Now applying cross multiplication rule,

$$\begin{matrix} 1 & & 1 & & 0 & & 1 \\ & \times & & \times & & \times & \\ -1 & & 1 & & 0 & & -1 \end{matrix}$$

$$\frac{x_1}{1+1} = \frac{x_2}{0-0} = \frac{x_3}{0}$$

$$\therefore \frac{x_1}{2} = \frac{x_2}{0} = \frac{x_3}{0}$$

ie $X_3 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

Now $\bar{X}_1 = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$; $\bar{X}_2 = \begin{bmatrix} 0 \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$; $\bar{X}_3 = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$

So $N = \begin{bmatrix} 0 & 0 & \sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$; $N^T = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \sqrt{2} & 0 & 0 \end{bmatrix}$

Now $N^TAN = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ \sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} N$

$$= \begin{bmatrix} 0+0+0 & 0+0+\frac{1}{\sqrt{2}} & 0+\frac{1}{\sqrt{2}}+0 \\ 0+0+0 & 0+0+\frac{1}{\sqrt{2}} & 0-\frac{1}{\sqrt{2}}+0 \\ \sqrt{2}+0+0 & 0+0+0 & 0+0+0 \end{bmatrix} N$$

$$= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \sqrt{2} & 0 & 0 \end{bmatrix} N$$

$$= \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ \sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \sqrt{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + \frac{1}{2} + \frac{1}{2} & 0 - \frac{1}{2} + \frac{1}{2} & 0 + 0 + 0 \\ 0 + \frac{1}{2} - \frac{1}{2} & 0 - \frac{1}{2} - \frac{1}{2} & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 2 + 0 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now $Q = Y^T [N^T A N] Y$ where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$= \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= y_1^2 - y_2^2 + y_3^2 \quad \text{(Ans)}$$

$\therefore$ rank $= 3$

index(p) $= 2$

Signature $= 2p - r = (2 \times 2) - 3 = 1$.

nature is indefinite.