Incomplete copy of de Quadratura Curvarum

Author: Isaac Newton

Source: MS Add. 9597/2/18/83-88, Cambridge University Library, Cambridge, UK

Published online: February 2013

<83r>

Tractatus de Quadratura Curvarum

Quo tempore incidi in methodum serierum interminatarum convergentium necesse habui mutare notationem quæ tunc in usu erat et pro $\sqrt{x}$, $\sqrt{x^3}$, $\sqrt[3]{x}$, $\sqrt[3]{x^2}$, $\frac{1}{x}$, $\frac{1}{xx}$, $\frac{1}{\sqrt{x}}$ **{,}** $\sqrt{aa-xx}$, $\frac{1}{\sqrt{aa-xx}}$ &c. scribere $x^{\frac{1}{2}}$, $x^{\frac{3}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{2}{3}}$, x^{-1}, x^{-2}, $x^{-\frac{1}{2}}$, $\overline{aa-xx}^{\frac{1}{2}}$, $\overline{aa-xx}^{-\frac{1}{2}}$ &c Hac enim ratione computationes magis uniformes et expeditæ, et theoremata magis generalia evaserunt. Qua de causa etiam exponentes dignitatum indefinite designavi cum Slusio in hunc modum x^{μ}, x^{ν}, $\overline{aa-xx}^{\nu}$ ponendo dignitatum exponentes μ et ν et similes pro numeris quibusvis integris an fractis, affirmativis an negativis. Quas quidam Notarum formulas, cum jam in usu esse cœperint non opus est ut fusius exponam.

Quantitates indeterminatas ut motu perpetuo crescentes vel decrescentes id est ut fluentes vel defluentes in sequentibus considero designoque literis z, y, x, v, et earum fluxiones seu celeritates crescendi noto iisdem literis punctatis $\dot{z}$, $\dot{y}$, $\dot{x}$, $\dot{v}$, sunt et harum fluxionum fluxiones seu mutationes magis aut min us celeres quas ipsorum z, y, x, v fluxiones secundus nominare licet et sic designare $\ddot{z}$, $\ddot{y}$, $\ddot{x}$, $\ddot{v}$; et harum fluxiones primas seu ipsarum z, y, x, v fluxiones tertias, sic $\dddot{z}$, $\dddot{y}$, $\dddot{x}$, $\dddot{v}$; et Quartas sic $\ddddot{z}$, $\ddddot{y}$, $\ddddot{x}$, $\ddddot{v}$. Et quemadmodum $\dddot{z}$, $\dddot{y}$, $\dddot{x}$, $\dddot{v}$ sunt fluxiones quantitatum $\ddot{z}$, $\ddot{y}$, $\ddot{x}$, $\ddot{v}$, et hæ sunt fluxiones quantitatum $\dot{z}$, $\dot{y}$, $\dot{x}$, $\dot{v}$; et hæ sunt fluxiones quantitatum primarum z, y, x, v; Sic hæ quantitates considerari possunt ut fluxiones aliarum quas sic designabo $\overset{\prime\prime}{z}$, $\overset{\prime\prime}{y}$, $\overset{\prime\prime}{x}$, $\overset{\prime\prime}{v}$, et hæ ut fluxiones aliarum $\overset{\prime\prime\prime}{z}$, $\overset{\prime\prime\prime}{y}$, $\overset{\prime\prime\prime}{x}$, $\overset{\prime\prime\prime}{v}$, et hæ ut fluxiones aliarum $\overset{\prime\prime\prime\prime}{z}$, $\overset{\prime\prime\prime\prime}{y}$, $\overset{\prime\prime\prime\prime}{x}$, $\overset{\prime\prime\prime\prime}{v}$. Designavit igitur $\overset{\prime\prime\prime}{z}$, $\overset{\prime\prime}{z}$, z, $\dot{z}$, $\ddot{z}$, $\dddot{z}$, $\ddddot{z}$, $\overset{\cdot\cdot\cdot\cdot\cdot}{z}$, &c seriem quantitatum quarum quælibet posterior est fluxio præcedentis et quælibet prior est fluens quantitas fluxionem habens subsequentem. Similis est series $\sqrt{\overset{\prime\prime\prime}{aa-zz}}$, $\sqrt{\overset{\prime\prime}{aa-zz}}$, $\sqrt{aa-zz}$ $\sqrt{\overset{\cdot}{aa-zz}}$, $\sqrt{\overset{\cdot\cdot}{aa-zz}}$, $\sqrt{\overset{\cdot\cdot\cdot}{az-zz}}$; ut et series $\sqrt{\overset{\prime\prime\prime}{\frac{aa+zz}{a-z}}}$, $\sqrt{\overset{\prime\prime}{\frac{az+zz}{a-z}}}$, $\frac{aa+zz}{a-z}$, $\sqrt{\overset{\prime\prime}{\frac{az+zz}{a-z}}}$, $\sqrt{\overset{\cdot}{\frac{az+zz}{a-z}}}$, $\sqrt{\overset{\cdot\cdot}{\frac{az+zz}{a-z}}}$, $\sqrt{\overset{\cdot\cdot\cdot}{\frac{az+zz}{a-z}}}$; Et Notandum est quod quantitatis quælibet prior in his seriebus est area figuræ Curvilineæ cujus ordinatim applicata rectangula est quantitas <84r> posterior et abscissa z: uti $\sqrt{\overset{\prime\prime}{aa-zz}}$ area Curvæ cujus ordinata est $\sqrt{aa-zz}$ et abscissa z. Hanc aream sic etiam designo $\left[\sqrt{aa-zz}\right]$ vel etiam sic z. Hanc aream sic etiam designo $\boxed{\sqrt{aa-zz}}$. Quo autem spectant hæc omnia patebit in Propositionibus quæ sequuntur.

