Topical note on Numerical Method lecture 1

Prepared by Debashish Chakraborty
Based on lecture by Professor Kazi Ashrafuzzaman
MAT-431

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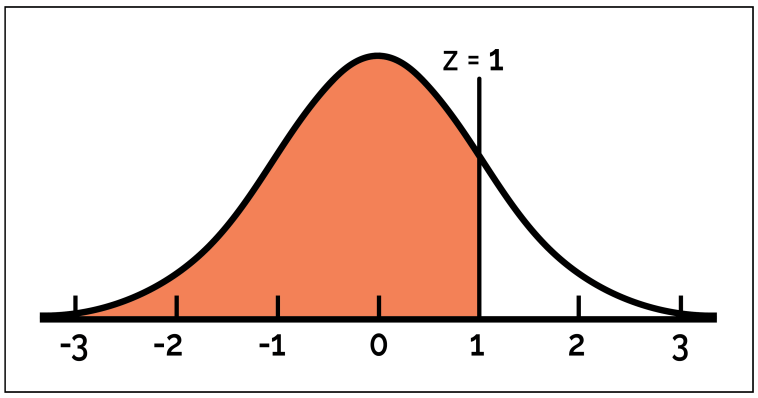
Numerical Method's alternative name is Scientific Computing.

1 Normal Distribution CDF

Normal distributions's PDF cannot be integrated in closed form.



Normal Distribution CDF,

$$f_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

We can compute this integral using various methods.

1. **Riemann Sum:** A Riemann sum approximates the definite integral of a function by summing the areas of rectangles under the curve
2. **Monte Carlo Method:** The Monte Carlo method for computing integrals approximates the value of a definite integral by using random sampling and averaging.
3. **Quadratic Equation:** Using random 3 points we can get a quadratic equation from which we can calculate the area more accurately.

Lab assignment 1: Write a program to calculate the normal distribution CDF using random points(Monte-Carlo method).

2 Modeling

In the realm of numerical methods, "modeling" refers to creating a mathematical representation of a real world physical system or phenomenon to predict its behavior.

- **Approximation:** Approximation in modeling refers to the practice of using simplified representations or techniques to analyze complex systems, while acknowledging that the model is not a perfect replica of the real-world situation.
- **Modeling Errors:**
  1. Formatting Error
  2. Quantization Error
  3. Rounding Error
  4. Absolute Error
  5. Relative Error

3 Catastrophic Cancellation:

When we substract a very small number(electron mass) from a very big number like Avogadro's number($6.023 * 10^{23}$), the result seems to be the big number and the smaller number is completely ignored, this phenomenon is called catastropic cancellation.

4 Floating point sum

We can compute the floating point sum using 3 methods.

1. Random sum
2. Increasing sum
3. Decreasing sum

Lab assignment 2: Write a program to calculate the sum of 10000 floating point numbers.

5 Linear and Non-linear equation

5.1 Linear Equation:

$f(x)$ is linear iff for scalars a and b with points x_1 and x_2,

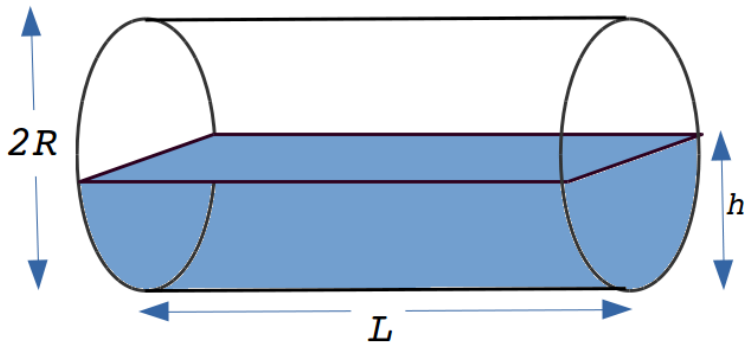
$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

5.2 Non-Linear Equation:

1. **Transcendental equation:** A transcendental nonlinear equation is an equation involving functions that are not algebraic, meaning they cannot be expressed as a solution to a polynomial equation. Examples include equations involving trigonometric, logarithmic, or exponential functions.
2. **Algebraic equation:** An equation of the type $f(x) = 0$ is algebraic if it contains power of x, that is, $f(x)$ is a polynomial.

6 Analytical Question

6.1



A cylinder is lying on the horizontal side and it is continuously filling with water through a pipe. What would be the height(h) of the water level from the ground to fill up quarter($\frac{1}{4}$) of the volume of that cylinder?

6.2

Represent the fibonacci number recurrence using 2*2 matrices.