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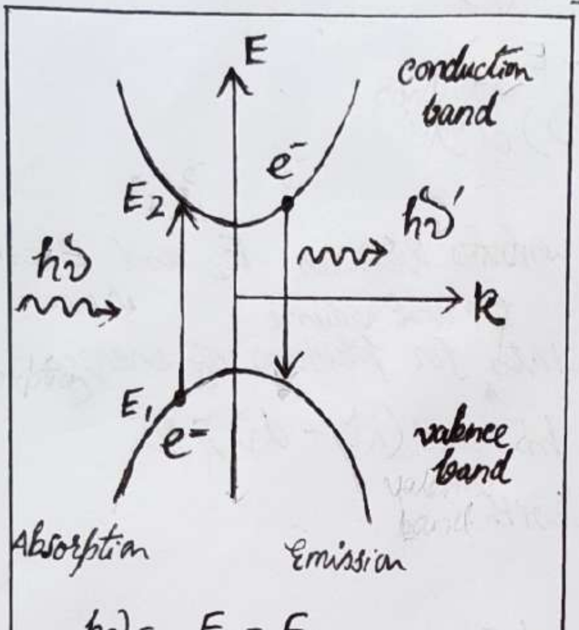
Q2> Derive an expression for

i> Optical joint density of states and.

ii> Density of states of photon.

Ans:- i>

Optical Joint Density of States



$h\nu = E_2 - E_1$

How many states are possible for photon interaction of energy $h\nu$ in valence and conduction band is given by optical joint density of states. To determine the density of state $\rho(\nu)$ with a photon of energy $h\nu$ interacts under a condition of energy and momentum conservation in a direct band gap semiconductor.

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} \quad \text{—(i)}$$

$$E_1 = E_v - \frac{\hbar^2 k^2}{2m_v} \quad \text{—(ii)}$$

$$\therefore h\nu = E_2 - E_1 = E_g + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_c} + \frac{1}{m_v}\right) \quad [\text{where } E_g = E_c - E_v]$$

$$= E_g + \frac{\hbar^2 k^2}{2m_r}$$

$$\therefore \boxed{k^2 = \frac{2m_r}{\hbar^2}(h\nu - E_g)}$$

(PTO)

Substituting value of k^2 in (i) & (ii)

$$E_2 = E_C + \frac{\hbar^2 k^2}{2m_C}$$

$$= E_C + \frac{\hbar^2}{2m_C} \cdot \frac{2m_r}{\hbar^2} \cdot (h\nu - E_g)$$

$$\Rightarrow E_2 = E_C + \frac{m_r}{m_e}(h\nu - E_g) \quad \text{—(iii)}$$

Similarly, $E_1 = E_V - (m_r/m_v)(h\nu - E_g)$

Now we know, $\varphi_c(E_2)dE_2 = \varphi(\nu)d\nu$

[where $\varphi_c(E_2)dE_2$ = no. of states between E_2 and $E_2 dE_2$ per unit volume

$\varphi(\nu)d\nu$ = no. of states for photons of energy between $h\nu$ and $(h\nu + d\nu)$ to interact with.]

$$\therefore \varphi(\nu) = \varphi_c(E_2) \cdot \frac{dE_2}{d\nu} \quad \text{— (iv)}$$

from (iii); $E_2 - E_C = (m_r/m_c)(h\nu - E_g)$

$$\therefore (E_2 - E_c)^{1/2} = (m_r/m_c)^{1/2}(h\nu - E_g)^{1/2} \quad \text{—(v)}$$

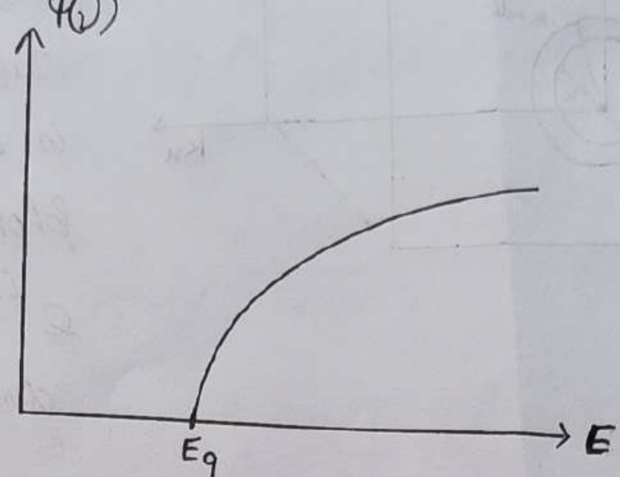
from (iv); $\varphi(\nu) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} \cdot (E_2 - E_c)^{1/2} \cdot h\left(\frac{m_r}{m_c}\right)$

$$= \frac{1}{\pi\hbar^2} \cdot (2m_c)^{3/2} \cdot \left(\frac{m_r}{m_c}\right)^{1/2} (h\nu - E_g)^{1/2} \left(\frac{m_r}{m_c}\right)$$

$$\boxed{\varphi(\nu) = \frac{1}{\pi\hbar^2} \cdot (2m_r)^{3/2} \cdot (h\nu - E_g)^{1/2}} \quad [\text{for } h\nu \geq E_g]$$

