Mathematical Modeling of Hydraulic Conductivity in Homogeneous Porous Media: Influence of Porosity and Implications in Subsurface Transport of Contaminants

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Abstract: *Fluid flow in porous media is an important process with significant impact on the environment. By using mathematical modeling technique, this work investigates the influence of porosity on the hydraulic conductivity of homogeneous porous media A simple analytic model is proposed which predicts the saturated hydraulic conductivity curves from experimental values of porosity by using the plot obtained from MATLAB. Data for the modeling were obtained from our previous laboratory experiments, which utilized constant head permeameter to determine volume flux and the porosities were determined by volumetric approach. The model was used to access the effects of new values of porosities on the hydraulic conductivities of porous media. The results showed that at porosity of greater than 0.1, the hydraulic conductivity increases linearly. However, it remains constant at values below 0.1. The proposed model is compared with the existing empirical model on the basis of the measured data of five (5) soils samples. It is clear here that the new model is in better agreement with the observations from previous work. The validity and applicability the proposed approach is briefly discussed.*

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Keywords: *Saturated Hydraulic Conductivity, Empirical formula, porosity.*

1. Introduction

Many day to day processes involve the movement of fluids across porous medium; for instance, the filtration of water by using sand or other porous materials. As commonly observed, when fluid flows through the media , portion of the fluid is stored in the pores present in the media. The knowledge of how the fluid permeates through these materials and the relevant factors affecting the permeation as well as storage or trapping is really useful in engineering practices (Wikipedia, 2015)

Darcy's law describes the one dimensional flow of water through a saturated soil profile whereas Darcy-Buckingham law describes the one dimensional flow of water through an unsaturated soil profile (Sanjit and Manoj, 2012). These formulations essentially indicate that the flow through the saturated porous media is directly proportional to the hydraulic gradient that is the driving force causing flow. In this case, the constant of proportionality, is known as hydraulic conductivity (K_s) of the saturated porous media (Lal, 2004; Sanjit et al., 2012). The K_s is strongly influenced by various properties of a porous media such as: structure, pore connectivity as well as the properties of the fluid such as viscosity and temperature. Porosity is the fraction of the total soil that is taken up by the pore space (Omojola et al, 2014).

In groundwater hydrology, K_s is one of the most important parameters for soil-water-plant interactions, water and solute movement and retention through the soil profile. The knowledge of K_s is necessary for modeling the water flow in the soil in the saturated (and the unsaturated zone) and transportation of water-soluble pollutants in the soil. It is also an important parameter for designing of the drainage of an area and in the construction of earth dam and levee. Furthermore, it is of paramount importance in relation to some geotechnical problems, including the determination of seepage losses, settlement computations, and stability analyses (Boadu, 2000).

Another similar term is permeability, which is defined as the property of the porous medium controlled only by the pore geometry (Richards, 1952). In geotechnical engineering projects and in environmental sites characterization, it is one of the most important soil properties of interest to engineers. It is the most important physical property of porous medium, which is a measure of the ability of a material to transmit fluid through it (Alabi, 2011). It influences soil erosion, seepage pollution, settlement, and stability of roads, foundation building and even crop production (Okagbue, 1995). It is no wonder therefore, that many researchers (Wallace, 1948; Bjerrum and Huder, 1957; Olson and Daniel, 1979; Green et al, 1998; Chapius et al, 1989; Okagbue,1995 amongst others) has shown significant interest. These researchers has shown that degree to which soils are permeable depends on a number of factors such as soil type, soil grain size and shape, void ratio (permeability increase in void ratio), degree of compaction, viscosity and density, to mention a few.

