Mathematical Notebook

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<2r>

Of the extraction of Pure Square Cubick. Square-square & square-cubick rootes &c.

Let the number whose roote is to bee extracted bee pointed makeing the first point under the **{**unite**}** & comprizeing soe many numbers under each point as the number hath dimensions as if the number be square-cube tis thus pointed 57086352410802

Then out of the figures of the first point next the left hand extract the greatest roote proper to the power of the number & set that downe in the Quotient which is the first side & is called A. (as the roote quintuplicate of 5708 is (5), & (5) quintuplicate is 3125) then takeing that roote duely multiplied out of the number (as 3125 out of 5708) with the rest of the numbers to the next point. seeke the seacond side which is found by divideing that number by another number made out of the first side (which is called the Divisor) & this second side I name E. (thus by divideing 258363524 by $5\mathrm{A}\,\mathrm{qq}+10\mathrm{A}\,\mathrm{c}+10\mathrm{A}\,\mathrm{q}+5\mathrm{A}$ after such a maner that $5\mathrm{A}\,\mathrm{qqE}+10\mathrm{A}\,\mathrm{cE}\,\mathrm{q}+10\mathrm{A}\,\mathrm{qE}\,\mathrm{c}+5\mathrm{AE}\,\mathrm{qq}+\mathrm{E}\,\mathrm{qc}$ may be conteined in the number the product of that division shall be E =

<2v>

The extraction of the square roote

The square to be resolved	29	16	(54 The Product
The square of y^e first side	25		be taken away.
The rest of y^e sqare to be	4	16	resolved.
The divisor for finding y^e seacond	1	0	sidie. which is y^e first side doubled
The rectangle by 2A & E	4	0	to be substracted
The square of E		16	
The sum̄e of y^e rectangles	0	16	to be subducted
	0	00	The remainder

The extraction of the cube roote

The cube to be resolved	157	464	(54
The cube to be subducted	125	whose roote is	A = 5
The remainder for y^e finding	32	464	of E
The divisors for y^e finding	7	5	3A q
of (E) y^e seacond side.		15	3A
The sume of y^e divisors	7	65	
Sollids to be substracted	30	0	3A qE
	2	40	3AE q
		64	E c
The sume of those	32	464	sollids
The remainder	00	000	

The extraction of the square square roote

The square-square	33	1776	(24
The square-squ: to be subduc:	16	= A q q	
Remainder.	17	1776	
Divisors for finding y^e	3	2	4A c
seacond side E.		24	6A q
		8	4A
Theire sum̄e	3	448	
Squ-Squares to be sub=	12	8	4A cE
=ducted	3	84	6A qE q
		512	4AE c
		256	E qq
Theire Sum̄e	17	1776	

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The extraction of the Square-Cube roote

The squ: cube to be resolved	79	62624	(24
Substract	32	A qc	
Remaind^e	47	62624	
Divisors	8	0	5A qq
		80	10A c
		40	10A q
		10	5A
The Sume of y^e divisors	8	8410	
Plano-Sollids to be substracted	32	0	5A qqE
	12	80	10A cE q
	2	560	10A qE c
		2560	5AE qq
		1024	E qc .
Theire Sum̄e	47	62624	
Remainder	00	00000	

Note that the 3^d 4^th 5^th & other figures are found by the same manner that the seacond figure is found onely makeing all the figures found to stand for A the first side & the figure sought for e or the 2^d side

And if roote is found inexpressible in whole numbers then adding ciphers & pointing them from the unite towards the right **{kind}** as was before explained & soe hold on the worke in decimalls.

As for the Divisors they are easily found by the 2^d Table of Powers from a Binomial roote.

If the Number bee of 6.7.8.9.10 &c dimensions The roote may be extracted after the same manner

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Of the Extraction of Rootes in Affected powers.

The manner of the extraction of rootes in pure & affected powers is very much alike, especially when the affected powers are decently prepared, that is, when theire affections are not over large & those altogether either affirmative or negative, & the power affirmative, affirmations & negations so mixt that there be noe ambiguity & all fractions & Asymmetry taken away

All the figures in the coefficients & affected power are to be pointed (after the manner before explained in the Analisis of pure powers) according to the degree of theire dimensions & the worke onely differs from that in pure powers in that the coefficients enter into the divisors

Let the first side be called A. the 2^d be called E. the Roote of the equation **{L}** the coefficients B . C q . D c . F qq . G qc . H cc &c the Power P . P q . P c . P qq &c & the Operation follows

The analysis of Cubick Equations.

The equation supposed $Lc^* + 30L = 14356197$. $Lc + CqL = Pc$

The square coëfficient					3	0				
The cube affected to be	1	4	3	5	6	1	9	7	(243	
Sollids to bee substracted		8							= A c	
					6	0			= AC q	
Theire sum̄e		8	0	0	6	0				
Rests		6	3	5	0	1	9	7	for finding y^e 2^d side	
The extraction of			y^e seac			ond side				
Coëfficient						3	0		or superior divisor	
The rest of y^e cube to be		6	3	5	0	1	9	7	resolved	
The inferior divisors		1	2						3A q	
				6		0			3A	
Theire sum̄e		1	2	6	0	3	0			
Sollids to be sub= stracted		4	8						= 3A qE	
			9	6					= 3AE q	
				6	4				= E c	
					1	2	0		EC q	
Theire sum̄e		5	8	2	5	2	0			

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The superior part of y^e divisor		3 0	or y^e square coefficient
The remainder for finding	5 2 4	9 9 7	y^e third side
The inferior part of y^e divisor	1 7 2	8	3Aq that is $3 \times 24 \times 24$
		7 2	3A or 3×24.
The sume of y^e divisor	1 7 3	5 5 0	
Sollids to be taken away	5 1 8	4	3AqE
	6	4 8	3AEq
		2 7	Ec
		9 0	ECq
Theire Sume	5 2 4	9 9 7	
Remaines	0 0 0	0 0 0	

But the Coëfficient maybe greater than the Power soe that it cannot be substracted from it which argues that the Cube more propperly affects than is affected. In this case the coëfficient must descend towards the unite soe many points untill it may be substracted, & soe many points as the coëfficient is devolved soe many pricks must be blotted out towards the left hand in the power affected. As the Example shews
$Lc + 95400L = 1819459$.

9	5 4 0	0	Coefficiens
1	8 1 9	4 5 9	The Power

Since 9 is greater y^n 1 make a devolution thus.

	9 5 4	0 0	The Coefficient
The Quote (19 1	8 1 9	4 5 9	The affected power
Sollids to be substracted	9 5 4	0 0	ACq
	1		Ac
Suma	9 5 5	0 0	substrahenda
Divisorū superior pars	9 5	4 0 0	Coefficiens Planum
	8 6 4	4 5 9	Potestas reliqua
Divisorū pars inferior		3	3Aq
		3	3A
Divisorū suma	9 5	7 3 0	
Sollida ablativa	8 5 8	6 0 0	ECq
	2	7	3AqE
	2	4 3	3AEq
		7 2 9	Ec
Eorū Summa	8 6 4	4 5 9	
Restat	0 0 0	0 0 0	

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To place the unite of the coefficient in its right place in respect of the power make so many pricks above as there are under the power begining at the unit, & if the coefficient be one dimension lesse than the power make a prick on every figure if 2 dimensions les than every other figure of 3 dimensions lesse make it one each third figure &c

If there be many coefficients in the equation each must be placed according to this rule.

Sometimes the coefficient is under a negative sine as $Lc - 10L = 13584$ & the Analysis is as follows

Coëfficiens planum	−	10	sublaterale
Cubus resolvendus	+ 13	584	(24
Solida ablativa	+ 8		Ac
	−	20	ACq
Suma	+ 7	80	
Restat	+ 5	784	resolvendum
Divisorū $\underline{p}^s$ superior		−10	coëfficiens planum
Divisorū $\underline{p}^s$ inferior	+ 1	2	+3AA
	+	6	+3A
Suma divisorū	+ 1	25	
Solida ablativa	+ 4	8	3AAE
	+	96	3AEE
	+	64	EEE
	−	40	ECC
Eorū suma	+ 5	784	

But sometimes the square coëfficient hath more paires of figures than the cube to be analysed, hath & then there is præfixing so many ciphers to the cube as figures are wanting, the first side will not much differ from the square roote of the coefficient. as $Lc - 116620L = 352947$

	− 1 1	6 6 2	0	Coefficiens planū
Cubus resolvendus	0 0	3 5 2	9 4 7	(343
Sollida Ablativa	+ 2 7			A c
	− 3 4	9 8 6		ACq
Restat auferendū	− 7	9 8 6		
Reliquum resolvendi	+ 8	3 3 8	9 4 7	Cubi

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Divisorū ps superior Coeff:	− 1	1 6 6	2 0	planum.
Reliquū resolvendi cubi	+ 8	3 3 8	9 4 7	negative affecti
Divisorū ps inferior	+ 2	7		3AA.
	+	9		3A.
	* *	* * *	* * *	************
Suma Divisorum	+ 1	6 2 3	8	3AA + 3A + Cq
Sollida ablativa	+ 1 0	8		3AAE
	+ 1	4 4		3AEE
	+	6 4		EEE
	− 4	6 6 4	8	CCE
Eorum summa	+ 7	6 3 9	2	
Restat Resolvend	+	6 9 9	7 4 7	pro 3o latere
Divisorum ps superior ~	−	1 1 6	6 2 0	CC
Divisorū ps inferior	+	3 4 6	8	3AA
	+	1	0 2	3A
***********	* * *	* * *	* * *	*
Eorum Summa		2 3 1	2 0 0	= 3AA + 3A + CC
Sollida ablativa	+ 1	0 4 0	4	3AAE
	+	9	1 8	3AEE
	+		2 7	EEE
	−	3 4 9	8 6 0	ECC
Eorum Summa	+	6 9 9	7 4 7	

Sometimes though there be as many 2 figures in the coefficient as 3 figures in the cube affected yet the coëfficient may be so greate as to deceive an unwary Analist As in this $\mathrm{L\,c} - 6400\mathrm{L} = 153000$. where the roote of 64 is 8 which cubed is 512 which added to 153 makes 665 then whose roote the number immediately greater is 9 which is the first side $= \mathrm{A}$.

But if the coefficient had beene affirmative, then not the aggregate of the facts but the difference must be taken as in this. $\mathrm{L\,c} + 64\mathrm{L} = 1024$.

Since the roote of 64 is 8 . which cubed is 512 . & $1024 - 512 = 512$. the roote of which is $8 = \mathrm{A}$. The like is observable in equations of higher powers

If the Cube be affected with a negative sine as $13{,}104\mathrm{L} - \mathrm{L\,c} = 155{,}520$. Then the Equation is expressible of 2 rootes: whereof the square of one is <6r> lesse & the square of the other is greater then $\frac{13104}{3}$. & therefore one roote is lesse the other greater then $\frac{155520}{13104}$. & in this equation $27755\mathrm{L} - \mathrm{L\,qq} = 217944$ are two rootes whereof one is greater the other lesse then $\frac{217944}{27755}$.

☞

Suppose in the former cubick equation the lesse roote be 12. then $\frac{155520}{12} = 12960$. or else $13104 - 12 \times 12 = 12960$. & $\mathrm{L\,q} + 12\mathrm{L} = 12960$. where $\mathrm{L} = 108$ is the greater roote.

And in the latter equation if the greater roote be 27. & $\frac{217944}{27} = 8072$, c. or $-27 \times 27 \times 27 + 27755 = 8072$. $27 \times 27 = 729$. If there be 4 cubes continually proportionall whose greate extreame is $27\mathrm{c} = 19683$. & the aggregate of the 3 rest is 8072 & $\mathrm{L\,c}$ the lesse extreame, therefore $\mathrm{L\,c} + 27\mathrm{L\,q} + 729\ \mathrm{L} = 8072$. the roote of which .is 8 the other roote of the equation

☞ Or haveing one roote of an equation the Equation may be lessened by division thus $13104\mathrm{l} - \mathrm{lc} = 155520$ or $\mathrm{l}^3 - 13\,1\,04\mathrm{l} + 155520 = 0$. & one roote is 12 . therefore divide this equation by $\mathrm{l} - 12$ & the Quote is an equation conteining the other roote viz: $\mathrm{lq} + 12\mathrm{l} = 12960$.

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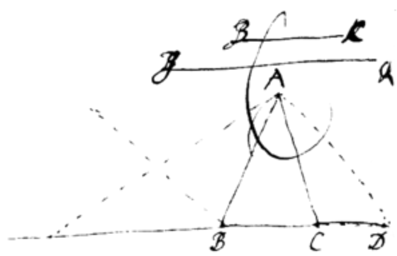
Propositiones Geometricæ. Franc: Vietæ.



prop 1

ab ∶ ac ∷ ce ∶ bd

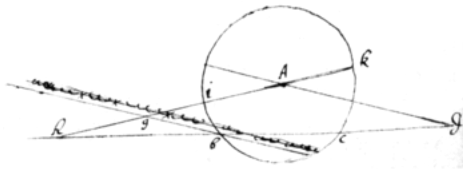
prop 2

& if ab ∶ ac ∷ ac ∶ bd : then ac ∶ ab ∷ ab ∶ ce

prop 3. If $\mathrm{ab} \times \mathrm{ac} = \mathrm{bd} \times \mathrm{ce}$. then bd ∶ ac ∷ ac ∶ ab ∷ ab ∶ ce ∺

prop 3. To find two meane proportionalls **{twixt}** Bc & IK . On the center a with the radius ai describe the circle ibck . inscribe $b\,c = cd$. draw da through the center & bg parallel to it. draw hk through A soe that $gh = ab = (= ai)$. & $ik : hb :: hb : hi :: hi : bc$. ∺

Prop: 4



If $ad = db = cb$. then the Angle c be is tripple to the Angle abd .

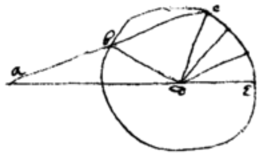
Prop 5

If $ab = bd = \text{Rad}.\ 3\ \text{Ang: } bad = cde$

Prop 6

If $3rpq = $ spq:recto. that is If $2qr = pr$. then $3or \times or = sp \times sp + op \times op + px \times px$

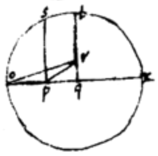
Prop 7



If $ad = dc = ce = ef$. then $ecf = efc = 3dac = 3dca$ & $AC^3 = 3AC \times ad^2 + Cf \times ad^2$. $\boxed{Z^3 = aaz + b^3. \quad \text{~ ~ ~}}$

prop 8. If $op = pr = qr = qs$. then, $prt = 3qsr$ &c and $sr^3 = 3sr \times qr^2 - or \times qr^2$. $\boxed{Z^3 = aaz - b^3}$

prop 9 If $ah = hb = bf = fd$. & $ch = 2eh$ or $ceh = 3hce$. then $\left.\begin{array}{ccccccc} & ac^3 & = & 3ac & \times & ah^2 & - & db \times ah^2 \\ \& & cb^3 & = & 3cb & \times & ah^2 & - & db \times ah^2 \end{array}\right\}$ $\boxed{Z^3 = aaz - b^3.}$



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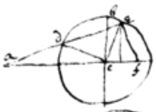
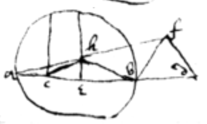
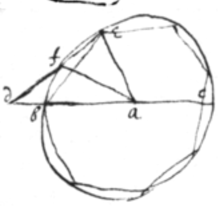
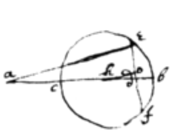
Prop 10

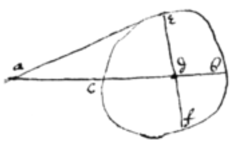
If $de = ea$ & $db : da :: ab \times ab : dc \times dc$. then be is a side of a 7 equall sided & angled figure. or $7eab = 4$right angles.





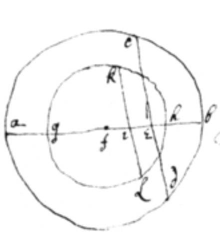


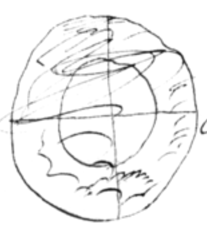


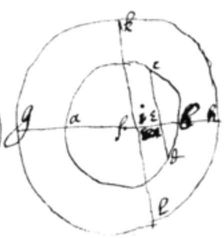


prop 10

If $ac = ef$. & aef a right angle & ab passes through the center then $cd : de :: de : df :: df : db$. And if $cd : de :: de : df :: df : db$ then ae is perpendicular to ef . 2hd is the difference of the extreames & 2do is the difference of the meanes. which given the proportionall lines may be found &c.

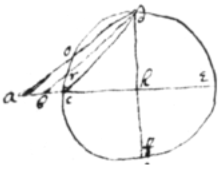






prop 11

Pseudomesolabium wherby To find 2 meane proportionalls. If, $ae : ec :: ec : ed :: ed : eb$. they be inscribed in the circle acbd the diam : being $ae + eb$. If twixt g i & i h two meane proportionalls are sought on the same center f with the Rad : $\frac{gi+ih}{2}$ describe gkhl & inscribe a line kl parallel to cd cutting ab in the point i & $gi : ki :: ki : il :: il : ih$. Examine it.



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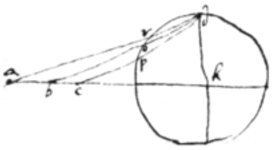
prop 12

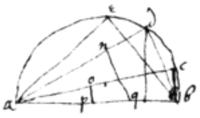
If $do = dh$ & ac bisected in b & bd bee drawne rd is the side of a pentagon which may be inscribed in defcro



prop 13.

If rd be the side of a $\left\{\begin{matrix}\text{octogon}\\ \text{decagon}\end{matrix}\right\}$ & pd the side of an $\left\{\begin{matrix}\text{hexagon}\\ \text{octogon}\end{matrix}\right\}$ the arch rp divided in o , od will be the side of an $\left\{\begin{matrix}\text{heptagon}\\ \text{enneagon}\end{matrix}\right\}$ to be inscribed in the circle ord & the arch RP is rightly divided by Bisecting the Line ac . Examine it

Of Angular sections.



prop 14

If $ead = cab$. Then $ab : ab^2 :: cb : eb \times ad - ae \times db :: ac : ae \times ad + eb \times db$ or, $ab : aq \times ab :: op : eb \times an - ae \times nq :: ao : ae \times an + eb \times nq$. But the angles anq , aop are right ones and e an $= oap = eab - dab$

prop 15

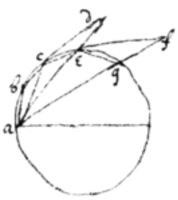
If the angle $cab + dab = eab$. or $naq + oaq = eab$. & anq , aop are right angles then $Ab : ab^2 :: Eb : ad \times bc + ac \times db :: ea : ad \times ac - db \times cb$. or the triang: unequall. $ab : ap \times aq :: eb : an \times op + ao \times nq :: ea : an \times ao - nq \times op$

prop 16.

In 2 rectang: triang: acb & aed , if the first have an acute angle cab submultiple to the acute angle eab of the 2d triang aeb the sides of the seacond have this proportion. Suppose the Hypoten of the first tri: be z . the base b . the Cathetus c .

If ye acute angle of ye seacond triangle be to ye acute angle of ye first triangle in a proportion

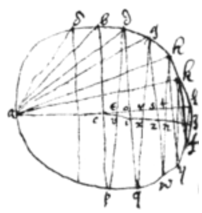
Hypoten:	Base	Perpendicular
Duple, Z^2.	B^2. $-C^2$.	$2BC$.
Triple, Z^3.	B^3. $-3DDC$.	$3BBC$. $-C^3$.
Quadruple, Z^4.	B^4. $-6B^2C^2$. $+C^4$.	$4B^3C$. $-4BC^3$.
Quintuple, Z^5.	B^5. $-10B^3C^2$. $+5BC^4$.	$5B^4C$. $-10B^2C^3$. $+C^5$.



Prop 17. If {∠} $ab = bc = ce = eg$ &c: & $ac = cd$. & $ae = ef$ &c then $ab : ac :: ac : ad :: ae : af$ &c & $ed = ab$ & $ac = gf$ &c. {nam} triangle cde & cba , efg & eac &c: = & sim.

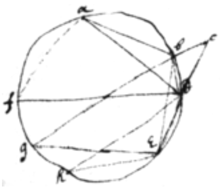
Prop.18.

If $bd = dg = gh = hk = pq = pw$ &c Then $al : ak :: pe : pc :: ed : do :: pd : pc + do ::$ $rg : gs :: rq : qo :: qg : qo + gs ::$ &c & if $\left(\frac{lf}{2} = lʒ\right)$ from ʒ to the center be drawne cʒ then $al : ak :: di : dv :: iq : qx :: dq : dv + qx :: gz : gx :: wz : wx$ &c Ergo $ac : ak :: ab :: a\delta + ad : ad : ab + ag : ag : ad + ah : ah : ag + ak$ &c.



Prop 19

If $fa = ab = be = eh$ &c. &. $af + ab + be + eh$ are greater than the semiperiphery: & dh is the greatest, db the least line drawn from d to these points a, b, e, h. then $rad : dh :: db : da - de$.

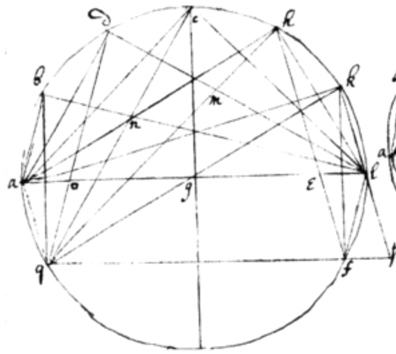


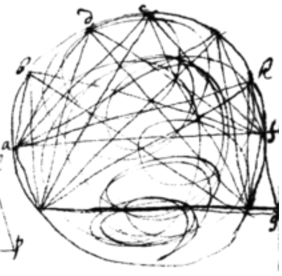
Prop 20

Out of the 18th & 19th Propositions To divide An angle into any number of points in the figure of the 18th prop: $al = diam = 2z$. ah is the greatest of the inscribed lines = B: now $z : B :: ah + 2z$. therfore $bb = ah$ in $z + 2z^2$. & $\frac{bb-2zz}{z} = ah$. And $z : B :: \frac{b^2-2z^2}{z} : b + ag$. therfore $\frac{b^3-2zzb}{zz} - b = ag$ Likewise $\frac{b^4-4zzbb^2+2z^4}{z^3} = ad$. & $\frac{B^5-5zzB^3+5z^4B}{z^4} = ab$ $\frac{B^6-6zzB^4+9z^4BB-2z^6}{zzzzz} = a\delta$
$\frac{B^7-7zzB^5+14z^4B^3-7z^6B}{z^6} =$ to a seaventh line
$\frac{B^8-8zzb^6+20z^4b^4-16z^6bb+2z^8}{z^7} =$ to an eight line $\frac{B^9-9zzb^7+27z^4b^5-30z^6b^6+9z^8b}{z^8} =$ a nineth line $\frac{B^{10}-10zzb^8+35z^4b^6-50z^6b^4+25z^8bb-2z^{10}}{z^9} =$ tenth &

Prop 21

out of the 17th Theor.: in the figure whereof if ab {:} the least inscribed line = z. & ac the next line bee B . then $z : B :: B : z + ae$. & $\frac{bb-zz}{z} = ae$ & $\frac{B^3-2z^2B}{zz} = ag$ &
$\frac{B^4-3z^2bb+z^4}{z^3} =$ to a fift line. $\frac{B^5-4zzb^3+3z^4bz^4}{z^4} =$ a sixt. & $\frac{b^6-5zzb^4+6z^4bb-z^6}{z^5} =$ seaventh $\frac{b^7-6zzb^5+10z^4b^3-4z^6b}{z^6} =$ to an eight line
$\frac{B^8-7zzb^6+15z^4b^4-10z^6bb+z^8}{z^7} =$ to a nineth line $\frac{B^9+8zzb^7+21z^4b^5-20z^6b^3+5z^8b}{z^8} =$ to a tenth line $\frac{B^{10}-9zzb^8+28z^4b^6-35z^6b^4+15z^8bb-z^{10}}{z^9} =$ eleventh.





Prop 22.

If $aq = ab = bd = dc = ch = hk = kl = lf$. Then GK Rad $: kl :: kl : el(= al - ah) :: hl : hm(= qh - qd = ak - ac) :: dl : do(= qd - qa = ac - ab) :: lc : cn(= qc - qb = ah - ad$ &c) Soe that the Periph: divided into any number of points.
$gl : lk :: lk : al - ak :: lh : ak - ac :: lc : ah - ad :: dl : ac - ab$ &c. & $gl : lk :: ah : lc - lk :: ac : ld - lh :: ad : lb - lc :: ab : al - ah$ &c.

hence Prop 23.

In the former scheame If $al = 2x = \mathrm{hypotenusa}. kl = b. x : b :: b : 2x - ah$ & $\frac{-bb+2xx}{x} = ah$ / $x : b :: \frac{-bb+2xx}{x} : \frac{2bxx-b^3}{xx} (= lc- b$) therefore $\frac{3bxx-b^3}{xx} = lc$. & $\frac{2x^4-4bbxx+b^4}{x^3} = ad$ the base of the 4^{th} triang: & $\frac{5bx^4-5b^3xx+b^5}{x^4} =$ the perpendicular (bl) of the 5^t triangle & $\frac{2x^6-9x^4bb+6xxb^4-b^6}{x^5} =$ base of the 6^t triang. $\frac{7x^6b-14x^4b^3+7x^2b^5-b^7}{x^6} =$ perpendicular of the 7^{th} triangle $\frac{2x^8-16x^6bb+20x^4b^4-8xxb^6+b^8}{x^7} =$ base of the 8^{th} tri. $\frac{9x^8b-30x^6b^3+27x^4b^5-9xxb^7+b^9}{x^8} =$ perp: of the 9^{th} tri:

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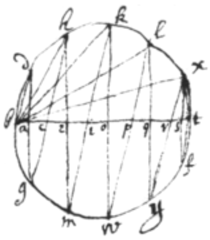
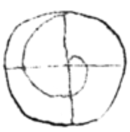


Prop 24:

If $bd = dh = hk = kl = b\ g$ &c: then $bh = gd$ & $bk = gh$ & $bl = hm$ &c: & then
$xt : bx :: ac : ag :: ce : eh :: ei : em :: io : ok :: op : on :: pq : ql :: qr : qy :: rs : sx :: st : sf :: ba : ad$ therefore
$xt : bx :: bt : dg + hm + kn + ly + xf$. againe $xt : bt :: ab : bd :: ac : cg :: ce : ch :: ei : im$ &c Therefore
$xt : bt :: bt : bd + gh + mk + ml + yx + ft$. & since, as $xt : bt :: bx : dg + hm + kn + ly + xf$ Therefore
$xt : bt :: bx + bt : bd + dg + gh + hm + mk + kn + \quad + nl + ly + yx + xf + ft$. And $xt : bt :: bx + bt + xt :$ to all the perpen dicular & transverse line $+bt$. that is
(5) $xt : bt :: xt + bt + bx : 2bd + 2bh + 2bk + 2bl + 2bx + 2bt$.

Prop 24

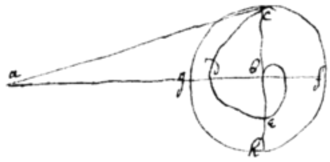


If in the circle cfgh be inscribed the helix bedc & ac touch it in the point c then $ab =$ to the circumference.

Prop 25

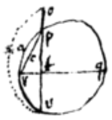
If apcr be les than halfe the circle. & $vt = tp$. & $vo =$ to vrap : then $\frac{rq \times po}{2} = 4$ times the section rapc

Prop 26



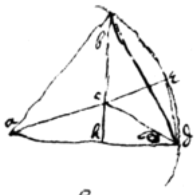
If $ab = bd = ad$ & bh perpendicular to ad from the angle b . $ce = ed$. then $aed = adi = 3dae$. & ed is the side of a heptagon

Prop 27.



If a line be cut by extreame & meane proportion the lesse segment almost is to the whole line as the diameter is to 5 times the periphery divided by 6 .

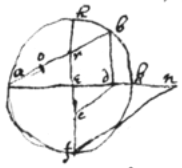
Prop 28



Si secetur linea per extremam & mediam proportionem erit proximè, ut tota linea plus minori segmento ad bis totam lineam, ita quæ potest quadrato sesquialterum semidiametri, ad latus quadrati circulo equalis.
linea secta sit 100,000 . minus segmentum 38,197 . Semidiametrum 100,000 , quæ potest quadrato sesquialterum semidiametri paulo maior est quam 122,474 . Radix Peripheriæ, 177,245 .

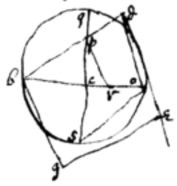
<11r>

Prop 28.



If $er = rh = or$. & $ao = fc =$ to the side of a decagon; & fn parallell to cd then en shall be almost equall to the fourth parte of a circle for ef is divided in extreame & meane propor in the point c . & $ec : ef :: ef : \frac{5}{12}\mathrm{Perimeterhbkfa} :: hr : \frac{5}{24}\mathrm{Perim} :: \frac{6}{10} ef (= de) :: \frac{1}{4}\mathrm{Perim}$; by the 27^{th} prop: & $ec : ef :: ed : en \left(= \frac{1}{4}\mathrm{Perim}\right)$.

Prop 29.

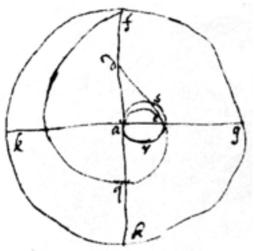


If $os = 2cp$. & co is divided by extreame & meane proportion in r . & od parallell to rp then db is the side of a square = to the area of the circle. for by the 28^{th} prop: As $br (= \text{to line} + \text{less } \overline{\text{segm}}) : bo (= \text{twice } y^e \text{ line}) :: bp$ ($\left(= \sqrt{\tfrac{3}{2}\text{of the square of } y^e \text{ semidiameter}}\right) : bd (= \text{to } y^e \text{ roote of a square equall to } y^e \text{ area of a circle}.$

Prop 30

If the line dc touch the helix in the line ag . & the line hf toucheth the beginning of it in the center a & $4ac = af$ then 2ad shall bee equall to perim: asr . & ac being the Diameter: the area of the triang acd = to the area of the circle asr

Prop 31



If bed be a square of one revolution of an helix & the angle $dbe = dba$ & through the points a , d , in the helix be drawne the line adk & through the points e , d in the Helix be drawne edg . & the angle kdg bisected by dh ; then dh shall almost touch the helix in d . & it shall be soe much the nigher a touch line by how much the angles
ebd dba are lesser.

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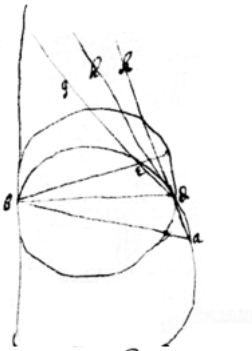
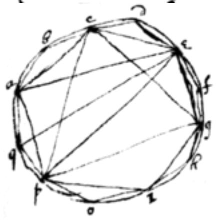
Prop 32



If many Polygons be inscribed in a circle the number of theire sides increaseing in a double proportion. & theire apotomies, or the base of a tri: whose cathetus is a leg of the Polygon & hypotenusa is the Diameter (as the apotome of the Polygon cgp is ce . of pacegi is ae &c) if the Apotome of the sides of the first Polygon be called b . of the $2^{d} = c$. of the $3^{d} = d$. of the $4^{th} = f$. of the $5^{t} = g$ of the Sixt $= h$. & the diameter be z And the first Polygon be $= p$. the $2^{d} = q$. the $3^{d} = r = abcdefghiopq$. the fourth $= s$. the $5^{t} = t$ the sixt $= v$. the $7^{th} = w$ &c then
$p : q :: b : z$. & $p : r :: bc : zz$. & $p : s :: bcd : zzz$. & $p : t :: bcdf : z^4$. & $p : v :: bcdfg : z^5$. & $p : w :: bcdfgh : z^6$ &c

To know how many divers ways things, whereof some of them are equall, may bee ordered. . as of . $a.b.b.c.c.c.d.d.$ doe thus **{** $\overset{a}{\frac{1}{1}} \times \overset{bb}{\frac{2\times3}{1\times2}} \times \overset{ccc}{\frac{4\times5\times6}{1\times2\times3}} \times \overset{dd}{\frac{7\times8}{1\times2}} = 112$ **} {** $\overset{a}{\frac{1}{1}} \times \overset{b\ b}{\frac{2\times3}{1\times2}} \times \overset{c\ c\ c}{\frac{4\times5\times6}{1\times2\times3}} \times \overset{d\ d}{\frac{7\times8}{1\times2}} = 112$ **}** the number of changes, in order.

To know how many elections may bee made doe thus $\overset{a}{2} \times \overset{bb}{3} \times \overset{ccc}{4} \times \overset{dd}{3} \times \overset{eeee}{5} = 360 = \overset{a}{2} \times \overset{b\ b}{\frac{2\times3}{2}} \times \overset{c\ c\ c}{\frac{2\times3\times4}{2\times3}} \times \overset{d\ d}{\frac{2\times3}{2}} \times \overset{e\ e\ e\ e}{\frac{2\times3\times4\times5}{2\times3\times4}}$ therefore there are $359 = 360 - 1$ elections in $abbc^3dde^4$.



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Propositiones Geometricae Ex Schootenij
Sectionibus miscellaneis.

Sectio 1ma

To know how many changes 6 Bells, $abcdef$ or how divers conjuctions the 7 planets can make ♄ ♃ ♂ ⊙ ♀ ☿ ☾. or how many divisors $abcde$ hath, or how man**{y}** divers compositions the 24 letters can make &c the examples following show.

1. a 1
2. b . ab 3
4. c . cb . cab . ac . 7
8. d . da . db . dab . dc . dac . dcb . dcab . 15
16. e . ea . eb . eab . ec . eac . ecb . ecab . ed . eda . edb . edab . edc . edac . edcb . edcab . 31
32. f . fa . fb . fab . fc . fcb . fac . fcab . fd . fda . fdb . fdab &c 63.
64. g . ga . gb . gab . gc . gcb . gac . gcab . gd . gda . gdb &c 127

which shows that in 7 letters 127 elections may be made. that 7 Planets may be conjoyned 120 divers ways. that $abcdefg$. hath 128 divisors for an unite is one of **{them}**& $1 \times 2 \times 3 \times 4 \times 5 \times 6. = 720$; are the number of changes in six bells.

Sec 2

To know how many things & of what sort they are which may be chosen 15 ways. $15 + 1 = 16$. $\frac{16}{2} = 8$. $\frac{8}{2} = 4$. $\frac{4}{2} = 2$. $\frac{2}{2} = 1$. &
$2-1$. $2-1$. $2-1$. $2-1$. $= 1$. 1 . 1 . 1 . $= 4$. that 4 things all unequall may be varyed 15 ways. also. $\frac{16}{4} = 4$. $\frac{4}{2} = 2$. $\frac{2}{2} = 1$
$4-1$. $2-1$. $2-1 = 5$ &. 5 things whereof 3 are equall viz: a . a . a . b . c . & $\frac{16}{4} = 4$. $\frac{4}{4} = 1$. $4-1$. $+4-1 = 3+3 = 6$. & 6 things whereof 3 & 3 are equall as $aaabbb$. may be varied 15 ways. & $\frac{16}{8} = 2$ $\frac{2}{2} = 1$. $8-1$. $2-1 = 8 = 7+1$. & 8 things whereof 7 are = may be varyed 15 ways. as $aaaaaaab$. $\frac{16}{16} = 1$. $16-1 = 15$. 2 wherefore 15 alike things &c as a 15. 2 what things vary 23 ways. $23+1 = 24$ 24 admitts a 7 fold divisor therefore the multitude of things sought may be 7 fold but since 43 is a primary number (viz which cannot bee divided) $42+1 = 43$. $\frac{43}{43} = 1$ $43-1 = 42$. therefore onely 42 like things can be varyed 42 ways as a^{42} .

<12v>

Sec 3

Every quantity hath one divisor more that it hath aliquote parts (that is parts of whole numbers.). How to find a quantity haveing a given multitude of divisors or aliquote parts: suppose its aliq: parts must be 15. $15+1 = 16$ & soe by the former section $abcd$. a^3bc . a^3b^3 . a^7b . a^{15} . may be varyed 15 ways. therefore they shall have 15 aliquote parts & 16 divisors. but since onely 42 like things (as a^{42}) can be varyed 42 ways therefore onely a^{42} hath 42 aliquote parts & 43 divisors. &c

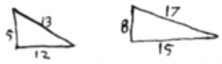
Sec 4

To find the least numbers haveing a given multitude of divisors & aliquote parts instead of soe many letters in the former sec: put soe many least primary numbers & take the least result from them. as from the former example: $abcd$. a^3bc . a^3b^3 . a^7b . a^{15} . that is 2 . 3 . 5 . 7 . or 2 . 2 . 2 . 3 . 5 . &c. now. $2 \times 3 \times 5 \times 7. = 210$. & $2 \times 2 \times 2 \times 3 \times 5 = 120$. &c therefore $2 \times 3 \times 5 \times 7.$ $= 210$ is the least number haveing 16 divisors.

Sec: 5 conteines a table of Primary numbers.

Sec 6

To find progressions constituteing rectangular triangles with sides rationall the examples following shew. take two numbers as 1 . 2 . then $1 \times 2 = 2$ since the product is eaven double it viz: $2 \times 2 = 4$. & 4 is the numerator then $1+2 = 3$ & since 3 is od multiply it by the difference of the termes: $1 \times 3 = 3$ & 3 is the denominator. & the first terme $\frac{4}{3}$. then since (1) the difference of the termes is od multiply it by 4 . $4 \times 1 = 4$ & $4\times$ per 2 majorem terminum. $4 \times 2 = 8$ $8+4$ (the former numerato**{r}**) $= 12 =$ numerator 2^{d}. then 3 (the former denom) added to. 2 (the double square of the diff: of the termes because the square (1) is odd) $= 5$ the 2^{d} denominator. I ad another example take 1 . 3 . then $1 \times 3 = 3 = 1^{st}$ numerator. then $1+3 = 4$ & since 4 is eaven $\frac{4\times2}{2}$ (diff: of the termes) $= 4$ & the first denom is 4. the first terme $\frac{3}{4}$. then becaus the diff of the termes is eaven 2 $2 \times 2 = 4$ & $4 \times 3 = 12$ & $12+3 = 15$. then $2 \times 2 = 4$. $4+4 = 8$. & $\frac{15}{8}$ the 2^{d} terme & now termes may be had by Arithmeticall proportion. thus. $\frac{4}{3}$. $\frac{12}{5}$ or $1\frac{1}{3}$. $2\frac{2}{5}$. $3\frac{3}{7}$. $4\frac{4}{9}$. $5\frac{5}{11}$. $6\frac{6}{13}$ $7\frac{7}{15}$ $8\frac{8}{17}$ $9\frac{9}{19}$ $10\frac{10}{21}$ &c & $\frac{3}{4}$. $\frac{15}{8}$ or
$\frac{3}{4}$ $1\frac{7}{8}$. $2\frac{11}{12}$. $3\frac{15}{16}$. $4\frac{19}{20}$. $5\frac{23}{24}$. $6\frac{27}{28}$. $7\frac{31}{32}$. $8\frac{35}{36}$ &c thus may other progressions be obteined. For the use take the numerator for one leg & the denom for another & the Hypoten: will be rationall as in $2\frac{2}{5}$ or $\frac{12}{5}$ $\sqrt{144+25} = \sqrt{169} = 13$. & in this $1\frac{7}{8}$ or $\frac{15}{8}$ $\sqrt{225+64} = 17$.



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If the suposed numbers be $2\ .\ 5$. then $2\times5=10.\ 10+10=20$. & $2+5=7.\ 3\times7=21$. so that $\frac{20}{21}$. then $4\times3=12.\ 12\times5=60.\ 60+20=80$. & $3\times3=9$. 9 doubled $=18.\ 18+21=39$. & the 2 first termes $\frac{20}{21}\ .\ \frac{80}{39}$ or $2\frac{2}{39}$. Againe, if the numbers be $3\ .\ 4$ $3\times4=12.\ 12\times2=24$. & $3+4=7.\ 1\times7=7$. therefore $\frac{24}{7}$. then $4\times1=4.\ 4\times4=16\ \ 16+24=40$. & $1\times1=1.\ 2\times1=2.\ 7+2=9$ therfore $\frac{40}{9}$ is the 2^{d} & the progres may be continued, as $\frac{20}{21}\ .\ 2\frac{2}{39}\ .\ 3\frac{5}{57}\ .\ 4\frac{8}{75}\ .\ 5\frac{11}{93}$. & $3\frac{3}{7}\ .\ 4\frac{4}{9}\ .\ 5\frac{5}{11}\ .\ 6\frac{6}{13}$ &c.

Sec 7

To find a **{**number**}** which divided by 7 leaves 2 . by 11 leaves 1 . by 13 leaves 9 . the least common divisor of $7\ .\ 11\ .\ 13\ .$ is $7\times11\times13\times=1001$. divide 1001 twice by each & consider the remainder of the seacond division thus.

1 Since more than 1 is left (viz 3) multiply 3 till it divided by 7 leavs 1 . $\frac{5\times3}{7}=2\frac{1}{7}$ therfore $5\times143=715$ the multiplier $\Big|\frac{1001}{7}\Big(\frac{143}{7}\Big(20\frac{3}{7}$.

2 Since more than 1 is left (viz: 3) $\frac{3\times4}{11}=1\frac{1}{11}$ therfore $4\times91==364$ the multipl: $\Big|\frac{1001}{11}\Big(\frac{91}{11}\Big(8\frac{3}{11}$.

3 If but 1 had beene left 77 had beene divisor but now $\frac{12\times12}{11}=13\frac{1}{11}$. therfore $12\times77=924$ is multiplyer. $\frac{1001}{13}\Big(\frac{77}{13}\Big(5\frac{12}{13}$. now the number sought is thus found.

< insertion from the center right of f 13r >

Divisor.	Reliq:		Multip.		
7 .	2	$\times$	715	$=$	1430.
11 .	1	$\times$	364	$=$	364.
13 .	9	$\times$	924	$=$	8316.
		The Sume			10110.

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Lastly divide
by the least com. divis: $\frac{10110}{1001}\Big(10\frac{100}{1001}$ wherefore 100 the number left is the number sought.

Sec 8.

Touching the Method of weights suppose a man have weights of $1.2.4.8.16.32$ pounds &c by them all intermediate pounds may be thus weighed

1.	2	3.	4	5	6	7	8	9	10	11	12	13	14
1.	2	$1+2$	4	$1+4.$	$2+4.$	$1+2+4.$	8	$8+1$	$8+2.$	$8+1+2.$	$8+4.$	$8+4+1.$	$8+4+2$

&c or if his weights be $1.3.9.27.81.$ all weights may be supplyed thus.

1.	2.	3.	4.	5	6	7	8	9	10	11	12	13	14
1	$3-1.$	$3.$	$3+1.$	$9-1-3.$	$9-3.$	$9+1-3.$	$9-1.$	$9.$	$9+1.$	$9+3-1.$	$9+3.$	$9+3+1.$	$27-9-3-1.$

&c Note that weight marked with − signifie the weight to be put in the opposite ballance.

<13v>

Sec. 9.

