Rough drafts of the Leibniz Scholium in the 2nd Edition of the Principia, and proposed additions to it

Author: Isaac Newton

Source: MS Add. 3968, ff. 20r-36v, Cambridge University Library, Cambridge, UK

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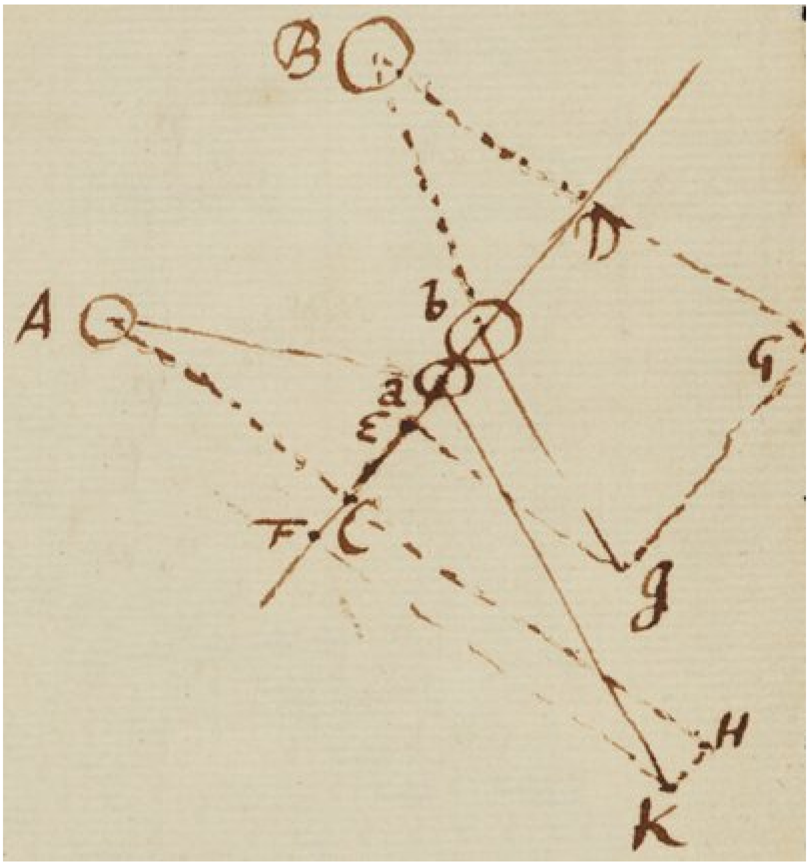
When two spherical bodies strike upon one another, not directly as in the foregoing experiment of Pendulums, but in any obliquity: to find the force of their shock & the motion which they will have afterward, we must first find the right line which passes through their centers at the moment of the shock, & then distinguish the motion of each body into two, the one perpendicular to that line the other parallel to it. The perpendicular motions will remain unaltered by the shock, the parallel will receive the same alteration as if the bodies had met directly with those motions alone. As for instance



B. Scholium

In literis quæ mihi cum Geometra peritissimo G. G. Leibnitio annis abhinc decem intercedebant, cum significarem me compotem esse methodi determinandi maximas et minimas, ducendi tangentes, quadrandi figuras curvilineas & similia peragendi quæ in terminis surdis æque ac in rationalibus procederet, methodumque exemplis illustrarem sed fundamentum ejus literis transpositis hanc sententiam involventalibus [Data æquatione quotcunque fluentes quantitates involvente, fluxiones invenire, et vice versa] [1]celarem: rescripsit Vir Clarissimus anno proximo, se quoque in ejusmodi methodum incidisse, & methodum suam communicavit a mea vix abludentem præterquam in verborum & notarum formulis. [2]Vtriusque fundamentum continetur in hoc Lemmate.

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✝ In eadem Epistola subjunxi — — — simul ederem. Sed antequam Tractatum illum absolvissem, Epistolá ad te missa qua breviter explicui conceptus meos de natura lucis, lites de coloribus **{illeg}** on subortæ sunt, quæ me quietis amantem a consilio detenuerunt. Epistola illa data fuit 6 Febr. $16\frac{71}{72}$ & in Transactionibus Philosophicis eodem mense impressa. Et in eadem scripsi me initio anni 1666 in Theoriam illam lucis et colorum incidisse. Inveni igitur hanc Theoriam eodem fere tempore cum methodo fluxionum, utramque simul edere constitueram, & ab itriusque editione simul destiti quietis gratia.

In eadem Epistola 24 Octob. 1676 ad Oldenburgum scripta, dixi me in Tractatu illo fundamentum aliquatenus posuisse solvendi Problemata

In my Analysis per æquat. communicated by Dr B. to Mr C. in Iuly 1669 n. L. inf. I said that my methods by series gave the areas of curvilinear figures exactly if it might be done & Mr Collins in his Letter to Mr Strode dated 26 Iuly 1672 afterward the same thing & thence it appears that when I wrote that Analysis I had the Method of fluxions so far at least as it is conteined in the first six Propositions of the Book of Quadratures. In that Tract I represented time by a line increasing uniformly & a moment of time by a particle of the line generated in a moment of time & thence called the particle a moment of the line, & the particles of all other quantities generated in the same moment of time I called the moments of those quantities, & the fact under the rectangular Ordinate & a moment of the Abscissa I considered as the moment of the Curvilinear area described by that Ordinate while it moves uniformly upon the Abscissa. And from the fluxion of time came the names of fluents & fluxions. And by considering how to deduce moments from quantities & quantities from moments I deduced the areas of figures from their Ordinates & their Ordinates from their areas: which is the same thing with deducing fluxions from fluents & fluents from fluxions. And in the end of the book I deonstrated by this calculus. the first of the three Rules set down in the beginning thereof. And applying this method not only to finite equations but also to converging Series considered as Equations consisting of an infinite number terms I gave to this Tract the name of <u>Analysis per æquationes numero terminorum infinitas</u>. And in my Letter of 13 Iune 1676 I said of this Analysis: Ex his videre est quantum fines Analyseos per hujusmodi infinitas æquationes ampliantur: Quippe quæ earum beneficio, ad omnia pene dixerim problemata (si numeralia Diophanti et similia excipias) sese extendit. And Mr Leibnitz in his Answer dated 27 Aug. 1676, replied Quod dicere videmini plerasque Difficultates (exceptis Problematibus Diophantæis) ad Series infinitas reduci; id mihi non videtur. Sunt enim multa usque adeo mira et implexa ut neque ab æquationibus pendeant neque ex quadraturis: Qualia sunt (ex multis alijs) Problemata mathodi tangentium inversæ. In the same Letter he placed the perfection of Analysis not in the Differential Method but in another Method composed of Analytical Tables of Tangents & the Combinatory Art. Nihil est, said he, quod norim . . . . . . cogitationum humarum. This was the top of his skill at that time.

Mr Collins in his Letter to Mr Strode dated 26 Iuly 1672 gave this account of the Method. Mense Septembri 1668 — — — — — haud integrum ducit. It appeared therefore to Mr Collins by the testimony of Dr Barrow grounded upon papers which I had communicated to him from time to time that I had the method conteined in the Analysis per Æquationes numero terminorum infinitas some years before the Dr sent that Tract to Mr Collins. And this is sufficient to justify what I said in the Introduction to the Book of Quadratures.

Mr Iames Gregory after a years study found the Method of Series proprio Marte but did not claim it because he knew that he was not the first invenentor

Nothing has been said to prove that Mr Leibnitz had the method before he came to London the second time. Then he met with Dr Barrows Lectures, & the Marquess de l'Hospital has said that where the Dr left off Mr Leibnitz proceeded, & that the improvement which Mr Leibniz <21r> made to the Doctors Methods consisted in shewing how to exclude fractions & radicals. But the Marquess did not know that by my Letter of 24 Octob. 1676 & a copy of my Letter of 10 Decem 1672 he had notice of this improvement & that it related to a very general method. He might afterwards find them proprio Marte but by that notice knew that I had them before him. For in his Letter of 21 Iune 1677 wherein he first began to communicate his method he acknowledged that I knew the improvement when I wrote my Letter of 24 Octob. 1676.

D^{r} Barrow then read his Lectures about motion & that might put me upon taking these things into consideration. In the beginning of the year 1666 I found also the Theory of colours. In the year 1671 I was preparing to publish this Theory with the methods of Series & fluxions, but for a reason mentioned in my Letter of 24 Octob. 1676 I desisted till the year 1704 & then printed the Tracts of Colours & Quadratures together.

That Tract was therefore written not only upon a method of finding Series, but upon a general method of Analysic by applying the method of moments to Equations both finite & infinite & to

<22r>

A Scholium.

In Literis

Scholium.

In literis quæ mihi cum Geometra peritissimo G. G. Leibnitio anno 1676 intercedebant, cum significarem me compotem esse methodi Analyticæ determinandi Maximas & Minimas, ducendi tangentes, quadrandi Figuras curvilineas, conferendi easdem inter se, & similia peragendi, quæ in terminis surdis æque ac in rationalibus procederet, & Tractatus duos de hujusmodi rebus scripsisse alterum anno 1671, ① alterum quem Barrovius noster anno 1669 Collinium misi, cumque literis transpositis hanc sententiam involventibus [Data æquatione quotcunque fluentes quantitates involvente, Fluxiones invenire, et vice versa] fundamentum hujus methodi celarem, specimen vero eusdem in Curvis quadrandis subjungerem & exemplis illustrarem; et cum Collinius Epistolam 10 Decem. 1672 datam a me accepisset in quo methodum hanc descripseram et exemplo Tangentium more Slusiano ducendarum illustrveram, & hujus Epistolæ exemplar mense Iunio anni 1676 in Galliam ad D. Leibnitium misisset; & Vir Clarissimus sub finem mensis Octobris, in reditu suo & Gallia per Angliam in Germaniam, epistolas meas in manu Collinij insuper consuluisset: incidit is tandem in methodum similem sub diversis verborum et notarum formulis, et mense Iunio sequente specimen ejus in Tangentibus more Slusiano ducendis ad me misit, & subjunxit se credere methodum meam a sua non abludere præsertim cum quadraturæ curvarum per utramque methodum faciliores redderentur. Methodi vero utriusque fundamentum continetur in hoc Lemmate.

et Tractatum scripsisse quem Barrovius, anno 1669 ad Collinium misit quique jam extat & methodum serierum ab momentorum **{troc}**tat, & alterum anno 1671 in qua hanc methodum fusius exposueram; cumque literis transpositis hanc

et Tractatus duas de hujusmodi scripsisse, alterum quem Barrovius anno 1669 ad Collinium misit, & alterum anno 1671 in quo hanc methodum fusius exposueram; cumque fundamentum hujus methodi literis transpositis hanc sententiam involventib [Data — — et vice versa] celarem, specimen vero ejusdem in Curvis quadrandis subjungerem & exemplis illustrarem; et cum Collinius Epistolam 10 Decem 1672 datam a me accepisset,

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In utique momenta quantitatum differentias vocavit & quantitates ipsas summas momentorum, & literis d et s præfixis differentias & summas notavit, & pro fluxionibus nulla habuit symbola: ego vero

plenius exponitur in Tractatu

<23v>

Scholium

Analysin meam per series et momenta. Barrovius noster anno 1669 ad Collinium misit. Easdem methodos annis 1665 et 1666 inventas in alio Tractatu plenius explicim**{illeg}** anno 1671; et in Epistola 10 Decem 1672 ad Collinium data [methodum meam generalem in Tractatu hocce explicatam verbis generalibus descripsi & exemplo Tangentium more Slusiano ducendarum illustravi dixique eandem ad quantitates surdas non hærere.

