Unpublished Appendix to 'methodus': Problem IX

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innititur: quæque magis perspicua et ornata evadet si fundamenta quædam pro more methodi syntheticæ præsternantur; qualia sunt haec.

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Axiomata.

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Axioma 1. Quæ fluxionibus æqualibus simul generantur sunt æqualia.

Axioma 2. Quæ fluxionibus in data ratione simul generantur, sunt in ratione fluxionum.

Nota , simul generari intelligo quæ tota eodem tempore generantur.

Axioma 3. Fluxio totius æquatur fluxionibus partium simul sumptis.

Ubi nota quod profluxiones affirmativè ac defluxiones negativè ponendæ sint.

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Axioma 4. Fluxiones sunt ut momenta contemporanea fluxionibus istis generata.

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Axioma 4. Momenta contemporanea sunt ut fluxiones.

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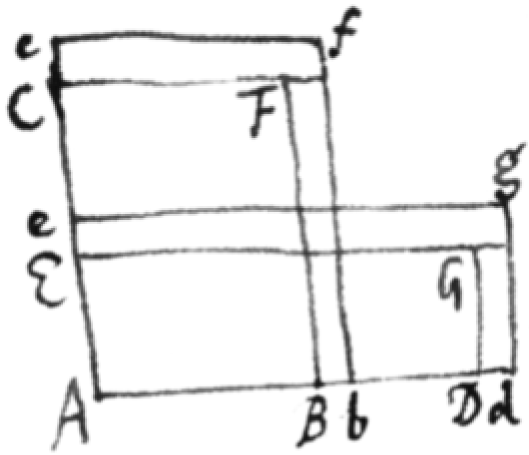
Theoremata.

Theorema 1. Positis quatuor perpetuò proportionalibus fluentibus quantitatibus: summa extremarum reciprocè ductarum in suas fluxiones æquatur summæ mediarum reciprocè ductarum in suas fluxiones. <34> /> **{.}** Sit $\mathrm{AB}\ .\ \mathrm{AD} :: \mathrm{AE}\ .\ \mathrm{AC}$, et erit $\mathrm{AB}\times$ fl: $\mathrm{AC} + \mathrm{AC}\times$ fl: $\mathrm{AB} = \mathrm{AD}\times$ fl: AE $+\mathrm{AE}\times$ fl: AD. Nam augeantur hæ lineæ momentis suis Bb, Dd, Ee, Cc fluendo, & propter perpetuam earum proportionalitatem, adeoque rectangula ab extremis et medijs constituta perpetuo æqualia nempe $\mathrm{AF} = \mathrm{AG}$ & $\mathrm{Af} = \mathrm{Ag}$; augmenta rectangulorum istorum BFCf & DGEg æqualia erunt: hoc est $\mathrm{Ab} \times \mathrm{Cc}$ (Cf)

$+\mathrm{AC} \times \mathrm{Bb}\ \ (\mathrm{bF}) = \mathrm{Ad} \times \mathrm{Ee}\ \ (\mathrm{Eg}) + \mathrm{AE} \times \mathrm{Dd}\ \ (\mathrm{dG})$. * < insertion from the left margin > * sive $\mathrm{Ab} + \mathrm{AC} \times \frac{\mathrm{Bb}}{\mathrm{Cc}} = \mathrm{Ad} \times \frac{\mathrm{Ee}}{\mathrm{Cc}} + \mathrm{AE} \times \frac{\mathrm{Dd}}{\mathrm{Cc}}$. Adeoque cùm fluxiones sint ut momenta quantitatum ab istis continuò generata, hoc est $\frac{\mathrm{Bb}}{\mathrm{Cc}} = \frac{\mathrm{fl\ AB}}{\mathrm{fl\ AC}}$ {,} $\frac{\mathrm{Ee}}{\mathrm{Cc}} = \frac{\mathrm{fl\ AE}}{\mathrm{fl\ AC}}$ & $\frac{\mathrm{Dd}}{\mathrm{Cc}} = \frac{\mathrm{fl\ AD}}{\mathrm{fl\ AC}}$, erit $\mathrm{Ab} + \mathrm{AC} \times \frac{\mathrm{fl\ AB}}{\mathrm{fl\ AC}} = \mathrm{Ad} \times \frac{\mathrm{fl\ AE}}{\mathrm{fl\ AC}} + \mathrm{AE} \times \frac{\mathrm{fl\ AD}}{\mathrm{fl\ AC}}$. Sive $\mathrm{Ab} \times \mathrm{fl\ AC} + \mathrm{AC} \times \mathrm{fl\ AB} = \mathrm{Ad} \times \mathrm{fl\ AE} + \mathrm{AE} \times \mathrm{fl\ AD}$. < text from p 34 resumes > Decrescant jam rectangula Af et Ag donec in prima rectangula AF & AG redierint, & tunc Ab evadet AB atque Ad evadet AD. Quare in ultimo istius infinitè parvæ defluxionis momento, hoc est in primo momento fluxionis quadrangulorum AF et AG quando incipiunt augeri vel diminui, erit $\mathrm{AB} \times \mathrm{fl{:}\ AC} + \mathrm{AC} \times \mathrm{fl\ AB} = \mathrm{AD} \times \mathrm{fl\ AE} + \mathrm{AE} \times \mathrm{fl\ AD}$. Q.E.D.



