Semi - Conductor Physics Homework (Assignment - 1) 01/11/20

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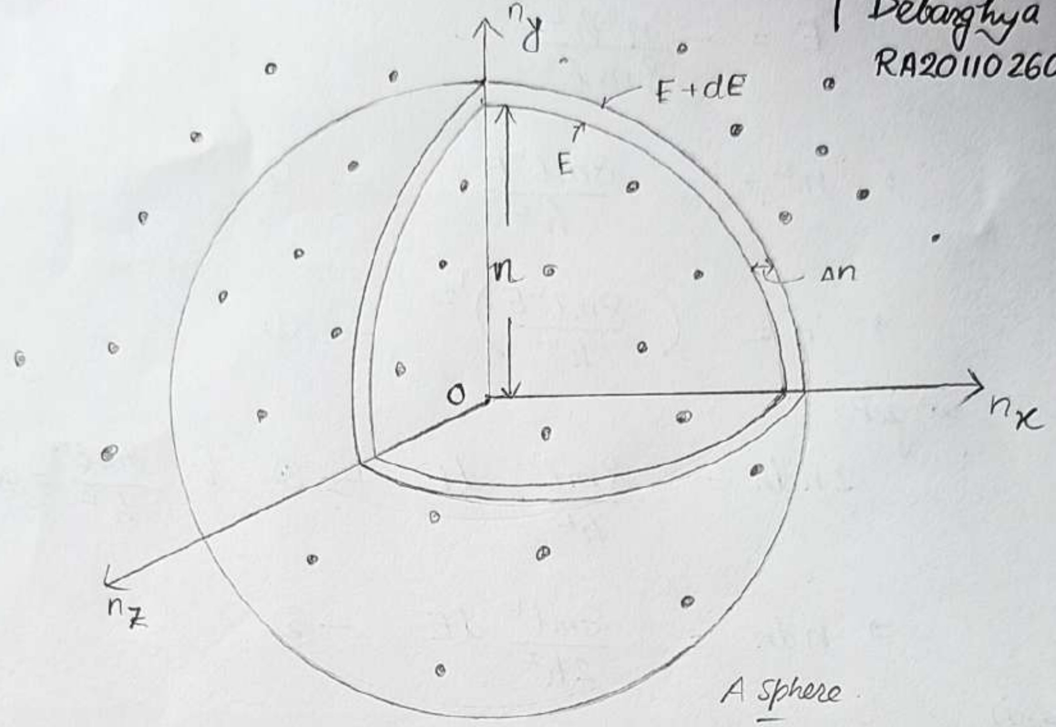
Ans 1:



A Sphere

Let us construct a sphere of radius 'n' in space. The sphere is further divided into many shells and each shell represents a particular combination of quantum number (n_x, n_y and n_z) and therefore represents a particular energy value. Now considering two energy values E and E+dE. The number of energy states between E and (E+dE) can be found by finding the number of energy states between the shells of radius n and n+Δn, from origin.

Considering only one octant of sphere ie. 1/8 th of the sphere volume, the no. of available energy states within the sphere of radius n is equal to:-

$$n = \frac{1}{8}\left[\frac{4}{3}\pi n^3\right]$$

Similarly, no. of available energy states within the sphere of radius n+dn is equal to :-

$$n+dn = \frac{1}{8}\left[\frac{4}{3}\pi(n+dn)^3\right]$$

∴ number of energy states shells of radius n & n+dn ie. between energy levels

E and E+dE $= \frac{1}{8}\left[\frac{4}{3}\pi\left\{(n+dn)^3 - n^3\right\}\right]$

ie. $Z(E)\,dE$

$$= \frac{1}{8}\left[\frac{4}{3}\pi \cdot 3n^2 dn\right] = \frac{\pi}{2} n^2 dn \quad \text{—} (1)$$

We know, the energy of electron in cubical metal piece of sides 'l'

$$E = -\frac{n^2h^2}{8ml^2}$$

$$\Rightarrow n^2 = -\frac{8ml^2E}{h^2} \quad — ②$$

$$\Rightarrow n = \left(\frac{8ml^2E}{h^2}\right)^{1/2} \quad — ③$$

Now diff. eq. ②, we get;

$$2n\,dn = \frac{8ml^2}{h^2}.\,dE \quad — ④ \quad \left[\because \frac{8ml^2}{h^2} = \text{constant}\right]$$