Prop. I. Prob: I

Data æquatione quotcunque fluentes Quantitates involvente
invenire fluxiones.

Solutio

Multiplicetur omnis æquationis terminus per Indicem dignitatis quantitatis cujusque fluentis quam involvit, et in singulis multiplicationibus mutetur dignitatis latus in suam fluxionem et aggregatum factorum omnium sub propriis signis exit nova æquatio.

Explicatio.

Sunto a, b, c, d, &c quantitates determinatæ et immutabiles et proponatur æquatio quavis quantitates fluentes z, y, x, &c involvens, uti $x^3 - xyy + aaz - b^3 = 0$ Multiplicentur termini primo per Indices Dignitatum x et in singulis multiplicationibus pro dignitatis latere seu x unius dimensionis scribatur $\dot{x}$ et summa factorum erit $3\dot{x}x^2 - \dot{x}yy$. Idem fiat in y et prodibit $-2xy\dot{y}$. Idem fiat in z et prodibit $aa\dot{z}$, ponatur summa factorum æqualis nihilo et habebitur æquatio $3\dot{x}x^2 - \dot{x}yy - 2xy\dot{y} + aa\dot{z} = 0$ Dico quod hac æquatione definitur relatio fluxionum.

Demonstratio.

Nam sit o quantitas infinite parva et sunto $o\dot{z}$, $o\dot{y}$, $o\dot{x}$ quantitatum z, y, x momenta, id est incrementa momentanea Synchrona. Et si quantitates fluentes jam sunt z, y, et x hæ post momentum temporis incrementis suis infinite parvis $o\dot{z}$, $o\dot{y}$, $o\dot{x}$ auctæ evadent $z + o\dot{z}$, $y + o\dot{y}$, $x + o\dot{x}$, quaæ in æquatione prima pro z, y, et x scriptæ dant æquationem $x^3 + 3xxo\dot{x} + 3xoo\dot{x}\dot{x} +$
$o^3\dot{x}^3 - xyy - o\dot{x}yy - 2xo\dot{y}y - 2\dot{x}oo\dot{y}y + xoo\dot{y}\dot{y} + \dot{x}o^3\dot{y}\dot{y} + aaz + aao\dot{z} \ -b^3 = 0$. Subducatur æquatio prior et residuum divisum per o erit $3\dot{x}x^2 + 3\dot{x}\dot{x}ox + \dot{x}^3oo - \dot{x}yy - 2x\dot{y}y - 2\dot{x}o\dot{y}y + aa\dot{z} = 0$. Minuatur Quantitas o in infinitum ut momenta fiant infinitissime parva, et <85r> neglectis terminis evanescentibus restabit $3\dot{x}x^2 - \dot{x}yy - 2x\dot{y}y + aa\dot{z} = 0$ Quod erat demonstrandum.