To find numeri amicabiles that is 2 numbers whose aliquote parts are mutually equall to theire wholes. take this Des-Cartes his rule

If (2), or any other number produced out of 2 as $2\times2\ .\ 2\times2\times2$ &c (viz $2\ .\ 4\ .\ 8\ .\ 16\ .\ 32$ &c) bee such a number that 1 taken out of it triple there rests a primary number**{,}** & that if 1 taken from it sextuple there rests a primary number, & if 1 taken from its square octodecuple a primary number rests: then multiply this last prime number by the assumed number doubled & the product is one amicable number & the aliquote points of it make the other Example. if 2 be taken. $2\times3-1=5$ numero primario primo. $2\times6-1=11$ numero primario secundo. $2\times2\times18-1=71$ numero primario tertio. $4\times71=284$, one amicable number, & the 2 former prime numbers × one another & the product $\times4$ the double of the assumed number viz $5\times11=5555\times4=220$. Thus from 8 . & 64 &c. may be deduced amicable numbers.

Sec 10

To find triangles whose sides, segments of theire bases, & Perpendiculars are expressible by rationall numbers

1^{st} if the perpendic: is without the tri: let $\mathrm{ac}=\mathrm{z}.\ \mathrm{bd}=\mathrm{x}\ \mathrm{cd}=\mathrm{y}.\ \mathrm{ad}=\mathrm{z}+\mathrm{y}.\ \mathrm{ad}=\mathrm{y}+\mathrm{b}.\ \mathrm{xx}+\mathrm{yy}=\mathrm{yy}+2\mathrm{by}+\mathrm{bb}.\ \mathrm{y}=\frac{\mathrm{xx}-\mathrm{bb}}{2\mathrm{b}}$. & $\mathrm{cd}=\mathrm{z}+\mathrm{y}+\mathrm{a}$. $\mathrm{xx}+\mathrm{zz}+2\mathrm{yz}+\mathrm{yy}=\mathrm{zz}+\ \mathrm{yy}+\mathrm{aa}+2\mathrm{zy}+2\mathrm{za}+2\mathrm{ay}.\ 2\mathrm{ay}=\mathrm{xx}-\mathrm{aa}-2\mathrm{za}=\frac{\mathrm{axx}-\mathrm{abb}}{\mathrm{b}}.\ \mathrm{bxx}-\mathrm{baa}-2\mathrm{zab}=\mathrm{axx}-\mathrm{abb}.\ \frac{\mathrm{bxx}-\mathrm{baa}-\mathrm{axx}+\mathrm{abb}}{2\mathrm{ab}}=\mathrm{z}$. puting any numbers for a , b , & x ; y & z may be found. then $\mathrm{ad}=\mathrm{z}+\mathrm{y}==\frac{\mathrm{xx}+\mathrm{bb}}{2\mathrm{b}}$. $\mathrm{cd}=\mathrm{z}+\mathrm{y}+\mathrm{a}=\frac{\mathrm{xx}+\mathrm{aa}}{2\mathrm{a}}$. which reduced to the common denominator 2ab; & that cast away. $\mathrm{cd}=\mathrm{bxx}+\mathrm{baa}.\ \mathrm{ad}=\mathrm{axx}+\mathrm{abb}.\ \mathrm{de}=2\mathrm{abx}.\ \mathrm{ae}=\mathrm{axx}-\mathrm{abb}.\ \mathrm{ce}=\mathrm{bxx}-\mathrm{baa}.\ \mathrm{ac}=\mathrm{bxx}-\mathrm{axx}+\mathrm{abb}\ \ -\mathrm{aab}$.

In like manner if the perpendicular fall within side. $\mathrm{ab}=\mathrm{bxx}+\mathrm{baa}.\ \mathrm{bd}=2\mathrm{abx}.\ \mathrm{ad}=\mathrm{bxx}-\mathrm{baa}.\ \mathrm{dc}=\mathrm{axx}-\mathrm{abb}.\ \mathrm{bc}=\mathrm{axx}+\mathrm{abb}$. $\mathrm{ac}=\mathrm{bxx}+\mathrm{axx}-\mathrm{abb}-\mathrm{baa}$.

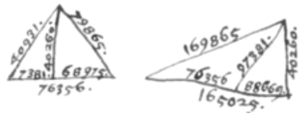
Also by the conjunction & disjunction of 2 triangles it may be found that $\mathrm{ab}=\mathrm{bbx}+\mathrm{aax}\ \ \mathrm{ad}=\mathrm{bbx}-\mathrm{aax}.\ \mathrm{ac}=\mathrm{bbx}-\mathrm{aax}-\mathrm{aax}\ \ +\mathrm{abb}.\ \mathrm{bc}=\mathrm{axx}+\mathrm{abb}$. $\mathrm{db}=2\mathrm{abx}.\ \mathrm{dc}=\mathrm{axx}-\mathrm{abb}$. For if $\mathrm{bd}=\mathrm{x}\ \mathrm{dc}=\frac{\mathrm{xx}-\mathrm{bb}}{2\mathrm{b}}$. $\mathrm{bc}=\frac{\mathrm{xx}+\mathrm{bb}}{2\mathrm{b}}$. that is $\mathrm{bd}=2\mathrm{bx}.\ \mathrm{dc}=\mathrm{xx}-\mathrm{bb}.\ \mathrm{bc}=\mathrm{xx}+\mathrm{bb}$. Likewise $\mathrm{bd}=2\mathrm{ab}.\ \mathrm{ad}=\mathrm{bb}-\mathrm{aa}$. $\mathrm{ab}=\mathrm{bb}+\mathrm{aa}$. 2abx the least quantity divisible by 2bx & 2ab, being divided by them, leaves a & x which must multiply the bases & hypotenusas. If the perpendic: fall without the legs may be thus exprest $\mathrm{cd}=\mathrm{acc}+\mathrm{ayy}.\ \mathrm{da}=\mathrm{yyc}+\mathrm{aac}$ <14r> $\mathrm{ca}=\mathrm{acc}-\mathrm{ayy}+\mathrm{cyy}-\mathrm{aac}.\ \mathrm{ae}=\mathrm{yyc}-\mathrm{aac}.\ \mathrm{ce}=\mathrm{acc}-\mathrm{ayy}.\ \mathrm{ad}=2\mathrm{acy}$.

Sec 11

To make that two such tri: be of the same base & altitude. Suppose an equation twixt the bases & perpendiculars of the 2 last tri: as $2\mathrm{abx}=2\mathrm{acy}.\ \mathrm{x}=\frac{\mathrm{cy}}{\mathrm{b}}.\ \mathrm{xx}=\frac{\mathrm{ccyy}}{\mathrm{bb}}$. $\mathrm{bbx}-\mathrm{aax}-\mathrm{axx}+\mathrm{abb}=\mathrm{acc}-\mathrm{ayy}+\mathrm{yyc}-\mathrm{aac}$ or $\frac{\mathrm{bbcy}-\mathrm{aacy}}{\mathrm{b}}-\frac{\mathrm{accyy}}{\mathrm{bb}}+\mathrm{abb}=\mathrm{acc}-\mathrm{ayy}+\mathrm{yyc}-\mathrm{aac}$ & $\mathrm{yy}=\frac{+\mathrm{b}^3\mathrm{cy}+\mathrm{aabbc}-\mathrm{aabc}+\mathrm{ab}^4-\mathrm{abbcc}}{\mathrm{bbc}+\mathrm{acc}-\mathrm{abb}}$. Suppose $\mathrm{aabbc}+\mathrm{ab}^4=\mathrm{abbcc}$. or $\mathrm{a}=\mathrm{c}-\frac{\mathrm{bb}}{\mathrm{c}}$. let $\mathrm{c}=3$ greater than $\mathrm{b}=2.\ \mathrm{a}=\frac{5}{3}.\ \mathrm{y}=\frac{22}{61}.\ \mathrm{x}=\frac{33}{61}$ & consequently

Sec 14 differs not from Cap 19: prob 18 Oughtred.

Sec: 15 Of Polygons or multangular numbers



The summe of all the tearmes in an arithmet: progres: increasing from an unite by 1 composeth triangles. by 2 , composes squares. by 3 , composes pentangles. by 4 , hexang: &c as $1 \ . \ 2 \ . \ 3 \ . \ 4 \ . \ 5 \ . \ 6 \ .$ compose the triangles

1 ● 3 ●●● 6 ●●●●●● 10 ●●●●●●●●●● 15 ●●●●●●●●●●●●●●● &c likewise $1 \ . \ 3 \ . \ 5 \ . \ 7 \ . \ 9 \ .$ compose

1 ● 4 ●●●● 9 ●●●●●●●●● 16 ●●●●●●●●●●●●●●●● 25 ●●●●●●●●●●●●●●●●●●●●●●●●● &c So $1 \ . \ 4 \ . \ 7 \ . \ 10 \ . \ 13 \ .$ compose the quintangles $1 \ . \ 5 \ . \ 12 \ . \ 22 \ . \ 35 \ . \ 51 \ . \ 70 \ .$ &c.

If $a = 1 =$ the first term{e} the excess of the progression $= x$. The summe of the termes $= z =$ to the polygon the multitude of the termes $= t =$ to the side of the Polygon. Suppose t given to find z. $z = \frac{1tt+1t}{2}$ or $z = \frac{tt+t}{2}$ in trigons. $z = tt$ in 4^{gons}. $z = \frac{3tt-t}{2}$ in 5^{gons}. $z = 2tt - t$ in 6^{gons} $z = \frac{5tt-3t}{2}$ in 7^{gons}. $z = 3tt - 2t$ in 8^{gons}. $z = \frac{7tt-5t}{2}$ in 9^{gons}. &{c} & z given t is found thus $t = \frac{-1+\sqrt{1+8z}}{2}$ in tri. $t = \frac{\sqrt{0+16z}}{4}$ in 4^{gons} $t = \frac{+1+\sqrt{1+24z}}{6}$, in 5^{gons}. {$t = +2 + \sqrt{4+32z}$ } { $t = \frac{+2+\sqrt{4+32z}}{8}$ } in 6^{gons} &c. As the side 12 of a tri given. the tri $= z = \frac{12\times 12+12}{2} = 78$ &c & if $z = 21$ be octangled. $t = \frac{+4+\sqrt{16+48z}}{12} = \frac{4+\sqrt{16+48\times 21}}{12}$ $t = \frac{4+\sqrt{1024}}{12} = 3$.

<14v>

July 4th 1699. By consulting an accompt of my expenses at Cambridge in the years 1663 & 1664 I find that in the year 1664 a little before Christmas I being then Senior Sophister, I bought Schooten's Miscellanies & Cartes's Geometry (having read this Geometry & Oughtred's Clavis above half a year before) & borrowed Wallis's works & by consequence made these Annotations out of Schooten & Wallis in winter between the years 1664 & 1665. At which time I found the method of Infinite series. And in summer 1665 being forced from Cambridge by the Plague I computed the area of the Hyperbola at Boothby in Lincolnshire to two & fifty figures by the same method. Is. Newton

<15r>

Annotations out of Dr Wallis his Arithmetica infinitorum.

1 A primanary series of quantitys is arithmetically proportionall, as $0 \ , \ 1 \ , \ 2 \ , \ 3 \ , \ 4 \ .$ & its index is 1

A Secundanary series are those whose rootes are arithmetically proportionall; as $0 \ , \ 1 \ , \ 4 \ , \ 9 \ , \ 16 \ .$ & its index is 2

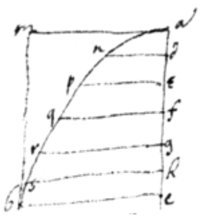
A Tertianary, quartanary, quintanary series of quantitys are those whose cube, square square, square cube rootes are Arithmetically Proportionall as $0 \ , \ 1 \ , \ 8 \ , \ 27 \ , \ 64 \ . / \ 0 \ , \ 1 \ , \ 16 \ , \ 81 \ , \ 156 \ . / \ 0 \ , \ 1 \ , \ 32 \ , \ 243 \ , \ 624 \ .$ &c Their indices being $3 \ , \ 4 \ , \ 5$ &c.

3 Subsecundanary, subtertianary, series &c: are those whose squares, cubes, &c are arithmetically proportionall, as $\sqrt{0} \ , \ \sqrt{1} \ , \ \sqrt{2} \ , \ \sqrt{3} \ .$ $. \ \sqrt{c:0} \ , \ \sqrt{c:1} \ , \ \sqrt{c:2} \ , \ \sqrt{c:3}$ &c. Theire indices being $\frac{1}{2} \ , \ \frac{1}{3} \ ,$ &c.

2 Primary Secundanary, tertianary series &c are said to bee reciprocally proportionall (that is to the same se increasing) which continually decrease as. $\frac{1}{0} \ , \ \frac{1}{1} \ , \ \frac{1}{2} \ , \ \frac{1}{3} \ , \ \frac{1}{4} \ , \ \ . \ \frac{1}{0} \ , \ \frac{1}{1} \ , \ \frac{1}{4} \ , \ \frac{1}{9} \ , \ \frac{1}{16} \ , \ \ . \ \frac{1}{0} \ , \ \frac{1}{1} \ , \ \frac{1}{8} \ , \ \frac{1}{27} \ , \ \frac{1}{64} \ ,$. Their indices being negative as $-1 \ , \ -2 \ , \ -3 \ .$

4 The indices of compound or mixt of rationall & irrati{onall} series, by multiplying or dividing the indices of the simple series may bee found as in a subsecundanary progression cubed $\sqrt{0} \ , \ \sqrt{1} \ , \ \sqrt{8} \ , \ \sqrt{27} \ , \ \sqrt{64}$ the index is $\frac{1}{2} \times 3 = \frac{3}{2}$. So in the cube rootes of a secundanary progression, $\sqrt{c:0} \ , \ \sqrt{c:1} \ , \ \sqrt{c:4} \ , \ \sqrt{c:9}$ &c. the index is $\frac{1}{3} \times 2 = \frac{2}{3}$. so in irrationall reciprocal progressions $\sqrt{qq:\frac{1}{0}} \ , \ \sqrt{qq:\frac{1}{1}} \ , \ \sqrt{qq:\frac{1}{2}} \ , \ \sqrt{qq:\frac{1}{3}}$, the index is $-1 \times \frac{1}{4} = -\frac{1}{4}$.

<15v>



Now suppose the line ac be divided into an infinite number of equall parts $ad \ , \ de \ , \ ef \ , \ fg$ &c, from each of which are drawne parallels $ndpe$ qf &c. which increase continually in some of the foregoeing progressions or in some progression compounded of them, all those lines may be taken for the surface bqnac , & to know what proportion that superficies hath to the superficies ambc that is what proportion all those lines have to soe may equal to the greatest of them, I say as the index of the progression increased by an unite is to an unite soe is the square abcm to the area of the crooked line. As if abc is a parabola the lines $ad \ , \ pe \ , \ qf \ ,$ &c are a subsecundanary series (for $y = \sqrt{rx}$) whose index is $\frac{1}{2}$ which added to an unite is $1 + \frac{1}{2} = \frac{3}{2}$ Therefore $\frac{3}{2} : 1 :: 3 : 2$ so is the square ambc to the area of the Parab. (the names of the lines are $(ad) \ , \ ae \ , \ af$ &c $= x \ . \ dn \ , \ pe \ , \ qf$ &c $= y \ . \ ac = p \ . \ bc = q$.) The case is the same if abc bee supposed a sollid, as suppose it a Parabolicall conoides. then since the nature of it is $rx = yy$. yy designes the squares $nd \ , \ pe \ , \ qf$ &c: all which taken together are equivalent to the Sollid. & those squares increase in the same proportion which rx. or x doth. that is they are a primanary series whose index is 1 to which (according to the rule I ad an unite & tis 2. Therefor $1 : 2 ::$ soe are all the squares of the Primary series to soe many squares equall to the greatest of that series. & soe is the conoides to a cilinder of the same altitude.

<16r>

Also if a superficies be compounded of 2 or more of these series, Their area is as easily found as if the nature of the line bee , $y = aa - xx$, or $y = a^4 - 2aaxx + x^4$ or $y = a^6 - 3a^4xx + 3aax^4 - x^6$. &c. Their areas will bee to the parallelograms {about} them as 2 to 3 , as 8 to 15 , as 48 to 105 &c. but if I put in the intermediate termes in these last named lines their order will bee $y = \sqrt{aa - xx}$, $y = aa - xx$, $y = \overline{aa - xx}\sqrt{aa - xx}$, $y = a^4 - 2aaxx + x^4$. $y = \overline{a^4 - 2aaxx + x^4}\sqrt{aa - xx}$. $y = a^6 - 3a^4xx + 3aax^4 - x^6$; &c: & since these lines observe a geometricall progression their areas must observe some kind of progression. of which every other terme is given viz $1 \ . \ \square \ . \ \frac{2}{3} \ . \ * \ . \ \frac{8}{15} \ . \ * \ . \ \frac{48}{105} \ . \ * \ . \ \frac{384}{945} \ . \ * \ . \ \frac{3840}{10395}$. Twixt which termes if the intermediate termes $\square \ . \ *$ can bee found the 2nd square will give the area of the line $y = \sqrt{aa - xx}$, the circle. Soe likewise in this progression of lines $y = 1$. $y = \sqrt{ax - xx}$. $y = aa - xx$ $y = \overline{ax - xx}\sqrt{ax - xx}$. $y = aaxx - 2ax^3 + x^4$ &c: the progression of their areas is $1 : \square : \frac{1}{6} : * : \frac{1}{30} : * : \frac{1}{140} : * \frac{1}{630} :$ &c. the 2nd if it can bee found gives the area of the circle for as its denominator to its numerator so is the square of the diameter to the area of a semicircle. If this last progression bee multiplyed by the respective termes in the progression $1 \ . \ 2 \ . \ 3 \ . \ 4$ & it may bee diminished the result being $1 \ . \ 2\square \ . \ \frac{1}{2} \ . \ 4* \ . \ \frac{1}{6} \ . \ 6* \ . \ \frac{1}{20} \ . \ 8* \ . \ \frac{1}{70}$ soe that in this progression $1 \ , \ b \ , \ \frac{1}{2} \ , \ c \ , \ \frac{1}{6} \ , \ d \ , \ \frac{1}{20} \ , \ e \ , \ \frac{1}{70} \ , \ f \ ,$ &c: if <16v> b can be found then, the square of the diameter to the area of the circle is as the denominator of b to its numerator. Likewise the 1st series of areas may be diminished by multiplying each terme by its correspondent terme in this progression $1 \ , \ 2 \ , \ 3 \ , \ 4 \ , \ 5 \ , \ 6 \ ,$ &c: & it will become, $1 \ , \ a \ , \ 2 \ , \ b \ , \ \frac{8}{3} \ , \ c \ , \ \frac{48}{15} \ ,$ $d \ , \ \frac{384}{105} \ , \ e \ , \ \frac{3840}{945}$. &c. In which if a can bee found then as the denominator of a to its numerator: so the square of the Radius is a semicircle, that making the radius $= q$. $2aq = \circ$. The same kinds of changes may bee performed by any other progressions, as by division by the geometricall progression $1 \ , \ 2 \ , \ 4 \ , \ 8 \ , \ 16 \ ,$ & the first series of areas becomes $1 \ , \ g \ , \ \frac{1}{6} \ , \ h \ , \ \frac{1}{30} \ , \ k \ , \ \frac{1}{140} \ , \ l \ , \ \frac{1}{630} \ ,$ &c viz the same with the 2d series.

Also these changes may be done by addition or substraction of mutuall termes in 2 proportions. Soe that the most convenient way may be chosen, wherby to reduce any series of proportions to the most convenient forme.

Now if it be propounded to find these middle termes, It will bee convenient to find how the given proportion may bee deduced from an Arithmeticall, Geometricall, or some other familiar proportion, viz whose meane termes may be found, as this progression $1\ .\ \frac{2}{3}\ .\ \frac{8}{15}\ .\ \frac{48}{105}$ deduceth its originall from this $A\times\frac{0\times2\times4\times6\times8}{1\times3\times5\times7\times9}$ & in which A is an infinite number $=\frac{1}{0}$.

It will also be convenient to find what relation all the other meanes have to the first soe that if the first bee had all the other may bee deduced thence. As in this case suppose the 1st meane to bee a . The progression will bee <17r> $\frac{1}{2}a : 1 : a : \frac{3}{2} : \frac{4a}{3} : \frac{15}{8} : \frac{8a}{5} : \frac{105}{48} : \frac{64a}{35} : \frac{945}{384} : \frac{640a}{315} :$ deducing its originall from $A\times\frac{0\times2\times4\times6\times8\times10}{1\times3\times5\times7\times9\times11}$ & from this $A\left(=\frac{1}{2}\right)\times\frac{2a\times4a\times6a\times8a\times10a}{1\times3\times5\times7\times9}$. &c {+} (note that the proportions of these meane termes to one another, or to (a), are found by finding the proportion of the circle $y=\sqrt{aa-xx}$ to the line $y=\overline{aa-xx}\sqrt{aa-xx}$ &c).

In this case to find the quantity a : Naming the termes in the progress: $\underset{b}{\frac{1}{2}a} : \underset{c}{1} : \underset{d}{a} : \underset{e}{\frac{3}{2}} : \underset{f}{\frac{4a}{3}} : \underset{g}{\frac{15}{8}} : \underset{h}{\frac{8a}{5}} : \underset{k}{\frac{35}{16}} :$. 1st observe that $\frac{d}{b}=2\ .\ \frac{e}{c}=\frac{3}{2}\ .\ \frac{f}{d}=\frac{4}{3}\ .\ \frac{g}{e}=\frac{5}{4}\ .\ \frac{h}{f}=\frac{6}{5}$ &c the proportions still decreasing & therefore that in $\frac{c}{b}\ .\ \frac{d}{c}\ .\ \frac{e}{d}\ .\ \frac{f}{e}\ .\ \frac{g}{f}\ .\ \frac{h}{g}\ .\ \frac{k}{h}$ &c: the latter terme is lesse than the former; & therefore $\frac{dd}{cc}$ is $\begin{cases}\text{less y}^n\ \frac{d}{c}\times\frac{c}{b}=\frac{d}{b}=2.\\ \text{greater y}^n\ \frac{d}{c}\times\frac{e}{d}=\frac{e}{c}=\frac{3}{2}.\end{cases}$ or a = d is $\begin{cases}\text{less y}^n\ 1\times\sqrt{2}=\sqrt{1+\frac{1}{1}}\\ \text{greater y}^n\ 1\times\sqrt{\frac{3}{2}}=\sqrt{1+\frac{1}{2}}\end{cases}$. Also

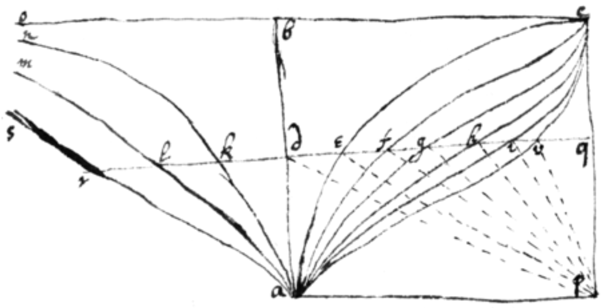
$\frac{ff}{ee}$ is $\left.\begin{cases}\text{lesse y}^n\ \frac{f}{e}\times\frac{e}{d}=\frac{f}{d}=\frac{4}{3}\\ \text{greater}\quad \bar{y}\quad \frac{f}{e}\times\frac{g}{f}=\frac{g}{e}=\frac{5}{4}\end{cases}\right\}=a:\ \frac{8a}{9}=\frac{2\times4a}{3\times3}$.

Therefore a is $\begin{cases}\text{lesse y}^n\ \frac{3\times3}{2\times4}\sqrt{\frac{4}{3}}\\ \text{greater y}^n\ \frac{3\times3}{2\times4}\sqrt{\frac{5}{4}}\end{cases}$. And So by the same reasoning. a is $\begin{cases}\text{less y}^n\ \frac{9\times25\times49\times81\times121\times169}{2\times16\times36\times64\times100\times144\times14}\sqrt{\frac{14}{13}}\ .\\ \text{greater y}^n\ \frac{3\times3\times5\times5\times7\times7\times9\times9\times11\times11\times13\times13}{2\times4\times4\times6\times6\times8\times8\times10\times10\times12\times12\times14}\sqrt{\frac{15}{14}}\ .\end{cases}$ &c. Thus Wallis doth it. but it may bee done thus. a is $\begin{cases}\text{greater y}^n\ 1\ .\\ \text{less then}\ \frac{3}{2}\ .\end{cases}$ | $\frac{4a}{3}$ is $\begin{cases}\text{greater then}\ \frac{3}{2}\\ \text{less then}\ \frac{15}{8}\end{cases}$ Therefore <17v> a is $\begin{cases}\text{greater y}^n\ \frac{3\times3}{2\times4}\\ \text{lesse then}\ \frac{3\times3\times5}{2\times4\times4}\end{cases}$. $\frac{8a}{5}$ is $\begin{cases}\text{greater y}^n\ \frac{15}{8}\\ \text{lesse y}^n\ \frac{35}{16}\end{cases}$ that is a is $\begin{cases}\text{greater then}\ \frac{3\times3\times5\times5}{2\times4\times4\times6}=\frac{3\times5\times5}{2\times4\times4\times2}\\ \text{lesse then}\ \frac{3\times3\times5\times5\times7}{2\times4\times4\times6\times6}=\frac{5\times5\times7}{2\times4\times4\times4}\end{cases}$ &c. By the same reasoning

a is $\begin{cases}\text{greater y}^n\ \frac{3}{2}\times\frac{3}{4}\times\frac{5}{4}\times\frac{5}{6}\times\frac{7}{6}\times\frac{7}{8}\times\frac{9}{8}\times\frac{9}{10}=\frac{5\times49\times81}{2\times16\times4\times64\times2}.\\ \text{lesse y}^n\ \frac{3}{2}\times\frac{3}{4}\times\frac{5}{4}\times\frac{5}{6}\times\frac{7}{6}\times\frac{7}{8}\times\frac{9}{8}\times\frac{9}{10}\times\frac{11}{10}=\frac{49\times81\times11}{2\times16\times4\times64\times4}.\end{cases}$ Or

a is $\begin{cases}\text{greater}\ \bar{y}\ \frac{11\times169\times225\times289\times369\times441}{2\times16\times4\times64\times4\times144\times4\times256\times4\times400\times2}\\ \text{lesse y}^n\ \frac{169\times225\times289\times369\times441\times23}{2\times16\times4\times64\times4\times144\times4\times256\times4\times400\times4}\end{cases}$. Note that a is greater than $\frac{1}{2}$ these two summes.

<18r>



Having the signe of any angle to find the angle or to find the content of any segment of a circle

Suppose the circle to be aec its semidiameter $ap=pc=1$. the given sine $pq=x$, viz: the signe of the angle epa . the segment sought eapq . abcp the square of its Radius. & that, $qi : qk : qg : qf : qe : qd : qh : ql : qr :$ &c are continually proportionall. Then is $eq=\sqrt{1-xx}$. $fq=1-xx$. $gq=\overline{1-xx}\ \text{in}\ \sqrt{1-xx}$. $hq=1-2xx+x^4$. $iq=1-2x^2+x^4\sqrt{1-xx}$ $dq=1$. $kq=\frac{1}{\sqrt{1-xx}}$. $lq=\frac{1}{1-xx}$. $rq=\frac{1}{1-xx\ \text{in}\ \sqrt{1-xx}}$. &c & since all the ordinately applyed lines in these figures $abcq$, $aecq$, $afcq$, $agcq$, &c are geometrically proportionall their areas $adqp$, $aeqp$, $afqp$, $agqp$, $ahpq$ &c will observe some proportion amongst one another. To find which proportion, 1st $adqp=1\times xx=x$. 2dly afc is a parab: therefore $afqp=x-\frac{x^3}{4}$. also since tis $qh=1-2xx+x^4$, therefore $ahqp=x-\frac{2}{3}x^3+\frac{1}{5}x^5$. Also $vq=1-3xx+3x^4-x^6$, therefore $avqp=x-x^3+\frac{3}{5}x^5-\frac{1}{7}x^7$. & by the same proceeding the proportion may bee still continued after this manner <18v> $x : x-\frac{1}{3}x^3 : x-\frac{2}{3}x^3+\frac{1}{5}x^5 : x-\frac{3}{3}x^3+\frac{3}{5}x^5-\frac{1}{7}x^7 :$
$x-\frac{4}{3}x^3+\frac{6}{5}x^5-\frac{4}{7}x^7+\frac{1}{9}x^9 : x-\frac{5}{3}x^3+\frac{10}{5}x^5-\frac{10}{7}x^7+\frac{5}{9}x^9-\frac{1}{11}x^{11} :$
$x-\frac{6}{3}x^3+\frac{15}{5}x^5-\frac{20}{7}x^7+\frac{15}{9}x^9-\frac{6}{11}x^{11}+\frac{1}{13}x^3 : x-\frac{7}{3}x^3+\frac{21}{5}x^5-\frac{35}{7}x^7+\frac{35}{9}x^9-\frac{21}{11}x^{11}+\frac{7}{13}x^{13}-\frac{1}{15}x^{15}$. &c.
And if the meane termes be inserted it will bee
$x : x- : x-\frac{1}{3}x^3 : x-\frac{3}{6}x^3+ : x-\frac{2}{3}x^3+\frac{1}{5}x^5 : x-\frac{5}{6}x^3+\frac{2}{5}x^5-$ The first letters x run in this progression $1\ .\ 1\ .\ 1\ .\ 1\ .\ 1$. &c. the 2d x^3 in this $\frac{-1}{3}\ .\ \frac{0}{3}\ .\ \frac{1}{3}\ .\ \frac{2}{3}\ .\ \frac{3}{3}\ .\ \frac{4}{3}\ .\ \frac{5}{3}$ &c the 3d x^5 in this $6\ .\ 3\ .\ 1\ .\ 0$. $0+1=1\ .\ 1+2=3\ \ 3+3=6\ .\ 6+4=10\ .\ 10+5=15$. the 4th x^7 this

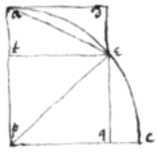
1st	$+x\times 1$	1	1	1	1	1	1	1	1	1	1
2d	$-\frac{x^3}{3}\times 0$	$0+1=1$	$1+1=2$	$2+1=3$	$3+1=4$	$4+1=5$	6	7	8	9	10
3d	$+\frac{x^5}{5}\times 0$	$0+0=0$	$0+1=1$	$1+2=3$	$3+3=6$	$6+4=10$	15	21	28	36	45
4th	$-\frac{x^7}{7}\times 0$	$0+0=0$	$0+0=0$	$0+1=1$	$1+3=4$	$4+6=10$	20	35	56	84	120
5	$+\frac{x^9}{9}\times 0$	$0+0=0$	$0+0=0$	$0+0=0$	$0+1=1$	$1+4=5$	15	35	70	126	210
6	$-\frac{x^{11}}{11}\times 0$	$0+0=0$	$0+0=0$	$0+0=0$	$0+0=0$	$0+1=1$	6	21	56	126	252
	1st.	* 2d.	* 3d	* 4th	* 5th	* 6th	1	7	28	84	210
							* 7th	1	8	36	120
								* 8th	1	9	45
									* 9th	1	10
										* 10th	1
											* 11th

Now if the meane termes in these progressions can bee calculated the first of them gives the area aeqp. Which is thus done

1st	$+x\times 1$	1	1	1	1	1	1	1	1	1	1	1	1	
2d	$-\frac{x^3}{3}\times 0$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	
3d	$+\frac{1}{5}x^5\times 0$	$-\frac{1}{8}$	0	$\frac{3}{8}$	1	$\frac{15}{8}$	3	$\frac{35}{8}$	6	$\frac{63}{8}$	10	$\frac{99}{8}$	15	
4	$-\frac{1}{7}x^7\times 0$	$+\frac{1}{16}$	0	$-\frac{1}{16}$	0	$\frac{5}{16}$	1	$\frac{35}{16}$	4	$\frac{105}{16}$	10	$\frac{231}{16}$	20	
5	$+\frac{1}{9}x^9\times 0$	$-\frac{5}{128}$	0	$\frac{3}{128}$	0	$-\frac{5}{128}$	0	$\frac{35}{128}$	1	$\frac{315}{128}$	5	$\frac{1155}{128}$	15	
6	$-\frac{1}{11}x^{11}\times 0$	$\frac{7}{256}$	0	$-\frac{3}{256}$	0	$\frac{3}{256}$	0	$-\frac{7}{256}$	0	$\frac{63}{256}$	1	$\frac{693}{256}$	6	
7	$\frac{1}{13}x^{13}\times 0$	$-\frac{21}{1024}$	0	$\frac{7}{1024}$	0	$-\frac{5}{1024}$	0	$\frac{7}{1024}$	0	$-\frac{21}{1024}$	0	$\frac{231}{1024}$	1	$\frac{3003}{1024}$

Soe that

$1\times x-\frac{1}{2}\times\frac{1}{3}\times x^3-\frac{1}{8}\times\frac{1}{5}\times x^5-\frac{1}{16}\times\frac{1}{7}\times x^7-\frac{5}{128}\times\frac{1}{9}\times x^9$ &c. is the area, apqe that is
$\frac{0}{0}\times x-\frac{0}{0}\times\frac{1}{2}\times\frac{1}{3}x^3-\frac{0}{0}\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{5}x^5-\frac{0}{0}\times\frac{1}{2}\times\frac{1}{4}\times\frac{3}{6}\times\frac{1}{7}x^7-\frac{1\times 3\times 5\quad x^9}{2\times 4\times 6\times 8\times 9}$ &c: The progression may be deduced from hence
$\frac{0\times 1\times -1\times 3\times -5\times 7\times -9\times 11}{0\times 2\times 4\times 6\times 8\times 10\times 12\times 14}$. &c <19r> Soe that if the given sine bee $pq=te=x$. & if the Radius $pc=1$. Then is the superficies
$ape=x-x\sqrt{1-xx}-\frac{1}{6}x^3-\frac{1}{40}x^5-\frac{x^7}{112}-\frac{5x^9}{1152}-\frac{7x^{11}}{2816}$ &c: And the area $ade=\frac{x^3}{6}+\frac{x^5}{40}+\frac{x^7}{112}+\frac{5x^9}{1152}+\frac{7x^{11}}{2816}+\frac{21x^{13}}{13312}$
$+\frac{11x^{15}}{10240}+\frac{429x^{17}}{557056}+\frac{715x^{19}}{1245184}+\frac{2431x^{21}}{5505024}$ &c. By which meanes the angle ape is easily found for $aecpa:\angle apc=90::ape:\angle ape$.



The same may bee thus done.

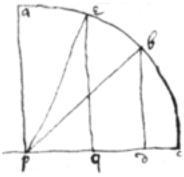
$adp=\frac{x}{2}$. Or $2adp=x$. $2afp=x+\frac{x^3}{3}$. $2ahp=x+\frac{2x^3}{3}-\frac{3x^5}{5}$. And $2avp=x+x^3-\frac{9x^5}{5}+\frac{5}{7}x^7$. &c. as in this order
x . $x+\frac{1}{3}x^3$. $x+\frac{2}{3}x^3-\frac{3}{5}x^5$. $x+x^3-\frac{9}{5}x^5+\frac{5x^7}{7}$. $x+\frac{4x^3}{3}-\frac{18x^5}{5}+\frac{20x^7}{7}-\quad-\frac{7}{9}x^9$. $x+\frac{5}{3}x^3-\frac{30}{5}x^5+\frac{50x^7}{7}-\frac{35x^9}{9}+\frac{9x^{11}}{11}$. $x+\frac{6x^3}{3}-\frac{45x^5}{5}+\frac{100}{7}x^7-\frac{105x^9}{9}+$
$+\frac{54x^{11}}{11}-\frac{11x^{13}}{13}$. $x+\frac{7x^3}{3}-\frac{63x^5}{5}+\frac{175x^7}{7}-\frac{245x^9}{9}+\frac{189}{11}x^{11}-\frac{77x^{13}}{13}+\frac{13x^{15}}{15}$. &c Which progression with their intermediate termes may bee thus exhibited. By which it may appeare that if $pe=1$. $pq=x$. then $aep=\frac{1}{2}x+\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{2304}$ &c. And the area aep given gives the angle ape for $apce:\angle apc=90^d::ape:\angle ape$

Likewise the angle ape given its sign may bee found hereby &c

$2\times adpa=$	$2\times aep=$	$2afp=$	$2agp=$	$2ahp=$	$2aip=$	$2avp=$
$+x$ in 1	1	1	1	1	1	1
$\frac{x^3}{3}$ in 0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$-\frac{3}{5}x^5$ in 0	$-\frac{1}{8}$	0	$\frac{3}{8}$	1	$\frac{15}{8}$	3
$+\frac{5}{7}x^7$ in 0	$\frac{1}{16}$	0	$-\frac{1}{16}$	0	$\frac{5}{16}$	1
$-\frac{7}{9}x^9$ in 0	$-\frac{5}{128}$	0	$\frac{3}{128}$	0	$-\frac{5}{128}$	0
$+\frac{9}{11}x^{11}$ in 0	$\frac{7}{256}$	0	$-\frac{3}{256}$	0	$\frac{3}{256}$	0

Note that $\sqrt{1-xx}=\frac{x}{2}-\frac{2x^3}{6}-\frac{5x^5}{80}-\frac{7x^7}{224}-\frac{45x^9}{2304}-\frac{77x^{11}}{5632}$ &c that is $\sqrt{1-xx}=\frac{x}{2}-\frac{x^3}{3}-\frac{x^5}{16}-\frac{x^7}{32}-\frac{5x^9}{256}-\frac{7x^{11}}{512}-\frac{21x^{13}}{2048}$ &c. According to this progression
$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}\times\frac{3\times5\times7\times9\times11\times13\times15\times17}{6\times8\times10\times12\times14\times16\times18\times20}$ &c. Note also that the segment $ae=\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{2304}$. &c.
$aep=\frac{x}{2}+\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{3304}+\frac{63x^{11}}{5632}+\frac{231x^{13}}{26624}+\frac{143x^{15}}{20480}+\frac{6435x^{17}}{1114112}+\frac{12155x^{19}}{2490368}+\frac{46189x^{21}}{11010048}$.

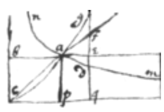
<19v>



If $pq=a$. $qd=x$. $pc=1=pb$. $db=\sqrt{1-aa-2ax-xx}$ then the areas of the lines in this progression. (supposeing also
$\overline{b-2ax-xx}\}^{\frac{3}{2}}$. $\begin{array}{c} bb-4abx-2bxx+4ax^3+x^4 \\ +4aa \end{array}$. * $\begin{array}{c} b^3-6abbx-3bbxx+8abx^3+3bx^4-6ax^5-x^6 \\ +12aabx^2-8a^3x^3-12aax^4 \\ +4abx^3 \end{array}$. &c

<20r>

To square the Hyperbola.



So if nadm is an Hyperbola. & $cp=1=pa$. $pq=x$. qd , qe , qf , qg &c $=y$. & $\frac{1}{1+x}=y=dq$. $1=y=eq$. $1+x=y=qf$.
$1+2x-xx=y=qg$. $1+3x+3xx+x^3$. $1+4x+6x^2+4x^3+x^4$. &c. Their squares are.
$*$. x . $x+\frac{xx}{2}$. $x+\frac{2xx}{2}+\frac{x^3}{3}$. $x+\frac{3xx}{2}+\frac{3x^3}{3}+\frac{x^4}{4}$. $x+\frac{4xx}{2}+\frac{6x^3}{3}+\frac{4x^4}{4}+\frac{x^5}{5}$. $x+\frac{5xx}{2}+\frac{10x^3}{3}+\frac{10x^4}{4}+\frac{5x^5}{5}+\frac{x^6}{6}$. &c As in the following

table. By whose first terme is represented the square of the Hyperbola, viz that it is

	a p q d =	a p q e =	a p q f =	a p q g =					
x	$\times 1.$	1.	1.	1.	1.	1.	1.	1.	1.
$\frac{x^2}{2}$	$\times -1.$	0.	1.	2.	3.	4.	5.	6.	7.
$\frac{x^3}{3}$	$\times 1.$	0.	0.	1.	3.	6.	10.	15.	21.
$\frac{x^4}{4}$	$\times -1.$	0.	0.	0.	1.	4.	10.	20.	35.
$\frac{x^5}{5}$	$\times 1.$	0.	0.	0.	0.	1.	5.	15.	35.
$\frac{x^6}{6}$	$\times -1.$	0.	0.	0.	0.	0.	1.	6.	21.
$\frac{x^7}{7}$	$\times 1.$	0.	0.	0.	0.	0.	0.	1.	7.

