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BACHELOR THESIS DRAFT

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A thesis submitted for the degree of Bachelor of Science (Research) with major in Mathematics

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December 2022

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Definition.
For $f \in \mathcal{S}$ and quasicharacters c with exponent > 0, we introduce a function $\zeta(f, c)$ as

$$\zeta(f, c) = \int_{K_{\mathfrak{p}}^{\times}} f(\alpha)c(\alpha)\mathrm{d}^{\times}\alpha$$

and call such a function a ζ-function of $K_{\mathfrak{p}}$.

Lemma 3.3.1.
A ζ-function is regular for all quasi-characters with of exponent greater than 0.

Proof. □

Theorem 3.3.2.
A ζ-function has an analytic continuation to the domain of all quasi-characters given by the functional equation of the type

$$\zeta(f, c) = \rho(c)\zeta(\hat{f}, c^{\vee})$$

The factor $\rho(c)$ is independent of the choice of f, is a meromorphic function of quasi-characters defined for $0\sigma < 1$ by the functional equation itself and for all quasi-characters by analytic continuation.

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