PH569: Tutorial III **Date: 22nd Oct, 2023**

1. (a) If the Lande g-factor (g_L) for a *f*-shell paramagnetic ion is equal to the electronic g-factor (= 2), how many electrons does the paramagnetic ion have in its *f*-shell?

(b) The effective number of Bohr magnetons for a d-shell paramagnetic material is $p = 3\left(\sqrt{7/5}\right)$. If the Lande g-factor of the paramagnetic ions constituting the material is $1 < g_L < 1.5$, how many electrons (total) are there in the d- shell of the paramagnetic ions? How many of them are unpaired?

2. Dysprosium ion (Dy^{3+}) has the following atomic configuration:

$$Dy^{3+}: (4f)^9(5s)^2(5p)^6$$

Calculate the spin, orbital and total angular momentum quantum numbers, and also the Lande g-factor. What is the molar susceptibility at room temperature?

3. The effect of magnetic field on an atom/ion can be understood by considering the Hamiltonian

$$H = H_0 - \boldsymbol{\mu}.\boldsymbol{B} = H_0 - \mu_z B$$

where H_0 is the atomic Hamiltonian in the absence of the magnetic field. Treating the term appearing due to the magnetic field as a perturbation, we can calculate the (additional) energy of the atom (in the state $|\alpha, J,\ m_J\rangle$) as

$$\Delta E = \langle \alpha, J, m_J | -\mu_z B | \alpha, J, m_J \rangle$$

Here

$$J_z |\alpha, J, m_J\rangle = m_J \hbar |\alpha, J, m_J\rangle$$

and

$$\boldsymbol{J}^2 |\alpha, J, m_J\rangle = J(J+1)\hbar^2 |\alpha, J, m_J\rangle$$

and the operator

$$\boldsymbol{\mu} = -\frac{\mu_B}{\hbar}(\boldsymbol{L} + 2\boldsymbol{S}) = -\frac{\mu_B}{\hbar}(\boldsymbol{J} + \boldsymbol{S})$$

Making use of the fact that (Wigner Eckert theorem)

$$\langle \alpha, J, m_J | \mu_z | \alpha, J, m_J \rangle = \frac{\langle \alpha, J, m_J | J_z | \alpha, J, m_J \rangle}{\hbar^2 J(J+1)} \langle \alpha, J, m_J | \boldsymbol{J}.\boldsymbol{\mu} | \alpha, J, m_J \rangle$$

(a) Show that

$$\Delta E = \mu_B B m_J \left(1 + \frac{\langle \alpha, J, m_J | \boldsymbol{J}.\boldsymbol{S} | \alpha, J, m_J \rangle}{J(J+1)\hbar^2}\right)$$

(b) Show that the factor within the parenthesis is the Lande g-factor.

4. Consider the anisotropic Heisenberg spin Hamiltonian (the exchange interaction coupling the

$$H = -\left(\sum_{\boldsymbol{R},\boldsymbol{R}'} [K(\boldsymbol{R}-\boldsymbol{R}')\boldsymbol{S}_{\boldsymbol{z}}(\boldsymbol{R})\boldsymbol{S}_{\boldsymbol{z}}(\boldsymbol{R}') + J(\boldsymbol{R}-\boldsymbol{R}')\boldsymbol{S}_{\perp}(\boldsymbol{R}).\boldsymbol{S}_{\perp}(\boldsymbol{R}')]\right)$$

with $K(\boldsymbol{R}-\boldsymbol{R}') > J(\boldsymbol{R}-\boldsymbol{R}') > 0$

(a) Show that the ground state ($|G\rangle$) and the one-spin wave state ($|k\rangle$ as defined in class) of the isotropic Heisenberg Hamiltonian remain eigenstates of H, but that the spin wave excitation energies are raised by $2S\sum_{\boldsymbol{R}}(K(\boldsymbol{R}) - J(\boldsymbol{R}))$.

5. Consider N spins, each of magnitude S, on a line or a ring, interacting with nearest neighbours by the Heisenberg interaction

$$U = -2J \sum_{p=1}^{N} S_p.S_{p+1}$$

(a) show that the time evolution of the angular momentum is governed by the equation

$$\frac{d\boldsymbol{S}_p}{dt} = \frac{2J}{\hbar}\left(\boldsymbol{S}_p \times \boldsymbol{S}_{p+1} + \boldsymbol{S}_p \times \boldsymbol{S}_{p-1}\right)$$

(b) If the amplitude of the excitation is small, i.e. if we can assume that $S_p^z \approx S$ and $S_p^x, S_p^y \ll S$, show that approximate solutions for S_p^x and S_p^y are given by

$$S_p^x = u\exp[i(pka - \omega t)] \text{ and } S_p^y = v\exp[i(pka - \omega t)]$$

where u, v, are constants, p is any integer and a is the lattice constant.

(c) By substituting the expressions of S_p^x and S_p^y into the corresponding equations of motion, find the relation between u and v and show that

$$\hbar\omega = 4JS(1 - \cos ka)$$