0000 . 00000 . 00000 .
3333 . 33333 . 33333 . 33333 . 33333 . 3
0000 . 00000 . 00000 . 00000 . 00000 . 0
8571 . 42857 . 14285 . 71428 . 57142 . 8
1111 . 11111 . 11111 . 11111 . 11111 . 1
9090 . 90909 . 09090 . 90909 . 09090 . 9

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + $$
$$+ \frac{x^7}{7} - \frac{x^8}{8} + \frac{x^9}{9} - \frac{x^{10}}{10}. \ \&c.$$

As if $x = \frac{1}{10}$. or $cq = \frac{11}{10} = 1{,}1$.

y^n is $dapq = \frac{1}{10} - \frac{1}{200} + \frac{1}{3000} -$
$- \frac{1}{40000} + \frac{1}{500000} - \frac{1}{6000000}$. &c

y^t is $apqd = 0{,}10000.00000.000000$
$\frac{1}{3}x^3 = +0{,}00033.33333.333333$
$\frac{1}{5}x^5 = +0{,}00000.20000.000000$
$\frac{1}{7}x^7 = +0{,}00000.00142.857142$
$\frac{1}{9}x^9 = +0{,}00000.00001.111111$
$\frac{1}{11}x^{11} = +0{,}00000.00000.009090$
$\frac{1}{13}x^{13} = +0{,}00000.00000.000076$
$\frac{1}{15}x^{15} = +0{,}00000.00000.000000$

0,10033.53477.310755.
Summa

9230 . 76923 . 07692 . 30769 . 23076 . 9230769230
6666 . 66666 . 66666 . 66666 . 66666 . 6666666666 .
0058 . 82352 . 94117 . 64705 . 88235 . 2941176470 .
0000 . 52631 . 57947 . 36842 . 10526 . 3157947368
0000 . 00476 . 19047 . 61904 . 76190 . 4761904761 .
0000 . 00004 . 34782 . 60869 . 56521 . 7391304347 .
0000 . 00000 . 04000 . 00000 . 00000 . 0000000000
0000 . 00000 . 000== ===== ===== .
37 . 03703 . 70370 . 3703703703
0000 . 00000 . 00000 . 34482 . 75862 . 0689655172 . 4137931034 . 48275862
0000 . 00000 . 00000 . 00=== ===== =
322 . 58064 . 5161290322 . 5806451612 . 90322580
0000 . 00000 . 00000 . 00003 . 03030 . 3030303030 . 3030303030 . 30303030
0 02857 . 1428571428 . 5714285714 . 28571428
0000 . 00000 . 00000 . 00000 00027 . 0270270270 . 2702702702 . 70270270
0000 . 00000 . 00000 . 00000 00000 . 2564010256 . 4010256401 . 02564010
0000 . 00000 . 00000 . 00000 00000 . 0024390243 . 9024390243 . 90243902

= 8063 . 57265 . 52007 . 40736 . 63159 . 415063 &c = summæ

−0 . 00500 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 0
2 . 50000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 0
1666 . 66666 . 66666 . 66666 . 66666 . 66666 . 66666 . 66666 . 66666 . 6
12 . 50000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 0
10000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 00000 . 0
83 . 33333 . 33333 . 33333 . 33333 . 33333 . 33333 . 33333 . 3
71428 . 57142 . 85714 . 28571 . 42857 . 14285 . 71428 . 5
630 . 55555 . 55555 . 55555 . 55555 . 55555 . 55555 . 5
5045 . 45454 . 54545 . 45454 . 54545 . 45454 . 5 .
41666 . 66666 . 66666 . 66666 . 66666 . 6
384 . 61538 . 46153 . 84615 . 38461 . 5
3 . 57142 . 85714 . 28571 . 42857 . 1
3364 . 58333 . 33333 . 33333 . 3
29411 . 76470 . 58823 . 5

−0 . 00502 . 51679 . 26750 . 72059 . 17744 . 28779 . 27385 . 30147 . 14044 . 12586 .

cui addendum

$$\begin{array}{r} -277.77777.77777.7 \\ -2.63157.89421.0 \\ -02523.80952.4 \\ -22727.3 \\ -217.4 \\ 2.1 \\ \hline -280.43459.71098.0 \\ \hline \end{array}$$

that is And so the summe will bee

$$\begin{array}{r} +0.10033.53477.31075.58063.57265.52007.40736.63159.41506.3 \\ -0.00502.51679.26750.72059.17144.28779.27385.30427.57503.8 \\ \hline 0.09530.01798.04324.86004.40121.23228.13351.32731.84002.5 \\ \hline \end{array}$$

which is the quantity of the area adpq . If $cpab = 1$. & $cp = ab = 10pq$ & $qde \parallel ap \parallel bc = ap$. In like manner if I make $x = \frac{1}{100} = pq$. The opperation followeth.

$$\begin{array}{rr} & 0,01000.03333.33333.33333.33333.33333.33333.33333.33333.3 \\ +\frac{1}{5}x^5 + \frac{1}{7}x^7 = & 33333.20001.42857.14285.71428.57142.85714.28571.40 \\ & == \;===== \;===== \;===== \;===== \;= \\ \frac{1}{9}x^9 + \frac{1}{11}x^{11} = & 11.11202.02020.20202.02020.20202.0 \\ \frac{1}{13}x^{13} + \frac{1}{15}x^{15} = & 769.23076.92307.69230.7 \\ \frac{1}{17}x^{17} = & 6666.66666.66666.6 \\ \frac{1}{19}x^{19} = & 58823.52941.1 \\ \frac{1}{21}x^{21} = & 5.26315.7 \\ & 47.6 \\ \hline & +0,01000.03333.53334.76201.58821.07551.40422.38870.97309. \\ \hline & -0,00005.00025.00166.66666.66666.66666.66666.66666.66666.6. \\ -\frac{1}{8}x^8 - \frac{1}{10}x^{10} - \frac{1}{12}x^{12} = & -1250.10000.83333.33333.33333.33333.3 \\ -\frac{1}{14}x^{14} = & -7.14285.71428.57142.8. \\ -\frac{1}{16}x^{16} - \frac{1}{18}x^{18} = & -62.50555.55555.5 \\ & ===== \;===== \;= \\ -\frac{1}{20}x^{20} = & -5000.4. \\ \hline & -0,00005.00025.00166.67916.76667.50007.14348.21984.17699. \\ \hline \\ & +0,00995.03308.53168.08284.82153.57544.26074.16886.79610 \\ \hline \end{array}$$

which is the quantity of the area apqd if $100p = cp$. and $abcp = 1$

<21r>

$y = db$. $x = ba$ $aay = x^3$. b + $z = y$ $z = bc$ $aab + aaz = x^3$. b − z= $y = z = bf$. $aab - aaz = x^3$. z− b= $y = z = bg$ aa z− a ab $= x^3$. $x = d + \xi$ $\xi = ah$. aay

$$-d^3 - 3dd\xi - 3d\xi\xi - \xi^3 = 0 \quad \left.\begin{array}{c} aab + aaz \\ aab - aaz \\ aaz - aab \end{array}\right\} = d^3 + 3dd\xi + 3d\xi\xi + \xi^3 . \; x = b - \xi \;. \; aq = \xi \quad \left.\begin{array}{c} aay \\ aab \pm aaz \\ aaz - aab \end{array}\right\} = d^3 - 3dd\xi + 3d\xi\xi - \xi^3 . \; x = \xi - b \;. \; ak = x$$

$$\left.\begin{array}{c} aay \\ aab \pm aaz \\ aaz - aab \end{array}\right\} = \xi^3 - 3d\xi\xi - 3dd\xi - d^3 .$$

$na = \xi$. $nd = z$. Or. $\xi^3 - a^2z + bb\xi$ $ab : an :: a : b$. &. $ab : nb :: a : c$ then $\xi^3 = \frac{b^3}{a}z - \frac{bbc}{a}\xi$. $\xi = d + \zeta$ $\zeta = mn$ $d^3 + 3d^2\zeta + bbc\zeta$ $+3d\zeta\zeta + \zeta^3 - \frac{b^3}{a}z + \frac{bbcd}{a}$

<21v>
<22r>

$$db = x.\; ba = y.\; aax = y^3 .\; x = b + z.\; z = bc.\; y = c + \xi.\; \xi = ah \quad \left.\begin{array}{c} aab + aaz = \\ (x = b - z \;.\; z = bf.) \\ aab - aaz = \\ (x = z - b \;.\; z = bg) \\ aaz - aab = \end{array}\right\} \begin{array}{ll} \xi^3 + 3c\xi^2 + 3cc\xi + \xi^3 . & y = c - \xi \;.\; \xi = aq \\ c^3 - 3cc\xi + 3c\xi^2 - \xi^2 . & y = \xi - c \;.\; \xi = ak . \\ \xi^3 - 3c\xi^2 + 3\xi c^2 - c^3 . & \end{array} \quad na = y.\; d\,n$$

$$= x.\; ab : an :: a : b.\; ab : nb :: a : c.\; \&\; y^3 = \frac{b^3}{a}x - \frac{bbc}{a}y.\; Or\; d^2x = \varepsilon\varepsilon y + y^3 . \quad \left.\begin{array}{c} (x = z + o \;.\; z = nc) \\ d^2z + d^2o = \\ (x = z - o \;.\; z = gn) \\ ddz - ddo = \\ (x = o - z \;.\; z = nf) \\ ddo - ddz = \\ ddz \end{array}\right\} \begin{array}{l} \varepsilon\varepsilon y + y^3 . \quad (y = n + \xi \;.\; \xi = na) \\ \xi^2 n + \varepsilon\varepsilon\xi + n^3 + 3nn\xi + 3n\xi^2 + \xi^3 . \\ (y = n - \xi \;.\; \xi = as) \\ \varepsilon\varepsilon n - \varepsilon\varepsilon\xi + n^3 - 3nn\xi + 3n\xi^2 - \xi^3 . \\ (y = \xi - n \;.\; \xi = av.) \\ \varepsilon\varepsilon\xi - \varepsilon\varepsilon n + \xi^3 - 3n\xi^2 + 3\xi n^2 - n^3 . \end{array} \quad al = y.\; dl = x.$$

$ab : al :: a : b$ &. $al : bl :: b : c$. whence $y^3 = \frac{xb^3 + yb^2c}{a}$ or $y^3 - \varepsilon\varepsilon y = ddx$ &c: as before onely varying the signes at εεn & εεξ . $ao = y$. $do = x$. $a : b :: bd : do$. $b : c :: do : ob$. $\frac{a^3}{b}x = y^3 - \frac{3cxyy}{b} + \frac{3ccxxy}{bb} - \frac{c^3x^3}{bb}$.

<22v>

D^r Wallis in a letter to S^r Kenelme Digby promiseth the squareing of the Hyperbola by finding a meane proportion twixt 1 , & $\frac{5}{6}$ in the progression $1, \frac{5}{6}, \frac{31}{30}, \frac{209}{140}, \frac{1471}{630}, \frac{10625}{2772}$ &c.

<23r>

The resolution of cubick equations out of D^r Wallis in his dedication before Meibomius confuted

suppose $x = \mho\, a \,\mho\, e$. then $x^3 = \mho\, a^3 \,\mho\, 3aae \,\mho\, 3aee \,\mho\, e^3$. or $x^3 = +\, 3aex \,\mho\, a^3 \,\mho\, e^3$. that is making $a^3 + e^3 = q$. & $3ae = p$. then $x^3 = +px \,\mho\, q$. Againe suppose $x = a - e$. then $x^3 = a^3 - 3aae + 3aee - e^3$. that is making $a^3 - e^3 = \mho\, q$, & $3ae = p$, then $x^3 = -px \,\mho\, q$.

Then in the first of these $p = 3ae$. or $\frac{p}{3e} = a$. or $\frac{p^3}{27e^3} = a^3 = q - e^3$. Therefore $e^6 = qe^3 - \frac{p^3}{27}$. & $e^3 = \frac{1}{2}q \,\mho\, \sqrt{\frac{1}{4}qq - \frac{p^3}{27}}$. & by the same reason $a^3 = \frac{1}{2}q \underset{\cap}{\mathrm{O}} \sqrt{\frac{1}{4}qq - \frac{p^3}{27}}$ where the irrationall quantitys have. divers signes otherwise $a^3 + e^3 = q$ would bee false. Soe that $x = \mho\, a \,\mho\, e = \mho \sqrt{c : \frac{1}{2}q \underset{\cap}{\mathrm{O}} \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}} \,\mho\, \sqrt{c : \frac{1}{2}q \,\mho\, \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}}$. is a rule for resolving the equation $x^3 * - p \underset{\cap}{\mathrm{O}} q = 0$, when it hath but one roote that is when it may be generated according to the supposition $x = \mho\, a \,\mho\, e$. &c. By the same reason $x^3 * + px \underset{\cap}{\mathrm{O}} q$. may be resolved by this rule $x = a - e = \sqrt{c : \frac{1}{2}q \,\mho\, \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}} - \sqrt{c : \frac{1}{2}q \underset{\cap}{\mathrm{O}} \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}}$.

But here observe that D^r Wallis would Argue that since in the first of these two cases sometimes (viz when the equation hath 3 reall rootes) the rule faileth as it were impossible for the equation to have rootes when yet it hath, therefore the fault is in Algebra. & therefore when Analysis leads us to an impossibility wee ought not to conclude the thing absolutely imposible, untill wee have tryed all the ways that may bee.

But let me answer that the fault is not in the Analysis in this example, but in his opperation. for when the equation $x^3 * + px \,\mho\, q = 0$, hath 3 roots hee supposeth it to have but one roote viz $x = \mho\, a \,\mho\, e$. but since the Equation cannot be then generated according to that supposition it is impossible it should be resolved by it.

<23v>

In like manner hee sayeth that Algebra representeth a thing possible when tis not so as in this example, in the triangle abc, make $ab = 1$. $bc = 2$ $ac = 4$. Then to find

$dc = x$, worke thus, $ad = 4 - x$. $bd \times bd = 1 - 16 + 8x - x^2 = 4 - x^2$ therefore $8x = 19$. or $x = \frac{19}{8}$. In which opperacon all things proceede as possible though they are not soe for ac is greater than $ab + bc$.

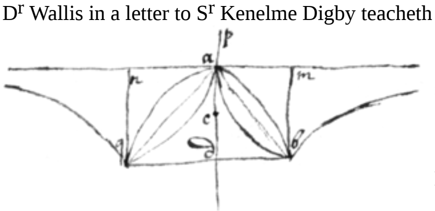
yet I answer that if the opperation & conclusion be compared together the absurdity will appeare. for in the equation $bd \times bd = 4 - xx = 4 - \frac{361}{64} = \frac{256 - 361}{64}$ or $bd \times bd = \frac{-105}{8}$. but it is impossible that a square number should be negative.

Thus $x = \sqrt{-b}$ is impossible. square it & tis $xx = -b$. Againe, & tis $x^4 = bb$. Extract the roote & tis $xx = b$ or $x = \sqrt{b}$. which is possible. The reason of this Event is that $x^4 - bb = 0$ hath two possible rootes viz $x = \sqrt{b}$. $x = -\sqrt{b}$. & two impossible viz: $x = \sqrt{-b}$. $x = -\sqrt{-b}$.

Thus the valors of $x^8 - a^8 = 0$ are $x = a$, $-a$, $\sqrt{-aa}$, $-\sqrt{-aa}$, $\sqrt{4 : -a^4}$, $-\sqrt{4 : -a^4}$, $\sqrt{-\sqrt{-a^4}}$, $-\sqrt{-\sqrt{-a^4}}$.

<24r>

D^r Wallis in a letter to S^r Kenelme Digby teacheth how to find the center of gravity in divers lines first when their position is as in this figure.

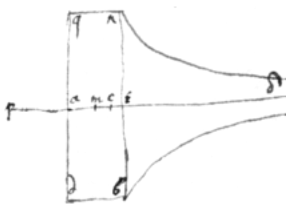


Suppose ad the Axis, a their vertex Then saying, as 1 to the index of the line increased by an unite (vide pag 2^dam) so cd to ca Then c is their center of gravity.

The Demonstration.

Let p bee the index of the series according to which the odinately aplyed lines (parallell to db) increase, then $1 : p + 1 ::$ area of the line : to nmbq . the distances of those ordinate lines from the vertex a are equall to the intercepted diameters & therefore a primanary series (whos index is 1. & since supposing a the center of the ballance the whole weight of the surface or figure is composed of its magnitude & distance from the center and therefore the index of all its moments or whole weight is $p + 1$, viz: the aggregate of the other two. Therefore as all its moments (or the weight of the figure in its site in respect of the center a are to soe many of the greatest (or to the weight of the rectangle nmbq hung on the point d) soe is 1, to $p + 2$. and if $ap : ad :: 1 : p + 2$, then nmbq hung on the point q shall counterballance the figure in its site &c therefore if $ac : cd :: p + 1 : 1$, c shall be the center of gravity of those figures.

Also as the figure is now put extending infinitely towards δ if $-2p + 1 : -$ {p }$+1 :: am : ac$. m being the center of qnbd then c shall bee the center of gravity of the whole figure qndbδ .



Demonstration

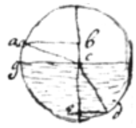
<24v>

since the lines parallell to aδ increase in series reciprocally proportionall their index is $-p$ & since the halfes of those lines increase in the same proportion their index is $-p$. whose extremitys or middle points of the whole lines (suposing a the center of the ballance) are theire centers of gravity, their distances from a being proportionall to the lines whose centers they are & consequently their index is $-p$ & since all the moments (or whole weight of the figure) increase in a proportion compounded of the proportion of the magnitudes & distances of the lines from the center a, they will be in a duplicate proportion of the lines magnitudes that is a reciprocall series whose index is $-2p$. Therefore the figure is to the inscribed parallelogram as 1 to $1 - p$. & all its moments or whole weight in this its site to the weight of the parallelogram as 1 to $1 - 2p$. Therefore if, $am : ap :: 1 - 2p : 1$, the parallelogram hanging on the point p shall counterballanc**{e}** the whole figure in its site &c: whence the point c may be found easily, viz $am : ac :: 1 - 2p : 1 - p$.

<26r>

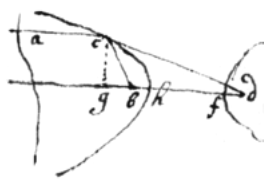
Of Refractions.

1 If the ray ac bee refracted at the center of the circle acdg towards d & $ab \perp be \perp gc \parallel ed$. Then suppose $ab : ed :: d : e$. See Cartes Dioptricks



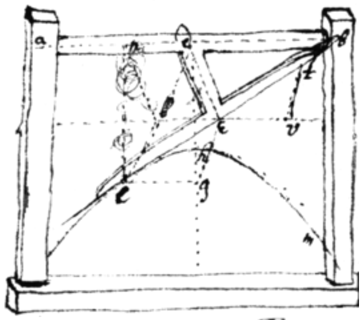
2 If there be an hyperbola the distance of whose foci bd are to its transverse axis hf as d to e . Then the ray $ac \parallel bd$ is refracted to the exterior focus (d). See C: Dioptr

3 Having the proportion of d to e, or. $bd : hf$. The Hyperbola may bee thus described.



1 Upon the centers a, b let the instrument adbtec bee moved in which instrument observe that ad ⊥de⊥ c et & that the beame cet is not in the same plane with adbe but intersects it at the angle tev soe that if $tv \perp ev$, then $d : e :: et : tv$. Or $d : e :: Rad : \text{sine of } \angle tev$. Also make $de = \frac{q}{2}$, i.e half the transverse diamet{er.} Then place the fiduciall side of plate chm in the same plaine with ab . & moving the instrument adbect to & fro its edge cet shall cut or weare it into the shape of the desired Parabola. Or the plate chm may bee filed away untill the edge cet exactly touch it everywhere.

2 By the same proceeding Des=Cartes concave Hyperbolicall wheele may bee described by beeing turned with a chissell d tec whose edge is a streight line inclined to the axis of the mandrill by the ∠ tev which angle is found by making $d : e :: et : tv :: Rad : \text{sine of } etv$.



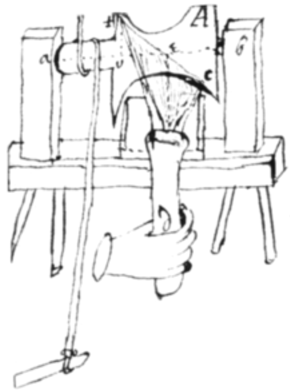
3 By the same reason a wheele may be turned Hyperbolically concave the Hyperbola being convex. Or a Plate may bee turned Hyperbolically concave

<26v>

Also Des=Cartes his Convex wheele B may be turned or {ground} trew a concave wheele A being made use of instead of a patterne

5 In turning the concave wheele A it will perhaps bee best to weare it with a stone p & let the streight edged chissell d serve for a patterne. And it may bee convenient to grind the stone (or iron &c) p into the fashion of a cone S That may fit the hollow of the wheele A. The angle of which cone being

<27r>



9 Halving such a cone smoothly pollished within & without, by the helpe of a square set the plate perpendicular to one side hae the fiduciall edge being distant from the vertex the length of $ae = \frac{edd - e^3}{dd + ee}$ & if the edge of the plaine every where touch the cone, tis trew.

10 The exact distance (ae) of the plate from the vertex of the cone neede not bee much regarded for that changeth onely the bigness not the shape of the figure.

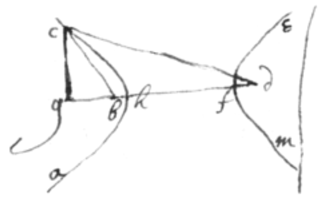
[By the broken lookinglasse I find in glasse refraction, that $d : e :: 43 : 28 :: 1000 : 651+ :: 1536 : 1000$. These are almost insensibly different from truth $d : e :: 20 : 13 :: 100 : 65 :: 153- : 1000$. Or $d : e :: 23 : 15 :: 100 : 652+$ $d : e :: 66 : 43 :: 100 : 651{,}5151+$. Or

For the Ellipsis $\frac{dd-ee}{d} x + \frac{ee-dd}{dd} xx = yy$

<27v>

The former $\left\{ \begin{array}{c} \text{descriptions} \\ \text{propositions} \end{array} \right\}$ **demonstrated.**

Lemma. If in the Opposite Hyperbolas abc



edf (one of which are to bee described) supposing $bd = d$. $hf = e$. $gh = x$ $gc = y$. $gc \perp ghd$ & gc terminated by the hyperbola Then is $\frac{dd-ee}{ee} xx + \frac{dd-ee}{e} x = yy$. b h $= \frac{d-e}{2}$. $dh = \frac{d+e}{2}$. b $g = \frac{2x-d+e}{2}$. $gd = \frac{2x+d+e}{2}$. $dc^2 = \frac{4xx+4dx+4ex+dd+2ed+ee+4yy}{4} = gd \times gd + gc \times gc$. $bc^2 = \frac{4xx-4dx+4ex+dd-2ed+ee+4yy}{4} = gb^2 + gc^2$. And since $dc = bc + hf$. Or $dc^2 = bc^2 + 2bc \times hf + hf^2$ Therefore $2dx + ed - ee = e\sqrt{4xx - 4dx + 4ex + dd - 2ed + ee + 4yy}$. Both parts of which squared & ordered the result is $4ddxx - 4eexx + 4eddx - 4e^3x - 4eeyy = 0$. That is $\frac{dd-ee}{e} x + \frac{dd-ee}{ee} xx = yy$.

Description the 1st demonstrated Synthetically. See that Scheame

Nameing the quantitys $ed = dh = \frac{e}{2}$. $gh = x$. $gc = be = y$. $dg = x + \frac{e}{2} = nc$. $cg \perp dhg$. $cd^2 = x^2 + ex + \frac{ee}{4} + y$ $ce^2 = xx + ex + yy$. $eg^2 = xx + ex$. Also $d : e :: et : tv :: ce : eg$, therefore $ddxx + ddex = eex^2 + e^3x + e^2y$ That is $\frac{dd-ee}{e} x + \frac{dd-ee}{ee} xx = yy$. As in the lem̄a

The Same demonstrated Analytically.

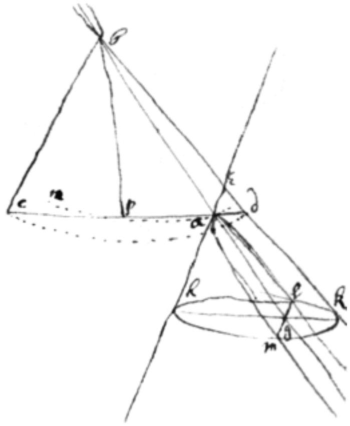
Nameing the quantitys, $de = dh = a$. $gh = x$. $gc = y$. $dg = a + x$ $dc^2 = aa + 2ax + xx + yy$. $ce^2 = 2ax + x^2 + yy$. $eg^2 = xx + ex$. Supose that $b : c :: et : tv :: ce : eg$.

<28r>

Then is $bbxx + bbex = 2ccax + ccxx + ccyy$. That is $\frac{bb-cc}{cc} xx + \frac{bbe-2cca}{cc} x = yy$. Therefore the line chm is a Conick Section & since (bb) is greater than (cc) tis an Hyperbola, which that it may bee the same with that in the lemma, Their correspondent termes are to bee compared together & soe I find that $\frac{bb-cc}{cc} xx = \frac{dd-ee}{ee} xx$. & $\frac{bbe-2cca}{cc} x = \frac{dd-ee}{e} x$ by the 1st equation $bb = \frac{ccdd}{ee}$. Or $b = \frac{cd}{e}$. that is $b : c :: d : e$. by the 2nd $ccee - ccdd + bbee = 2ccea$. And by substituting $\frac{ccdd}{ee}$ into the place of bb And ordering it tis $ccee = 2ccea$. Or $\frac{e}{2} = a$. Therefore if I take $\frac{e}{2} = a = de$. & $d : e :: b : c :: ct : tv$. then shall chm bee the Hyperbola desired Q:E:D.

The 2d 3d 4th & 5th Propositions are manifest from this

☞ Instead of the 6th & 7th Descriptions which are false use these



6 Draw 2 concentrick circles (na & cd) with the Radij e & d. Then from the common center b draw 2 lines bc & $\left\{\begin{matrix}\mathrm{ba}\\ \mathrm{bd}\end{matrix}\right\}$ at the given angle $\left\{\begin{matrix}\mathrm{bae}=\mathrm{abc}\\ \mathrm{aed}=\mathrm{cbd}\end{matrix}\right\}$ of $\left\{\begin{matrix}\text{section}\\ \text{y}^{\text{e}}\ \text{Cone}\end{matrix}\right\}$ then draw a line cad from c by the end of the Rad $\left\{\begin{matrix}\mathrm{ba}\\ \mathrm{bd}\end{matrix}\right\}$ & to the intersection of that line with the circle $\left\{\begin{matrix}\mathrm{cd}\\ \mathrm{na}\end{matrix}\right\}$ draw $\left\{\begin{matrix}\mathrm{bd}\\ \mathrm{ba}\end{matrix}\right\}$ & so the angle of $\left\{\begin{matrix}\text{y}^{\text{e}}\ \text{Cone},\ \mathrm{hek}=\mathrm{cbd}\\ \text{section},\ \mathrm{eab}=\mathrm{abc}\end{matrix}\right\}$ is found.

<28v>

Or which is the same make $\mathrm{ab}=\mathrm{e}.\ \mathrm{bd}=\mathrm{d}$ & then if that cone is sought the angle cba being given, make $\mathrm{ac}=\mathrm{a}$. Then is $\mathrm{cd}=\frac{\mathrm{dd-ee+aa}}{\mathrm{a}}$. & soe the $\angle\ \mathrm{cbc}=\mathrm{aed}$ is knowne & also $\mathrm{ae}=\mathrm{ed}=\frac{\mathrm{d}^3-\mathrm{dee}}{\mathrm{dd-ee+aa}}$, & $\mathrm{ad}=\frac{\mathrm{dd-ee}}{\mathrm{a}}$. But if the $\angle\ \mathrm{bae}=\mathrm{abc}$ of the section is sought the cone being given than make $\mathrm{cd}=2\mathrm{b}$. And it will bee $\mathrm{ac}=\mathrm{b}+\sqrt{\mathrm{ee-dd+bb}}$. & soe $\angle\ \mathrm{abc}=\mathrm{bae}$ is given also $\mathrm{ad}=\mathrm{b}-\sqrt{\mathrm{ee-dd+bb}}$. & $\mathrm{ae}=\frac{\mathrm{db-d}\sqrt{\mathrm{ee-dd+bb}}}{2\mathrm{b}}$

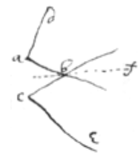
In generall observe that in any cone cut any ways $\mathrm{bd}=\mathrm{be}+\mathrm{ea}=\mathrm{d}$. & $\mathrm{ba}=\mathrm{e}$.

7. DesCartes his wheele thus described cut by any plaine produceth one of the Conick=Sections.

Description the 6th Demonstrated. Synthetically.

Call, $\mathrm{bd}=\mathrm{d}.\ \mathrm{ba}=\mathrm{e}.\ \mathrm{cp}=\mathrm{pd}=\mathrm{a}.\ \mathrm{bp}=\sqrt{\mathrm{dd-aa}}.\ \mathrm{ag}=\mathrm{x}\ \mathrm{ap}=\sqrt{\mathrm{ee-dd+aa}}.\ \mathrm{ac}=\mathrm{a}+\sqrt{\mathrm{ee-dd+aa}}.\ \mathrm{ad}=\mathrm{a}-\sqrt{\mathrm{ee-dd+aa}}.$ $\mathrm{ba}:\mathrm{ac}::\mathrm{ag}:\mathrm{gh}=\frac{\mathrm{ax+x}\sqrt{\mathrm{ee-dd+aa}}}{\mathrm{e}}$. $\mathrm{ba}:\mathrm{ad}::\mathrm{bg}:\mathrm{gk}=\frac{\mathrm{ea+ax}}{\mathrm{e}}\quad\frac{-\mathrm{e-x}}{\mathrm{e}}\sqrt{\mathrm{ee-dd+aa}}$. $\mathrm{gk}\times\mathrm{gh}=\mathrm{gm}^2=\mathrm{y}^2$. Therefore $\frac{\mathrm{dd-ee}}{\mathrm{ee}}\mathrm{xx}+\frac{\mathrm{dd-ee}}{\mathrm{e}}\mathrm{x}=\mathrm{yy}$. by ordering the result of $\mathrm{gk}\times\mathrm{gh}$. which is like that in the lemma.

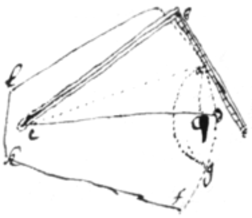
The 7th Proposition may be easyly demonstrated after the same manner.



If the two equall cones bad bcd intersect the one the other soe that $\mathrm{ab}=\mathrm{bc}$ their intersection (bf) shall bee one of the Conick sections as they had each beene intersected by the plane bf .

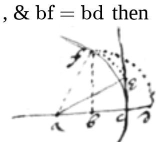
<29r>

To describe the Parabola (& other figures after the same manner) pretty exactly.

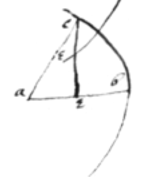


Take a squire cbe , soe that $\mathrm{cb}=\frac{\mathrm{r}}{2}$ (for then the circle described by (bc) will bee as crooked as the Parabola at the vertex d). Divide the other leg (be) of the Squire into any number of points, Then get a plate of Brasse &c: lkfd streight & eaven. And taking one point d for the vertex of it & another point c for the Squire to moven soe that $\mathrm{cd}=\mathrm{cb}=\frac{\mathrm{r}}{2}$, & weareing away the edge of the plate untill (the Squire being erected) $\mathrm{ab}=\mathrm{qd}$. the squire touching the plate at a . thus shall the edge adf become Parabolicall. the Rad: ab describe a circle it may bee knowne when $\mathrm{ab}=\mathrm{a}$. Instead of the leg <u>be a</u> circle may be used Demonstracon. $\mathrm{qd}=\mathrm{x}.\ \mathrm{cd}=\frac{\mathrm{r}}{2}=\mathrm{cb}.\ \mathrm{cq}=\frac{\mathrm{r}}{2}-\mathrm{x}.$ $\mathrm{aq}=\sqrt{\mathrm{rx}}=\mathrm{y}.\ \mathrm{ac}^2=\frac{\mathrm{rr}}{4}+\mathrm{xx}\ \ \mathrm{ab}^2=\frac{\mathrm{rr}}{4}+\mathrm{xx}-\frac{\mathrm{rr}}{4}$ & $\mathrm{ab}=\mathrm{x}$. Q.E.D.

Another description of the Parabola with the compasses. Make $\mathrm{ab}=\mathrm{bc}=\frac{\mathrm{r}}{4}$. Make $\mathrm{ce}=\mathrm{cd}$ & $\mathrm{ce}\perp\mathrm{bd}$. Make $\mathrm{af}=\mathrm{ae}$, & $\mathrm{bf}=\mathrm{bd}$ then shall f be a point in the Parabola.



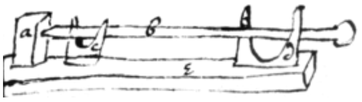
Another. Make $\mathrm{ab}=\frac{\mathrm{r+x}}{2}=\mathrm{ac}.\ \mathrm{eb}=\mathrm{x}\perp\mathrm{ce}$ & the point c shall bee in the parabola. This like the first by calculation may bee made use of in other lines.



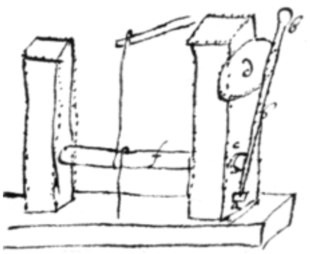
<29v>

The manner whereby any kind of little lines may be described very accurately. And that the same Instrument serve for all lines (though never so small) differing in quantity but not in quality.

Make the plate d of the figure required (by some of the former meanes) the larger the better. Then hold the streight steele staffe b against the center a & **{roule}{route}** it to & fro it shall grind c into the same figure but soe much lesse as ac is lesse than ad.



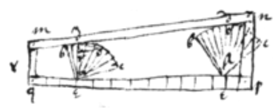
Soe if the glass c bee fastened upon the mandrill f, it may be ground acording to the sollid figure d by the helpe of a stick of steele (as a cone) whose cuspis is in the hole a upon which it is moved as on a center. when the cone b leanes uppon the vertices of d & c it must be perpendicular to the mandrill f. Perhaps it may be convenient to cause the cone b to turne about its axis. Or it may bee better instead of the nutt at a with a hole in it to make a sharpe pointed nutt, & instead of the cone b to make use of a broad plate to cover a, c & d & move every way upon them



<30r>

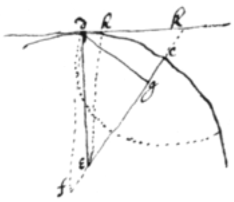
Another way to describe lines on plates

Suppose the plate bee abc, whose edge boc is to be made into the fashion of a given crooked line suppose (o) is its vertex & that a circle described with the Radius eo would bee as crooked as the given line at its vertex. Againe suppose two streight rulers mn & pq to bee very trew & steddyly fastened together which must a very little incline the one to the other, soe as that being produced they would meete at ar. Then are the lines $\mathrm{pn}=\mathrm{a}$, & $\mathrm{pr}=\mathrm{b}$ given.



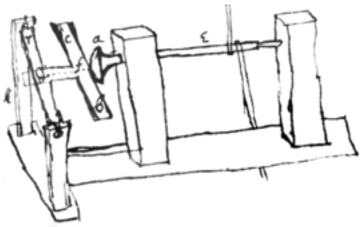
Suppose then the point d in the crooked line is to bee found then is dc given by supposition, & consequently (supposing dk to bee a tangent) $\mathrm{dg}=\mathrm{y}.\ \mathrm{gc}=\mathrm{x}.\ \mathrm{fg}=\mathrm{v}.\ \mathrm{fd}=\mathrm{s}\ \mathrm{ec}=\mathrm{c}.\ \mathrm{fk}=\mathrm{v}+\frac{\mathrm{yy}}{\mathrm{v}}$. $\mathrm{ef}=\mathrm{v}+\mathrm{x}-\mathrm{c}.\ \mathrm{ek}=\mathrm{c}-\mathrm{x}+\frac{\mathrm{yy}}{\mathrm{v}}$. & (if $\mathrm{eh}\perp\mathrm{dk}\perp\mathrm{df}$) then is $\mathrm{eh}=\frac{\mathrm{cv-xv+yy}}{\sqrt{\mathrm{vv+yy}}}=\mathrm{d}$. (eh) being thus found, supposing that $\mathrm{pn}=\mathrm{a}=\mathrm{ec}$, then I take $\mathrm{re}=\frac{\mathrm{bd}}{\mathrm{a}}$. that is $\mathrm{pe}=\mathrm{b}-\frac{\mathrm{bd}}{\mathrm{a}}$. haveing thus found the point e lay the plate twixt the two rulers so that the point of it, fall upon the point e then should the line mn touch the plate in d. But note that $\mathrm{pn}\perp\mathrm{mn}$.

In both telescopes & microscopes tis most convenient to have a convex glasse next the eye for by that meanes the angle of vision will bee much greater than it will bee with a concave one (though both doe magnifie alike). If the convex glasse be Hyperbolicall (&c) make it soe bigg that the penecilli may crosse in the pupill; that is, the exterior focus will be as far distant from the vertex as the eye is. let the glass bee as thinn as may bee that the eye bee not too far from the vertex that it should bee about as thick as the distance of the interior focus from the vertex.

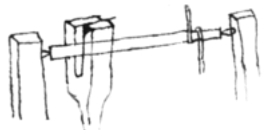


And by this meanes also, (the focus of the objectglasse being within the telescope twixt the glasses) there may bee placed at that focus the edge of <30v> a steele ruler accurately divided into equall parts (to measure the diameters or distances of starrs &c) which should bee soe made that by a pinne or handle it may be placed in any posture & in any parte of the focus, without otherwise altering the Telescope in observations.

Note that were not the glasses faulty they would not onely magnify objects but render vision more distinct; each of the penicilli passing through (perhaps but) the 10^{th}, 20^{th} or 100^{th} parte of the pupill must bee more exactly refracted to one point of the Tunica Retina than in ordinary vision in which each of the penicilli spreads over all the pupill.

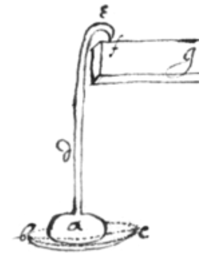


☞ Note also that that the glasse a may be ground Hyperbolicall by the line cb, if it turne on the mandrill e whilst cb turnes on the axis rd being inclined to it as was shewed before. If the edge (cb) bee not durable enough, inough instead thereof use a long small cilinder: which I conceive to bee the best way, of all. For a Cilinder of all sollids is most easily made exact (being turned, as in the figure, by a gage untill its thicknesse bee every where equall). 2 the Cilinder may bee made to slip up & downe & turne round whereby it will not onely grinde the glase crosse wise to take of all hubbes, but also the glasse & cilinder will grinde the one the other truer & truer. All the difficulty is in placing the axis rd perpendicular to the Mandrill ae & vertex to vertex, which yet may bee done exactly severall ways. & untill then the glasse & Cilinder will not fit. & should the axis not intersect the glasse would bee still Hyperbolicall except a point at the vertex of it. The same instrument may also serve for severall glasses onely making df longer or shorter. Let the Cilinder han**{g}** over the glasse.



<31r>

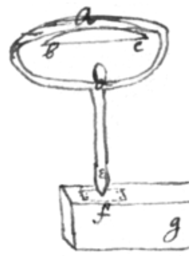
To Grinde Sphæricall optick Glasses



If the glasse (bc) is to bee ground sphærically hollow: naile a steele plate to the beame (fg), on the upper side: In which make a center hole for the steele point (f) of the shaft (def): to which shaft fasten a plugg (a) of stone or leade or leather &c: (with which you intend to grinde the glasse (bc)): which shaft & plugg being swung to & fro upon the center f will grind the glasse bc sphærically hollow.

The manner whereby glasses may bee ground sphærically convex may appeare by the annexed figure (being the former way inverted). Also the plugg (a), in the first / second figure, is ground sphærically convex. / concave.

But if this way bee not exact enough yet hereby may bee **{grownd}{ground}** plates of mettall well nigh sphæricall, And by those plates may bee ground glasses after the usual manner; If a circular hoope of steele (abc) bee put about the edge of the glasse (d) to keepe it from grinding away at the edges faster than in the middle.



But the best way of all will bee to turne the glass circularly upon a mandrill whilest the plate is steadily rubbed upon it or else <31v> to turne the plate upon a mandrill whilest the glasse is rubbed upon it or let sometimes the one, sometimes the other bee turned.: & by this meanes they will either of them weare the other to a truely sphericall forme. but however let there bee a hoope or of some mettall which weares more difficultly then glasse to defend the glasse from wearing more at its edges then in the middle. Perhaps it may doe well first to weare the plate sphæricall by the hoope alone without the glasse.

The same meanes may bee used for grinding plaine glasses.

Let not an object glasse bee ground sphærically convex on both sides, but sphaerically convex on one side & plane or but a little convex ~ on the other, & turne the convexest side towards the object.

<32r>

<32v>

If the Glasses of a Telescope bee not truely ground Theire errors may bee thus found.

Because an error is much more easily discernable in the object glasse than in the eye glasse let us first suppose the eye glasse to bee ground true towards its center, (tis exact enough if it be sphericall, & not Hyperbolicall), & so wee may find & rectifie the errors of the object glasse.

First make a thin plate (A) of brasse & in the center of it a Small hole (whose diameter perhaps may bee about the 50^{th} or 100^{dth} parte of an inch. With which plate cover the eye glass the center of it respecting the center of the glasse.

Secondly make two other plates the one B with two holes as neare to its edge as may bee their**{e}{}** distance being about the 5^{th} parte of an inch or lesse, & the other C with one hole close to the midst of its edge. Let the diameters of these 3 holes bee about a 20^{th} parte of an inch or lesse. And theire edges must bee true that they may slide one upon another, & yet not let the suns rays passe through, to which purpose make them oblique. with these two plates cover the object glasse (first stopping the hole of C the holes of the other plate respecting the center of the glasse & looke at a stare (or the edge of the sunne &c) & if the object appeare double (like two starrs &c) make the Tube longer or shorter untill it appeare single. Then open the hole of C , & the plate B being fixed, slide the plate C up & downe still looking at the starre, When then appeares <33r> but one starre that part of the glasse under the hole of C is truely ground in respect of the 2 parts of the glasse under the two holes of B. But **{no}** when the starre appeares double. And the position of the starre caused by the hole of C in respect of the starre caused by the holes of B, shews which way the glasse under the hole of C is erroneously inclined; the distance of the two starres giving the quantity of that error.

Thus the errors of the object glasse bein**{g}** found in every place of it they may bee all rectified, & found againe, & againe rectified, untill they almost or altogether vanish.

Then may the eye=glasse bee rectified much after the same manner, in every parte of it, & if it bee necessary the object glasse may bee againe rectified & againe the eye=glasse untill the Telescope bee as perfect as the workeman can make: Whome perhaps experience may teach by this & the former rules to make telescopes as perfect as men can hope to make them.

These glasses may also bee rectified whilst on the Mandrill by observing the images made by reflection from the vertex & all other parts of the glasse what proportion they have one to another & how much they are longer than broader in one place then another. &c.

<33v>

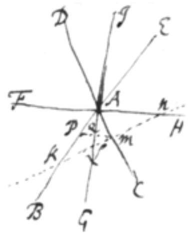
	Aere	water	Glasse	christall
The sines measuring refractions are in	42	56	65	70
The proportions of the motions of the extreamely heterogeneous rays are in	39,4 . 40,4 .	$70\frac{3}{8}$. $71\frac{3}{8}$.	$95\frac{1}{10}$. $96\frac{1}{10}$	$110\frac{1}{3}$. $111\frac{1}{3}$
The proportions of ye sines of refraction of the extreamely heterogeneous rays into aire out of	c	$90\frac{2}{3}$. $91\frac{2}{3}$	68 . 69	$61\frac{4}{5}$. $62\frac{4}{5}$
Their common sine of incidence		$68\frac{1}{3}$	$44\frac{1}{4}$	$36\frac{4}{5}$
Which substracted the difference is		$22\frac{1}{3}$. $23\frac{1}{3}$	$23\frac{3}{4}$. $24\frac{3}{4}$	24 . 25
The like proportions for refractions made into water out of			$275\frac{4}{5}$. $276\frac{4}{5}$ $238\frac{2}{5}$ $37\frac{2}{5}$. $38\frac{2}{5}$	$196\frac{1}{3}$. $197\frac{1}{3}$ $157\frac{4}{9}$ $39\frac{1}{9}$. $40\frac{1}{9}$

of the method of infinite series

I Newton

<35r>

Theoremata varia. Circa angulorum æqualitates.

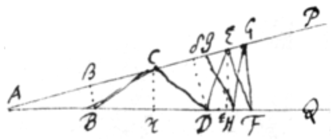


si ang DAB & DAE bisecentur a rectis FH et IG et ducatur quævis KLMN . Erit
1. $AK\ .\ AM :: KL\ .\ LM ::\ KN\ .\ MN$ Euclid 6 3
2. $AK \times AM = AL\,q + KL \times LM = KL \times LM - AL\,q$. Scho{o}ten de {concis} {æqu{is}}
3. $AM + AK\ .\ PK :: AQ\ .\ AL$ posito $AP = AM$.

Si in angulo quovis PAQ inseribantur æquales AB, BC, CD, DE, EF, FG, GH &c anguli BA P erit angulus CBQ duplus DCP tripl, ED Q quadr FEQ quint, GFQ sext, HGP sept. IHQ oct &c. Horum vero angulorum posito radio AB sinus erunt Bβ , Cχ &c cosinus AB , Bχ , Cδ &c. Ergo si $AB = r$, & $AB = x$ erit $AC = 2x$ $A\chi = \frac{2xx}{r}$. . $AD = (2A\chi - AB) = \frac{4xx - rr}{r}$. $A\delta = \frac{4x^3 - rrx}{rr}$ &c

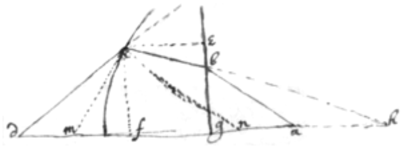
<79r>

To find the sume of the squares cu{bes} &c. of the rootes of an equation



If a , bg , c , d , e , f &c be the rootes of the equation $x^6 + px^5 + qx^4 + rx^3 + sxx + tx + v = 0$. then is
$a + b + c + d + e + f = p\,(= g)$ $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = pp - 2q$. $(= pg - 2q\ \ = h)$
$a^3 + b^3 + c^3 + d^3 + e^3 + f^3 = p^3 - 3pq + 3r$. $(= ph - qg + 3r = k)$ $a^4 + b^4$ &c
$= p^4 - 4ppq + 4pr + 2qq - 4s$. $(= pk - qh + rg - 4s = l)$ a^5 &c $= p^5 - 5p^3q + 5pqq + 5ppr - 5ps - 5qr + 5t$. $(= pl - qk + rh - sg + 5t\ \ =$ { m) } a^6 &c:
$= p^6 - 6p^4q + 9ppqq + 6p^3r - 12pqr + 6pps + 6pt - 2q^3 + 3rr + 6qs - 6v$.