Corollarium 1. Positis tribus continuè proportionalibus, summa extremarum reciproce ductarum in suas fluxiones æquatur duplo mediæ ductæ in suam fluxionem. Sit $\mathrm{A}\ .\ \mathrm{B} :: \mathrm{B}\ .\ \mathrm{C}$ et erit $\mathrm{A} \times \mathrm{fl{:}\ C} + \mathrm{C} \times \mathrm{fl{:}\ A}$ (= *[1] $\mathrm{B} \times \mathrm{fl{:}\ B} + \mathrm{B} \times \mathrm{fl\ B}) = 2\mathrm{B} \times \mathrm{fl\ B}$.

Corollarium 2. Positis tribus continuè proportionalibus $\mathrm{A}\ .\ \mathrm{B}\ .\ \mathrm{C}$ si summa extremarum sit data quantitas, fluxio minoris extremæ erit ad fluxionem mediæ ut duplum mediæ ad differentiam extremarum. $2\mathrm{B}\ .\ \mathrm{C} - \mathrm{A} :: \mathrm{fl\ A}\ .\ \mathrm{fl\ B}$. Nam cùm $\mathrm{A} + \mathrm{C}$ ex Hypothesi non fluat, *[2] erit $\mathrm{fl{:}\ A} + \mathrm{fl{:}\ C} = 0$, sive <35> /> $\mathrm{fl\ C} = -\ \mathrm{fl\ A}$. adeoque $\mathrm{A} \times \mathrm{fl{:}\ C} = -\mathrm{A} \times \mathrm{fl\ A}$. Quare $\overline{\mathrm{C} - \mathrm{A}} \times \mathrm{fl\ A}\ (= \mathrm{A} \times \mathrm{fl{:}\ C} + \mathrm{C} \times \mathrm{fl\ A})$ *[3] $= 2\mathrm{B} \times \mathrm{fl\ B}$, hoc est $2\mathrm{B}\ .\ \mathrm{C} - \mathrm{A} :: \mathrm{fl\ A}\ .\ \mathrm{fl\ B}$.

Corollarium 3{.} Sin differentia extremarum detur, fluxio alterutrius extremæ erit ad fluxionem mediæ, ut duplum mediæ ad summam extremarum. $2\mathrm{B}\ .\ \mathrm{A} + \mathrm{C} :: \mathrm{fl\ A}\ .\ \mathrm{fl\ B}$. Demonstratur ut Corollarium 2.

Corollarium 4. Quod si summa primæ et secundæ quantitatis detur, erit fluxio secundæ ad fluxionem tertiæ ut prima ad duplum secundæ auctum tertia. $\mathrm{A}\ .\ 2\mathrm{B} + \mathrm{C} :: \mathrm{fl\ B}\ .\ \mathrm{fl\ C}$. Nam cùm ex Hypothesi $\mathrm{A} + \mathrm{B}$ non fluat, *[4] erit $\mathrm{fl\ A} + \mathrm{fl\ B} = 0$ sive $\mathrm{fl\ B} = -\ \mathrm{fl\ A}$, adeoque $\mathrm{C} \times \mathrm{fl\ B} = -\mathrm{C} \times \mathrm{fl\ A}$. Quare $\mathrm{A} \times \mathrm{fl\ C}$ $(= 2\mathrm{B} \times \mathrm{fl\ B} - \mathrm{C} \times \mathrm{fl\ A}) = 2\overline{\mathrm{B} + \mathrm{C}} \times \mathrm{fl\ B}$; hoc est $\mathrm{A}\ .\ 2\mathrm{B} + \mathrm{C} :: \mathrm{fl{:}\ B}$ { . } $\mathrm{fl{:}\ C}$.

Corollarium 5. Si denique differentia primæ et secundæ datur erit fluxio alterutrius ad fluxionem tertiæ ut prima ad duplum secundæ diminutum tertia. $\mathrm{A}\ .\ 2\mathrm{B} - \mathrm{C} :: \mathrm{fl\ B}\ .\ \mathrm{fl\ C}$. Demonstratur ut Corollarium 4.

Corollarium 6. Positis quotcuncue continuè proportionalibus, quarum una sit data quantitas & cæteræ fluentes: fluxiones fluentium erunt inter se ut fluentes illæ ductæ in numerum terminorum quibus distant a dato illo termino. Sint $\mathrm{A}\ .\ \mathrm{B}\ .\ \mathrm{C}\ .\ \mathrm{D}\ .\ \mathrm{E}\ .\ \mathrm{F}$. continuè proportionales et si datur C, erit $-2\mathrm{A}\ .\ -\mathrm{B}\ .\ \mathrm{D}\ .\ 2\mathrm{E}\ .\ 3\mathrm{F} :: \mathrm{fl\ A}\ .\ \mathrm{fl{:}\ B}\ .\ \mathrm{fl\ D}\ .\ \mathrm{fl\ E}$ { . } $\mathrm{fl\ F}$. Nam propter $\mathrm{C}\ .\ \mathrm{D}\ .\ \mathrm{E}$ ∺, est $\mathrm{C} \times \mathrm{fl\ E} + \mathrm{E} \times \mathrm{fl\ C} = 2\mathrm{D} \times \mathrm{fl\ D}$, per Corollarium 1. At ex Hypothesi $\mathrm{fl\ C} = 0$. Ergo $\mathrm{C} \times \mathrm{fl{:}\ E} = 2\mathrm{D} \times \mathrm{fl\ D}$. hoc est $\mathrm{fl\ D}\ .\ \mathrm{fl\ E}\ (:: \mathrm{C}\ .\ 2\mathrm{D}) :: \mathrm{D}\ .\ 2\mathrm{E}$.