Collinius vero exemplar hujus Epistolæ mense Iunio anni 1676 ad D. Leibnitium hinc in Gallia agentem misit.] In literis insuper quæ mihi cum D. Leibnitio anno 1676, intercedebant, cum Tractatibus prædictis verba facerem & significarem me compotem esse methodi Analyticæ determinandi maximas & minimas, ducendi Tangentes, quadrandi figuras curvilineas, conferendi easdem inter se, et similia peragendi quæ in terminis surdis æque ac in rationalibus procederet, , & literis transpositis hanc sententiam involventibus [Data æquatione quotcunque quantitates involvente, fluxiones invenire, et vice versa] fundamentum hujus methodi celarem, specimen vero ejusdem in curvis curvilineis quadrandis subjungerem et exemplis illustrarem, et Vir clarissimus eadem anno in reditu suo e Gallia per Angliam in Germaniam, sub finem mensis Octobris Epistolas meas in manu Collinij consuleret: incidit is non **{mu}**lo post in methodum similem sub diversis verborum et notarum formulis, et mense Iunio sequente specimen ejusdem in Tangentibus more Slusiano ducendis ad me misit; & subjunxit se credere methodum meam a sua non abludere, præsertim cum quadraturæ curvarum per utramque methodum faciliores redderentur. Methodi vero utriusque fundamentum continetur in hoc Lemmate.

methodum momentorum exemplo tangentium more Slusiano ducendarum illustravi dixique eandem ad Quæstiones de curvitatibus, areis, longitudinibus, centris gravitatum Curvarum & Curvilinearum &c se extendere, et esse general**{e}**m et ad quantitatis surdas non hærere. Et Collinius exemplar

— ad Collinium data per methodum generalem quam methodo serierum [in Tractatu novissimo] intertexui, intellexi methodum momentorum, eamque verbis generalibus ibi descripsi, et exemplo Tangentium more Slusiano ducendarum illusstravi, dixique eandem ad quantitates surdas non hærere.

— ad Collinium data, & exemplo Tangentium more Slusiano ducendarum illustravi, dixique eandem ad quantitates surdas non hærere. Et Collinius exemplar hujus epistolæ mense Iunio anni 1676 ad D. Leibnitium tunc in Gallia agentem misit.

Scholium

per series et momenta a me scriptam Barrovius noster anno 1669 ad Collinium misit. Methodos ibi expositos an alio Tractatu plenius explicui anno 1671 in Epistola 10 Decem. 1672 ad Collinium data, methodum momentorum exemplo tangentium more Slusiano ducendarum illustravi, dixique eandem etiam ad quæstiones de curvitatibus, areis, longitudinibus, centris gravitatum Curvarum & Curvilinearum &c sese extendere et esse generalem & ad quantitates surdas non hærere; et Collinius exemplar hujus epistolæ mense Iunio anni 1676 ad D. Leibnitium tunc in Gallia agentem misit.

In literis insuper quæ mihi cum D. Leibnitio anno 1676 intercedebant, cum verba facerem de Tractatibus prædictis, & Significarem me compotem esse methodi Analyticæ determinandi maximas et minimas, ducendi tangentes, quadrandi figuras curvilineas, conferendi easdem inter se, et similia peragendi quæ in terminis surdis æque ac in rationalibus procederet; & literis transpositis hanc sententiam involventibus [Data æquatione quotcunque fluentes quantitates involvente fluxiones invenire, et vice versa] fundamentum hujus methodi celarem, specimen vero ejusdem in curvilineis quadrandis subjungerem & exemplis illustrarem; et Vir celeberrimus eodem anno in reditu suo e Gallia per Angliam in Germaniam, sub finem mensis Octobris, epistolas meas in manu Collinij etiam consuleret: incidit is non multo post in methodum momentorum sub diversis verborum et notarum formulis, et mense Iunio sequente specimen ejusdem in Tangentibus more Slusiano ducendis ad me misit, & subjunxit se credere methodum meam a sua non abludere, præsertim cum quadraturæ curvarum per utramque methodum faciliores redderentur. Methodi hujus fundamentum continetur in hoc Lemmate, et hæc methodus

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The Logarithmotechnia of M^r^ Mercator was published in Autumn 1668 & a few months after D^r^ Barrow received a copy thereof from M^r^ Collins & sent back to M^r^ Collins a small Tract of mine in M.S. entituled <u>Analysis per series numero terminorum infinitas</u> in which were these words concerning the joint method of series & fluxions: <u>Denique ad Analyticam merito pertinere censeatur cujus beneficio curvarum areæ & longitudines &c (ID MODO FIAT) exacte et Geometrice determinentur: sed ista narrandi non est locus</u>. And by the testimony of D^r^ Barrow & M^r^ Collins I had this Method some years before the Logarithmotechnia came abroad: For M^r^ Collins in a Letter to M^r^ Strode dated 26 Iuly 1672 & published by order of the R. S.

wrote thus Mense Septembri 1668, Mercator Logarithmotechniam edidit suam, quæ specimen hujus methodi (i.e. Serierum) in unica tantum figura, nempe quadraturam Hyperbolæ, continet. Haud multo postquam in publicum prodierat liber, exemplar ejus — Barrovio Cantabrigiam misi, qui quasdam Newtoni chartas — extemplo remisit: e quibus et ALIIS, quæ OLIM ab Auctore cum Barrovio communicata fuerant, patet illam methodum a dicto Newtono aliquot annis antea excogitatam et modo universali applicatam fuisse: ita ut ejus ope in quavis Figura Curvilinea proposita, quæ una vel pluribus proprietatibus definitur, Quadratura vel Area dictæ Figuræ, ACCVRATA SI POSSIBILE SIT, sin minus infinite vero propinqua, Evolutio vel longitudo lineæ curvæ, centrum gravitatis figuræ, Solida ejus rotatione genita, et eorum superficies,sine ulla radicum extractione, obtineri queant. How this is done I explained in a larger Tract written by me A.C. 1671 as I mentioned in a letter to Mr Oldenburg dated 24 Octob 1676 & p ublished by Dr Wallis. And in that Letter I set down the foundation of the method in these words <u>Data Æquatione quotcunque fluentes quantitates involvente fluxiones invenire et vice versa</u>, & added that upon this foundation I had endeavoured to advance the Theory of Quadratures & obteined some general Theorems for that purpose, & there set down the first of those Theorems & illustrated it with examples. This is a Theoreme for squaring Curves whose Ordinates are Binomial & I added that I had other Rules for Trinomials & some other figures. And this is the Method for squaring of figures by series which break of & become finite equations when the Curve admits of an exact & Ge**{**o**}**metric quadrature. Now to do this requires the knowledge of the method of fluxions so far as it is described in the first five or six Propositions, of the <24v> following book of Quadratures. And therefore by the testimony of Dr Barrow & Mr Collins, I knew thus much of the method some years before the Logarithmotechnia came abroad & by consequence in the year 1666. And this may suffice to justify what I said in the Preface of this book.

Mr Iames Gregory in a Letter to Mr Collins dated 5 Sept 1670, wrote that by comparing Dr Barrows methods of drawing Tangents with his own he had found a method of drawing Tangents to all Curves without calculation. And upon notice thereof & that Mr Slusius had such another method which he intended to communicate to Mr Oldenburg I wrote to Mr Collins the following Letter dated 10 Decem 1672. Ex animo gaudeo — — — — reducendo eas ad series infinitas. you have — — — — in the third volume of his works.

For from the Tract which I wrote in the year 1671, I extracted in the year 1676 the following Book of Quadratures & in my aforesaid Letter of Octob 24 1676 I set down the first Proposition thereof verbatim, & copied without any alteration the two Tables set down in the Scholium upon the tenth Proposition. And in my Letter to Mr Collins dated 8 Novem. 1676 & published by Mr Iones, I had relation to the tenth Proposition of this Book in saying: <u>Nulla extat Curva</u> — — — — was known to me in the year 1676 when I wrote this Letter.

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In the year 1691 this Book in MS was in the hands of Dr Halley & Mr Raphson as the ones has left attested in print & the other still attests. And hte next year at the request of Dr Wallis that I would explain how I found the Theoremes set down in my Letter of 24. Octob. 1676 for squaring Curvilineas which — — — — — — & is still the shortest the clearest & the best. In March 1695 Dr Wallis upon notice from Holland that the Method of fluxions was celebrated there by the name of the Differential Method of Mr Leibnitz, inserted into the Preface of the first Volume of his works (which came abroad after the second Volume) the following Paragraph. Quæ in secundo Volumine habentur — — — — ab ipso excogitatam. And as soon as the Volume was printed off wrote to me the following Letter. dated Apr. 10th. 1695.

