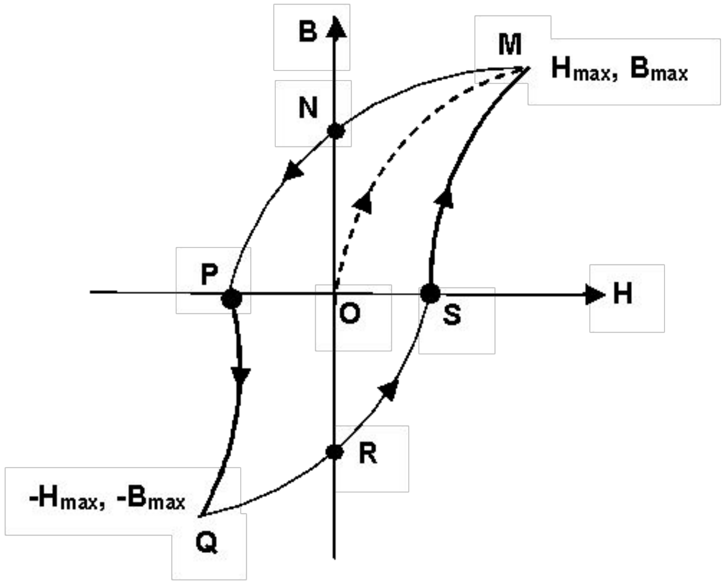


Fig. 7 – Hysteresis loop

Consider an iron bar which can be magnetized as shown in Fig. 6. Magnetizing force, H can be varied by controlling the current through the coil. Corresponding values of flux density B can be noted. First the B-H curve will follow OM shown if Fig. 7. Now if H is decreased gradually, B will not decrease along MO. Instead it will decrease along MN. Even when H is zero, B has a definite value ON. This implies that even on removing the magnetizing force, H, the iron bar is not getting demagnetized completely. The value of ON measures the retentivity of the material.

To demagnetize the iron bar, the magnetizing force has to be applied in the reverse direction. Flux density, B becomes zero at P. The value of H as measured by OP is known as coercive force. If H is further increased, the curve will follow the path PQ. By taking H back from – H_{max}, a similar curve QRSM is obtained. It is seen that B always lags behind H. This lagging character of B with respect to H is called hysteresis and the complete loop is called hysteresis loop. Different magnetic material will have different hysteresis Fig. 8 shows the hysteresis loop of cast steel and alloyed steel.

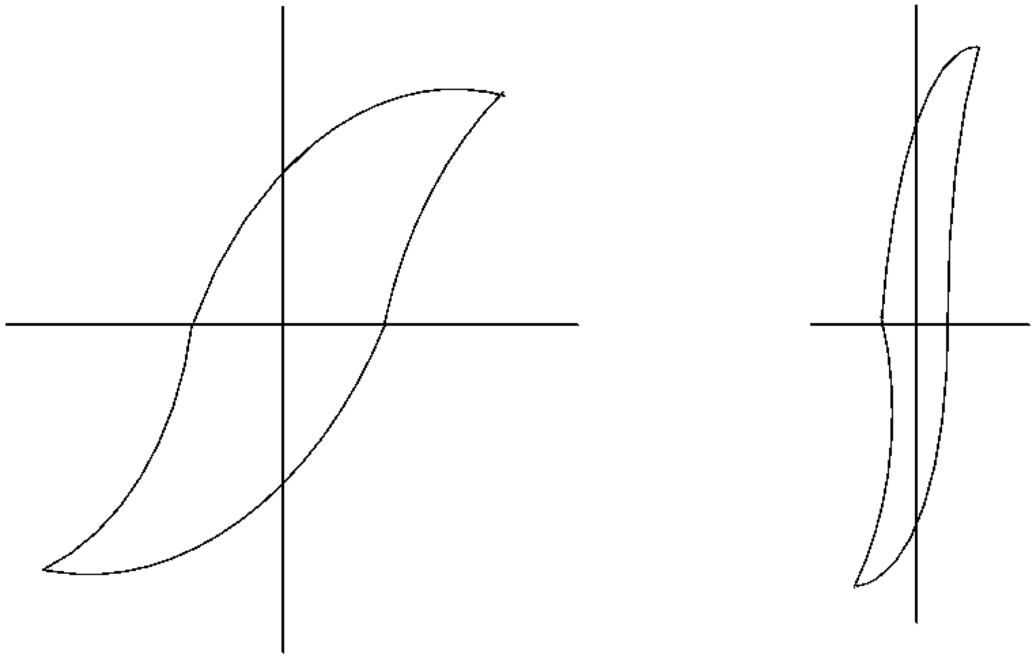


Fig. 8 – Hysteresis loop of cast steel and alloyed steel

Area of hysteresis loop gives the hysteresis loss per unit volume of the material. About 4% addition of Slican to steel give rise to reduction in hysteresis loop area and hence hysteresis loss.

Whenever a conducting material cuts the magnetic flux (armature core in the case of rotating machines) an emf is induced in the core. This emf sets up large current through the solid mass. Such current is known as eddy current. Flow of eddy current results in eddy current loss.

The eddy current loss is proportional to square of the thickness of the material. This loss can be minimized by using a laminated core, which offers high resistance for the flow of eddy current.

Faraday's Laws of Electromagnetic Induction

When a current flows in a conductor, magnetic field is produced. The reverse phenomenon whereby an Electro Motive Force (EMF) and hence current is produced in an electric circuit by some action of magnetic field, is called electromagnetic induction.

Consider the setup shown in Fig. 9.

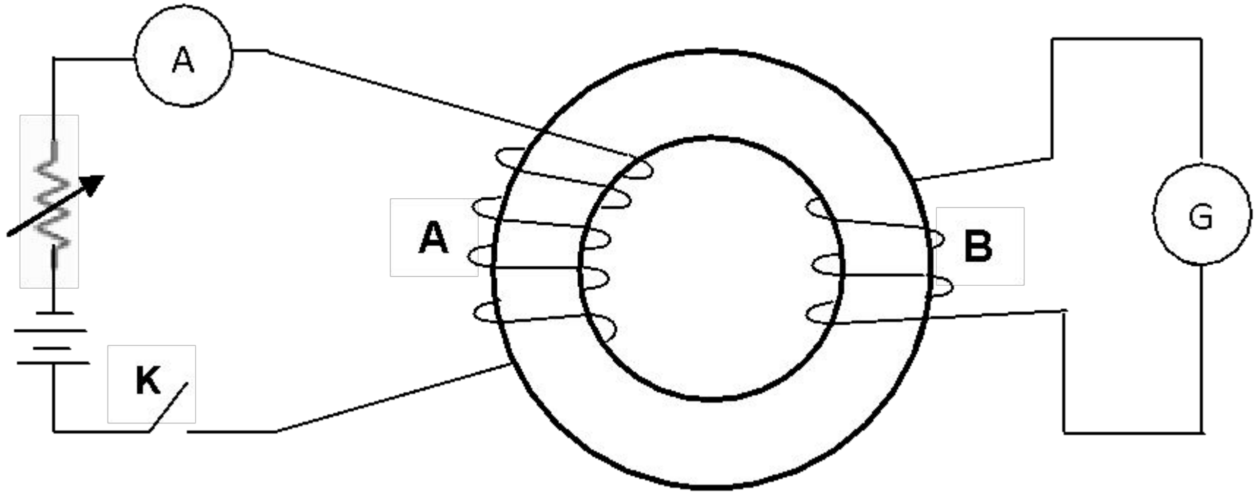


Fig. 9 – Static induced emf

When the switch, K is closed from the open position, there will be induced voltage and hence current in coil B as indicated by the galvanometer G. When the key is opened from the closed position, the current flow will be in the opposite direction. This illustrates the production of static induced emf.

Consider the setup shown in Fig. 10.

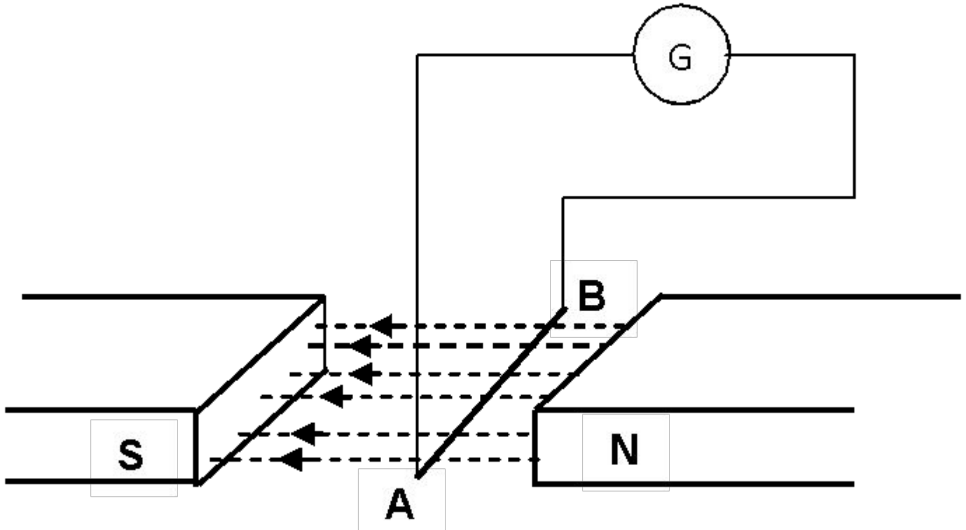


Fig. 10 – Dynamic induced emf

When the conductor AB is moved from the top position in the downward direction, it cuts the magnetic field at right angle. An emf is induced in the conductor resulting current flow as indicated by the galvanometer. When the conductor is moved from the bottom position in the upward direction, there will be current flow in the opposite direction.

The results of the above two experiments can be summed up into two laws, known as Faraday’s Laws of Electromagnetic Induction.

First Law: Whenever the flux linking with a coil changes, a static emf is induced in it and as such the emf lasts only for the time the change is taking place.

OR

When a moving conductor cuts the magnetic field, an emf induced in it which is called as dynamic emf.

Second Law: The magnitude of the induced emf is equal to the rate of change of flux linkage.

Induced emf

An emf is induced in a coil or conductor whenever there is a change influx linkages. The change in flux linkages can occur in two ways.

(i) The coil is stationary and the magnetic field is changing. Resulting induced emf is known as static induced emf. Transformer works on this principle.

(ii) The conductor is moved in a stationary magnetic field in such a way that there is change in flux linkage. Resulting induced emf is known as dynamic induced emf. Generator works on this principle.

Static induced emf

In this case, the coil is held stationary and the magnetic field is varied. The self induced emf may be self induced or mutually induced.