Iterum quia $\mathrm{D}\ .\ \mathrm{E}\ .\ \mathrm{F}$ ∺, erit $\mathrm{D} \times \mathrm{fl{:}\ F} + \mathrm{F} \times \mathrm{fl{:}\ D} = \mathrm{E} \times 2\ \mathrm{fl\ E}$. sive $\mathrm{D} \times \mathrm{fl\ F} = \mathrm{E} \times 2\ \mathrm{fl{:}\ E} - \mathrm{F} \times \mathrm{fl{:}\ D}$. Sed e jam ostensis est $\mathrm{fl\ D}\ .\ \mathrm{fl\ E}\ (:: \mathrm{D}\ .\ 2\mathrm{E}) :: \mathrm{E}\ .\ 2\mathrm{F}$. Ergo $\mathrm{F} \times \mathrm{fl\ D} = \frac{1}{2}\mathrm{E} \times \mathrm{fl\ E}$. adeoque $\mathrm{D} \times \mathrm{fl\ F} = \frac{3}{2}\mathrm{E} \times \mathrm{fl\ E}$. Et $\mathrm{fl{:}\ E}\ .\ \mathrm{fl{:}\ F}\ (:: 2\mathrm{D}\ .\ 3\mathrm{E}) :: 2\mathrm{E}\ .\ 3\mathrm{F}$. Atque ita in cæteris.

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Corollarium 7. Si fluentes duæ quantitates se multiplicant fluxio Facti componitur ex fluxionibus factorum alterne ductis in factores. $\mathrm{Fl{:}\ AB} = \mathrm{B} \times \mathrm{fl\ A} + \mathrm{A} \times \mathrm{fl\ B}$ {.} Nam $1\ .\ \mathrm{A} :: \mathrm{B}\ .\ \mathrm{AB}$. Ergo per Theorema 1.

Corollarium 8. Si fluens quantitas per fluentem quantitatem dividitur: fluxio Quoti prodit auferendo fluxionem divisoris multiplicatam per dividuum, a fluxione dividui multiplicata per divisorem & dividendo residuum per quadratum divisoris. $\mathrm{Fl{:}}\ \frac{\mathrm{B}}{\mathrm{A}} = \frac{\mathrm{A} \times \mathrm{fl{:}B} - \mathrm{B} \times \mathrm{fl{:}A}}{\mathrm{AA}}$. Nam $\mathrm{A}\ .\ 1 :: \mathrm{B}\ .\ \frac{\mathrm{B}}{\mathrm{A}}$. Ergo per Theorema 1,

$A \times \mathrm{fl}\,\frac{B}{A} + \frac{B}{A} \times \mathrm{fl}\,A = 1 \times \mathrm{fl{:}}\,B$, nam $B\times$ fl: 1 nihil est. Aufer utrobique $\frac{B}{A} \times \mathrm{fl}\,A$ et residuum divide per A, et prodibit fl: $\frac{B}{A} = \frac{A\times \mathrm{fl}\ B - B\times \mathrm{fl}\ A}{AA}$.

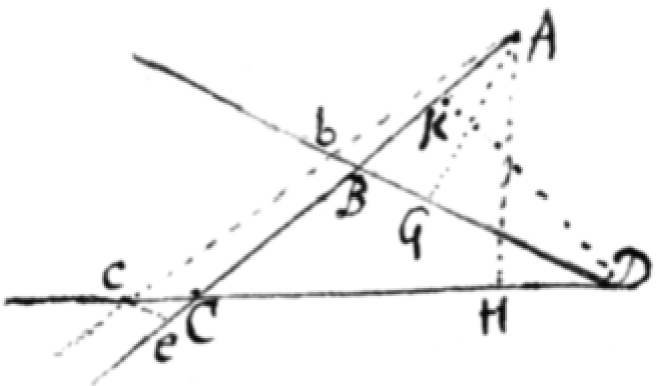
Corollarium 9. Fluxio radicis est ad fluxionem potestatis alicujus ut radix ad potestatem illam multiplicatam per numerum dimensionum fl: $A\ .\ \mathrm{fl}\,A^3 :: A\ .\ 3A^3$. vel fl: $\sqrt{}3 : A\ .\ \mathrm{fl{:}}\,A :: \sqrt{}3 : A\ .\ 3A$ & sic in alijs potestatibus. Patet per Corollarium 6.

