Draft response from Newton to John Collins's letter of 5 July 1671

Author: Isaac Newton

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Sir

I purposed to have given you a visit at the late solemnity of our Chancellors creation; but I was prevented in that Journey by the suddain surprisall of a fit of sicknesse, which (God bee thanked) I have now recovered. But since I am prevented from making a verball acknowledgment of your undeservd favors, I must bee yet contented to do it in writing. In which respect I find by your last letter, that I still become more your debtor both for the care you take about my concernes & for Borrellius de motionibus. The last winter I reveiwed the introduction & made some few additions to it. & partly upon D^{r} Barro**{ws}** instigation, I began to new methodise the discourse of infinite series, designing to illustrate it with such problems as may (some of them perhaps) be more acceptable then the invention of working by such series. But being suddeinly diverted by some buisiness I have not yet had leisure to return to thos**{e}** thoughts & I feare I shall not before winter. But since you informe me there needs no hast I hope I may get into the humour of completing them before the impression of the introduction because if I must helpe to fill up its title page I had rather annex somthing which I may call my owne & which may bee acceptable to Artists, as well as the other to Tyro's.

There haveing some things past betweene us concerning musicall progressions, & as I remember you desiring me to communicate somthing which I had hinted to you about it, which I then had not (nor have yet) adjusted to practise: I shall in its stead offer you somthing else which I think more to the purpose.

Any musicall progression $\frac{a}{b} \cdot \frac{a}{b+c} \cdot \frac{a}{b+2c} \cdot \frac{a}{b+3c} \cdot \frac{a}{b+4c}$ &c being propounded whose last terme is $\frac{a}{d}$. Suppose e is a meane proportion twixt $b - \frac{1}{2}c$ & $d - \frac{1}{2}c$ or any integrall or broken number that it is convenient by guesse which differs not considerably from it. suppose it intercede the limits $\sqrt{}bd$ & $\sqrt{}\overline{b-c} \times \overline{d-c}$ very near it. & this proportion will give you the summe of all the termes very nearely.

As the Logarithm $\frac{e+\frac{1}{2}c}{e-\frac{1}{2}c}$ to the logarithm of $\frac{d+\frac{1}{2}c}{b-\frac{1}{2}c}$, so is $\frac{a}{e}$ to the desired summe.

Examp suppose the progression bee $\frac{100}{5} \cdot \frac{100}{6} \cdot \frac{100}{7} \cdot \frac{100}{8} \cdot \frac{100}{9} \cdot \frac{100}{10}$. Then is $a = 100$ $b = 5.$ $c = 1.$ $d = 10.$ & $6\frac{1}{2}$ intercedes **{**$\frac{2b-c\times d+\frac{1}{2}c}{b+d}$**}** & $\sqrt{b - \frac{c}{2} \times d + \frac{c}{2}}$ (that is **{** $\begin{matrix} 6 & \& & \sqrt{}50 \\ 6\frac{3}{10} & \& & \sqrt{}47\frac{1}{4} \end{matrix}$ **}**) which I therefore put for e. And work as follows.

$$\begin{array}{rcrl}
45 & & 47{,}25 & \\
2\ 25 & (\ 6{,}3 & 1{,}75 & \\
\underline{94{,}5} & & 1\ 41 & \\
15 & & 34 & .
\end{array}
\qquad
\left(\begin{array}{r} 14 \\ 7 \\ -\ 12 \end{array}\right.$$

<2v>

$\frac{e+\frac{1}{2}c}{e-\frac{1}{2}c} = \frac{7}{6}$ its log. is 0,066917 & The Log of that Logarithm, 4,825731 , substract

$\frac{d+\frac{1}{2}c}{b-\frac{1}{2}c} = \frac{10\frac{1}{2}}{4\frac{1}{2}}$ its log. is 0,367977 & The Log: of that Logarithm 5,565820 } add.

$\frac{a}{e} = \frac{100}{6\frac{1}{2}}$ its logarithm is ______________ 1,187087 }

1,927176 the result, which is

the Logarith of the desired aggregate 84|562

The same by adding the severall termes together will bee found more justly to bee 84|5636

But note that were there more termes inserted into the progression, (as suppose it was $\frac{100}{5} \cdot \frac{100}{5\frac{1}{2}} \cdot \frac{100}{6} \cdot \frac{100}{6\frac{1}{2}} \cdot \frac{100}{7}$. &c) the rule would still more approach to truth.

Secondly that the difference of the denominators ought to be

$\frac{1}{x^3} \cdot \frac{1}{2xx} - \frac{1}{2yy} \cdot y - x \;::\; \frac{1}{2zz} - \frac{1}{2zz+4oz} \cdot o \cdot$

$\frac{oyy-oxx}{2xxyy} = \frac{\overline{y-x} \times oz}{z^4+2oz^3}$

$yy - xx \times z^3 = y - x \times 2xxyy$

$z^3 = \frac{2xxyy}{y+x}$

61 . 71

4,5 . 10,5

47,25

3010300

1.3064250

2.0423786

3,6498436

2 4637523

82125

0731

0,0659285 . 4,81907965

0,3679767 . 5,56582007 }

1,1804561 }

6,74627617

84566 1,92719652

845674 1,92720302

566

6746275

18512583

17853298 7198

659285

To Mr Isaac
Newton fellow of
Trinity Colledge
In
Cambridge