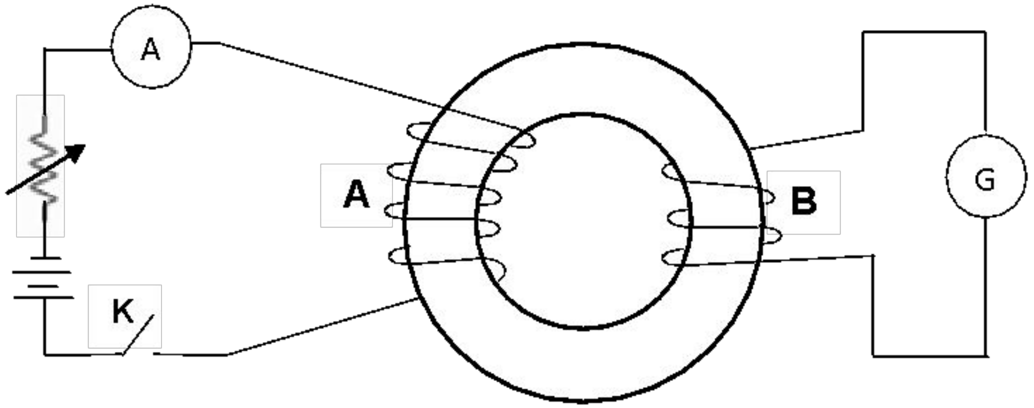


Fig. 9 – Static induced emf

Two coils are wound over a magnetic specimen. Coil A is energized using a battery. If switch K is initially closed, then a steady current of I ampere will flow through the coil A. It produces a flux of φ Wb. Let us assume that the entire flux links coils A and B. When the switch is suddenly opened, the current reduces to zero and the flux linking both the coils becomes zero. As per Faraday's law, emf is induced in both the coils A and B. Such emfs are known as static induced emfs. Static induced emf can be classified into two categories, namely self induced emf and mutually induced emf.

Self induced emf

If a single coil carries a current, flux will be set up in it. If the current changes, the flux will change. This change in flux will induce an emf in the coil. This kind of emf is known as self induced emf. In other words, self induced emf is the emf induced in a circuit when the magnetic flux linking it changes because of the current changes in the same circuit.

The magnitude of this self induced emf $e = N \frac{d\varphi}{dt}$

Mutually induced emf

Mutually induced emf is the emf induced in one circuit due to change of flux linking it, the flux being produced by the current in another circuit.

Referring to Fig. 9, when a change in current though coil A occurs, we find the flux linking coil B changes. Hence, an emf is induced in coil B and it is called as mutually induced emf.

Dynamic induced emf

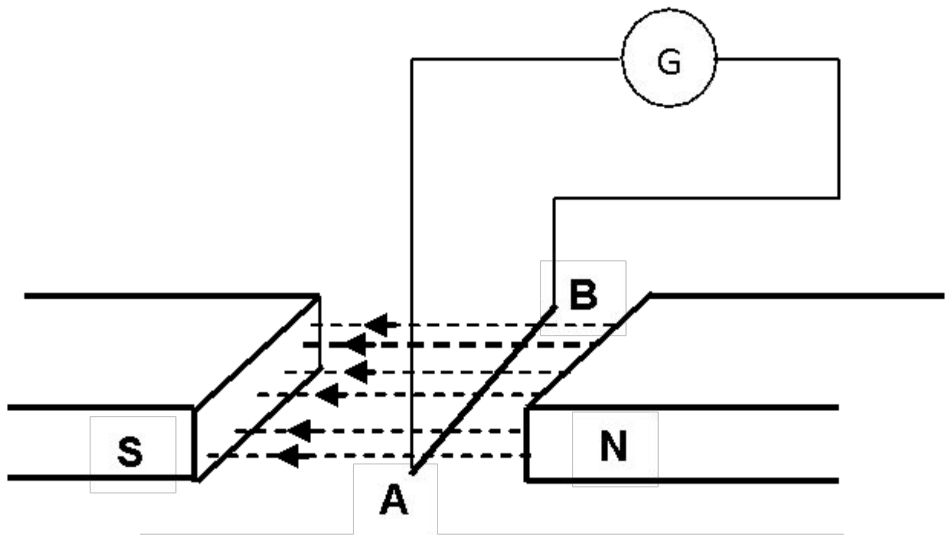


Fig. 10 – Dynamic induced emf

Consider the experiment set shown in Fig. 10.The magnetic poles, produce a stationary flux density of B Wb. / m^2. Let the conductor length be ℓ meters. The conductor is moved at right angle to he field. Let the distance moved in *dt* second be *dx* meters.

Area swept by the conductor in *dt* sec. = $\ell\ dx$ m^2

Magnetic flux cut by the conductor = $B\ \ell\ dx$ Wb.

Taking the conductor has one turn, corresponding

flux linkage, $\psi = B \ell \, dx$ Wb Turn

Rate of change of flux linkage = $B \ell \dfrac{dx}{dt}$

According to Faraday's Law, this is the induced emf, e in the conductor.

Thus induced emf, $e = B \ell v$ volts

where v = linear velocity = $\dfrac{dx}{dt}$

Let the conductor be moved with velocity v m / sec. in an inclined direction, making an angle θ to the direction of field. Then

Induced emf, $e = B \ell v \sin \theta$ volts

This is the basic principle of working of a generator.

Force on current carrying conductor

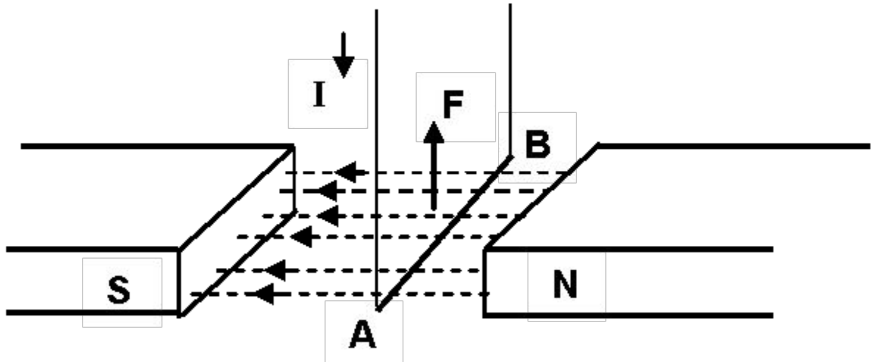


Fig. 11 – Force on current carrying conductor

Consider the setup shown in Fig. 11. When a current of I ampere flows in the conductor from A to B, it will experience a force, F given by

$$F = B \ell I \text{ Newton}$$

This relation is true if the conductor is at right angle to the magnetic field. In case if the conductor is an inclined direction, making an angle θ to the direction of field, then

$$F = B \ell I \sin \theta \text{ Newton}$$

This is the basic principle of working of a motor.

Self inductance, L

Self inductance of a coil, L is the rate of change of flux linkages with respect to the current in it. Its unit is Henry. Thus

$$L = \frac{d\psi}{dI} = N\frac{d\phi}{dI} \text{ Henry}$$

Equation for self inductance

Consider a magnetic circuit shown in Fig. 12.

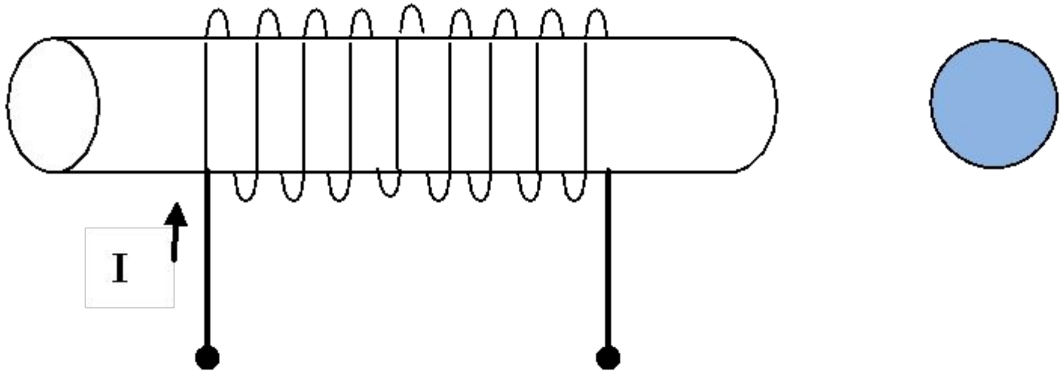


Fig. 12 Self inductance

With usual notations

Magnetizing force, $H = \dfrac{NI}{\ell}$ AT / m

Flux density, $B = \mu_0 \mu_r H = \mu_0 \mu_r \left(\dfrac{NI}{\ell}\right)$ Wb. / m^2

Magnetic flux, $\varphi = \mu_0 \mu_r \left(\dfrac{NI}{\ell}\right) a$ Wb.

Flux linkage = $N \varphi = \mu_0 \mu_r \left(\dfrac{N^2 I}{\ell}\right) a$ Wb. Turns

Self inductance, $L = N \dfrac{d\varphi}{dI} = N \dfrac{\varphi}{I} = \dfrac{\mu_0 \mu_r N^2 a}{\ell} = \dfrac{N^2}{(\ell / a \mu_0 \mu_r)}$

$$= \frac{N^2}{\text{Reluctance}}$$

Expression for self induced emf in terms of self inductance

The magnitude of self induced emf, $e = N\frac{d\varphi}{dt}$

Thus self induced emf, $e = N\frac{d\varphi}{dI} \times \frac{dI}{dt}$

$$= L\frac{dI}{dt}$$

Mutual inductance

When there are two are more coils, there will be mutual inductance between any two coils. If the two coils are far apart, then the there will not be any common flux linking both the coils and hence mutual inductance will be zero.

Consider two air core coils having self inductances L_1 and L_2 that are closer to each other as shown in Fig. 12. When current passes through coil 1, flux φ_{11} is produced in coil 1. Only a part of this flux links with coil 1 and the remaining flux links both the coils 1 and 2. Generally, the flux linking both the coils is useful and it is called mutual flux and represented by φ_{21}. The other part of the flux is called leakage flux represented by $\varphi_{\ell 1}$. When the coil 2 carries current, flux produced in it is φ_{22} and leakage flux is $\varphi_{\ell 2}$ and the mutual flux is φ_{12}. Fluxes $\varphi_{\ell 1}$, φ_{21}, $\varphi_{\ell 2}$ and φ_{12} are shown in Fig. 13.

