History of the Method of Fluxions

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The History of the Method of Series Fluxions & Moments.

D[r] Wallis in his Opus Arithmeticum published A.C. 1657 cap 33 Prop. 68 reduced the fraction $\frac{A}{1-R}$ by perpetual division into the series $AR^2 + AR^3 + AR^4 +$ &c

In the winter between the years 1664 & 1665 M[r] Newton (now Sir Isaac Newton) by trying to interpole a series of D[r] Wallis, for squaring the circle found out the series for squaring the circle & Hyperbola & their segments. Let the radius of a circle be 1 & the Abscissa x & the Ordinate $\sqrt{1-xx}$ & the segment described by the Ordinate while the Abscissa increases will be $x - \frac{1}{6}x^3 - \frac{1}{40}x^5 - \frac{1}{112}x^7 - \frac{5}{1152}x^9 -$ &c . And if $\sqrt{1+xx}$ be the rectangular Ordinate of the Hyperbola the segment will be $x + \frac{1}{6}x^3 - \frac{1}{40}x^5 + \frac{1}{112}x^7 - \frac{5}{1152}x^9 +$ &c . And at the same time he found out also the reducing of the dignities of Binomials into converging series, & the extraction of roots both single & affected in such series, & before the next winter he found, the solution of this Probleme <u>Data Æquatione fluentes quotcunque quantitates involvente fluxiones invenire</u>. And the next winter he found his Theory of refractions & colours & the next year he found how to proceed in his method of fluxions without stopping at fractions or surds.

And that time Vicount Brounker squared the Hyperbola by this series $\frac{1}{1\times2} + \frac{1}{3\times4} + \frac{1}{5\times6} + \frac{1}{7\times8} +$ &c , that is by this $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} +$ &c , conjoyning every two terms into one. And the Quadrature was published in the Philosophical Transactions for April 1668.

Ejus anni mense Septembri D. Mercator published his Logarithmotechnia with a Demonstration of this Quadrature by the Division of D[r] Wallis. And soon after that M[r] Iames Gregory published a Geometrical Demonstration thereof. And these Books were a few months after sent by M[r] Iohn Collins to D[r] Barrow at Cambridge & by D[r] Barrow communicated in May or Iune 1669 to M[r] Newton. Whereupon D[r] Barrow mutually sent to M[r] Collins a Tract of M[r] Newton entituled <u>Analysis per series numero terminorum</u> infinitas. And in this Analysis the Author shews first how to reduce the Ordinates of Curvilinear figures into converging series & how by those series to square the Figures. And then he shews how by his method of fluxions to apply this method of Series to the solution of other Problems And then adds. (Nec <u>quicquam hujus modi scio ad quod hæc Methodus idque varijs modis se non extendit Et quicquid vulgaris Analysis per Æquationes ex finito terminorum numero constantes, (quando id sit possibile) perfici, hæc per æquationes infinitas semper perficiat: ut nil dubitaverim momen Analyseos huic etiam tribuere. Ratiocinia nempe in hac non minus certo sunt quam in illa, nec æquationes minus</u> <146v> exactæ. Denique ad Analytican merito pertinere censeatur, cujus beneficio Curvarum areæ & longitudines &c (id modo fiat) exacte et Geometrice determinentur: Sed ista narrandi non est locus. These last words refer to a method, which is explained in a Letter of M[r] Newton to M[r] Oldenburg dated 24 Octob. 1676 & more fully in the 5[th] Proposition of M[r] Newtons Tract de Quadratura Curvarum, & this method is not to be attained without the knowledge & conteined in the four preceding Propositions of that Book. And therefore, M[r] Newton in the year 1669 under the Method of fluxions so far as it is conteined in the first four or five Propositions of that Book of Quadratures.

And by the testimony of Dr Barrow & Mr Collins he had it some years before Mercators Logarithmotechnia came abroad. For Mr Collins in a Letter to Mr Strode dated 26 Iuly 1672 writes thus. Haud multo postquam in publicum prodierat [Mercatoris Logarithmotechnia,] exemplar ejus — Barrovio Cantabrigiam misi, qui quasdam Newtoni chartas [sc. Analysin prædictam per series] extemplo remisit: e quibus et ex alijs quæ OLIM ab auctore cum Barrovio communicata fuerant, patet illam Methodum a dicto Newtono ALIQVOT ANNIS ANTEA excogitatam et modo universali applicatam fuisse; ita ut ejus ope in quavis Figura curvilinea proposita quæ una vel pluribus proprietatibus definitur Quadratura vel Area dictæ figuræ, ACCVRATA si possibile sit, sin minus infinite vero propinqua; evolutio vel longitudo lineæ curvæ Centrum gravitatis figuræ, Solida ejus rotatione genita et eorum superficies; sine ulla radicum extractione obtineri queant. Mr Newton therefore in the year 1676 had the Method of fluxions so far at the least as is is conteined in the first four or five Propositions of the Book of Quadratures.

In the above mentiond Analysis Mr Newton sometimes represents fluents by the Areas of Curves, fluxions by their ordinates, time by their common Abscissa, uniformly increasing, a moment of time by a small part of the Abscissa & moments of the fluents by the Ordinates drawn into the moment of time, & sometimes by the Ordinates alone, the moment of time being not exprest but understood. And sometimes he denotes the fluent by the fluxion encompassed with a square. As if a be a given quantity & x be the time & $\frac{aa}{64x}$ the fluxion, he represents the fluent by $\boxed{\frac{aa}{64x}}$

Pag. 182 lin 16 add: except that he doth not confine himself to any set form of symbols.