<80r>



$ag = a$. $ab = x$. $bh = \frac{dx}{c}$. $bc = y$. $bg = \sqrt{xx - aa}$ $gh = \sqrt{\frac{ddxx - eexx + eeaa}{ee}}$. $dg = b$.
$ce = \frac{y}{dx}\sqrt{ddxx - eexx + eeaa} = fg$. $cf = \frac{dx + ey}{dx}\sqrt{xx - aa}$. $df = b - \frac{y}{dx}\sqrt{ddxx - eexx + eeaa}$.

$$z^2 = dc^2 = \begin{cases} xx - aa + \frac{2eyx}{d} - \frac{2eyaa}{dx} + bb + yy \\ \frac{-2by}{dx}\sqrt{ddxx - eexx + eeaa} \end{cases} \quad \frac{2by}{dx}\sqrt{ddxx - eexx + eeaa} = xx + yy - zz + bb - aa + \frac{2eyxx - 2eyaa}{dx} .$$

$$\begin{array}{lllllll} ddx^6 & + 4deyx^5 & + 2ddyyx^4 & - 4deaayx^3 & - 4bbddyyxx & - 4dey^3aax & + 4e^2a^4y^2 \\ & & - 2ddzzx^4 & + 4dey^3 & + 4bbeeyy & + 4deyz^2a^2 & \\ & & + 2ddbb & - 4deyzz & - 8aaeeyy & - 4deybba^2 & \\ & & - 2ddaa & + 4deybb & + 2ddy^4 & + 4deya^4 & - 4bbe^2a^2y^2 \\ & & + 4eeyy & - 4deyaa & & & \end{array}$$

Ad constructionem Canonis angularis.

$\frac{90^{gr}}{5} = 18^{gr}$. $\frac{18^{gr}}{5} = 3^{gr} + 36'$. Et $\frac{60^{gr}}{3} = 20^{gr}$. $\frac{20^{gr}}{3} = 6^{gr} + 40'$ $\frac{6^{gr} + 40'}{2} = 3^{gr} + 20'$. $3^{gr} + 36' - 3^{gr} - 20' = 16'$. $\frac{16'}{2} = 8'$. $\frac{8'}{2} = 4'$. $\frac{4'}{2} = 2'$ $\frac{2'}{2} = 1'$.

If r = radius. Then

$$\text{y}^{\text{e}}\ \text{sine of} \begin{cases} 78^{degr} \text{ is, } \dfrac{r\sqrt{5} - r + r\sqrt{30 + 6\sqrt{5}}}{8} . \\ 66^{degr} \text{ is, } \dfrac{r\sqrt{5} - r + r\sqrt{30 - 6\sqrt{5}}}{8} . \\ 42^{degr} \text{ is, } \dfrac{-\sqrt{5rr} + r + \sqrt{30rr + 6rr\sqrt{5}}}{8} . \\ 6^{degr} \text{ is, } \dfrac{\sqrt{30rr - 6rr\sqrt{5}} - \sqrt{5rr} - r}{8} . \end{cases}$$

<80v>

Suppose $gh = x.\ nh = r.$ Then

$$\begin{aligned}
gh &= x \ \text{. unisectio}\\
ab \times r &= 2rr - xx \ \text{. bisectio}\\
h^2b \times r^2 &= 3rrx - x^3 \ \text{. trisectio}\\
a^3b \times r^3 &= 2r^4 - 4rrxx + x^4 \ \text{. quadrisec}^{o}.\\
h^4b \times r^4 &= 5r^4x - 5rrx^3 + x^5 \ \text{. quintusect}^{o}.\\
a^5b \times r^5 &= 2r^6 - 9r^4x^2 + 6rrx^4 - x^6 \ .\\
hb \times r^6 &= 7r^6x - 14r^4x^3 + 7rrx^5 - x^7 \ .\\
ab \times r^7 &= 2r^8 - 16r^6x^2 + 20r^4x^4 - 8rrx^6 + x^8 \ .\\
hb \times r^8 &= 9r^8x - 30r^6x^3 + 27r^4x^5 - 9rrx^7 + x^9 \ .\\
ab \times r^9 &= 2r^{10} - 25r^8x^2 + 50r^6x^4 - 35r^4x^6 + 10rrx^8 - x^{10} \ .\\
hb \times r^{10} &= 11r^{10}x - 55r^8x^3 + 77r^6x^5 - 44r^4x^7 + 11rrx^9 - x^{11} \ .
\end{aligned}$$

< insertion from the top right of f 80v >

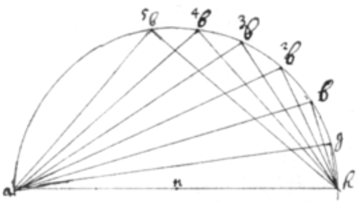
As on the other leafe excepting some signes here changed.

< text from f 80v resumes >

If $gh = x.\ bh = y.$Then

$$\begin{aligned}
&y = bh \ \text{. duplicatio anguli hag}\\
&yy - xx = x \times h^2b \ \text{. triplicatio anguli hag .}\\
&y^3 - 2xxy = xx \times h^3b \ \text{. quadruplicatio.}\\
&y^4 - 3xxyy + x^4 = x^3 \times {}^4bh \ \text{. quint}^{o}\\
&y^5 - 4xxy^3 + 3x^4y = x^4 \times h^5b \ \text{. sext}^{o}\\
&y^6 - 5xxy^4 + 6x^4yy - x^6 = x^5 \times hb \ \text{. sept}^{o}\\
&y^7 - 6xxy^5 + 10x^4y^3 - 4x^6y = x^6 \times hb \ \text{. oct}^{o}\\
&y^8 - 7xxy^6 + 15x^4y^4 - 10x^6yy - x^8 = x^7 \times hb \ \text{. nonc}\\
&y^9 - 8xxy^7 + 21x^4y^5 - 20x^6y^3 + 5x^8y = x^8 \times hb \ \text{. dec}\\
&y^{10} - 9xxy^8 + 28x^4y^6 - 35x^6y^4 + 15x^8y^2 - x^{10} = x^9 \times hb \ \text{. und,}\\
&y^{11} - 10xxy^9 + 36x^4y^7 - 56x^6y^5 + 35x^8y^3 - 6x^{10}y = x^{10} \times hb \ \text{. duod}
\end{aligned}$$

<81r>

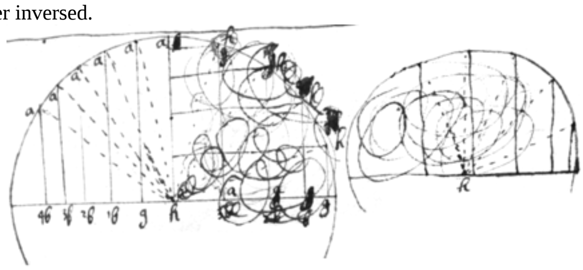


Of Angular sections

Suppose $ab = q.\ \frac{ah}{2} = r.$ & $ag = x.$ & that the arches hg, gb, bb are equall. By the following Equations an angle bah may bee divided into any number of partes.

$$\begin{aligned}
&x = q \ \text{. unisectio.}\\
&x^2 - 2rr = rq \ \text{. bisectio.}\\
&x^3 - 3rrx = rrq \ \text{. trisectio.}\\
&x^4 - 4rrxx + 2r^4 = r^3q \ \text{. quadrisectio.}\\
&x^5 - 5rrx^3 + 5r^4x = r^4q \ \text{. quintusectio.}\\
&x^6 - 6rrx^4 + 9r^4xx - 2r^6 = r^5q \ \text{. sextusectio.}\\
&x^7 - 7rrx^5 + 14r^4x^3 - 7r^6x = r^6q \ \text{. septusectio.}\\
&x^8 - 8rrx^6 + 20r^4x^4 - 16r^6x^2 + 2r^8 = r^7q \ .\\
&x^9 - 9rrx^7 + 27r^4x^5 - 30r^6x^3 + 9r^8x = r^8q \ .\\
&x^{10} - 10rrx^8 + 35r^4x^6 - 50r^6x^4 + 25r^8x^2 - 2r^{10} = r^9q \ .\\
&x^{11} - 11rrx^9 + 44r^4x^7 - 77r^6x^5 + 55r^8x^3 - 11r^{10}x = r^{10}q \ .\\
&x^{12} - 12rrx^{10} + 54r^4x^8 - 112r^6x^6 + 105r^8x^4 - 36r^{10}x^2 + 2r^{12} = r^{11}q \ .\\
&x^{13} - 13rrx^{11} + 65r^4x^9 - 156r^6x^7 + 182r^8x^5 - 91r^{10}x^3 + 13r^{12}x = r^{12}q \ .\\
&x^{14} - 14rrx^{12} + 77r^4x^{10} - 210r^6x^8 + 294r^8x^6 - 196r^{10}x^4 + 49r^{12}x^2 - \ \&c\\
&x^{15} - 15rrx^{13} + 90r^4x^{11} - 275r^6x^9 + 450r^8x^7 - 318r^{10}x^5 + 140r^{12}x^3 - \ \&c\\
&x^{16} - 16rrx^{14} + 104r^4x^{12} - 352r^6x^{10} + 660r^8x^8 - 672r^{10}x^6 + 336r^{12}x^4 - \ \&c\\
&x^{17} - 17rrx^{15} + 119r^4x^{13} - 442r^6x^{11} + 935r^8x^9 - 1122r^{10}x^7 + 714r^{12}x^5 - \ \&c\\
&x^{18} - 18rrx^{16} + 135r^4x^{14} - 546r^6x^{12} + 1287r^8x^{10} - 1782r^{10}x^8 + 1386r^{12}x^6 - \ \&c\\
&x^{19} - 19rrx^{17} + 152r^4x^{15} - 665r^6x^{13} + 1729r^8x^{11} - 2717r^{10}x^9 + 2508r^{12}x^7 - \ \&c\\
&x^{20} - 20rrx^{18} + 170r^4x^{16} - 800r^6x^{14} + 2275r^8x^{12} - 4604r^{10}x^{10} + 4290r^{12}x^8 - \ \&c
\end{aligned}$$

This scheame is the former inversed.



<81v>

Suppose the perifery bgh to bee a & the whole perifery to bee p. The line bh subtends these arches. a. $p - a.\ p + a.\ 2p - a.\ 2p + a.\ 3p - a.\ 3p + a.\ 4p - a.\ 4p + a.\ 5p - a.\ 5p + a.\ 6p - a.\ 6p + a.$ &c: All which are bisected, trisected, quadrisected, quintusected &c after same manner. As for example

The rootes of the equation $h^2b \times rr = 3rrx - x^3$. are 3. The first whereof subtends the arches $\frac{a}{3}.\ \frac{3p-a}{3}.\ \frac{3p+a}{3}.\ \frac{6p-a}{3}.\ \frac{6p+a}{3}.\ \frac{9p-a}{3}.\ \frac{9p+a}{3}$ &c. The second subtends the arches $\frac{p-a}{3}.\ \frac{2p+a}{3}.\ \frac{4p-a}{3}.\ \frac{5p+a}{3}.\ \frac{7p-a}{3}.$ &c. The 3d $\frac{p+a}{3}.\ \frac{2p-a}{3}.\ \frac{4p+a}{3}.\ \frac{5p-a}{3}.\ \frac{7p+a}{3}$ &c.

Soe the rootes of the equation ~ $hb \times r^4 = 5r^4x - 5rrx^3 + x^5$, doe the first subtend the arches $\frac{a}{5}.\ \frac{5p-a}{5}.\ \frac{5p+a}{5}$ &c: the 2d $\frac{p-a}{5}.\ \frac{4p+a}{5}.\ \frac{6p-a}{5}$. the 3d $\frac{p+a}{5}.\ \frac{4p-a}{5}.\ \frac{6p+a}{5}$ &c. the 4th $\frac{2p-a}{5}.\ \frac{3p+a}{5}$. &c the 5t $\frac{2p+a}{5}.\ \frac{3p-a}{5}.\ \frac{7p+a}{5}$. &c.

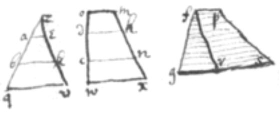
Hence may appeare the reason of the number of rootes in these equations & that the points of the circumference to which they are extended æquidistant. & by the lower scheme may bee known which rootes are affirmative & which negative.

The numerall cöefficients of the afforesaid equations may bee deduced from this progression (if $\angle : \angle :: 1 : n$.)
$1 \times \frac{-0+n \ \times \ -1+n}{1 \ \times \ 1-n} \times \frac{n-2 \ \times \ n-3}{2 \ \times \ 2-n} \times \frac{n-4 \ \times \ n-5}{3 \ \times \ 3-n} \times \frac{n-6 \ \times \ n-7}{4 \ \times \ 4-n} \times \frac{n-8 \ \times \ n-9}{5 \ \times \ 5-n} \times \frac{n-10 \ \times \ n-11}{6 \ \times \ 6-n}$ &c. As if $n = 10$. the progression
$1 \times -10 \times \frac{-7}{2} \times \frac{-10}{7} \times \frac{-1}{2} \times \frac{2}{25} \times 0$. And the coefficients $1\ .\ -10\ .\ +35\ .\ -50\ .\ +25\ .\ -2$.

1663 /4 January.

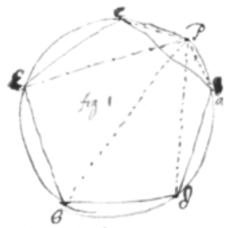
All the parallell lines which can be understoode to bee drawne uppon any superficies are equivalent to it, as all the lines drawne from (ao) to (co) may be used instead of the superficies $(aco.)$

If all the parallell lines drawne uppon any superficies be multiplied by another line they produce a Sollid like that which results from the superficies drawne into the same line as if either all the lines in the superficies (oac) or if the superficies oac be drawne into the line (b) they both produce the same sollid (d) whence All the parallell superficies which can bee understoode to bee in any sollid are equivalent to that Sollid. And If all the lines in any triangle, which are parallell to one of the sides, be squared there results a Pyramid. if those in a square, there results a cube. If those in a crookelined figure there results a sollid with 4 sides terminated & bended according to the fashion of the crookelined figure**{.}**



If each line in one superficies bee drawne into each correspondent line in another superficies as in aebk, & omnc if $ae \times dh$. $bk \times cn$. $qv \times wx$. &c. they produce a sollid whose opposite sides are fashioned by one of the superfic as Sollid fpsrg. where all the lines drawne from fr to ps are equall to all the correspondent lines drawne from ow to mx. & those drawne from fg to fr are equall to the correspondent lines drawne from qz to vz.

Theorema. 1

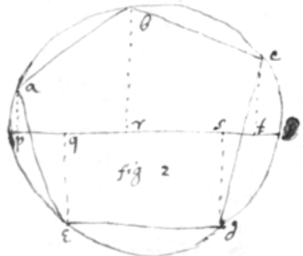


If in the Circle abcdeP there be inscribed any Poligon abcde with an odd number of sides, & from any point in the circumference P there bee drawne lines Pe, Pa, Pb, Pc, Pd to every corner of the Polygon: the summ of every other line is equall to the summ of the rest, $Pa + Pb + Pc = Pd + Pe$. & soe are their cubes $Pa^3 + Pb^3 + Pc^3 = Pd^3 + Pe^3$. unless the figure be a Trigon

Theorema 2

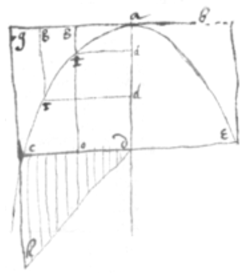
If from the points of the Polygon then bee drawne perpendicular ap, br, ct, ds, eq to any Diameter pt: the summe of the Perpendiculars on one side the Diameter is **{**equall**}{**equal**}** to their summe on the other $ap + br + ct = eq + ds$. & soe is the summe of their cubes (unlesse when the figure is a Trigon), $ap^3 + br^3 + ct^3 = eq^3 + ds^3$. & of theire square cubes (except when the figure is a Trigon or Pentagon. &c.

Theorema 3



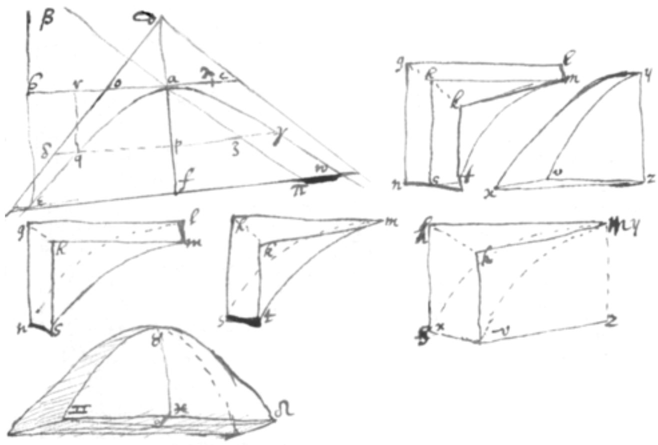
If the 2 circles (fig 1 & 2) be equall with like Poligo**{illeg}{**ns**}** inscribed, & Pa in fig 1 be assumed double to pa in fig 2. then are all the other corresponding lines in fig 1 double to those in fig 2 viz $Pb = 2rb$, $Pc = 2tc$, $Pd = 2sd$, $Pe = 2qe$.

To square the Parabola



In the Parabola cae suppose the Parameter $ab = r$. $ad = y$. $dc = x$. & $ry = xx$ or $\frac{xx}{r} = y$. Now suppose the lines called x doe increase in arithmeticall proportion all the x's taken together make the superficies dch which is halfe a square let every line drawne from cd to hd be square & they produce a Pyramid equall to every $xx = \frac{x^3}{3}$. which if divided by r there remaines $\frac{x^3}{3r} = = \frac{yx}{3}$ equall to every $\frac{xx}{r}$ equall to every (y) or all the lines drawne from ag to accc equall to the superficies ag c equall to a 3d parte of the superficies adcg & the superficie $acd = \frac{2yx}{3}$.

Otherwise. suppose $ce = b$. $co = x$. $to = y$. & $ry = bx - xx$ the lines x increasing in arithmeticall proportion every x is equall to 4 times the superficies $cdh = \frac{bb}{2}$ which drawne into b produceth the sollid $\frac{b^3}{2}$ but if every x be squared they produce a pyramid equall to $\frac{b^3}{3}$. wherefore every $bx - xx = \frac{b^3}{6}$ equall to every ry equall to the superficies adce drawne into r & $\frac{b^3}{6r} =$ to cade as before.



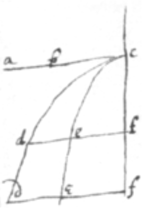
* $p\delta = pd = \frac{q+y}{5}$ & $\frac{qy+yy}{5} = xx$ ⊛

To Square the Hyperbola

In the Hyperbola eqaw. suppose $ef = a$. $fa = b$. $ap = rq = y$ $\{a\lambda = d\}$ $pq = ar = x$. $ad = q = 5oa = 5ac = 5$ $r =$. & $da + ar : ar :: ar : rq$. $xx = dy + yx$. In which equation Every x taken together is equall to the triangle aβb equall to $\frac{aa}{2}$ & every xx taken together is a pyramid $= \frac{a^3}{3}$. Every y taken together is equall to the superficies $eba = mkt$ If then $gh = lm = ns = a\lambda = d$. every dy is equall to the solid nglmhs. If the angle mhk is a right one & if $mh = gl = ba = ef = a$ that is if the triangle $mhk = ab\beta$. every y x will be equall to the sollid mhstk Joyne these two sollids together as in $lmtng = \frac{a^3}{3}$. *

⊛ Againe Suppose every x taken together to be equall to the superficies aef , the line $q\gamma$ squared is 4xx every 4xx composeth a Sollid like (♊♉♌) an eighth parte whereof (which is equall to every $\frac{xx}{2}$) being like ♊♉♓$o = xyzv$; xv will be equall to ♊ ♓ $= ef = a = st = xz = o$♓ $= hm$. & $vz =$ ♊♓ $= a\beta km$. whence the convexe superficies xyv of the figure xyvz will fitly joyne with the concave superficies mst of the figure shmkt . If every x is equall to the superficies aef , every y shall be equall to the triangle $af\pi = \frac{bb}{2}$. every $yy = \frac{bbb}{3}$ every $qy = \frac{qbb}{2}$ & therefor the Sollid $yxzv = \frac{b^3}{30} + \frac{qbb}{20} = \frac{2b^3+3qbb}{60}$. Joyne the Sollid shmkt to yxvz & there resulteth $shmztk = = \frac{aab}{2}$ from which againe substract $xvzy = \frac{2b^3+3qbb}{60}$ & there remaines the sollid $mhstk = \frac{30aab-2b^3-3qbb}{60}$ which substract from the sollid $ntmlg = \frac{a^3}{3}$ & there remaines $nglmhs = \frac{20a^3+2b^3+3qbb-30aab}{60}$ which being divided by $\theta = \sqrt{\frac{5rr}{4}}$. there remaines $\frac{40a^3+4b^3+6qbb-60aab}{r\sqrt{5}}$ = to the superficies abe

The squareing of severall crooked lines of the Seacond kind.

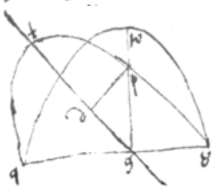


In any two crooked lines I call the Parameter or right side of the greater. (r). but of the lesse (s) Transverse side (q). the right axis as cf (x) or $ef = v$. y Transverse axis as fe y, or fd z.

Suppose in the Parab: ddc: $ac = r$. & in eec: $bc = s$ $rx = zz = df^2$. $sx = yy = fe^2$. $\sqrt{rx} - \sqrt{sx} = de = = p$. $rx = sx + pp + 2p\sqrt{sx}$.
$rx - sx - pp = 2p\sqrt{sx}$ $rrxx - 2rsxx + ssxx - 2pprx - 2ppsx + p^4 = 4ppsx$. Or $p^4 - 2rxpp - 6sxpp + rrxx - 2rsxx + ssxx = 0$. if $p = y$.
$xx = \frac{\begin{smallmatrix}+2ryy \\ +6syy\end{smallmatrix}\; x - y^4}{rr-2rs+ss}$. make $cf = a$. $fd = b$. $fe = c$. $ceeff = \frac{2ac}{3}$. & $cddeff = \frac{2ab}{3}$ therefore $\frac{2ab-2ac}{3} = cddee$ the square of the crooked line cdd (when the line cee is supposed too close with the line cf) whose nature is exprest by the foregoing Equation.

2 $lk = b$. $li = x$. $qi = y$. $in = z$. $+bx - xx = ry$ $bx - xx = sz$. $\frac{-xx+bx}{r} = y$. $\frac{-xx+bx}{s} = z$ $qn = \frac{-sxxs+bsx+rxxr-brx}{rs} = v = y$ or, r
$rx - sxx + bsx - brx - rsy = 0$. Or $xx - bx - \frac{rsy}{r-s} = 0$

3 $tg = x$. $dg = z$. $gp = y$. $rx - rz = dp^2$. $rx - rz + zz = y^2$. $zz = rz - rx + yy$



$r : a :: rx - rz : zz$. $zz = arx - arz$. $zz = -az + ax$. $z = -\frac{1}{2}a + \sqrt{\frac{1}{4}aa + ax}$. $ax - az = rz - rx + yy$ Or

$-yy + rx + ax = -\frac{1}{2}aa - \frac{1}{2}ar + \frac{a}{r}\sqrt{\frac{1}{4}aa + ax}$.

$$\begin{array}{rr} & 354 \\ 186 & 59 \\ \hline 31 & 944 \end{array}$$

$$\begin{array}{llllllll} y^4 & -\ 2rx & yy & +\ rrxx & + & \frac{1}{4}a^4 & + & 2aarx \\ & -\ 2ax & & +\ 2arxx & + & \frac{1}{2}a^3 & + & arrx \\ & -\ aa & & +\ aaxx & + & \frac{1}{4}aarr & + & a^3x \\ & -\ ar & & & & & & \end{array}$$

$\frac{1}{4}a^4 + a^3x + \frac{1}{4}rraa + rrax$

$$\begin{array}{llllll} y^4 & -\ 2rx & yy & +\ rrxx & & 0= \\ & -\ 2ax & & +\ 2arxx & & \\ & -\ aa & & +\ aaxx & & \\ & -\ ar & & +\ \frac{1}{2}a^3r & & . \\ & & & +\ 2aarr & & \end{array}$$

$$\frac{\begin{array}{llll} xx & +\ 2aar\ x & +\ y^4 & = 0 \\ & -\ 2ryy & -\ aayy & \\ & -\ 2ayy & -\ aryy & . \\ & & +\ \frac{1}{2}a^3r & \end{array}}{rr + 2ar + aa}$$

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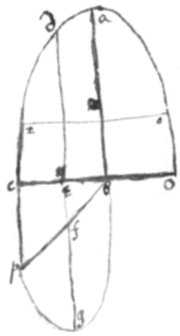
$$\begin{array}{l} 51\ )\ 358\ (\ 7,019608\ . \\ \underline{357} \qquad -58\ . \\ \underline{100} \\ 490 \\ \underline{459} \\ 310 \\ \underline{306} \\ 400 \end{array}
\qquad
\begin{array}{l} 51\ )\ 197\ (\ 3,862745 \\ \underline{153} \qquad +392 \\ 440 \\ \underline{408} \\ 320 \\ \underline{306} \\ 140 \\ \underline{102} \\ 380 \\ \underline{357} \\ 230 \\ \underline{204} \\ 26 \end{array}
\qquad
\begin{array}{l} \underline{93} \\ 53\ )\ 372\ (\ 7018868 \\ \underline{371} \qquad +682\ . \\ \underline{100} \\ 470 \\ \underline{424} \\ 460 \\ \underline{424} \\ 36 \\ \underline{318} \\ 42 \end{array}
\qquad
\begin{array}{l} 212\ )\ \underline{819}\ (\ 3863207 \\ 636 \qquad 70 \\ 1830 \\ \underline{1696} \\ 1340 \\ \underline{1272} \\ 680 \\ 636 \\ 44 \\ \underline{424} \\ 160 \end{array}$$

$\dot{0} \cdot 0 \cdot 9 \cdot 7 \cdot 5 \cdot 4 \cdot 5 \cdot \dot{5} \cdot \dot{5} \cdot \dot{9} \cdot \dot{7} \cdot \dot{7} \cdot \dot{7} \cdot \quad \cdot \quad \cdot\ 3 \cdot \quad \cdot \quad \cdot$

$\cdot \quad \cdot \quad \cdot\ \dot{0} \cdot \dot{0} \cdot \dot{0} \cdot \dot{2} \cdot \dot{0} \cdot \dot{0} \cdot \dot{2} \cdot \dot{0} \cdot \dot{0} \cdot$

<87r>

4 In the Parabola $cb = a$. $be = x$. $2aa - 2ax \; -aa + 2ax - xx = ed^2 \;\; aa - xx = ed^2 = yy$. $\frac{aa-xx}{r} = y$. $cp = cb \;\; eb \times df = fg \;\{ \times c \}$ = zc. $\frac{aax-x^3}{r} + xx = zc \;\; x^3 - rxx - aax + rcz = 0$. Since all $eb \times df = \frac{1}{8}$ all $co^2 = \frac{1}{4} ab \times ab \times r$. $ab = b$. all $eb^2 = \frac{a^3}{3}$ therefore bgpf $\frac{1}{4} bbr + \frac{a^3}{3}$.



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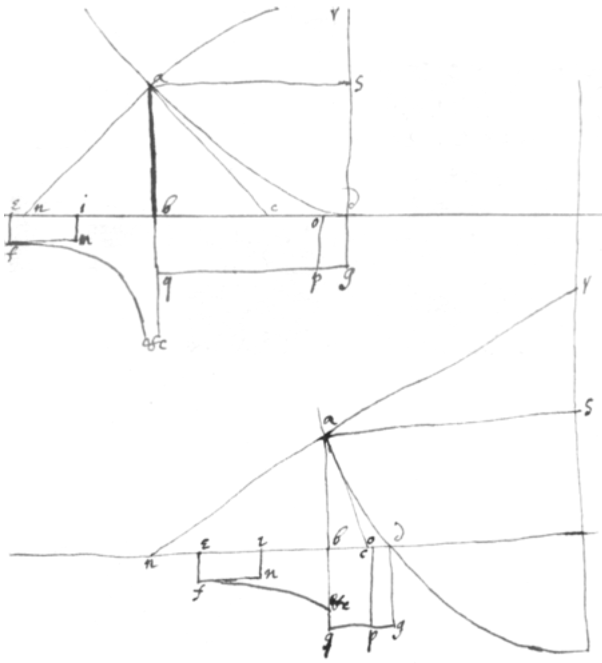
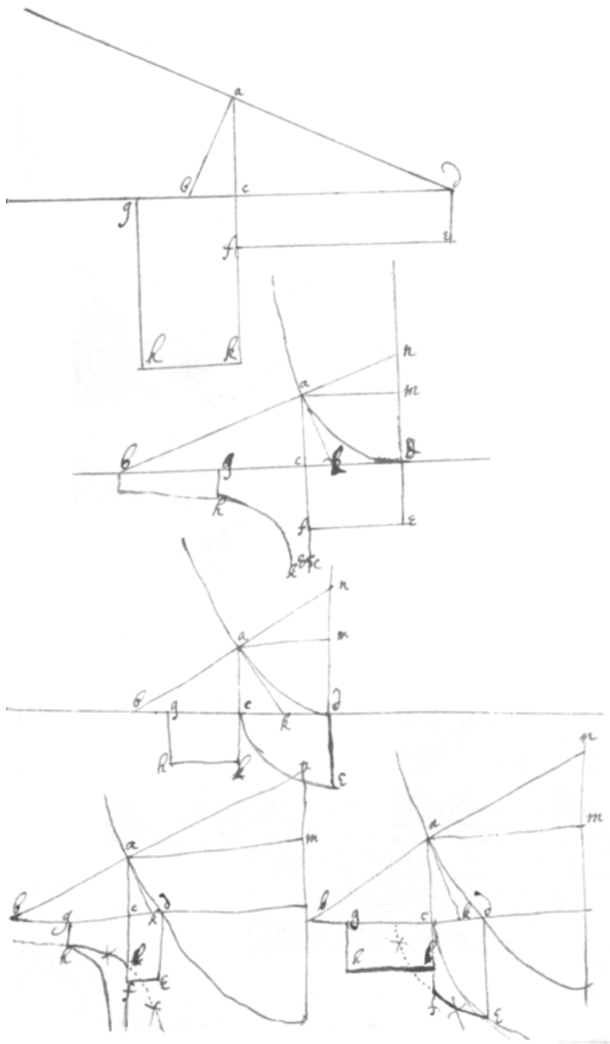


<88r>

$ab = b\,e = y$. $bd = x$. $bq = dg = b$. $nb = c$. $\frac{ybx}{yc} = \frac{bx}{c} = ef$ Then shall bq&c: be the axis of gravity in feb&c & bqgd.

<88v>



<89r>

In the 1st figure.

$gc : cd :: cfed : ckhg = \frac{cd \times cfed}{gc}$. $ac = gc$. $x : z :: za : xy$. $\frac{zza}{x} = ckhg$. or $\frac{xxy}{z} = cdef$. Suppose $cd : ca :: ac : bc ::$ the swiftnesse of de : to the swiftnesse of gh . $de \times$ its swiftnes : $gh \times$ its swiftness $:: gc : cd$. $de \times cd : gh \times ca :: de \times ac : gh \times bc :: gc \times cd$.

Fig 2d. 3d.

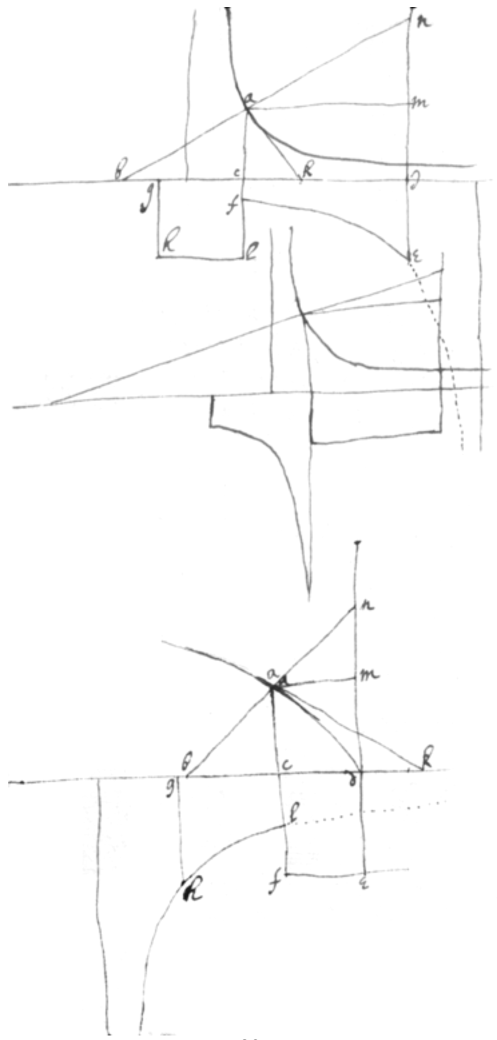
$c\,\theta : ca :: ac : bc :: nm : am ::$ swiftness θe : swiftnesse gh . $de \times$its swiftnesse : $gh \times$ its swiftness $:: c\,k \times de : gh \times ac :: de \times ac : gh \times bc$ $de \times k : gh \times ac :: de \times ac : gh \times bc$ $:: de \times nm : gh \times am :: gc : cd$. $de \times ck \times cd = gh \times ac \times gc$. $de \times cd = gh \times bc$. $de \times nm = gh \times gc$.

Fig 4

$ck : ca ::$motion of the point a from c : motion of the point a from m $:: ck : ca ::$increasing of $ac = gc$:increasing of cd ::motion of gh : motion of de. &c as before.

These are to find such figures cghk, cfed, as doe equiponderate in respect of the axis acfk.

<89v>



<90r>

Reasonings concerning chance.

If

1 If p is the number of chances by one of which I may gaine a, & q those by one of which I may gaine b, & r those by one of which I may gaine c; soe that those chances are all equall & one of them must necessarily happen: My hopes or chance is worth $\frac{pa+qb+rc}{p+q+r} = A$. The same is true if p, q, r signify any proportion of chances for a, b, c.

2. If I bargaine for more than one chance (viz: that after I have taken the gaines by my first chance, from the stake $a+b+c$; I will venter another chance at the remaining stake &c) my second lott is worth $A\frac{-AA}{a+b+c} = A - \frac{AA}{a+b+c} = B$. My third lot is worth $A\frac{-AA-AB}{a+b+c} = C$. My Fourth lot is worth $A\frac{-AA-AB-AC}{a+b+c} = D$. My Fift lot is worth $A\frac{-AA-AB-AC-AD}{a+b+c} = E$. My sixt lot is worth $A - A \times \frac{A+B+C+D+E}{a+b+c}$. &c

As if 6 men $(1\ .\ 2\ .\ 3\ .\ 4\ .\ 5\ .\ 6\ .\ )$ cast a die soe that he gaines a who throws a cise first: since there is but one chance to gaine a & 5 to gaine nothing at each cast, I make $b = 0 = c = r.\ p = 1$ & $q = 5$. Therefore by the <90v> The first mans lot is worth $\frac{a}{6}$ The seconds is worth $\frac{a}{6} - \frac{a}{36} = \frac{5a}{36}$. The thirds is worth $\frac{5a}{36} - \frac{5a}{216} = \frac{25a}{216}$. The fourths is $\frac{25a}{216} - \frac{25a}{1296} = \frac{125a}{1296}$ The fifts lot is worth $\frac{125a}{1296} - \frac{25a}{7776} = \frac{625a}{7776}$. The Sixts lot is $\frac{625a}{7776} - \frac{625a}{46656} = \frac{3125a}{46656}$. &c. Soe that their lots are as $7776 : 6480 : 5400 : 4\ 5\,00 : 3\ 950 : 3\ 1\ 25$.

Soe that if I cast a die two or more times tis $\dot{1}$ to 5 that I cast a cise at the first cast & 11 to 25 that I throw it at two casts, & 91 to 125 that I cast it at thrice, & 671 to 625 that I cast it once in 4 trialls, & 4651 to 3125 that I cast it once in 5 times. &c

3. If I bargaine to cast severall sorts of lots successively at the same stake the valor of each lot is thus found viz: The first prop: gives the valor of the first lot; which valor being destructed from the stake, the remainder is the stake of the 2d lot which therefore may bee also found by the first prop: &c.

As if I gaine a by throwing 12 at the first cast, or 11 at the 2d or 10 at the 3d &c with two dice. Since at the first cast there is but one chance for a (viz 12) & 35 for nothing Therefore its valor is $\frac{a}{36}$ (by Prop 1). & the stake for the 2d cast is $a - \frac{a}{36} = \frac{35a}{36}$. Now since there are two chances for it (viz: ⚅⚄ & ⚄⚅) & 34 for 0 at the 2d cast therefore its valor is $\frac{2\times35a}{36\times36} = \frac{35a}{648}$. as the stake for the 3d lot is $\frac{595a}{648}$ for which there are 3 chances (viz ⚄⚄, ⚅⚃, ⚃⚅) & 33 for nothing Therefore its valor is $\frac{595a}{7776}$.

<91r>

4 If I bargaine with one or two more to cast lots in order untill one of us by an assigned lott shall win the stake a: Since the chances may succede infinitly I onely consider the first revolution of them The valor of each mans whole expectation being in such proportion one to another as the valors of their lots in one revolution. & the valors of each mans first lot being to the valor of his whole expectation as the summe of the valors of their first lots to the stake a.

As if I contend with another that who first throws 12 with 2 dice shall have a, I haveing the dice. My first lot is worth $\frac{a}{36}$ (by prop 1), The 2d his first lot is worth $\frac{35a}{36\times36}$. And $\frac{a}{36} : \frac{35a}{36\times36} :: 36 : 35 ::$ my expectation : to his. for the two first lots make one revolution because I have the same lot If I throw a 2d time that I had at the first. Therefore $\left(36 + 35 = 71 : a :: 36 : \frac{36a}{71}\right)$ $\frac{36a}{71}$ is my interest in the stake.

If our bargaine bee soe that there is some lott at the beginning of our play which returnes not in the after revolutions, detract the valor of those irregular lotts from the stake & the rest shall bee the stake of the lots which follow & revolve successively. As if I contend with another that who first casts 11 must have a , onely I have **{the}** first cast for 12. My first lot is worth $\frac{a}{36}$. & the stake for our after throws is $\frac{35a}{36}$. his firts lot being $\frac{35a}{648}$. & my next lot $\frac{595a}{11664}$. soe that his share in the stake $\frac{35a}{36}$ is to mine as $\frac{35a}{648} : \frac{595a}{11664} :: 18 : 17$. Soe that my share in it is $\frac{17a}{36}$. To which adding the valor of my first lot viz: $\frac{a}{36}$, the summe is $\frac{18a}{36} = \frac{a}{2}$, my interest in the stake a at the begining.

5 If the Proportion of the chances for any stake bee irrationall the interest in the stake may bee found after the same manner. As if the Radij ab , ac , divide the horizontall circle bcd into two points <91v> abec & abdc in such proportion as 2 to $\sqrt{5}$. And if a ball falling perpendicularly upon the center a doth tumble into the portion abec I winn (a): but if into the other portion, I win b . my hopes is worth $\frac{2a+b\sqrt{5}}{2+\sqrt{5}}$.

Soe if a die bee not a Regular body but a Parallelipipedon or otherwise unequall sided, it may bee found how much one cast is more easily gotten then another.

☞ 6 Soe that the facility of the chances & the stake belonging to each chance being knowne the worth of the lott may bee ever found by the precedent precepts. And if they bee not both immediatly known they must bee sought before the valor of the lott can bee found.

As if I want two games at Irish & my adversary three to win a , & I would know my interest in the stake (a.) my first lot can gaine me nothing but the advantage of another lot, & therefore to know its vallue I must first find the value of that other lot &c. First therefore if wee each wanted one lot to win a our interest in it would bee equall viz my lot worth $\frac{a}{2}$. Secondly If I want one game & my adversary two, & I gaine the next game then I gaine a but if I loose it I onely gaine an equall lot for a at the next game which is worth $\frac{1}{2}$a, Therefore my interest in the stake is $\frac{a+\frac{1}{2}a}{2} = \frac{3a}{4}$. Thirdly If I want one game & my adversary three & I gaine the next game I get a; but if I loose it, then I want one game & my adversary but two, that is I get $\frac{3a}{4}$: Therefore (there being one chance for a & one for $\frac{3a}{4}$) my interest in the stake is $\frac{a+\frac{3a}{4}}{2} = \frac{7a}{8}$. Fourthly If I want 2 games & my adversary 3; & I win I get $\frac{7a}{8}$. but if I loose I get $\frac{1}{2}$a for our chances <92r> will then bee equall; Therefore my interest in the stake is $\frac{11a}{16}$. Soe if I want 1 games & my adversary 4 my interest in a is $\frac{15a}{16}$. If I want two and hee 4 , it is $\frac{13a}{16}$. If I want 3 and hee 4 it is $\frac{21a}{32}$. If I 1 and hee 5 , it is: $\frac{31a}{32}$. If I 2 and hee 5 it is $\frac{57a}{64}$. If I 3 and hee 5 it is $\frac{99}{128}$a. If I 4 and hee 5 , it is: $\frac{163}{256}$a. (The like may bee done if 3 or more play together. (as if one wants one game, another 3 a third 4: Their lots are as 616 : 82 : 31 . &c.) As also if their lots bee of divers sorts.)

By this meanes also some of the precedent questions may bee resolved. as if I have two throws for a cise to win a , with one die; If I have missed my first lot already, I have at my second cast five chances for nothing. & one for a . therefore that cast is worth $\frac{a}{6}$. Soe that in my first cast I had five chances for $\frac{1}{6}$a & one for a , which therefore (with my 2d cast) is worth $\frac{11}{36}$a. That is tis 11 to 25 that I cast a cise once in two throws. as before

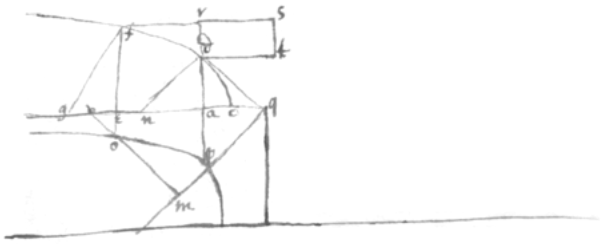
By this meanes also my lot may bee known if I am to draw 4 cards of severall sorts out of 40 cards 1 0 of each sort.

Or if out of two white & 3 black stones I am blindfold to chose a white & a black one.

<92v>

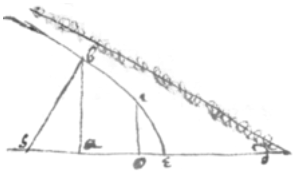
⌖ Equation

An equation given; if both x , y , have divers dimensions, try if the roote of one of them may be extracted: & If a quantity wherein y is not is divided by x in the line equall to x . that crooked cannot be squared.