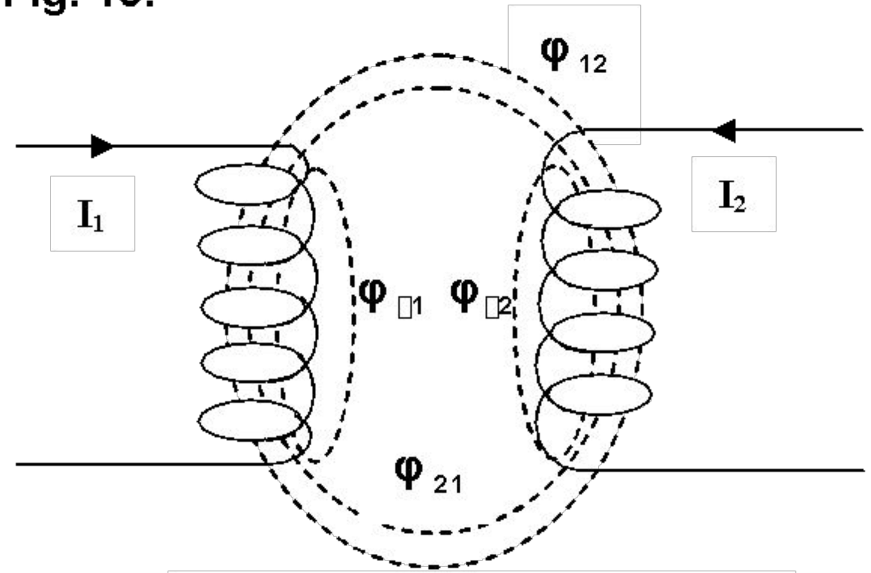


Fig. 13 Two coils in proximity

The operation of many useful devises which utilizes mutual inductance phenomenon depends upon how close the coils are coupled to each other. A fraction of total flux produced by a coil links both the coils and this coefficient represented by k. The coefficient of coupling depends on the relative position of coils 1 and 2. Thus, coefficient of coupling, $k = \frac{\varphi_{21}}{\varphi_{11}} = \frac{\varphi_{12}}{\varphi_{22}}$. It is to be noted that coefficient of coupling is always ≤ 1. If both the coils are far apart, then k = 0. On the other hand if both the coils are wound over the same core, then k = 1. Similar to the definition of self inductance, mutual inductances can be written as

$$M_{12} = N_1 \frac{d\varphi_{12}}{dI_2} \quad \text{and} \quad M_{21} = N_2 \frac{d\varphi_{21}}{dI_1}$$

Using energy criteria, it can be proved that $M_{12} = M_{21} = M$

$$\text{Then } M^2 = N_1 N_2 \frac{d\varphi_{12}}{dI_2} \frac{d\varphi_{21}}{dI_1} = N_1 N_2 \, k \frac{d\varphi_{22}}{dI_2} \, k \frac{d\varphi_{11}}{dI_1}$$

$$= k^2 \, N_1 \frac{d\varphi_{11}}{dI_1} N_2 \frac{d\varphi_{22}}{dI_2} = k^2 L_1 L_2$$

Thus $M = k\sqrt{L_1 L_2}$

Certain formulae

Static induced emf:

$$e = N \frac{d\varphi}{dt} \text{ volts} \qquad e = L \frac{dI}{dt} \text{ volts}$$

Dynamic induced emf:

$$e = B \ell v \sin\theta \text{ volts}$$

Force on a current carrying conductor:

$$F = B \ell I \sin\theta \text{ Newton}$$

Self inductance:

$$L = N \frac{d\varphi}{dI} \text{ Henry} \qquad L = \frac{N^2}{\text{Reluctance}} \text{ Henry}$$

Mutual Inductance:

$$M = k \sqrt{L_1 L_2}$$

Example 7

A coil of resistance 150 Ω is placed in a magnetic flux of 0.1 m Wb. The has 500 turns and a galvanometer of 450 Ω resistance is connected in series with it. The coil is moved from the given field to another field of 0.3 m Wb. In 0.1 sec. Find the average induced emf and the average current through the coil.

Solution

Given $R_c = 150\ \Omega$; $\varphi_1 = 0.1 \times 10^{-3}$ Wb.; $N = 500$ turns; $R_g = 450\ \Omega$; $\varphi_2 = 0.3 \times 10^{-3}$ Wb.;

$t = 0.1$ sec.

Induced emf, $e = N\dfrac{d\varphi}{dt} = 500 \times \dfrac{(0.3 \times 10^{-3} - 0.1 \times 10^{-3})}{0.1} = 500 \times 2 \times 10^{-3} = 1.0$ Volt

Current, I = induced emf / total resistance = 1.0 / (150 + 450) = 0.001667 A

Example 8

A conductor of length 100 cm moves at right angle to a uniform magnetic field of flux density 1.5 Wb. / m^2 with a velocity of 30 m / sec. Calculate the emf induced in it. Find also the value of induced emf when the conductor moves at an angle of 60^0 to the direction of the magnetic field.

Solution

Given $\ell = 1.0$ m; B = 1.5 Wb. / m^2; v = 30 m / sec.; $\theta = 60^0$

Induced emf, $e = B \ell v = 1.5 \times 1.0 \times 30 = 45$ V

With $\theta = 60^0$. Induced emf, $e = B \ell v \sin \theta = 45 \times \sin 60^0 = 38.9711$ V

Example 9

A conductor of 10 cm long lies perpendicular to a magnetic field of strength 1000 AT / m., Find the force acting on it when it carries a current of 60 A.

Solution

Given $\ell = 0.1$ m; $H = 1000$ AT / m; $I = 60$ A

Flux density, $B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 = 0.001257$ Wb. / m^2

Force, $F = B \ell I = 0.001257 \times 0.1 \times 60 = 0.00754$ Newton

Example 10

An air cored toroidal coil has 480 turns, a mean length of 30 cm and a cross-sectional area of 5 cm^2. Calculate (a) the inductance of the coil and (b) the average induced emf, if a current of 4 A is reversed in 60 m sec.

Solution

Given N = 480 turns; ℓ = 0.3 m; a = 5 x 10^{-4} m^2; dI = 8 A; dt = 60 x 10^{-3} sec.

Inductance, L = N^2 / Reluctance

$$\text{Reluctance, } S = \ell / (a\,\mu_0) = \frac{0.3}{5 \times 10^{-4} \times 4\pi \times 10^{-7}} = 0.4775 \times 10^{9}\ \text{AT/Wb}$$

$$\text{Inductance, } L = \frac{480^2}{0.4775 \times 10^{9}} = 0.4825 \times 10^{-3} = 0.4825\,\text{mH}$$

$$\text{Induced emf, } e = L\,\frac{dI}{dt} = 0.4825 \times 10^{-3} \times \frac{8}{60 \times 10^{-3}} = 0.06433\,\text{V}$$

Example 11

A current of 5 A when flowing through a coil of 1000 turns establishes a flux of 0.3 m Wb. Determine the self inductance if the coil.

Solution

Given $I = 5$ A; $N = 1000$ turns; $\varphi = 0.3 \times 10^{-3}$ Wb.;

Self inductance, $L = \frac{d\psi}{dI} = 1000 \times \frac{0.3 \times 10^{-3}}{5} = 0.06\,H$

Example 12

A coil has a self inductance of 30 mH. Calculate the emf in the coil when the current in the coil (a) increases at the rate of 300 A / sec. (b) raises from 0 to 10 A in 0.06 sec.

Solution

Given $L = 30 \times 10^{-3}$ H;

(a) Induced emf, $e = L \dfrac{dI}{dt} = 30 \times 10^{-3} \times 300 = 9\,V$

(b) Induced emf, $e = L \dfrac{dI}{dt} = 30 \times 10^{-3} \times \dfrac{10}{0.06} = 5\,V$

Example 13

The number of turns in a coil is 250. When a current of 2 A flows in this coil, the flux in the coil is 0.3 m Wb. When this current is reduced to zero in 2 m sec., the voltage induced in another coil is 63.75 V. If the coefficient of coupling between the two coils is 0.75, find the self inductances of the two coils, mutual inductance and the number of turns in the second coil.

Solution

Given $N_1 = 250$; $I_1 = 2$ A; $\varphi_1 = 0.3 \times 10^{-3}$ Wb.; $dI_1 = 2$ A ; $dt_1 = 2$ m sec; $e_2 = 63.75$ V;

$k = 0.75$

Given $N_1 = 250$; $I_1 = 2$ A; $\varphi_1 = 0.3 \times 10^{-3}$ Wb.; $dI_1 = 2$ A ; $dt_1 = 2$ m sec; $e_2 = 63.75$ V;

$k = 0.75$

Self inductance, $L_1 = N_1 \dfrac{d\varphi}{dI} = 250 \times \dfrac{0.3 \times 10^{-3}}{2} = 0.0375\,\text{H}$

Induced emf in coil 2, $e_2 = M \dfrac{dI_1}{dt_1} = M \times \dfrac{2}{0.002} = 63.75$

Thus mutual inductance, M = 63.75 mH

Since $M = k \sqrt{L_1 L_2}$

$0.06375^2 = 0.75^2 \times 0.0375 \times L_2$

Thus self inductance of coil 2, $L_2 = 0.1927$ H

Flux $\varphi_2 = k\varphi_1 = 0.75 \times 0.3 \times 10^{-3}$ Wb $= 0.225 \times 10^{-3}$ Wb

Also, $e_2 = N_2 \times \dfrac{d\varphi_2}{dt} = N_2 \times \dfrac{0.225 \times 10^{-3}}{2 \times 10^{-3}} = 63.75$

Thus $N_2 = 567$

Working principle, construction and applications of DC Generator

The dc generator is rotating electrical machine which converts mechanical energy into electrical energy. The generator is usually driven by a steam turbine or water turbine which is called as prime mover.

The dc generator operates on the principle based on the Faraday's Law of electromagnetic induction. The generator should have (i) magnetic field (ii) conductors capable of carrying current (iii) movement of conductors in the magnetic field. Necessary magnetic field is produced by field coil. The set of conductors is called the armature.

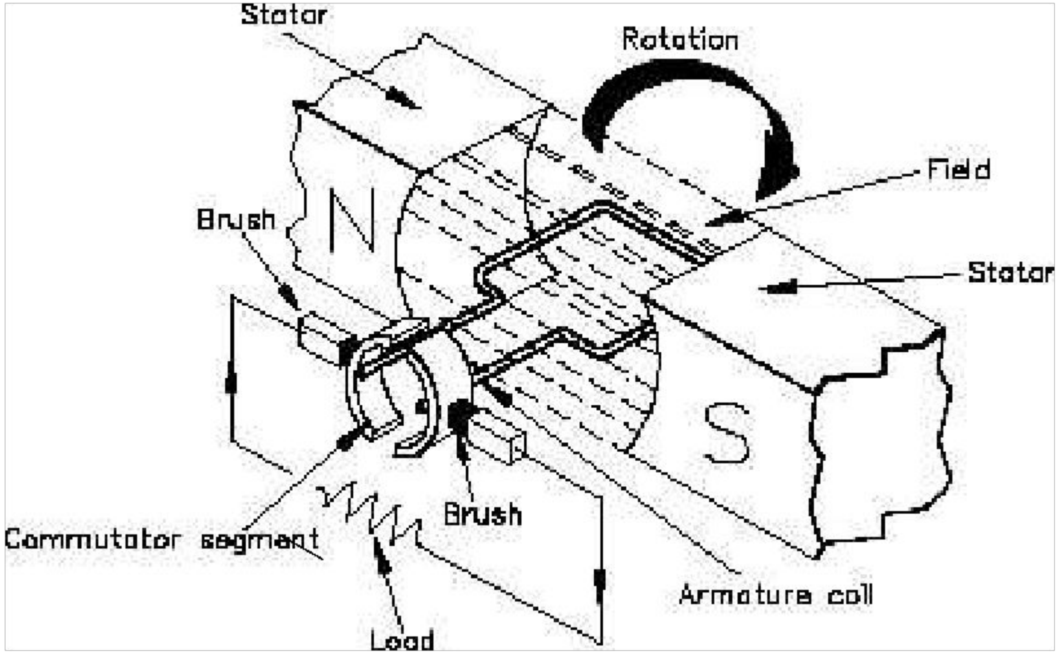


Fig. 14 Principle of operation of DC Generator

The voltage induced in the coil will be as shown in Fig. 15.

