

Printed Copy of a Letter from John Bernoulli to Sir Isaac Newton, with Newton's Observations on it

Author: Isaac Newton

Source: MS Add. 3968, ff. 473r-485v, Cambridge University Library, Cambridge, UK

<34:i(r)>

29. Iulii 1713.

L us nunc Viennæ Austriæ agens ob distantiam locorum nondum vidit libellum in Anglia nuper editum, quo N o primam inventionem Calculi differentialis vindicare quidam conantur. Ne tamen commentum mora invalescat, quam primum retundi debere visum est. Equidem negare non potuerunt novam hanc Analyticam Artem primum a L o fuisse editam (cum diu satis pressisset) & publice cum, amicis excultam; & post complures demum annos a N o aliis notis & nominibus, quendam quem vocat Calculum Fluxionum, Differentiali similem, fuisse productum; qui tamen tunc nihil contra L um movere ausus est. Nec apparet quibus argumentis nunc velint L um hæc a N o didicisse, qui nihil tale unquam cuiquam quod constet communicavit, antequam ederet. L us tamen ex suo candore alios æstimans, libenter fidem habuit Viro talia ex proprio ingenio sibi fluxisse dictanti; atque ideo scripsit N um aliquid calculo differentiali simile habuisse videri. Sed cum postremo intelligeret, facilitatem suam contra se verti, & quosdam in <34:i(v)> Anglia præpostero gentis studio eousque progressos, ut non N um in communionem inventi vocare, sed se excludere non sine vituperii nota vellent, & N um ipsum (quod vix credibile erat) illaudabili laudis amore contra conscientiae dictamen tandem figmento favere; re attentius considerata, quam alias præoccupato in N ni favorem animo examinaturus non fuerat, ex hoc ipso processu a candore alieno suspicari coepit, Calculum Fluxionum ad imitationem Calculi Differentialis formatum fuisse. Sed cum ipse per occupationes diversas rem nunc discutere non satis posset, ad iudicium primarii Mathematici, & harum rerum peritissimi, & a partium studio alieni recurrendum sibi putavit. Is vero omnibus excussis ita pronuntiavit literis 7. Iunii 1713. datis:

Videtur N us occasionem nactus serierum opus multum promovisse per Extractiones Radicum, quas primus in usum adhibuit, & quidem in iis excolendis ut verisimile est ab initio omne suum studium posuit, nec credo tunc temporis vel somniavit adhuc de Calculo suo fluxionum & fluentium, vel de reductione ejus ad generales operationes Analyticas ad instar Algorithmi vel Regularum Arithmeticarum aut Algebraicarum. Ejusque meæ conjecturæ [primum] validissimum indicium est, quod de literis x vel y punctatis, uno, duobus, tribus, &c. punctis superpositis, quas pro dx, ddx, d³x; dy, ddy, &c. nunc adhibet, in omnibus istis Epistolis [Commercii Epistolici Collinsiani, unde argumenta ducere volunt] nec volam, nec vestigium invenias. Imo ne quidem in principiis Naturæ Mathematicis N i, ubi calculo suo fluxionum utendi tam frequentem habuisset occasionem, eius vel verbulo fit mentio, aut notam hujusmodi unicam cernere licet, sed omnia fere per lineas figurarum sine certa Analysis ibi peraguntur more non ipsi tantum, sed & Hugenio, imo jam antea [in nonnullis] dudum Torricellio, Robervallio, Cavallerio, aliis, usitato. Prima vice hæc literæ punctatæ comparuerunt in tertio Volumine Operum Wallisii, multis annis postquam Calculus differentialis iam ubique

locorum invaluisse. Alterum indicium, quo coniecere licet Calculum fluxionum non fuisse natum ante Calculum differentialem, hoc est, quod veram rationem fluxiones fluxionum capiendi, hoc est differentiandi differentialia, N us nondum cognitam habuerit, quod patet ex ipsis Principiis <34:iv(r)> Phil. Math. ubi non tantum incrementum constans ipsius x, quod nunc notaret per x punctatum uno puncto, designat per o [more vulgari, qui calculi differentialis commoda destruit] sed etiam regulam circa gradus ultiores falsam dedit [quemadmodum ab eminente quodam Mathematico dudum notatum est] Saltem apparet, N o rectam Methodum differentiandi differentialia non innotuisse longo tempore, postquam aliis fuisset familiaris &c. Haec ille.

Ex his intelligitur N um, cum non contentus laude promotæ synthetice vel linealiter per infinite parva, vel (ut olim minus recte vocabant,) indivisibilia Geometriæ; etiam inventi Analytici seu calculi differentialis a L o in Numeris primum reperti, & (excogitata *Analysi infinitesimalium*) ad Geometriam translatis, decus alteri debitum affectavit, adulatoribus rerum anteriorum imperitis nimis obsecutum fuisse, & pro gloria, cujus partem immeritam aliena humanitate obtinuerat, dum totam appetit, notam animi parum æqui sincerique meruisse: de quo etiam Hookium circa Hypothesin planetariam, & Flamstedium circa usum observationum, questos ajunt.

Certe aut miram ejus oblivionem esse oportet, aut magnam contra conscientiae testimonium iniquitatem, si accusationem (ut ex indulgentia colligas) probat, qua quidam ejus asseclæ etiam seriem, quæ arcus circularis magnitudinem ex tangente exhibet, a Gregorio hausisse L um volunt. Tale quiddam Gregorium habuisse, ipsi Angli & Scoti, Wallisius, Hookius, Newtonus & junior Gregorius, prioris credo ex fratre nepos, ultra triginta sex annos ignorarunt, & L i esse inventum agnoverunt. Modum quo L us ad seriei Nicolai Mercatoris (primi talium inventoris) imitationem invenit seriem suam, ipse statim Hugenio B. Lutetiæ agenti communicavit, qui & per Epistolam laudavit. Eundem sibi communicatum laudavit ipse mox N us, fassusque est in litteris hanc novam esse Methodum pro Seriebus, ab aliis quod sciret nondum usurpatam, Methodum deinde generalem series inveniendi, pro curvarum etiam transcendentium ordinatis in Actis Lipsiensibus editam, non per Extractions dedit, quibus N us usus est, sed ex ipso <34:iv(v)> fundamento profundiore Calculi differentialis L us deduxit. Per hunc enim calculum etiam res serierum ad majorem perfectionem deducta est. Ut taceam *Calculi exponentialis*, qui transcendentis perfectissimus est gradus, quem L us primus exercuit, Johannes vero Bernoullius proprio Marte etiam assecutus est, nullam N o aut ejus discipulis notitiam fuisse: & horum aliquos, cum etiam ad Calculum differentialem accedere vellent, lapsus subinde admisisse, quibus eum parum sibi intellectum fuisse prodiderunt, quemadmodum ex junioris Gregorii circa Catenariam paralogismo patet. Cæterum dubium non est, multos in Anglia præclaros viros hanc N ianorum Asseclarum vanitatem & iniquitatem improbaturos esse; nec vitium paucorum genti imputari debet.