$$\Rightarrow n\,dn = \frac{8ml^2}{2h^2}\,dE \quad — ⑤$$

Now from eq. (i),

$$Z(E)dE = \frac{\pi}{2}n^2dn$$

$$= \frac{\pi}{2}n.\,ndn$$

$$= \pi/2.\,n.\,\frac{8ml^2}{2h^2}dE. \text{ [from eq. ⑤]}$$

$$= \frac{\pi}{2}.\left(\frac{8ml^2E}{h^2}\right)^{1/2}.\,\frac{8ml^2}{2h^2}.\,dE.$$

$$= \frac{\pi}{4}.\left(\frac{8ml^2}{h^2}\right)^{3/2}.\,E^{1/2}dE$$

$$= \frac{\pi}{4}.\left(\frac{8m}{h^2}\right)^{3/2}.\,l^3.\,E^{1/2}dE.$$

[where l^3 = volume of sphere]

Now when $l^3 = 1$, $Z(E)dE = \frac{\pi}{4}.\left(\frac{8m}{h^2}\right)^{3/2}.\,E^{1/2}dE.$ — ⑥

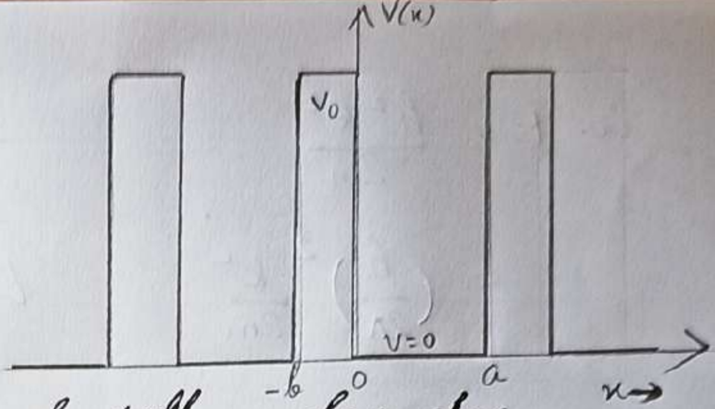
Each energy level provides two electron states.

$$\text{So} \quad Z(E)dE = \frac{\pi}{4}.\,2.\left(\frac{8m}{h^2}\right)^{3/2}E^{1/2}dE$$

$$\Rightarrow \boxed{Z(E)dE = \frac{\pi}{2}.\left(\frac{8m}{h^2}\right)^{3/2}E^{1/2}dE} \quad \text{(Ans)}$$

[Where $Z(E)dE$ = density of states, m = mass of e^-, h = Planck's constant,
E = energy of an electron]

Ans ②: With the Kronig Penny model, it is assumed that the potential energy of an e^- when it moves in one dimensional crystal lattice



is represented in forms of rectangular wells as shown above.
In the graph, region $0<x<a$ the potential energy is assumed to be Zero and in region, $-b<x<0$, the potential energy is V_0. The 1D Schroedinger wave equation for the two regions are,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E-0]\psi = 0 \quad \text{for } 0<x<a \quad -①$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \alpha\psi = 0 \quad \text{where } \alpha = \frac{2m}{\hbar^2}.E \quad -②$$

and;

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E-V_0]\psi = 0 \quad \text{for } -b<x<0 \quad -③$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \beta\psi = 0 \quad \text{where } \beta = \frac{2m(E-V_0)}{\hbar^2} \quad -④$$

For both the regions, the appropriate solution as suggested by Bloch is;

$$\psi = e^{ikx}U_K(x) \quad -⑤$$

Now diff. eq. ⑤ and substituting in eq. ② & ④ and solving, we get;

$$\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka \quad -⑥$$

$$\left[\text{where } P = \frac{mV_0ba}{\hbar^2} \ \& \ \alpha = \frac{\sqrt{2mE}}{\hbar}\right]$$
(Scattering power)

For $P \to 0$, $\cos\alpha a = \cos ka \Rightarrow \alpha = k$

ie. $\alpha^2 = k^2 \Rightarrow \frac{2mE}{\hbar^2} = k^2 \Rightarrow E = \frac{k^2\hbar^2}{2m}$

i.e. $E = \frac{\hbar^2 k^2}{2m}$

$= \left(\frac{h}{2\pi}\right)^2 \frac{k^2}{2m}$ $\quad [\because \hbar = \frac{h}{2\pi}]$

$= \frac{h^2 k^2}{8\pi m}$

Thus energy of electron moving in constant and periodic potential is;

$$\boxed{E = \frac{h^2 k^2}{8\pi m}}$$

(Ans)

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x