Various standard techniques are available for the direct measurement of the saturated hydraulic conductivity K_s on field or laboratory (Todd and Mays, 2005; Ishaku et al, 2011). However, this methods remain expensive and time-consuming (Aimrum and Amin, 2009). As a result, indirect methods have been developed where K_s is expressed as a function of other soil parameters (Vassailis et al, 2013).These soil parameters include particle/grain size distribution, pore size distribution, porosity, shape of particles and other factors (Legrand,1971; Rasmusen, 1964). For instance, the Pedotransfer Functions (PTFs) model which happens to be the most popular of the equation to determine Ks have their independent variables based on particle size distribution; physically based equations developed based on the Kozeny-Carman approach used porosity attributes such as the total porosity Φ as function of K_s; also the modified Kozeny-Carman equation by Timlin et al. (1999) and Han et al. (2008) used λ parameter of Brooks-Corey equation as exponent of ϕ_e which is a function of pore size distribution (Vassilis et al, 2013).

Considering the importance of the parameter K_s, the challenges in its measurement and the efforts already expended by researchers in providing mathematical models, with which its value can be easily obtained, one can readily assume that there is no room for further improvement in the mathematical modeling for determination of K_s. However, close scrutiny reveals that the permeabilities predicted by Kozeny-Carman are valid for laminar flow and holds for Reynolds numbers up to approximate 1.0, after which the kinetic energy losses affects the result. Also, the formulas are valid within more or less a factor of three of the observed values (Schlueter and Witherspoon, 1994). Hazen formula has range limitations and the level of error when the Hazen formula is used outside its data ranges is significant (Liou, 1998). Apart from above mentioned limitations, the proper use of the Hazen and Kozeny-Carman required expertise in grain size analysis and there is high possibility of transferring of error from one step to other in the processes of determination of hydraulic conductities via-a-vis permeabilities. Thus, there is a need for a simple equation with the ability for quick estimation of hydraulic conductivity.

The objective of this study is to propose new model to determine K_s based on the simplified Kozeny-Carman approach (Ahuja et al., 1984; Flint and Selker, 2003; Aimrum et al., 2004; Regalado and Muñoz-Carpena, 2004; Henderson et al., 2010) given by: $Ks = A\phi$ and $Ks = B\phi_e$ .The validity and reliability of the model will be evaluated using data sets of Alabi, (2011) in order to make comparism with the model.

2. Theoretical Review

Henry Darcy a French Hydraulic Engineer made the first systematic study of movement of water in a porous media. He published the work in which he described a series of experiments he had performed in an attempt to quantify the amount of water that would flow through a column of sand. The experimental apparatus used was similar to the one displayed below *(Freeze and Cherry, 1979)*. From the experiment carried out, he proposed an expression known as Darcy's law. Water was introduced to a soil column at a constant rate Q and leaves the column at the same rate Q, thereby creating a steady-state flow. The fluid pressure is measured at two points on the column by vertical piezometer inserted at both ends, separated by a column distance Δl and a vertical distance The upper and lower distances of the piezometer gives the vertical distance (z_1-z_2) and hydraulic head (Δ h). The formulations give the general expression as:

$$q = Ki \tag{1}$$

where k is permeability; q is the flux and i, the hydraulic gradient

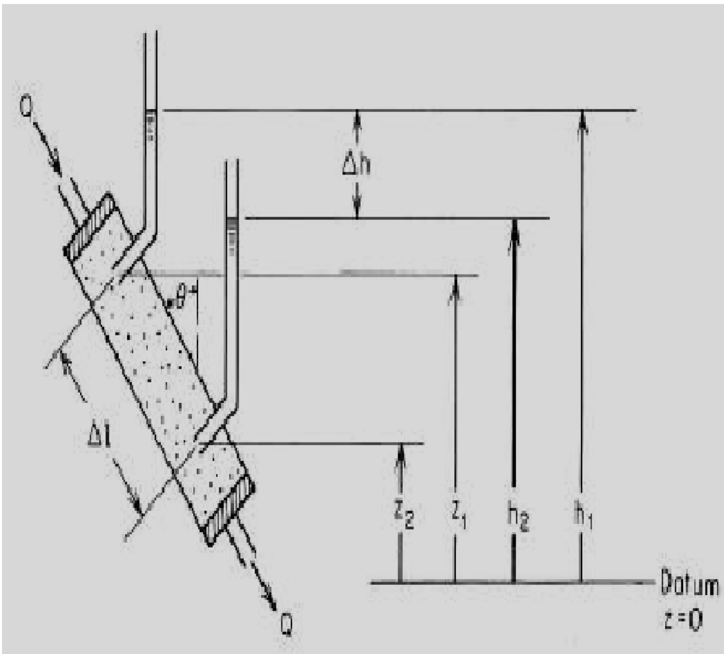


Figure 1 *Experimental apparatus for the illustration of Darcy's Law.*

2.1 Previous Established Empirical Formulae from Grain Size Distribution of Soil Samples.

Hydraulic conductivity (K_s) can be estimated as a function of several soil parameters by particle size analysis of the sediment of interest using empirical equations relating either K to some size property of the sediment. Several empirical methods from previous studies were summarized by Vukovic and Soro, (1992) and presented a general formula:

$$K = \frac{g}{v} \cdot C \cdot f(n) \cdot d_e^2 \tag{2}$$

K= hydraulic conductivity

g = acceleration due to gravity

v = kinematic viscosity

C = sorting coefficient

f(n) = porosity function

d_e = effective grain diameter.

The kinematic viscosity (v) is related to dynamic viscosity (μ) and the fluid (water) density (ρ) as follows:

$$v = \frac{\mu}{\rho} \tag{2}$$

The values of C, f(n) and d_e are dependent on the different methods used in the grain size analysis. Porosity (n) may be derived from the empirical relationship with the coefficient of grain uniformity (U) as follows:

$$n = 0.255\left(1 + 0.83^{U}\right) \tag{3}$$

Hazen formula is given as:

$$k = \frac{g}{v} \times 6 \times 10^{-3}\left[1 + 10(n - 0.26)\right]d_{10}^{2} \tag{4}$$

Hazen formula was developed originally to determine hydraulic conductivity of uniformly graded sand but is also useful for fine sand to gravel range, provided the sediment has a uniform coefficient less than 5 and effective grain size between 0.1 and 3mm.

Kozeny-Carmen formula is given as:

$$K = \frac{g}{v} \times 8.3 \times 10^{-3}\left[\frac{n^{3}}{(1-n)^{2}}\right]d_{10}^{2} \tag{5}$$

The Kozeny-Carman equation is one of the most widely accepted and used derivations of permeability as a function of the characteristics of the soil medium. This equation was originally proposed by Kozeny (1927) and was then modified by Carmen (1937, 1956) to become the Kozeny-Carman equation. It is not appropriate for either soil with effective size above 3mm or for clayey soils (Carrier, 2003).

Breyer formula is given as:

$$K = \frac{g}{v} \times 6 \times 10^{-4} \log \frac{500}{U} d_{10}^{2} \tag{6}$$

Slitcher's formula is given as:

$$K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} d_{10}^{2} \tag{7}$$

This formula is most applicable for grain-size between 0.01 and 5mm.

Terzaghi's formula is given as:

$$K = \frac{g}{v} \cdot C_t \cdot \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^{2} \tag{8}$$

Where;

C_t = sorting coefficient and $6.1 \times 10^{-3} < C_t < 10.7 \times 10^{-3}$. Terzaghi formula is most applicable for large-grain sand (Cheng and Chen, 2007).

Alyamani & Sen:

$$K = 1300\left[I_0 + 0.025\left(d_{50} - d_{10}\right)\right]^2 \tag{9}$$

Where;

K = hydraulic conductivity (m/day)

I_0 = intercept (mm) of the line formed by d_{50} and d10 with the grain-size axis

d_{10} = effective grain diameter

d_{50} = median grain diameter (mm)

NB: The terms in the formula above bear the stated units for consistency. Therefore, the formula is exceptionally different from those that take the general form of equation (1a) above. It is however, one of the well known equations that also depend on grain-size analysis. The method considers both sediment grain sizes d_{10} and d_{50} as well as the sorting characteristics.