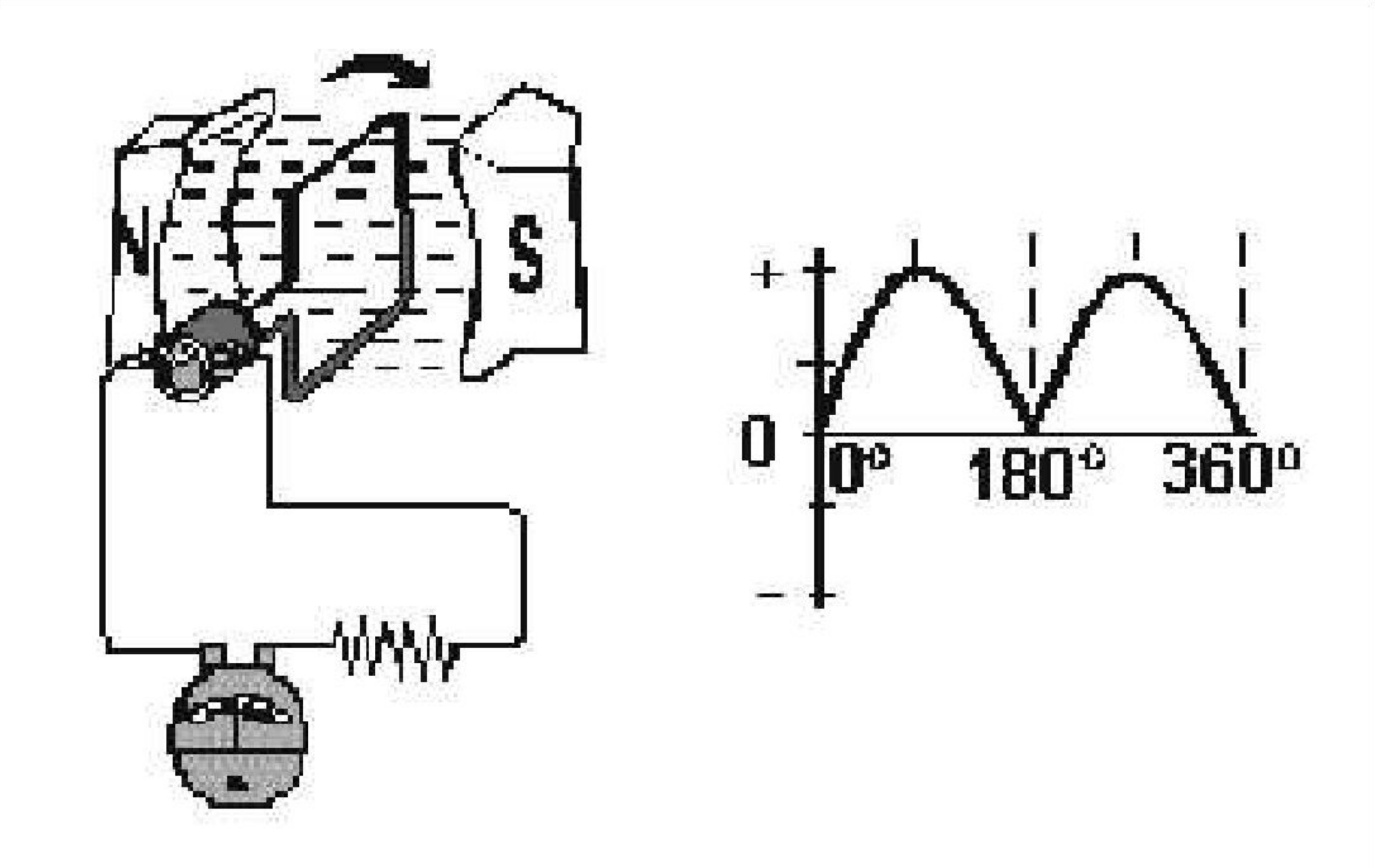
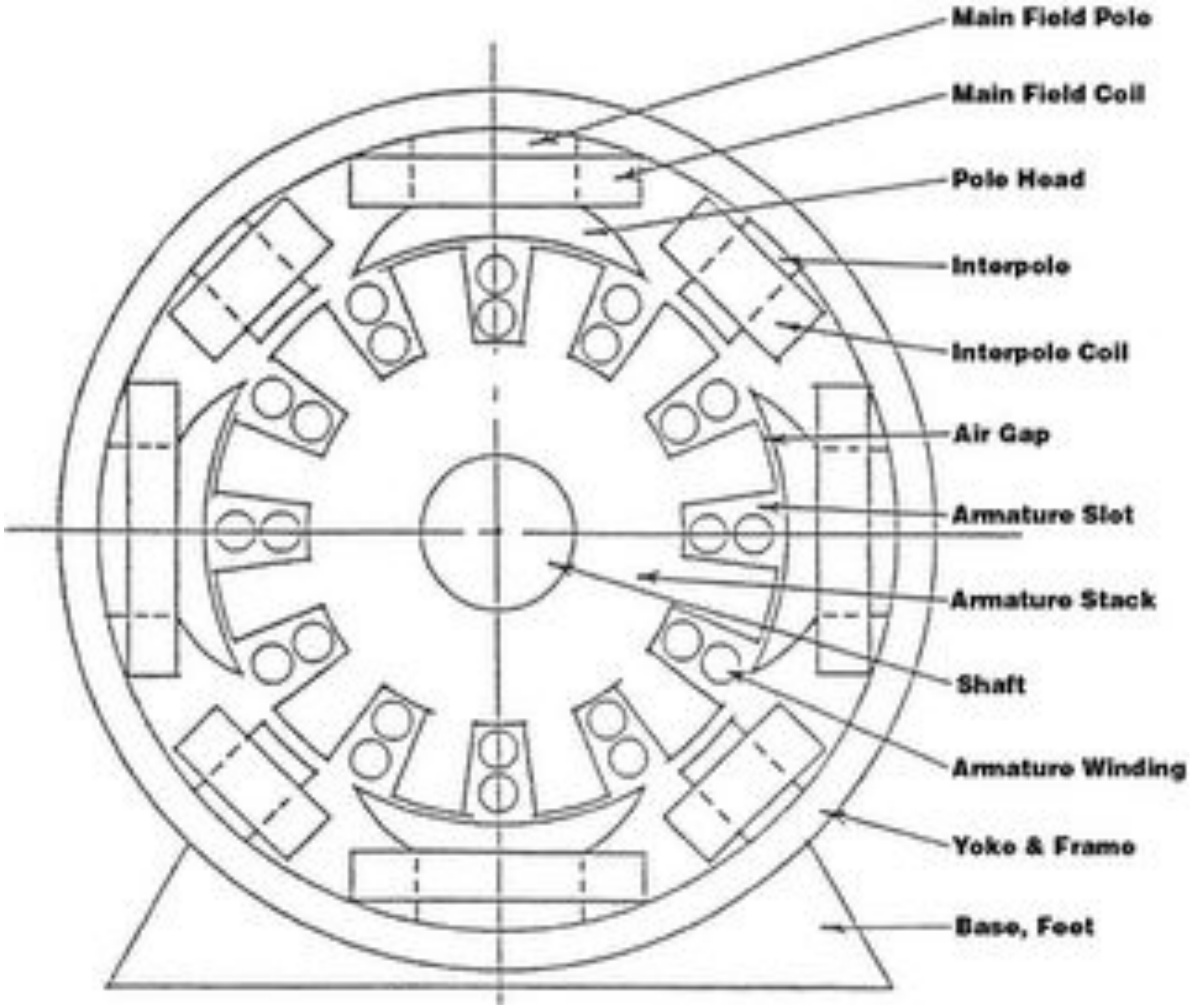
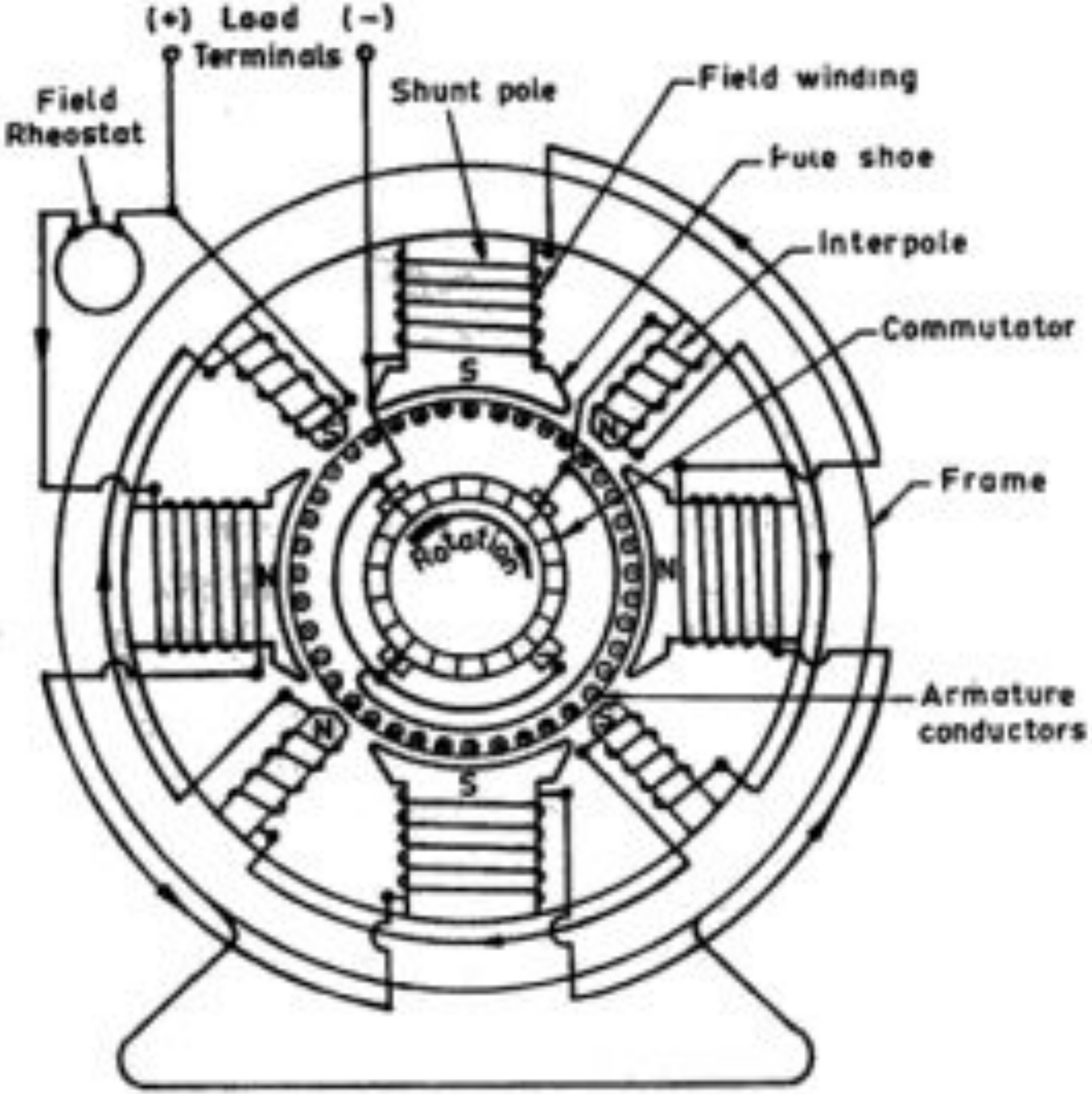


Fig. 15 EMF induced in an armature coil


Main Field Pole
Main Field Coil
Pole Head
Interpole
Interpole Coil
Air Gap
Armature Slot
Armature Stack
Shaft
Armature Winding
Yoke & Frame
Base, Feet


(+) Load (−)
Terminals
Field Rheostat
Shunt pole
Field winding
shoe
Interpole
Commutator
Frame
Armature conductors
S
N
N
S
S
N
N
S
Rotation

Depending on how the Armature and Field windings are connected, we have different types of dc generators. They are shown in Fig. 18.