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And the next year the Editor of the Acta Eruditorum in gaving an Account of this Volume cited some words out of this Paragraph & Dr Wallis in a Letter dated 1 Decem. 1696 gave notice to Mr Leibnitz of the same Paragraph in the Leters which ensued between him & Dr W. And yet neither the Editor of the Acta nor Mr Leibnitz denyed what Dr Wallis had affirmed. On the contrary the Marquess de L'Hospital in the Preface to his book de Infinite parvis published this year allowed that the Book of Principles was almost wholy of the Differential calculus, & Mr Leibnitz himself three years before in a letter to me dated $\frac{7}{17}$ March 1693 (the

Original of which is in the Archives of the R. S.) acknowledged the same thing in these words Quantum tibi Scientiam rerum Mathematicarum totiusque Naturæ debere arbiter, occasione data etiam publice sum professus. Mirifice ampliaveras Geometriam tuis seriebus, sed edito Principiorum opere ostendisti patere Tibi, etiam quæ Analysi receptæ non subsunt. Conatus sum ego quoque Notis commodis adhibitis quæ differentias & summas exhibent, Geometriam illam quam Transcendentem appello Analysi quodammodo subjicere nec res male processit. Here he gives me the Preference. And in the Acta Eruditorum for May 1700, he acknowledged that I was the first who had manifested by a specimen made publick that I had the method of maxima & minima in infinitesimals or moments, meaning the specimen in the Scholium upon the 34th Proposition of the Book of Math. Principles. And And in general this Book of Principles is the first specimen made publick of applying the Method of Fluxions & Moments to the difficulter Problems. And Mr Leibnitz in the Acta Eruditorum for 1684 affirmed that such Problems were not to be solved Without the differential method or another like it, meaning the method of fluxions. For when he wrote to Mr Oldenburg his Letter of 21 Iune 1677 wherein he first began to write of the Differential method, & deduced it from the method of Tangents of Dr Barrow, he aknowledged that I had such another method. Clarissimi Slusij (saith he) Methodum Tangentium nondum esse absolutam Celeberrimo Newtono assentior. And after he had shewn how to deduce the method of Tangents of Slusius from that of Dr Barrow (as Gregory had done before) & how to make this method proceed without taking away surds, he added: Arbitror quæ celare voluit Newtonus de Tangentibus ducendis ab his non abludere. Quod addit, ex hoc edem fundamento quadraturas quoque reddi faciliores, me in sententia hac confirmat. He knew therefore in those days by my Letters that I had a method like the Differential, & by the same Letters he was told that in the year 1671 I wrote a Tract upon this method & the Method of Series together.

In the second Lemma of the second Book of Principles I demonstrated the Elements of the Method of fluxions synethetically. And because Mr Leibnitz had published those Elements in another form two years before, without acknowledging the correspondence which had been between us eight years before, I added a Scholium not to give away that Lemma but to put Mr Leibnitz in mind of making a publick acknowledgment of that correspondence.

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In composing these Books I was much assisted by the Book of Quadratures. At the request of Dr Wallis I sent to him the first Proposition of this Book in the year 1692 Aug. 27, & he printed it in the second Volume of his works before the end of the year & the Book came abroad in Spring 1693. & this was the first time that any rule for finding second third & fourth fluxions was published, This is a proof that the Book of Quadratures was then in Manuscript. Dr Halley & Mr Ralpson saw it in my hands at Cambridge in summer 1691 as Mr Ralphson has left attested in print, & Dr Halley a living evidence still attests. And therefore it was in MS in the year 1691, & continued in MS thirteen years at the least before it was published. In my Letter of 24 Octob 1676 I cited many things out of it, Particulary I cited (in an Enigma) the very words of the first Proposition Data æquatione fluentes quotcunque quantitates involvente invenire fluxiones, I mentioned also the substance of the fift & sixt Propositions & gave a solution of the fift with some examples. These two are the inverse of the third & fourth Propositions & these two are exampels of the second & all of them are deduced from the first , & therefore I was in those days no stranger to the first six Propositions of this Book.

In the same Letter of 24 Octob. 1676, I wrote thus. Seriei a D. Leibnitio pro Quadratura Conicarum Sectionum propositæ, affinia sunt Theoremata quædam quæ pro comparatione Curvarum cum Conicis sectionibus in Catalogum dudum retuli. The series for squaring the Conic sections Mr Leibnitz had twice from Mr Oldenburg. The Theorems for comparing other Curves with the Conic Sections, I reduced into a Catalogue in the Tract above mentioned which I wrote in the year 1671, & thenc I copied it into the Book of Quadratures & the Ordinates of the Curves in the more intricate part of the Table I set down in the said Letter in the very same order & in the very same letters & symbols in which you will now find them in the scholium upon the 10th Proposition of the Book of Quadratures. And therefore that Table was composed before I wrote that Letter; & the 7th 8th 9th 10th Propositions upon which it depends were then known to me.

Between the years 1671 & 1676 I medled not with these studies being tyred with them before: but in the year 1676 I extracted the Book of Quadratures from the Tract which I wrote in year 1671 & from other older papers. And soon after I had finished it I wrote to M^r Collins a Letter dated Novem. 8. 1676 a part of which is here set down. Nulla extat Curva cujus Æquatio ex tribus constat terminis, in qua, licet quantitates incognitæ se mutuo afficiant, vel Indices dignitatum sint surdæ quantitates (v. g. $ax^{\lambda} + bx^{\mu}y^{\sigma} + cy^{\tau} = 0$, x designat basin, y Ordinatam, λ, μ, σ, τ Indices dignitatum ipsius x et y, & a, b, c quantitates cognitas una cum signis suis + vel −) nulla inquam hujusmodi est Curva, de qua, an quadrari possit necne, vel quænam sint figuræ simplicissimæ quibusquam comparari possit, sive sint Conicæ Sectiones sive aliæ magis complicatæ, intra horæ octantem respondere non possim. Deinde methodo directa & brevi, imo methodorum omnium generalium brevissima, eas modo comparari possint, comparo. To do all this is the inverse method of fluxions so far as that method is carried on in the Book of Quadratures. And by the help of this method I composed the Book of Principles, & therefore in this Edition have added the Book of Quadratures to the end of it.

[I made use also of the method of maxima & minima in Infinitesimals & by the confession of M^r Leibnitz was the first who shewed by a specimen made publick that I had this method. I made use also of the method by me called the differential Method, & for that reason have annexed this Method to the Book of Quadratures. In my Letter of Octob. 24 1676 I said that the Tract which I was writing in the year 1671 I never finished, & that that part of it was wanting in which I intended to teach the manner of resolving <26v> Problems which cannot be reduced to Quadratures. But what I then intended to write is now gone out of my mind through long disuse of these methods]

In the Introduction to the Book of Quadratures I said that I invented the Method of fluxions gradually in the years 1665 & 1666. & D^r Wallis (who received from M^r Oldenburg copies of my Letters in the year 1676 & corresponded also with M^r Collins in those days & was very inquisitive in things of this nature, published the same things nine years before me in the Introduction to the first Volume of his works which came abroade in April 1695. His words are. Quæ in secundo Volumine [ante biennium edito] habentur, in Præfatione eidem præfixa dicitur. Vbi (inter alia) habetur Newtoni Methodus de Fluxionibus (ut ille loquitur) consimilis naturæ cum Leibnitij (ut hic loquitur) Calculo Differentiali, (quod qui utramque methodum contulerit, satis animadvertat, utut sub loquendi formulis diversis) quam ego descripsi (Algebræ cap. 91 &c præsertim cap. 95) ex binis Newtoni Literis (aut earum alteris) Iunij 13 & Octob. 24 1676 ad Oldenburgum datis, cum Leibnitio tum communicandis (ijsdem fere verbis, saltem leviter mutatis, quæ in illis literis habentur) ubi methodum hanc Leibnitio exponit, tum ante decem annos nedum plures, ab ipso excogitatam.

In Iuly 1669 D^r Barrow sent to M^r Collins a small Tract written by me under the Title of Analysis per series numero terminorum infinitas. This title implies that the scope of the Book was not only to reduce finite quantites into converging series but also to apply æquations involving such series as well as finite æquations to the resolution of Problems by means of the Analysis there proposed And in this Analysis I considered quantities as generated by motion & their parts generated in moments of time I call their moments, & shew how to compute the moments of lines, superficies, & solids & to find as many Curves as I please which may be squared which is (in substance) the first & second Propositions of the Book of Quadratures. And after I had shewed how to find the Ordinate Areas & Lengths of Mechanical lines, I describe the universality of this method in these words. Nec quicquam hujusmodi scio ad quod hæc methodus, idque varijs modis sese non extendit. Imo tangentes — — — — sed ista narrandi non est locus. These last words relate to the method described in the fift & sixt Propositions of the book of Quadratures. And therefore the Methods conteined in the first six Propositions of this book were then known to me. For the fift & sixt Propositions are the inverse of the third & fourth. [In the beginning of this Tract I call this Method methodum generalem quam olim excogitaveram. This Tract is the Compendium methodi serierum mentioned in my Letter of Octob. 24. 1719, where I say that it was communicated by D^r Barrow to M^r Collins at that time when M^r Mercators Logarithmotechnia came abroad. And M^r Collins in his Letter to M^r Tho. Strode dated 26 Iuly 1672 has testified that by this & other Papers communicated before to D^r Barrow it appeared that I had invented this method & applied it generally some years before he sent it to M^r Collins.

In my Letter of 24 Octob. 1676 at the request of M^r Leibnitz I descri < insertion from the left margin > be < text from f 26v resumes > how I found out the method of converging series a little before the plague which raged in London in the year 16 65, & particularly how I then invented the reduction of any power of any

Binomial into such a Series by the Rule set down in teh beginning of my Letter of 13 Iune 1676. Let $\overline{x+o}|^{n}$ be any dignity of any binomial $x+o$ & by this Rule $\overline{x+o}|^{m}$ will be equal to $x^n + oox^{n-1} + \frac{nn-m}{2}oox^{n-2} + \&c$. And here if x be an increasing quantity of fluents & o its moment, then nox^{n-1} will be the first moment of the fluent x^n. And this Theoreme quickly gave me the first Proposition of the Book of Quadratures & the inverse thereof gave me the two first Rules in the Analysis per æquationes numero terminorum infinitas. And this Relation between the Methods of Series & moments made me joyne them together as two branches of one very <27r> general method. And all this I hope may suffice to justify me in saying in the Introduction to the Book of Q. that I found the method of fluxions gradually in the years 1665 & 1666.

