$= 1 - xy + xy + y$

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Assignment:

(1) $u = \dfrac{x+y}{1-xy}$; $v = \tan^{-1} x + \tan^{-1} y$

$$J\left(\frac{u,v}{x,y}\right) = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \dfrac{1+y^2}{(1-xy)^2} & \dfrac{1+x^2}{(1-xy)^2} \\ \dfrac{1}{1+x^2} & \dfrac{1}{1+y^2} \end{vmatrix}$$

$$= \left[\frac{(1+y^2)}{(1-xy)^2} \times \frac{1}{(1+y^2)}\right] - \left[\frac{(1+x^2)}{(1-xy)^2} \times \frac{1}{(1+x^2)}\right]$$

$$= \frac{1}{(1-xy)^2} - \frac{1}{(1-xy)^2} = 0$$

So the function are functionally dependent.

$v = \tan^{-1}x + \tan^{-1}y$

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$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}u$

Thus ~~from~~ $\underline{\underline{u = \tan v}}$. (relation)

② $f(x,y) = x^4 - y^4 - 2x^2 + 2y^2$

$\therefore f_x = 4x^3 - 4x$

$f_y = -4y^3 + 4y$

Now $f_x = 0 \Rightarrow 4x^3 - 4x = 0$

$\Rightarrow 4x(x^2-1) = 0$

$\Rightarrow x = 0, \pm 1$

& $f_y = 0 \Rightarrow -4y^3 + 4y = 0.$

$\Rightarrow 4y^3 - 4y = 0$

$\Rightarrow 4y(y^2-1) = 0$

$\Rightarrow y = 0, \pm 1$

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So pts are :- $(0,0); (0,-1); (0,1); (1,0); (1,-1); (1,1);$
$(-1,0); (-1,-1); (-1,1)$

Now $f_{xx} = 12x^2 - 4 \rightarrow A$

$f_{xy} = 0 \rightarrow B$

$f_{yy} = -12y^2 + 4 \rightarrow C.$

	(0,0)	(0,-1)	(0,1)	(1,0)	(1,-1)	(1,1)	(-1,0)	(-1,-1)	(-1,1)
A	-4	-4	-4	8	8	8	8	8	8
B	0	0	0	0	0	0	0	0	0
C	0	-8	-8	4	-8	-8	4	-8	-8
$AC-B^2$	0	32	32	32	-64	-64	32	-64	-64

$AC-B^2 > 0$ & $A > 0$

The minimum pts are :- $(1,0)$ & $(-1,0)$
" maximum " " $(0,1)$ & $(0,-1)$
$AC-B^2 > 0$ & $A < 0$.

Saddle pts :- ~~(0,0)~~, $(1,-1), (1,1), (-1,-1), (-1,1)$

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③ e^{xy} at (1, 1) upto 2nd degree

Ans: $f(x,y) = \cancel{\theta}\, e^{xy}$ Here $a = b = 0$

at (1, 1)

$f_x = \cancel{ye^{xy}}\ ye^{xy}$ | e

$f_{xx} = \cancel{\theta}\ y^2 e^{xy}$ | e

$f_y = xe^{xy}$ | e

$f_{yy} = x^2 e^{xy}$ | e

$f_{xy} = ye^{xy}$ | e

$$\therefore f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x(a,b) + k f_y(a,b)\right]$$

$$+ \frac{1}{2!}\left[h^2 f_{xx}(a,b) + 2hk f_{xy}(a,b) + \cancel{\theta}\, k^2 f_{yy}(a,b)\right]$$

⇒ Substituting the value:-

$$e^{xy} = e + \left[(x-1)e + \cancel{k}\,(y-1)e\right]$$

$$+ \frac{1}{2}\left[(x-1)^2 \cdot \cancel{\theta}\, e + 2(x-1)(y-1) \cdot e + (y-1)^2 \cdot e\right] + \ldots$$

i.e. $e^{xy} = e + [e(x+y+2)] + \frac{1}{2}\left[(x-1)^2 e + 2e(x-1)(y-1) + (y-1)^2 e\right] + \ldots$

④ $(D^2+6D+8)y = e^{-2x} + \cos^2 x$

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Ans:: $\phi(D) = 0.$

The aux eq :- $m^2+6m+8 = 0$

$\Rightarrow m^2+4m+2m+8=0$

$\Rightarrow m(m+4)+2(m+4)=0$

$\Rightarrow (m+4)(m+2)=0$

$\therefore m = -2, -4$ root are real & distinct.

so $CF = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$= C_1 e^{-2x} + C_2 e^{-4x}$

PI 1 : $\frac{1}{\phi(D)} \cdot F(x)$

$= \frac{1}{D^2+6D+8} \cdot e^{-2x}$

Now replacing D by a :- where $a = -2$,

$\frac{1}{(-2)^2+6(-2)+8} \cdot e^{-2x} = \frac{1}{0}.$

So diff. $\phi(D)$.