<34:ii(r)>

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<473r>

Whereas a Letter dated 7 June 1713 & published in July or August 1673 without the name of the Author or Printer or Place in Germany where it was printed was then dispersed by M^r Leibnitz & his correspondents & is now reprinted in the Novells Literairs at Hage under the title of a Letter written by M^r Iohn Bernoulli of Basil, & to

Whereas a Libel dated 7 Iune 1713 was pretended to be written to M^r Leibnitz; & was inserted into another Libel dated 29 Iuly {17}13 & both of them by the procurement of M^r Leibnitz were published {wi}thout the names of the authors or writer or place in Germay where they were printed & were secretly dispersed by M^r Leibnitz & his correspondents & {wolu}{illeg}{an}s & the said Libell of 7 Iun. 1713 is newly reprinted in the Nouvelles Litterairs at the Hage under the title of a Letter written by M^r Iohn Bernoulli of Basil: these are to give notice that the true Author of the said Letter speaks of M^r I. Bernoulli as distinct from himself in these words [quemadmodum ab eminente quodam Mathematico dudum notatum est], which words {we}re fraudulently omitted in the Nouvelles Litterairs The aforesaid two Libells are both of them written in the style of M^r Leibnitz & if he pretends that he did not write them himself he knows the authors & i{t} lies upon him to discover his confederates It lies upon M^r Bernoulli also to cleare himself from being the author {of the} Letter here laid to his charge or else to justify it against {the a}nswer made to it by D^r Iohn Keill

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Charge

I received of E{u^d} Nicolas Esque at one time 250^{li} at another 125. Total received — } 375.00.00

Vpon recconing with the Princes Administrators I paid back the ballance of the Account the same being 25^{li}. 3^s. 00^d & took his Receipt in full of all Accounts 18 Apr. 171{8} } 25. 3. 0

The total expence — 349. 17. 0

Discharge

Paid to M^r Churchil for Paper & Printing — 194. 13 {illeg}

To M^r Flamsteed for his Copy — 125. 00. 00

To M^r Machin for correcting the copy by the Minute book & examining some calculations — } 30. 00. 00

349. 17. 00

Sometime after this, D^r Halley undertook to finish the book & the Referees of the Prince acted no further & after the work was finished & the Accounts stated, moneys were impressed to me without Account to pay them o{ff}

Charge

Received — 364^{li}. 15^s. 00^d

Discharge

Paid to M^r Churchil for paper & printing — 98. 11. 00

Paid for designing & graving the draughts & rolling off the Plates 116. 4. 7 $\frac{1}{2}$

Paid to D^r Halley 150. 0. 0

364. 15. 7 $\frac{1}{2}$

Bendes 20^{li} paid to Sen^r Catenaro which I did not bring to account.

Whereas it has been pretended that M^r Leibnitz desired a Mathematician of the first rank to examin the *Commercium Epistolicum* published by order of the R. S. & that the Mathematician wrote back his opinion in a Letter dated 7 Iuly 1713. And whereas the said Mathematician in that Letter quotes the authority of M^r Iohn Bernoulli in these words [quemadmodum ab eminente quodam Mathematiko dudum notatum est] those are to give notice that the said Letter is newly reprinted in French in the *Novelles Litteraires* pag. 414 & there said to be written by M^r Iohn Bernoulli of Basil & the words [quemadmodum ab eminente quodam Mathematiko dudum notatum est] are omitted that the fraud may not be perceived.

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Cum Charta quædam rixis & accusationibus plena impressa fuerit per D. Menkenium in Actis Eruditorum mense Iulio anni 1716 pro D. Ioanne Bernoullio contra D. Keill sine nomine Authoris; et in eadem D. Bernoullius vocatus fuerit excelsum ingenium & vir ad abstrusa & abdita detegenda natus, quasi is non fuisset ejusdem Author, & Author tamen formulam quandam æquationis Bernoullianæ formulam meam vocando, Chartam illam ipsi D. Bernoullio attribuerit, & D. Bernoullius eandem a se scriptam fuisse nondum publice negaverit: D. Bernoullius & D. Menkenius postulantur ut quis fuerit chartæ illius verus author notum faciant.

Et cum D. Bernoullius in Epistola quadam ad D. Libnitium 7 Iunij 1713 data citetur nomine eminentis cujusdam Mathematici quasi ipse non esset author istius epistolæ, et hoc non obstante D. Leibnitius in Epistolis pluribus affirmaverit D. Bernoullium ejusdem authorem fuisse: postulatur D. Bernoullius ut ipse declaret quis sit ejusdem author.

In Synopsi Libri de Quadratura Curvarum a D. Menkenio in Actis Eruditorum pro mense Ianuario anni 1705, liber ille plagarij accusator his verbis: Ingeniosissimus deinde Author [Newtonus] antequam ad Quadraturas Curvarum (vel potius Figurarum Curvilinearum) veniat, præmittit brevem Isagogem: Quæ, VT MELIVS INTELLIGATVR, sciendum est, cum magnitudo aliqua continuo crescit, incrementa illa momentanea appellari differentias, nempe inter magnitudinem quæ antea erat & quæ per mutationem momentaneam est producta; atque hinc natum esse Calculum Differentialem eique reciprocum summatoorium cujus elementa ab INVENTORE D. Godefrido Guilielmo Leibnitio in his Actis sunt tradita variique usus tum ab ipso tum a D. Marchione Hospitalio — sunt ostensi. Pro diffentijs IGITVR Leibnitianis D. Newtonus adhibet semperque [pro ijsdem] adhibuit fluxiones — ijsque tum in suis Principijs Naturæ Mathematicis tum in alijs postea editis [pro differentijs Leibnitianis] eleganter est usus, QVEMADMODVM et Honoratus Fabrius in sua Synopsi Geometrica, motuum progressus Cavallerianæ methodo SVBSTITVIT. Hæc accusatio initium dedit controversiæ huic de auctore hujus methodi, proindeque D. Menkenius qui eandem edidit rogatur ut auctorem accusationis hujus prodatur.

In epistola prædicta septimo Iunij 1713 data quam D. Leibnitius D. Bernoullio attribuit, accusatio eadem repetitur, & affirmatur quod Newtonus ubi scripsit antiquas Epistolas in Commercio epistolico impressas ne quidem somniavit de calculo suo fluxionum & fluentium, propterea quod in ijsdem nullæ sunt literæ punctis notatæ. Imo ne quidem in Principijs naturæ Mathematicis: sed prima vice hæ literæ punctatæ Comparuerunt in tertio Volumine Operum Wallisij, multis annis postquam Calculus differentialis jam ubique locorum <474v> invaluisse. Affirmatur etiam Newtono rectam Methodum differentiandi differentia non innotuisse, longo tempore postquam alijs fuisset familiaris. Rogatur igitur D. Bernoullius ut consulat Volumen secundum operum Wallisij pag. 391, 392, 393 &c et tunc agnoscatur publice, literas punctatas lucem vidisse anno 1693, annis nimirum sex antequam Volumen tertium prodijt. Agnoscat etiam Propositionem primam libri de Quadratura Curvarum & solutionem ejus cum exemplis in fluxionibus primis & secundis ibi impressam fuisse, idque propemodum verbatim atque adeo Librum illum tunc in M.S. latuisse. Agnoscat etiam Regulam ibi traditam pro inveniendis fluxionibus secundis tertijs quartis alijsque in infinitum veram esse et lucem vidisse annis tribus antequam Regula aliqua pro inveniendis differentijs secundis tertijs quartis alijsque lucem videre