Explicatio plenior.

Ad eundem modum si æquatio esset $x^3 - xyy + aa\sqrt{ax - yy} - b^3 = 0$ produceretur

$3x^2\dot{x} - \dot{x}yy - 2x\dot{y}y + aa\sqrt{\dot{\overline{ax-yy}}} = 0$ Ubi si fluxionem $\sqrt{\dot{\overline{ax-yy}}}$ tollere velis, pone $\sqrt{ax - yy} = z$, et erit

$ax - yy = z^2$ et per hanc propositionem $a\dot{x} - 2\dot{y}y = 2\dot{z}z$ seu $\frac{a\dot{x}-2\dot{y}y}{2z} = \dot{z}$, hoc est $\frac{a\dot{x}-2\dot{y}y}{2\sqrt{ax-yy}} = \sqrt{\dot{\overline{ax-yy}}}$ et inde

$3x^2\dot{x} - \dot{x}yy - 2x\dot{y}y + \frac{a^3\dot{x}-2aa\dot{y}y}{2\sqrt{ax-yy}} = 0$.

Et per operationem repetitam pergitur ad fluxiones secundas, tertias et sequentes. Sit æquatio $zy^3 - z^4 + a^4 = 0$ et fiat per operationem primam $\dot{z}y^3 + 3z\dot{y}y^2 - 4\dot{z}z^3 = 0$. per secundam $\ddot{z}y^3 + 6\dot{z}\dot{y}y^2 +$

$+6z\dot{y}^2y - 4\ddot{z}z^3 - 12\dot{z}^2z^2 = 0$. per Tertiam $\dddot{z}y^3 + 9\ddot{z}\dot{y}y^2 + 9\dot{z}\ddot{y}y^2 +$

$+18\dot{z}\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - 4\dddot{z}z^3 - 36\ddot{z}\dot{z}z^2 - 48\dot{z}^3z = 0$

Ubi vero sic pergitur ad fluxiones secundas tertias et sequentes convenit quantitatem aliquam ut uniformiter fluentem considerare et pro ejus fluxione prima unitatem scribere, pro secunda vero et sequentibus nihil. Sit æquatio $zy^3 - z^4 + a^4 = 0$ ut supra et fluat z uniformiter sitque ejus fluxio Unitas et fiet per operationem primam $y^3 + 3z\dot{y}y^2 - 4z^3 = 0$, per secundam $6\dot{y}y^2 + 3z\ddot{y}y^3 + 6z\dot{y}^2y - -12z^2 = 0$, per tertiam

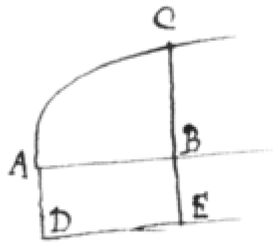
$9\dddot{y}y^2 + 18\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - -48z = 0$

In hujus autem generis æquationibus concipindum est quod fluxiones in singulis terminis sint ejusdem ordinis, id est, vel omnes primi ordinis $\dot{y}$, $\dot{z}$ vel omnes secundi $\ddot{y}$, $\dot{y}^2$, $\dot{y}\dot{z}$, $\dot{z}^2$, vel omnes tertii $\dddot{y}$, $\ddot{y}\dot{y}$, $\ddot{y}\dot{z}$, $\dot{y}^3$, $\dot{y}^2\dot{z}$, $\dot{y}\dot{z}^2$, $\dot{z}^3$ &c. Et ubi res aliter se habet complendus est ordo per subintellectas fluxiones quantitatis uniformiter fluentis. Sic æquatio novissima complendo ordinem tertium sit

$$9\dot{z}\ddot{y}y^2 + 18z\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3y - 48z\dot{z}^3 = 0$$