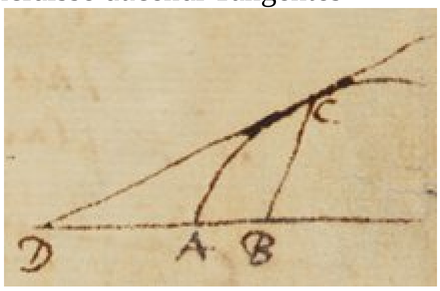
<28r>

In writing the Book of Principles I made very much use of the following Book of Quadratures, & therefore have subjoyned it. And because my saying in the Introduction, that I found the Method of fluxions gradually in the years 1665 & 1666 has been called in question, it will not be amiss to justify what I said

Upon notice from M^r Collins in Decemb. 1672 that M^r Iames Gregory had improved the method of Tangents of D^r Barrow so as to draw tangents without calculation & that M^r Slusius had such another method which he intended to communicate to M^r Oldenburg, I wrote the following. Letter to M^r Collins dated 10 Decem 1672. "Ex animo gaudeo D. Barrovij, amici nostri reverendi lectiones Mathematicas exteris adeo placuisse, neque parum me juvat intelligere eos [Slusium & Gregorium] in eandem mecum incidisse ducendi Tangentes Methodum. Qualem eam esse conjiciam, ex hoc exemplo percipies. Pone CB applicatam ad AB in quovis angulo dato, terminari ad quamvis Curvarum AC; et dicatur AB x & BC y, habitudoque inter x et y exprimatur qualibet æquatione, puta $x^3 - 2xxy + bxx - bbx + byy - y^3 = 0$, qua ipsa determinatur Curva. Regula ducendi Tangentem hæc est: Multiplica æquationis terminos per quamlibet progressionem arithmeticam juxta dimensiones y, puta

$$\begin{array}{c} x^3 - 2xxy + bxx - bbx + byy - y^3 \\ 010023 \end{array}$$

; ut et juxta dimensiones x, puta



$x^3 - 2xxy + bxx - bbx + byy - y^3 = 0$, qua ipsa determinatur Curva. Regula ducendi Tangentem hæc est: Multiplica æquationis terminos per quamlibet progressionem arithmeticam juxta dimensiones y, puta

$$\begin{array}{c} x^3 - 2xxy + bxx - bbx + byy - y^3 \\ 322100 \end{array}$$

Prius productum erit Numerator, & posterius divisum, per x Denominator Fractionis quæ exprimet longitudinem BD æd cujus extremitatem D ducenda est Tangens CD. Est ergo longitudo BD $= \frac{-2xxy+2byy-3y^3}{3xx-4xy+2bx-bb}$. Hoc est unum particulare vel corollarium potius Methodi generalis quæ extendit se, citra molestum ullum calculum, non ad ducendum Tangentes ad quasvis Curvas, sive Geometricas, sive Mechanicas, vel quomodocunque rectas lineas aliasve Curvas respicientes; verum etiam ad resolvendum alia abstrusiora Problematum genera de Curvitatibus, Areis, Longitudinibus, centris gravitatis Curvarum &c. Neque (quemadmodum Huddenij methodus de Maximis et Minimis) ad solas restringitur æquationes illas quæ quantitatibus surdis sunt immunes. Hanc methodum intertexui alteri isti, qua Æquationum Exegesin instituo reducendo eas ad Series infinitas. &c. In this Letter you have an example of the Method of Fluxions in drawing of Tangents & a description of the large extent of it in this & other more difficult Problems without stopping at surds; & the last words of the Letter relate to a Tract which I wrote the year before & in which the method here described as interwoven with another Method wherein æquations are reduced to converging series, both which methods together constitutde the general Method described in my Letters to M Oldenburg dated 13 Iune & 24 Octob. 1676 which Letters were published by D^r Wallis in the third volume of his works.

For in the year 1671 I wrote a Tract concerning the method of converging series & Fluxions joyntly but did not finish it, that part of it being wanting in which I intended to explain the manner of solving such problems as cannot be reduced to quadratures, as I mentioned in my Letter of 24 Octob. 1676 aforesaid. And from this Tract I extracted in the year 1676 the following Book of Quadratures & therein copied from the former Tract without any alteration the two Tables set down in the Scholium upon the tenth Proposition, the one for squaring some Curves & the other for comparing others with the Conick Sections. And In my Letter to M^{r} Collins dated 8 Novem. 1676 & published by M^{r} Iones, I had relation to the 10th Proposition of this Book in saying: <u>Nulla extat Curva cujus Æquatio ex tribus constat terminis, in qua, licet quantitates incognitæ se mutuo afficiant, vel indices dignitatum sint surdæ quantitates (v. g. $ax^{\lambda} + bx^{\mu}y^{\sigma} + cy^{\tau} = 0$, x designat Basin, y Ordinatam & λ, μ, σ, τ indices dignitatum ipsius x & y & a, b, c quantitates cognitas una cum signis suis + vel −) nulla inquam hujusmodi est Curva de qua an quadrari possit, necne, vel quænam sint Figuræ simplicissimæ quibuscum comparari possit, sive sint Conicæ Sectiones sive aliæ magis complicatæ, intra horæ octantem respondere non possim. Deinde methodo directa et brevi, imo methodorum omnium generalium brevissima [de qua vide Coroll. 2 Prop. 10 Libri sequentis] eas, modo comparari possint, comparo. Affirmatio quidem videri potest temeraria, propterea quod perdifficile sit dictu an Figura quadrari vel cum alia comparari possit, necne; mihi autem manifestum est, ex eo unde deduxi, quanquan id alijs demon <28v> strare in me suscripere nollem. Eodem methodus Æquationes quatuor terminoru{m} aliasque complectitur, haud tamen adeo generaliter.</u> All this relates to the method of squaring figures set down in the Book of Quadratures, & chiefly to the tenth Proposition of that Book: & the methodus directa & brevis mentioned in this Letter, by which I compare such trinomial figures as may be compared is that mentioned in Corol 2 Prop. 10; & this Proposition with its Corollaries is deduced from the 5^{t} 6th 7th 8th & 9th Propositions of this Book & these are deduced from the 1st 2^{d} 3^{d} & 4th Propositions of the same Book: & therefore the method of Quadratures so far as it is conteined in the ten first Propositions of this Book was known to me in the year 1676 when I wrote this Letter.

This Book I made use of in the year 1679 when I found the demonstration of Keplers Proposition that the Planets moved in Ellipses, & again in the year 1684, 1685 & 1686 when I wrote the Book of Mathematical Principles of Philosophy & for that reason I have now subjoyned it to that Book In the year 1691 it was in the hands of M^{r} Ralpson & D^{r} Halley as one of them attested in print before his death & the other still attests. And in the year 1692 at the request of D^{r} Wallis that I would explain how I found the Theorems set down in my Letter of 24 Octob 1676 for squaring of Curves by series which break off & become finite equations when the Curves can be squared by finite equations, I sent to him in a Letter dated Aug 27 the fift Proposition of the Book of Quadratures as a more general Theoreme which comprehended the Theorems mentioned by him & at the same time I sent to him also the first Proposition of this book illustrated with examples in sending first & second fluxions. And these two Propositions were printed befor the end of the year in the second Volume of his works pag. 391, 392, 393 which Volume came abroad in August 1693. And this was the first time that any Rule was published for finding second third fourth & other fluxions or differences & is still the shortest the clearest & the best. But the Book of Quadratures continued in which I affirmed that I found the Method of fluxions gradually in the years 1665 & 1666. For I thought that I might safely write this because D^{r} Wallis, in the Preface to the first Volume of his works which came out after the second & was published in April 1695, had without ever being contradicted, inserted the following Paragraph. <u>Quæ in secundo Volumine habentur in Præfatione eidem præfixa dicitur. Vbi (inter alia) habetur Newtoni Methodus de Fluxionibus (ut ille loquitur) consimilis naturæ cum Leibnitij (ut hic loquitur) Calculo Differentiali, (quod qui utramque methodum contulerit, satis animadvertat, utut sub loquendi formulis diversis,) quam ego descripsi (Algebræ cap. 91 &c præsertim cap. 95) ex binis Newtoni literis (aut earum alteris) Iunij 13 & Octob. 24, 1676 ad Oldenburgum datis, cum Leibnitio tum communicandis (ijsdem fere verbis, saltem leviter mutatis, quæ in illis literis habentur;) ubi methodum hanc Leibnitio exponit tum ante decem annos, nedum plures, ab ipso excogitatam.</u>

At the request of M^{r} Leibnitz I described in my said Letter of 24 Oct 1676, how, before the plague which raged in London in the years 1665 & 1666, by interpoling the series of D^{r} Wallis, I found the method of converging series together with the Rule for resolving the dignities of Binomials into such series. I there I mentioned also that upon the publication of Mercator's Logarithmotechnia D^{r} Barrow sent to M^{r} Collins a Compendium of these series. A copy of this Compendium in the handwriting of M^{r} Collins was found by M^{r} Iones in the Archive of M^{r} Collins & published after it had been collated with the original which M^{r} Iones borrowed of me for that purpose. The litle thereof was <u>Analysis per Series numero terminorum infinitas</u>. And in this Tract the method of series is interwoven with that of fluxions. For after I had found the method of Series it quickly led me into the method of fluxions, & their affinity made me write of them both together as composing one general method of Analysis. In this Tract I affirmed that this method extends to all Problemes

& that ejus beneficio <29r> Curvarum areæ et longitudines &c (id modo fiat) exacte et Geometrice determinantur. Which I could not have said without understanding at that time so much of the method as is conteined in the first five or six Propositions of the Book of Quadratures.

And by the testimony of D^{r} Barrow & M^{r} Collins I understood thus much of the method some years before Mercators Logarithmotechnia came abroad For M^{r} Collins in a Letter to M^{r} Strode dated 26 Iuly 1672 & published by order of the R. Society, wrote thus. Mense Septembri 1668 Mercator Logarithmotechniam edidit suam, quæ specimen hujus methodi (i.e. serierum infinitarum) in unica tantum figura, nempe Quadraturam Hyperbolæ continet Haud multo postquam in publicum prodierat Liber, exemplar ejus Cl. Wallisio Oxonium misi, qui suum de eo judicium in Actis Philosophicis statim fecit; aliumque Barrovio Cantabrigiam, qui quasdam Newtoni chartas extemplo remisit: e quibus et ALIIS, quæ OLIM ab Auctore cum Barrovio communicata fuerant, patet illam methodum a dicto Newtona ALIQVOT ANNIS ANTEA excogitatam & modo universali applicatam fuisse: ita ut ejus ope in quavis Figura Curvilinea proposita quæ una vel pluribus proprietatibus definitur Quadratura vel Area dictæ figuræ, ACCVRATA SI POSSIBILE SIT, sin minus infinite vero propinqua, Evolutio vel Longitudo lineæ curvæ, Centrum gravitatis Figuræ; solida ejus rotatione genita, & eorum superficies; sine ulla radicum extractione, obtineri queant. /

And all this may suffice to justify my saying in the Introduction to the Book of Quadratures, that I found this Method gradually in the years 1665 & 1666. However, the Method is capable of improvements, & the improvements are theirs who make them.

Now since By the help of this Method I wrote the Book of Principles I have therefore subjoyned the book of Quadratures to the end of it. And for the same reason I have subjoyned also the Differential Method. And because several Problems proposed in the Book of Principles to be solved concessis Figurarum Quadraturis I have added to the end of the Book of Quadratures some Propositions taken from my Letters already published for reducing quantities into converging series & thereby squaring the figures. For Quadratures by such series have the same place in Arithmetick & Algebra with operations in decimal numbers.