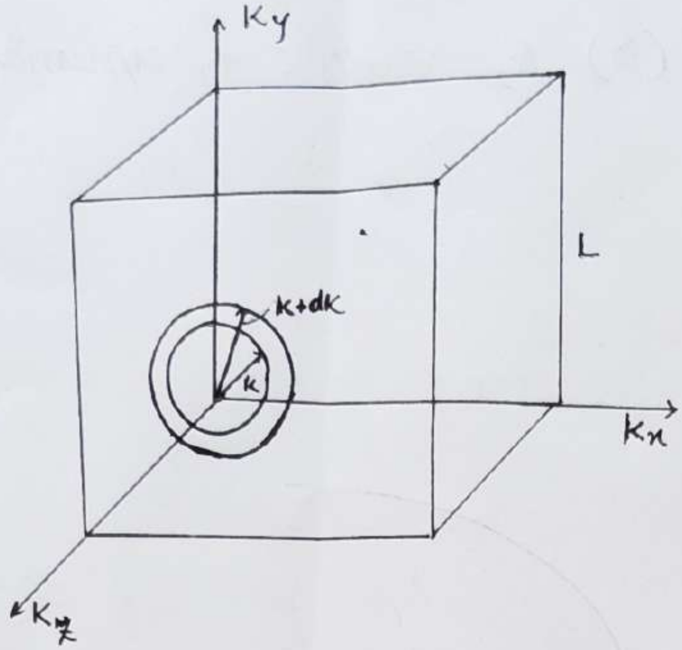
There is an one-to-one correspondence between E_2 and $\varphi(\nu)$ in the equation. PTO

The density of states which a photon of energy $h\nu$ interact increases with $h\nu \geq E_g$ in accordance with a square root law. Together with $\varphi(\nu)$ E_2 results in an expression for $\varphi(\nu)$ identical.



$\varphi(\nu)$ vs E graph.

Q2) ii) Density of States for Photons :-



To define the density of states of p for photons, we assume that photon is enclosed in a large cube of side L, such that the volume is L^3. The wave function of photon is a plane wave $e^{i\vec{k}\vec{r}}$ with periodic L in direction x, y & z.

Because the wave function has to be zero at boundaries, we have.

Quantisation of wave number

$$LK = n2\pi$$

$$K_x = l\frac{2\pi}{L} \quad ; \quad K_y = l\frac{2\pi}{m} \quad ; \quad K_z = l\frac{2\pi}{n}$$

The volume of state in K space is $\left(\frac{2\pi}{L}\right)^3$

Now $\cancel{dL}^3 \; \dfrac{d^3K}{\left(\frac{2\pi}{L}\right)^3} = \dfrac{K^2 dK\, d\Omega}{\left(\frac{2\pi}{L}\right)^3}$ [Where $d\Omega$ is differential solid angle]

$$\therefore \quad N(E_{21}) = \frac{2}{V}\sum_K \delta(E_2 - E_1 - E_k)$$

$$= 2\int \frac{K^2 dK d\Omega}{(2\pi)^3}\,\delta(E_2 - E_1 - E_K)$$

PTO

$$\therefore E_K = \hbar \omega_K = \frac{\hbar K C}{n_r}$$

where, C/n_r is speed of light in medium with r.i of n_r.

$$\therefore K = \frac{n_r E_K}{\hbar C}$$

$$dK = \frac{n_r 2\pi}{hC} \cdot dE_K$$

$$N(E_{21}) = 2\int \frac{K^2 dK d\Omega}{(2\pi)^3} \delta(E_{21} - E_K)$$

$$= 2\int \frac{K^2}{(2\pi)^3} \cdot \frac{n_r 2\pi}{hC} dE_K (4\pi) \delta(E_{21} - E_K)$$

$$= 2\int \frac{1}{(2\pi)^3} \left(\frac{E_K n_r 2\pi}{hC}\right)^2 \frac{n_r 2\pi}{hC} dE_K (4\pi) \delta(E_{21} - E_K)$$

$$= \frac{2 \times 4\pi \times (2\pi)^3 (n_r)^3}{(2\pi)^3 (hC)^3} \int (E_K)^3 dE_K \, \delta(E_{21} - E_K)$$

$$\therefore \boxed{N(E_{21}) = \frac{8\pi (n_r)^3}{(hC)^3} \cdot E_{21}^2} \quad \left(\hbar = \frac{h}{2\pi} \;\therefore h = \hbar 2\pi\right)$$

This is the number of states with photon energy E_{21} per unit volume per energy interval.

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