Termini æquationis primæ ut Genitores terminorum æquationis cujus vis alterius quæ per hanc propositionem ex prima prodit et termini ex eodem Genitore nati, ut fratres vel Socii considerari possunt. Innotescunt fratres et ex fratribus Genitor, mutando fluxiones in fluentes quantitates. Sic in æquatione Novissima termini omnes præter ultimum mutando fluxiones in fluentes quantitates evadunt mzy^3, ideoque fratres sunt et genitorem habent in mzy^3, et terminus ultimus qui solitarius est, eadem ratione <86r> migrat in Genitorem nz^4; hic m et n pro coefficientibus determinatis indefinite ponuntur, et inveniri possunt quærendo fratres ex his genitoribus et comparando cum fratribus datis: Sic ex genitore mzy^3 prodeunt fratres $9m\dot{z}\ddot{y}y^2$, $18m\dot{z}\dot{y}^2y$, $3mz\dddot{y}y^2$, $18mz\ddot{y}\dot{y}y$, $6mz\dot{y}^3$, $48mz\dot{z}^3$ qui cum fratribus datis collati dant $m = 1$.

Prop: II. Prob: II.
Invenire curvas quæ quadrari possint



Sit ABC figura invenienda, BC ordinatim applicata rectangula et AB abscissa; producatur BC ad E, ut sit $BE = 1$ et compleatur parallelogrammum ABDE. et arearum ABC, ABDE fluxiones erunt ut BC ad BE; Assumatur igitur æquatio quævis qua relatio arearum definiatur et inde dabitur relatio ordinatarum BC et BE per prop. I Q.E.I. Hujus rei exempla habentur in propositionibus duabus sequentibus.

Prop. III. Theor. I

Si pro abscissa AB et area AE seu $AB \times 1$ promiscuo scribatur z, et si pro $e + fz^\eta + gz^{2\eta} + hz^{3\eta} + \&c$ scribatur R sit autem area Curvæ $z^\theta R^\lambda$ erit ordinatim applicata BC =
$$\theta e + \underset{\lambda\eta}{\theta} fz^\eta + \underset{2\lambda\eta}{\theta} gz^{2\eta} + \underset{3\lambda\eta}{\theta} hz^{3\eta} + \&c \quad \text{in} \quad z^{\theta-1}R^{\lambda-1}$$

Demonstratio.

Nam si sit $z^\theta R^\lambda = 0$ erit per Prop: 1 $\theta\dot{z}z^{\theta-1} + \lambda z^\theta\dot{R}R^{\lambda-1} = \dot{v}$ Pro R^λ in primo æquationis termino et z^θ in secundo scribe $RR^{\lambda-1}$ et $zz^{\theta-1}$ et fiet $\theta\dot{z}R + \lambda z\dot{R}$ in $z^{\theta-1}R^{\lambda-1} = \dot{v}$. Erat autem $R = e + fz^\eta + gz^{2\eta} + hz^{3\eta}$ &c et inde per prop I fit $R = \eta f\dot{z}z^{\eta-1} + 2\eta g\dot{z}z^{2\eta-1} + 3\eta h\dot{z}z^{3\eta-1} + \&c$ huic ductæ in λz adde $\theta\dot{z}R$ et summa ducta in $z^{\theta-1}R^{\lambda-1}$, si modo pro $\dot{z}$ scribatur BE sive 1, fiet
$$\theta e + \underset{\lambda\eta}{\theta} fz^\eta + \underset{2\lambda\eta}{\theta} gz^{2\eta} + \underset{3\lambda\eta}{\theta} hz^{3\eta} + \&c \quad \text{in} \quad z^{\theta-1}R^{\lambda-1} = \dot{v} = BC$$. Q.E.D.