$\frac{x}{\phi'(D)} \cdot e^{-2x}$

$= \frac{x}{2D+6} e^{-2x}$

$= \frac{x}{2(-2)+6} \cdot e^{-2x} = \frac{x}{2} \cdot e^{-2x}$

$PI_2 : \dfrac{1}{\phi(D)} \cdot F(x)$

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$$= \frac{1}{(D^2+6D+8)} \cdot \cos^2 x.$$

$$= \frac{1}{(D^2+6D+8)} \left(\frac{1+\cos 2x}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^2+6D+8} + \frac{\cos 2x}{D^2+6D+8}\right]$$

$$= \frac{1}{2}\left[\frac{e^{0x}}{D^2+6D+8} + \frac{\cos 2x}{D^2+6D+8}\right]$$

Replacing D by a in $\dfrac{e^{0x}}{D^2+6D+8}$ where $a=0$

& " D^2 by $-a^2$ in $\dfrac{\cos 2x}{D^2+6D+8}$. where $a=2$

$$= \frac{1}{2}\left[\frac{1}{0+(6\times 0)+8} \cdot e^{0x} + \frac{\cos 2x}{-4+(6\times D)+8}\right]$$

$$= \frac{1}{2}\left[\frac{e^{0x}}{8} + \frac{\cos 2x}{6D+4}\right]$$

$$= \frac{1}{2}\left[\frac{1}{8} + \left(\frac{6D-4}{36D^2-16}\right)\cos 2x\right]$$

$$= \frac{1}{2}\left[\frac{1}{8} + \left(\frac{6D-4}{-160}\right)\cos 2x\right]$$

$$= \frac{1}{2}\left[\frac{1}{8} + \frac{6D(\cos 2x) - 4\cos 2x}{-160}\right]$$

$$= \frac{1}{2}\left[\frac{1}{8} + \frac{12 \sin 2x - 4\cos 2x}{-160}\right]$$

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$$= \frac{1}{2}\left[\frac{1}{8} + \frac{3\sin 2x + \cos 2x}{40}\right]$$

$$PI_2 = \frac{1}{16} + \frac{3\sin 2x + \cos 2x}{80}$$

So $y = CF + PI_1 + PI_2$

$$= c_1 e^{-2x} + c_2 e^{-4x} + \frac{x}{2} e^{-2x} + \frac{3\sin 2x + \cos 2x}{80} + \frac{1}{16}$$

(Ans)

⑤ Solve $(D^2 - 3D + 2)y = \cos 3x \cos 2x$

$$= \frac{\cos 5x + \cos x}{2}$$

~~Ans:-~~

Ans:- $\phi(D) = 0$

$\Rightarrow D^2 - 3D + 2 = 0$

Aux. eq :- $m^2 - 3m + 2 = 0$

$\Rightarrow m^2 - 2m - m + 2 = 0$

$\Rightarrow m(m-2) - 1(m-2) = 0$

$\Rightarrow (m-1)(m-2) = 0$

$\Rightarrow m = 1, 2$ roots are distinct.

So $CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$= c_1 e^{x} + c_2 e^{2x}$$

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$$PI_1 :- \frac{1}{\phi(D)} . F(x)$$

$$= \frac{1}{2} \frac{1}{D^2 - 3D + 2} . \cos 5x$$

Replacing $D^2 \rightarrow -a^2$ where $a = 5$.

$$= \frac{1}{2} . \frac{1}{-25 - 3D + 2} . \cos 5x$$

$$= \frac{1}{2} . \frac{1}{-3D - 23} . \cos 5x$$

$$= -\frac{1}{2} . \left(\frac{3D - 23}{9D^2 - 529} \right) \cos 5x$$

$$= -\frac{1}{2} . \left(\frac{3D - 23}{9.(-25) - 529} \right) . \cos 5x$$

$$= -\frac{1}{2} . \left[\left(\frac{3D - 23}{-225 - 529} \right) . \cos 5x \right]$$

$$= -\frac{1}{2} \left[\frac{3D(\cos 5x) - 23(\cos 5x)}{-754} \right]$$

$$= \frac{1}{1508} \left[3(-\sin 5x) . 5 - 23 \cos 5x \right]$$

$$PI_1 = -\frac{1}{1508} . \left[15 \sin 5x + 23 \cos 5x \right]$$

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$PI_2 : \frac{1}{\phi(D)} \cdot F(x)$

$= \frac{1}{2} \frac{1}{D^2 - 3D + 2} \cdot \cos x$

Replacing D^2 by $-a^2$ where $a = 1$

$= \frac{1}{2} \cdot \frac{1}{-1 - 3D + 2} \cdot \cos x$

$= \frac{1}{2} \left(\frac{1}{-3D + 1} \right) \cdot \cos x$

$= -\frac{1}{2} \cdot \left(\frac{3D + 1}{9D^2 - 1} \right) \cdot \cos x$ [multiplying numerator & denominator by $3D+1$]

$= -\frac{1}{2} \cdot \left(\frac{3D + 1}{9(-1) - 1} \right) \cos x$

$= -\frac{1}{2} \cdot \left[\frac{3D(\cos x) + \cos x}{-10} \right]$

$= \frac{1}{20} \left[3(-\sin x) + \cos x \right]$

$PI_2 = \frac{1}{20} (\cos x - 3 \sin x)$

So $y = CF + PI_1 + PI_2$

$= c_1 e^x + c_2 e^{2x} - \frac{1}{1508} [15 \sin 5x + 23 \cos 5x]$

$+ \frac{1}{20} [\cos x - 3 \sin x]$ (Ans)