Theorema 2. In triangulo quovis perpetim rectangulo cujus latera quomodocunque fluunt summa laterum ductorum in suas fluxiones æquatur hypotenusæ ductæ in fluxionem suam. Sit $AA + BB = CC$ et erit $A\times \mathrm{fl{:}}\,A + B\times \mathrm{fl{:}}\,B = C\times \mathrm{fl{:}}\,C$ Nam per Corollarium 9 Theorematis 1 est fl: $A\ .\ \mathrm{fl{:}}\,AA\ \ (:: A\ .\ 2AA)$ {::}$1\ .\ 2A$, adeoque fl: $AA = 2A \times \mathrm{fl}\,A$. Eadem ratione $\mathrm{fl}\,BB = 2B \times \mathrm{fl}\,B$ & $\mathrm{fl}\,CC = 2C\times \mathrm{fl{:}}\,C$. Quare cum $AA + BB = CC$ atque adeo per Axiomata 1 & 3 fl: $AA+$ fl: $BB =$fl: CC erit $2A \times \mathrm{fl}\,A + 2B \times \mathrm{fl}\,B = 2C \times \mathrm{fl}\,C$. Quod dimidiatum fit $A \times \mathrm{fl}\,A + B \times \mathrm{fl}\,B = C \times \mathrm{fl}\,C$. Q.E.D.

Corollarium 1. Si crus alterutrum sit data quantitas, erit fluxio alterius cruris ad fluxionem hypotenusæ ut hypotenusa ad crus illud alterum. Detur A, et erit $C\ .\ B ::\mathrm{fl{:}}\,B\ .\ \mathrm{fl{:}}\,C$, nam $B \times \mathrm{fl}\,B = C \times \mathrm{fl}\,C$ propterea quòd $A \times \mathrm{fl}\,A$ nihil sit.

Corollarium 2. Si hypotenusa datur, erit profluxio unius <37> /> cruris ad defluxionem alterius ut illud alterum crus ad crus primum. Detur C, et erit $B\ .\ A ::\mathrm{fl{:}}\,A\ .\ \mathrm{fl{:}}\,B$ propterea quod $C\times \mathrm{fl{:}}\,C$ nihil sit.

Theorema 4. Si recta circa datum punctum gyrans, secet alias duas positione datas & ad commune punctum terminatas rectas: fluxiones earum quæ positione dantur, erunt ut illæ rectæ ductæ in conterminas partes lineæ gyrantis. Circa datum punctum A gyret recta AC, & inter gyrandum secet ea rectas positione datas DC ac DB in punctis C et B. Dico esse $DB \times AB\ .\ DC \times AC :: \mathrm{fl}\,DB\ .\ \mathrm{fl}\,DC$. Sit enim Abc positio rectæ gyrantis in proximo temporis momento, hoc est Cc momentum rectæ DC et Bb contemporaneum momentum rectæ DB; et ipsi DB parallela agatur ce occurrens AC in e: et propter similia triangula CBD, Cec, erit $DC\ .\ DB :: Cc\ .\ ce$. Dein propter similia triangula Aec, ABb, erit $Ac\ .\ Ab :: ec\ .\ Bb$, et additis rationibus $DC \times Ac\ .\ DB \times Ab :: Cc\ .\ Bb ::$ (per axioma 4) fl: $DC\ .\ \mathrm{fl{:}}\,DB$. Coeant jam lineæ infinitè parùm distantes Ac & AC, et in momento concursus evadet $DC \times AC\ .\ DB \times AB ::\mathrm{fl{:}}\,DC\ .\ \mathrm{fl{:}}\,DB$. Q.E.D.

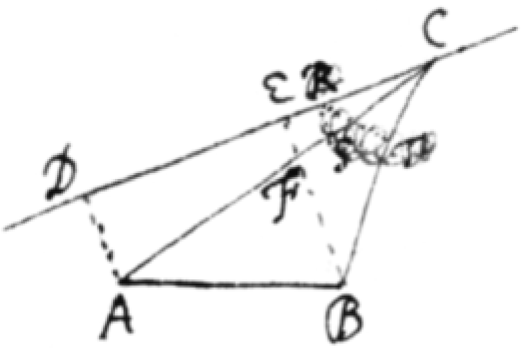


Corollarium 1{.} Iisdem positis, et ab A demissis ad DB et DC normalibus AG et AH: erit primo $DC \times AC\ .\ DB \times BG ::\mathrm{fl{:}}\,DC\ .\ \mathrm{fl{:}}\,AB$. Nam per Corollarium 1 Theorematis 2, est $AB\ .\ BG ::\mathrm{fl{:}}\,DB\ .\ \mathrm{fl{:}}\,AB$. sive $DB \times AB\ .\ DB \times BG ::\mathrm{fl{:}}\,DB\ .\ \mathrm{fl}\,AB$, et supra erat $DC \times AC\ .\ DB \times AB ::\mathrm{fl{:}}\,DC\ .\ \mathrm{fl}\,DB$. Ergo ex æquo $DC \times AC\ .\ DB \times BG :: \mathrm{fl{:}}\,DC\ .\ \mathrm{fl{:}}\,AB$.

