Copy letter from Newton to John Smith, dated 24 July 1675

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Sir,

I received your former Letter as well as your later, and should have written to you sooner, but that I stay'd to think of something that might satisfy your Desire; But though I can not hitherto doe it to my owne liking, yet that I may not wrack your patience too much I have here writt you what occurrs to mee, which is only about facilitating the Extraction of √. The former Method might be applyed to determin all by every 1000th, as well as by every 100th √, but not with advantage, for it will require the Extraction of √ to 14 or 15 places, besides a greater number of Additions, Subductions & Divisions in those greater numbers: And therefore I have rather sent you the following Notes about Extracting √.

1.) When you have extracted any √ by common Arithmetick to 5 Decimal places, you may get the figures of the other 6 places by Dividing only the Residuum by

$$\left.\begin{array}{l} \text{double the Quotient} \\ \text{triple the } q \text{ of the Quotient} \\ \text{quadruple the } c \text{ of the Quotient} \end{array}\right\} \text{ for the } \sqrt{} \left\{\begin{array}{l} \text{square} \\ \text{cube} \\ \text{square square} \end{array}\right.$$

Suppose B. the Quotient or √ extracted to 5 Decimal places, and C. the last Residuum, by the Division of which you are to get the next figure of the Quotient, and D the Divisor (that is 2B or 3BB or $4B^{.c.} = D$ & $B + \frac{C}{D}$ shall be the √ desired. That is, the same Division, by which you would finde the 6th decimal figure, if prosecuted, will give you all to the 11th decimal figure.

2) You may seek the √ if you will, to 5 Decimal places by the logarithm's, But then you must finde the rest thus. Divide the propounded number $\left.\begin{array}{l}\text{once}\\\text{twice}\\\text{thrice}\end{array}\right\}$ by that √ prosecuting the Division alwayes to 11 Decimal places, and to the Quotient add

$$y^e \text{ said } \sqrt{} \left\{\begin{array}{l} \text{once, \& halfe} \\ \text{twice, \& a third part} \\ \text{thrice, \& a quarter} \end{array}\right\} \begin{array}{l}\text{of the summ}\\\text{shall be the}\end{array} \left\{\begin{array}{l} \text{square} \\ \text{Cube} \\ \text{square square} \end{array}\right\} \sqrt{} \text{ desired.}$$

<div align="center">For instance</div>

$$\text{let A be the number, and B. its } \left\{\begin{array}{l} Q \\ C \\ QQ \end{array}\right. \sqrt{} \text{ extracted by Logarithms unto 5 decimal places:}$$

$$2)\ \mathrm{B} + \frac{\mathrm{A}}{\mathrm{B}},\qquad\qquad\qquad\qquad\left.\begin{array}{l} \mathrm{Q} \\[6pt] \mathrm{C} \\[6pt] \mathrm{QQ} \end{array}\right.$$

and

$$3)\ 2\mathrm{B} + \frac{\mathrm{A}}{\mathrm{B}^2},\ \text{shall be the}\qquad\qquad \mathrm{C}\quad\text{root desired}$$

$$4)\ 3\mathrm{B} + \frac{\mathrm{A}}{\mathrm{B}^3},$$

Note that you have according to my former Direction but 76 Q ℞ & 88 C ℞ & 94 QQ ℞ to extract, whereof 10 are exact ℞. But I think you will doe well to lett the Table of QQ ℞ alone, til you have done the other two, and then, if you finde your time too short, print the Q. & C. ℞ without troubling your selfe any further.

Sir, I am,   your humble Servant

Is: Newton

< insertion from the left margin >

Cambridge July 24th 1675

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For Mr Collins Copie Mr Newtons 2^d Letter concerning extraction of ℞