In the Analysis per series abovementioned were several instances of squaring the Circle & Conic Sections & other Figures & of finding the lengths of Curve lines, & on the contrary of finding the Abscissas & Ordinates of Figures whose Areas or lengths of the Curve lines are given & M^{r} Collins was very free in communicating to Mathematicians the series there set down. M^{r} Leibnitz was in London in the begining of the year 1673 & conversed with D^{r} Pell &c above numeral series & carried with him to Paris Mercator's Logarithmotechnia but did not yet understand the higher Geometry. The next year he studied this Geometry & in two Letters dated 15 Iuly & 26 Octob. he wrote to M^{r} Oldenburg that he had found the circumference of a circle in a series of rational numbers, & that by the same method, any arch of a circle whose sine was known might be exhibited in a like series without knowing its proportion to the whole circumference. Which is as much as to say that he had found a series of rational numbers expressing the length of any arch of a circle whose sine was given, which gave the whole circumference if its proportion to the Arch was known, or at least it gave the arch. The next year M^{r} Oldenburg in a Letter dated 15 Apr. 1675 sent to M^{r} Leibnitz <29v> these two series which M^{r} Collins had received from me Posita pro Radio Vnitate, datoque x pro sinu, ad inveniendum z Arcum Series hæc est: $\mathrm{z} = \mathrm{x} + \frac{1}{6}\mathrm{x}^3 + \frac{3}{40}\mathrm{x}^5 + \frac{5}{112}\mathrm{x}^7 + \frac{35}{1152}\mathrm{x}^9$ &c et si dederis z pro arcu ad inveniendum x sinum series hæc est; $\mathrm{x} = \mathrm{z} - \frac{1}{6}\mathrm{z}^3 + \frac{1}{120}\mathrm{z}^5 - \frac{1}{5040}\mathrm{z}^7 + \frac{1}{362880}\mathrm{z}^9 - \&\mathrm{c}$ And these two which M^{r} Collins had received from M^{r} Iames Gregory. Pone formula $= \mathrm{r}$, Arcum a, Tangentem t; erit $\mathrm{t} = \mathrm{a} + \frac{\mathrm{a}^3}{3\mathrm{r}^2} + \frac{2\mathrm{a}^5}{15\mathrm{r}^4} + \frac{17\mathrm{a}^7}{315\mathrm{r}^6} + \frac{62\mathrm{a}^9}{2835\mathrm{r}^8} + \&\mathrm{c}$. Et conversim ex Tangente invenire Arcum ejus $\mathrm{a} = \mathrm{t} - \frac{\mathrm{t}^3}{3\mathrm{r}^2} + \frac{\mathrm{t}^5}{5\mathrm{r}^4} - \frac{\mathrm{t}^7}{7\mathrm{r}^6} + \frac{\mathrm{t}^9}{9\mathrm{r}^8} - \&\mathrm{c}$. And M^{r} Leibnitz acknowledged the receipt of these series by a Letter dated May 20th 1675. Literas tuas multa fruge Algebraica refertas accepi, pro quibus tibi & doctissimo Collinio gratias ago. Cum nunc præter ordinarias Curas Mechanicis imprimis negotijs distrahar, non potui examinare series quas misistis ac cum meis comparare. Vbi fuero perscribam tibi sententiam meam: nam aliquot jam anni sunt quod inveni meas. The next year M^{r} Leibnitz wrote the following Letter to M^{r} Oldenburg dated 12 May 1676. Cum Georgius Mohr Danus in Geometria et Analysi versatissimas, nobis attulerit communicatam sibi a Doctissimo Collinio vestro expressionem relationis inter Arcum et Sinum per infinitas series sequentes: Posito sinu x, arcu z, Radio 1. erit $\mathrm{z} = \mathrm{x} + \frac{1}{6}\mathrm{x}^3 + \frac{3}{40}\mathrm{x}^5 + \frac{5}{112}\mathrm{x}^7 + \frac{35}{1152}\mathrm{x}^9 + \&\mathrm{c}$ et

$x = z - \frac{1}{6}z^3 + \frac{1}{120}z^5 - \frac{1}{5040}z^7 + \frac{1}{362880}z^9 - \&c$. Hæc inquam Cum nobis attulerit ille, quæ mihi valde ingeniosa videntur, et posterior imprimis series elegantiam singularem habeat, ideo rem gratam mihi feceris, Vir clarissime, si Demonstrationem transmiseris. Habebis vicissim mea ab his longe diversa circa hanc rem meditata, de quibus jam aliquot abhinc annis ad te perscripsisse credo, demonstratione tamen non addita quam nunc polio. Oro ut Cl. Collilnio multam a me Salutem dicas: is facile materiam suppeditabit satisfaciendi desiderio meo. By the Demonstration of these series M^r^ Leibnitz meant the Method of finding them & since he sent M^r^ Oldenburg to M^r^ Collins for the same he had heard of the Analysis per series numero terminorum infinitas, & wanted it. But M^r^ Collins being unwilling to let him have a copy of it he & M^r^ Oldenburg joyned in desiring that I would send him the Demonstration & thereupon I wrote to M^r^ Oldenburg the following letter dated 13 Iune 1676 to be communicated to M^r^ Leibnitz. And M^r^ Leibnitz sent back by way of recompence the series of M^r^ Gregory above mentioned, viz^t^
$\text{arc} = t - \frac{t^3}{3} + \frac{5^5}{5} - \frac{t^7}{7} + \frac{t^9}{9} - \&c$ & five years after published it in the Acta Eruditorum as his own without taking any notice of the correspondence between him & M^r^ Oldenburgh by which he had received it from London.

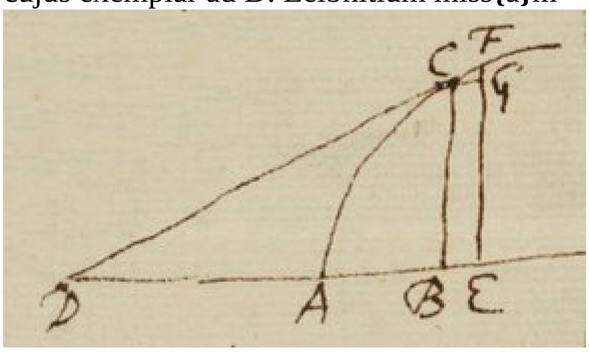
And all this may suffice to justify my saying in the Introduction to the following Book of Quadratures, that I found the method gradually in the years 1665 & 1666 However, the method is capable of improvements & the improvements & theirs who make them.

<30r>

C In the end of the Scholium in Princip. Philos. pag. 227 after the words Vtriusque fundamentum continetur in hoc Lemmate, add

Sunto quantitates datæ a, b, c; fluentes x, y, z; fluxiones p, q, r; & momenta op, oq, or: et proponatur æquatio quævis $x^3 - 2xxy + bxx - bbx + byy - y^3 + czz = 0$. Et per hoc Lemma, Si sola fluat x, erit fluxio totius $3xxop - 4xopy + 2bxop - bbop$; si sola fluat y, erit fluxio totius $-2xxoq + 2byoq - 3yyoq$; si sola fluat z, erit fluxio totius $2czor$; si fluant omnes, erit fluxio totius $3xxop - 4xopy - 2boxp - bbop - 2xxoq + 2byoq - 3yyoq + 2czor$. Et quoniam totum semper æquale est nihilo, erit fluxio totius æqualis nihilo. Dividatur totum per momentum o, et prodibit æquatio quæ ex fluentibus dat fluxiones, viz^t^ $3xxp - 4xpy + 2bxp - bbp - 2xxq + 2byq - 3yyq + 2czr = 0$. Exhibet igitur hoc Lemma solutionem Propositionis præfatæ, Data æquatione fluentes quotcunque quantitates involvente fluxiones invenire. ☉ < insertion from f 30v > ☉ Dixi vero in Epistola mea 24 Octob. 1676 ad Oldenburgium quo mediante commerci**{um}** tunc habui cum D. Leibnitio, quod hæc Propositio fundamentum esset methodi generalis de qua scripseram anno 1671. < text from f 30r resumes > < insertion from f 30v > Et hanc Propositionem esse fundamentum methodi generalis de qua scripseram anno 1671 dixi in Litteris prædictis < text from f 30r resumes > Et hanc Propositionem esse fundamentum methodi generalis de qua scripseram anno 1671 dixi in Literis præfatis, anno 1671.

In Epistola mea 24 Octob 1676 ad Oldenburgium missa & cum D. Leibnitio communicata, posui hanc Propositionem ut fundamentum methodi de qua scripseram anno 1671. Et hæc methodus facile colligitur etiam ex Epistola quam ad Collinium 10 Decem 1672 scripsi, et cujus exemplar ad D. Leibnitium miss**{a}**m fuit anno 1676. Sit ACF linea quævis Curva, AB ejus Abscissa, & BE momentum Abscissæ, et sint BC, EF Ordinatæ duæ ad Curvam in C et F terminatæ, CF momentum Curvæ, & FG momentum Abscissæ existentæ BCGE parallelogrammo. Agatur chorda CF et producatur eadem donec Abscissæ occurrat in D: et similia erunt triangula CGF, DBC. Et ubi momentum CF diminuitur in infinitum, recta CD curvam tanget in C. Hoc omnibus notum est. Iam dicatur AB x, & BC y, et sint earum fluxiones p et q, & habitudo inter x & y exprimatur qualibet æquatione puta



$x^3 - 2xxy + bxx - bbx + byy - y^3 = 0$. Et per Epistolam illam Regula ducendi Tangentem hæc erit. Multipliæ æquationes terminos per quamlibet progressionem Arithmeticam juxta dimensiones y, puta

$$x^3 - 2xxy + 6xx - bbx + byy - y^3 = 0$$
$$010023$$
. ut et juxta dimensiones x, puta

$$x^3 - 2xxy + bxx - bbx + byy - y^3$$
$$322100$$
. Prius productum erit Numerator & posterius divisum per x denominator fractionis quæ exprimet longitudinem BD ad cujus extremitatem D. ducenda est Tangens. Ducet ur autem BD ad eandem p**{**lag**}**am cum Abscissa AB ubi valor ejus affirmativus est et ad contrariam ubi negativus. Est ergo, per Regulam in hac Epistola positam; summa omnium terminorum multiplicatorum per indices dignitatum x ac divisorum per x ad summam omnium multiplicatorum per indices dignitatum y ac divisorum per y, ut est y ad — BD, et ita est FG ad — CG seu oq ad — op. Ducantur extrema et media in invicem & Æquatio factorum per o divisa evadet $3xxp - 4xpy + 2bxp - bbp = 2xxq - 2byq + 3yyq$, seu $3xxp - 4xpy + 2bxp - bbp - 2xxq + 2byq - 3yyq = 0$. Hæc equatio producetur etiam per Lemma ubi duce tantum sunt fluentes quantitates. Vbi vero sunt plures, eadem operatio applicari debet ad omnes. Et sic habebitur æquatio fluxiones involvens. Et in hac operatione fundatur methodus generalis, uti dixi in eadem Epistola, quæ extendit se citra <31r> molestum ullum calculum non modo ad ducendum Tangentes ad quasvis Curvas sive Geometricas sive mechanicas vel quomodocunque rectas lineas aliasve Curvas respicientes; verum etiam ad resolvendum alia abstrusiora Problematum genera de Curvitatibus, Areis, Longitudinibus, centris gravitatis Curvarum &c. Neque quemadmodum Huddenij methodus de Maximis et Minimis) ad solas restringitur æquationes illas, quæ quantitatibus surdis sunt immunes. Hanc methodum intertexui alteri isti quæ Æquationum Exegesin instituo, reducendo eas ad series infinitas: sc. in Tractatu illo quem scripsi anno 1671.

