Theoretical Framework and Physical Model of the AeroStruct3D Simulation Tool

December 11, 2025

Contents

1 Introduction

This document outlines the theoretical underpinnings of the `AeroStruct3D` MATLAB tool. The software is designed to perform a coupled static aeroelastic analysis of wing-like structures provided as STL files. It integrates aerodynamic force calculations with structural deformation analysis in an iterative framework to find a converged solution. Furthermore, it assesses dynamic instabilities such as flutter and divergence and provides a clear failure analysis based on material properties.

The core philosophy is to provide a robust, dependency-free tool that combines several classic theories from aerodynamics and solid mechanics into a cohesive simulation.

2 Structural Modeling

The structural properties of the 3D model are not assumed but are rigorously calculated from the geometry itself. The model assumes the STL represents a thin-walled, closed-section monocoque structure.

2.1 Geometry Slicing and Pre-processing

The 3D model is discretized into a series of 2D cross-sections (slices) along its spanwise axis (y-axis). For each slice, the following structural properties are calculated.

2.2 Cross-Sectional Property Calculation

A key innovation of the tool is the use of Green's Theorem for calculating sectional properties from the slice's polygonal outline, which is determined via a convex hull.

2.2.1 Enclosed Area and Torsional Constant (Bredt-Batho Theory)

For a closed, thin-walled section, the torsional constant, J, is given by the Bredt-Batho formula:

$$J = \frac{4A_e^2}{\oint \frac{ds}{t}} \tag{1}$$

where A_e is the area enclosed by the midline of the section wall, t is the wall thickness, and the integral is taken over the perimeter. Assuming a constant thickness, this simplifies to:

$$J = \frac{4A_e^2 t}{P} \tag{2}$$

where P is the perimeter of the cross-section. The enclosed area A_e is calculated using the shoelace formula, a direct application of Green's Theorem for a polygon with vertices (x_i, z_i):

$$A_e = \frac{1}{2}\left|\sum_{i=1}^{n-1}(x_i z_{i+1} - x_{i+1} z_i)\right| \tag{3}$$

2.2.2 Area Moment of Inertia

The area moment of inertia, I_{xx}, about the section's centroidal axis is calculated using the Parallel Axis Theorem applied to each small segment of the thin wall. The contribution dI_{xx} of a small segment of length ds and thickness t located at a vertical distance $(z - z_c)$ from the centroid is:

$$dI_{xx} = (z - z_c)^2 \, dA = (z - z_c)^2 (t \, ds) \tag{4}$$

Integrating over the entire perimeter gives the total area moment of inertia:

$$I_{xx} = \oint (z - z_c)^2 t \, ds \tag{5}$$

For a discrete polygon, this is computed as a summation over the line segments.

2.2.3 Mass Properties

The mass per unit length, ρ_L, and the polar mass moment of inertia per unit length, I_α, are calculated for use in the dynamic analysis:

$$\rho_L(y) = \rho_m \oint t \, ds = \rho_m P(y) t \tag{6}$$

$$I_\alpha(y) = \rho_m \oint r^2 t \, ds = \rho_m t \sum_i r_i^2 \Delta s_i \tag{7}$$

where ρ_m is the material density, r is the radial distance from the elastic axis (approximated as the centroid) to a segment on the perimeter.

3 Aerodynamic Modeling

The tool employs a hybrid aerodynamic model that adapts to the flight regime.

3.1 Subsonic Regime: Lifting-Line Theory (LLT)

For subsonic flight ($M < 0.85$), a robust implementation of Prandtl's Lifting-Line Theory is used. The lift per unit span $L'(y)$ is related to the circulation $\Gamma(y)$:

$$L'(y) = \rho U_\infty \Gamma(y) \tag{8}$$

The circulation is represented by a Fourier sine series to automatically satisfy the boundary condition $\Gamma(\pm b/2) = 0$:

$$\Gamma(\theta) = 4bU_\infty \sum_{n=1}^{N} A_n \sin(n\theta) \tag{9}$$

where $y = -\frac{b}{2}\cos(\theta)$. The fundamental equation of LLT relates the local effective angle of attack $\alpha_{eff}(y)$ to the circulation:

$$\alpha_{eff}(y) = \frac{\Gamma(y)}{\pi U_\infty c(y)} + \frac{1}{4\pi U_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma/d\eta}{(y - \eta)} d\eta \tag{10}$$

This is solved as a system of linear equations for the Fourier coefficients A_n at discrete control points along the span. The effective angle of attack includes the aircraft's root AoA, geometric twist, and the crucial aeroelastic twist from the structural solver:

$$\alpha_{eff}(y) = \alpha_{root} + \theta_{geo}(y) + \theta_{ae}(y) \tag{11}$$

3.2 Supersonic Regime: Linearized Panel Method

For supersonic flight ($M > 1.0$), the pressure coefficient C_p on each panel is approximated using Linearized Supersonic Theory. The pressure on a surface is primarily dependent on its inclination to the freestream.

$$C_p = \frac{2\delta}{\sqrt{M^2 - 1}} \tag{12}$$

where δ is the angle between the panel's normal vector and the freestream velocity vector. This provides a fast and effective method for estimating supersonic loads.