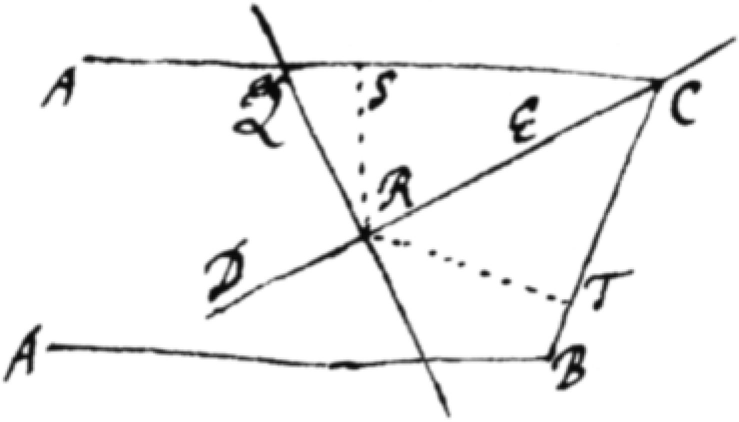
Corollarium 2. Erit secundo $DC \times CH\ .\ DB \times BG :: \mathrm{fl}\,DC\ .\ \mathrm{fl}\,AB$. Nam per Corollarium 1 Theorematis 2 est $CH\ .\ AC$ (vel $DC \times CH\ .\ DC \times AC$)$:: \mathrm{fl}\,AC\ .\ \mathrm{fl}\,DC$. Et supra erat $DC \times AC\ .\ DB \times BG :: \mathrm{fl}\,DC\ .\ \mathrm{fl}\,AB$. Ergo ex æquo $DC \times CH\ .\ DB \times BG :: \mathrm{fl}\,AC\ .\ \mathrm{fl}\,AB$.

Theorema 3{.} Si Trianguli alicujus Basis et longitudine et positione detur, vertex autem sit ad rectam positione datam; demisso ab alterutro termino basis ad rectam illam positione datam perpendiculo, quod occurrat opposito cruri trianguli, erit fluxio ejus oppositi cruris ad fluxionem alterius cruris ut illud alterum crus ad partem hujus cruris inter verticem trianguli et perpendiculum illud situm. Sit AB basis trianguli, C vertex, DE locus verticis, et <38> /> BE perpendiculariter demissa ad DC occurrat AC in F, eritque $BC\ .\ FC :: \mathrm{fl}\,AC\ .\ \mathrm{fl}\,BC$. Nam ab A ad DE demisso perpendiculo AD, erit (per Corollarium 1. Theorematis 2) $BC\ .\ EC :: \mathrm{fl}\,EC\ .\ \mathrm{fl}\,BC$, et $DC\ .\ AC$ (sive $EC\ .\ FC$) $:: \mathrm{fl}\,AC\ .\ \mathrm{fl}\,BC = \mathrm{fl}\,EC$. Ergo ex æquo perturbatè $BC\ .\ FC :: \mathrm{fl}\,AC\ .\ \mathrm{fl}\,BC$. Q.E.D.

Corollarium 1{.} Est fl: AC ad fl: BC ut cosinus anguli ACD ad cosinum angulum BCD. Nam cosinus isti sunt ut BC ad FC.

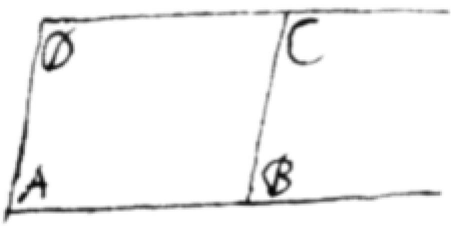


Corollarium 2. Si punctum A infinitè distet a B, hoc est si AC sit ipsi AB parallela, age quamvis RQ occurentem AC in Q, sitque RQ positione data, et demisso ad BC normali RT, erit $\mathrm{TC\ .\ QC} ∷ \text{fl: } \mathrm{BC\ .\ fl\ QC}$. Nam demisso insuper ad QC normali RS, erit per Corollarium 1, $\mathrm{TC\ .\ SC} ∷ \text{fl: } \mathrm{BC\ .\ fl\ AC} = \text{fl: } \mathrm{SC}$, & propter datam rationem SC ad QC erit per axioma 2 $\mathrm{SC\ .\ QC} ∷ \text{fl: } \mathrm{SC\ .\ fl: QC}$. Ergo additis rationibus $\mathrm{TC\ .\ QC} ∷ \text{fl: } \mathrm{BC\ .\ fl: QC}$. Q.E.D.



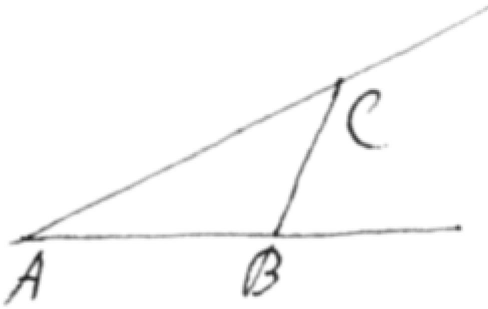
Hujusmodi alia Theoremata non inutilia proponi possent: sed ad fluxiones superficierum festinamus.

Theorema 5. Si recta quævis motu parallelo per aream aliquam, a duabus parallelis & positione datis rectis terminatam transferatur: erit fluxio areæ ut fluxio alterutrius rectæ parallelæ. Sint AB, DC rectæ parallelæ, & BC recta per spatium interjectum ADCB in data inclinatione ABC translata et AD terminus a quo incipit transferri; et erit fl: ADCB ut $\mathrm{fl\,AB}$. Nam area BD est ut longitudo AB Quare per Axioma 2 fl: BD ut fl: AB.