<93r>

The line cdf is a Parab. 4 ac =2 ad = r =2 na =2 ge. ce = x. ef = y. rx = yy.
$ge : ef :: \frac{1}{2}r : \sqrt{rx} :: eo : ap$. $eo = z$. $ap = a$. $\frac{1}{2}ra = z\sqrt{rx}$. $\frac{1}{4}rraa = rzzx$. $\frac{raa}{4} = zzx$. or supposeing ea =y. $x = y + \frac{1}{4}r$ & $\frac{raa}{4} = zzy + \frac{1}{4}zzr$. which shews the nature of the crooked line po. now if dt = ap . then drst = eoap . for supposeing eo moves uniformely from ap , rs moves from dt with motion decreaseing in the proportion that the line eo doth shorten. Suppos $aq = ap = \frac{r}{2} = a$ & $eq = y$. $x = y - \frac{1}{4}r$. then $\frac{r^3}{16} = zzy - \frac{1}{4}zzr$. suppose $z = a + y$. then $r^3 = aax + 2ayx + yyx$. Or
$aax + 2ayx + yyx + \frac{1}{4}aar + \frac{1}{8}ayr + \frac{1}{4}yyr = \frac{r^3}{16}$. Or $aax + 2ayx + yyx - \frac{1}{4}aar - \frac{1}{8}ayr - \frac{1}{4}yyr = \frac{r^3}{16}$. Or suppose $z = y - a$. then $\frac{r^3}{16} = aax - 2ayx + yyx$. Or $aax - 2ayx + yyx + \frac{1}{4}aar - \frac{1}{8}ayr + \frac{1}{4}yyr = \frac{r^3}{16}$ Or $aax - 2ayx + yyx - \frac{1}{4}aar + \frac{1}{8}ayr - \frac{1}{4}yyr = \frac{r^3}{16}$. Or, if $x = a - r$. $\frac{r^3}{16} = zza - zz\,x$. &
$a^3 + 2\,aay + ayy - aax - 2ayx - y\,yx = \frac{r^3}{16}$. Or $a^3 - 2aay + aay - aax + 2ayx - yyx = \frac{r^3}{16}$. $o\mathfrak{b}^2 = 2z^2$. $mp = \mathfrak{z}$. $mq = a + \mathfrak{z} = m\mathfrak{b}$. $mo = y = a + \mathfrak{z} - \sqrt{2z^2}$
$pq\ (= a) : aq\ (= \frac{1}{2}r) :: \mathfrak{b}q\ (= x + z) : m\mathfrak{b} = y + \sqrt{2zz}$ $ay + a\sqrt{2zz} = \frac{1}{2}rx + \frac{1}{2}rz$. $\frac{2ay + 2z\sqrt{2aa}}{r} - \frac{1}{2}z = x = \frac{r^3 + 4zzr}{16zz}$.
$32zzay + 32z^3a\sqrt{2} - 8z^3r - r^4 - 4zzrr = 0$ $mv = \xi = a + \mathfrak{z} - z\sqrt{2}$. $\frac{\xi - a - \mathfrak{z}}{\sqrt{2}} = z$. $\frac{\xi^2 - 2\xi a - 2\xi\mathfrak{z} + aa + 2a\mathfrak{z} + \mathfrak{z}^2}{2} = zz$. $\frac{\xi^3 - 3\xi^2 a - 3\xi^2\mathfrak{z} + 3aa\xi + 6a\mathfrak{z}\xi + 3\mathfrak{z}\mathfrak{z}\xi - a^3 - 3aa\mathfrak{z} - 3a\mathfrak{z}\mathfrak{z} - \mathfrak{z}^3}{\sqrt{8}} = z^3$



$32\sqrt{2aa} - 8r = c$ $\begin{array}{c} cz^3 + 32a\xi zz - r^4 \\ -4rr \end{array} = 0$ <93v> <94r>

$$\begin{array}{cccccccccl}
 & c\xi^3 & - & 3ca\xi^2 & + & 3caa\xi & - & ca\mathfrak{z} & & \\
+ & 32a\xi^3 & - & 3c\mathfrak{z}\xi^2 & + & 6ca\mathfrak{z}\xi & - & 3ca^2\mathfrak{z} & & 0= \\
 & & & & + & 3c\mathfrak{z}\mathfrak{z}\xi & - & 3ca\mathfrak{z}^2 & & \\
 & & - & 64aa\xi^2 & + & 32a^3\xi & - & c\mathfrak{z}^3 & & \text{Or} \\
 & & - & 64a\mathfrak{z}\xi^2 & + & 64aa\mathfrak{z}\xi & - & 4rraa & & \\
 & & - & 4rr\xi^2 & + & 32a\mathfrak{z}\mathfrak{z}\xi & - & 8rra\mathfrak{z} & & \\
 & & & & + & 8rra\xi & - & 4rr\mathfrak{z}\mathfrak{z} & & \\
 & & & & + & 8rr\mathfrak{z}\xi & - & r^4 & &
\end{array}$$

$$\begin{array}{cccccccl}
\xi^3 & - & \frac{d}{e}\mathfrak{z}\xi^2 & + & gg\xi & - & f\mathfrak{z} & \\
 & - & d & + & h\mathfrak{z} & - & \mathfrak{z}m^2 & 0= \\
 & & & + & \frac{i}{g}\mathfrak{z}\mathfrak{z} & - & n\mathfrak{z}^2 & . \\
 & & & & & - & \frac{o}{n}\mathfrak{z}^3 &
\end{array}$$

$$x^3 - 2vx^2 \begin{array}{l} +\ vv \\ -\ ss\,x \\ +\ aa \end{array} + a^3 = 0$$

Let $ed = a$. $ae = x$ $ab = y$ $se = v$. $sb = s$.

2 1 0 − 1

$\mathrm{aax} = \mathrm{yyx} + \mathrm{yya}\,.\ \mathrm{y}^2 = \mathrm{ss} - \mathrm{vv} + 2\mathrm{vx} - \mathrm{xx}\ \ \mathrm{aax} = \mathrm{ssx} - \mathrm{vvx} + 2\mathrm{vxx} - \mathrm{x}^3 + \mathrm{ssa} + 2\mathrm{avx}\ - \mathrm{axx} - \mathrm{avv}.\ \ \mathrm{x}^3 \begin{array}{l} -2\mathrm{v} \\ +\mathrm{a} \end{array} \mathrm{x}^2 \begin{array}{l} -\mathrm{ss} \\ +\mathrm{vv} \\ -2\mathrm{av} \\ +\mathrm{aa} \end{array} \mathrm{x} \begin{array}{l} -\mathrm{ssa} \\ +\mathrm{avv} \end{array} = 0\,.$

$\mathrm{x}^2 - 2\mathrm{ex} + \mathrm{ee} \quad \times \quad \mathrm{x} + \mathrm{f}$

$\mathrm{x}^3 \begin{array}{l} -2\mathrm{e} \\ +\mathrm{f} \end{array} \mathrm{x}^2 \begin{array}{l} +\mathrm{eefx} \\ +2\mathrm{eex} \end{array} + \mathrm{eef} = 0 \qquad -2\mathrm{v} + \mathrm{a} = \mathrm{f} - 2\mathrm{e}\,.\ \mathrm{f} = \mathrm{a} + 2\mathrm{e} - 2\mathrm{v}.\ \mathrm{eef} = \mathrm{avv} - \mathrm{ass}\ \ \frac{-\mathrm{eea} - 2\mathrm{e}^3 + 2\mathrm{eev} + \mathrm{avv}}{\mathrm{a}} = \mathrm{ss}.$

$$\begin{array}{llllll} \mathrm{vvx} & -2\mathrm{vvx} & +\mathrm{x}^3 & & & \\ \mathrm{vva} & -2\mathrm{vax} & +\mathrm{ax}^2 & = & \begin{array}{l} \mathrm{a} \\ \mathrm{x} \end{array} \Bigg\} & \frac{2\mathrm{eev} + \mathrm{aav} - 2\mathrm{e}^3 - \mathrm{eea}}{\mathrm{a}} \\ & & +\mathrm{aax} & & & \end{array}$$

<94v>

To square those lines in which is y onely

If y is in but one terme onely of the Equation (as $\mathrm{xx} = \mathrm{ay}$. or, $\mathrm{a}^3 = \mathrm{xxy}$) resolve the Eq: into the proport $\mathrm{y} : \mathrm{a}$ (as $\mathrm{y} : \mathrm{a} :: \mathrm{xx} : \mathrm{aa}$. or, $\mathrm{y} : \mathrm{a} :: \mathrm{aa} : \mathrm{xx}$.) If the line hath Assymptotes
$\mathrm{x}^3 = \mathrm{aay}.\ \mathrm{v} = \frac{3\mathrm{x}^5}{\mathrm{a}^4} + \mathrm{x}.$

<95r>

$$\begin{array}{llll} & & -2\mathrm{v}\dot{\mathrm{x}}^2 & +\dot{\mathrm{x}}^3 \\ \mathrm{a} & \mathrm{vv} & -2\mathrm{vax} & +\mathrm{a}\dot{\mathrm{x}}^2 \\ \mathrm{x} & & -2\mathrm{ve}\dot{\mathrm{e}} & +\mathrm{a}\dot{\mathrm{a}}\mathrm{x} \\ -\mathrm{a} & & -\frac{2\mathrm{x}}{\mathrm{a}}\mathrm{eev} & +2\dot{\mathrm{e}}^3 \\ -\mathrm{x} & & & +\mathrm{e}\dot{\mathrm{e}}\mathrm{a} \\ & & & +\frac{2\mathrm{x}}{\mathrm{a}}\dot{\mathrm{e}}^3 \\ & & & +\mathrm{xe}\dot{\mathrm{e}} \end{array} = 0$$

$\mathrm{v} = \dfrac{4\mathrm{x}^3 + 2\mathrm{axx} + \mathrm{a}\dot{\mathrm{a}}\mathrm{x} + 2\frac{\mathrm{x}^4}{\mathrm{a}}}{4\mathrm{xx} + 2\mathrm{ax} + 2\frac{\mathrm{x}^3}{\mathrm{a}}}$

$\mathrm{v} = \frac{4\mathrm{ax}^2 + 2\mathrm{aax} + \mathrm{a}^3 + 2\mathrm{x}^3}{4\mathrm{ax} + 2\mathrm{aa} + 2\mathrm{xx}}$ $\quad \frac{2\mathrm{a}^4\mathrm{x}}{\mathrm{x}+\mathrm{a}} : \frac{2\mathrm{a}^6}{4\mathrm{xx} + 8\mathrm{ax} + 4\mathrm{aa}} : 2\mathrm{aax} : \frac{2\mathrm{a}^6}{8\mathrm{x}^3 + 24\mathrm{ax} + 24\mathrm{aax} + 8\mathrm{a}^3} : 2\,\mathrm{x}^3 + 2\mathrm{ax}^2\ \ \mathrm{ad.}\ \frac{2\mathrm{a}^4\mathrm{x}^2 + 2\mathrm{a}^5\mathrm{x}}{8\mathrm{x}^3\ \ \&}\ \frac{\mathrm{a}^4\mathrm{x}}{4\mathrm{xx} + 8\mathrm{ax} + 4\mathrm{aa}}$

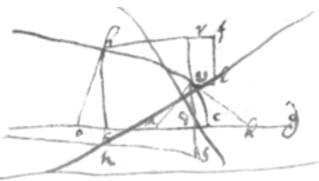
$\mathrm{sa} = \dfrac{\begin{array}{c} 4\mathrm{ax}^2 + 2\mathrm{x}^3 + 2\mathrm{aax} + \mathrm{a}^3 - \\ -4\mathrm{ax}^2 - 2\mathrm{x}^3 - 2\mathrm{aax} \end{array}}{4\mathrm{ax} + 2\mathrm{aa} + 2\mathrm{xx}}$

$\mathrm{sa} = \frac{\mathrm{a}^3}{2\mathrm{x}^2 + 4\mathrm{ax} + 2\mathrm{aa}}$

$2\mathrm{x}^3 + 4\mathrm{ax}^2 + 2\mathrm{aax} - \mathrm{a}^3 = 0\quad \frac{\sqrt{\mathrm{aax}}}{\mathrm{a}+\mathrm{x}} : \frac{\mathrm{a}^3}{2\mathrm{x}^2 + 4\mathrm{ax} + 2\mathrm{aa}} :: \mathrm{p} : \mathrm{z}\ \ \frac{\mathrm{z}^2\mathrm{aax}}{\mathrm{a}+\mathrm{x}} = \dfrac{\mathrm{a}^6\mathrm{pp}}{4\mathrm{x}^4 + 16\mathrm{ax}^3 + \begin{array}{c} 8\mathrm{aa} \\ +16\mathrm{aa} \end{array} \mathrm{xx} + 16\mathrm{a}^3\mathrm{x} + 4\mathrm{a}^4}$
$\mathrm{oe} = \mathrm{p}$

$\mathrm{a}^5\mathrm{pp} + \mathrm{a}^4\mathrm{xpp} = 4\mathrm{zzx}^5 + 16\mathrm{zzax}^4 + 24\mathrm{aazzx}^3 + 16\mathrm{zza}^3\mathrm{x}^2 + 4\mathrm{a}^4\mathrm{z}^2\mathrm{x}$ divided by $\mathrm{x} + \mathrm{a}$ it produceth. $4\mathrm{zzx}^4 + 12\mathrm{zzax}^3 + 12\mathrm{zzaax}^2 + 4\mathrm{zza}^3\mathrm{x} - \mathrm{a}^4\mathrm{pp} = 0$

<95v>



By the Squares of the simplest lines to square lines more compound. 1st those whein y.

find the valor of y. If the number of the termes in the denominator thereof be neither $1\ .\ 3\ .\ 6\ .\ 10\ .\ 15\ .\ 21\ .\ 28\ .$ &c. the line cannot be squared If it have but one terme tis squared by finding the square of each particular terme in the valor of y & then adding all those squares together. Example 1st. $3\mathrm{x}^4 + \mathrm{a}^4 = \mathrm{yaxx}$. & $\mathrm{y} = \frac{3\mathrm{x}^4 + \mathrm{a}^4}{\mathrm{axx}}$. Then makeing y equall to each particular terme. $\frac{3\mathrm{xx}}{\mathrm{a}} = \mathrm{y}$. $\frac{\mathrm{a}^3}{\mathrm{xx}} = \mathrm{y}$ or $3\mathrm{xx} = \mathrm{ay}$ whose square is $\frac{\mathrm{x}^3}{\mathrm{a}}$. & $\mathrm{a}^3 = \mathrm{xxy}$. whose square is $\frac{\mathrm{a}^3}{\mathrm{x}}$ Add these 2 squares together & they (viz: $\frac{\mathrm{x}^4 + \mathrm{a}^4}{\mathrm{ax}}$) are the square of the line $3\mathrm{x}^4 + \mathrm{a}^4 = \mathrm{ayxx}$. Againe $2\mathrm{a}^7 - 2\,\mathrm{b}\,\mathrm{x}^6 + \mathrm{x}^7 = \mathrm{a}^3\mathrm{x}^3\mathrm{y}$. Or $\mathrm{y} = \frac{2\mathrm{a}^7 - 2\mathrm{bx}^6 + \mathrm{x}^7}{\mathrm{a}^3\mathrm{x}^3}$. then disjoynting the valor of y. $\mathrm{y} = \frac{2\mathrm{a}^4}{\mathrm{x}^3}$. $\mathrm{y} = \frac{\mathrm{x}^4}{\mathrm{a}^3}$. $\mathrm{y} = \frac{2\mathrm{bx}^3}{\mathrm{aaa}}$ Or $\mathrm{x}^3\mathrm{y} = 2\mathrm{a}^4$, whose square is $\frac{\mathrm{a}^4}{\mathrm{xx}}$. $\mathrm{ya}^3 = \mathrm{x}^4$, whose square $\frac{\mathrm{x}^5}{5\mathrm{a}^3}$. $\mathrm{y}\ \mathrm{a}^3 = -2\mathrm{bx}^3$, whose square $-\frac{\mathrm{x}^4\mathrm{b}}{2\mathrm{a}^3}$. which 3 squares (viz $\frac{10\mathrm{a}^7 + 2\mathrm{x}^7 - 5\mathrm{bx}^6}{10\mathrm{a}^3\mathrm{xx}}$) taken together are the square sought for. And these lines may bee ever squared unless in the valor of y there bee found $\frac{\mathrm{aa}}{\mathrm{x}}$, $\frac{\mathrm{ab}}{\mathrm{x}}$, $\frac{\mathrm{cc}+\mathrm{de}}{\mathrm{x}}$, &c. for the Squareing of that line depends on the squareing of the Hyperbola. As in the line $\mathrm{x}\ \mathrm{xxy} = \mathrm{x}^4 + \mathrm{a}^3\mathrm{x} + \mathrm{a}^4$.

<96r>

Secondly. If it have 3 termes See if it may be reduced to $\left\{ \begin{array}{c} \text{one or} \\ \text{fewer} \end{array} \right\}$ dimensions by adding or subtracting a knowne quantity to or from x . Example.
$2\mathrm{bax} + \mathrm{axx} = \mathrm{bby} + 2\mathrm{bxy} + \mathrm{xxy}$. which (makeing $\mathrm{x} + \mathrm{b} = \mathrm{z}$) is thus reduced
$\mathrm{zzy} = -\,\mathrm{bba} + \mathrm{azz}$. Or $\frac{-\mathrm{bba} + \mathrm{azz}}{\mathrm{zz}} = \mathrm{y}$

<97r>

$\mathrm{aax} + \mathrm{bby} = \mathrm{y}^3.\ \mathrm{x} = \mathrm{v} - \sqrt{\mathrm{ss} - \mathrm{yy}}\,.\ \mathrm{aav} - \mathrm{aa}\sqrt{\mathrm{ss} - \mathrm{y}^2} + \mathrm{bby} = \ \mathrm{y}^3\ \ \mathrm{aav} + \mathrm{bby} - \mathrm{y}^3 = \mathrm{aa}\sqrt{\mathrm{ss} - \mathrm{y}^2}\,.$

$$\begin{array}{llllll} \mathrm{a}^4\mathrm{v}^2 & +2\mathrm{aavbby} & -2\mathrm{aavy}^3 & +\mathrm{b}^4\mathrm{y}^2 & -2\mathrm{bby}^4 & +\mathrm{y}^6 \\ -\mathrm{a}^4\mathrm{ss} & & & +\mathrm{a}^4\mathrm{y}^2 & & \\ 0 & 1 & 3 & 2 & 4 & 6 \end{array} \qquad \frac{6\mathrm{y}^5 - 8\mathrm{bby}^3 + 2\mathrm{b}^4\mathrm{y} + 2\mathrm{a}^4\mathrm{y}}{-2\mathrm{aabb} + 6\mathrm{aayy}} = \mathrm{v}$$

$$\mathrm{v} - \mathrm{x} = \begin{array}{lll} -2\mathrm{b}^4\mathrm{y} & +6\mathrm{bby}^3 & -6\mathrm{y}^5 \\ +2\mathrm{b}^4\mathrm{y} & +2\mathrm{bby}^3 & +6\mathrm{y}^5 \\ +2\mathrm{a}^4\mathrm{y} & -8\mathrm{bby}^3 & \end{array} \quad \mathrm{v} - \mathrm{x} = \frac{2\mathrm{a}^4\mathrm{y}}{6\mathrm{aay}^2 - 2\mathrm{aabb}}$$

$\mathrm{v} - \mathrm{x} = \frac{\mathrm{aay}}{3\mathrm{y}^2 - \mathrm{bb}}.\ \frac{-\mathrm{bby}+}{\mathrm{aa}}\Big|\frac{\mathrm{y}}{} : \frac{\mathrm{aay}}{3\mathrm{y}^2 - \mathrm{bb}} : \frac{\mathrm{y}^3 - \mathrm{bby}}{\mathrm{aa}} : \&\ \mathrm{ad.}\ \frac{\mathrm{aay}^4 - \mathrm{aabbyy}}{3\mathrm{aay}^3 - \mathrm{aabby}} : \frac{\mathrm{y}^3 - \mathrm{bby}}{3\mathrm{y}^2 - \mathrm{bb}}.$

$\frac{\mathrm{aay}}{3\mathrm{y}^2 - \mathrm{bb}} = \frac{\mathrm{y}^3 - \mathrm{bby}}{\mathrm{aa.}}\ \ \mathrm{a}^4 = 3\mathrm{y}^4 - 4\mathrm{bbyy} + \mathrm{b}^4$
$\mathrm{y}^4 = \frac{4\mathrm{bbyy} - \mathrm{b}^4 + \mathrm{a}^4}{3}\ \ \mathrm{a} = \mathrm{b}\ \ \mathrm{y}^2 = \frac{4\mathrm{bb}}{3}.\ \mathrm{y} = \frac{2\mathrm{b}}{\sqrt{3}} = \mathrm{dm} = \mathrm{dv}.\ \frac{8\mathrm{b}^3}{3\sqrt{3}} - \frac{2\mathrm{bbb}}{\sqrt{3}} - \mathrm{aax}.\ \frac{2\mathrm{b}}{3\sqrt{3}} = \mathrm{dc} = \mathrm{ds}.\ \mathrm{y} : \frac{\mathrm{aay}}{3\mathrm{y}^2 - \mathrm{bb}} :: \frac{2\mathrm{b}}{3\sqrt{3}} : \mathrm{z}.\ \mathrm{yz} = \frac{2\mathrm{aaby}}{9\mathrm{y}^2\sqrt{3} - 3\mathrm{bb}\sqrt{3}}\ \ 9\mathrm{yyz} - 3\mathrm{bb}\ \ \mathrm{z} = \frac{2\mathrm{aab}}{\sqrt{3}}$ An Equation expressing the nature of the line ns .

<98r>

$\mathrm{aax} + \mathrm{byx} = \mathrm{y}^3.\ \mathrm{x} = \mathrm{v} - \sqrt{\mathrm{ss} - \mathrm{yy}}\,.$
$\mathrm{aav} + \mathrm{byv} - \mathrm{y}^3 = \begin{array}{c} \mathrm{aa} \\ \mathrm{by} \end{array} \sqrt{\mathrm{ss} - \mathrm{yy}}$

$$\begin{array}{lllllll}
a^4v^2 & +\ 2aav^2by & -\ 2aavy^3 & +\ bbvvy^2 & -\ 2bvy^4 & +\ y^6 \\
-\ a^4ss & & & +\ a^4yy & +\ bb & \\
 & & & -\ bbss & & & ss=\frac{2a^4vv+2aavvby+aavy^3+4bvy^4-2bby^4-4y^6}{2a^4} \\
-2 & -1 & +1 & 0 & 2 & 4 \\
0 & 1 & 3 & 2 & 4 & 6
\end{array}$$

$$\left.\begin{array}{llllll}
-12a^4y^6 & -\ 16a^4bvy^4 & & & +\ 4a^6vvby \\
-\ 8b^3vy^6 & +\ 8a^4bby^4 & -\ 12a^6vy^3 & +\ 4a^8yy & \\
+\ 4b^4y^6 & -\ 2aabbvy^5 & -\ 4aavvb^3 & & \\
+\ \underline{8y^8bb} & & & &
\end{array}\right\}0$$

$$12a^4y^6 - 16a^4bvy^4 + 4a^4bby^4 + 4bba^4vvy^4 - 12a^6vy^3 + 4a^6vvby - \begin{array}{l} +\ 4a^6by \\ +\ 4aab^3y^3 \end{array} \left\{ vv = \frac{\begin{array}{ll} 12a^6y^3av & -\ 8a^4bby^4 \\ 2aab^2y^5 & -\ 8y^8bb \\ 16a^4by^4 & -\ 4b^4y^6 \\ 8b^3y^6 & -\ 12a^4y^6 \end{array}}{4a^6by - 4aab^3y^3} \right.$$

$$+\ 4a^8yy$$

$$v = \frac{\begin{array}{l} 6a^6y^2 \\ +\ a^2b^2y^4 \\ +\ 8a^4by^3 \\ +\ 4b^3y^5 \end{array}}{4a^6b - 4aab^3y^2}$$ ♉ $$\sqrt{\frac{\begin{array}{lll} 36a^{12}y^4 & +\ 8aab^5y^9 & -\ 8a^4bby^3 \\ +\ 12a^8b^2y^6 & +\ 64a^8b^8y^6 & -\ 8y^7bb \\ +\ 16a^{10}by^5 & +\ 16bby^{10} & -\ 4b^4y^5 \\ +\ 24a^8b^3y^7 & & +\ 12a^4y^5 \\ +\ a^4b^4y^8 & & \\ b^5 & & \end{array}}{\begin{array}{ll} 16a^{12}b^2-32a^8b^4y^2 & +\ 4a^6b \\ +16a^4b^6y^4 & -\ 4aab^3y^2 \end{array}}}$$

<99r>

$$a^3x=y^4\,.\ x=v-\sqrt{ss-yy}\,.\ a^3v-y^4=a^3\sqrt{ss-yy}\ -\ \begin{array}{llll} abv^2 & -\ 2a^3vy^4 & +\ y^8 & +\ a^6yy \\ abss & & & \\ 0 & 4 & 8 & 2 \end{array}\ .\ v=\frac{4y^6+a^6}{4a^3y^2}\ \ v-x=\frac{4y^6+a^6-4y^6}{4a^3y^2}=\frac{a^3}{4y^2}\,.$$

$$y\cdot\frac{a^3}{4y^2}::\frac{y^4}{a^3}:\frac{a^3y^4}{4a^3y^3}=\frac{y}{4}$$

<100r>

$$aax-aay=y^3\,.\ aav-aay-y^3=aa\sqrt{ss-yy}\ -\ \begin{array}{llllll} a^4v^2 & -\ 2a^4vy & -\ 2aavy^3 & +\ 2a^4y^2 & +\ 2aay^4 & +\ y^6 \\ a^4ss & & & & & \\ 0 & 1 & 3 & 2 & 4 & 6 \end{array}\qquad v=\frac{3y^5+4aay^3+2a^4y}{a^4+3aayy}$$

$$v-x=\frac{3y5+4aay^3+2a^4y-aay^3-3y^5-ya^4-3aay^3}{a^4+3aayy}\quad v-x=\frac{aay}{aa+3yy}\,.\ y:\frac{aay}{a^2+3y^2}::\frac{a^2y+y^3}{aa}:\frac{a^4y^2+a^2y^4}{a^4y+3a^2y^3}=\frac{aay+y^3}{aa+3yy}$$

<101r>

$$aax=by^2+y^3\,.\ aav-by^2-y^3=aa\sqrt{ss-yy}\ -\ \begin{array}{llllll} a^4v^2 & -\ 2aabyyv & -\ 2aavy^3 & +\ bvy^4 & +\ 2by^5 & +\ y^6 \\ a^4ss & +\ a^4 & & & & \\ 0 & 2 & 3 & 4 & 5 & 6 \end{array}\qquad v=\frac{3y^4+5by^3+2bby^2+a^4yy}{2aab+3aay}$$

$$v-x=\frac{3y^4+5by^3+2bby^2-2bbyy-3by^3-2by^3-3y^4+a^4}{2aab+3aay}\quad v-x=\frac{a^4}{2aab+3aay}\quad a=b\,.\ a^3+3aay=byy+bby\ \ aa+3ay-yy-ay=0\ \ yy=2ay+aa\ \ y=a+\sqrt{aa+aa}\,.$$

$$y=a+a\sqrt{2}=dm=vd\,.\ \frac{a^3\ +\ 3a^3\sqrt{2}\ +\ 6a^3\ +\ 2a^3\sqrt{2}}{a^3\ +\ 2a^3\sqrt{2}\ +\ 2a^3}\over aa=10a+7a\sqrt{2}=dc\ \ y:\frac{y^3+by^2}{bb+3by}::10b+7b\sqrt{2}:z\,.\ yz=\frac{\begin{array}{l}10y^3\ +\ 10by^2\ +\ 7y^3\sqrt{2} \\ \qquad +\ 7by^2\sqrt{2}\end{array}}{b+3y}$$

$$bz+3yz=10yy+10by+7yy\sqrt{2}+7by\sqrt{2}\,.$$

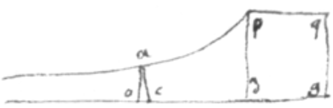
<102r>

$$axy=y^3+b^3\,.\ ayv-y^3-b^3=ay\sqrt{ss-yy}\,.\ -\ \begin{array}{llllll} aavvy^2 & -\ 2avy^4 & -\ 2avb^3y & +\ y^6 & +\ 2b^3y^3 & +\ b^6 \\ aass & +\ aa & & & & \\ 0 & 2 & -1 & 4 & 1 & -2 \end{array}\qquad v=\frac{4y^6+2b^3y^3+2aay^4-2b^6}{4ay^4-2ab^3y}$$

$$v-x=\frac{\begin{array}{l}4y^6\ +\ 2b^3y^3\ +\ 2aay^4\ -\ 2b^6\ -\ 4y^6\ +\ 2b^3y^3\ -\ 4b^3y^3 \\ \qquad +\ 2b^6\end{array}}{4ay^4-2ab^3y}\quad v-x=\frac{ay^3}{2y^3-b^3}\,.\ y:\frac{ay^3}{2y^3-b^3}:\frac{y^3+b^3}{ay}:\frac{y^4+b^4}{2y^3-b^3}$$

<102v>

<103r>



$a^3=xyy.\ a=dg=dp\ .\ go=x.\ oa=y.\ cg=v.\ ca=s.\ ss=y^2+xx-2vx+v^2\,.$

$$\begin{array}{lllll} a^3 & -\ xss & +\ x^3 & -\ 2vx^2 & +\ vvx=0 \\ -1 & 0 & 2 & 1 & 0 \end{array}\quad \frac{2x^3-a^3}{2x^2}=v.\ \frac{a^3}{x}:\frac{4x^6-4a^3x^3+4a^6}{4x^4}::x.\ \{Ad\}\ \frac{4x^6-4a^3x^3+a^6}{4xxa^3}.\ x-v=\frac{a^3}{2x^2}\,.$$

$\sqrt{\frac{a^3}{x}}:\frac{a^3}{2xx}::p:z\ \ \frac{zza^3}{x}=\frac{a^6pp}{4x^4}\ \ 4x^3zz=a^3pp\ \ go=y.\ oa=x.$ &c: $a^3=xyy.\ s\,s=xx+yy+vv-2vy\ \ xx=ss-yy+2vy+vv\,.$

$$\begin{array}{llllll} a^6 & -\ ssy^4 & +\ y^6 & -\ 2vy^5 & -\ vvy^4 & =\ 0 \\ -4 & 0 & 2 & 1 & 0 & \end{array}\quad v=\frac{2y^6-4a^6}{2y^5}\ \ y-v=\frac{2a^6}{y^5}\ \ \frac{a^3}{y^2}:\frac{2a^6}{y^5}::p:z\ \ z=\frac{2a^3p}{yyy}\,.\ zy^3=2a^3p.$$ which shewes the nature of another crooked line that may be squared.

<104r>

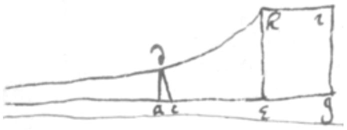
$axx = y^3 .\ x = v - \sqrt{ss - yy}\ .\ x^2 = v^2 - 2v\sqrt{ss - y^2} + ss - y^2\ .\ av^2 + ass - ay^2 - y^3 = 2\ a\ v\sqrt{ss - yy}\ .$

$$\begin{array}{llllll} a^2v^4 & & & - 2av^2y^3 & + aay^4 - 2ay^5 & + y^6 \\ & + aas^4 & - 2aassy^2 & - 2assy^3 & & \\ & - 2a^2v^4ss & & & & \\ & & + 2vvaay^2 & & & \\ & +3 & 1 & -0 & -1 \quad -2 & -3 \end{array}$$

$$\begin{array}{l} 3a^2s^4 - 2a^4y^2ss + 3a^2v^4\ \&c + 2v^2a^2y^2 - a^2y^4 + 4ay^5 - 3y^6 \\ \qquad - 6aavv \end{array}$$

$$ss = \begin{array}{l} \frac{2yy}{3} \\ + \frac{6vv}{3} \end{array} + \sqrt{\frac{4y^4}{9}\ \frac{-v^4+y^4}{3}\ \frac{+y^6-2ay^5}{3aa}}\quad ss = \frac{2yy+6vv}{3} - \sqrt{\frac{\begin{array}{l} 4\dot{y}^4a^2+24aay\dot{y}vv+36aav^4 \\ -9aav^4-6aay\dot{y}vv-12a\dot{y}^5 \\ +3aa\dot{y}^4+9\dot{y}^6 \end{array}}{9aa}}\quad ss = \frac{2y^2+6v^2}{3} - \sqrt{\frac{\begin{array}{l} 9y^6-12ay^5+7aay^4+18aavvy^2 \\ +27aav^4 \end{array}}{9aa}}$$

<104v>



$a = b - A = C - B = D - d = e - D = F - E = G - f = 1{,}117313\ .\ A - a = d - C = ff - F = 0{,}921787\ .$
$B - b = E - e = 0{,}706724\ .\ A = b - a = d - B = D - C = f - E = G - F = 2{,}039100\ .$
$B - A = C - b = E - D = F - e = 1{,}824037\ .\ e - d = aa - f = 2{,}234626\ .$
$b = D - B = e - C = G - E = aa - F = AA - f = 3{,}156413\ .\ C - A = E - d = F - D = 2{,}941350\ .$
$B - a = d - b = f - e = 2{,}745824\ \ B = C - a = d - A = D - b = E - C = f - D = G - e = 3{,}863137\ .$
$e - B = aa - E = bb - f = 4{,}273726\ .\ F - d = 4{,}058663\ .\ A^2 - F = 4{,}078200\ .$
$C = D - A = e - b = E - B = F - C = f - d = G - D = aa - e = B^2 - f = 4{,}980450\quad A^2 - E = bb - F = 5{,}195513\ .\ d - a = 4{,}784924\ .$
$d = D - a = f - C = A^2 - e = B^2 - F = 5{,}902237\ .\ bb - e = 6{,}312826\ .\ e - A = F - B = G - d = aa - D = C^2 - f = 6{,}097763\ .\ E - b = 5{,}687174\ .$
$D = e - a = f - B = G - C = A^2 - D = bb - e = B^2 - E = C^2 - F = dd - f = 7{,}01955\quad E - A = F - b = 6{,}804487\ .\ aa - d = 7{,}215076\ .$
$e = G - B = aa - C = A^2 - d = bb - D = C^2 - E = D^2 - f = 8{,}136863\quad E - a = f - b = B^2 - e = 7{,}726274\ .\ F - A = 7{,}921800\ .\ dd - F = 7{,}941337\ .$
$E = F - a = f - A = G - b = B^2 - D = C^2 - e = 8{,}843587\ .\ A^2 - d = dd - E = D^2 - F = 9{,}058650\ .\ aa - B = bb - d = ee - f = 9{,}254176\ .$
$F = G - A = aa - b = B^2 - d = C^2 - D = E^2 - f = 9{,}960900\ .\ A^2 - B = bb - C = D^2 - E = ee - F = 10{,}175963\ .\ f - a = dd - e = 9{,}765374\ .$
$f = G - a = A^2 - b = B^2 - C = dd - D = D^2 - e = E^2 - F = 10{,}882687\ .\ aa - A = C^2 - d = F^2 - f = 11{,}078213\ .\ B^2 - C = ee - E = 11{,}293276\ .$

This table shews the distance of any two notes As the distance of C & E is B , or a third, or 3,863137 halfe notes. Of B & E tis a fourth, or 4,98045 halfe notes. of B & F tis 6,097763 halfe notes, or greater than a fifth ♭, by 0,095526 halfe notes &c.

<105r>

$aa = xy .\ eg = eh = a .\ ga = x .\ ad = y .\ d\,c = s .\ cg = v\ y^2 = ss - xx + 2vx - vv .$

$$\begin{array}{llll} - a^4 & + x^2s^2 & - x^4 & + 2vx^3 = 0 \\ & - x^2v^2 & & \\ +2 & 0 & -2 & -1 \end{array}$$

$\frac{2x^4-2a^4}{2x^3} = v\ x - v = \frac{a^4}{x^3} .$

$\frac{aa}{x} : \frac{a^4}{x^3} :: p : z .\ zxx = a^2p .$

$8^{th} - 5^{t} = 4^{th} = G - D = C = 6^{t} - 3^{d} = E - B$
$5^{t} - 4^{th} = 5^{t} + 5^{t} - 8^{th} = 2^{d} = A\ .$
$4^{th} + 4^{th} = 8^{th} + 8^{th} - 5^{t} - 5^{t} = 7^{th} = F\ .$
$4^{th} - 3^{d} = 2^{d}\flat = a$
$8^{th} - 3^{d} = 6^{t}\flat = e$
$4^{th} + 3^{d} = 6^{t} = E$
$5 - 3^{d} = 3^{d}\flat = 8^{th} - 6^{t} = b$
$3^{d} + 5^{t} = 7^{th}\sharp = f$
$7^{th}\sharp - 4^{th} = 2^{d} + 3^{d} = 5^{t}\flat = d = 3^{d} + 5^{t} - 4^{th} = -2^{d}\flat + 5^{t}\ .$

By the helpe of concordant notes all the notes in the Gam ut may bee thus tuned viz:

First tune the eighths, G , G^2 , G^3 , G^4 &c.

Seacondly tune fifts to them both above them D , D^2 , D^3 , D^4 . & below them ${}^2C$, C , C^2 , C^3 .

Thirdly tune thirds to them both above them B , B^2 , B^3 , B^4 , & below them ${}^2E\flat$, $E\flat$, $E^2\flat$, $E^3\flat$.

Fourthly from each B , B^2 , B^3 , B^4 rise a fift for $F\sharp$, $F^2\sharp$, $F^3\sharp$, $F^4\sharp$ & fall a fift for ${}^2E$, E , E^2 , E^3 .

Fiftly from ${}^2E\flat$, $E\flat$, $E^2\flat$, $E^3\flat$ rise a fift for $B\flat$ $B^2\flat$, $B^3\flat$, $B^4\flat$. & fall a fift for ${}^2A\flat$, $A\flat$, $A^2\flat$, $A^3\flat$.

Sixtly from D , D^2 , D^3 , D^4 . rise a fift for A^2 , A^3 , A^4 A^5 . & from ${}^2C$, C , C^2 , C^3 fall a fift for ${}^3F$, ${}^2F$, F , F^2 .

Seaventhly from each $F\sharp$, $F^2\sharp$, $F^3\sharp$, $F^4\sharp$. rise a fift for $D^2\flat$, $D^3\flat$, $D^4\flat$, $D^5\flat$. The rest as A , $D\flat$ are supplyed by eighths viz to A^2 , $D^2\flat$ &c.

<105v>

November 20. 1665.

$\frac{1}{2}$	360	2,55630247	2,5563025	360,00000,0	12,00000.	G
$\frac{8}{15}$	384	2,58433118	2,5813883	381,40667,8.	10,88268,7	f
$\frac{9}{16}$	405	2,60745497	2,6064742	404,08640,6.	9,96090,0	F
$\frac{3}{5}$	432	2,63548369	2,6315600	428,11458,1.	9,84358,7.	E
$\frac{5}{8}$	450	2,65321247	2,6566458	453,57157,8.	8,13686,3.	e
$\frac{2}{3}$	480	2,68124123	2,6817317	480,54236,7.	7,01955,0	D
$\frac{32}{45}$	512	2,70926992	2,7068175	509,11688,24543.	5,90223,7.	d
$\frac{3}{4}$	540	2,73239371	2,7319033.	539,39055,9.	4,98045,0.	C
$\frac{4}{5}$	576	2,76042244	2,7569892	571,46447,4.	3,86313,7.	B
$\frac{5}{6}$	600	2,77815121	2,7820750	605,44546,7.	3,15641,3.	b
$\frac{8}{9}$	640	2,80617993	2,8071608	641,44697,3.	2,03910,0.	A
$\frac{15}{16}$	675	2,82930373	2,8322467	679,589514,9.	1,11731,3.	a
1	720	2,857332447.	2,8573325	720,00000,0.	0,00000,0.	G
	How y^e string 1 or 720 is to bee di= =vided y^t it may sound all y^e musicall notes & halfe notes in an eight	The proportion w^{ch} those musicall notes & $\frac{1}{2}$ notes beare y^e one to y^e other (viz y^e logarithmes of y^e string sounding them)	Twelve exact or equidistant $\frac{1}{2}$ notes (or y^e logarithmes of a cord divided into 12 geome tricall partes) y^e distāce of each $\frac{1}{2}$ note being 0,025085833333 &c. A just note being 0,050171666666 &c.	A string (720) divided into 12 (geometrically progressionall) parts, y^t it may sound y^e 12 exact $\frac{1}{2}$ notes in an eight	The proportion of all y^e 12 musicall $\frac{1}{2}$ notes in a eight; An exact halfe note being a unite.	

<106r>

$2{,}635483\ 69\ a^4 = xy^3 \,.\, ad = x.\quad \begin{array}{ccccccccccc} - & a^8 & + & y^6ss & - & y^8 & + & 2vy^7 & - & vvy^6 & = 0 \\ & +6 & & 0 & & -2 & & -1 & & 0 & \end{array}\quad \frac{2y^8-6a^8}{2y^7} = v.\ v - y = \frac{3a^8}{2y^7}.\ \frac{a^4}{y^3} :: \frac{3a^8}{2y^7} :: p \quad z\ \frac{za^4}{y^3} = \frac{3a^8p}{3y^7}.\ zy^4 = \frac{3a^4p}{2}$

$0{,}921787 = A - a = d - C = f - F$. $2{,}039100 = A = b - A = d - B = D - C = f - E = b - F$ $\quad 0{,}706724 = B - b = E - e$.
$F - e = 1{,}824037 = B - A = C - b = E - D$. $e - d = 2{,}234626$ $aa - f = 2{,}234626$ $2{,}745824 = B - a = d - b = f - e$. $aa - E = bb - f = = 4{,}273726 = e - B$.
$F - d = 4{,}058663$ $2941350 = C - A = E - d = F - D$. $4{,}078200 = A^2 - F$ $3{,}156413 = b = D - B = e - C = G - E = aa - F = AA - f$. $bb - e = 6312826$
$3863137 = B = C - a = d - A = D - b = E - C = f - D = G - e$ $\quad 4980450 = C = D - A = e - b = E - B = F - C = f - d = G - D = aa - e = B^2 - f$
$4784924 = d - a$ $5{,}195513 = A^2 - E = bb - F$ $5{,}902237 = d = D - a = f - C = A^2 - e = B^2 - F$. $7{,}9218 = F - A$
$6{,}097763 = e - A = F - B = G - d = aa - D = C^2 - f$ $7941337 = dd - F$ $5{,}687174 = E - b$ $6{,}804487 = E - A = F - b$
$7{,}019550 = D = e - a = f - B = G - C = A^2 - D = bb - e = B^2 - E = C^2 - F = dd - f$ $\quad 5{,}215076 = aa - d$. $7{,}726274 = E - a = f - b = B^2 - e$
$8{,}136863 = e = G - B = a^2 - C = A^2 - d = bb - D = C^2 - E = D^2 -$ f $9{,}254176 = aa - B = bb - d = ee - f$. $D^2 - F = 9{,}058650 A^2 - d = dd - E$.
$9{,}765374 = f - a = dd - e$ $10{,}175963 = A^2 - B = bb - C = D^2 - E = ee - F$ $11{,}078213 = aa - A = C^2 - d = F^2 - f$ $11{,}293276 = B^2 - C = ee - E$ perhaps

$d = E - b$ is better than $d = 3^d + 5^t - 4^{th}$.

0	1,1	2	3,1	3,9	5	5,9	7	8,1	8,9	10	109	12
G	a	A	b	B	C	d	D	e	E	F	f	G
0 .	1,2 .	2 .	3,2 .	3,8 .	5 .	5,8 .	7 .	8,2 .	8,8 .	10 .	108 .	12
0 .	6 .	10 .	16 .	19 .	25 .	29 .	35 .	41 .	44 .	50 .	54 .	60 .