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Whereas a squabbling defamatory Paper was printed by D^r Menkenius in favour of M^r I. Bernoulli against D^r Keill in the *Acta Eruditorum* for Iuly 1716 without the name of the Author, & therein M^r Bernoulli is

called excelsum ingenium & vir ad abstrusa & abditas detegenda natus, as if he were not the Author of that Paper & yet the Author of the Paper by naming M^r Bernoulli's formula of an Equation meam formulam (pag. 314) he ascribed that Paper to M^r Bernoulli himself, & M^r Bernoulli has not yet disowned it publicly: he & D^r Menkenius are desired to tell the world who was the author thereof, & And whereas in another defamatory paper dated 29 July 1713 & printed without the name of the author a Letter to M^r Leibnitz is inserted dated 7 July 1713 in which M^r Bernoulli is cited by the name of an eminent Mathematician as if he were not the author of that Letter & yet M^r Leibnitz has in several Letters affirmed that M^r Bernoulli was the author of that Letter: M^r Bernoulli is desired to tell the world who was the author of that Letter. And whereas in another paper published by D^r Menkenius in the Acta Eruditorum for Ian. 1705, in giving an Account of the book De quadratura Curvarum the Book is accused as a piece of Plagiarism, in these words; Ingeniosissimus deinde author [Newtonus] antequam ad Quadraturas Curvarum vel potius Figurarum Curvilinearum) veniat, præmittit brevem Isagogem. Quæ ut MELIVS intelligatur, sciendum est, cum magnitudo aliqua continuo crescit, incrementa illa momentanea appellari differentias, nempe inter magnitudinem quæ antea erat & quæ per mutationem momentaneam est producta; atque hinc natum est Calculum Differentialem, eique reciprocum summatorium; cujus elementa ab INVENTORE D. Godefrido Guilielmo Leibnitio in his Actis sunt tradita, variique usus tum ab ipso tum a D. D fratribus Bernoullijs tum a D. Marchione Hospitalio — sunt ostensi. Pro differentijs IGITUR Leibnitianis D. Newtonus adhibet semperque [pro Differentiis illis] adhibuit Fluxiones — ijsque tum in suis Principijs Naturæ Mathematicis tum in alijs postea editis [pro differentiis Leibnitianis] eleganter est usus, QVEMADMODVM et Honoratus Fabrius in sua synopsi Geometrica, motuum progressus Cavallerianæ methodo SVBSTITVIT. D^r Menkenius who published this defamatory Paper is desired to tell the world who was the Author.

And whereas in the aforesaid Letter of 7 June 1713 ascribed by M^r Leibnitz to M^r John Bernoulli, this accusation of plagiarism is pursued, & the world is told that M^r Newton did not so much as dream of the Calculus of fluxions when he wrote the ancient Letters & Papers published in the commercium Epistolicum, ② no nor when he wrote his Principia Naturæ Mathematica, ④ because there are no prickt letters in them ③ because these Letters First appeared in the third Volume of Wallises works many years after the Differential Calculus was every where known, & that M^r Newton did not understand how to find second differences till it was familiar to others: M^r Bernoulli is desired to look into the second Volume of the Works of D^r Wallis pag. 391 392, 393 &c & then tell the world whether prickt Letters did not come abroad in the year 1693 six years before the third Volume was published & whether the first Proposition of the book of Quadratures with the solution & examples in first & second fluxions be not there published almost verbatim & whether the Rule there given for finding second third & fourth fluxions &c be not genuine & did not come abroad some years before any Rule for finding second third & fourth differences <476v> & whether the method of fluxions be not taught in the Introduction to the very book of Quadratures without the use of prickt letters: For as he has publicly accused M^r Newton of Plagiarism by feigned pretences so he ought to make a publick Recantation if his accusation be not true.

And whereas it is pretended that an Italian named Ricatti has made many experiments with Lenses & Prisms of crystal of the Rock, which destroy the Hypothesis of M^r Newton about apparent colours: & yet M^r Newton assumes no hypothesis nor meddles with apparent colours & Prisms of Crystall by reason of a double refraction are unfit for such experiments & M^r Ricatti instead of sending an experiment to prove his assertion, has sent a challenge to solve a mathematical Probleme & thereby discovered that he is in a confederacy with M^r Bernoulli, & upon these considerations he is looked upon here by some as a Pretender & a bully: for clearing himself from this imputation he is desired to communicate to the world any one of his experiments which contradicts M^r Newton's Theory.

M^r Raphson a little before his death published that the Book of Quadratures which came abroad in the year 1704, was about the year 1676 written from a former Tract, & that about the year 1691 D^r Halley & he had it in their hands at Cambridge in order to bring it up to London; & D^r Halley remembers that this was presently after the election of M^r Raphson into the R. Society, which was in 1690. And therefore the Book was written before M^r Bernoulli knew any thing of the Differential Method, & for him to accuse it of Plagiarism is to tell

the world that he has a mind to something that is in it. It is easy to add to inventions, & if he has improved the method the improvements are his own, & he is at liberty to publish them when ever he pleases. But the vanity introduced by him & M^r Leibnitz of challenging every body to solve their Problems, has not yet obtained in England, & we look up it with contempt.

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Whereas a wrangling Paper was published in the Acta Eruditor{um} for Iuly 1716 — who is the author thereof. And whereas in the same paper M^r Iohn Bernoulli is said to have invented Rules for integrating differential quantities, & given copies of his MS to the Marquess de l'Hospital, & to M^r Herman & other : if he has improved the inverse method of fluxions, the improvements will be allowed to be his when ever he pleases to publish them. But it will not follow from them that the book of Quadratures published by M^r Newton A.C. 1704 is a piece of plagiarism as in his Letter to M^r Leibnitz dated 13 Iune 1713 he as represented, & as the author of another scandalous Paper printed in the Acta Eruditorum A.C. 1705 pag. (suspected by some {to} be M^r Iohn Bernulli or one of his disciples) has also feigned.

The first Proposition of this Book with it solution & examples in first & second fluxions was published almost verbatim by D^r Wallis in the second Volume of his works A.C. 1693, being sent to the Doctor & printed off the year before; & therefore the Book was then in manuscript. M^r Raphson has published that D^r Halley & he had it in their hands at Cambridge about the year 1691 in order to bring it up to London & D^r. Halley remembers that it was in A.C. 1690 & thence it may be understood that it was in MS before the Rules invented & composed by M^r Bernoulli. In M^r Newtons Letters of Iune 13 Octob. 24th & Novem 8th 1676 there are many things relating to this Book & therefore it was in Manuscript before M^r Leibnitz knew any thing of the Differential Method.