<31v>

Et hæc Quæstio est: Vtrum D. Leibnitius sit inventor methodi, & pro differentijs igitur Leibnitianis Newtonus adhibet semperque [ex quo usus est hac methodo] adhibuit fluxiones, quemadmodum Honoratus Fabrius motuum progressus Carallerianæ methodo substituit.

Gregorius ad Collinium scripsit 5 Sept. 1670 se ex Barrovij methodis tangentes ducendi & suis invenisse Methodum generalem & Geometricam tangentes ducendi ad omnes Curvas sine calculo. Newtonus ad eundem Collinium scripsit 10 Decem 1672 in hæc verba Ex animo gaudeo D. Barrovij nostri reverendi Lectiones Mathematicas exteris adeo placuisse, neque parum me juvat — — — reducendo eas ad series infinitas. Et harum duarum literarum exemplaria ad Leibnitium missa sunt ab Oldenburgo inter excerpta ex Epistolis Gregorij 26 Iunji 1676. Et Leibnitius mense novembri proximo Lectiones Barrovij secum tulit in Germaniam.

Sunto jam ipsarum x et y fluxiones pet q & momenta op et oq et in AB producta capiatur $BE = op$, erigatur ordinata EF parallela BC & occurrens Curvæ in F et compleatur parallelogrammum BCGE et erit $FG = oq$, et ex methodo tangentium Barrovij erit BD. CB∷CG.FG∷op. oq∷p.q. adeoque $\frac{-py}{q} = BD = \frac{-2xxy+2byy-3y^3}{3xx-4xy+2bx-bb}$. Et facta reductione $3pxx - 4pxy + 2bbx - 2pbb + 2xxq - 2byq - 3yyq = 0$. Vnder prodit hujusmodi Regula si detur æquatio fluentes duas quantitates x et y involves, invenientur æquatio fluxiones p et q involvens multiplicando omnem æquationes data terminum per indicem dignitatis quantitatis cujusque fluentis quam in volvit et in singulis multiplicationibus mutando dignitatis latus in fluxionem suam. Nam factorum omnium aggregatum sub proprijs signis erit æquatio nova. Et si plures sint fluentes, eadem operatio instituenda est in singulis. [Hæc est Propositio prima libri Newtoni de Quadraturis. Hanc Propositionem Newtonus posuit in Epistola 24 Octob 1676.] Et hæc Regula illud omne comprehendit quod Leibnitius anno 1677 de methodo differentiali ad Newtonum rescripsit estque Propositio primæ Libri de quadraturis.

In Hæc Regula habetur Algorithmus hujus methodi. eam**{**que**}** Wallisius anno 1693 in secundo Volumine operum ejus pag 392. in lucem edidit, cum exemplis in fluxionibus primis et secundis inveniendis. Estque Regula antiquissima quæ lucem vidit pro differentialibus differentiandis. Eandem Newtonus demonstravit Synthetice in secundo Lemmate Libri Principiorum cum Propositionem prius posuisset in Epistola sua 24 Octob. 1676, ad Oldenburgum pro Leibnitio missa, et ibi significasset eandem esse fundamentum methodi de qua scripserat tum ante annos quinque id est anno 1671. Demonstratur vero in Lemmate illo in hunc modum.

⊕< insertion from f 31r >⊕Sunto quantitates datæ a, b, c, fluentes x, y, z fluxiones p, q, r momenta op, oq, or, & proponatur æquatio quævis — — — — — — — exhibet igitur hoc Lemma solutionem Propositionis data æquatione fluentes quotcunque quantitates involvente invenire fluxiones. Et hanc Propositionem esse fundamentum methodi generalis de qua scripserat anno 1671 dixit Newtonus in literis prædictis.

Propositionem quintam — < text from f 31v resumes > Propositionem quintam libri de Quadraturis, Wallisius edidit anno 1693 in secundo operum suorum Volumine pag. 391. Hac Propositione quadrantur Figuræ curviline accurate & Geometrice si fieri potest. Partem hujus Propositionis Newtonus posuit in Epistola 24 Octob 1676 ad Oldenburgum scripta. In Analysi sua per series dixit quod Analysen illius beneficio Curvarum areæ & longitudie, &c (id modo fiat) exacte & Geometrice determinetur: ideoque quinta illa Propositio tunc ipsi innotuit. Hanc Analysen Leibnitio{s} videre potuit in secundo suo in Angliam itinere ubi Collinius ips. monstravit plures Newtoni Gregorij & aliorum Literas quæ circa series præcipue versæbantur. Collinius in Epistola sua ad Thomam Strode 26 Iuly 1672 data Scripsit in hæc verba: Mense Septembri 1668 Mercator Logarithmotechniam edidit suam — Haud multo postquam in publicum prodierat liber exemplar ejus Cl. Wallisio Oxonium misi — aliumque Barrovio Cantabrigiam, qui quasdam Newtoni chartas [sc. Analysin per series] extemplo remisit: e quibus et EX ALIIS quæ OLIM ab auctore communicata fuerant, patet illam Methodum a dicto Newtono aliquot annis antea excogitatam & modo universali applicatam fuisse: ita ut ejus ope in quavis Figura Curvilinea proposita quæ una vel pluribus Proprietatibus definitur, Quadratura vel Area: dictæ figuræ ACCVRATA SI POSSIBILE SIT, sin minus infinite vero propinqua &c — obtineri queat: id est accurata si series abrumpitur, sin minus infinite vero propinqua. Testibus igitur Barrovio et Collinio Newtonus aliquot annis antea quam prodiret Logarithmotechnia illa adeoque quadrandi Curvilineas anno 1666 aut antea, methodum habuit per series accurate si series abrumpitur et finita evadit; sin minus infinite vero propinqua. Et hoc fit per Propositionem illam quintam. Hæc autem Propositio pendet a quatuor prioribus: et propterea Method i serierum et fluxionum quatenus continentur in Propositionibus quinque primis libri de Quadratura Curvilinearum Newtono innotuere anno 1666 aut antea testibus Barrovio et Collinio; ut et teste etiam Wallisio, qui in Præfatione ad Volumen primum operum suorum scripsit quod Newtonus in literis suis 13 Iunij & 24 Octob. 1676 datis methodum hanc [fluxionum vel differentialem] Leibnitio exponit tum ante decem annos nedum plures ab ipso excogitatam, i. e. anno 1666 aut antea. < insertion from f 31r > plures ab ipsa excogitam; id est anno 1666 aut antea. Hoc Leinitius et Menkenius legerunt & per ea tempora non negarunt, ut ex Actis Eruditorum & Epistolis a Wallisio editis patet. Paulo post D. Nicolas Fatio in Tractatu de investigatione solidi in quod minima fiat resistentia, scripsit se anno 1687 in Calculum differentiali similem incidisse, Newtonum tamen primum ac pluribus annis vetustissimum hujus Calculi inventorem ipsa rerum evidentia se coactum agnoscere, visis scilicet Newtoni manuscriptis codicibus. Et Le bneitius in Actis Eruditorum respondendo nondum cœpit hoc negare, sed conatus est tantum se defendere quasi anno 1684 cum elementa calculi sui edidit, ne constabat quidem ipsi aliud de Inventis Newtoni in hoc genere quam quod ipse olim significaverat in literis, posse se Tangentes invenire non sublatis irrationalibus: quod Hugenius quoque se posse ipsi significavit postea etsi cæterorum istius calculi adhuc expers; & ipsum Fatium agressus est proponendo Problema solvendum. Tandem anno 1703 Wallisius mortuus est et subinde Leibnitius audacior esse cœpit. Nam cum libri Newtoni de coloribus de numero Curvarum secondi generis de quadratura Figurarum anno proximo prodierent & , Newtonus in Actis Eruditorum pro Ianuario anni 1705 accusari cæpit quasi Methodum a Leibnitio dedicisset. Ejus Elementa, inquiunt, ab INVENTORE D. G. Leibnitio in his Actis sunt tradita, varijsque usus tum ab ipso tum a D. D. Fratribus Bernoullijs tum & D. Marchione Hospitalio — sunt ostensi. Pro differentijs igitur Leibnitianis D. Newtonus adhibet semperque [ex quo usus est {hu}c methodo]] adhibuit fluxiones — quemadmodum & Honoratus Fabrius in sua Synopsi Geometrica, motuum progressus Cavallerianæ methodo substituit

Quæstio est igitur utrum Newtonus hanc methodum didicita a Leibnitio. Ad hanc Quæstionem dirimandam editam sunt anno 1712 ex antiquis monumentis Commercium Epistolicum & ineunte anno 1615 Recensio ejusdem Libri. Hæc Recensio edita fuit Anglicæ in Actis Philosophicis R. Societatis, et jam latine versa præponitur Commercio & ad veritatem stabiliendam Commercium citatur Commercio opposuit. Et hæc < text from f 31v resumes > et his præmissis, et quod pro symbolis $\boxed{\frac{aa}{64x}}$ & oυ, vel oy quibus Newtonus utitur in Analysi per series Leibnitius utatur symbolis $\int \frac{aa}{64x}$ & dz, & Analysin illam videre potuit <31r> in secundo suo in Angliam itinere ubi Collinius ipsi monstravit plures Newtoni Gregorij & aliorum Literas quæ circa Series præcipue versabantur: his inquam præmissis, legatur jam Commercium Epistolicum.