Scholium. Si angulus ABC rectus sit, tum quemadmodum statui solet $\mathrm{BD} = \mathrm{AB} \times \mathrm{BC}$, sic nos statuemus <39> /> $\mathrm{fl\,BD} = \mathrm{fl\,AB} \times \mathrm{BC}$ hoc est (per Corollarium 7 Theorematis 1) $= \mathrm{BC} \times \mathrm{fl\,AB}$. Sed hic sicut per $\mathrm{AB} \times \mathrm{BC}$ non intelligitur linea sed productum arithmeticum quod exprimit numerum unitatum in area BD ex Hypothesi quod unitas superficialis sit quadratum cujus latera sunt unitates lineares: sic in hoc Scholio per $\mathrm{BC} \times \mathrm{fl\,AB}$ non intelligitur fluxio linearis sed fluxio generans productum arithmeticum quod exprimit numerum unitatum superficialium in BD ex Hypothesi quod momentum basis fluentis ductum in datam altitudinem parallelogrammi facit momentum parallelogrammi.

Theorema 6. Si recta quævis motu parallelo transferatur per aream alijs duabus positione datis et non parallelis rectis terminatam, erit fluxio areæ ut fluxio alterutrius rectæ positione datæ ducta in rectam mobilem. Transferatur BC per spatium CAB rectis AC AB positione datis terminatum: et erit fl: CAB ut $\mathrm{BC} \times \text{ fl: } \mathrm{AB}$. Etenim triangulum CAB est ut AB^q. ergo, per Axioma 2, fl: trianguli CAB est ut $\mathrm{fl\,AB}^q$. Sed per Corollaria 6 & 9, Theorema 1, $\mathrm{fl\,AB}^q$ est ut $2\mathrm{AB} \times \mathrm{fl\,AB}$ hoc est cùm 2AB et BC sint in data ratione, ut $\mathrm{BC} \times \mathrm{fl\,AB}$.

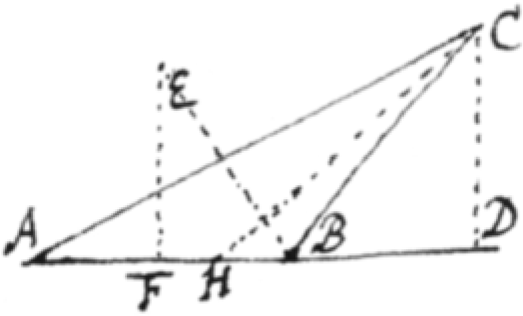


Scholium. Si angulus ABC rectus sit, potest juxta Scholium præcedens, poni $\mathrm{fl\,CAB} = \frac{1}{2}\mathrm{BC} \times \text{ fl: } \mathrm{AB}$.

Theorema 7{.} Si recta circa datum punctum gyrans, continuò terminetur ad aliam rectam positione datam: erit fluxio spatij a gyrante recta descripti, ut fluxio alterius rectæ. Gyret recta CB circa punctum C sitque CA terminus a quo incipit gyrare, et AB recta ad quam terminatur, et erit area ABC ut recta AB; adeoque (per Axioma 2) $\mathrm{fl\,ABC}$ ut $\mathrm{fl\,AB}$.

Scholium. Demisso ad AB normali CD, erit (juxta Scholium Theorematis 5) $\frac{1}{2}\mathrm{CD} \times \text{ fl: } \mathrm{AB} = \mathrm{fl\,ABC}$, quia $\frac{1}{2}\mathrm{CD} \times \mathrm{AB} = \mathrm{ABC}$.

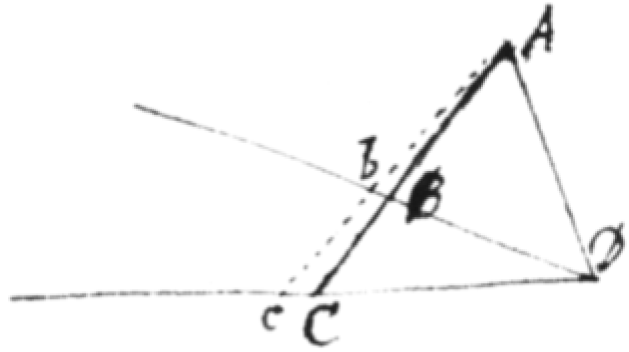
Theorema 8. Iisdem positis, si ab alio insuper quovis dato puncto recta perpetim ducatur ad concursum priorum rectarum, et a primo puncto ad hanc rectam demittatur linea perpendicularis terminata ad rectam positione datam: erit fluxio hujus novæ rectæ ducta in lineam perpendicularem, ut fluxio areæ a prima recta descriptæ.