In this Book are many things which had they been proposed as Problems to be solved, might have puzzled all the Mathematicians in Europe, as for instance to reduce the integration of \dot{y} \dot{y} in the following equations

$\frac{dz z^{2n-1}}{e+fz^n+gz^{2n}} = \dot{y} \cdot \frac{dz z^{\frac{1}{2}n-1}}{e+fz^n+gz^{2n}} = \dot{y} \cdot \frac{dz z^{\frac{3}{2}n-1}}{e+fz^n+gz^{2n}} = \dot{y} \cdot \frac{dz}{z} \sqrt{2+fz^n+gz^{2n}} = \dot{y}$. Or to reduce the integration of \dot{y} to the simplest cases of quadratures in the following equations $az^{\dot{p}+q}z^m + bz^q\dot{z}\dot{y}^p = cz^p\dot{y}^q$.

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Sir

I received your Letter with the Proposals inclosed & desire you to acquaint Mons^r Bernsdorf that

The Longitude is already found at Land by Astronomy & good clockwork together & the method is practised every day with good success for rectifying Geography & I am of opinion that there is no other way to find it at sea then by improving this. This has been my opinion above these th{e}rty years & I reported it to the Committee of the House of Commons to whom the buisines of the {l vas} & now repeat the report . If any serious man hath any other method to propose, he would endeavour that it should not be referred {t}o me because I have made a report already against his project. & am now too old to change my opinions. Proposals relating to sea affairs are usually referred to Trinity House [& the proper way to obtain such a Reference is to apply to the board of the Admiralty] And if any Projector declines such a Reference his business is something else then the Longitude .

Sir

I received your Letter with the Paper which Mon^r de Bernsdorf ordered you to send to me, & desire you to acquaint him that the Longitude is already found at Land by Astronomy & good Clockwork together, & the method is practised every-day with success for rectifying Geography: & I have been many years of opinion that there is no other way of finding it by sea then by pursuing & improving this method. [This has been my

opinion above these thirty years & I reported it to the Committee of the House of Common who summoned me to attend them about this matter & I can make no other report then what I have made already . for I am too old to change my opinions or to take these matters into fresh consideration.] And having reported this opinion I am incapable of meddling with the Paper which you sent me it being contrary to my report. But if other Gentlemen have a mind that it should be examined, I will not oppose them.

Sir

I received the Proposals which Mon^r de Bernstorff desired you to send to me, & beg the favour of you to acquaint him that I do not approve of them & desire that the person who makes them may not be sent to me. I am

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29 Iulij 1713.

L us nunc Viennæ Austriæ agens ob distantiam locorum nondum vidit libellum in Anglia nuper editum, quo N o primam inventionem Calculi differentialis vindicare quidam conantur. Ne tamen commentum mora invalescat, quam primum retundi debere visum est. Equidem negare non poterunt novam hanc Analyticam Artem primum a L o fuisse editam (cum diu satis pressisset) & publice cum amicis excultam; et post complures demum annos a N o alijs notis & nominibus, quendam quem vocat calculum fluxionum, Differentiali similem, fuisse productum; qui tamen tunc nihil contra L um movere ausus est. Nec apparet quibus argumentis nunc velint L um hæc a N o didicisse qui nihil tale unquam cuiquam quod constet communicavit, antequam ederet. L us tamen ex suo candore alios æstimans, libenter fidem habuit Viro talia ex proprio ingenio sibi fluxisse dictanti; atque ideo scripsit N um aliquid calculo differentiali simile habuisse videri. Sed cum postremo intelligeret, facilitatem suam contra se verti, & quosdam in Anglia præpostero gentis studio eousque progressos, ut non N um in communionem inventi vocare, sed se excludere non sine vituperij nota vellent, et N um ipsum (quod vix credibile erat) illaudabili laudis amore contra conscientiæ dictamen tandem figmento favere; re attentius considerata, quam alias præoccupato in N ni favorem animo examinaturus non fuerat, ex hoc ipso processu a candore alieno suspicari cœpit, Calculum Fluxionum ad imitationem Calculi Differentialis formatum fuisse. Sed cum ipse per occupationes diversas rem nunc discutere non satis posset, ad iudicium primarij Mathematici, et harum rerum peritissimi et a partium studio alieni recurrendum sibi putavit. Is vero omnibus excussis ita pronuntiavit literis 7 Iunij 1713 datis:

Videtur N us occasionum nactus serierum opus multum promovisse per extractiones Radicum, quas primus in usum adhibuit, & quidem in ijs excolendis ut verisimile est omne suum studium posuit, nec credo tunc temporis vel somniavit adhuc de calculo suo fluxionum et fluentium, vel de reductione ejus ad generales operationes analyticas ad instar Alorithmi vel regularum Arithmeticarum aut Algebraicarum. Ejusque meæ conjecturæ [primum] validissimum indicium est, quod de literis x vel y punctatis, uno, duobus, tribus, &c punctis superpositis quas pro dx, ddx, d³x; dy, ddy, &c nunc adhibet, in omnibus istis Epistolis <478v> [Commercij Epistolici Collinsiani, unde argumenta ducere volunt] nec volam nec vestigium invenias. Imo ne quidem in Principijs Naturæ Mathematicis N i, ubi calculo suo fluxionum utendi tam frequentem habuisset occasionem, ejus vel verbulo fit mentio, aut notam hujusmodi unicam cernere licet sed omnia fere per lineas figurarum sine certa Analysis ibi peraguntur more non ipsi tantum, sed et Hugenio imo jam antea [in nonnullis] dudum Torricellio, Robervallio, Cavallerio, alijs, usitato. Prima vice hæc literæ punctatæ comparverunt in tertio Volumine Operum Wallisij, multis annis postquam Calculus differentialis jam ubique locorum invaluisse. Alterum indicium quo conjicere licet Calculum fluxionum non fuisse natum ante Calculum Differentialem, hoc est, quod veram rationem fluxiones fluxionum capiendi hoc est differentiandi differentialia, N us nondum cognitam habuerit, quod patet ex ipsis Principijs Phil. Math. ubi non tantum incrementum constans ipsius x quod nunc notaret per x punctatum uno puncto, designat per o [more vulgari qui calculi differentialis commoda destruit] sed etiam regulam circa gradus ultiores falsam dedit [quemadmodum ab eminente quodam Mathematico dudum notatum est] Saltem apparet, N o rectam methodum differentiandi differentialia non innotuisse longo tempore, postquam alijs fuisset familiaris. &c. Haec ille.

Ex his intelligitur N um, cum non contentus laude promotæ synthetice vel linealiter per infinite parva, vel (ut olim minus recte vocabant) indivisibilia Geometriæ; etiam inventi Analytici seu calculi differentialis a L o in numeris primum reperti, & (excogitata Analysis infinitesimalium) ad Gemetriam translati, decus alteri debitum affectavit, adulatoribus rerum anteriorum imperitis nimis obsecutum fuisse, et pro gloria, cujus partem immeritam aliena humanitate obtinuerat, dum totam appetit, notam animi parum æqui sincerique meruisse: de quo etiam Hookium circa Hypothesin Planetariam, et Flamstedium circa usum observationum, quæstos aiunt.