<32r>

Scholium

In literis quæ mihi cum Geometra peritissimo G. G. Leibnitio anno 1676 intercedebant, cum significarem me compotem esse methodi determinandi Maximas et Minimas, ducendi Tangentes, quadrandi Figuras curvilineas, conferendi easdem cum se mutuo & similia peragendi quæ in terminis surdis æque ac in rationalibus procederet, & Tractatum de eadem anno 1671 scripsisse, ut et Tractatum alium quem Barrovius noster anno 1669 ad Collinium atque literis transpositis hanc sententiam involventibus [Data æquatione quotcunque Fluentes quantitates involvente, Fluxiones invenire, et vice versa] eandem celarem: rescripsit Vir Clarissimus mense Iunio anni 1677 se quoque in ejusmodi methodum incidisse; & methodum suam communicavit a mea vix abludent em præterquam in verborum & notarum formulis & Idea generationis quantitatum. Incidi paulætim in hanc methodum, annis 1666 & 16667 Et extant ejusdem specimina quædam in Analysi mea per Series quam Barrovius mense Iunio anni 1669 ad Collinium misit: ut et in Epistola quam de hac methodo 10 Decem. 1672 ad Collinium misi, et cujus exemplar Collinius mense Iunio anni 1676, una cum exemplaribus plurium Ia. Gregory, epistolarum D. Leibnitium in Gallia tunc agentem misit. Mense a**{u}**tem Octobri ejusdem anni ad finem vergente, D. Leibnitius E Gallia in Angliam veniens, ✝ < insertion from f 32r > ✝ vidit Epistolam meam Octob. 24 ad Oldenburgum datam ubi locutis sum de Analysi per series, ad Collinium missa deque Tractatu quem anno 1671 conscripsi, ac de methodo qua maximas et minimas, et areas curvarum inveniro: Et ubi Theorema etiam posui quadrandi figuras hac methodo inventem et exemplis illustravi Quinetiam D. Leibnitius epistolas autographas et Gregorij et meas in manu Collinij consuluit. Et subinde in Germaniam rediens ut negotijs <32v> publicis interesset, incidit in hanc methodum < text from f 32r resumes > epistolas et Gregorij et meas in manu Collinij consulavit. Et subinde ut negotijs publicis interesset, in Germaniam rediens incidit in hanc methodum anno 1677; et mense Iunio ejusdem anni specimen ejus ad me misit, ut supra. Hujus vero methodi fundamentum continetur in hoc Lemmate.

<32v>

et vice versa] fundamentum hujus methodi celarem, specimen vero ejusdem subjungerem et exemplis illustravem; cumque vir clarissimus in reditu suo e Gallia per Angliam in Germaniam, epistolas **{et}** meas in manu Collinij insuper consuluisset: incidit is in methodum similem sub diversis verborum et notarum formulis. Methodi utriusque fundamentum continetur in hoc Lemmate.

Et cum Collinius Epistolam 10 Decem 1672 datam a me accepisset in qua methodum hanc descripsissem & exemplo Tangentium ducendarum illustrassem, et hujus Epistola exemplar mense Iunio anni 1676 in Galliam ad D. Leibnitium misisset; & vir clarissimus sub finem mensis Octobris in reditu suo e Gallia per Angliam in Germaniam Epistolas meas in manu Collinij insuper consuluisset: incidit is finito itinere, in methodum similem sub diversis verborum et notarum formulis, et mense Iunio sequente specimen ejusdem in Tangentibus ducendis ad me misit, eandemque a methodo mea non abludere subjunxit. Methodi utriusque fundamentum continetur in hoc Lemmate.

<33r>

[3]ad Spatium quod corpus in Medid non resistente e queste cadendo eodem tempore describere potest, ut arearum prædictarum differentia ad $\frac{BD\times V^2}{AB}$.

In epistola quadam ad D. I. Collinium nostratem 10 Decem. 1672 data, cum descripsissem methodum tangentium quam suspicabar eandem esse cum methodo Slusij tum nondum communicata; subjunxi: Hoc est unum particulare, vel corollarium potius Methodi generalis, quæ extendit se, citra molestum ullum calculum, non modo ad ducendum tangentes ad quasvis Curvas sive Geometricas verum etiam ad resolvendum alia abstrusiora problematum genera de curvitatibus, Areis, longitudinibus, centris gravitatis curvarum &c. Neque (quemadmodum Huddenij methodus de Maximis et Minimis) ad solas restringitur æquationes illas quæ quantitatibus surdis sunt immunes. Hanc methodum intertexui alteri isti, qua æquationum exegesin instituo, reducendo eas ad series infinitas. Hactenus Epistola Et Hæc ultima verba spectant ad Tractatum quem anno 1671 de his rebus scripseram. Methodi vero hujus generalis fundamentum continetur in Lemmate præcedente.

Coroll. Quare si longitudo quæ oritur applicando aream DET ad lineam BD dicatur M Et si longitudo alia V sumatur in ea ratione ad longit. M quam habet linea DA ad lineam DE Spatium quod corpus — — — illarum differentia ad $\frac{BD\times V^2}{AB}$, ideoque ex dato tempore datur. Nam spatium in Medio non resistente est in duplicata ratione temporis sive ut V^2, et ob datas BD et AB, ut $\frac{BD\times V^2}{AB}$. Hæc area æqualis est areæ $\frac{DA^q\times BD\times M^2}{DE^q\times AB}$ et ipsius M momentum est m et propterea hujus areæ momentum est $\frac{DA^q\times BD\times {}^2Mm}{DE^q\times AB}$. Et hoc momentum est ad momentum differentiæ arearum DET et AbNK viz^t ad $\frac{AP\times BD\times m}{AB}$ ut $DA^q \times BD \times MDE^q$ ad $\frac{1}{2}AP \times BD$ sive ut $\frac{DA^q}{DE^q}$ in DET ad DAP adeoque ubi areæ DET et DAP quam minimæ sunt, in ratione æqualitatis. Æqualis est igitur area quam minima $\frac{BD\times V^2}{AB}$ differetiæ quam minimæ arearum DET & AbNK. Vnde cum spatia in Medio utroque <33v> ut $\frac{DA^q\times BD\times 2M\times m}{DE^q\times AB}+\frac{DA^q\times BD\times mm}{DE^q\times AB}$ ad $\frac{AP\times BD\times m}{AB}$ hoc est ut $\frac{DA^q\times BD\times 2M+m}{BE^q}$ ad AP, BD. sive ut $\frac{DA^q}{DE^q}$ in ${}^2DET+DTV$ ad $2DAP$; adeoque ubi areæ DET, DAP quam minimæ sunt, et area DET evanescit, in ratione æqualitatis. [Æqualis est igitur area quam minima] seu $\frac{DA^q}{DE^q}$ in $DTV=2DAP$

$U=\frac{2DET\times DA}{BD\times DE}.\ \frac{BD,VV}{4AB}=\frac{DET\times DET\times DA^q\times BD}{AB\times DE^q\times BD^q}=\frac{DET^q\times DA^q}{AB,DE^q\times BD}=\frac{DET\times DET\times DA^q}{AB\times DB^{cub}}=\frac{BD\times VV}{4AB}.\ M=\frac{DET}{BD}.$ $\frac{BD\times VV}{4AB}=\frac{MM\times DA^q}{AB\times BD}.$

$V=\frac{2DA\times M}{DE}.\ \frac{BD,VV}{4AB}=\frac{4DA^q\times M^q\times BD}{4AB,DE^q}=\frac{DA^q\times M^q\times BD}{AB\times DE^q}$

Momentum
$M^q=2DEV\times DVT+DVTq=\frac{2DET\times DVT}{D^q}+\frac{DVT^q}{BD^q}=\frac{2DET\times DVT}{BD^q}+\frac{DVT^q}{BD^q}=\frac{2DET\times BD\times m}{BD^q}+\frac{BD^q\times m^q}{BD^q}$.
$m=\frac{1}{2}TV.\ M=\frac{DET}{BD}=\frac{DE\times\frac{1}{2}ET}{DE}=\frac{1}{2}VT.\ ET=x+o.\ ET^q=xx+2ox$

lin 1. sive huic æqualis $\frac{DA^q\times BD\times M^2}{DE^q\times AB\times BD}$ et ad momentum differentiæ arearum DET et AβNK ut $\frac{DA^q\times M\times m}{DE^q\times AB}$ ad $AP\times BD\times mAB$, hoc est ut $\frac{DA^q\times BD\times M}{DE^q}$ ad $\frac{AP\times BD}{2AB}$, sive ut $\frac{DA^q\times DET}{DE^q}$ ad $DAP\times BD^q=BD\times 2DAP$ sive ut $\frac{DA^q}{BD^q}DET$ ad DAP.

P. 252. lin. 1. sive huic æqualis $\frac{DA^q\times BD\times M^2}{DE^q\times AB}$, est ad Momentum differentiæ arearum DET et AβNK, ut $\frac{DA^q\times BD\times M\times m}{DE^q\times AB}$ ad $AP\times BD\times mAB$, hoc est, ut $\frac{DA^q\times BD\times M}{DE^q}$ ad $AP\times BD$, sive ut $\frac{DA^q}{DE^q}$ in DET ad $2DAP$

<33r>

$$\frac{DA^qM^2}{AB,BD}\begin{aligned}&=DA^q\times M^2+2Mo+oo\\&=BA^q\times M^2+M\times m\end{aligned}\ .\ o=\frac{1}{2}VT=\frac{DTV}{2BD}$$

$\frac{DET}{DB}=M.\ \frac{DET\times DET}{DB^q}=MM.$

Ejus M momentum $=\frac{DVT}{DB}=m$

<33v>

Corol. Quare si longitudo aliqua V sumatur in ea ratione ad longitudinem M quæ oritur applicando aream DET ad BD quam habet linea DA ad lineam DE: spatium quod corpus ascensu vel descensu tota in Medio resistente describit erit ad spatium quod in Medio non resistente eodem tempore describere posset, ut arearum illarum differentia ad $\frac{BD\times VV}{AB}$ ideoque ex dato tempore datur. Nam spatium in Medio non resistente est in duplicata ratione temporis sive ut VV, et ob datas BD et AB ut $\frac{BD\times V^q}{AB}$. Momentum hujus areæ, sive areæ huic æqualis $\frac{DA^q\times BD\times M^2}{DE^q\times AB}$ est ad momentum d

<34r>

Scholium.

