MA 353: Elliptic Curves
Assignment-1

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1. $V/\mathbb{Q}$ is the variety $V : 5X^2 + 6XY + 2Y^2 = 2YZ + Z^2$

   Claim: $V(\mathbb{Q}) = \emptyset$.

   Proof. $\square$

2. For each prime $p \geq 3$, let $V_p \subseteq \mathbb{P}^2$ be the variety corresponding to the curve

   $$V_p : X^2 + Y^2 = pZ^2$$

   (a) **Claim**: $V_p \cong \mathbb{P}^1$ over $\mathbb{Q}$ iff $p \equiv 1 \pmod 4$

   Proof. $\square$

   (b) **Claim**: For $p \equiv 3 \pmod 4$, no two V_ps are isomorphic.

   Proof. $\square$

3. Let $F(x, y, x) \in k[x, y, z]$ be a homogeneous polynomial polynomial of degree $d \geq 1$ and the curve corresponding to F is non-singular.

   Claim:

   $$\mathfrak{g}(C) = \frac{(d-1)(d-2)}{2}$$

   Proof. $\square$

4. (a) $L : 2x + 5y - 1 = 0$

   (b) $C : x^2 - 4xy + 3y^2 - 3x + 5y - 10 = 0$

5. Given $f(x, y) = y^2 - x^3 - ax^2 - bx$

6. Suppose E is an elliptic curve given by the Weierstrass equation $y^2 = x^3 + ax^2 + bx + c$ and $P = (x, y)$ a point on E.

   (a)

(b)

7. Given $E : y^2 = x^3 + 17$ is an elliptic curve over $\mathbb{Q}$

   (a) $P = (-1, 4), Q = (2, 5)$. We wish to find $P + Q$

   (b) $P = (-2, 3), Q = (2, 5)$. We wish to find $-P + 2Q$