Certe aut miram ejus oblivionem esse oportet, aut magnam contra conscientiæ testimonium iniquitatem, si accusationem (ut ex indulgentia colligas) probat, qua quidam ejus asseclæ etiam seriem, quæ arcus circularis magnitudinem ex tangente exhibet, a Gregorio hausisse L um volunt. Tale quiddam Gregorium habuisse ipsi Angli & Scoti, Wallisius Hookius Newtonus & junior Gregorius, prioris credo ex fratre nepos, ultra triginta sex annos ignorarunt, & L i esse inventum agnoverunt. Modum quo L us ad seriei Nicolai Mercatoris (primi talium inventoris) imitationem invenit seriem suam, ipse statim Hugenio B. Lutetiæ agenti communicavit, qui et per Epistolam laudavit. Eundem sibi communicatum laudavit ipse mox N us fassusque est in Literis hanc novam esse Methodum pro Seriebus, ab alijs quod sciret nondum usurpatam. Methodum deinde generalem series inveniendi, pro curvarum etiam transcendentium ordinatis <479r> in Actis Lipsiensibus editam, non per extractiones dedit, quibus N us usus est, sed ex ipso fundamento profundiore Calculi differentialis L us deduxit. Per hunc enim Calculum etiam res serierum ad majorem perfectionem deducta est. Vt taceam calculi exponentialis, qui transcendentis perfectissimus est gradus, quem L us primus exercuit, Iohannes vero Bernoullius proprio Marte etiam assecutus est, nullam N o aut ejus discipulis notitiam fuisse: & horum aliquos, cum etiam ad calculum differentialem accedere vellent, lapsus subinde admisisse, quibus eum parum sibi intellectum fuisse prodiderunt, quemadmodum ex junioris Gregorij circa Catenariam paralogismo patet. Cæterum dubium non est, multos in Anglia præclaros viros hanc N ianorum Asseclarum vanitatem & iniquitatem improbaturos esse; nec vitium paucorum genti imputari debet.

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The following Letter to M^r Leibnitz was written originally in Latin, & we have met with the ensuing Observations upon it

Videtur N us occasionum nactus serierum opus multum promovisse per Extractions Radicum, quas primus in usum adhibuit, et quidem in ijs excolendis ut verisimile est ab initio omne suum studium posuit, nec credo tunc temporis¹ vel somniavit adhuc de calculo suo fluxionum et fluentium, vel de reductione ejus ad generales operationes Analyticas ad instar Algorithmi vel regularum Arithmeticarum aut Algebraicarum. Ejusque meæ conjecturæ [primum] validissimum indicium est,² quod de literis x vel y punctatis, uno, duobus, tribus, &c punctis superpositis, quas pro dx, ddx, d³x; dy, ddy, &c nunc adhibet, in omnibus istis Epistolis [Commercij Collinsiani, unde argumenta ducere volunt] nec volam, nec vestigium invenias. Imo ne quidem in Principijs Naturæ Mathematicis N i,³ ubi calculo suo fluxionum utendi tam frequentem habuisset occasionem, ejus vel verbulo fit mentio, aut notam hujusmodi unicam cernere licet, sed omnia fere per lineas figurarum sine certa Analysis ibi peraguntur more non ipsi tantum, sed et Hugenio imo jam antea [in nonnullis] dudum Torricellio, Robervallio, Cavallerio, alijs, usitato. Prima⁴ vice hæ literæ punctatæ comparuerunt in tertio Volumine Operum Wallisij, multis annis postquam Calculus differentialis jam ubique locorum invaluisse. Alterum indicium, quo conjicere licet Calculum fluxionum non fuisse natum ante Calculum differentialem, hoc est,⁴ quod veram rationem fluxiones fluxionum capiendi, hoc est differentiandi differentialia, N us nondum cognitam habuerit, quod patet ex ipsis Principijs Phil. Math. ubi⁶ non tantum incrementum constans ipsius x, quod nunc notaret per x punctatum uno puncto, designat per o [more vulgari, qui calculi differentialis commoda destruit] sed etiam regulam circa gradus ultiores falsam dedit [quemadmodum⁷ ab eminente quodam Mathematico dudum notatum est] Saltem apparet N o rectam methodum differentiandi differentialia non innotuisse longo tempore, postquam alijs fuisset familiaris &c.

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Observatons upon the foregoing Letter.

D^r Wallis an older & abler Mathematician has attested in the Preface to the first Volume of his works that M^r Newton explained this Method to M^r L z in his Letters of 13 Iune & 24 Octob. 1676 & invented it ten years before or above In the second Lemma of the second Book of Mathematical Principles the Elements of this Method are taught & demonstrated; & in the Introduction to the book of Quadratures the Method it self is expresly taught & illustrated by examples, & in M^r Newtons Letter of 10 Decem 1672 the Method is plainly described & illustrated with an example of Drawing Tangents thereby: & all this is done without the use of prickt letters. The Book of Principles was writ by Composition & therefore there was no occasion of using the calculus of fluxions in it. Prickt letters appeared in the second Volume of the works of D^r Wallis, & this Volume was almost all printed in the year 1692 & came abroad the next spring before the Differential method began to make a noise. The Manuscript of the Book of Quadratures was in the hands of D^r Halley & M^r Raphson in the year 1691 as both of them have attested. And this Book is sufficiently described in M^r Newtons Letters of Octob. 24 & Novem 8 1676, & the Quadratures there cited out of it are not to be attained without the Method in dispute. The constant fluxion of x M^r Newton denotes by \dot{x} with a point above it; but the constant increment or moment of x M^r Newton denotes not by \dot{x} but by o & still uses this Notation as convenient: M^r Leibnitz hath no Notation for fluxions, & therein his Method is defective. The Only Rule which M^r Newton has given for finding first second third fourth & other fluxions is contained in the first Propoosition of the Book of Quadratures & is a very true one & was published with examples in the second Volume of the works of D^r Wallis before any other Rule for finding second third & fourth Differences came abroad. And the very words of the Proposition were set down in the Scholium upon the second Lemma of the second Book of Principles, & in M^r Newtons Letter of 24 Octob. 1676, as the foundation of the method of fluxions. And the inverse of this Rule is the first Rule in the Analysis per series numero terminorum infinitas communicated by D^r Barrow to M^r Collins in Iuly 1669 & that Rule is demonstrated by the method of fluxions in the end of that Analysis. And without this Rule the Series for Quadratures which break off & become finite when the Curve can be squared by a finite equation, & which are mentioned in the said Analysis, are not to be attained 4. In translating this Letter of 7. Iune 1713 into French & printing it in the Nouvelles Literairs Decem. 28 1715, we are told that the Author of this Letter was M^r Iohn Bernoulli, & to make this credible, the citation of the Eminent Mathematician is omitted in the body of the Letter. For if M^r I. Bernoulli be the Eminent Mathematician there cited by the author of the Letter he cannot be the Author himself. And M^r I. Bernoulli in a Letter to M^r Newton hath positively declared that he was not the author thereof. And if he had been the author thereof he would not have said that the Eminent Mathematician charged M^r Newton with a false Rule. That Mathematician noted only that there was an error in the Solution of Prob. III Lib. II Princip. Philos. & suspected that it lay in second differences. M^r Nicolas Bernoulli told M^r Newton what his Vnkle had observed. M^r Newton upon examining the Solution found that the error lay in drawing the Tangent of the Arch GH from the wrong end of the arch, & corrected the error himself & told him that the solution should be set right in the new edition of the Principles. The Tangents of the Arcs GH & HI are the first moments of the arcs FG & FH & should have been drawn the same way with the motion of the body describing the Curve FGHK, whereas through in advertency the tangent of the Arc FG had been drawn the contrary way from the point of contact.. There is an error of greater consequence committed in second dif{f}erences by M^r Leibniz in his Tentamen de motuum cœlestium causis, sect 15. which tho complained of is not yet set right, nor so much as acknowledged.