3.0 Modelling Approach

Various methods have been employed to determine hydraulic conductivity. These methods are grouped into three basic classes. They include: laboratory methods, Field Methods (in-situ) and empirical methods. As pointed out earlier, the difficulties encountered in laboratory/experimental investigations are many. Also, they are often laden with inconsistencies. For the models already in existence, it has been shown that there is a need for simplicity and ease of evaluation of embedded parameters. Thus, this section describes new formulation for determining hydraulic conductivity of homogenous porous media. The new model is intended for easy application in the field of porous media and contaminant transport.

3.1. **Theory**

The technique used in developing the model comes under the curve fitting technique. This technique can utilize the principles of linear or nonlinear regression, interpolation, local smoothening regression or custom equations. Tools for this technique exist in MATLAB (The Mathworks, Inc. 2011) with which multiple fits can be created, plotted and compared. The tool creates a curve or surface fit to the elements of predictor and response variables. Weights can be assigned to these elements with a view to creating proportionality in the effects of the predictor on the response or output variable.

Results using these tools can provide a view of goodness-of-fit statistics, display confidence intervals and residuals, remove outliers and assess fits with validation data.

3.2 **Data Source**

The data used in the curve fitting procedure for this work were obtained from Alabi (2011). The data are related to experimental investigations of fluid flow in homogenous porous media. Alabi (2011) utilized constant-head permeameter technique in the experimental procedure.

3.3 **Custom equation**

In this work, custom equation in the curve fitting tool of MATLAB was employed to derive the model targeted. Following a series of trial and error, the exponential form of custom equation was arrived at, showing low residual, low mean square error and very correlation coefficient. The form of the equation is,

$$K = a\exp\left(b\Phi^{8}\right) + c$$

where K is the hydraulic conductivity and Φ is the porosity. Having chosen the above equation format, the task remains to evaluate the parameters 'a', 'b' and 'c'. In doing this the fitting session was run several times until the goodness-of-fit statistics were satisfactory.

3.4 Empirical Method of Determination of Hydraulic Conductivity.

In predicting, an analytical model to determine the hydraulic conductivity of a porous media based on its porosity. The graph of the experimental result for hydraulic conductivity was plotted against porosity using MATLAB (version) and an equation was obtained from the plot. This equation was used to determine new values of hydraulic conductivities using the experimental values of porosity. Another graph was plotted using the values obtained by applying the equation.

These two graphs (that is the experimental and the model) were then plotted to determine the deviation of the predicted model from the experimental values. Furthermore, new sets of porosity lower than the experimental porosities were used to determine a new set of hydraulic conductivity. Then, a plot of these hydraulic conductivities against porosity was prepared. The equation is given by:

$$K = a\exp\left(b\Phi^{8}\right) + c$$

K = hydraulic conductivity; ϕ = porosity; a = -32.25; b = -0.03252; c = 32.25. The correlation was seen to have the best fit of $R^2 = 0.9812$, SSE = 1.32 x 10-8 and RMSE = 8.123 x 10-5.

4.0 Results

From experimental approach in determining hydraulic conductivity, the volume flux for five different homogenous samples labeled A-E (Table 1) was plotted against the hydraulic gradient, i. The slopes of the line of volume flux, q against hydraulic gradient, i indicate the hydraulic conductivity for each sample (Figures 4 -8). The results (Table 2) were then used as a model for the empirical approach.

The original data was curve fitted using MATLAB version, 2011 to obtain an equation (Fig. 10). The best fitting curve with $R^2 = 0.9812$ was obtained. The equation was used to determine values of hydraulic conductivity by substituting the original values of porosity. These two values (experiment and predicted) were then plotted (Figure 9a shows the relationship between the experimental hydraulic conductivity and the model hydraulic conductivity). Smaller values for porosity (ϕ) were considered and the correlation obtained in figure 9 above was used to determine the model values corresponding to these new values of porosities.

Figure 9 shows the equation from the curve fitting of the experimental values

Table 2 gives the experimental values for Porosity and Hydraulic conductivity

Figure 10 shows the relationship between the experimental hydraulic conductivity and the model hydraulic conductivity. It can be seen that the points of the model K_s are closer to the experimental K_s values. This shows that the correlation predicts well the experimental values.