Prop IV. Theor II

Si Curvæ Abscissa AB sit z et si pro $e + fz^\eta + gz^{2\eta} + hz^{3\eta} + \&c$ scribatur R et pro $k + lz^\eta + mz^{2\eta} + nz^{3\eta} + \&c$ scribatur S: sit autem area curvæ $z^\theta R^\lambda S^\mu$: erit ordinatim applicata.

<87r>

$$\left.\begin{array}{llllllll}
\theta ek & \begin{matrix}+\theta\\+\lambda\eta\end{matrix} fkz^{\eta} & \begin{matrix}+\theta\\+2\lambda\eta\end{matrix} gkz^{2\eta} & \&c & & \\[2ex]
 & \begin{matrix}+\theta\\+\mu\eta\end{matrix} elz^{\eta} & \begin{matrix}+\theta\\+\lambda\eta\\+\mu\eta\end{matrix} flz^{2\eta} & \begin{matrix}+\theta\\+2\lambda\eta\\+\mu\eta\end{matrix} glz^{3\eta} & \&c & \\[2ex]
 & & \begin{matrix}+\theta\\+2\mu\eta\end{matrix} emz^{2\eta} & \begin{matrix}+\theta\\+\lambda\eta\\+2\mu\eta\end{matrix} fmz^{3\eta} & \begin{matrix}+\theta\\+2\lambda\eta\\+2\mu\eta\end{matrix} gmz^{4\eta} &
\end{array}\right\} \quad \text{in} \quad z^{\theta-1}R^{\lambda-1}S^{\mu-1}.$$

Demonstratur ad modum præcedentis propositionis cum scilicet ordinatim applicata sit $\theta\dot{z}RS+\lambda z\dot{R}S+\mu zR\dot{S} \quad \text{in} \quad z^{\theta-1}R^{\lambda-1}S^{\mu-1}$.

Prop V. Theor. III.

Si Curvæ Abscissa AB sit z, et pro $e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}+\&c$ scribatur R. Sit autem ordinatim applicata $z^{\theta-1}R^{\lambda-1} \quad \text{in} \quad a+bz^{\eta}+cz^{2\eta}+dz^{3\eta}+\&c$ et ponatur $\frac{\theta}{\eta}=r.\ r+\lambda=s,\ s+\lambda=t,\ t+\lambda=v$ &c erit area

$$z^{\theta}R^{\lambda} \quad \text{in} \quad \frac{\frac{1}{\eta}a}{re}+\frac{\frac{1}{\eta}b-sfA}{\overline{r+1}\ ,\ e}z^{\eta}+\frac{\frac{1}{\eta}c \begin{matrix}-s\\-1\end{matrix} fB-tgA}{\overline{r+2}\ ,\ e}z^{2\eta}+\frac{\frac{1}{\eta}d \begin{matrix}-s\\-2\end{matrix} fC \begin{matrix}-t\\-1\end{matrix} gB-ukA}{\overline{r+3}\ ,\ e}z^{3\eta}+\frac{\begin{matrix}-s\\-3\end{matrix} fD \begin{matrix}-t\\-2\end{matrix} gC \begin{matrix}-v\\-1\end{matrix} hB}{\overline{r+4}\ ,\ e}z^{4\eta}+\&c.$$

Ubi A, B, C, D, &c. denotant totas Coefficientes datas terminorum singolorum in serie cum signis suis + et −, nempe A primi termini coefficientem $\frac{\frac{1}{\eta}a}{re}$, B secundi coefficientem $\frac{\frac{1}{\eta}b-sfA}{\overline{r+1}\ ,\ e}z^{\eta}$, C tertii coefficientem $\frac{\frac{1}{\eta}c \begin{matrix}-s\\-1\end{matrix} fB-tgA}{\overline{r+2}\ ,\ e}z^{2\eta}$ et sic deinceps.

Demonstratio.

