# SUDOKU GAME SOLUTION USING GRAPH COLORING

By

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# INTRODUCTION

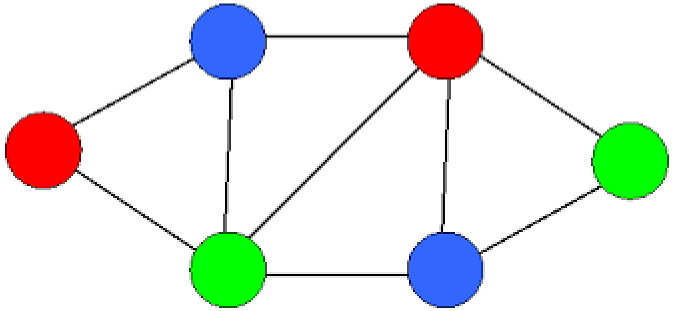
- The Sudoku puzzle has become a very popular puzzle.
- The puzzle consists of a 9×9 grid in which some of the entries of the grid have a number from 1 to 9.
- Filling the table with the numbers must follow these rules:
  - Numbers in rows are not repeated
  - Numbers in columns are not repeated
  - Numbers in 3 × 3 blocks are not repeated
  - Order of the numbers when filling is not important

# A SAMPLE SUDOKU PUZZLE

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

# WHAT IS GRAPH COLORING?

- Graph Coloring is the assignment of colors to vertices of a graph such that no two adjacent vertices have the same color.



# CONVERTING SUDOKU TO COLORING PROBLEM

- The graph will have 81 vertices with each vertex corresponding to a cell in the grid.
- Two distinct vertices will be adjacent if and only if the corresponding cells in the grid are either in the same row, or same column, or the same sub-grid.
- Each completed Sudoku square then corresponds to a k-coloring of the graph.

# CONTINUED..

- Consider an $n2 \times n2$ grid, To each cell in the grid, we associate a vertex labeled $(i, j)$ with $1 \leq i, j \leq n2$.
- We will say that $(i, j)$ and $(i', j')$ are *adjacent* if $i = i'$ or $j = j'$

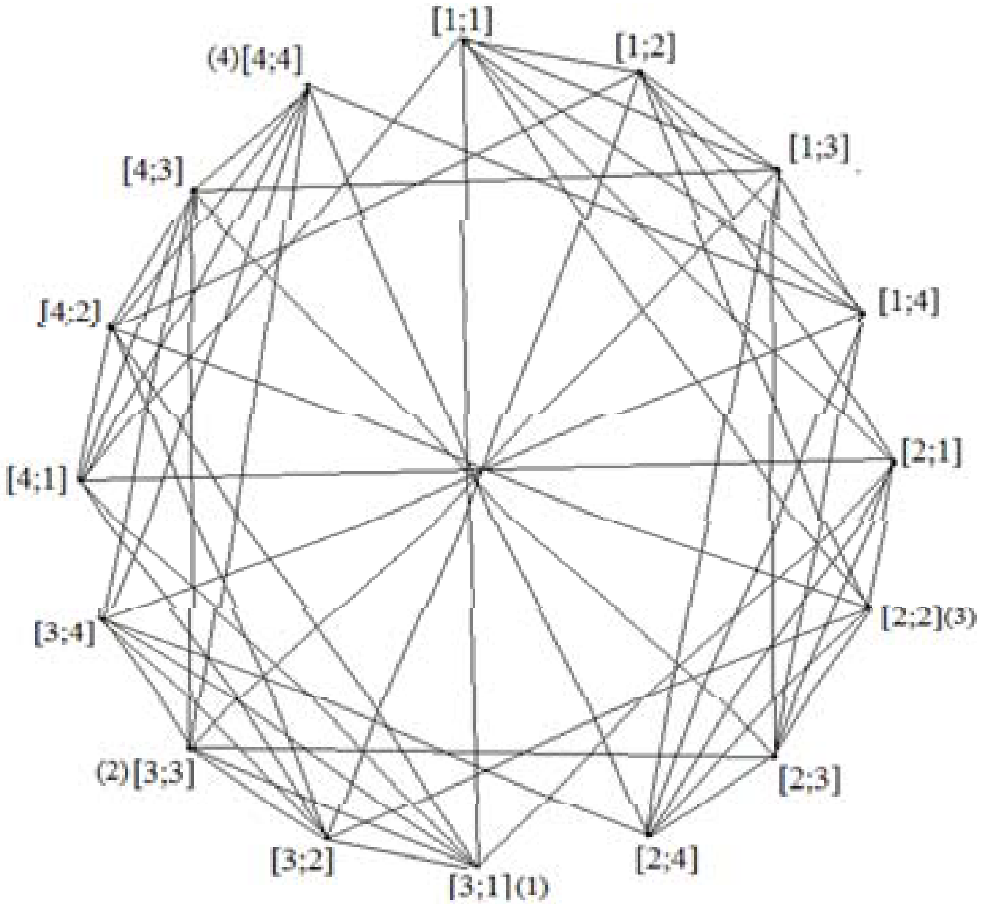
  or $\lceil i/n \rceil = \lceil i'/n \rceil$ *and* $\lceil j/n \rceil = \lceil j'/n \rceil$.