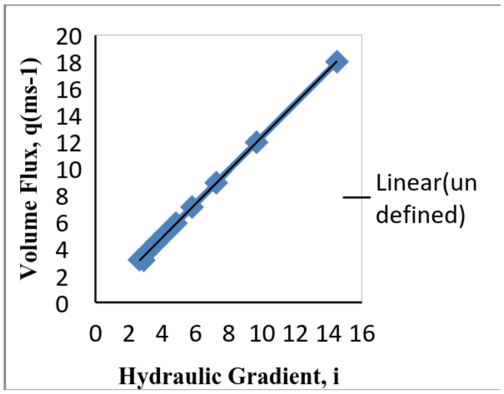
Table 3 shows the relationship between new porosities and model prediction. Smaller values for porosity (ϕ) were considered and the correlation obtained in above was used to determine the model values corresponding to these new values of porosities.

Figure 11 shows the variation in the hydraulic conductivity with these new porosities.

It is clear from **figure 10** that the points of the K_M are closer to the experimental K_E. This shows that the correlation predicts well the experimental values.

Figure 11 gives the variation in the hydraulic conductivity with these new porosities. It is shown that at porosity of greater than 0.1, the hydraulic conductivity increases linearly. However, it remains constant at values below 0.1. This can be applied in physical terms, say in unconsolidated soil that the value 0.1 is the threshold porosity value. Furthermore, it is also inferred that there are no noticeable variations in the hydraulic conductivity of very small values of porosity.

Graphs



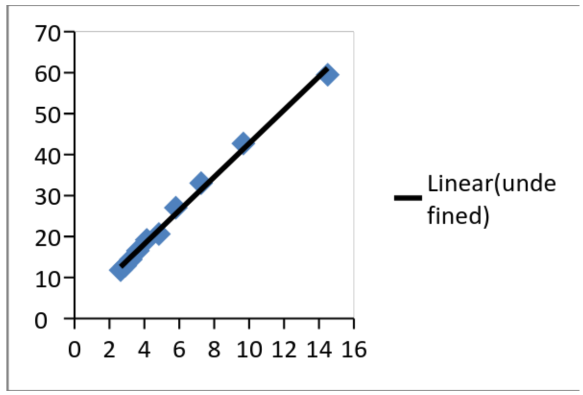
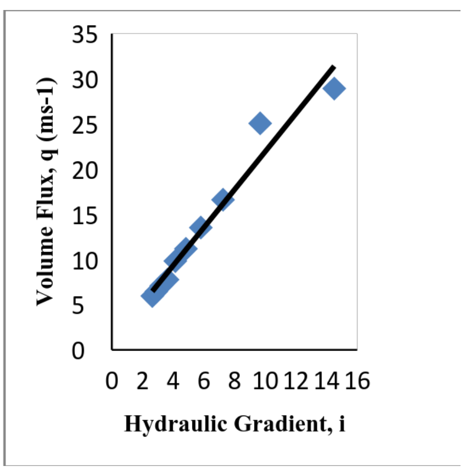


Fig 4: Plot of volume flux, q against hydraulic gradient for sample A

Fig 5 Plot of volume flux, q against hydraulic against hydrauli ic gradient for sample B