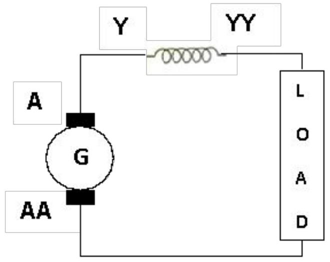


Fig. 18 (a) Series generator

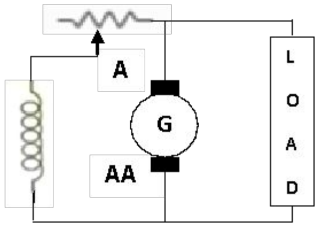


Fig. 18 (b) Shunt generator

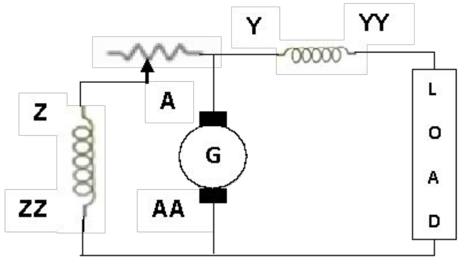


Fig. 18 (c) Short shunt compounded generator

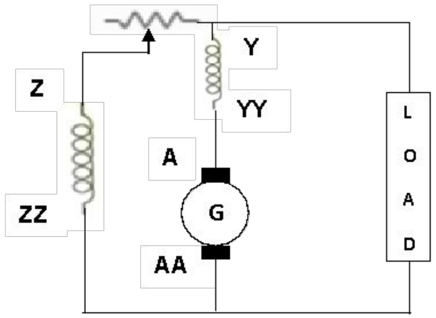


Fig. 18 (d) Long shunt compounded generator

Application of dc generators

Shunt generators are used in supplying nearly constant loads. They are used for charging batteries and supplying the fields of synchronous machines.

Series generators are used to boosters for adding voltage to transmission lines to compensate for the line drop.

Cumulative compound generators are used for drives which require constant dc voltage supply.

Differential compound generators are used in arc welding.

Working principle, construction and applications of DC motor

Whenever a current carrying conductor is kept in a stationary magnetic field, an electromotive force is produced. This force is exerted on the conductor and hence is moved away from the field. This is the principle used in dc motors.

Construction of dc motor is exactly similar to dc generator.

In a dc motor, both the armature and the field windings are connected to a dc supply. Thus, we have current carrying armature conductors placed in a stationary magnetic field. Due to electromagnetic torque exerted on the armature conductors, the armature starts revolving. Thus, electrical energy is converted into mechanical energy in the armature.

When the armature is in motion, we have revolving conductors in a stationary magnetic field. As per Faraday's Law of electromagnetic induction, an emf is induced in the armature conductors. As per Lenz's law, this induced emf opposes the voltage applied to the armature. Hence it is called back emf. There will be small voltage drop due to armature resistance. Thus, the applied voltage has to overcome the back emf in addition to supplying the armature voltage drop. The input power is used to produce necessary torque for the continuous rotation of the armature.

Depending on how the Armature and Field windings are connected, we have different types of dc motors. They are shown in Fig. 19.

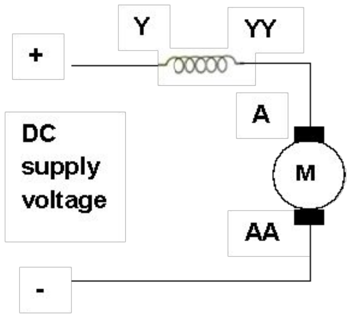


Fig. 19 (a) Series motor

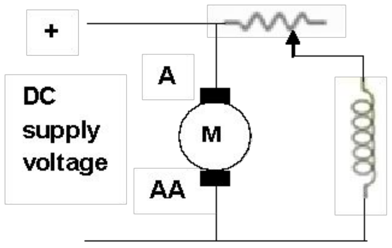


Fig. 19 (a) Shunt motor

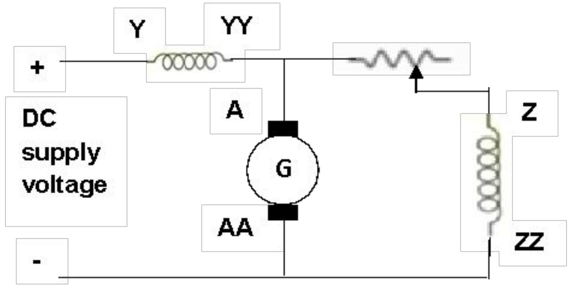


Fig. 19 (c) Short shunt compounded motor

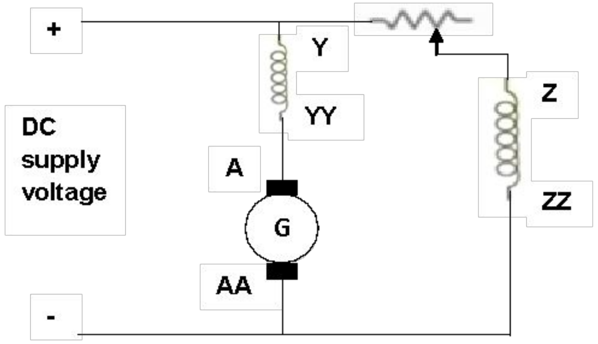


Fig. 19 (d) Long shunt compounded motor

Application of dc motors

DC series motors are used in electric trains, cranes, hoists, conveyors etc. where high starting torque is required.

Shunt motors are used where the speed has to remain constant under loaded condition.

Compound motors are used for driving heavy tools for intermittent heavy loads such as rolling mills, printing machines etc.

Working principle, construction and applications of 1- phase transformer

The transformer works on the principle of electromagnetic induction. In this case the coils are stationary. The magnetic flux is produced by ac voltage and hence it varies with respect to time. Thus the induced emf comes under the classification of statically induced emf.

The transformer is a static apparatus used to transfer electrical energy from one circuit to another. The two circuits are magnetically coupled. One of the circuits, namely Primary, is energized by connecting it to an ac supply at specific voltage magnitude, frequency and waveform. Then we have a mutually induced voltage available across the second circuit, namely Secondary, at the same frequency and waveform but with a desired voltage magnitude. These aspects are indicated in Fig. 20.

EMF induced in primary side $E_1 = N_1 \dfrac{d\varphi}{dt}$

Since same flux is linking both the primary and secondary coils

EMF induced in primary side $E_2 = N_2 \dfrac{d\varphi}{dt}$

Voltage ratio $\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2}$

Since losses in the transformer are very less

$E_1 I_1 = E_2 I_2$

Then the current ratio $\dfrac{I_1}{I_2} = \dfrac{E_2}{E_1} = \dfrac{N_2}{N_1}$

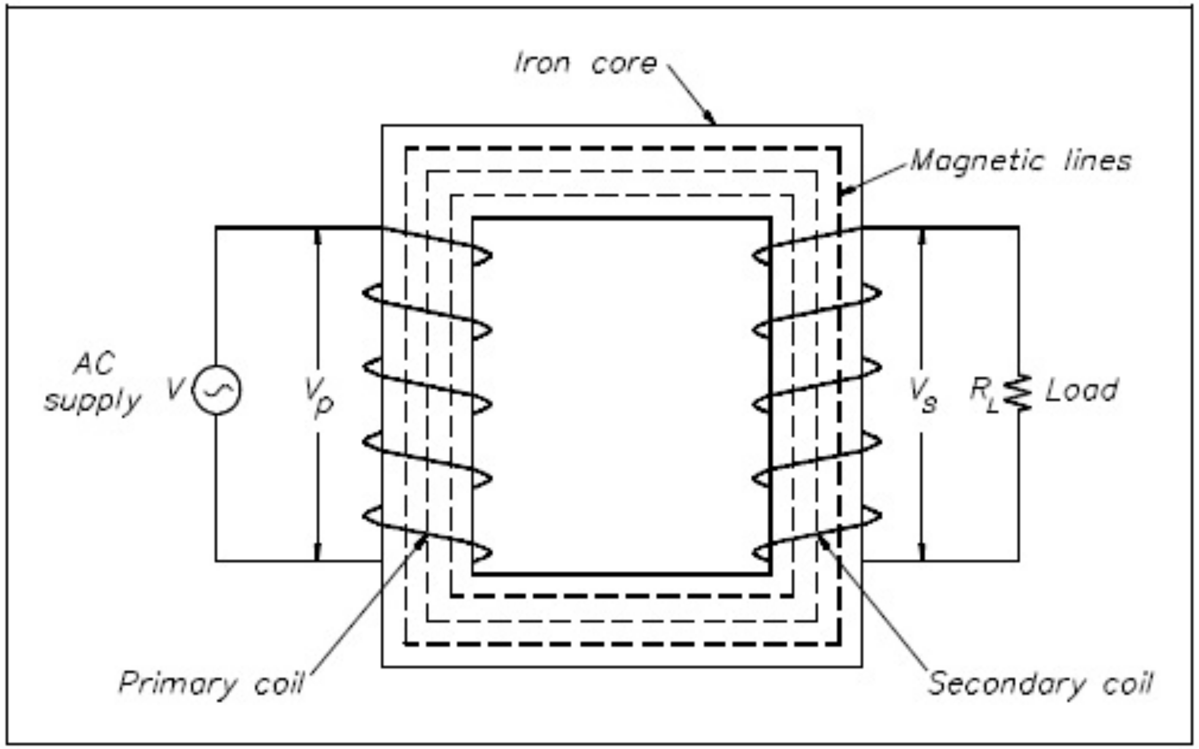


Figure 1 Core-Type Transformer

When alternating voltage is applied to the primary winding, an alternating current will flow that will magnetize the magnetic core, first in one direction and then in the other direction. This alternating flux flowing around the entire length of the magnetic circuit induces a voltage in both the primary and secondary windings. Since both windings are linked by the same flux, the voltage induced per turn of the primary and secondary windings must be the same value and same direction. This voltage opposes the voltage applied to the primary winding and is called counter-electromotive force (CEMF).

The transformer mainly consists of a good magnetic core and primary and secondary windings.

The transformer core is generally laminated and is made out of a good magnetic material such as transformer steel or silicon steel. Such a material has high relative permeability and low hysteresis loss. There are two types of transformer cores. They are known as Core Type and Shell type. In core type, L – shaped stampings as shown in Fig. 21 are used. One core type transformer is shown in Fig. 22.