- Graph is called *regular* if the degree of every vertex is the same.
- Each vertex has degree 20, thus the number of edges is:
- $| H | = 20 * 81 / 2 = 810$

# SAMPLE VERSION OF 4X4 SUDOKU

| | | | |
|---|---|---|---|
| [1;1] | [1;2] | [1;3] | [1;4] |
| [2;1] | [2;2] 3 | [2;3] | [2;4] |
| [3;1] 1 | [3;2] | [3;3] 2 | [3;4] |
| [4;1] | [4;2] | [4;3] | [4;4] 4 |

# GRAPH CORRESPONDING TO THE 4*4 SUDOKU TASK




The above mentioned graph has 16 vertices and 56 edges.

# GRAPH COLORING TECHNIQUE

- The whole algorithm can be divided into the following steps:

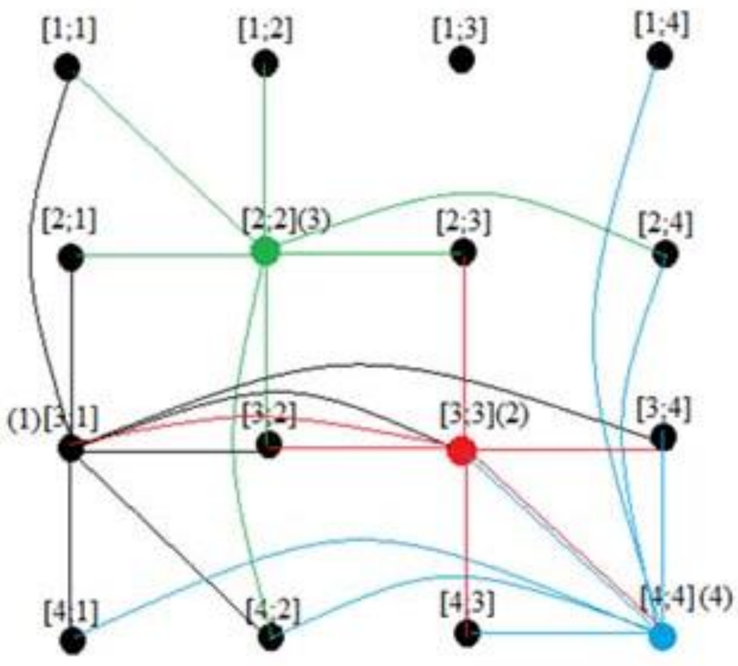
1. The vertex that is already colored is selected and linked by edges of same color with all other vertices of sets in which the vertex is located. These vertices can no longer be colored with the same color. This is repeated for all the vertices for which hints are given.

2. The vertices where the largest number of colored edges converge are found (it is most likely that there will be only one candidate).