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M^r Leibniz appealed from the commercium Epistolicum to the judgment of the anonymous Author of this Letter: but D^r Wallis an older & abler Mathema-

5. M^r Nicholas Bernoulli told M^r Newton in his Vnkles name in autumn 1712 that there was an error in the conclusion of the solution of Prob. III Lib. II Princip. Phil. Math. M^r Newton corrected the error himself, shewed him the correction & told him that the Proposition should be reprinted in the new Edition which was

then coming abroad. The Tangents of the Arcs GH & HI are first moments of the Arcs FG & FH & should have been drawn the same way with the motion describing those arcs, whereas through inadvertency one of them had been drawn the contrary way, & this occasioned the error in conclusion.

There is an error of much greater consequence committed in second Differences by M^r L z A.C. 1689 in his Tentamen de motuum cœlestium causis sect 15: which tho often complained, of is not yet corrected nor so much as acknowledged. Nor doth it appear by any instance that M^r L z knew how to work in second differences before M^r Newtons Book of Principles came abroad: whereas it plainly appears by Prop. XIV. Lib II Princip. that M^r Newton when he wrote that Book knew how to work in second differences which he there calls Differentia Momentorum id est momentum differentię. And M^r Leibnitz himself in the Acta Eruditorum for May 1700 has acknowledged that M^r N. was the first who shewed by a specimen made publick, that he had the method of maxima & minima in infinitesimals, & there called it a method of the highest moment & greatest extent. And in the Analysis per series numero terminorum infinitas communicated by D^r Barrow to M^r Collins in the year 1669 M^r Newton mentions that by that method of Analysis Curvarum arę & longitudines &c id modo fiat) exacto et Geometrice determinantur. And how this done by the Method of F{illeg} is explained in the first six Propositions of the Book of Quadratures. And without that Method it cannot be done.

M^r Collins in a Letter to M^r Tho. Strobe dated 26 Iuly 1672, & printed from the Original in the commercium Epistolicum; mentioning the Papers which he had received from D^r Barrow in Iuly 1669, subjoyns: Ex quibus [chartis] et alijs quę olim ab autore cum Barrovio communicata fuerant, patet illam methodum a dicto Newtono aliquot annis antea excogitatam et modo universali applicatam fuisse: ita ut ejus ope in quavis Figura Curvilinea proposita quę una vel pluribus proprietatibus definitur, Quadratura vel Area dictę figurę, ACCVRATA SI POSSIBILIS SIT, sin minus infinite vero propinqua; Evolutio vel longitudo lineę curvę, Centrum gravitatis Figurę, Solida ejus rotatione genita, & eorum superficies: sine ulla radicum extractione [lege exterminatione] obtineri queant. Thus by the testimony of D^r Barrow & M^r Collins, as well as by that of D^r Wallis, M^r Newton had the method described in the first six Propositions of the Book of Quadratures, some years before Iuly 1669. And these three ancient & able Mathematicians knew what they wrote: but the author of the Letter of 13 Iune 1713 above mentioned, wrote only by conjecture. His words are Ejusque conjecturę meę primum validissimum indicium est &c. He accused M^r Newton of plagiary without any better proof then conjecture & therefore is guilty of calumny, even by the concession of M^r Leibnitz in the end of his Letter of 9 April 1716 to M. Conti printed above pag

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M^r Newton in the Introduction to the Book of Quadratures said that he invented the Method of fluxions gradually in the years 1665 & 1666. which was not so much as D^r Wallis had published in the Preface to the first Volume of his works 1695 & notified to M^r Leibnitz in a Letter to him dated 1 Decem. 1696 without being then contradicted [as I understand by the Acta Eruditorum for 169 . This Book was published by M^r Newton in the year 1704 & the next year in giving an Account of it in the Acta Eruditorum for the month of pag M^r Leibnitz is called the Inventor of the Method & the improvement there of is ascribed to M^r Bernoullj & the Marquess de l'Hospital, & thence is inferred that M^r Newton always used fluxions instead of the Leibnitian Differences just as Honoratus Faber substituted the progress of motions for the indivisibles in the Philosophical Transactions A.C. 170 represented that M^r Leibnitz had the Method from M^r Newton. In the Acta Eruditorum for Iune 1696 an Account is given of the two first Volumes of the works of D^r Wallis, & notice is taken of what is published in the Preface to those two Volumes relating to this meethod, without denying what was there said or complaining of the Doctor for saying it. The Author of that Account in relation to what is said in the Preface hath indeed these words. Cæterum ipse Newtonus non minus candore quam præclaris in rem Mathematicam meritis insignis, publice et privatim agnovit Leibnitium tum cum (inter veniente celeberrimo Viro Henrico Oldenburgo Bremensi Societatis Regiæ Anglicanę tunc Secretario) inter ipsos (ejusdem jam tum Societatis Socios) commercium intercederet, id est, jam fere ante annos viginti et amplius, Calculum suum Differentialem, Seriesque Infinitas & pro ijs quoque Methodos

generales habuisse; quod Wallisius in præfatione Operum factæ inter eos communicationis mentionem faciens, præterijt, quoniam deo eo fortasse non satis ipsi constabat. And upon these words there is this Observation in the *Commercium Epistolicum*. Methodum Differentialem Moutoni — — — N . . . s nondum agnovit publice. M^r L . . . z published the Elements of his Method A.C. 1684 without making any mention of the correspondence he had formerly had with me. M^r N . . . n demonstrated the Elements of this Method of moments the next year in the second Lemma of the second book of Principles & added a Scholium not to give away this Lemma to M^r L . . . z but to assert it to himsel in a civil manner, by putting him in mind of that correspondence & of making that acknowledgment publickly which he made privately in his Letter of Iune 21 1677 wherein he first began to communicate his Differential method, & acknowledg that M^r Newton's Method mentioned in his Letters of 13 Iune & 24 Octob 1676 as a part of a Treatise wrote by him in 1671, did the same things.