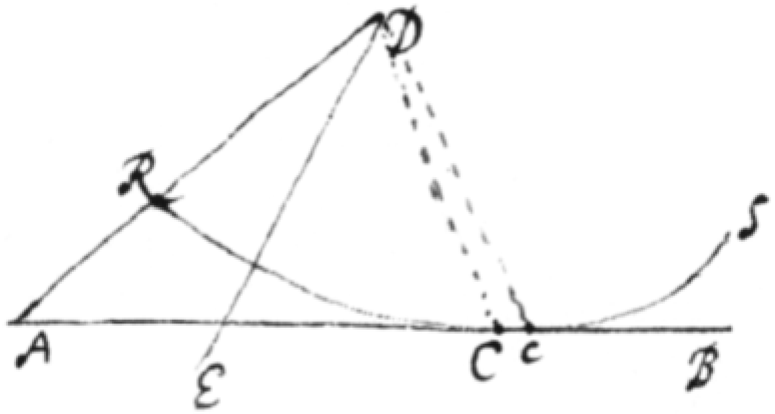
A dato E agatur EB, et ad hanc perpendicularis CH occurrens AB in H, eritque $\mathrm{CH}\times$ fl: EB, ut fl: ABC. Nam demisso ad AB normali EF, erit per Corollarium 1 Theorematis 2,
$\mathrm{fl\ AB\ (fl{:}\ FB)}\ .\ \mathrm{fl{:}\ EB} :: \mathrm{EB}\ .\ \mathrm{FB}$. (hoc est propter similia triangula EBF, HCD) $:: \mathrm{HC}\ .\ \mathrm{CD}$. Ergo $\mathrm{CD}\times \mathrm{fl{:}\ AB} = \mathrm{HC}\times \mathrm{fl{:}\ EB}$. Quare cùm CD datum sit, adeoque $\mathrm{CD}\times \mathrm{fl{:}\ AB}$ ut fl: AB, sitque etiam (per Theorema 7) $\mathrm{fl\ AB}$ ut $\mathrm{fl\ ABC}$, erit $\mathrm{HC} \times \mathrm{fl\ EB}$ ut $\mathrm{fl\ ABC}$.

Scholium. Est et per Scholium superius $\frac{1}{2}\mathrm{HC} \times \mathrm{fl\ EB} = \mathrm{fl\ ABC} \left(= \frac{1}{2}\mathrm{CD} \times \mathrm{fl\ AB}\right) = \mathrm{fl\ ABC}$. Et insuper in eo temporis momento quo contingit angulum EBC rectum esse, est $\frac{1}{2}\mathrm{BC}\times \mathrm{fl{:}\ EB} = \mathrm{fl{:}\ ABC}$ quia tunc HC et BC coincidunt.

Theorema 9. Si recta circa datum punctum gyrans, secet alias duas positione datas rectas: superficierum inter datum punctum et rectas positione datas isto motu generatarum fluxiones erunt ut quadrata longitudinum generantium. Sit A datum punctum circa quod AC gyrat, sintque BD et CD rectæ positione datæ, & AD principium a quo AC incipit gyrare, et erit $\mathrm{fl{:}\ ADB}\ .\ \mathrm{fl\,ADC} :: \mathrm{AB}^{q}\ .\ \mathrm{AC}^{q}$. Sit enim Abc positio rectæ gyrantis in proximo temporis momento, et triangula infinitè parva ABb, ACc erunt momenta superficierum ADB, ADC, adeoque ut ipsarum fluxiones. Sed per 15. 6. Elementarum ista triangula sunt ut $\mathrm{AB} \times \mathrm{Ab}$ ad $\mathrm{AC} \times \mathrm{Ac}$: Quæ ratio, si Abc retro volvatur donec redeat in AC, in ultimo ejus regressûs momento, hoc est in primo momento progressûs ubi AC incipit <41> pergere ad Ac evadit AB^{q} ad AC^{q}. Q.E.D.



Theorema 10. Si recta positione data tangat curvam positione datam et utraque secentur ab alia utcunque motâ rectâ: fluxiones curvæ illius & tangentis ejus in eo temporis momento æquales erunt, quo mota illa linea secat utramque in puncto contactû;s. Esto curva RS, Tangens ejus AB punctum contactûs C et linea mobilis DE: dico fluxiones linearum RC et AC æquales evadere quando DE pertingit ad C. Nam in RC sumatur arcus infinitè parvus Cc, & cum hæc juxta Hypothesin Archimedeam pro recta haberi possit, produc eam utrinque in directum, sitque ea producta AB propterea quod ipsa AB tantùm tangat curvam. RS itaque et AB commune habent momentum Cc, & proinde eandem fluxionem dum DE transit per illud momentum. Q.E.D.



Corollarium. Hinc omnia quæ in Theorematibus 3 et 4 de fluxionibus rectarum positione datarum demonstrata sunt, conveniunt etiam fluxionibus curvarum quas rectæ illæ tangunt in intersectione cum linea mobili.

Theorema 12. Si recta illa DE circa datum punctum D convoluta, describat duas superficies quarum una DRC terminatur ad curvam RC, altera DAC ad tangentem curvæ AC: fluxiones illarum superficierum æquales erunt in eo temporis momento quo recta circumacta transit per punctum contactus C . Nempe
$\mathrm{fl\ DRC} = \mathrm{fl\ DAC}$ quia tunc commune est utriusque momentum CDc.

Corollarium**{.}** Hinc omnia quæ in Theorematibus 7, 8 & 9 de fluxionibus superficierum rectis positione datis terminatarum demonstrata sunt, conveniunt etiam fluxionibus superficierum terminatarum curvis positione datis quas rectæ illæ tangunt.