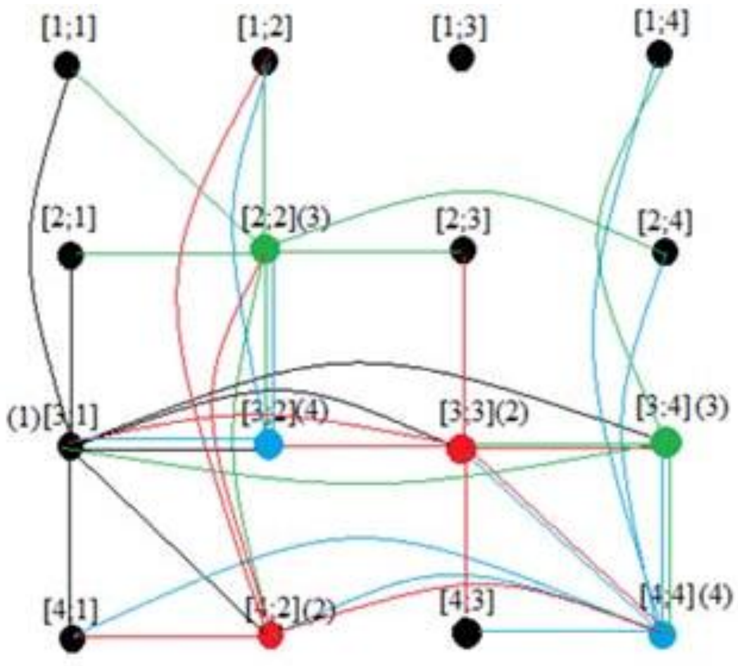
# CONTINUED..

3. If there are vertices among them that can be colored only by one color, then they are colored with it and the procedure continues from the first step (there is no need to draw those edges into the graph that lead to a vertex where there is already another edge of the same color). If there are no such vertices, the procedure continues with the fourth step.

4. From the set of those selected vertices, the one that is adjacent to the largest number of uncolored vertices is chosen and colored to the color with the lowest value that is not used for its neighbors. If there are more such vertices one of them is selected randomly. In the next step the procedure continues from the first step.




Sudoku graph after step one




Sudoku graph after step two

# MATRIX REPRESENTATION

| List of vertices | Neighboring vertices | | | | | |
|---|---|---|---|---|---|---|
| [1;1] | [1;2] | [1;3] | [1;4] | [2;1] | [2;2] (3) | [3;1] (1) | [4;1] |
| [1;2] | [1;1] | [1;3] | [1;4] | [2;1] | [2;2] (3) | [3;2] | [4;2] |
| [1;3] | [1;1] | [1;2] | [1;4] | [2;3] | [2;4] | [3;3] (2) | [4;3] |
| [1;4] | [1;1] | [1;2] | [1;3] | [2;3] | [2;4] | [3;4] | [4;4] (4) |
| [2;1] | [1;1] | [1;2] | [2;2] (3) | [2;3] | [2;4] | [3;1] (1) | [4;1] |
| [2;2] (3) | [1;1] | [1;2] | [2;1] | [2;3] | [2;4] | [3;2] | [4;2] |
| [2;3] | [1;3] | [1;4] | [2;1] | [2;2] (3) | [2;4] | [3;3] (2) | [4;3] |
| [2;4] | [1;3] | [1;4] | [2;1] | [2;2] (3) | [2;3] | [3;4] | [4;4] (4) |
| [3;1] (1) | [1;1] | [2;1] | [3;2] | [3;3] (2) | [3;4] | [4;1] | [4;2] |
| [3;2] | [1;2] | [2;2] (3) | [3;1] (1) | [3;3] (2) | [3;4] | [4;1] | [4;2] |
| [3;3] (2) | [1;3] | [2;3] | [3;1] (1) | [3;2] | [3;4] | [4;3] | [4;4] (4) |
| [3;4] | [1;4] | [2;4] | [3;1] (1) | [3;2] | [3;3] (2) | [4;3] | [4;4] (4) |
| [4;1] | [1;1] | [2;1] | [3;1] (1) | [3;2] | [4;2] | [4;3] | [4;4] (4) |
| [4;2] | [1;2] | [2;2] (3) | [3;1] (1) | [3;2] | [4;1] | [4;3] | [4;4] (4) |
| [4;3] | [1;3] | [2;3] | [3;3] (2) | [3;4] | [4;1] | [4;2] | [4;4] (4) |
| [4;4] (4) | [1;4] | [2;4] | [3;4] | [4;1] | [4;2] | [4;3] | [3;3] (2) |

# ANOTHER WAY – INDEPENDENT SETS

- Definition : Set A subset of V(G) is called an independent set of graph G if no two vertices of set A are linked by an edge.

- Each solution of Sudoku (9 × 9 version) can be described by nine independent subsets of the graph.
- Each of them has nine vertices labeled with the same number. Finding such a solution that would correspond to the task is, laborious and complicated.


SUDOKU
Thank you.