4 Iterative Static Aeroelastic Coupling

The core of the simulation is the two-way coupling between aerodynamics and structures. This is an iterative process to find the equilibrium state where the deformed structure is in balance with the aerodynamic loads it generates.

The algorithm proceeds as follows:

1. **Initialization:** Start with an undeformed wing geometry ($\theta_{ae} = 0$, $w = 0$).

2. **Aerodynamic Solution:** Calculate the aerodynamic load distribution $L'(y)$ over the current wing shape using the appropriate aerodynamic model (LLT or Supersonic).

3. **Structural Statics:** Integrate the load distribution to find the shear force $V(y)$ and bending moment $M(y)$ along the span.

$$V(y) = \int_y^{b/2} L'(\eta)\, d\eta \tag{13}$$

$$M(y) = \int_y^{b/2} V(\eta)\, d\eta \tag{14}$$

   Simultaneously, calculate the torsional moment $T(y)$ caused by the aerodynamic force acting at the aerodynamic center, which is offset from the elastic axis.

4. **Deformation Calculation:** Solve the governing equations for beam bending and torsion to find the new structural deformation.

$$\frac{d^2w}{dy^2} = \frac{M(y)}{EI_{xx}(y)} \quad \implies w(y) = \iint \frac{M}{EI_{xx}}\, dy^2 \tag{15}$$

$$\frac{d\theta_{ae}}{dy} = \frac{T(y)}{GJ(y)} \quad \implies \theta_{ae}(y) = \int \frac{T}{GJ}\, dy \tag{16}$$

   where $w(y)$ is the vertical deflection and $\theta_{ae}(y)$ is the aeroelastic twist.

5. **Update and Converge:** The wing geometry is updated with the new twist $\theta_{ae}(y)$. The process repeats from Step 2. The loop terminates when the change in twist between iterations falls below a specified tolerance:

$$\max |\theta_{ae}^{(k)} - \theta_{ae}^{(k-1)}| < \epsilon_{tol} \tag{17}$$

   A relaxation factor is used to aid convergence.

5 Structural Failure and Instability Analysis

After finding the converged aeroelastic solution, the tool assesses the structural integrity.

5.1 Static Failure (Yielding)

The maximum bending stress σ_{max} at any point along the span is calculated using the classical flexure formula:

$$\sigma(y) = \frac{M(y)c(y)}{I_{xx}(y)} \tag{18}$$

where $c(y)$ is the maximum distance from the neutral axis to the outer fiber of the cross-section. Failure is predicted if this stress exceeds the material's yield strength σ_{yield}.

$$\text{Safety Factor (SF)} = \frac{\sigma_{yield}}{\sigma_{max}} \tag{19}$$

A SF less than 1.0 indicates structural failure.

5.2 Dynamic Instability Estimation

The tool estimates the critical speeds for two primary aeroelastic instabilities.

5.2.1 Natural Frequency Estimation (Rayleigh's Method)

The fundamental bending (ω_h) and torsional (ω_α) natural frequencies are estimated using Rayleigh's energy method. The maximum potential (strain) energy is equated to the maximum kinetic energy, assuming the static deformation shape approximates the fundamental mode shape.

$$U_{bend,max} = \frac{1}{2}\int_0^{b/2} EI_{xx}\left(\frac{d^2w}{dy^2}\right)^2 dy \tag{20}$$

$$T_{bend,max} = \frac{1}{2}\omega_h^2 \int_0^{b/2} \rho_L w^2 dy \tag{21}$$

$$\implies \omega_h = \sqrt{\frac{\int EI_{xx}(w'')^2 dy}{\int \rho_L w^2 dy}} \tag{22}$$

A similar energy balance is used for torsion to find ω_α.

5.2.2 Flutter and Divergence Speed

A simplified, classical binary (bending-torsion) flutter model is used to estimate the flutter speed, V_f. This model is highly dependent on the mass ratio μ, the radius of gyration r_α, and the ratio of bending to torsional frequencies.

Divergence occurs when the aerodynamic pitching moment overcomes the wing's torsional stiffness, leading to catastrophic twisting. The divergence dynamic pressure q_d is approximated using strip theory:

$$q_d \approx \frac{GJ}{ec\frac{dC_L}{d\alpha}} \implies V_d = \sqrt{\frac{2q_d}{\rho}} \tag{23}$$

The tool reports the lowest of these instability speeds as the "Aeroelastic Speed Limit".