Sunto juxta propositionem tertiam Curvarum Ordinatæ et earundum Areæ

$$\begin{array}{llllll}
1 & \theta eA & \begin{matrix}+\theta\\+\lambda\eta\end{matrix} fAz^{\eta} & \begin{matrix}+\theta\\+2\lambda\eta\end{matrix} gAz^{2\eta} & \begin{matrix}+\theta\\+3\lambda\eta\end{matrix} hAz^{3\eta} & \&c \\[2ex]
2 & & \theta+\eta\ eBz^{\eta} & \begin{matrix}+\theta+\eta\\+\lambda\eta\end{matrix} fBz^{2\eta} & \begin{matrix}+\theta+\eta\\+2\lambda\eta\end{matrix} gBz^{3\eta} & \&c \\[2ex]
3 & & & \theta+2\eta\ eCz^{2\eta} & \begin{matrix}+\theta+2\eta\\+\lambda\eta\end{matrix} fCz^{3\eta} & \&c \\[2ex]
4 & & & & \theta+3\eta\ eDz^{3\eta} & \&c
\end{array}\left.\vphantom{\begin{matrix}1\\1\\1\\1\\1\\1\\1\\1\end{matrix}}\right\} \quad z^{\theta-1}R^{\lambda-1} \quad \left\{\begin{array}{l} Az^{\theta}R^{\lambda} \\[3ex] Bz^{\theta+\eta}R^{\lambda} \\[3ex] Cz^{\theta+2\eta}R^{\lambda} \\[3ex] Dz^{\theta+3\eta}R^{\lambda}\end{array}\right.$$

Et si summa ordinatarum ponatur æqualis ordinatæ $a+bz^{\eta}+cz^{2\eta}+dz^{3\eta}$ &c $\quad \text{in} \quad z^{\theta-1}R^{\lambda-1}$, summa Arearum $z^{\theta}R^{\lambda} \quad \text{in} \quad A+Bz^{\eta}+Cz^{2\eta}+Dz^{3\eta}$ &c æqualis erit Areæ Curvæ cujus ista est Ordinata; Æquentur igitur Ordinatarum termini correspondentes et fiat $a=\theta eA,\ b=\begin{matrix}\theta\\\lambda\eta\end{matrix} fA \begin{matrix}+\theta\\+\eta\end{matrix} eB$,

$c=\begin{matrix}\theta\\2\lambda\eta\end{matrix} gA \begin{matrix}+\theta+\eta\\+\lambda\eta\end{matrix} fB + \theta+2\eta\ eC$ &c . et inde $\frac{a}{\theta e}=A.\ \frac{b-\overline{\theta+\lambda\eta}fA}{\theta+\eta \quad \text{in} \quad e}=B$, ac pariter $\frac{c-\overline{\theta+2\lambda\eta}gA-\overline{\theta+\eta+\lambda\eta}fB}{\overline{\theta+2\eta}\ ,\ e}=C$ et sic deinceps in infinitum. Pone jam $\frac{\theta}{\eta}=r.\ r+\lambda=s.\ s+\lambda=t$ &c et in Area $z^{\theta}R^{\lambda} \quad \times \quad A+Bz^{\eta}+Cz^{2\eta}+Dz^{3\eta}$ &c scribe ipsorum A, B, C, &c valore inventos et prodibit series proposita. Quod erat Demonstrandum.

Et nota quod Ordinata omnis duplici modo in seriem resolvitur. Nam Index η vel affirmativus esse potest vel negativus. Proponatur Ordinata $\frac{3\mathrm{k}-\mathrm{lzz}}{\mathrm{zz}\sqrt{\mathrm{kz}-\mathrm{lz}^3+\mathrm{mz}^4}}$. Hæc vel sic scribi potest