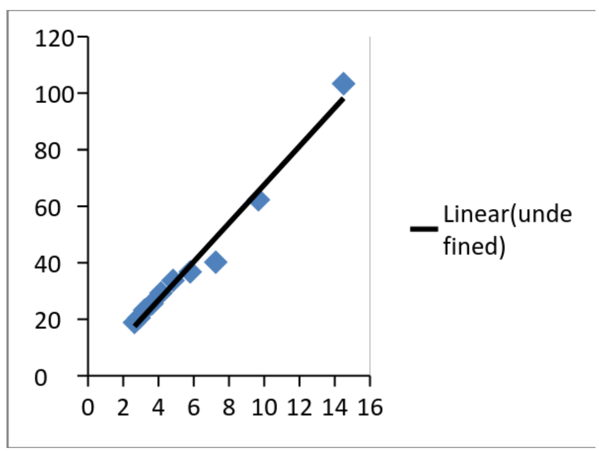
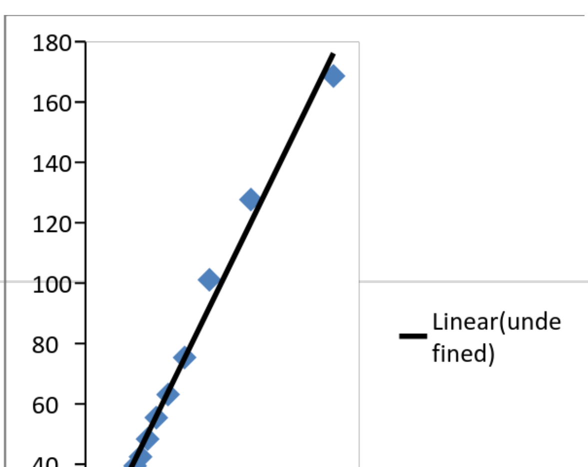


Fig 6 Plot of volume flux, q against hydraulic **Fig 7** Plot of volume flux,
q against hydraulic gradient, i for sample gradient, i for sample C D



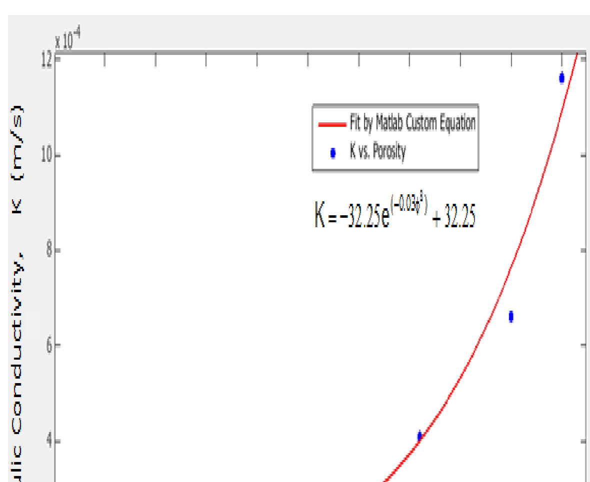
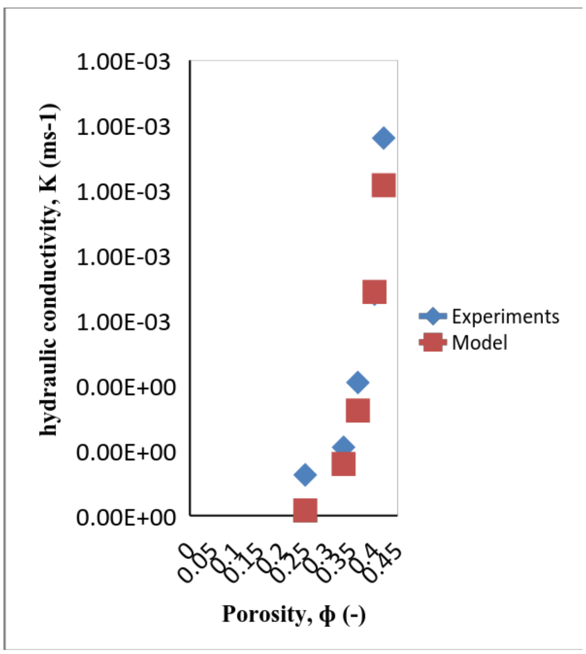