<106v>

	2,00000	g	$2 = 2,0000.$
	1,88774		$\frac{15}{8} = 1,8750.$
	1,7818	a	$\frac{16}{9} = 1,777777$ &c
	1,6818		$\frac{5}{3} = 1,66666$ &c
	1,5874	b	$\frac{8}{5} = 1,6000$
	1,4983	c	$\frac{3}{2} = 1,5000$
$\frac{64}{45} = 14222$ &c	1,4142136		$\frac{10}{7} = 1,428571428571$ &c
	1,3349	d	$\frac{4}{3} = 1,33333$ &c
	1,2599		$\frac{5}{4} = 1,2500.$
	1,18920	e	$\frac{6}{5} = 1,2000.$
	1,12245	f	$\frac{9}{8} = 1,1250.$
	1,05946		$\frac{16}{15} = 1,066666$ &c
	1,00000	g	$1 = 1,0000.$
	A string divided in a geometricall progres=sion		A string divided by a musicall progressi̅

By this table it may appeare that a

Second minor, Second _____, Third minor } is higher by y^e { $17^{th}+$, $51^{th}+$, $13^{th}-$

Third _____, Fourth _____ } is lower by y^e { $15^{th}-$, 102^{th}

Fift min^e is lower by y^e 20^{th} $\frac{1}{2}$ or higher by y^e 20^{th} $\frac{1}{2}$

Fift _____, Sixt minor } is higher by y^e { 102^{th}, 15^{th}

Sixt, Seaventh, Eight minor } is lower by y^e { 13^{th}, 51^{th}, 17^{th}

} parte of a note then bee were y^e musica divided in geome progression

$$1,\ \frac{15}{16},\ \frac{8}{9},\ \frac{5}{6},\ \frac{4}{5},\ \frac{3}{4},\ \frac{32}{45},\ \frac{2}{3},\ \frac{5}{8},\ \frac{3}{5},\ \frac{9}{16},\ \frac{8}{15}$$

$$\frac{24}{25},\ \frac{7}{8},\quad \frac{18}{25},\ \frac{5}{7},\ \frac{45}{64},\ \frac{7}{10},\ \frac{25}{36},\quad \frac{4}{7},\ \frac{25}{48}$$

$$\frac{20}{21},\ \frac{9}{10},\quad \frac{5}{9},\ \frac{21}{40}$$

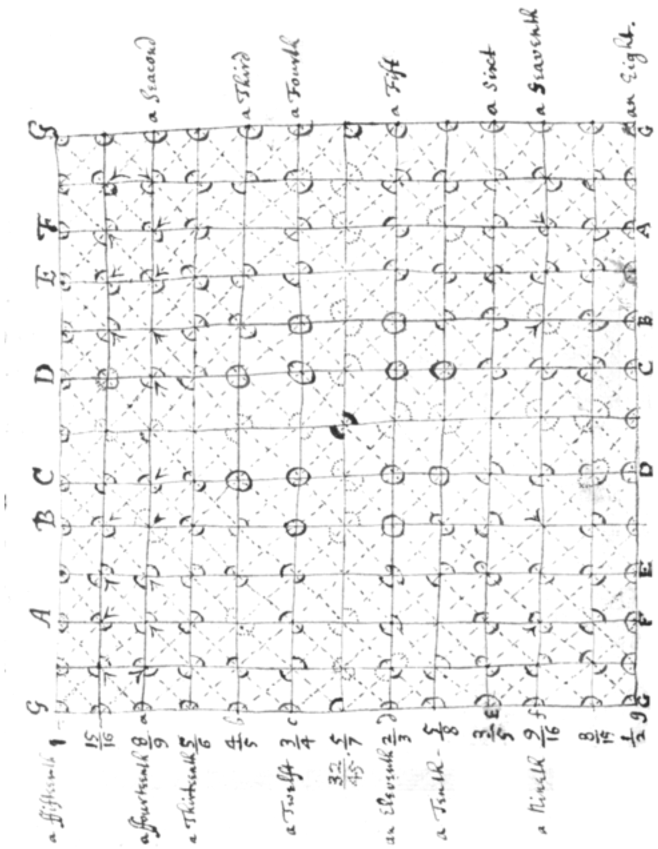
$$\frac{1}{2},\ \frac{15}{32},\ \frac{4}{9},\ \frac{5}{12},\ \frac{2}{5},\ \frac{3}{8},\ \frac{16}{45},\ \frac{1}{3},\ \frac{5}{16},\ \frac{3}{10},\ \frac{9}{32},\ \frac{4}{15}$$

$$\frac{1}{4},\ \frac{15}{64},\ \frac{2}{9},\ \frac{5}{24},\ \frac{1}{5},\ \frac{3}{16},\ \frac{8}{45},\ \frac{1}{6},\ \frac{5}{32},\ \frac{3}{20},\ \frac{9}{64},\ \frac{2}{15}$$

$$\frac{1}{7}$$

<107r>

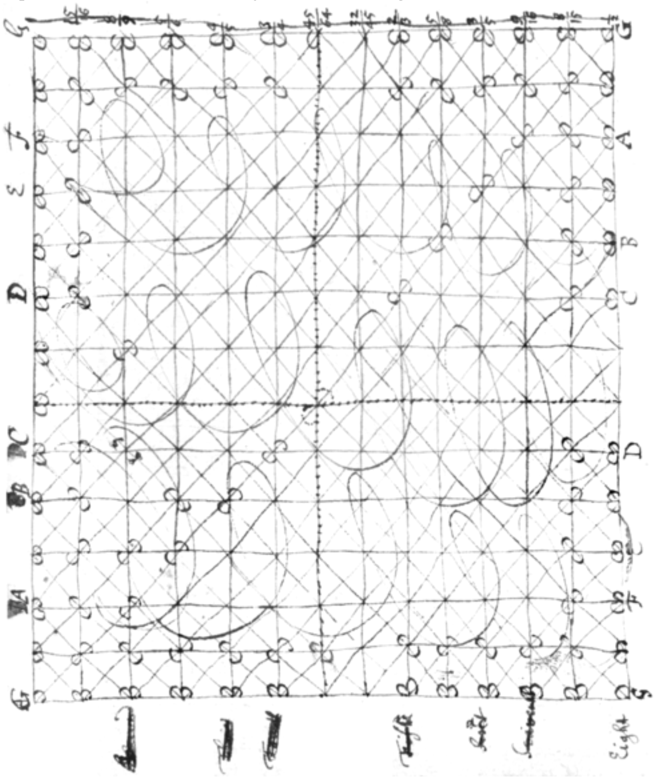
$a^5 = xxy^3$



By this table may bee knowne the distance of any two notes whither a **{trew}** second of the lesse, second, third f the lesse, a third fourth &c: As to know the distance twixt A re & B sol re I follow the pricked stroke from A to D or from D to A where I find it crossed by a black crooked line & against it, a Fourth written, therefore I conclude A re & D la sol distant a true fourth.

<107v>

And Thus to find the distance of B mi & D la sol re I follow the prick line from the top B to the right hand side thence to the bottom B thence towards the left hand side untill I come **{over}** D. Or (which is the same) I follow the prick**{t}** line from the top D to the left hand side thence to the bottom D, thence toward the right hand side untill I come just over B, where I find the pricked line to be crossed by a ⌖ stroke & against it to bee written on the upper line a tenth , on the lower

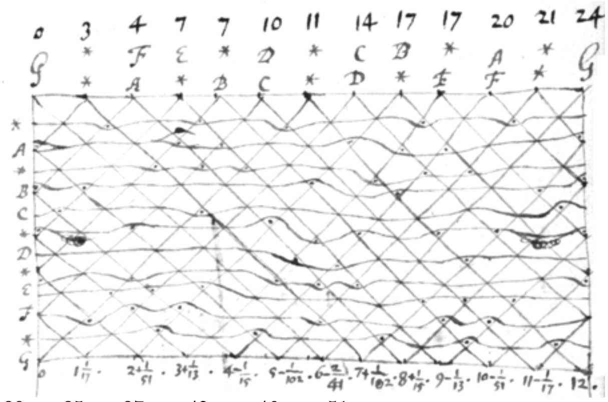


a ninth therefore tis a tenth minor exactly. But if it

<108r>

$a^3 = xxy.\ x = v + \sqrt{ss - yy}\ .\ a^3\ vyy = yy\sqrt{ss - yy}$ $\begin{array}{cccccccc} a^6 & - & 2a^3vyy & + & vvy^4 & + & y^6 & = 0 \\ & & & & - ss & & & \\ -4 & & -2 & & 0 & & 2 & \end{array}$ $\frac{-2y^6 + 4a^6}{2a^3y^2}\ .\ x - v = \frac{2y^6 - 2a^6}{2a^3y^2}\ .\ 2a^3y^3 : 2y^6 - 2a^6 : p : z$

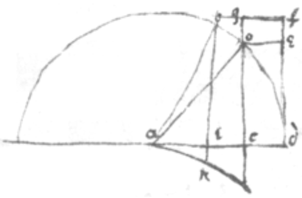
$2a^3y^3z = 2y^6p - 2a^6p\ \ py^6 - a^3y^3 - a^6p$



0	5	9	14	16	21	25	30	35	37	42	46	51
0	5	9	14	17	22	26	31	36	39	44	48	53
g	a	A	b	B	C	d	D	e	E	F	f	G
0	4	7	11	13	17	20	24	28	30	34	37	41
0	3	5	8	9	12	14	17	20	21	24	26	29
0	4	6	10	11	15	17	21	25	26	30	32	36
0	11	20	31	39	50	59	70	81	89	100	109	120
0	4	2	6	1	5	3	7	11	6	10	8	12

0	2	5	7	8	10	13	15	17	18	20	23	25
0	1	4	5	7	8	11	12	13	15	16	19	20

<108v>



$0\ .\ a\ .\ \ \ .\ b\ \ \ .\ b + a\ \ \ .$

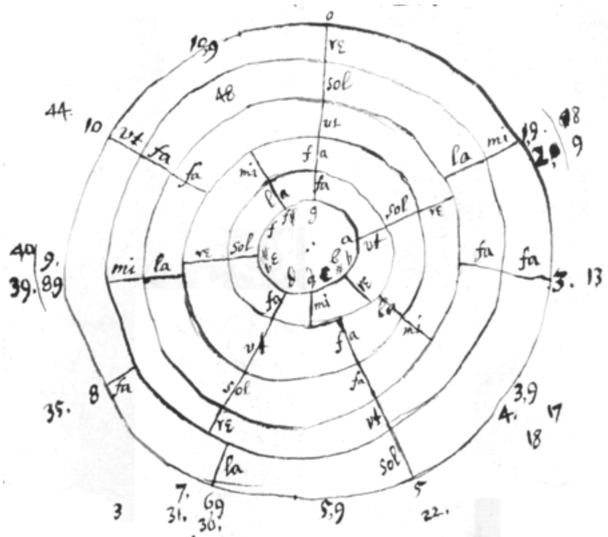
$0\ .\ a\ .\ a + b\ .\ a + b + c\ .\ 2a + b + c\ .\ 2a + 2b + c\ .$

The notes	The proportion of their sounds	or thus	or thus	How they may bee otherwise distinguished by figures thus	thus	or thus		or thus	or thus
G	53	612	59	100	100	36	29	12	12 .
f	48	555		90	89	32	26	$10\frac{2}{3}$	10,9
F	44	508		82		30	24	10	10
E	39	451		72		26	21	$8\frac{2}{3}$	8,9
e	36	415	40	69		21	17	7	7
D	31	358		59		25	20	$8\frac{1}{3}$	8,1
d	26	301		49	48	17	14	$5\frac{2}{3}$	5,9
C	22	254		41	41	15	12	5	5
B	17	197	19	31	30	11	9	$3\frac{2}{3}$	3,9
b	14	161		28	29	10	8	$3\frac{1}{3}$	3,1
A	9	104		18	18	6	5	2	2
a	5	57		10	11	4	3	$1\frac{1}{3}$	1,1
G	0	0	0	0	0	0	0	0	0

<109r>

$ao = a = ad$. $dc = p$. $ai = x$. $oi = y$. $aa - xx = y^2$ $in = z$. $xx : aa - xx :: pp : zz$. $zzxx = aapp - p^2x^2$. $id = x$. $oi = 2ax - xx$. $aa - 2ax + x^2 : 2ax - x^2 :: pp : zz$

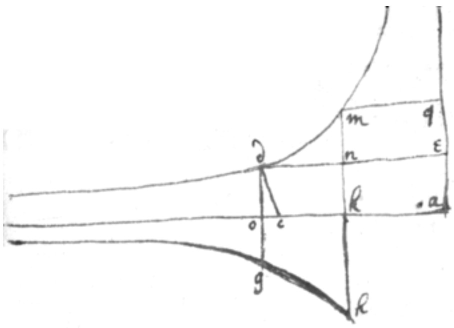
$zzx^2 - 2azzx + aazz = 0$
$pp \quad - 2app$.

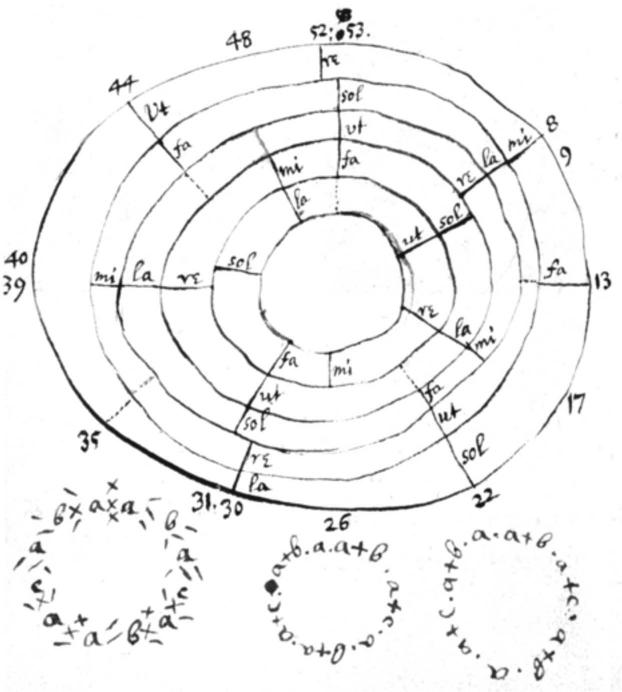


The 3 meanes are best there being an imperfect fift in the outward extreame & a tritonus in the inmost.

1 . 6 . 4 . 3 . 2 . 5 . / 10 . 7 . 2 . 5 . . 6 . 8 . 4 . 6 . 11 . 8 . 3 . 10 . 2 . 3 . 9 . 10 . 12 . 7 .
1 . 6 . 11 . 8 . 4 . 3 : 9 : 10 : 12 . 7 . 2 . 5 .

<109v>





<110r>

In the Hyperbola dm . suppose $ak = a = kh$ $ao = x.$ $od = y.$ $dc =$ a secant. ca =v $og = z.$ $xy = aa.$ $y\,y = ss - xx + 2vx - vv$

$- \ a^4 \ + \ xxss \ - \ x^4 \ + \ 2vx^3 \ = \ 0$ double route equall

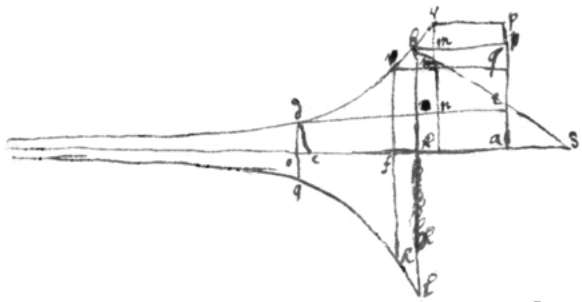
$- \ vvxx$ $\frac{x^4 - a^4}{x^3} = v$ $.\ x - v = \frac{a^4}{x^3} = oc.$ $od : oc :: kh : og.$ $\frac{aa}{x} : \frac{a^4}{x^3} :: a : z.$ $\frac{aaz}{x} = \frac{a^5}{x^3}.$ $zxx = a^3.$ which equation

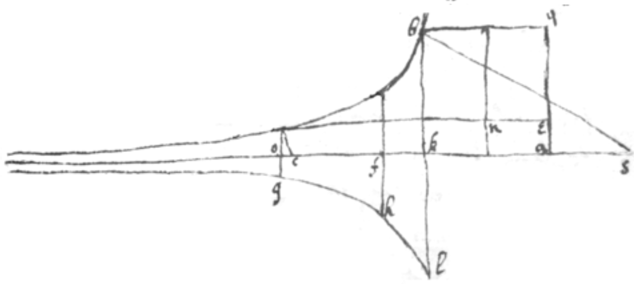
$+ \ 2 \quad 0 \quad - \ 2 \quad - \ 1$

continues the nature of the crooked line gh. Now supposeing the line og always moves over the same superficies in the same time, it will increase in motion from kh in the same proportion that it decreaseth in lenght & the line ne will move uniformely from (mq), soe that the space $mqen = gokh.$ suppose $ok = a.$ $ao = 2a.$ $od = \frac{a}{2} = nm.$ & $mqen = \frac{1}{2}aa = ogkh.$

	1		1		1	1		1		1	1		1	
		2		2		2	2		2		2	2		
	3		3		3		3	3		3		3	3	
	4	4		4		4		4	4		4		4	
		5	5		5		5		5	5		5		
Modi	6		6	6		6		6		6	6		6	1 . 6 .
		7		7	7		7		7		7	7		
	8		8		8	8		8		8		8	8	
	9	9		9		9	9		9		9		9	
		10	10		10		10	10		10		10		
	11		11	11		11		11	11		11		11	
		12		12	12		12		12	12		12		

<110v>





In the order of the musicall tones the 2 halfe notes may not be together 1st because every note would then bee distant 3 tones from some other which is most ungratefull Secondly whole notes ought to bee interposed to moderate their harshnesse. Thirdly since there must bee a Fift to the ground: these $\frac{1}{2}$ notes must bee either next the ground or its Fift which would make them harsh & that wee could not gradually passe to or from them.

Neither ought they to be distant but one tone for the second reason **{afforesd}** & because they will bee more consonant by the absense of more 3 tones &c if they be distant 2 tones yet perhaps they may not bee wholly uselesse. See the last modes.

A catalogue of the 12 Musicall modes in theire order of gratefulnesse.

1	G	.	a	.	b	c	.	d	.	e	f	.	g
3	c	.	d	.	e	f	.	g	.	a	.	b	c
2	d	.	e	f	.	g	.	a	.	b	c	.	d
4	a	.	b	c	.	d	.	e	f	.	g	.	a
5	e	f	.	g	.	a	.	b	c	.	d	.	e
6	f	.	g	.	a	.	b	c	.	d	.	e	f
	b	c	.	d	.	e	f	.	g	.	a	.	b
	.	b	c	.	d	.	e	f	.	g	.	a	.
	.	a	.	b	c	.	d	.	e	f	.	g	.
	.	d	.	e	f	.	g	.	a	.	b	c	.
	.	g	.	a	.	b	c	.	d	.	e	f	.
	.	e	f	.	g	.	a	.	b	c	.	d	.
	0 .	1 .	2 .	3 .	4 .	5 .	6 .	7 .	8 .	9 .	10 .	11 .	12
2	G .	.	a	.	b	c	.	d	e	.	f	.	g
1	c	.	d	e	.	f	.	g	.	a	.	b	c
3	d	e	.	f	.	g	.	a	.	b	c	.	d

0 . 9 . 17 . 22 . 31 . 39 . 44 . 53 .
0 . 9 . 17 . 22 . 31 . 40 . 48 . 53 .
0 . 9 . 14 . 22 . 31 . 40 . 45 . 53 .
0 . 9 . 14 . 22 . 31 . 36 . 45 . 53
0 . 5 . 14 . 22 . 31 . 36 . 45 . 53 .
0 9 17 . 26 . 31 . 40 . 48 . 53

G . a . b . cde . f . g
de . f . g . a . b . cd

f . g . a . bc . de . f .

<111r>

suppose the line last found to be md . $mk = kh = a = ka$ $ao = x$. $od = y$. $dc = s$. $ca = v$. $yxx = a^3$. $yy = ss - vv + 2vx - xx$.

$$\begin{array}{ccccccccc} & ssx^4 & + & 2vx^5 & - & x^6 & - & a^6 & = 0 \\ - & vv & & & & & & & \\ & 0 & & -1 & & -2 & & +4 & \end{array}$$

$v = \frac{x^6 - 2a^6}{x^5}$. $x - v = \frac{2a^6}{x^5}$. to find at what point $do = oc$: $\frac{a^3}{xx} = \frac{2a^6}{x^5}$. $x^3 = 2a^3$. $x = a\sqrt{c:2} = af$. $mq = fh = a$. $og = z$.

$od : oc :: fh : og$. $\frac{a^3}{xx} : \frac{2a^6}{x^5} :: a : z$. $a^3zx^5 = 2a^7xx$. $zx^3 = 2\,a^4$. which shews the nature of the line (gh). & $mneq = gofh$ or $nbpe = gokl$. suppose $ko = ka = a$.
$oa = 2a = x$ $od = \frac{a^3}{xx} = \frac{a^3}{4aa} = \frac{a}{4}$. $bn = \frac{3a}{4}$. $bpen = \frac{3aa}{4} = gokl$.

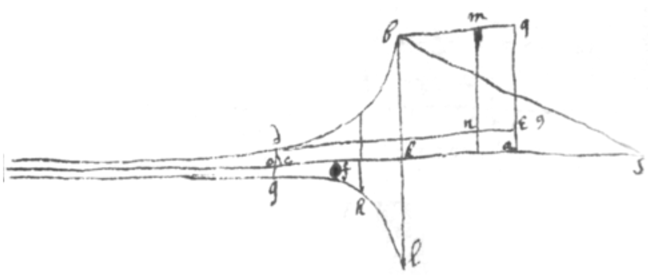
$kl = ka = a = kb$. Suppose $kl = ka = kb = a$ $ak = x = a$. $bk = y = a$. $bs = s$. $as = v$. $yy = ss - vv - 2vx - xx$. $yyxxxx = a^6$.

$$\begin{array}{cccccccc} & ssx^4 & - & 2vx^5 & - & x^6 & - & a^6 \\ - & vv & & & & & & . \\ & 0 & - & 1 & - & 2 & + & 4 \end{array}$$

$\frac{-2x^6 + 4a^6}{2x^5} = v$ $v + x = \frac{2a^6}{x^5} = ks = 2a$. $ks : bk :: kl : fh$. $2a : a :: a : \frac{a}{2}$. $fh = \frac{a}{2} = ne = mq = rp$. $mn = \frac{3a}{4} = qe$ $\frac{3aa}{8} = mneq = lkog$. $ao = 2a = x$. $\frac{a^3}{4aa} = do$ $do = \frac{a}{4}$.
$oc = \frac{2a^6}{32a^5} = \frac{a}{16}$. $\frac{a}{4} : \frac{a}{16} :: \frac{a}{2} : og = \frac{a}{8}$ $\frac{2a^6}{x^5} :: \frac{a^3}{xx} : z :: \frac{a}{2}$. $a^4 = zx^3$.

<111v>

a , b , c = $\frac{1}{2}$ tone max; medî: minimus. $a + a = d$, $a + b = e$, $a + c = f$ ~ tone **{maj me:mi.}** $a + a + b = a + e = 3^d\flat$
$a + a + b + c = e + f = 3^d\sharp$.
$3a + b + c = a + e + f = f + 3^d\flat = 4^{th}$



$$2r + s + t = 5^t . \begin{array}{l} 3^d \text{ maj} = r + s \text{ . } r + t = 3^d \text{ mi} \\ 6^t \text{ min} = 2r + s + 2t \\ 6^{th} \text{ maj} = 2r + 2s + t \\ 4^{th} = r + s + t \end{array}$$ hath 8 Fifts

r . s . t . r . s . t . r , r s t r s t r .
1 2 3 4 5 fifts
1 2 third maj^s **2**
1 2 third min.
1 2 . 6^{ts} maj
1 1 2 sixt minors
1 2 3 4 5 forths

s r t r s t r , s . r . t . r s t
1 2 3 4 5 fourths 3^{d} mode
1 2 3 4 5 fifts
1 2 3 3^{d} maj. **1**
1 2 3 third min
1 2 3 Sixt maj.
1 2 3 Sixt min.

r s t r r t s , r s t r r t s .
1 2 3 4 fourths
1 2 third maj. **4**
1 2 third minor

s r t r r t s , s r t .
1 2 fourths **5**
1 3^{d} maj
1 2 3 3^{d} min.

r . r . t . s s t r , r r t s
1 2 fourths **6**
0 3^{d} maj
1 2 3^{d} min

r r t s r t s , r r t s
1 2 3 4 5 fourths
1 2 third maj. **3**
1 2 third min.

6^{t}. mode
r s r t s r t , r s
1 2 3 4 5 . 4^{ths}
1 2 3 3^{d} maj
1 . 2 . 3 . 3^{d} min

s r r t r s t , s
ac , ab , a , ab , ac , a , ab : ac , ab
ac , ba , aaa ba
ab . ac . aab : a c a b a . a
. . . b . c . b . c . b
a b a a c a b a a c a b , a
a b a c a b a a c a b a

9×4^{ths} . $7 \times 3^{ds}\sharp$. 6 $3^{ds}\flat$.

<112r>

Suppose againe the last line whose nature is comprised in this equation $y\,x^3 = a^4$. $ak = bk = lk = a$ $ao = x$. $ac = v$. $do = y$. $dc = s$. $og = z$.

$ssx^6 + 2vx^7 - x^8 - a^8 = 0$
$- vv$
$0 \quad -1 \quad -2 \quad +6$

$v = \frac{2x^8 - 6a^8}{2x^7}$ $x - v = \frac{3a^8}{x^7}$. to find where $do = dc$

$\frac{a^4}{x^3} = \frac{3a^8}{x^7}$ $a^4x^4 = 3a^8$. $x^4 = 3a^4$. $x = \sqrt{qq{:}\,3}$ $af = a\sqrt{qq{:}\,3{:}}$ $bk = y = a$. $bs = s$ $as = v$ $ak = x = a$ $x + v = \frac{3a^8}{x^7} = x + v = 3a$.

$3a : a :: ki\,(= a) : fh = \frac{a}{3} = mq = ne$ $oa = x = 2\,a$. $\frac{a^4}{x^3} = do = y = \frac{a^4}{8a^3} = \frac{a}{8} = do$. $mn = \frac{7a}{8} = qe$. $mqen = \frac{7aa}{24} = lkog = \frac{7aa}{24}$.

r s t r s t r ; r s $2 \times 3^{d}\flat$. $2 \times 3^{d}\sharp$. 5×4^{th}. no 4^{th} 1^{st} mode 2^{d} mode. 3^{d}.	s r t r s t r ; s r $3 \times 3^{d}\flat$. $3 \times 3^{d}\sharp$. 5×4^{th}. 6^{t}, 4^{th} & 5^{t} $\overline{m}$ode 3^{d} $\overline{m}$ode. 1^{st} $\overline{m}$ode. no 2^{d}	r s t s r t r , r s $2 \times 3^{d}\flat$. $2 \times 3^{d}\sharp$. 3×4^{th}.	s r t s r t r , s r $3 \times 3^{d}\flat$. $3 \times 3^{d}\sharp$. 5×4^{th}. no 1^{st} 5^{t}. 4^{th} $\overline{m}$ode. 3^{d} mode 6^{t} $\overline{m}$ode. 2^{d} $\overline{m}$ode
r r t s s t r ; r r $2 \times 3^{d}\flat$. $0 \times 3^{d}\sharp$. 2×4^{th}.	s s t r r t r ; s s $3 \times 3^{d}\flat$. $1 \times 3^{d}\sharp$. 2×4^{th}.	r s t r r t s ; r s $2 \times 3^{d}\flat$. $2 \times 3^{d}\sharp$. 2×4^{th}. 2^{d} mod.	s r t r r t s ; s r . $3 \times 3^{d}\flat$. $1 \times 3^{d}\sharp$. 2×4^{ths}.
r r t r s t s ; r r . $2 \times 3^{d}\flat$. $2 \times 3^{d}\sharp$. 3×5^{t}.	r r t s r t s , r r $2 \times 3^{d}\flat$. $2 \times 3^{d}\sharp$. 5×4^{ths}. 2^{d} . $\boxed{\text{noe } 3^{d} .}$ 4^{th} mode. 6^{t} mode		

<112v>

1.	e	f		g		a		b	c		d		e	5
2.	a		b	c		d		e	f		g		a	4
3.	d		e	f		g		a		b	c		d	2
4.	g		a		b	c		d		e	f		g	1
5.	c		d		e	f		g		a		b	c	3
6.	f		g		a		b	c		d		e	f	6

r = ton. maj. s = ton min. t = semit maj. v = semit min.

1st. $gd = cg = da = 5^{t} = 2r + s + t$. $dg = gc = ad = r + s + t$. $cd = ga = r$. $ac = s + t$. $ca = 3r + s + t$. $ab = s$. $bc = t$. $dc = 2r + 2s + 2t$. $de = s$. $ef = t$. per sup.

et fg
3 | s t r r s t r ♭
4 | r s t r s t r   Modus ⊕ harum vocum respectu fundamenti.
5 | r s t r r s t ♯

2d. $gd = da = ae = 5^{tæ} = 2r + s + t$. $dg = ad = ea = r + s + t$. $ga = da - dg = de = ae - ad = r$. $ge = 3r + s + t = gd + de$. $eg = s + t$. $ef = t$. $fg = s$. And if $ab = s$. $bc = t$. Then $cd = r$ & the voyces in respect of their {ground} are best
2 | s t r r t s r . ♭
3 | r t s r s t r .   ⊕
4 | r s t r r t s . ♯

If in the 1st case $de = r$. then
3. r t s r s t r. ♭
4. r s t r r s t.   ⊕
5. r r s t r s t. ♯

If in the 2d case $ab = r$ then
2. r t s r t s r. ♭
3. r t s r r t s.   ⊕
4. r r t s r t s. ♯

2 s t r r t s r.	1. t s r r t s r.	3. s t r r s t r.
3 r t s r s t r.	2. r t s r t s r.	4. r s t r s t r.
4 r s t r r t s.	3. r t s r r t s.	5. r s t r r s t.
5 r r t s r s t.	4. r r t s r t s.	6. r r s t r s t.

<113r>

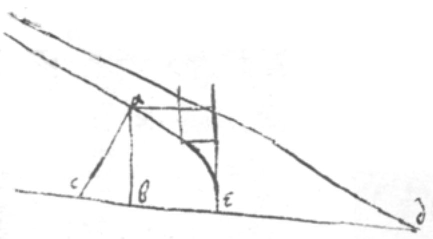
Likewise supposing the line $yx^4 = a^5$. $x - v = \frac{4a^{10}}{x^9} = oc$. $af = a\sqrt{qc: 4}$. $ka\ (= x) + v = 4a$ $fh = \frac{a}{4}$. $do = \frac{a}{16}$. $mq = ne = \frac{a}{4}$. $\frac{15a}{16}$ $\frac{15aa}{64}$. &c whence supposeing x to be a line increasing in arithmeticall proportion from the quantity of the line (a) untill it be as long as b. the superfices resulting out of $\frac{a^3}{xx}$. $\frac{a^4}{x^3}$ &c is found as follows.

$\frac{a^3}{xx} = aa - \frac{a^3}{b}$. $\frac{a^6}{x^5} = \frac{aa}{4} - \frac{a^6}{4b^4}$
$\frac{a^4}{x^3} = \frac{aa}{2} - \frac{a^4}{2bb}$. $\frac{a^7}{x^6} = \frac{aa}{5} - \frac{a^7}{5b^5}$.
$\frac{a^5}{x^4} = \frac{aa}{3} - \frac{a^5}{3b^3}$. $\frac{a^8}{x^7} = \frac{aa}{6} - \frac{a^8}{6b^6}$. &c

1 t r s r t s r
2 r t s r t r s
3 r t r s r t s

1 Of the Key or Ground sound. Secondly, Of its Eighths. Thirdly, of their divisions into Fifts & Fourths Sixts & Thirds, illustrated by the division of a corde. Fourthly, The order of the concords in respect of gratefulnes deduced thence & from other considerations. Fifthly the degrees deduced thence & of the proportion of the concords & degrees i.e. the logarrithmes of their strings. 6 Of the various ordering of the degrees & distance of the halfe notes , the keys fift being onely stable 7 Of the moodes ariseing thence & their dignity; explained by one line, o . p . qr . s . tv . o . p . qr . s . tv . o . p . &c. Eighthly, How the tones major & minor are best ordered in every Moode. Ninthly of passing from one moode to another explained by 3 lines
g . a . bc . d . ef . g
c . d . ef . g . a . bc
f . g . a . bc . d . ef
10 How the notes major and minor to be ordered for that purpose.

<114v>



<115r>

in the Hyperb: $ed = q$. $be = x$. $ab = y$. $\begin{array}{ccccccccccccc} rx & + & \frac{r}{q}xx & = & yy & = & ss & - & xx^2 & + & 2vx & - & vv \\ 1 & & 2 & & & & 0 & & -2 & & - & & 0 \end{array}$ $\frac{r}{2} + \frac{r}{q}x + x$ $= v\frac{r}{2} + \frac{r}{q}x = v - x$ $rx + \frac{r}{q}xx : \frac{rr}{4} + \frac{rrxx}{qq} + \frac{rrx}{q} :: p : z$. $rxz + \frac{r}{q}xxz - \frac{rrp}{4} - \frac{prrxx}{qq} - \frac{rrxp}{q}$
$zz = \frac{p^2qqrr + 4rrxxp^2 + 4qrrxxp}{4qqrx + 4qrxx}$ $zz = \frac{bbb + 4cxx + 4dxb}{qx + xx}$

<115v>

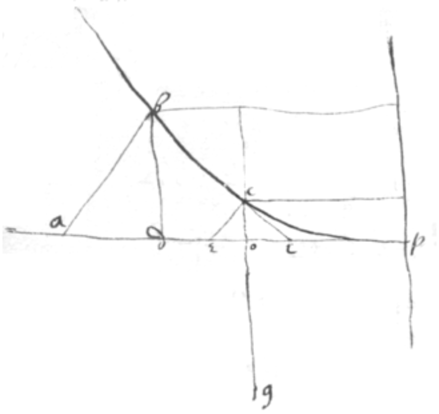
<116r>

$pd = x$. $db = y$. $ayy = x^3$. $ap = v$. $ab = s$. $op = og = b$. $\begin{array}{ccccccccccc} ass & - & avv & + & 2avx & - & axx & - & x^3 & = & 0 \\ 0 & & 0 & & 1 & & 2 & & 3 & & \end{array}$ $\frac{2axx + 3x^3}{2ax} = v$ $v - x = \frac{3xx}{2a} = ad$: $ad^2 = \frac{9x^4}{4aa}$.
$\frac{x^3}{a} : \frac{9x^4}{4a^2} :: bb : zz$. $\frac{z^2x^3}{a} = \frac{9x^4b}{4aa}$. $4az^2 = 9b^2x$.

<120r>

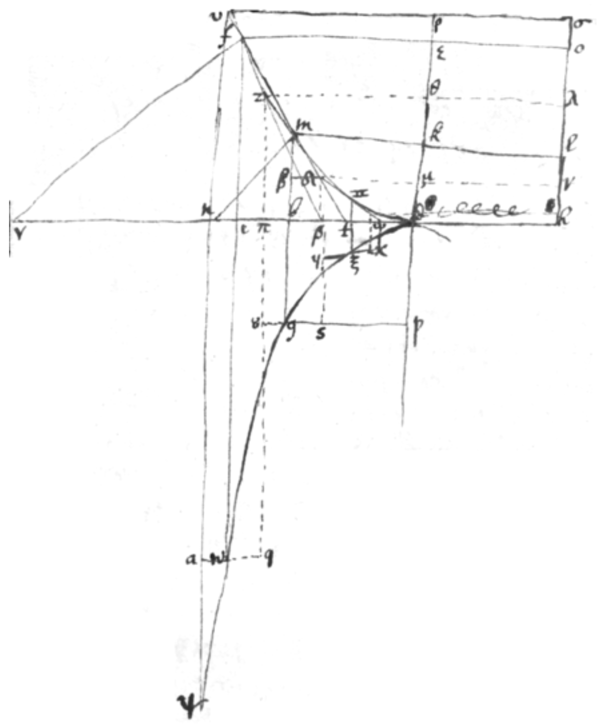
A Method whereby to square those crooked lines which may be squared.

That a line may be squared Geometrically tis required that its area may be expressed in generall by some equation in which there is an unknowne quantity, so that this quantity being determined the area thereof (comprehended by the crooked line, the two lines to which all the points in the crooked line are referred) is limited & may bee found by the same equation. Also every such equation must be of two dimensions because it expresseth the quantity of a superficies.



That an equation expresse the area of a crooked line tis required that the superficie{s} increase in an unequall proportion, when the line (considered as unknowne) increaseth in arithmeticall proportion, wherefore (suppos ing *x* always to signifie the unknowne quantity: *a*, *b*, *c*, &c; to signifie the quantitys given) *ax*, or *xx* either alone or added to any other supperficies, serve not to find the area of any crooked line which may not be found with out them.

<120v>



<121r>

Prop:

Haveing an equation of 2 dimensions to find what crooke line it is whose area it doth expresse, suppose the equation is $\frac{x^3}{a}$. nameing the quantitys; *a* = *dh* = *kl*. *bg* = *y*. *db* = *mk* = *x* = *gp*. the superficies $dbg = \frac{x^3}{a}$ supose the square *dkhl* is equall to the superficies *gbd*; then $dk = z = bm = lh = \frac{x^3}{aa}$, & $aaz = x^3$. which is an equation expressing the nature of the line *fmd*.

$$\begin{array}{ccccccccc} ss & - & vv & + & 2vx & - & vv & = & \frac{x^6}{a^4} & = \\ 0 & & 0\,. & & 1\,. & & 2\,. & & 6\,. & \end{array}$$

mb squared. w^{ch} is an equation haveing 2 equall rootes & therefore multiplyed $2vx = 2xx + 6\frac{x^6}{a^4}$ according to Huddenius his method, produceth another.

Next making *nm*=*s* a line which cutteth *dmf* at right angles. *nd*=*v*.

$v = x + \frac{3x^5}{a^4}$. & $nb = v - x = \frac{3x^5}{a^4}$. Now supposeing, *mb*:*bn*::*dh*:*bg*. that is, $\frac{3x^5}{a^4} : \frac{x^3}{aa} :: y : a$. $\begin{cases} 3 \quad x \quad x = a \quad y\,. \\ 3 \quad x \quad x = a^2 \quad y\,. \end{cases}$ Which is the nature of the line *dgw* & the area $dbg = dklh = \frac{x^3}{a}$, makeing *db*=*x*. *dh*=*a*. or. $diw = deoh = \frac{x^3}{a}$, determining (*di*) to be (*x*). &c

The Demonstration whereof is as followeth

Suppose *ωΠ♌, ♌mz, zfv;* &c are tangents of the line *dmf*. & from theire intersections *z, ♌, v,* draw *va, zq. ♌s. ωx*, & from theire touch points draw *fw, mg, Πξ*. all parallell to *kp*. also from the same point of intersection draw *vσ, zλ, ♌v. ωh.*

<122r>

And *mb:nb::bt:bm::♌B:Bm::kl:bg*. wherefore *♌B×bg=Bm×kl*. that is the rectangle *klvμ=bρsg*. And. *πρs♉=θλvμ*. in like manner it may be demonstrated that *;aqπn=θλσρ*, & *ρωxy=μdvh*. &c so that the rectangle *ρshd* is equall to any number of such like squares inscribed {twixt} the line *ny* & the point *d*, which squares if they bee infinite in number, they will bee equall to the superficies *dnywgξ*.

This being demonstrated that I may shunne confusion in squareing the lines of every sort I shall use this method in. distinguishing them. viz: first such lines whose area is exprest by equations in which the unknowne quantity is numerator, & that 1[st] all the sines being affirmative, 2[d]ly mixed.

2dly lines whose area is exprest by quantitys in which the unknowne quantity is divisor, & those 1[st] under affirmative sines, 2[d] under mixt one's 3 lines squared by equations mixt of the 2 former kinds, whose quantitys are all 1[s] affirmative 2dly mixt.

<124r>

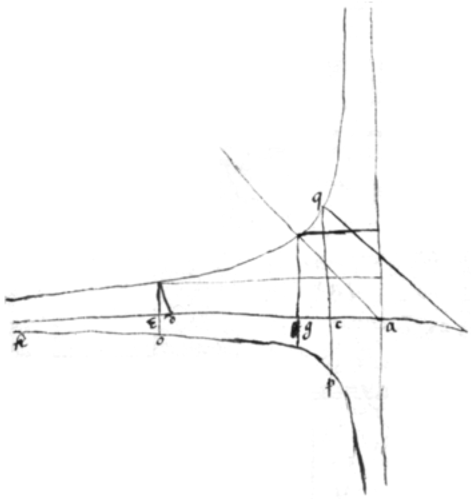
The squareing of those lines whose area is exprest by affirmative quantitys in which the unknowne quantity is {n}umeral{e}

The equations expressing the nature of y^e lines.		Theire square.	
$3xx = ay$.	Parab:	$\frac{x^3}{a}$.	
$4x^3 = aay$.	$-----$	$\frac{x^4}{aa}$	& soe infinitely.
$5x^4 = ya^3$	$-----$	$\frac{x^5}{a^3}$	
$6x^5 = ya^4$	$-----$	$\frac{x^6}{a^4}$	
$7x^6 = ya^5$	$-----$	$\frac{x^7}{a^5}$.	
$4x^3 + 3bx^2 = bay$		$\frac{x^3}{a} + \frac{x^4}{ab}$	
$5x^4 + 3b^2x^2 = bbay$		$\frac{x^3}{a} + \frac{x^5}{abb}$	
$6x^5 + 3b^3x^2 = b^3ay$		$\frac{x^3}{a} + \frac{x^6}{ab^3}$	
$7x^6 + 3b^4x^2 = b^4ay$		$\frac{x^3}{a} + \frac{x^7}{ab^4}$	

Soe that the nature of every crooked line, whose area is compounded of the area of 2 or more of the former lines, or of the difference of the area of 2 or more of the former lines, is exprest by an equation compounded of the equations expresing the nature of those lines.

$4x^3 - 3bx^2 = bay$	$\frac{x^4 - x^3b}{ab}$
$5x^4 - 3bbx^2 = bbay$	$\frac{x^5 - x^3bb}{abb}$
$6x^5 - 3b^3x^2 = b^3ay$	$\frac{x^6}{ab^3} - \frac{x^3}{a}$.

<124v>



<125r>

The squareing those lines whose area is exprest by an equation in which the unknown quantity is denominator.