The *Acta Eruditorum* for Iune 1696 came to the hands of D^r Wallis in the end of November following, & the Doctor in a letter writen to M^r Leibnitz the next day Decem. 1st, has these words. Neque Calculi Differentialis vel Nomen audivisse me memini — — — — id monitum inseruerim. Thus M^r Leibnitz had fresh notice of the Paragraph inserted into the said Preface of the two first Volumes of the Doctors Works, & yet in the Letters which afterwards passed between them denied not what the Doctor had there published nor exprest himself affended at it,

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In autumn 1713 I received from M^r Chamberlain (who then kept a correspondence with M^r Leibnitz) a flying Paper in Latin dated 29 Iuly 1713 in which it was pretended that M^r Leibnitz being then at Vienna had not then the *Commercium Epistolicum* nor had time to examin it himself but had referred it to the judgment of a very famous Mathematician who was impartial & very able to judge of it, & had received his judgment in a Letter dated Iune 7th 1713. And this Letter was inserted into the flying Paper, & in the end of the Letter M^r Iohn Bernoulli was cited by the author of the Letter as a Person different from himself, in these words [quem admodum ab enminente quodam Mathematico dudum notatum est. For these words referred to a Paper publisheed by M^r Iohn Bernoulli in the *Acta Eruditorum* of Feb. et Mar. 1713.

This flying Paper was translated into French & inserted into nother Letter of the same stile of the former & printed at the Hague in Holland in M^r Iohnsons Iournal Litteraire of Novem & Decem 1713 pag 448, 449, 450 & 451. And about two years after M^r Leibnitz began to father the said Letter upon M^r Bernoulli & for tht end to omitt the said citation in the copies of that Letter which he then sent to his friends. For

In November or December 1715 he wrote a Letter to Sen^r Conti with a Postscript in which were these words suivant ce que M. Bernoulli a tres-bien jugé.

And About the same time he sent a letter with the aforesaid flying paper to the author of the *Novelles Litteraires* in Holland who printed them both Decem 28 1715 pag 414: & the sentence [quemadmodum ab eminente quodam Mathematico dudum notatum est] was now left out, & the world was told that M^r Iohn Bernoulli was the author of the aforesaid Letter of 7 1713. And soon after in a Letter written by himself Apr 18 1716 & sent to Madam Kilmanseg he inserted a copy of the same Letter of Iune 17 1713 & ascribed it to M^r Bernoulli omitting the aforesaid sentence by which the Author of the Letter had cited M^r Bernoulli as a man different from himself. And again in the Postscript of a Letter written about the same time to Count Bathmarke he affirmed that M^r Bernoulli was the author. <482v> But these two last Letters were not published till about two years after, & I had no hand in publishing them. The first four or five sheets of the second part of M. Desmaizeans collection in which these letters are contained, were printed off in Holland before I knew any thing of the designe to publish them.

About the time that those Letters went into the Press, M^r Des Maizeaus received from Se^r Conti then at Paris several manuscript pieces of M^r Leibnit as he mentions in his Letter to Sen^r Abbe Conti dated 21 Aug. 1718, & printed in the said second part pag. 362, 363. In that Letter he said that he would add these pieces to the

other writings of M^r Leibnitz which he was going to print in Holland. And you have them in the said second Part. M^r Des-Maizeaus had a correspondence with M^r Leibnitz & was his friend & on that account published this collection of his Remains.

About eleven months after the writing of this Letter, when the Collection was almost printed off except the preface, I received from you M^r Bernoullis Letter dated Iulij 5. 1719 in which he assured me that he wrote no such Letter to M^r Leibnitz as that dated 7 Iune 1713, & in my Answer I acquiesced in that Declaration & have ever since told my friends that I am satisfied that M^r Iohn Bernoulli was not the author of that Letter, [notwithstanding what M^r Leibnitz had written in the affirmative.] The author of the Letter in citing M^r I. Bernoulli as a person different from himself is a witness that M^r I. Bernoulli was not the author thereof. M^r Leibnitz himself in sending that Letter to the Press both in Germany & in Holland in the year 1713 with that citation in it, is witness that he then knew that M^r Iohn Bernoulli was not the author M^r Iohn Bernoulli in affirming per omnia humanitatis sacra that he wrote no such anonymous Letter is a third witness, & the leaving out that citation in order to father the Letter upon M^r I. Bernoulli is a falsification of the Letter. And for these reasons I have been & am still of opinion that M^r I. Bernoulli was not the author.^[1]

Sir

The author of the Letter of Iune 7th 1713, as it was at the first sent to the Press by M^r Leibnitz, cited M^r Iohn Bernoulli as a person different from himself in these words [quemadmodum ab eminente quodam mathematico dudum notatum est.]

In the winter between the years 1715 & 1716 M^r Leibnitz began to father the said Letter upon M^r Leibnitz & for that end to leave out the citations abovementioned in the copies of it which he then sent to his friends: as I find by his Letter to Sen^r Conti writ in November or December 1715 where he has this expression suivant ce que M. Bernoulli a tres bien juge. & by his Letter sent at the same time to the author of the Nouvelles Litteraires in Holland & his Letter of Apr 18 1716 sent to Madam Kilmanseg & his Letter sent about the same time to Count Bothmar.

But by consid

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A Letter dated 7 Iune 1713 being mentioned above as written by M^r Iohn Bernoulli; to set that matter right we will here set down that Letter with the following Observations upon it.

Videtur N us occasionem nactus, serierum opus multum promovisse per Extractiones Radicum, quas primus in usum adhibuit, & quidem in ijs excolendis ut verisimile est ab initio omne suum studium posuit, nec credo tunc temporis ¹ vel somniavit adhuc de Calculo suo fluxionum & fluentium vel de reductione ejus ad generales operationes Analyticas ad instar Algorithmi vel Regularum Arithmeticarum aut Algebraicarum. Ejusque meæ conjecturæ [primum] validissimum indicium est² quod de literis x vel y punctatis uno duobus tribus &c punctis superpositis, quas pro dx, ddx, d³x; dy, ddy, &c. nunc adhibet, in omnibus istis Epistolis [Commercij Epistolici Collinsiani unde argumenta ducere volunt] nec volam nec vestigium invenias. Imo ne quidem in Principijs Naturæ Mathematicis N i, 3 ubi calculo suo fluxionum utendi tam frequentem habuisset occasionem, ejus vel verbulo fit mentio, aut notam hujusmodi unicam cernere licet, sed omnia fere per lineas figurarum sine certa Analysis peraguntur more non ipsi tantum, sed & Hugenio, imo jam antea [in nonnullis] dudum Torricellio, Robervallio, Cavallerio, alijs, usitato. Prima⁴ vice hæ literæ punctatæ comparuerunt in tertio Volumine operum Wallisij, multis annis postquam Calculus differentialis jam ubique locorum invaluisset. Alterum indicium, quo conjicere licet Calculum fluxionum non fuisse natum ante Calculum differentialem, hoc est,⁵ quod veram rationem fluxiones fluxionum capiendi, hoc est differentiandi

differentialia, N us nondum cognitam habuerit, quod patet ex ipsis Principijs Phil. Math. ubi⁵ non tantum incrementum constans ipsius x quod nunc notaret per x punctatum uno puncto, designat per o [more vulgari qui calculi differentialis commoda destruit] sed etiam⁵ regulam circa gradus ultiores falsam dedit [quemadmodum⁷ ab eminente quodam Mathematico dudum notatum est:] Saltem apparet, N o⁵ rectam Methodum differentiandi differentialia non innotuisse longo tempore postquam alijs fuisset familiaris. &c.