Fig 8: Plot of volume flux, q against hydraulic gradient, i for sample E

Fig 9: Plots of Curve fitting to experimental data



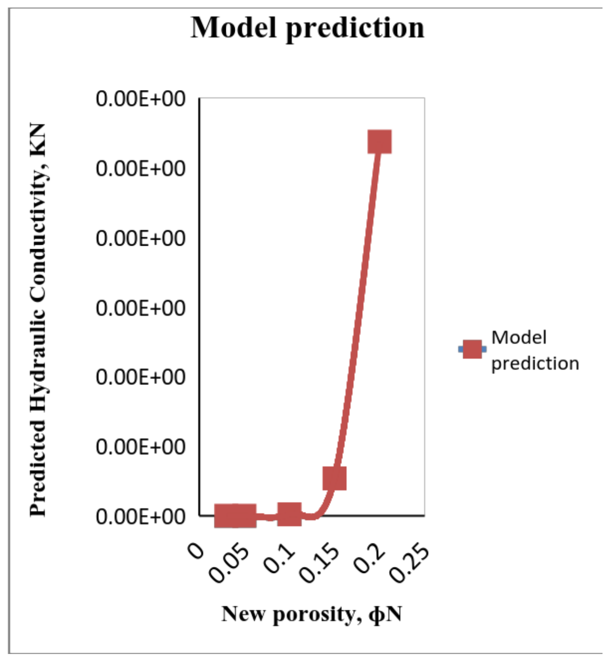


Fig. 10: Comparison between experimental and model

Fig. 11: Plots of Predicted K_s against porosity(Model Prediction)

Porosity	Expt. $K_s(ms^{-1})$	Model $K_s(ms^{-1})$

0.230	1.25 x 10-4	8.21 x 10-6
0.330	2.10 x 10-4	1.48 x 10-4
0.364	4.09 x 10-4	3.23 x 10-4
0.400	6.61 x 10-4	6.87 x 10-4
0.420	11.61 x 10-4	10.15 x 10-4

Table 2: Experimental values for Porosity and Hydraulic conductivity

Table 3: Relationship between New Porosities and Model Prediction

Porosity(ϕ)	Model K_M(ms^{-1})
0.03	6.89 X 10^{-13}
0.05	4.10 x 10^{-11}
0.10	1.05 x 10^{-8b}
0.15	2.69 x 10^{-7}
0.20	2.68 x 10^{-6}

Discussion

In investigating the influence of porosity on the hydraulic conductivity of the porous medium, a simple analytical model is proposed, which predicts the saturated hydraulic conductivity curves in homogenous porous media by using the plot obtained from MATLAB. The MATLAB was used to obtain an equation of the best fitted curves from the experimental values obtained in the laboratory (**Table 1**). This equation was used to determine new values of hydraulic conductivities using the experimental values of porosity. Another graph was plotted using these new set of values obtained.

Figure 13 gives the variation in the hydraulic conductivity with these new porosities. It is shown that at porosity of greater than 0.1, the hydraulic conductivity increases linearly. However, it remains constant at values below 0.1. This can be applied in physical terms, say in unconsolidated soil that the value 0.1 is the threshold porosity value. Furthermore, it is also inferred that there are no noticeable variations in the hydraulic conductivity of very small values of porosity.

Soils of high porosity, many pores and good interconnectivity between them, have high K values. The dependency of K on porosity is reflected in the obtained formula. However, the plot shows that for sand of porosity ranging from 0.23-0.42, the minimum porosity required for a significant flow that can be called a discharge is 0.1.

It has been established that Darcy's law fails in flow through sand of high porosity such as soil cracks and gravels, where the flow rate increases less rapidly than the hydraulic gradient (Swartzendruber, 1962, 1968).While the law is usually true for flow in fine grained sands. However, the present result shows that there is exception to Darcy's that in fine grained sand with porosity less than 0.1. This is true because there would not be a significant flow until the porosity is beyond 0.1. Thus, the result suggests that Darcy's law fails at two extremes, rather than just one usual extreme of high porosity.

In physical sense, sand serve as a good filter for contaminated fluid. It is apparent that the inclusion of fine-grained sand of porosity less than 0.1 can drastically reduce the hydraulic conductivity and this should be recognized in drain and filter design.

5.0 Conclusion

Methods of determination of Ks should be simple and easy with fewer numbers of parameters. The saturated hydraulic conductivity estimated by the model was compared with laboratory measured K_s and analyzed statistically. From this paper it is inferred that the empirical model gives reasonable estimate of K_s. Due to its simplicity, and better estimating capabilities the empirical equation is recommended for field use.

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