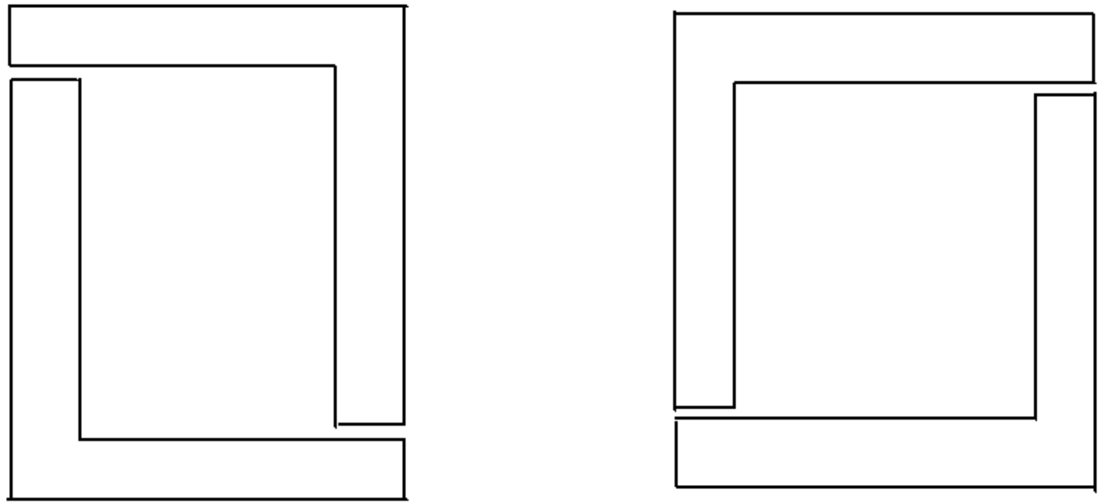
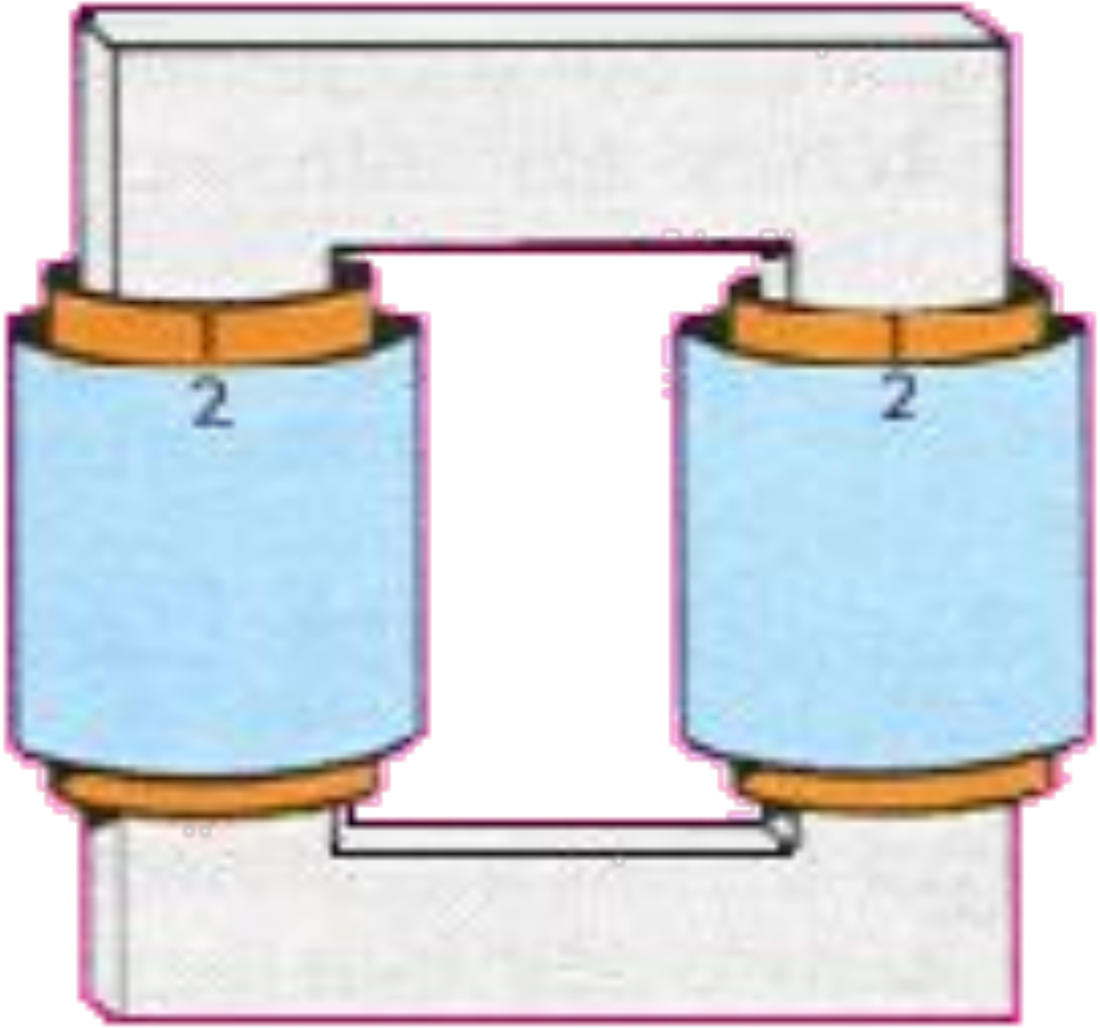


Fig. 21 L – type stampings


2
2

Laminated core of a shell type transformer is shown in Fig. 23. In this E – type and I type laminations are used. Fig. 24 shows a shell type transformer.

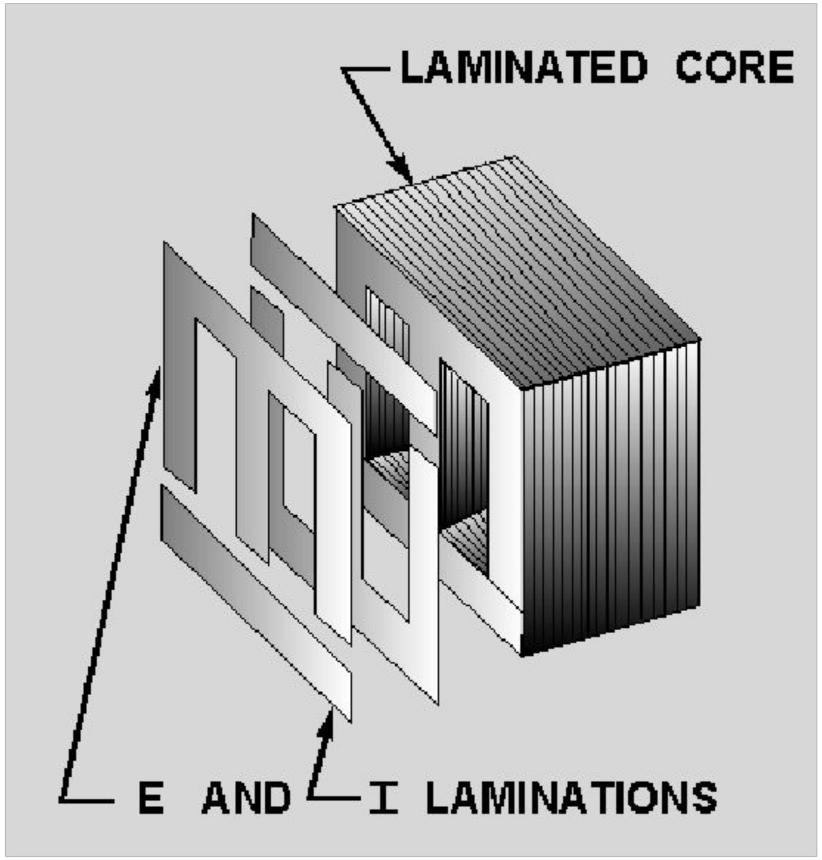
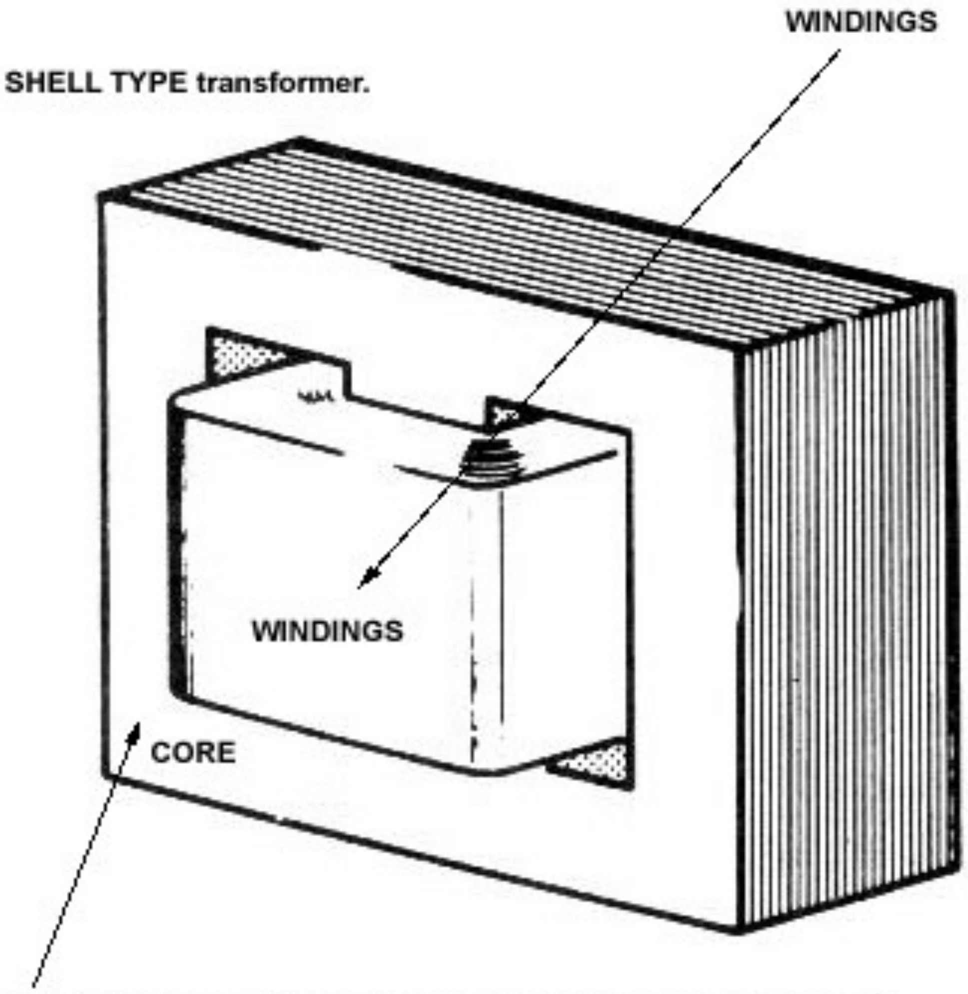


Fig. 23 Laminated core of shell type transformer

SHELL TYPE transformer.



CORE material made up of thin laminate iron sheets, each sheet is coated with an insulating varnish and the entire core is then pressed together.

Application of transformers

The transformers are classified as Step-up transformers and Step-down transformers. When the secondary voltage is more than the primary voltage, transformer is called a step-up transformer. In step-down transformer, the secondary voltage is less than the primary voltage.

Transformers are used in the following applications:

(i) Power transformers located in Power Plants are used to step-up the generated voltage to a high transmission voltage.

(ii) Transformers are used in distribution circuit to step-down voltages to the desired level.