The Equations expressing y^e nature of y^e lines.		The square thereof when
$xxy = a^3$	$---------$	$\frac{a^3}{x}$
$x^3y = 2a^4$	$---------$	$\frac{a^4}{xx}$
$x^4y = 3a^5$	$---------$	$\frac{a^5}{x^3}$
$x^5y = 4a^6$	$---------$	$\frac{a^6}{x^4}$
$x^6y = 5a^7$	$---------$	$\frac{a^7}{x^5}$.
$4yy = 9a^8x$	Parab.$---$	$x\sqrt{ax}$
$4ayy = 25x^3$	$---------$	$\frac{xx}{a}\sqrt{ax}$
$4a^3yy = 49x^5$	$---------$	$\frac{x^3}{aa}\sqrt{ax}$
$4a^5yy = 81x^7$	$---------$	$\frac{x^4}{a^3}\sqrt{ax}$
$4a^7yy = 121x^9$	$---------$	$\frac{x^5}{a^4}\sqrt{ax}$

$4xyy = a^3 \qquad -------- \qquad a\sqrt{ax}$
$4x^3y^2 = a^5 \qquad -------- \qquad \frac{aa}{x}\sqrt{ax}$
$4x^5y^2 = 9a^7 \qquad -------- \qquad \frac{a^3}{xx}\sqrt{ax}$
$4x^7y^2 = 25a^9 \qquad -------- \qquad \frac{a^4}{x^3}\sqrt{ax}$
$4x^9y^2 = 49a^{11} \qquad -------- \qquad \frac{a^5}{x^4}\sqrt{ax}$

<126r>

$9ax^2 + 12aax + 4a^3 = 4yyx + 4ayy. \qquad x\sqrt{ax+aa}$
$25x^4 + 40ax^3 + 16aax^2 = 4axy^2 + 4aay^2. \qquad \frac{x^2}{a}\sqrt{ax+aa}$
$49x^6 + 84ax^5 + 36a^2x^4 = 4a^3xy^2 + 4a^4y^2. \qquad \frac{x^3}{aa}\sqrt{ax+aa}$
$81x^8 + 144ax^7 + 64aax^6 = 4a^5xy^2 + 4a^6y^2. \qquad \frac{x^4}{a^3}\sqrt{ax+aa}$

$a^3 = 4xy^2 + 4ayy. \qquad a\sqrt{ax+aa}$
$a^5x^2 + 4a^6x + 4a^7 = 4x^5yy + 4ax^4y^2. \qquad \frac{aa}{x}\sqrt{ax+aa}$
$9a^7x^2 + 24a^8x + 16a^9 = 4x^7y^2 + 4ax^6y^2. \qquad \frac{a^3}{x^2}\sqrt{ax+aa}$
$25a^9x^2 + 60a^{10}x + 36a^{11} = 4x^9y^2 + 4ax^8y^2. \qquad \frac{a^4}{x^3}\sqrt{ax+aa}$

$9ax^2 - 12aax + 4a^3 = 4xy^2 - 4ayy. \qquad x\sqrt{ax-aa}$
$25x^4 - 40ax^3 + 16aax^2 = 4axy^2 - 4aayy. \qquad \frac{x^2}{a}\sqrt{ax-aa}$

$a^3 = 4xy^2 - 4ay^2. \qquad a\sqrt{ax-aa}$
$a^5x^2 - 4a^6x + 4a^7 = 4x^5yy - 4ax^4yy. \qquad \frac{aa}{x}\sqrt{ax-aa}$

$9ax^2 - 12aax + 4a^3 = 4ayy - 4xyy. \qquad x\sqrt{aa-ax}$
$25x^4 - 40ax^3 + 16aax^2 = 4a^2yy - 4axyy. \qquad \frac{x^2}{a}\sqrt{aa-ax}$

$a^3 = 4ay^2 - 4xy^2. \qquad a\sqrt{aa-ax}$
$a^5x^2 - 4a^6x + 4a^7 = 4ax^4yy - 4x^5yy. \qquad \frac{aa}{x}\sqrt{aa-ax}.$

Note that the lines whose nature is exprest by the 4 latter sorts of equations, are the same with the lines of the 2 former sorts. Doubtfull.

<127r>

$9axx + 24axx + 16x^3 = 4xyy + 4ayy. \qquad x\sqrt{ax+xx}$
$25aax^3 + 60ax^4 + 36x^5 = 4aaxyy + 4a^3yy. \qquad \frac{xx}{a}\sqrt{ax+xx}$
$49aax^5 + 112ax^6 + 64x^7 = 4a^4xyy + 4a^5yy = \qquad \frac{x^3}{aa}\sqrt{ax+xx}$
$81aax^7 + 180ax^8 + 100x^9 = 4a^6xyy + 4a^7yy = \qquad \frac{x^4}{a^3}\sqrt{ax+xx}$

$a^4 + 4a^3x + 4aaxx = 4axyy + 4xxyy. \qquad a\sqrt{ax+xx}.$
$a^6 = 4ax^3yy + 4x^4yy. \qquad \frac{aa}{x}\sqrt{ax+x^2}.$
$9a^8 + 12a^7x + 4a^6x^2 = 4ax^5yy + 4x^6y^2. \qquad \frac{a^3}{xx}\sqrt{ax+xx}.$
$25a^{10} + 40a^9x + 16a^8x^2 = 4ax^7yy + 4x^8yy. \qquad \frac{a^4}{xx}\sqrt{ax+xx}.$

$9aax - 24axx + 16x^3 = 4ayy - 4xyy. \qquad x\sqrt{ax-xx}.$

<128r>

$4x^4 + 4aaxx + a^4 = xxyy + aayy \qquad x\sqrt{aa+xx}$
$9x^6 + 12aax^4 + 4a^4x^2 = aaxxy^2 + a^4y^2. \qquad \frac{xx}{a}\sqrt{aa+xx}$
$16x^8 + 24aax^6 + 9a^4x^4 = a^4x^2y^2 + a^6y^2. \qquad \frac{x^3}{a^2}\sqrt{aa+xx}$
$25x^{10} + 40aax^8 + 16a^4x^6 = a^6x^2y^2 + a^8y^2. \qquad \frac{x^4}{a^3}\sqrt{aa+xx}.$

$aaxx = aayy + xxyy. \qquad a\sqrt{aa+xx}$
$a^8 = aax^4yy + x^6yy \qquad \frac{aa}{x}\sqrt{aa+xx}$
$4a^{10} + 4a^8xx + a^6x^4 = aax^6yy + x^8yy. \qquad \frac{a^3}{xx}\sqrt{aa+xx}$
$9a^{12} + 12a^{10}xx + 4a^8x^4 = aax^8y^2 + x^{10}yy. \qquad \frac{a^4}{x^3}\sqrt{aa+xx}.$

$4x^4 - 4aaxx + a^4 = aayy - xxyy. \qquad x\sqrt{aa-xx}$

$aaxx = aayy - xxyy. \qquad a\sqrt{aa-xx}.$
$a^8 = aax^4yy - x^6yy \qquad \frac{aa\sqrt{aa-xx}}{x}$
$a\sqrt{\frac{a^3-axx}{x}}.$

<129r>

$a^4 + 3a^2bx + 4aax^2 + \frac{9}{4}bbx^2 + 6bx^3 + 4x^4 = a^2y^2.$ $\qquad x\sqrt{aa+bx+xx}$

$+bx$

$+xx$

$aabb + 4aabx + 4aaxx = 4ccyy + 4bxy^2 + 4xxy^2$ $\qquad a\sqrt{cc+bx+xx}$

$\sqrt{a^3x+x^4}$

$\sqrt{a^4+ax^3}$

$9ax^4 + 6a^3x^2 + a^5 = 4aaxyy + 4x^3yy$ $\qquad \sqrt{a^3x+ax^3}.$

$\sqrt{a^4+x^4}$

<130r>

$\frac{a^3}{a+x}$
$\frac{a^2x}{a+x}$
$\frac{ax^2}{a+x}$
$\frac{x^3}{a+x}$

$\frac{a^3}{a-x}$

$\frac{a^3}{x-a}$

<131r>

.

$a^3 = bby + 2bxy + xxy$ ----- $\frac{a^3}{b+x}$

$2b^4x + b^4a = x^4y + 2ax^3y + aaxxy$ ----- $\frac{b^4}{ax+x^2}$

$3b^5x + 2b^5a = x^5y + 2ax^4y + a^2x^3y$ ----- $\frac{b^5}{ax^2+x^3}$

$4b^6x + 3b^6a = x^6y + 2ax^5y + a^2x^4y$ ----- $\frac{b^6}{ax^3+x^4}$

$\frac{x^3}{a+x}$
$\frac{x^4}{aa+ax}$
$\frac{x^5}{a^3+xa^2}$
$\frac{x^6}{a^4+xa^3}$

$aab = bby + 2bxy + xxy.$ ----- $\frac{a^2x}{b+x}$

$2bax + axx = bby + 2bxy + xxy.$ ----- $\frac{ax^2}{b+x}$

<132r>

$2a^4x = b^4y + 2bbxxy + x^4y$

$$\frac{a^4}{bb+x^2} \quad \frac{a^5}{bbx+x^3} \quad \frac{a^6}{bbx^2+x^4} \quad \frac{a^7}{bbx^3+x^5}$$

$$\frac{x^4}{bb+x^2} \quad \frac{x^5}{b^3+x^2b} \quad \frac{x^6}{b^4+bbx^2} \quad \frac{x^7}{b^5+bbx^2}$$

$$\frac{a^3x}{bb+x^2} \quad \frac{aaxx}{bb+xx} \quad \frac{ax^3}{bb+xx}$$

<132v>

$27x^2y^3 = a^5$ ——— $a\sqrt{c : aax}\,.$
$27x^5y^3 = 8a^8$ ——— $\frac{aa}{x}\sqrt{c : aax}$
$27x^8y^3 = 125a^{11}$ ——— $\frac{a^3}{x^2}\sqrt{c : aax}$
$27x^{11}y^3 = 512a^{14}$ ——— $\frac{a^4}{x^3}\sqrt{c : aax}$

$27xy^3 = 8a^4$ ——— $a\sqrt{c : axx}\,.$
$27x^4y^3 = a^7$ ——— $\frac{aa}{x}\sqrt{c : axx}\,.$
$27x^7y^3 = 64a^{10}$ ——— $\frac{a^3}{xx}\sqrt{c : axx}\,.$
$27x^{10}y^3 = 343a^{13}$ ——— $\frac{a^4}{x^3}\sqrt{c : axx}\,.$

$27y^3 = 64aax$ ——— $x\sqrt{c : aax}\,.$
$27ay^3 = 343x^4$ ——— $\frac{xx}{a}\sqrt{c : aax}\,.$
$27a^4y^3 = 1000x^7$ ——— $\frac{x^3}{aa}\sqrt{c : aax}\,.$

$27y^3 = 125ax^2$ ——— $x\sqrt{c : axx}$
$27a^2y^3 = 512x^5$ ——— $\frac{xx}{a}\sqrt{c : axx}$
$27a^5y^3 = 1331x^8$ ——— $\frac{x^3}{a}\sqrt{c : axx}\,.$

<133r>

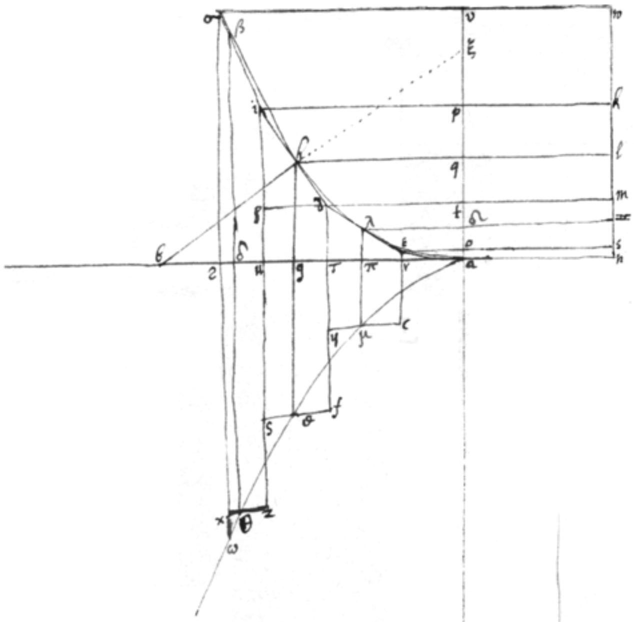
$$\frac{a^5}{b^3+x^3} \quad \frac{a^6}{b^3x+x^4} \quad \frac{a^7}{b^3x^2+x^5} \quad \frac{a^8}{b^3x^3+x^6} \quad \frac{x^5}{b^3+x^3} \quad \frac{x^6}{b^4+x^3b} \quad \frac{x^7}{b^5+bbx^3} \quad \frac{x^8}{b^6+b^3x^3} \quad \frac{a^4x}{b^3+x^3} \quad \frac{a^3x^2}{b^3+x^3} \quad \frac{a^2x^3}{b^3+x^3} \quad \frac{ax^4}{b^3+x^3}$$

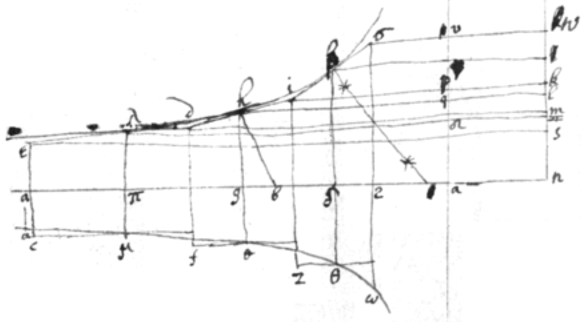
<133v>

$256x^3y^4 = a^7.$ ——— $a\sqrt{qq : aaax}\,.$
$256x^7y^4 = 81a^{11}.$ ——— $\frac{aa}{x}\sqrt{qq : aaax}\,.$
$256x^{11}y^4 = 2401a^{15}$ ——— $aaa\sqrt{qq : aaax}\,.$

$256xy^4 = 81a^5$ ——— $a\sqrt{qq : ax^3}$
$256x^5y^4 = a^9$ ——— $\frac{aa}{x}\sqrt{qq : ax^3}$
$256x^9y^4 = 625a^{13}$ ——— $\frac{a^3}{xx}\sqrt{qq : ax^3}$
$256x^{13}y^4 = 6561a^{17}$ ——— $\frac{a^4}{x^3}\sqrt{qq : ax^3}$

<134v>





<135r>

A Method whereby to square such crooked lines as may be squared.

If the crooked lines *σha* & *aoθ* are of such a nature that (supposeing [*gh*] parallell to [*qa*], & [*bh*] perpendic: to *σha* & [*an*] a given line) *gh*∶*bg*∷*an*∶*ge*. Then the area [*age*]=[*qlna*] the rectangle made by [*an*] & [*gh*].

Demonstration.

Suppose *σi, id, de,* &c; are tangents of *σha*, from whose intersections or ends are drawne, *ec, df, iz, σw,* &{c}**{illeg}** & from whose touch points are drawne *βθ, ho, λμ,* &c: all parallel to *av*. From the said intersections draw *sw, ik, dm, es,* &c. parallel to *bn*. Since *gh*∶*bg*∷*pd*∶*ip*∷*an*∶*ge*. *pd*×*ge*=*iv*×*an*. that is □*pkmt*=□*uτfs*. by the same reason *tmso*= =*τvcy*; & *vpkw*=*ζuzx* &c: Thus also it may be prooved that the ▭vwna is equall to anyy number of such like ▭s inscribed twixt the line *ζω* & the point a, which if they be infinite are equall to superficies *ζaω*=*vwna* .also *gπμo*. =*qlΠΩ*. &c.

Prop 1

To find the line whose area is exprest by any given equation. Suppose the equatiō is $\frac{x^3}{a}$. nameing the quantitys *a*=*an*. *x*=*ag*. $\frac{x^3}{a}=qlna=goa$ $gh=qa=\frac{x^3}{aa}$. *bh*=*s*. . *ba*=*v*. $\begin{array}{ccccccccc} ss & - & vv & + & 2vx & - & xx & = & \frac{x^6}{a^4} \\ 0 & & 0 & & 1 & & 2 & & 6 \end{array}$ equa hath 2 equall rootes & is therefore multiplied according to Huddenius his Meth**{illeg}**

$vx=x^2+3\frac{x^6}{a^4}$. $gb=v-x=\frac{3x^5}{a^4}$. Wherefore if $\frac{x^3}{aa}:\frac{3x^5}{a^4}::a:\frac{3xx}{a}=ge$. therefore *aoω* is a Parab: & $age=\frac{x^3}{a}=qlna$

<135v>

Also if the Equation be $\frac{a^3}{x}$. Then makeing $a=an.x=ag.\ \frac{a^3}{x}=qlna=gea.\ qa=\frac{aa}{x}=gh$. $bh=s.ba=v$. $\begin{array}{ccccccccc} ss & - & vv & + & 2vx & - & xx & = & \frac{a^4}{x}. \\ 0 & & 0 & & -1 & & -2 & & +2 \\ & & & & -2vx & - & +2xx & = & \frac{2a^4}{xx}. \\ & & & & v=x-\frac{a^4}{x^3}. & & & & \end{array}$. which multiplied by Huddenius his method by reasō of z equall rootes. $x-v=gb=\frac{a^4}{x^3}$. Lastly, $\frac{aa}{x}:\frac{a^4}{x^3}::a:\frac{a^3}{xx}:\frac{a^3}{xx}=ge=y$. & $a^3=xxy$. which last equation expresseth the nature of the line *aθo*, whose surface $aeg=\text{qlan}=\frac{a^3}{x}$.

Note that I call that line [*x*] to which both the lines σha & aoω have respect as *πα, ga,* &c. but that line to which but one line hath respect I call [*y* as *go, πμ*: or [*z*] as *gh, πλ,* &c.

If $ax^m=by^n$.(*m* & *n* being numbers that signifie the dimensions of *x* & *y*), then $\frac{nxy}{n+m}=ago$, the area of the line aμo. And if $a=b\times x^m\times y^n$. y^n is $\frac{nxy}{x-m}=ago$. the area of that line.

<136r>

The squareing of the simplest lines in which *y* is but of one dimension.

Equations expressing y^e nature of y^e lines.		Theire squares.	Lines.		$\square$.
$3xx = ay.$	Parab:——	$\frac{x^3}{a}.$	$xxy = a^3$	————	$\frac{a^3}{x}.$
$4x^3 = aay$	————	$\frac{x^4}{aa}.$	$x^3y = 2a^4$	————	$\frac{a^4}{xx}.$
$5x^4 = ya^3$	————	$\frac{x^5}{a^3}.$	$x^4y = 3a^5$	————	$\frac{a^5}{x^3}.$
$6x^5 = ya^4$	————	$\frac{x^6}{a^4}.$ &c.	$x^5y = 4a^6$	————	$\frac{a^6}{x^7}.$ &c

The square of the simplest lines in which *y* is of 2 dimensions.

The lines squared.		Theire Squares.	Lines squared.		$\square$s.
$4yy = 9ax.$	Parab:——	$x\sqrt{ax}.$	$4xyy = a^3$	————	$a\sqrt{ax}.$
$4ayy = 25x^3$	————	$\frac{xx}{a}\sqrt{ax}.$	$4x^3yy = a^5$	————	$\frac{aa}{x}\sqrt{ax}.$
$4a^3yy = 49x^5$	————	$\frac{x^3}{aa}\sqrt{ax}.$	$4x^5yy = 9a^7$	————	$\frac{a^3}{xx}\sqrt{ax}.$
$4a^5yy = 81x^7$	————	$\frac{x^4}{a^3}\sqrt{ax}.$	$4x^7yy = 25a^9.$	————	$\frac{a^4}{x}\sqrt{ax}$ &c
		&c.			

The square of those $\sqrt{\text{lines}}$ where *y* is of 3 dimensions onely.

The lines squared.		Their squares	Lines squared.		Theire squares
$27xy^3 = 8a^4$	————	$a\sqrt{c : axx}.$	$27y^3 = 64aax$	————	$x\sqrt{c : aax}.$
$27x^4y^3 = a^7$	————	$\frac{aa}{x}\sqrt{c : axx}.$	$27ay^3 = 343x^4$	————	$\frac{xx}{a}\sqrt{c : aax}.$
$27x^7y^3 = 64a^{10}$	————	$\frac{a^3}{xx}\sqrt{c : axx}.$	$27a^4y^3 = 1000x^7$	————	$\frac{x^3}{aa}\sqrt{c : aax}.$
$27x^{10}y^3 = 343a^{13}$	————	$\frac{a^4}{x^3}\sqrt{c : axx}.$	$27a^7y^3 = 2197x^{10}.$	————	$\frac{x^4}{a^3}\sqrt{c : aax}$
		&c			&c
$27xxy^3 = a^5$	————	$a\sqrt{c : aax}.$	$27y^3 = 125ax^2$	————	$x\sqrt{c : axx}.$
$27x^5y^3 = 8a^8$	————	$\frac{aa}{x}\sqrt{c : aax}.$	$27aay^3 = 512x^5$	————	$\frac{xx}{a}\sqrt{c : aax}.$
$27x^8y^3 = 125a^{11}$	————	$\frac{a^3}{xx}\sqrt{c : aax}.$	$27a^5y^3 = 1331x^8$	————	$\frac{x^3}{aa}\sqrt{c : axx}.$
$27x^{11}y^3 = 512a^{14}$	————	$\frac{a^4}{x^3}\sqrt{c : aax}.$	$27a^8y^3 = 2744x^{11}$	————	$\frac{x^4}{a^3}\sqrt{c : axx}$
		&c			&c

<138r>

Of Musick.

1. First some one sound must bee pitched upon, to which all the musick must bee more especially refered than to any other sound, (as number to an unit) let this sound be called the Cliffe or Key of the song.

2. Then consider the sound which is one or two or thre 8^{ths} above or below that key (for Musick seldome takes a larger compasse than 3 8^{ths}) The cheife of which is the 8^{th} next above the Key. 3.Each of these Eights are alike divided into parts, for the parts of the higher eight are an Eight above their correspondent parts of the lower eight. so that the parts of one Eight knowne give all the rest, the other Eights being but a repetition of that. in {a}more base or treble sound. (Hence some call an 8^{th} the largest consonant.)

4. This Eight is first divided into a 5^t & 4^{th}, the fift being next above the Key; to which it adds so much sweetnesse that should this fift bee omitted in any song, the Key would imparte its name & nature to some sound which hath a fift above it. And since all harmony without a fift is flat, therefore the key must necessarily have a fift above it. ✝ < insertion from f 137v > ✝ here annex a discourse of the motion of strings sounding an 8^t 5^t & 4^{th} & of the Logarithmes of those strings, or distances of the notes.

< text from f 138r resumes >

5. An 8^{th} is next divided into a third major & 6^t minor, & lastly into a 3^d minor & 6^t major. * < insertion from f 137v > * these are all the concords conteined in an Eight. Hereto annex a discourse of the 3^{ds} & 6^{ts}

The notes in order of concordance

Eight. 5^t. 3^d maj. 4^{th}. 6^t maj. 3^d min. 6^t min. 2^d maj. 7^{th} maj 7^{th} min. 2^d min. 5^t min. < text from f 138r resumes > But as too suddaine a change from lesse to greater light offfends the eye by reason that, the spirits rarified by the augmented motion of the light too violently stretch the optick nerve: soe the suddaine passing from grave to acute sounds is not so pleasant as if it were done by degrees, because of too greate a change of motion made thereby in the auditory spirits ♀.✝ < insertion from f 137v > ✝And as a man suddainely cooming from greater to lesse light, cannot discerne objects thereby so well, as if he came to it by degrees or as when hee hath staid some while in the lesser light (by reason that the motion of the spirits in the optick nerve caused by the greater light, doth, untill it bee allayed; disturbe & as it were drowne the motion of the weaker light) soe if the slower motion of the lower sound immediately succede the much more smart motion of the higher its impression on the auditory spirits — being then less perceptible, the lower sound must bee less pleasant that if the step had beene graduated, Thus a little heate is least perceptible to one newly come from a greater. Coroll: 1. The distance of sounds adds to the imperfection of their concordance. Cor: 2: Tis better to descend than ascend by leapes the first makeing the highest sound harsher, the seacond makeing the lower onely lesse perceptible. < text from f 138r resumes > Which graduation may be thus don.

6. The prime parts of an 8^{th} are a 5^t & 4^{th}: of a fift are a 3^d major & 3^d minor: which two consist the first of a tone major & tone minor, the 2^d of a tone major & semitone. A 4^{th} consists of a tone major, minor & semitone. Soe that an eight consists of thre <139r> tone majors, 2 tone minors, & 2 semitones. [The tones might be againe divided into $\frac{1}{2}$ tones & $\frac{1}{4}$ tones, but they would bee of noe use for tones $\frac{1}{2}$ tones & $\frac{1}{4}$ tones being discords can onely serve to move by from concord to concord which if done by $\frac{1}{2}$ tones & $\frac{1}{4}$ tones the number of discords twixt each concord would much more bee harsh than the concord would bee pleasant, besides $\frac{1}{2}$ tones & $\frac{1}{4}$ tones are harsher discords by far than tones, & experience speakes that an 8^{th} run over by $\frac{1}{2}$ notes is unpleasant. Yet perhaps $\frac{1}{2}$ or $\frac{1}{4}$ notes passed over very hastily with a larger stay upon the concords twixt which they are, might bee delightfull. But since they are such discords, inserted as 'twere by accident onely to

graduate concords, & soe quickly slipt over, the sence cannot perceive any error or exactnesse in them, & therefore bee they usefull yet to treate of them would be lost labor]

7. The degrees (viz 2 tone majors, a tone minor & semitone in the 5^t & a tone major, a tone minor & semitone in a 4^th) are 12 severall ways ordered in the 8^th which orders are called Modes, generally, because they much limit the partes of the tune from discord sounds of one with another particularly because tunes framed by divers of them differ in their aires or Modes.

8. These modes are 3 fold, viz: 6 in which the $\frac{1}{2}$ notes are distant 2 tones: foure in which they are distant one tone: & 2 in which they are together. The last two are of small or noe use, because every sound is distant 3 tones from some other excepting that there are but 2 fifts. Also thos $\frac{1}{2}$ notes are two harsh to come together much more to bee annext to the Key or its fift. Neither is the seacond sort very useful for one of the $\frac{1}{2}$ notes are annexed either to the Key or its 5^t or 8^t, also 4 of its sounds are distant 3 notes & but 4 of them are distant a fift from some other: whereas there are but 2 in those of the first sort distant **{**those**}** notes & six of them distant fifts from other sounds.; the harshnes of the $\frac{1}{2}$ notes being there also more moderated by their distance. And therefore the first 6 are yet in use.

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9. The following table may expresse the 12 Modes in their order of Elegancy. In which the tone major & minor are not distinguished, their difference being too little to make new modes by their order changed, though thereby they may add much grace or harshnesse to any particular mode.

	y^e key		2^d	3^d minor	3^d major	4^th	Tritonus	5^t	6^t minor	6^t major	7^th			8^th
4.	O	.	p	.	q	r	.	s	.	t	v	.	o	1
3.	s	.	t	v	.	o	.	p	.	q	r	.	s	2
5	r	.	s	.	t	v	.	o	.	p	.	q	r	3
2	p	.	q	r	.	s	.	t	v	.	o	.	p	4
1	t	v	.	o	.	p	.	q	r	.	s	.	t	5
6	v	.	o	.	p	.	q	r	.	s	.	t	v	6
	o	.	p	.	q	r	.	s	x	.	v	.	o	7
	r	.	s	x	.	v	.	o	.	p	.	q	r	8
	s	x	.	v	.	o	.	p	.	q	r	.	s	9
	v	.	o	.	p	.	q	r	.	s	x	.	v	10
	o	.	p	.	q	.	y	s	x	.	v	.	o	11
	s	x	.	v	.	o	.	p	.	q	.	y	s	12

This order may be thus evinced. The first Mode excells the 2^d, by reason of the $\frac{1}{2}$ Note's more convent place twixt the Key & its fift, it lesse detracting from the fift because of its greater distance from it. Also the key hath its 3^d major & the fift its 3^d minor in the 1^st mode, but contrarily in the 2^d mode the key hath its 3^d minor & the 5^t its 3^d major. The sweetness of the key in the 3^d mode is still more diminished by haveing the $\frac{1}{2}$ note imediateely below it & its 8^ts. The 4^th Mode succedes as partakeing of the 3^ds defect; the sweetnesse of its key's 5^t, & consequently of its key, being also diminished by the $\frac{1}{2}$ note immediately above it. The 5^t mode succeds because to the imperfections of the 4^th this is added that its first $\frac{1}{2}$ note is next above the key & its fifts have tritones. The 6^t mode is yet more unpleasant <141r> for both the key, its 5^ts, & eights have a $\frac{1}{2}$ note next below them: Also the key & its eights have tritones above & below them. Other reasons might bee added for this order, & also for the order of the sixt last modes; & it might perhaps bee shown that the 7^th mode may bee as usefull as the Sixt, but that would bee tedious. Note, that sometime a note is put out of its place for some particular reason (as to prevent a greater discord &c) but that seemes soe rare & accidentall to the song as not to change its aire or constitute a new mode.

10. The tones major & minor may bee six severall ways ordered in each mode & but 10 severall ways in all the six first modes. . the first is by makeing the distances, *pq, rs, vo*, to bee tone majors *op*, & *st, to* bee tone minors. In this order there are five 5^ts, 3 third majors, & 3 third minors in an 8^th. Thus is the 3^d 5^t & first mode best ordered, & thus may the 4^th & 6^t moode bee ordered but not the 2^d well for its keys fift will thenbee oo flat. The 2^d way is by putting the tone minor twixt, *o* & *p*, *r* & *s*. This order makes also 5 fifts, thre 3^d majors & 3 3^d minors, in each 8^th. And thus may the 4^th, 6^t, & 2^d Moode bee best ordered; the 3^d & 5^t moode may bee also ordered thus, but the first not well, for the Keys 5^t will then bee too flatt. The 3^d way is by putting the minor note betwixt *r* & *s*, *v* & *o*. & thus each 8^th will have five fifts, 2 third majors & 2 minor thirds. The 4^th 6^t & 2^d moode may bee well thus ordered the 1^st & 5^t not so well & the 3^d worst of all. The 4^th Order is by putting the minor tone twixt *p* & *q*, *s* & *t* & thus each 8^th hath 5 fifts, 2 minor 3^ds, & 2^d moode bee ordered well, but the 6^t & 4^th moode not well. The other six orders are lesse convenient to the Moodes. Note that, In every 8^th there are 6 5^ts, 3 major thirds & 4 minor thirds whereof one or more of them are mad**{**e**}** too flat or sharpe by about the 10^th parte of a note, but in this computation I onely reckon the exact concords <141v> Esteeming that order more perfect whose sounds agree in more of the exact concords. Note also that every Eight hath soe many exact 4^ths, 6^t minors & third majors as it hath 5^ts, 3^d majors & 3^d minors their complements to an 8^th.

12. It may bee required sometimes to raise or let fall the voyce in singing which is best done by raising or depressing the key of the song a fift, (if an 8^t be too greate), for that will bee consonant with the former sound which is now become (for the present) gratefull to the eare. Also instruments are usually tuned one a fift above another if the keys of severall parts be a fift one above another; & a tune might bee pricked for too high a voyce in one parte of the Gamut & too base a voyce if removed an 8^th lower. Hence ariseth a comparison of the same moode with it selfe placed a fift higher. The precedent scheme may serve to represent any of the six modes repeated six times with the distance of a fift twixt each, according to the order of the left hand figures. But they cannot bee soe repeated more than 3 times, unlesse with more discord than harmony.

1 . o . p . q r . s . t v . o . p . q r . s . t v . o . p . q r . s .
2 . r . s . t v . o . p . q r . s . t v . o . p . q r . s . t v . o .
3 . v . o . p . q r . s . t v . o . p . q r . s . t v . o . p . q r .

f G , a , b , c , d , e , f , g , a b c d e f g a b c
3

Any of the 6 Moodes with its eights may bee represented by any of these 3 orders of letters for the key being *o* they re present the first Moode, & the second it being, *s*, & the 3d if it be *r* &c: Also the first ranke being lowest the 2d a fift above it & the 3d a fift above that,this scheame may represent any of the Modes with the same mode one or 2 fifts above or below it.

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11. These degrees have of old beene expressed by the Six notes, *vt, re, mi, fa, sol, la,* the 7th note being omitted as being a discord to the key in the first moode. But of late the usuall notes are sol, la, mi, fa, sol, la, fa, hitherto expressed by the letters o, p. q. r. s.t. v. Tis generally best (by see 10) to make the distance from sol to la, to be a minor tone, from la to mi & fa to sol a major tone, & a semitone from la to fa & mi to fa. Onely in the 2d Mode make sol & la & mi to bee distant a major tone, fa a minor tone from sol els the fift to the key will bee too flat. Or thus if the key bee f, a, b, or c make the distances twixt g & a, c & d to bee a minor tone if the key bee d or e make the distances from f to g & c to d a minor tone, but if it be g make $a-g=d-c=g-f$.

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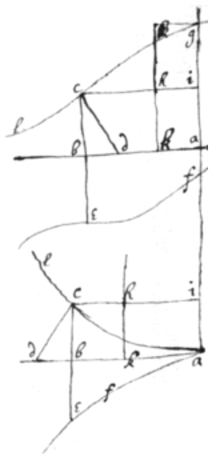
13. Tis usuall to passe from one moode to another in the midst of a song which how & to what moode it may be done will appeare by the precedent scheme. For the 3 rankes may signifie any three Moodes which have one common key, as F is the key of the first third & sixt Moode, G the key of the first 2d & 4th mood &c: And wee may passe from any of those Moodes to another which in that scheme have the same key. But this transition is better done from one key to the key next it, than to the remoter key. Neither may it bee done twixt any other Moodes as twixt the first & fift or 3d & 4th by reason of their great difference, which would soe change the aire of the song as to make the parts of it rather seeme divers songs.

14. It may app10) that if the key bee f, or b, or e, the the transition may be best done the degrees of the Moode being ordered the first way. If the key bee a or d the 2d order is best. If the key bee g the 3d order is best, & the fourth the key being c. But in generall, if the degrees bee ordered the 4th way in the 2d Moode & the 1st way in all the rest, this transition may bee well done.

15. from the consideration of passing from one moode to another in the same song two other moodes may bee usefull the one whereof wants the key the other its fift, but these defects are parly supplyed by the eares retaining the impression of their sweetness made by the former parte of the song. q is the key of one moode & v the key's 5t in the other moode.

<147r>

A Method whereby to find the areas of Those Lines which can bee squared.



Prop: 1st. If *ab=x ⊥y=be. cb=z. bd=v* secant=*cd. m* & *n* are numbers expressing the dimensions of *x, y,* or *z. a, b, c, d,*&c:are knowne quantitys, & $\frac{ax^{\frac{m}{n}}}{b} = z$. then $\frac{mazx^{\frac{m}{n}}}{nbx} = = \frac{mazx^{\frac{m-n}{n}}}{nb} = v$. And in generall what ever the relation twixt *x* & *z* bee, make all the termes equall to nothing, multiply each terme by so many times *zz* as *x* hath dimensions in that terme, for a Numerator: then multiply each terme by soe many times $-x$ as *z* hath dimensions in that terme for a denominator in the valor of *v*.

Prop: 2d. If *hi=r*. & *rv=zy.* then *hi* & *be* describe equall spaces *higk*, or *hiak* & *abef.* that is *abef=aik{h}*

Prop: 3d. If $a^n x^m = b^n y^m$. Or $\frac{ax^{\frac{m}{n}}}{b} = y$. then is $\frac{nxy}{n+m} = \frac{n\times a\times x^{\frac{m+n}{n}}}{nb+mb} = abef$ the area of the line *aef.* And if $\frac{a}{bx^{\frac{m}{n}}} = y$: then is $\frac{nxy}{n-m} = \frac{na}{\overline{n-m}\times bx^{\frac{-n+m}{n}}} = abef = \frac{na}{\overline{n-m}\times bx^{\frac{m-n}{n}}}$

Demonstracion.

For Suppose *akhi* is a parallelogram & equall to $\frac{nax^{\frac{m+n}{n}}}{nb+mb}$. then is $\frac{nax^{\frac{m+n}{n}}}{nbr+mbr} = ai = z$. & (prop i) $\frac{az^2x^{\frac{m+n}{n}}}{brxz} = \frac{azx^{\frac{m}{n}}}{br} = v$. & (prop 2d) *rv=zy.* that is $\frac{ax^{\frac{m}{n}}}{b} = y$.

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Prop: 4th. If $y = ax^m + bx^n$. then is $\frac{ax^{m+1}}{m+1} + \frac{bx^{n+1}}{n+1} = abef$

And in generall if the valor of *y* consists of severall termes so that *x* is not of divers dimensions in the denominator of any terme, then multiply each terme by *x* & divide it by the number of the dimensions of *x*, all those products shall bee the area of the given line: supposeing also that either none or all the signes of those termes are changed by this operation. For if some bee changed & others bee not they proceed divers ways & joyne not, & then the quantitys *y* or *x* must be increased or diminished or otherwise altered.

The reason of this prop: is, that the area described by *y* is also described by its parts that is by the termes of its valor, & what areas those termes describe appeares by prop 3d.

Prop 5t. The progressions in this Table may bee designed by these geomet: lines. Whereby also any intermediate termes may bee found.

		a	b	b			b		b		b	
1	1	1	1	1	1	1	1	1	1	1	1	$\sim\ 1=y.$
−2	−1	0	1	2	3	4	5	6	7	8	9	$\sim\ x=y.$
3	1	0	0	1	3	6	10	15	21	28	36	$\sim\ xx-x=2y.$
−4	−1	0	0	0	1	4	10	20	35	56	84	$\sim\ x^3-3xx+2x=6y.$
5	1	0	0	0	0	1	5	15	35	70	126	$\sim\ x^4-6x^3+11xx-6x=24y.$
−6	−1	0	0	0	0	0	1	6	21	56	126	$\sim\ x^5-10x^4+35x^3-50x^2+24x=120y.$
7	1	0	0	0	0	0	0	1	7	28	84	$\sim\ x^6-15x^5+85x^4-225x^3+274xx-120x=720y.$
−8	−1	0	0	0	0	0	0	0	1	8	36	$\sim\ x^7-21x^6+175x^5-735x^4\quad \&c=5040y.$

The distance of the terme *b* from the terme *a* being called *x*. & the quantity of that terme being *y*. & each terme being distant an unit from the next. The nature of which table is such that the summe of any figure & the figure above it is equall to the figure after it. & the nature of the lines are such that any figure; multiplyed by the number of dimensions of *x* in the first terme, being substracted from the figure following it, is equall to the figure under that following figure. And that the numbers of *y* may be deduced hence 1×2×3×4×5×6×7 &c.

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Prop 6t. If $\frac{m}{n}=x$. This Progression $\frac{n\times m\times\overline{m-n}\times\overline{m-2n}\times\overline{m-3n}\times\overline{m-4n}\times\overline{m-5n}}{n\times n\times 2n\times 3n\times 4n\times 5n\times 6n}$ &c gives all the quantitys downward, in the preceding table. As if *m=3. n=1.* the quantitys downward are $\frac{1}{1}.\frac{m}{n}.\frac{m\times\overline{m-n}}{n\times 2n}.\frac{m\times\overline{m-n}\times\overline{m-2n}}{n\times 2n\times 3n}.\frac{m\times\overline{m-n}\times\overline{m-2n}\times\overline{m-3n}}{n\times 2n\times 3n\times 4n}$. &c that is 1. 3. 3. 1. 0. &c. So if $\frac{m}{n}=\frac{1}{2}=x$. that $1.\frac{1}{2}.\frac{-1}{8}.\frac{1}{16}.\frac{-5}{128}.\frac{7}{256}$. &c. are the terms downward.

Prop 7th. $\overline{a+b}\}^{\frac{m}{n}}=a^{\frac{m}{n}}+\frac{m}{n}\times\frac{b}{a}\times a^{\frac{m}{n}}+\frac{m}{n}\times\frac{m-n}{2n}\times\frac{bb}{aa}\times a^{\frac{m}{n}}\ .+\frac{m}{n}\times\frac{m-n}{2n}\times\frac{m-2n}{3n}\times\frac{b^3}{a^3}\times a^{\frac{m}{n}}$. &c As may bee deduced from $a^{\frac{m}{n}}\times\frac{mb}{na}\times\frac{\overline{m-n}\times b}{2na}\times\frac{\overline{m-2n}\times b}{3na}\times\frac{\overline{m-3n}\times b}{4na}\times\frac{mb-4nb}{5na}$ &c.

The truth of this Prop: appeareth by compareing it with the two former as also by calculation if $\frac{m}{n}$ is a whole & affirmative number, or *b* lesse than *a*

Prop 8*th*. $\overline{a+b}\}^{\frac{m}{n}}=\frac{1}{a^{\frac{m}{n}}}-\frac{m}{n}\times\frac{b}{a}\times\frac{1}{a^{\frac{m}{n}}}.-\frac{m}{n}\times\frac{-m-n}{2n}\times\frac{bb}{aa}\times\frac{1}{a^{\frac{m}{n}}}-\frac{m}{n}\times\frac{\overline{-m-n}}{2n}\times\frac{-m-2n}{3n}\times\frac{b^3}{a^3}\times\frac{1}{a^{\frac{m}{n}}}$. &c. As may bee deduced from $\frac{1}{a^{\frac{m}{n}}}\times\frac{-mb}{na}\times\frac{-mb-nb}{2na}\times\frac{-mb-2nb}{3na}\times\frac{-mb-3nb}{4na}$ &c.

The truth of this appeares also by the 5t & 6t proposition, or by calculation If *a>b*.

The truth of these two prop: is also thus demonstrated If $\overline{a+b}\}^{\frac{1}{1}}=\frac{1}{a+b}$ I divide, 1 by *a+b* as in decimall fractions & find the quote $\frac{1}{a}-\frac{b}{aa}+\frac{bb}{a^3}-\frac{b^3}{a^4}+\frac{b^4}{a^5}$ &c as appeareth also by multiplying both parts by *a+b*. So I extract the **{**note**}** of a^2+b as if they were decimall numbers & find $\sqrt{a^2+b}=a+\frac{b}{2a}+\frac{-bb}{8a^3}+\frac{b^3}{16a^5}$ &c, as also may appeare by squareing both parts

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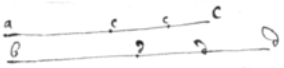

1.If two bodys *c*, *d* describe the streight lines *ac*, *bd*, in the same time, (calling *ac=x*, *bd=y*, *p*=motion of *c*, *q*=motion of *d*) & if I have an equation expressing the relation of *ac=x* & *bd=y* whose termes are all put equall to nothing. I multiply each terme of that equation by so many times *py* or $\frac{p}{x}$ as *x* hath dimensions in it, & also by soe many times *qx* or $\frac{q}{y}$ as y hath dimensions in it. the summe of these products is an equation expresing the relation of the motions of c & d. Example if $ax^3+a^2yx-y^3x+y^4=0$ then $3apxx+a^2py-py^3+aaqx-3qyyx+4qy^3=0$.

2. If an equation expressing the relation of their motions bee given, tis more difficult & sometimes Geometrically impossible, thereby to find the relation of the spaces described by those motions.

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If $apx^{\frac{m}{n}}=q$. ________then $\frac{na}{m+n}x^{\frac{m+n}{n}}=y$.

— — — — — — — — — — — — — — — — — —

As if *m=3. n=2.* then $apx^{\frac{3}{2}}=q$, & $\frac{2a}{3}x^{\frac{5}{2}}=y$. Soe if $apx^{\frac{-3}{2}}=q=\frac{ap}{x^{\frac{3}{2}}}$, then *m=-3. n=2.* & $\frac{2a}{-1}x^{\frac{-1}{2}}=\frac{-2a}{x^{\frac{1}{2}}}=y$. If the valor of q consisteth of severall such termes, consider each terme severall *y*. as if $ax+bxx=y$. the first terme gives $\frac{ax^2}{2}$ the 2d $\frac{bx^3}{3}$. therefore $\frac{axx}{2}+\frac{bxxx}{3}=y$.

In generall multiply the valor of *y* by *x* & divide each terme of it by the logarithme of *x*, in that terme: if that valor of *q* consist of simple termes.