Analysin per series et momenta a me scriptam, Barrovius noster anno 1669 ad Collinium misit. Methodos ibi expositas in alio Tractatu plenius explicui anno 1671, et ex hoc Tractatu Tractatum de quadratura

Curvilinearum anno 1676 extraxi. Interea Et In Epistola 10 Decem. 1672 ad Collinium data, methodum momentorum Exemplo tangentium more Slusiano ducendarum illustravi, dixique eandem etiam ad quæstiones de curvitatibus & longitudinibus Curvarum & areis ac centris gravitatis Curvilinearum &c sese extendere, & esse generalem et ad quantitates surdas non hærere & methodo serierum in scriptis meis intertextam esse: et Collinius exemplar hujus Epistolæ mense Iunio anni 1676 ad D. Leibnitium tunc in Gallia agentem misit.

In Literis insuper quæ mihi cum D. Leibnitio anno 1676 intercedebant, cum verba facerem de Tractatibus prædictis, & significarem me compotem esse methodi. Analyticæ determinandi maximas et minimas, ducendi tangentes, quadrandi figuras curvilineas, conferendi easdem inter se, et similia peragendi quæ in terminis surdis æque ac in rationalibus procederet; et literis transpositis hanc sententiam involventibus [<u>Data æquatione quotcunque fluentes quantitates involvente, fluxiones invenire, et vice versa</u>] fundamentum hujus methodi celarem, specimen vero ejusdem in curvilineis per seriem quadrandis subjungerem et exemplis illustrarem; ⊙ < insertion from the bottom of the page > — et exemplis illustrarem, ⊙ sed in lucem me quietis gratia hæc non edere dicerem; et cum Vir clarissimus eodem anno < text from f 34r resumes > et Vir clarissimus eodem anno in reditu suo e Gallia per Angliam in Germaniam, sub finem mensis Octobris, epistolas meas in manu Collinij etiam consuleret: incidit is non multo post in methodum momentorum sub diversis verborum et notarum formulis, et mense Iunio sequente specimen ejusdem in Tangentibus more Slusiano ducendis idque non obstantibus irrationalibus, ad me misit, et subjunxit se credere methodum meam a sua non abludere, præsertim cum quadraturæ per methodum meam faciliores redderentur. Methodi hujus fundamentum continetur in hoc Lemmate, & in Tractatu de Quadratura curvilinearum fusius exponitur.

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Schol.

Schol.

Per momenta quantitatum hic intelligo earum incrementa momentanea seu particulas momento temporis genitas; easque indefinite parvas & in infinitum divisibiles. Et designando fluxionem temporis per unitatem & fluxionis aliarum quantitum ut x y z per x, y, z vel etiam per alia symbola uti p, q, r: denoto momentum temporis per 1 × o seu o et momenta aliarum quantitatum per $\dot{x}$o, $\dot{y}$o, $\ddot{z}$o &c vel po, qo, ro. Et habita quantitatis fluxione ut p, q vel r quantitate fluentem, designo quandoque per eadem symbola vel in imponendo lineolam in hunc modum p′, q′, r′ vel præfigendo aut circumscribendo rectangulum ut in his ▯p ▯q vel p , q . Et hujusmodi symbolis adhibitis Analysin instituo in fluentibus et eorum fluxionibus & momentis adhibendo unicam tantum quantitatem indefinite parvam o. Sed hujusmodi Analysin docere non est hujus loci.

Pag. 226. In literis quæ mihi cum Geometra peritissimo G. G. Leibnitio Anno 1676 intercedebant, cum significarem me compotem esse methodi determinandi Maximas & Minimas, ducendi Tangentes, quadrandi figuras curvilineas conferendi easdem cum Sectionibus Conicis & similia peragendi, quæ in terminis surdis æque ac in rationalibus procederet, & Tractatum de eadem anno 1671 scripsisse; atque literis transpositis hanc sententiam involventibus [Data æquatione — — — — et vice versa] eandem celarem: rescripsit vir clarissimus mense Iunio anni proximi, se quoque — — — quantiatum. Mense Iunio anni 1676 Collinius exemplar Epistolæ meæ de hac methodo 10 Decem. 1672 ad ipsum datæ, una cum exemplaribus Epistolarum plurium Iacobi Gregorij ad D. Leibnitium in Gallia agentem misit. Mense Octobri ejusdem anni ad finem vergente D. Leibnitius E Gallia in Angliam veniens, epistolas autographas et Gregorij et meas in manu Collinij consuluit. ⊙ < insertion from lower down f 35r > ⊙ Vidit etiam Epistolam meam 24 Octob. 1676 ad Oldenburgum datam. Et subinde in Germaniam rediens ut negotijs publicis interesset, incidit in hanc methodum, et mense Iunio **{**anni**}** < text from higher up f 35r resumes > & subinde in Germaniam rediens ut negotijs publicis interesset, incidit in hanc methodum: ‡ < insertion from lower down f 35r > ‡ Et mense Iunio anni 1677 specimen ejus ad me misit. Ejus vero fundamentum continetur in hoc Lemmate. < text from higher up f 35r resumes > Ejus vero fundamentum continetur in hoc Lemmate.

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Phænomenon VII.

Planetæ circum axes suos uniformi cum motu revolvuntur respectu fixarum: Iupiter horis 9. 56′ Mars horis 24.40′ Venus horis 23, Sol diebus $25\frac{1}{2}$ Luna diebus 27. horis 7. minutis primis 43′; et Terra horis 23 minutis primis 56′

Revolutiones Iovis, Martis, Veneris & Solis per revolutiones macularum in eorum corporibus observatæ fuerunt. Maculæ Solis non sunt permanentes. Post mensem unum aut altera evanescere solent, ideoque tempus revolutionis corporis ejus nondum satis accurate definiri potuit. Redeunt ad eundem situm in disco Solis Spatio dierum $27\frac{1}{2}$ circiter respectu Terræ et inde colligitur quod respectu fixarum redeunt ad eundem situm diebus $25\frac{1}{2}$ circiter. Eadem Lunæ facies Terram semper respicit; ideoque respectu fixarum Luna revolvitur circa axem suum semel in mense periodico. Eadem vero facies ejus Terram semper respicit non accurate sed cum motu quodam libratorio tam in Longitudinem quàm in Latitudinem. Librando in Longitudinem faciem eandem semper vertita in umbilicum superiorem, Orbis sui **{**qu**}**amproxime, ideoque revolutio ejus respectu facerum uniformis est quoad motum in longitudinem. Librando in Latitudinem faciem ejus vertit in boream in Latituduine boreali & in austrum in Latitudine australi idque in angulo latitudinem æquante [ideoque axis area quem revolvitur plano Eclipticæ perpendicularis est quamproxime. Hæc omnia, a me accepta D. Mercator in Astronomia sua initio anni 1676 edita plenius exposuit plenius exposuit, Simili motu Satelles extimus Saturnius circa axem suum revolvi videtur, eadem sui facie Saturnum perpetuo respiciens. Nam circa Saturnum revolvendo, quoties ad orbis sui partem orientalem accedit ægerrime videtur & plerumque videri cessat: id quod evenire potest per maculas quasdam in ea corporis sui parte quæ Terræ tunc advertitur, Simili etiam motu Satelles extimus Ioviales circa axem suum revolvi videtur, propterea quod in parte corporis Iovi aversa: maculam habeat, qua tanquam in corpore Iovis cernitur ubi Satelles inter Iovem & oculos nostros transit.

Hæc omnia a me anno 1675 accepta D. Mercator in Astronomia sua initio anni sequentis edita plenius exposuit

— — idque in angulo latitudinem æquante nisi quatenus hic angulus nunc augetur nunc diminuitur aliquantulum per inclinationem axis Lunæ ad planum Eclipticæ Hæc Librationis rationem a me accepta**{**m**}**, D. Mercator in Astronomia sua initio anni 1676 edita plenius exposuit. Simili moto Satelles extimus Saturnius circa axem suum revolvi videtur, eadem sui facie — — —

In Analysi mea per series, quam Barrovius mense Iunio anni 1669 ad Collinium misit, extant methodi hujus specimina quadam, tum ante annos aliquot inventæ Mense Iunio anni 1676 Collinius

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Pag. 226.

Scholium.

Analysin per series et momenta a me scriptam, Barrovius noster anno 1669 ad Collinium misit. Methodos ibi expositos an alio Tractatu plenius explicui anno 1671, inde Tractatum de Quadratura Curvilinearum anno 1676 extraxi. In Epistola 10 Decem 1672 ad Collinium data, methodum momentorum exemplo tangentium more Slusiano ducendarum illustravi, dixique eandem etiam ad quæstiones de curvitatibus et longitudinibus Curvarum ac de areis & centris gravitatis &c sese extendere, & esse generalem, et ad quantitates surdas non hærere, & methodo serierum in scriptis meis intertextam esse: et Collinius exemplar hujus Epistolæ mense Iunio anni 1676 ad D. Leibnitium tunc in Gallia agentem misit.

In literis insuper quæ mihi cum D. Leibnitio anno 1676 intercedebant, cum verba facerem de Tractatibus prædictis & significarem me compotem esse methodi Analyticæ determinandi maximas et minimas, ducendi tangentes, quadrandi figuras curvilineas, conferendi easdem inter se, et similia peragendi quæ in terminis surdis æque ac in rationalibus procederet; & literis transpositis hanc sententiam involventibus [<u>Data æquatione quotcunque fluentes quantitates involvente, fluxiones invenire, et vice versa</u>] fundamentum hujus methodi celarem, specimen vero ejusdem in curvilineis per seriem quadrandis subjungerem et exemplis illustrarem, sed me quietis gratia hæc in lucem non edere dicerem; et cum D. Leibnitius eodem anno in reditu suo E Gallia per Angliam in Germaniam, sub finem mensis Octobris, Commertium meum in manu Collinij etiam consuleret: incidit is in methodum momentorum sub diversis verborum et notarum formulis, et mense Iunio {anni} sequentis specimen ejusdem in Tangentibus more Slusiano ducendis ad me misit, et subjunxit methodum suam ad irrationales non hærere & se credere methodum meam a sua non abludere, præsertim cum quadraturæ per methodum utramque faciliores redderentur. Fluxiones utique sunt velocitates quibus fluentium momenta generantur, et momenta a D. Leibnitio vocantur differentiæ: et inde Methodus fluentium et Methodus fluxionum & Methodus momentorum & Methodus differentialis eandem methodum significare possunt. Methodi hujus fundamentum continetur in Lemmate præcedente, et in prædicto de Quadraturis Tractatu fusius exponitur.

<36v>

Octob 5. 1722. Apog ☽ in ♌ 23$\frac{1}{2}$. ☉ in ♎ 23. ☽ in ♑ 16. $\left|\begin{array}{c}1722.\\574\end{array}\right.$ 1656. 3907. 2251. 563.

[1] {Pander}

[2] not addi{to} in 2^{d} {Edit}

[3] Pag. 226