(iii) Almost all electronic circuits use transformers.

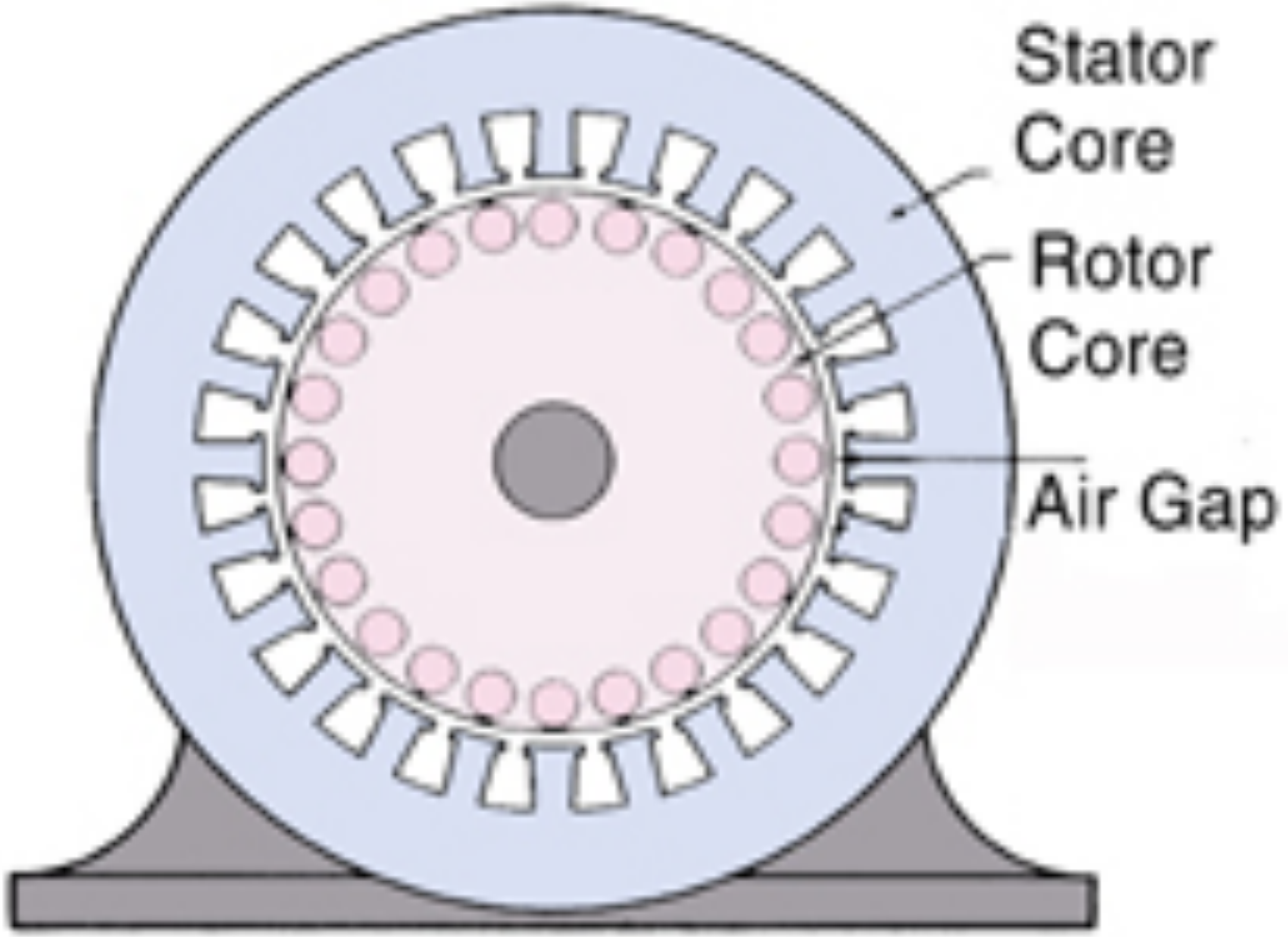
(iv) Potential transformers are used to measure high voltages and current transformers are used to measure high currents.

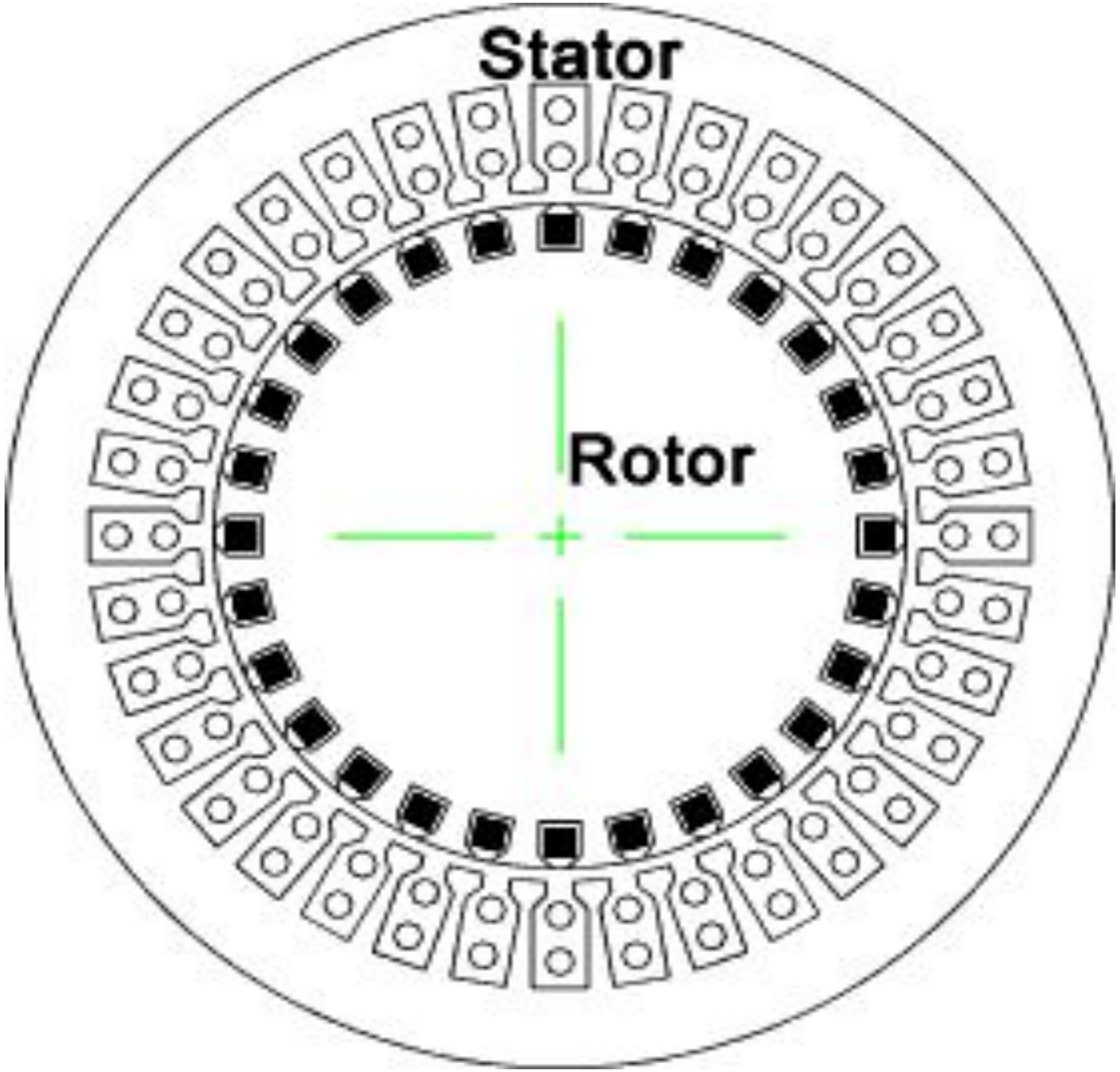
(v) Furnace transformers and welding transformers are some special applications of transformers.

Working principle, construction and applications of 3- phase induction motor

When a three phase balanced voltage is applied to a three phase balanced winding, a rotating magnetic field is produced. This field has a constant magnitude and rotates in space with a constant speed. If a stationary conductor is placed in this field, an emf will be induced in it. By creating a closed path for the current to flow, an electromagnetic torque can be exerted on the conductor. Thus the conductor is put in rotation.

The important parts of a three phase induction motor are schematically represented in Fig. 25. Broadly classified, they are stator and rotor which are described below.


Stator
Core
Rotor
Core
Air Gap


Stator
Rotor

Stator is the stationary part of the motor. The stator core consist of high grade, low loss electrical sheet-steel stampings assembled in the frame. Slots are provided on the inner periphery of the stator to accommodate the stator conductors. Required numbers of stator conductors are housed in the slots. These conductors are arranged to form a balanced three phase winding. The stator winding may be connected in star or delta.

Rotor is the rotating part of the induction motor. The air gap between the stator and rotor is as minimum as possible. The rotor is also in the form of slotted cylindrical structure. There are to types of rotors, namely Squirrel Cage rotor and Slip-ring or Wound rotor.

Fig. 26 shows the construction of a squirrel cage rotor.

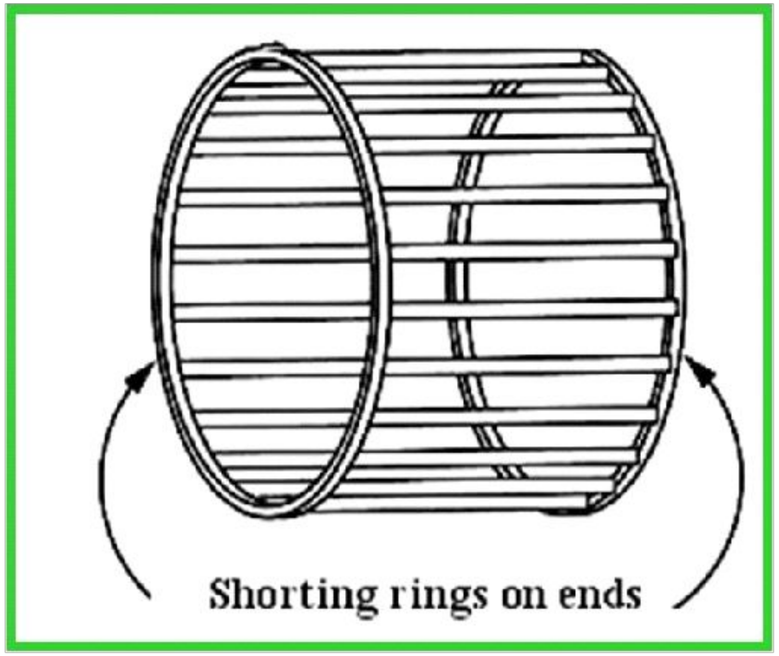


Fig. 26 Squirrel cage rotor of three phase induction

In this type, each rotor slot accommodates a rod or bar made of good conducting material. These rotor bars are short circuited at both ends by means of end rings made of the same metal as that of rotor conductors. Thus the rotor circuit forms a closed path for any current to flow through.