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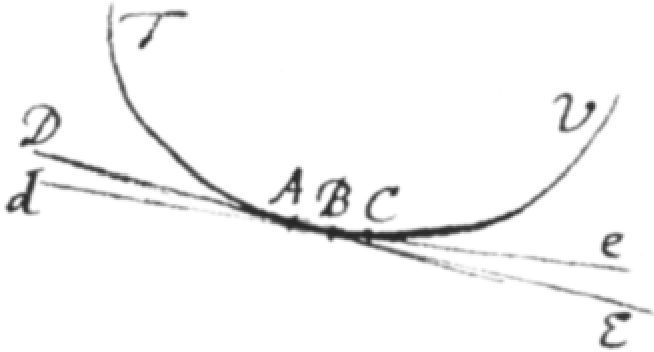
Theorema 13. Si recta mobilis perpetuò tangat curvam, punctum contactûs in omni temporis momento erit centrum circa quod recta in illo momento volvitur.



Concipe Curvam TV lineolis parvitate et multitudine infinitis constare quarum duæ sunto AB & BC. Hasce produc utrinque in directum, nempe AB ad D et E et BC ad d et e, et manifestum est quod tangens mobilis in eo temporis momento quo volvitur de loco DE in locum de, convertitur circa punctum contactus B, propterea quod istud B sit communis intersectio locorum DE ac de.

Corollarium 1. Hinc omnia quæ in Theorematibus 3, 4, 7, 8, 9, 10, 11 de recta circa datum punctum ceu centrum volvente demonstrata sunt, conveniunt etiam rectæ perpetuò tangenti curvam lineam positione datam, si modò punctum contactûs circa quod recta illa in momento contactûs illius convolvitur, vicem centri dati gerere concipiatur. Et proinde sigillatim**{.}**

Corollarium 2. S

<45>

cruris ad defluxionem alterius ut illud alterum crus ad crus primum. Detur C, et erit $B\ .\ A :: \text{fl: } A\ .\ \text{fl}\, B$ propterea quod $C \times \text{fl: } C$ nihil sit.

<46> diametri A, turn ab altero diametri termino B demisso BC normali ad AC, erit fluxio segmenti ACE ut dimidium fluxionis perpendiculi BC ductum in AC: vel juxta Scholium fl: $ACE = \frac{1}{2}AC \times \text{fl}\, BC$.

Jactis hisce demonstrationum fundamentis, methodus tenendi demonstrationes, uno et altero exemplo constabit. Proponatur itaque constructio in Exemplo secundo demonstranda. Per Corollarium 2 Theorematis 1, est fl: ID ad fl: IP ut AI ad ID. Estque AI ad ID ut ID ad CE ex natura Curvæ AGE. Et proinde $CE \times \text{fl: } ID = ID \times \text{fl: } IP$. Sed (per Scholium Theorematis 3) $CE \times \text{fl}\, ID =$ fluxioni areæ ACEG, et $ID \times \text{fl: } IP =$ fluxioni areæ PDI. et proinde areæ illæ per Axioma 1 æquantur. Q.E.D.

Proponatur denuò constructio qua Cissoidis area in Exemplo 3 determinatur. Ad hanc autem demonstrandam, lineæ punctim notatæ in schemate deleantur et agantur DQ, AE et Cissoidis Asymptoton QR. Iam propter AQ, DQ, CQ ∺, est (per Corollarium 2 Theorematis 1) $\text{fl}\, DQ\ .\ \text{fl}\, CQ :: DQ\ .\ 2CQ$. Et propter similia triangula QDC, DEA, est $DQ\ .\ 2CQ :: ED\ .\ 2AD$. Ergo $\text{fl}\, DQ\ .\ \text{fl}\, CQ :: ED\ .\ 2AD$. & $ED \times \text{fl}\, CQ = 2AD \times \text{fl: } DQ$ sive $= 4 \times \frac{1}{2}AD \times \text{fl}\, DQ$. Sed per Corollarium 1, Theorema 3, est $\frac{1}{2}AD \times \text{fl}\, DQ =$ fluxioni generanti aream ADOQ, est et ejus quadruplum $ED \times \text{fl}\, CQ =$ fluxioni generanti Cissoidalem aream QREDO. Et proinde per Axioma 2 area illa infinite longa QREDO generatur quadrupla alterius ADOQ. Q.E.D.

Denique ad demonstrandam constructionem areæ Conchoidalis in Exemplo 4; Ubi demonstraveris ut supra quod sit $AG\ .\ AP :: DE\ .\ MK$, sic procede. Est autem (per Corollarium 2, Theorema 3) $MK \times \text{fl}\, PC = \text{fl: areæ } PKC$ & $DE \times \text{fl}\, PC = \text{fl areæ } DPE$. Ergo $\text{fl}\, PKC\ .\ \text{fl}\, DPE\ (:: MK\ .\ DE) :: AP\ .\ AG$. Adeoque per Axioma 2 Areæ PKC et DPE sunt in eadem ratione.

[1] * Theorema 1.

[2] * Axioma 3

[3] * Corollarium 1.

[4] * Axioma 3.