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Observations

1. M^r Newton is here accused of plagiarism by a mere conjecture & the whole Letter tends to make it probable contrary to the testimony of an abler & older Mathematician D^r Wallis in the Preface to the first Volume of his works.

2 In the second Lemma of the second Book of Principles the Elements of the Method of fluxions are taught, & in the Introduction to the book of Quadratures the Method it self is expresly taught & illustrated by examples, & all this is done without the use of prickt Letters.

3 In the Book of Principles there was not occasion of using the calculus of fluxions. For the book was writ by the Method of Composition.

4 The third Volume of the works of D^r Wallis came abroad in Spring 1699. And prickt letters appeared in the second Volume of his works which was almost all printed in the year 1692, & came abroad the next Spring before the Differential method began to make a noise. The Manuscript of the book of Quadratures was in the hands of D^r Halley & M^r Raphson in the year 1691 as both of them have attested And this book is sufficiently described in M^r Newtons Letters of Octob 24 & Novem 8. 1676, & the Quadratures there cited out of it are not to be attained without the Method in dispute.

5. The first Proposition of the Book of Quadratures contains the true Rule of finding first second third fourth & other fluxions and was published with examples in the second Volume of the Works of D^r Wallis some years before any other Rule for finding second third & fourth differences came abroad. And the very words of the Rule were set down in the Scholium upon the second Lemma of the second Book of Principles, & in M^r Newtons Letter of 24 Octob 1676, as the foundation of the Method of fluxions.

6 M^r Newton denotes fluents by any letters, their fluxions by the same letters with pricks set over them, & their moments by their fluxions multiplieed by the letter o & its powers o^2 o^3 o^4 . And where he puts the letter o for the constant moment of x he puts an unite for \dot{x} that is for the constant fluxion of x , & still uses this notation as very advantageous. M^r Leibnitz to denote the moments of fluents (which he calls their differences) præfixes d , dd , d^3 , d^4 &c to the fluent but has no notation for their fluxions; which is a defect in his method.

7 By the eminent Mathematician here cited, the Author of the Letter meant a Mathematician different from himself. And M^r Leibnitz could not take them for one & the same person when he first received this Letter & sent it to the Press. And therefore if M^r Iohn Bernoulli was the eminent Mathematician, he was not the author of the Letter. However, in printing this Letter translated into French, in the Nouvelles Litteraires Decem 28 1715, we are told that the Author of this Letter was M^r Iohn Bernoulli, & to make this credible, the citation of the eminent Mathematician is omitted in the body of the Letter. But M^r Iohn Bernoulli is a Letter to M^r Newton hath positively declared that he was not the author thereof.

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$$16'.57\frac{1}{3} \times 60 :: 1.14\frac{1}{3} \times 15 = 143\frac{1}{3} = 215.$$

1935	$2114 \times \frac{10000}{790}$	79)2114000(26759 $\frac{1}{2}$
$71\frac{2}{3}$	$107\frac{1}{2}$	158
	<hr style="width: 100%;"/>	
	2042 $\frac{1}{2}$	534
	$81\frac{1}{2}$	474
	<hr style="width: 100%;"/>	
	2114.	600
		553
		470
		395
		750
		711
		39

$$32.60,60,4 = 7200,64$$

$$57600$$

$$460800$$

$$\text{Diam } \odot = 17^{\text{IV}}.$$

$$\text{Diam Orbis M.} = 30'''$$

$$\text{As the semidiameter } \square \text{ of Saturns Orb } 953800\square$$

$$\text{To the semidiameter } \square \text{ of the } \odot \quad 942\square$$

$$\text{So is the density of the Suns light at his surface } 10000000000 \text{ or } 1000\square$$

$$\text{To the density of the Sun's light at Saturns Orb. } 10000 \quad 1\square$$

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On the Charta Volans of 1713

Since the printing of the foregoing Letters the folloing Observations upon a flying paper dated have

Observations upon the foregoing Letter

1. The author of the Letter cites M^r Iohn Bernoulli as a person different from himself, & doth it with an Elogium by the title of a certain Eminent Mathematician, & thereby denied that M^r Iohn Bernoulli was the Author.

2 Two years & an half after the publishing of this Letter a translation of it into French was published in the Nouvelles Literaires in Holland & there (as also in a Letter of M^r Leibtz to Madam Kilmansegg) this Letter was ascribed to M^r Iohn Bernoulli & to make this credible the aforesaid citation was omitted. Which act of omitting it is not justifiable but savours of falsification

3 M^r Iohn Bernoulli could not but know that the Elements of the Method of fluxions were set down without the use of prickt Letters in the second Lemma of the second book of math. Principles of Philos., & that there was no occasion to use prickt Letters in that book because it was written by the method of Composition, & that the first Proposition of the Book of Quadratures was published with prickt letters in the second Volume of the works of D^r Wallis which came abroad in Spring 1693 at which time the Differential Method was only begining to make a noise {As} that that this Proposition contains the true Rule of finding first second third fourth & other fluxions & was there illustrated with examples in first & second fluxions & that th{is} was the

first Rule made publick for finding second & third differences. And all this makes it highly improbable that M^r Bernoulli should be the author of the said Letter.

4 M^r Iohn Bernoulli in a letter written M^r Newton 5 Iuly N. S. 1719 hath declared that he was not the Author of the said Letter. His words are.

it was pretended In a flying paper dated 29 Iuly 1713 that the Leibnitz had not seen the book but had written to a Mathematician of the first rank, very skilfull in these matters who upon examining all things had given his opinion of the matter in a Letter dated 7 Iune 1693 as follows.

Videtur N . . . s — postquam alijs fuisset familiaris.

This Letter therefore conteins the judgment or pretended judgment of a Mathematician to whom M^r Leibnitz appealed from the report of the Committee of the R. S. It was printed in Germany without the name of the author or printer or city where it was printed & it was dispersed over Europe two years & an half before we were told that M^r Iohn Bernoulli was the author of it. That he was the Author we were told in the *Novelles Literaires* for the month of 1716 & afterwards in some Letters of M^r L. & to make it probable, the citation of M^r Iohn Bernoulli in the body of the Letter was omitted.

But in that citatio we have the testimony of the author of the Letter that M^r Iohn Bernoulli was not the author. And The omission of the citation amounts to a falsification of the Letter for suppressing this testimony

M^r Iohn Bernoulli could not but know — — — be the author of the said Letter.

M^r Iohn Bernoulli in a letter — — — his words are — And so we have two witnesses the author the Letter & M^r Iohn Bernoulli that M^r Iohn Bernoulli was not the author.

The Letter therefore is of no credit the Author of it being still unknown.

[1] Written in 1720

[2] See above pag. 37