Fig. 27 shows the rotor of slip-ring induction motor. In this case conductors are housed in rotor slots. These conductors are connected to form a star connected balanced three phase winding. The rotor is wound to give same number of poles as the stator. The three ends of the rotor winding are connected to the brushes riding over the slip-rings. Slip-rings are short circuited at the time of starting. External resistances can be connected to control the speed of the motor. Although the wound rotor motor costs more than a squirrel cage motor, it has the features of controlling the torque and the speed.

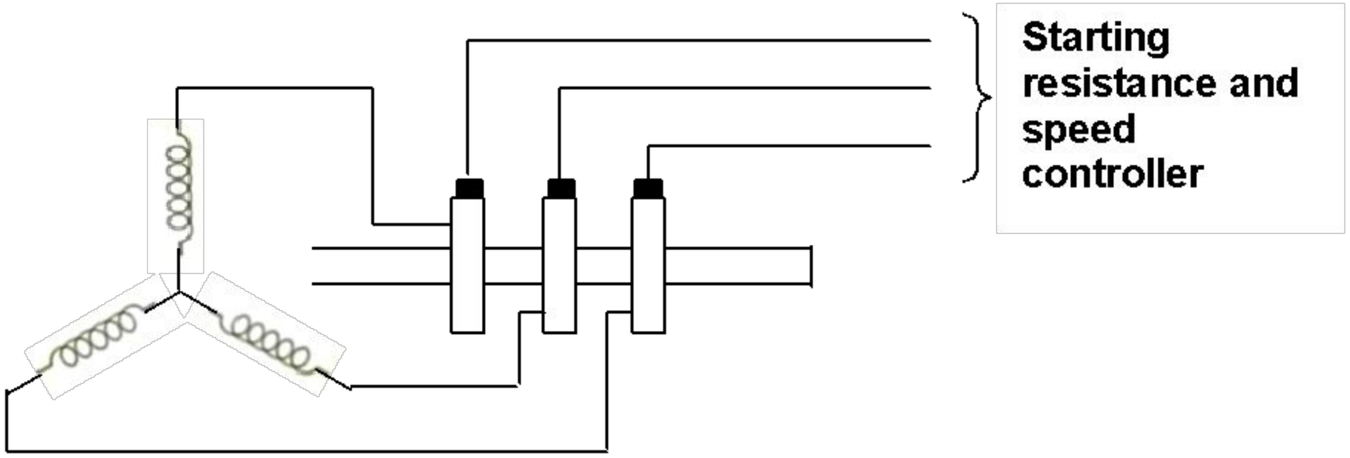


Fig. 27 Rotor of slip-ring induction motor

A three phase balanced voltage is applied across the three phase balanced stator winding. A rotating magnetic field is produced. This magnetic field completes its path through the stator, the air gap and the rotor. The rotor conductors, which are stationary at the time of starting, are linked by time varying start magnetic field. Therefore emf is induced in the rotor conductors. Since the rotor circuit forms a closed path, rotor current is circulated. Thus the current carrying conductors are placed in a rotating magnetic field. Hence an electromotive force is exerted on the rotor conductors and the rotor starts rotating.

According to Lenz's law, the nature of the induced current is to oppose the cause producing it. Here the cause is the relative motion between the rotor conductors and the rotating magnetic field. Hence the rotor rotates in the same direction as hat of the rotating magnetic field.

In practice, the rotor speed never equals to the speed of the rotating magnetic field. The difference in the two speeds is called the slip. The current drawn by the stator gets adjusted according to the load on the motor.

Three phase induction motors are used in industry for very many purposes. They are used in lathes, drilling machines, agricultural and industrial pumps, compressors and industrial drives. They are also used in lifts, crane and conveyors.

Working principle, construction and applications of single phase induction motor

Single phase induction motors are used in variety of applications at home, factory, office and business establishments. Single phase induction motor is not self starting. Additional arrangement has to be made to make it self-starting. This could be achieved by using two windings, main winding and starting winding, with large phase difference between the currents carried by them. This kind of split-phase motor produces a revolving flux and hence makes the motor self starting. Depending on the circuit element connected in series with the starting winding, the split-phase motors are classified into

(i) Resistance-start induction motor

(ii) Capacitance-start induction motor

(iii) Capacitance-start-and-run motor

Resistance-start induction motor

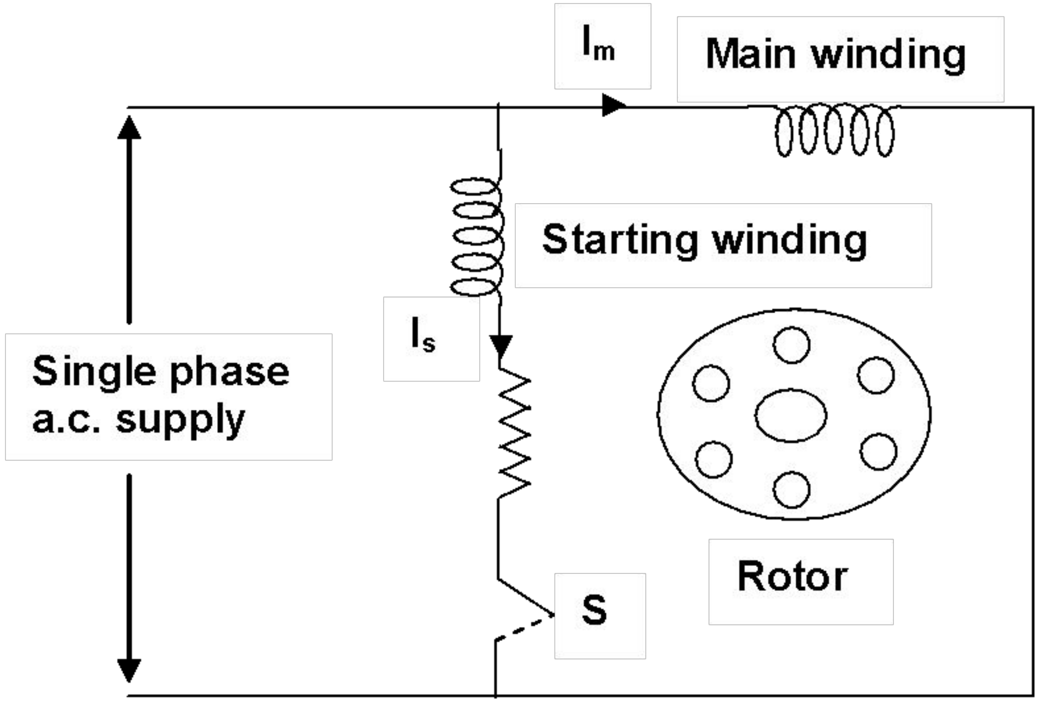


Fig. 28 Resistance start induction motor

Resistance start induction motor is shown in Fig. 28. The starting winding has a high resistance connected in series with it. The current flowing through it is given by I_s. The centrifugal switch S disconnects the starting winding when the motor speed reaches 80% of full load speed. The main winding has low resistance and high reactance and it carries current I_m. Current in starting winding is I_s. The torque developed by the motor is proportional to $\sin\alpha$ where α is the angle between I_m and I_s as shown in Fig. 29. For obtaining high torque, α should be as high as possible. Here θ is the power factor angle.

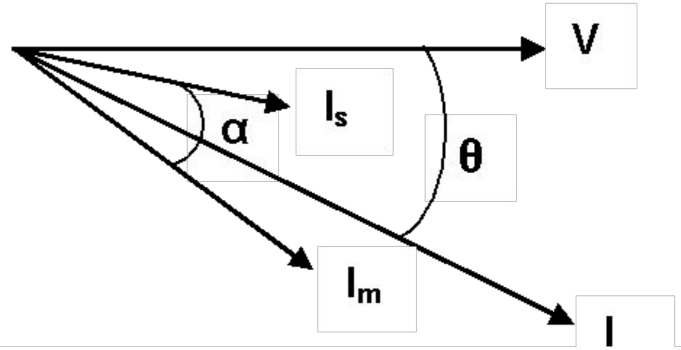


Fig. 29 Phasor diagram of Resistance start induction motor

Capacitor-start induction motor

In the capacitor-start induction motor, a capacitor is connected in series with the starting winding as shown in Fig. 30.

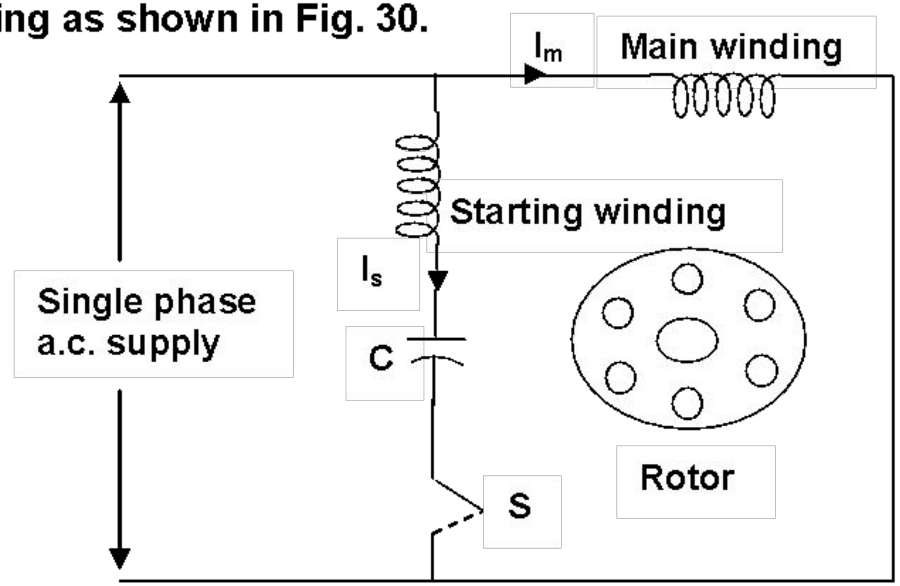


Fig. 30 Capacitor start-induction motor

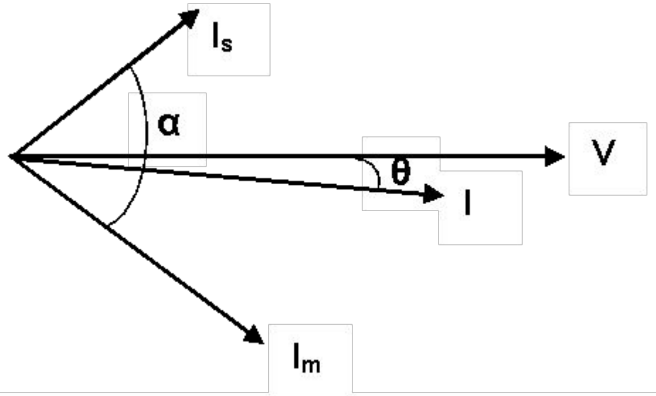


Fig. 31 Phasor diagram of capacitor-start induction motor

The phasor diagram of capacitor-start induction motor is shown in Fig. 31.

The following are the advantages of capacitor-start induction motor:

(i) Increase in starting torque
(ii) Better starting power factor

Capacitor-start-and- run motor

Capacitor-start-and-run motor is similar to that of the capacitor-start motor except that the capacitor in the starting winding circuit remains there through out the operation of the motor. The advantages of this type of motor are:

(i) Low noise in the motor while running
(ii) Higher power factor
(iii) Higher efficiency
(iv) Improved over-load capacity