$\mathrm{z}^{-\frac{5}{2}} \times \overline{3\mathrm{k}-\mathrm{lzz}} \times \overline{\mathrm{k}-\mathrm{lzz}+\mathrm{mz}^3\,|}^{-\frac{1}{2}}$ vel sic $\mathrm{z}^{-2} \times \overline{-\mathrm{l}-3\mathrm{kz}^{-2}} \times \overline{\mathrm{m}-\mathrm{lz}^{-1}+\mathrm{kz}^{-3}}^{-\frac{1}{2}}$. In casu priori est $\mathrm{a}=3\mathrm{k}$**{.}** <88r> $\mathrm{b}=0$, $\mathrm{c}=-\mathrm{l}$, $\mathrm{e}=\mathrm{k}$, $\mathrm{f}=0$, $\mathrm{g}=-\mathrm{l}$, $\mathrm{h}=\mathrm{m}$, $\lambda-1=-\frac{1}{2}$ $\lambda=\frac{1}{2}$, $\eta=1$, $\theta-1=-\frac{5}{2}$ $\theta=-\frac{3}{2}=\mathrm{r}$. $\mathrm{s}=-1$. $\mathrm{t}=-\frac{1}{2}$. $\mathrm{v}=0$. In posteriori est $\mathrm{a}=-\mathrm{l}$. $\mathrm{b}=0$ $\mathrm{c}=3\mathrm{k}$. $\mathrm{e}=\mathrm{m}$. $\mathrm{f}=-\mathrm{l}$. $\mathrm{g}=0$. $\mathrm{h}=\mathrm{k}$. $\lambda=\frac{1}{2}$**{.}** $\eta=-1$. $\theta-1=-2$. $\theta=-1$ $\mathrm{r}=1$. $\mathrm{s}=$—. $\mathrm{t}=$—. $\mathrm{v}=$—. Tentandus est casus uterque et si Serierum alterutra ob terminos tandem deficientes abrumpitur ac terminatur, habebitur area Curvæ in terminis finitis. Sic in exemplo hujus priore casu scribendo in serie Valores ipsorum a, b, c, e, f, g, h, λ, θ, r, s, t, u, termini omnes post primum evanescunt in infinitum et Area Curvæ prodit $-2\sqrt{\frac{\mathrm{k}-\mathrm{lzz}+\mathrm{mz}^3}{\mathrm{z}^3}}$. Et hæc Area ob signum negativum adjacet abscissæ ultra ordinatam productæ. Nam area omnis affirmativa adjacet t**{**a**}**m abscissæ quam ordinatæ, negativa vero cadit ad contrarias partes ordinatæ et adjacet abscissæ productæ. Hoc modo Series alterutra et nonnumquam utraque semper terminatur & finita evadit, si curva Geometrice quadrari potest. At si Curva talem Quadraturam non admittit, series utraque continuabitur in infinitum, et earum altera converget et aream dabit approximando, præterquam ubi r (propter aream infinitam) vel nihil est vel numerus integer et negativus, vel ubi $\frac{\mathrm{z}}{\mathrm{e}}$ equalis est unitati. Si $\frac{\mathrm{z}}{\mathrm{e}}$ minor est unitate converget Series, in qua index η affirmativus est: Sin $\frac{\mathrm{z}}{\mathrm{e}}$ unitate major est converget series altera. In uno casu area adjacet abscissæ in altero adjacet abscissæ ultra ordinatam productæ

Nota insuper quod si Ordinata contentum est sub factore Rationali Q et factore surdo irreducibili R^{π}, et factoris surdi latus R non dividit factorem Rationalem Q; erit $\lambda-1=\pi$ et $\mathrm{R}^{\lambda-1}=\mathrm{R}^{\pi}$. Sin factoris Surdi latus R dividit factorem rationalem semel erit $\lambda-1=\pi+1$ et $\mathrm{R}^{\lambda-1}=\mathrm{R}^{\pi+1}$: Si dividit bis erit $\lambda-1=\pi+2$ et $\mathrm{R}^{\lambda-1}=\mathrm{R}^{\pi+2}$. Si ter erit $\lambda-1=\pi+3$ et $\mathrm{R}^{\lambda-1}=\mathrm{R}^{\pi+3}$ et sic deinceps.