$\frac{-rdx^{r-1}}{ddx^{2r}+2dex^r+ee}=\frac{q}{p}.\ \frac{2}{dx^r+e}=y.\ \frac{\overline{m-r}\times adx^{m+r}+\overline{m-s}\times aex^{m+s}+\overline{n-r}\times bdx^{n+r}+\overline{n-s}\times bex^{n+s}}{x\text{ in }ddx^{2r}+2dex^{r+s}+eex^{2s}}=\frac{q}{p}.\ \frac{ax^m+bx^n}{dx^r+ex^s}=y.$ ____________ or thus $\frac{ma+\overline{3n+m}\times bx^n}{2x}\times\sqrt{ax^m+bx^{n+m}}=\frac{q}{p}.\overline{a+bx^n}\sqrt{ax^m+bx^{n+m}}=y$. $\frac{\overline{mm+8mn+15nn}\times ddx^{2n}-\overline{2mn-mm}\times ee}{x}\sqrt{ex^m+dx^{m+n}}=\frac{q}{p}$. And ij $\overline{\overline{2m+6n}\times ddx^{2n}+2ndex^n-\overline{2m-4n}\times ee}\times\sqrt{ex^m+dx^{m+n}}=y$. Or thus $\frac{\overline{3m-2n}\times maax^{m-n}+\overline{3n-2m}\times nbbx^{n-m}}{2x}\sqrt{ax^m+bbx^n}=\frac{q}{p}$. $\overline{maax^{m-n}+\overline{m-n}\times ab-nb^nx^{n-m}}\sqrt{ax^m+bx^n}=y$.

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+

$\overline{maax^{m-n}+\overline{m-n}\times ab-nbbx^{n-m}}\sqrt{ax^m+bx^n}=y$. And $\frac{3m-2n\times maax^{m-n}+3n-2m\times ndbx^{n-m}}{2x}\sqrt{ax^m+bx^n}=\frac{q}{p}$.

$mac+\overline{3r-2m}\times adx^{r-m}+\overline{3m+2n}\times bcx^{m+n}+3r+2n\times bdx^{n+r}\frac{\text{ in }\sqrt{cx^m+dx^r}}{2x}=\frac{q}{p}$. $\overline{ac+adx^{r-m}+bcx^{m+n}+bdx^{r+n}}\times\sqrt{cx^m+dx^r}=y$ $\frac{\overline{3m-2n}\times\overline{2n-m}\times md^3x^{m-n}+\overline{3n-2m}\times\overline{5n-4m}\times ne^3x^{2n-2m}}{2x}$ in $\sqrt{dx^m+ex^n}=\frac{q}{p}$ $\overline{2n-m}\times md^3x^{m-n}+\overline{2n-m}\times\overline{m-n}\times edd+\overline{n-m}\times needx^{n-m}+\overline{3n-2m}\times ne^3x^{2n-2m}$ × × $\sqrt{dx^m+ex^n}=y$.

Or more generally, $\frac{\overline{3m-2n}\times mcddx^{m-n}+\overline{2m-3n}\times nceex^{n-m}+\overline{3m+2p}\times mbddx^{m+p}+\overline{3n+2p}\times bedmx^{n+p}}{2x}$ inin $\sqrt{dx^m+ex^n}=\frac{q}{p}$. And $\overline{mddcx^{m-n}+\overline{m-n}\times cde-nceex^{n-m}+mbd^2x^{p+m}+mbdex^{n+p}}\sqrt{dx^m+ex^n}=y$. $\frac{\overline{5m-2n}\times\overline{2m+n}}{m}\times e^3x^{2m-n}\frac{\overline{+6n-3m}}{4n-m}\times 9nd^3x^{2n-m}$ in $\sqrt{\frac{2m+n}{m}\times eex^m+}$

$\overline{+\frac{9ndd}{4n-m}x^{2n-m}-3dex^{n}}=\frac{2qx}{p}$. And $\overline{\frac{\overline{2m+n}}{m}\times e^3x^{2m-n}\frac{\overline{+n-m}}{m}\times eedx^m+}\ \overline{\frac{+m-n}{4n-m}\times 3ddex^n\frac{+9nd^3}{4n-m}\times x^{2n-m}}$ in $\sqrt{\frac{2m+n}{m}\times eex^m-3dex^n+\frac{9ndd}{4n-m}\times x^{2n-m}}==y$.

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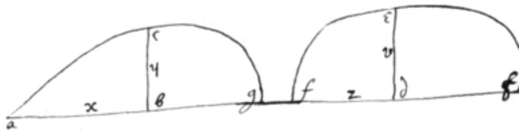
$\frac{\overline{5n-2m}\times\overline{2m-5n}\times 3mnnb^5}{\overline{2n+m}\times\overline{16n^4-8nnmm+m^4}}\times x^{3m-2n}\ \frac{\overline{+2m-5n}\times 3mnb^4c}{\overline{2n+m}\times\overline{4nn-mm}}x^{2m-n}\ \frac{\overline{+2n-2m}\times\overline{5n-2m}\times nb^3cc}{\overline{2n+m}\times\overline{4nn-mm}}x^m\ \frac{\overline{+2n-2m}}{2n+m}\times bbc^3x^n\ \frac{\overline{+m-n}}{5n-2m}\times bc^4x^{2n-m}$
$\frac{+3m-6n}{5n-2m}\times c^5x^{3n-2m}$ in $\sqrt{\frac{5n-2m\times nbb}{4nn-mm}x^m+bcx^n+ccx^{2n-m}}==y$. And, $\frac{\overline{7m-4n}\times\overline{5n-2m}\times\overline{2m-5n}\times 3mnnb^5}{\overline{2n+m}\times\overline{16n^4-8mmnn+m^4}}\times x^{3m-2n}\ \frac{\overline{+4m-n}\times\overline{2m-5n\times 3mnb^4c}}{\overline{2n+m}\times\overline{4nn-mm}}x^{2m-n}\ \frac{\overline{+8n-5m}\times\overline{3m-6n}}{5n-2m}\times c^5x^{3n-2m}$
in $\sqrt{\frac{\overline{5n-2m}\times nbb}{4nn-mm}x^m+bcx^n+ccx^{2n-m}}=\frac{2qx}{p}$.

<156r>



sit *ab=x. bc=y. df=z, de=v.* ______

The area abc of y^e line whose nature is	is equall to y^e area fde of y^e line whose nature is	supposeing y^e re= lation twixt ab & fd to bee
$2xx\sqrt{c+dxx}=y.$	$\sqrt{cz+dzz}=v.$	$xx=z.$
$\sqrt{cx+dxx}=y.$	$a\sqrt{caz+daazz}=v.$	$ax=z.$
$\frac{-1}{x^3}\sqrt{cx+d}=y.$	$\sqrt{cz+dzz}=v.$	$1=zx.$
$\frac{-2}{x^5}\sqrt{cxx+d}=y.$	$\sqrt{cz+dzz}=v.$	$1=zx^2.$
$\frac{-3}{x^7}\sqrt{cx^3+d}=y.$	$\sqrt{cz+dzz}=v.$	$1=zx^3.$
$3x^3\sqrt{cx+dx^4}=y.$	$\sqrt{cz+dzz}=v.$	$x^3=z.$
$4x^5\sqrt{c+dx^4}=y.$	$\sqrt{cz+dzz}=v.$	$x^4=z.$
$\frac{1}{2x}\sqrt{cx^{\frac{3}{2}}+dxx}=y.$	$\sqrt{cz+dzz}=v.$	$x=zz.$
In generall		
$nx^{n-1}\times\sqrt{cx^n+dx^{n+n}}=y.$	$\sqrt{cz+dzz}=v.$	$x^n=z.$

$2x\sqrt{c+dx^4}=y.$	$\sqrt{c+dzz}=v.$	$xx=z.$
$3xx\sqrt{c+dx^6}=y.$	$\sqrt{c+dzz}=v.$	$x^3=z.$
$\frac{-1}{x^3}\sqrt{cxx+d}=y.$	$\sqrt{c+dzz}=v.$	$1=zx.$
$\frac{-2}{x^5}\sqrt{x^4+d}=y.$	$\sqrt{c+dzz}=v.$	$1=zxx.$
$\frac{1}{2x}\sqrt{cx+dxx}=y.$	$\sqrt{c+dzz}=v.$	$x=zz.$
$\frac{3}{2}\sqrt{cx+dx^4}=y.$	$\sqrt{c+dzz}=v.$	$x^3=zz.$
$\frac{-1}{2xx}\sqrt{cx+d}=y.$	$\sqrt{c+dzz}=v.$	$1=xzz.$
$\frac{-3}{2x^4}\sqrt{cx^3+d}=y.$	$\sqrt{c+dzz}=v.$	$1=x^3zz.$
In generall		
$nx^{n-1}\sqrt{c+dx^{n+n}}=y.$	$\sqrt{c+dzz}=v.$	$x^n=z.$

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The area abc of y^e line $\sim$.	is equall to y^e area of y^e line $\sim$.	Supposeing y^t
$2x\sqrt{c+dxx+ex^4}=y.$	$\sqrt{c+dz+ezz}=v.$	$xx=z.$
$3xx\sqrt{c+dx^3+ex^6}=y.$	$\sqrt{c+dz+ez^2}=v.$	$x^3=z.$
$4x^3\sqrt{c+dx^4+ex^8}=y.$	$\sqrt{c+dz+ezz}=v.$	$x^4=z.$
$\frac{-1}{x^3}\sqrt{cxx+dx+e}=y.$	$\sqrt{c+dz+ezz}=v.$	$1=zx.$
$\frac{-2}{x^5}\sqrt{cx^4+dxx+e}=y.$	$\sqrt{c+dz+ezz}=v.$	$1=zxx.$
$\frac{-3}{x^7}\sqrt{cx^6+dx^3+e}=y.$	$\sqrt{c+dz+ezz}=v.$	$1=zx^3.$
$\frac{1}{2x}\sqrt{cx+dx^{\frac{3}{2}}+exx}=y.$	$\sqrt{c+dz+ezz}=v.$	$x=zz.$
$\frac{-1}{2xx}\sqrt{cx+dx^{\frac{1}{2}}+e}=y.$	$\sqrt{c+dz+ezz}=v.$	$1=zzx.$
	In generall.	
$nx^{n-1}\sqrt{c+dx^n+ex^{n+n}}=y.$	$\sqrt{c+dz+ezz}=v.$	$x^n=z.$

$\frac{b}{a+bx}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx=z.$
$\frac{2bx}{a+bxx}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx^2=z.$
$\frac{3bxx}{a+bx^3}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx^3=z.$
$\frac{4bx^3}{a+bx^4}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx^4=z.$
$\frac{-b}{axx+bx}=y.$	————	$\frac{1}{z}=v.$	————	$ax+b=zx.$
$\frac{-2b}{ax^3+bx}=y.$	————	$\frac{1}{z}=v.$	————	$ax^2+b=zx^2.$
$\frac{-3b}{ax^4+bx}=y.$	————	$\frac{1}{z}=v.$	————	$ax^3+b=zx^3.$
$\frac{1}{2a\sqrt{x}+2bx}=y.$	————	$\frac{1}{z}=v.$	————	$a+b\sqrt{x}=z.$
$\frac{3xb}{2a\sqrt{x}+2bxx}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx^{\frac{3}{2}}=z.$
$\frac{-b}{ax\sqrt{x}+bx}=y.$	————	$\frac{1}{z}=v.$	————	$a+\frac{b}{\sqrt{x}}=z.$
$\frac{-3b}{axx\sqrt{x}+bx}=y.$	————	$\frac{1}{z}=v.$	————	$a+\frac{b}{x\sqrt{x}}=z.$

$\frac{a+2bx}{ax+bxx}=y.$	$\frac{1}{z}=v.$	$ax+bx^2=z.$
$\frac{a+3bxx}{ax+bx^3}=y.$	$\frac{1}{z}=v.$	$ax+bx^3=z.$

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$\frac{axx\,z-b}{ax^3+bxx}=y.$	————	$\frac{1}{z}=v.$	————	$aax+b=zx.$
$\frac{2a+3bx}{ax+bxx}=y.$	————	$\frac{1}{z}=v.$	————	$ax^2+bx^3=z.$
$\frac{2a+4bxx}{ax+bx^3}=y.$	————	$\frac{1}{z}=v.$	————	$axx+bx^4=z.$
$\frac{2a+5bx3}{ax+bx^4}=y.$	————	$\frac{1}{z}=v.$	————	$ax^2+bx^5=z.$
$\frac{3a+4bx}{ax+bx^2}=y.$	————	$\frac{1}{z}=v.$	————	$ax^3+bx^4=z.$
$\frac{3a+5bxx}{ax+bx^3}=y.$	————	$\frac{1}{z}=v.$	————	$ax^3+bx^5=z.$
$\frac{4a+5bx}{ax+bxx}=y.$	————	$\frac{1}{z}=v.$	————	$ax^4+bx^5=z.$
$\frac{4a+6bxx}{ax+bx^3}=y.$	————	$\frac{1}{z}=v.$	————	$ax^4+bx^5=z.$
$\frac{-a+bxx}{ax+bx^3}=y.$	————	$\frac{1}{z}=v.$	————	$a+bx^2=xz.$
$\frac{-a+2bx^4}{axx+bx^5}=y.$		$\frac{1}{z}=v.$	————	$a+bx^3=xxz.$
$\frac{-2a-bx}{ax+bxx}=y.$		$\frac{1}{z}=v.$	————	$a+bx=xxz.$

	In generall.	
$\frac{madx^{m-1}+nbdx^{n-1}}{ax^m+bx^n}=y.$	$\frac{d}{z}=v.$	$ax^m+bx^n=z.$

note that these are compounded onely of the first simplest Areas:

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The area abc of y^e line	is equall to y^e area fde of y^e line.	Supposeing that
$\frac{xx}{\sqrt{dxx-dc}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{xx}{d}-\frac{c}{d}}=z.$
$\frac{x}{2\sqrt{dxx-dcx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{x-c}{d}}=z.$
$\frac{-1}{2xx\sqrt{d-cdx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{1-cx}{dx}}=z.$
$\frac{-1}{x^3\sqrt{d-cdxx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{1-cxx}{dxx}}=z.$
	Or generally.	
$\frac{sx^{3s-1}}{\sqrt{dx^{2s}-cd}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{x^{2s}-c}{d}}=z.$
$\frac{bcc+2bbcdx+b^3ddxx}{\sqrt{2bcx+bbdxx}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2cbx+dbbxx}=z.$
$\frac{2bcc+4bbcdxx+2b^3ddx^4}{\sqrt{2bc+bbdxx}}=y.$	$\sqrt{cc+dzz}=v.$	$x\sqrt{2bc+bbdxx}=z.$
$\frac{-bccxx-2bbcdx-b^3dd}{x^3\sqrt{2bcx+bbd}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2cbx+bbd}=zx.$
$\frac{-2bccx^4-4bbcdxx-2b^3dd}{x^5\sqrt{2bcxx+bbd}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2bcxx+bbd}=zxx.$
	In generall	
$\frac{mbccx^m+2mbbcdx^{2m}+mb^3ddx^{3m}}{x\sqrt{2bcx^m+dbbx^{2m}}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2bcx^m+bbdx^{2m}}=z.$
$\frac{b\sqrt{c+ad+bdx}}{2\sqrt{a+bx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+bx}=z.$
$\frac{bx\sqrt{c+ad+bdxx}}{\sqrt{a+bxx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+bxx}=z.$
$\frac{-b\sqrt{cx^2+adxx+bdx}}{2xx\sqrt{axx+bx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+\frac{b}{x}}=z.$
$\frac{-b\sqrt{cxx+adxx+bd}}{x^3\sqrt{axx+b}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+\frac{b}{xx}}=z.$
	In generall.	
$\frac{max^m+nbx^n}{2x\sqrt{ax^m+bx^n}}\times\sqrt{c+adx^m+bdx^n}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{ax^m+bx^n}=z.$

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$\frac{eb\sqrt{ac+dee+cbx}}{\overline{2a+2bx}\times a+bx}=y.$	$\sqrt{c+dzz}=v.$	$\frac{e}{\sqrt{a+bx}}=z.$
$\frac{ebx\sqrt{ac+dee+cbxx}}{\overline{a+bxx}\ a+bxx}=y.$	$\sqrt{c+dzz}=v.$	$\frac{e}{\sqrt{a+bxx}}=z.$
$\frac{-eb\sqrt{acxx+deexx+cbx}}{\overline{2ax^2+2bx}\ axx+bx}=y.$	$\sqrt{c+dzz}=v.$	$\frac{ex}{\sqrt{axx+bx}}=z.$
$\frac{-eb\sqrt{acxx+deexx+cb}}{\overline{ax^3+bx}\ axx+b}=y.$	$\sqrt{c+dzz}=v.$	$\frac{ex}{\sqrt{axx+b}}=z.$
$\frac{4a^4cdxx+4aabcd+bbcd}{\overline{2aaxx+2bx}\ 4a^4ddxx+4aabddx}=y.$	$\sqrt{cc+ddzz}=v.$	$\frac{cb}{2ad\sqrt{aax^2+bx}}=z.$
$\frac{a^4cdx^4+4aabcdxx+bbcd}{\overline{aax^3+bx}\times 2ad\ aax^4+bxx}=y.$	$\sqrt{cc+ddzz}=v.$	$\frac{cb}{2adx\sqrt{a^2x^2+b}}=z.$
$\frac{-4a^4cd-4aabcdx-bbcdxx}{\overline{2aax+2bxx}\times 2ad\ aa+bx}=y.$	$\sqrt{cc+ddzz}=v.$	$\frac{cbx}{2ad\sqrt{aa+bx}}=z.$
$\frac{-4a^4cdx^2-4aabcdx^4-bbcdx^6}{\overline{aa+bx^2}\times 2ad\ aa+bxx}=y.$	$\sqrt{cc+ddzz}=v.$	$\frac{cbxx}{2ad\sqrt{aa+bxx}}=z.$
	In generall.	
$\frac{\overline{emax^m+enbx^n}\sqrt{cax^m+cbx^n+dee}}{2ax^{m+1}+2bx^{n+1}\times ax^m+bx^n}=y.$	$\sqrt{c+dzz}=v.$	$\frac{e}{\sqrt{ax^m+bx^n}}=z.$
$\frac{emax^{m-1}+enbx^{n-1}\sqrt{cax^m+cbx^n+dee}}{2aax^{2m}+4abx^{m+n}+2bbx^{2n}}=y.$	$\sqrt{c+dzz}=v.$	$\frac{e}{\sqrt{ax^m+bx^n}}=z.$
$\frac{ceb^3xx}{aa+2abxx+bbx^4}=y.$	$\sqrt{cc-\frac{acc}{ee}zz}=v.$	$\frac{e}{\sqrt{a+bbxx}}=z.$
	In generall.	
$\frac{nb^3cex^{\frac{3n-2}{2}}}{2aa+4abx^n+2bbx^{2n}}=y.$	$\sqrt{cc\frac{-acc}{ee}zz}=v.$	$\frac{e}{\sqrt{a+bbx^n}}=z.$

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$$\mathrm{cbx}\frac{\mathrm{cb}}{\mathrm{a+bx}} = \mathrm{y}. \quad \text{————————} \quad \mathrm{c + zv = 0}. \quad \text{————————} \quad \frac{1}{\mathrm{a+bx}} = \mathrm{z}.$$

$$\frac{\mathrm{2bcx}}{\mathrm{a+bxx}} = \mathrm{y}. \qquad \mathrm{c + zv = 0}. \qquad \frac{1}{\mathrm{a+bxx}} = \mathrm{z}.$$

$$\frac{\mathrm{3bcxx}}{\mathrm{a+bx^3}} = \mathrm{y}. \qquad \mathrm{c + zv = 0}. \qquad \frac{1}{\mathrm{a+bx^3}} = \mathrm{z}.$$

$$\frac{\mathrm{cb}}{\mathrm{axx+bx}} = \mathrm{y}. \quad \text{————————} \quad \mathrm{c = zv}. \quad \text{————————} \quad \frac{\mathrm{x}}{\mathrm{ax+b}} = \mathrm{z}.$$

$$\frac{\mathrm{cb}}{\mathrm{2ax^3+2bx}} = \mathrm{y}. \quad \text{————————} \quad \mathrm{c = zv}. \quad \text{————————} \quad \frac{\mathrm{xx}}{\mathrm{ax^2+b}} = \mathrm{z}.$$

As before, In generall,

$$\frac{\mathrm{cmax^{m-1}+ncbx^{n-1}}}{\mathrm{ax^m+bx^n}} = \mathrm{y}. \quad \text{————————} \quad \mathrm{c + zv = 0}. \quad \text{————————} \quad \frac{1}{\mathrm{ax^m+bx^n}} = \mathrm{z}.$$

Also

$$\frac{\overline{\mathrm{rdx^{r-1}+sex^{s-1}}} \times \overline{\mathrm{ax^m+bx^n}}}{\mathrm{ax^m+bx^n} \times \mathrm{dx^r+ex^s}} \quad \frac{\mathrm{-max^{m-1}-nbx^{n-1}}}{\mathrm{ax^m+bx^n}} = \mathrm{y}. \qquad \mathrm{c = zv}. \qquad \frac{\mathrm{dx^r+ex^s}}{\mathrm{ax^m+bx^n}} = \mathrm{z}.$$

$$\frac{\mathrm{crdx^{r-1}+csex^{s-1}}}{\mathrm{dx^r+ex^s}} \quad \frac{\mathrm{-cmax^{m-1}-cnbx^{n-1}}}{\mathrm{ax^m+bx^n}} = \mathrm{y}. \qquad \mathrm{c = zv}. \qquad \frac{\mathrm{dx^r+ex^s}}{\mathrm{ax^m+bx^n}} = \mathrm{z}.$$

$$\frac{\mathrm{-9ac}}{\mathrm{2bx^4+2cx}} = \mathrm{y}. \qquad \mathrm{a = zv}. \qquad \overline{\mathrm{bx^3 + c}} \times \sqrt{\mathrm{bx^4 + cx}} = \mathrm{x^5z}.$$

$$\frac{\mathrm{-3ac}}{\mathrm{bx^3+cx}} = \mathrm{y}. \qquad \mathrm{a = zv}. \qquad \overline{\mathrm{bxx + c}}\sqrt{\mathrm{bxx + c}} = \mathrm{x^3z}.$$

$$\frac{\mathrm{-3ac}}{\mathrm{2bxx+2cx}} = \mathrm{y}. \qquad \mathrm{a = zv}. \qquad \overline{\mathrm{bx + c}}\sqrt{\mathrm{bxx + cx}} = \mathrm{xxz}.$$

$$\frac{\mathrm{3ac}}{\mathrm{2b+2cx}} = \mathrm{y}. \qquad \mathrm{a = zv}. \qquad \mathrm{b + cx}\sqrt{\mathrm{b + cx}} = \mathrm{z}.$$

$$\frac{\mathrm{3acx}}{\mathrm{b+cxx}} = \mathrm{y}. \qquad \mathrm{a = zv}. \qquad \mathrm{b + cxx}\sqrt{\mathrm{b + cxx}} = \mathrm{z}$$

As before was found, In generall

$$\frac{\overline{\mathrm{3m+2r}} \times \mathrm{abx^{m+r}} + \overline{\mathrm{3n+2r}} \times \mathrm{acx^{n+r}}}{\mathrm{2bx^{m+r+1}+2cx^{n+r+1}}} = \mathrm{y}. \qquad \mathrm{a = zv}.$$

And $\overline{\mathrm{bx^{m+r} + cx^{n+r}}} \times \sqrt{\mathrm{bx^m + cx^n}} = \mathrm{z}.$

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$$\mathrm{xx = ay}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{x^3 = 3az}.$$

$$\mathrm{x^3 = ay}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{x^4 = 4az}.$$

$$\mathrm{x^4 = ay}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{x^5 = 5az}.$$

$$\mathrm{a = xxy}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{-a = xz}.$$

$$\mathrm{a = x^3y}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{-a = 2xxz}.$$

$$\mathrm{x^3 = ayy}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{4x^5 = 25azz}.$$

$$\mathrm{a = x^3yy}. \quad \text{————————} \quad \mathrm{1 = v}. \quad \text{————————} \quad \mathrm{4a = zzx}.$$

In generall

$$\mathrm{ax^m = y}. \quad \text{————————} \quad \mathrm{1 = v} \quad \text{————————} \quad \frac{\mathrm{a}}{\mathrm{m+1}}\mathrm{x^{m+1}} = \mathrm{z}.$$

That is.
multiply the valor of *y*. by *x*, & divide each terme in that valor by soe many units as *x* hath dimensions in that terme, the product is the area.

$$\frac{\mathrm{c}}{\mathrm{bb+2bcx+ccxx}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{b+cx}} = \mathrm{z}.$$

$$\frac{\mathrm{2cx}}{\mathrm{bb+2bcxx+ccx^4}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{b+cxx}} = \mathrm{z}.$$

$$\frac{\mathrm{3cxx}}{\mathrm{bb+2bcx^3+ccx^6}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{b+cx^3}} = \mathrm{z}.$$

$$\frac{\mathrm{-c}}{\mathrm{bbxx+2bcx+cc}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{\mathrm{x}}{\mathrm{bx+c}} = \mathrm{z}.$$

$$\frac{\mathrm{-2cx}}{\mathrm{bbx^4+2bcxx+cc}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{\mathrm{xx}}{\mathrm{bxxx+c}} = \mathrm{z}.$$

In generall

$$\frac{\mathrm{ncx^{n-1}}}{\mathrm{bb+2bcx^n+ccx^{2n}}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{b+cx^n}} = \mathrm{z}.$$

$$\frac{\mathrm{b+2cx}}{\mathrm{bbxx+2bcx^3+ccx^4}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{bx+cxx}} = \mathrm{z}.$$

$$\frac{\mathrm{b+3cxx}}{\mathrm{bbxx+2bcx^4+ccx^6}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{bx+cx^3}} = \mathrm{z}.$$

$$\frac{\mathrm{2b+3cx}}{\mathrm{bbx^3+2bcx^4+ccx^5}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{bxx+cx}} = \mathrm{z}.$$

In generall

$$\frac{\mathrm{mbx^{m-1}+ncx^{n-1}}}{\mathrm{bbx^{2m}+2bcx^{m+n}+ccx^{2n}}} = \mathrm{y}. \qquad \mathrm{1 = v}. \qquad \frac{1}{\mathrm{bx^m+cx^n}} = \mathrm{z}.$$

cui eodem modo

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$$\frac{-b+cxx}{bb+2bcxx+ccx^4} = y. \qquad 1 = v. \qquad \frac{x}{b+cxx} = z.$$
$$\frac{-bxx-2cx}{bbxx+2bcx+cc} = y. \qquad 1 = v. \qquad \frac{xx}{bx+c} = z.$$
$$\frac{-bx^4-3cxx}{bbx^4+2bcxx+cc} = y. \qquad 1 = v. \qquad \frac{x^3}{bxx+c} = z.$$
$$\frac{-2bx^3-3cxx}{bbxx+2bcx+cc} = y. \qquad 1 = v. \qquad \frac{x^3}{bx+c} = z.$$
$$\frac{-2bx^5-4cx^3}{bbx^4+2bcx^2+cc} = y. \qquad 1 = v. \qquad \frac{x^4}{bxx+c} = z.$$
$$\frac{cd-eb}{dd+2edx+eexx} = y. \qquad 1 = v. \qquad \frac{b+cx}{d+ex} = z.$$
$$\frac{cd-2ebx}{dd+2edxx+eex^4} = y. \qquad 1 = v. \qquad \frac{b+cx}{d+exx} = z.$$
$$\frac{2cd-2ebxx}{dd+2edxx+eex^4} = y. \qquad 1 = v. \qquad \frac{b+cxx}{d+exx} = z.$$

In generall

$$\frac{\overline{m-r}\times bdx^{m+r}+\overline{m-s}\times bex^{m+s}+\overline{n-r}\times cdx^{n+r}+\overline{n-s}\times cex^{n+s}}{ddx^{2r+1}+2edx^{r+s+1}+eex^{2s+1}} = y. \quad 1 = v. \qquad \frac{bx^m+cx^n}{dx^r+ex^s} = z.$$

$$\frac{-3c}{2xx\sqrt{bx^4+cx}} = y. \qquad 1 = v. \qquad bx^3+c = z^2x^3.$$
$$\frac{-c}{xx\sqrt{bxx+c}} = y. \qquad 1 = v. \qquad bxx+c = z^2xx.$$
$$\frac{-c}{2x\sqrt{bxx+cx}} = y. \qquad 1 = v. \qquad bx+c = z^2x.$$
$$\frac{c}{2\sqrt{b+cx}} = y. \qquad 1 = v. \qquad b+cx = zz.$$
$$\frac{cx}{\sqrt{b+cxx}} = y. \qquad 1 = v. \qquad b+cxx = zz.$$
$$\frac{3cxx}{\sqrt{b+cx^3}} = y. \qquad 1 = v. \qquad b+cx^3 = zz.$$
$$\frac{bx^4-3c}{2xx\sqrt{bx^5+cx}} = y. \qquad 1 = v. \qquad bx^4+c = z^2x^3.$$
$$\frac{bx^3-2c}{2xx\sqrt{bx^3+c}} = y. \qquad 1 = v. \qquad bx^3+c = z^2xx.$$

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$$\frac{bxx-c}{2x\sqrt{bx^3+cx}} = y. \qquad 1 = v. \qquad bxx+c = z^2x.$$
$$\frac{b+2cx}{2\sqrt{bx+cxx}} = y. \qquad 1 = v. \qquad bx+cxx = zz.$$
$$\frac{b+3cxx}{2\sqrt{bx+cx^3}} = y. \qquad 1 = v. \qquad bx+cx^3 = zz.$$
$$\frac{b+4cx^3}{2\sqrt{bx+cx^4}} = y. \qquad 1 = v. \qquad bx+cx^4 = zz.$$
$$\frac{2bx^3-c}{2x\sqrt{bx^4+cx}} = y. \qquad 1 = v. \qquad bx^3+c = zzx.$$
$$\frac{2b+3cx}{2\sqrt{b+cx}} = y. \qquad 1 = v. \qquad bxx+cx^3 = zz.$$
$$\frac{b+4cxx}{2\sqrt{b+cxx}} = y. \qquad 1 = v. \qquad bxx+cx^4 = z^2.$$

In generall

$$\frac{mbax^m+nacx^n}{2x\sqrt{bx^m+cx^n}} = y. \qquad a = v. \qquad \sqrt{bx^m+cx^n} = z.$$

Also more generally.

$$\frac{mabx^m+nacx^n+radx^r}{2x\sqrt{bx^m+cx^n+dx^r}} = v. \qquad a = v. \qquad \sqrt{bx^m+cx^n+dx^r} = z.$$

$$\frac{-ac}{\overline{bxx+c}\sqrt{bxx+c}} = y. \qquad a = v. \qquad \frac{x}{\sqrt{bxx+c}} = z.$$
$$\frac{-ac}{\overline{2bx+2c}\sqrt{bxx+cx}} = y. \qquad a = v. \qquad \frac{\sqrt{x}}{\sqrt{bx+c}} = z.$$
$$\frac{ac}{\overline{2b+2cx}\sqrt{b+cx}} = y. \qquad a = v. \qquad \frac{1}{\sqrt{b+cx}} = z.$$
$$\frac{acx}{\overline{b+cxx}\sqrt{b+cxx}} = y. \qquad a = v. \qquad \frac{1}{\sqrt{b+cxx}} = z.$$
$$\frac{ab+2acx}{2bx+2cxx\sqrt{bx+cxx}} = y. \qquad a = v. \qquad \frac{1}{\sqrt{bx+cxx}} = z.$$

In generall

$$\frac{mabx^{m-1}+nacx^{n-1}}{2bx^m+2cx^n\times\sqrt{bx^m+cx^n}} = y. \qquad a = v. \qquad \frac{1}{\sqrt{bx^m+cx^n}} = z.$$

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$\frac{3}{2}\sqrt{b+cx}=y.$	$1=v.$	$\overline{b+cx}\sqrt{b+cx}=z.$
$3acx\sqrt{b+cxx}=y.$	$a=v.$	$\overline{b+cxx}\sqrt{b+cxx}=z.$
$\frac{9acxx}{2}\sqrt{b+cx^3}=y.$	$a=v.$	$\overline{b+cx^3}\times\sqrt{b+cx^3}=z.$
$\frac{-3ac}{2x^3}\sqrt{bxx+cx}=y.$	$a=v.$	$\overline{bx+c}\sqrt{bxx+cx}=zx.$
$\frac{-3ac\sqrt{bxx+c}}{x^4}=y.$	$a=v.$	$\overline{bxx+c}\sqrt{bxx+c}=zxxx.$

In generall

$\frac{3nacx^{n-1}}{2}\times\sqrt{b+cx^n}=y.$	$a=v.$	$\overline{b+cx^n}\times\sqrt{b+cx^n}=z.$

$\frac{\overline{2ba+5acx}}{2}\sqrt{b+cx}=y.$	$a=v.$	$\overline{bx+cxx}\sqrt{b+cx}=z.$
$\overline{ab+4acxx}\sqrt{b+cxx}=y.$	$a=v.$	$\overline{bx+cx^3}\times\sqrt{b+cxx}=z.$
$\frac{2abx-ac}{2xx}\times\sqrt{bxx+cx}=y.$	$a=v.$	$\overline{bx+c}\times\sqrt{bx^2+cx}=zx$

In generall

$$\frac{\overline{3m+2r}\times bx^{m+r}+\overline{3n+2r}\times cx^{n+r}}{2x}\times a\sqrt{bx^m+cx^n}=y. \qquad a=v.$$

and

$$\overline{bx^{m+r}+cx^{n+r}}\times\sqrt{bx^m+cx^n}=z$$

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And more generally

$$\frac{\overline{2m+r}\times bdx^{m+r}+\overline{2m+s}\times bex^{m+s}+\overline{2n+r}\times cdx^{n+r}+\overline{2n+s}\times cex^{n+s}}{2x\sqrt{dx^r+ex^s}}\times a=y.$$

$$a=v. \qquad \overline{bx^m+cx^n}\times\sqrt{dx^r+ex^s}=z.$$

$\frac{3cdx}{2\sqrt{dx+e}}=y.$	$1=v.$	$\overline{\frac{-2ce}{d}+cx}\times\sqrt{dx+e}=z.$
$\frac{3cdx^3}{\sqrt{dxx+e}}=y.$	$1=v.$	$\overline{\frac{-2ce}{d}+cxx}\sqrt{dxx+e}=z.$
$\frac{-3cd}{2xx\sqrt{dx+exx}}=y.$	$1=v.$	$\overline{\frac{-2cex}{d}+c}\times\sqrt{dx+exx}=zxx.$
$\frac{-3cd}{x^4\sqrt{d+exx}}=y.$	$1=v.$	$\overline{\frac{-2cexx}{d}+c}\times\sqrt{d+exx}=zx.$

In generall

$\frac{\overline{3m+3n}\times cdx^{3m+2n}}{2x\sqrt{dx^{3m+n}+ex^{2m}}}=y.$	$1=v.$	$\overline{cx^n-\frac{2ce}{d}x^{-m}}\times\sqrt{dx^{3m+n}+ex^{2m}}=z.$

$\frac{\overline{5ab+5bbx}\times\sqrt{a+bx}}{2}=y.$	$1=v.$	$\overline{aa+2abx+bbxx}\sqrt{a+bx}=z.$
$\overline{5abx+5bbx^3}\sqrt{a+bxx}=y.$	$1=v.$	$\overline{aa+2abx^2+bbx^4}\sqrt{a+bxx}=z.$
$\frac{\overline{5aax+10abxx+5bbx^3}\sqrt{ax+bxx}}{2}=y.$	$1=v.$	$\overline{a^2x^2+2abx^3+bbx^4}\sqrt{ax+bxx}=z.$
$\frac{\overline{-5abx-5bb}\sqrt{axx+bx}}{2x^3}=y.$	$1=v.$	$\overline{aaxx+abx+bb}\sqrt{axx+bx}=zx^2.$

In generall

$$\frac{5maax^{2m}+\overline{5m+5n}\times abx^{m+n}+5nbbx^{2n}}{2x}\sqrt{ax^m+bx^n}=y. \qquad 1=v.$$

And $\overline{aax^{2m}+2abx^{m+n}+bbx^{2n}}\times\sqrt{ax^m+bx^n}=z.$

$ax\sqrt{b+cx}=y.$	$\frac{a}{5c}=v.$	$\overline{bccxx+2bcx-4bb}\times\sqrt{b+cx}=z.$

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$$\frac{+15aee}{x^6\sqrt{dxx+e}} = y. \qquad a = v. \qquad -3ee + 4dex^2 - 8d^2x^4\sqrt{dxx+e} = x^5ze.$$

$$\frac{+15aee}{2x^3\sqrt{dxx+ex}} = y. \qquad a = v. \qquad -3ee + ex - 8d^2xx\sqrt{dxx+ex} = x^3ze.$$

$$\frac{+15aeexx}{2\sqrt{d+ex}} = y. \qquad a + v = 0. \qquad -3eex^2 + 4dex - 8dd\sqrt{d+ex} = ze.$$

$$\frac{+15aeex^5}{\sqrt{d+exx}} = y. \qquad a + v = 0. \qquad -3eex^4 + 4dex^2 - 8d^2\sqrt{d+exx} = ze.$$

In generall

$$\frac{15naeex^{3n-1}}{2\sqrt{d+ex^n}} = y \qquad a = v. \qquad \text{And}$$

$$-3eex^{2n} - 4edx^n + 8dd\sqrt{d+ex^n} = ze.$$

$$15ddx\sqrt{dx+e} = y. \qquad 1 = v. \qquad \overline{6ddxx + 2dex - 4ee}\sqrt{dx+e} = z.$$

$$60ddx^3\sqrt{dxx+e} = y. \qquad 1 = v. \qquad \overline{12ddx^4 + 4dexx - 8ee}\sqrt{dxx+e} = z.$$

$$\frac{-15dd}{x^4}\sqrt{dx+exx} = y. \qquad 1 = v. \qquad \frac{\overline{-6dd-2dex+4eexx}\sqrt{dx+exx}}{x^3} = z.$$

$$\frac{-15dd\sqrt{d+exx}}{x^6} = y. \qquad 1 = v. \qquad \frac{\overline{-3dd-dexx+2eex^4}}{x^5}\sqrt{d+exx} = z.$$

In generall

$$\frac{15nddx^{2n}}{x}\sqrt{dx^n+e} = y. \qquad 1 = v. \qquad \overline{6ddx^{2n} + 2dex^n - 4ee}\sqrt{dx+e} = z.$$

$$\frac{24ddxx-3ee}{x}\sqrt{dxx+ex} = y. \qquad 1 = v. \qquad \left.\begin{matrix} 8ddxx \\ +2dex \\ -6ee \end{matrix}\right\} \times \sqrt{dxx+ex} = z.$$

$$77ddx^4 - 5ee\sqrt{dx+\frac{e}{x}} = y. \qquad 1 = v. \qquad \left.\begin{matrix} 14ddx^4 \\ +4dexx \\ -10ee \end{matrix}\right\} \times \sqrt{dx^3+ex} = z.$$

$$\frac{8dd+eexx}{x^3}\sqrt{d+ex} = y. \qquad 1 = v. \qquad \overline{-4dd - 2dex + 2eex^2}\sqrt{d+ex} = xxz.$$

$$\frac{45dd+3eex^4\sqrt{dx+ex^3}}{x^6} = y. \qquad 1 = v. \qquad \left.\begin{matrix} -10dd \\ -8dexx \\ +6eex^4 \end{matrix}\right\} \sqrt{dx+ex^3} = x^5z.$$

$\frac{32dd+4eex^4}{x^5}\sqrt{d+exx}=y.$	$1=v.$	$\left.\begin{matrix}-8dd\\ -4dexx\\ +4eex^4\end{matrix}\right\}\sqrt{d+exx}=x^4z.$
$\overline{35ddxx-8ee}\sqrt{dx+e}=y.$	$1=v.$	$\left.\begin{matrix}10ddxx\\ +2dex\\ -6ee\end{matrix}\right\}\sqrt{dx+e}=z.$
$\overline{96ddx^4-12ee}\sqrt{dxx+e}=y.$	$1=v.$	$\left.\begin{matrix}\overline{16ddx^4+4dex^2}\\ \underline{-12ee}\end{matrix}\right\}\sqrt{dxx+e}=z.$
$\frac{21dd+3deex^4}{x^5}\sqrt{dx+ex^3}=y.$	$1=v.$	$\left.\begin{matrix}-12dd\\ -4dexx\\ \underline{+2eex^4}\end{matrix}\right\}\sqrt{dx+ex^3}=zx^4.$
$\overline{48ddxx-15ee}\sqrt{dxx+ex}=y.$	$1=v.$	$\left.\begin{matrix}\overline{8ddxx}\\ +2dex\\ \underline{-10ee}\end{matrix}\right\}\sqrt{dx^4+ex^3}=z.$
$\overline{117ddx^4-21ee}\sqrt{dx^3+ex}=y.$	$1=v.$	$\left.\begin{matrix}18ddx^4+4dexx\\ \underline{-14ee}\end{matrix}\right\}\sqrt{dx^5+ex^3}=z.$
$\frac{12d}{x^4}\sqrt{d+exx}=y.$	$v=1.$	$\overline{-4d-4exx}\sqrt{d+exx}=zx^3.$
$\frac{-dd-8eexx}{xx}\sqrt{dx+exx}=y.$		$\overline{2dd-2dex-4eex^2}\sqrt{dx+ex^2}=z.$
$\overline{63ddx^3-24eex}\sqrt{dx+e}=y.$	$v=1.$	$\left.\begin{matrix}14ddxx\\ +2dex\\ -12ee\end{matrix}\right\}\sqrt{dx^5+ex^4}=z.$
$\overline{80ddx^3-35eex}\sqrt{dxx+ex}=y.$	$v=1.$	$\left.\begin{matrix}16ddx\\ +2dex\\ -14ee\end{matrix}\right\}\sqrt{dx^6+ex^5}=z.$
$\frac{3dd-24eexx}{x}\sqrt{dx+exx}=y.$	$v=1.$	$\overline{6dd-2dex-8eex^2}\sqrt{dx+exx}=z.$
$\overline{99ddx^4-42eexx}\sqrt{dx+e}=y.$	$v=1.$	$\left.\begin{matrix}18ddx^5+2dex^4\\ -16eexxx\end{matrix}\right\}\sqrt{dx+e}=z.$
$\overline{8dd-35eexx}\sqrt{d+ex}=y.$	$v=1.$	$\overline{8ddx-2dexx-10eex}\sqrt{d+ex}=z.$
$\overline{120ddx^4-63eexx}\sqrt{dxx+ex}=y.$	$v=1.$	$\overline{20ddx^5+6dex^4-18eex^3}\sqrt{xx+ex}=z.$
$\frac{-4dd-32eex^4}{xx}\sqrt{d+exx}=y.$	$v=1.$	$\overline{4dd-4dexx-16eex^4}\sqrt{d+exx}=zx.$
$\frac{\overline{24dd-3eexx}\sqrt{d+ex}}{x^4}=y.$	$v=1.$	$-8dd-2dex-6eexx\sqrt{d+ex}=zx^3.$

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In Generall.

$\overline{mm+8mn+15nn}\times ddx^{2n-1}-\overline{mm-2mn}\;eex^{-1}$ in $\sqrt{dx^{m+n}+ex^m}=y.\ 1=v.$ &, $\overline{2m+6n}\times ddx^{2n}+2ndex^n-\overline{2m-4n}\times ee$ in $\sqrt{dx^{m+n}+ex^m}=z.$

[1] prop 12. 13 & I think 11 are trew onely mechanically.