Pag. 194. lin. 6. This letter of Mr Newton dated Octob. 24 1676 was seen by Mr Leibnitz as soon as it came to London that before the end of that month For Mr Leibnitz was then at London in his way from Paris into Germany: But he did not stay to take a copy of that long Letter with him. Yet he had time to visit Mr Collins who shewed him many Letters of Mr Newton Mr Gregory & others which ran principally upon series. And it doth not yet appear that he did not then see the Demonstration of the two series which he desired Mr Oldenburg to procure from Mr Collins & by consequence the <u>Analysis per series numero terminorum infinitas</u>. And that time he procured Dr Barrows Lectures & carried them with him into Germany. And when Mr Oldenburg heard that he was arrived at Hannover he sent to him a copy of Mr Newtons Letters of October 24.

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Pag. 186. lin. 3. add. For by the Demonstration of these two series which he desired Mr Oldenburg to procure from Mr Collins, saying that Mr Collins could easily supply him therewith, he meant the method of finding them conteined in the Analysis per series numero terminorum infinitas

Pag. 197. lin. 15. add. And while he knew by Mr Newtons Letters that Mr Newton had such a method before the year 1677, he ought not to have published the differential method as his own without mentioning that correspondence & making a candid an acknowledgement of Mr Newtons having before those days a method like the differential, as he made thereof in that Letter.

In Feb. 1682 Mr Leibnitz published the series of Mr Iames Gregory in the Acta Eruditorum as his own without making any mention of his having received it from Mr Oldenburg & Mr Collins In NovemberOctober 1682 —— & 1676. In the Acta Eruditorum of Iune Pag 198 lin. 32. post <u>Lemmate</u> add. The designe of this Scholium was not to give away Lemma but to put Mr Leibnitz in mind of making a publick & candidacknowledgment of this correspondence which he had with with Mr Newton in the year 1696 by means of Mr Oldenburg [& of what he had learnt by that correspondence], before he proceeded any further to claim the differential mathod exclusively of Mr Newton For in these Letters which in the year 1676 passed between them, & in another Letter dated 10 Decemb 1672, a copy of which was sent to Mr Leibnitz in the year 1676, as is mentioned above

Pag. 199. lin.5. Insert. D^{r} Halley & M^{r} Ralpson had the Book of Quadratures in MS in their hands in the year 1691 as M^{r} Ralpson has attested publickly & D^{r} Halley still attests

Pag. [198. lin 1]. or Pag. 197 lin 15 insert. In spring 1684 M^{r} Newton made known to some Mathematicians that he had demonstrated from the Principle of Gravity the Proposition of Kepler that that the Planets move in Ellipsis & with rays drawn to the Sun placed in the lower focus of the Ellipsis describe areas proportionall to the times. And in Autum following he sent the Demonstration to D^{r} Halley who communicated it to the R. Society with some other Propositions concerning the heaven: & the R. S. desired that the D^{r} Hook said there upon to **{**apermigh**}** have be printed; & M^{r} Newton thereupon began to write his Book of Principles M^{r} Hook is contended with M^{r} Newton about this matter: but he never produced any Demonstration of the Proposition. For he was not skilled in Mathematicks & the Demonstration of Keplers was not to be found without the Method of fluxions.

In November 1684 M^{r} Leibnitz published in his letters of 1672 & 1676. But it was impossible from foreigners to understand this by the words here published. M^{r} Leibnitz ought to have acknowledged in express words that he understood by his late correspondence with M^{r} Oldenburg that M^{r} Newton had a methodus similis. ⊙+ < insertion from f 147r > ⊙+ On the contrary He < text from f 147v resumes > < insertion from f 146v > ⊙+ On the contrary he published in the Acta Eruditorum for May 1700 pag 203, that when he published the Elements of his Calculus < text from f 147v resumes > He

In the Acta Eruditorum of Iune 1686 pag. 297 M^{r} Leibnitz added more brevity.

Anno 1686 mense Maio Newtonus Philosophiæ naturalis PrincipioMathematica ad Societatem Regiam misit ut imprimeretur. Et liber ille Mense Martio anni proximi lucem vidit. This book is full of Problems he acknowledged the same thing. Certe, saith he, cum edidi calculi mei edidi anno 1684 ne constabat quidem mihi aliud de inventis ejus in hoc genere quod olim ipse significaverat in literis, posse se tangentes <147r> invenire non sublatis irrationalibus, quod Hygenius quoque se posse mihi significavit postea, etsi cæterorum illius Calculi ad huc expers: sed majore multo consecutum Newtonum, viso denum libro Principiorum ejus satis intellexi. And a little after, pag. 206 lin. 5. Non hic de problemate menti valdeque diffusa circa maxima et minima fuit actum: quam ante Newtonum et me nullus, quod sciam, Geometra habuit, uti ante hunc maximi nominis Geometram nemo specimine publice dato se habere probavit This method is a principal part of the Method of fluxions being the method by which the Probleme de Curva celerimi descensus proposed by M^{r} Bernoulli to all the world, was solved, & M^{r} Leibnitz here acknowledges that M^{r} Newton by finding the solid of least resistance had proved that he had this method when he wrote his book of Principles. Vide Scholium in Prop. XXXIV Lib.II.

Pag. 199. lin 4. add. D^{r} Halley & M^{r} Ralpson had the Book of Quadratures in MS in their hands in the year 1691 as M^{r} Ralpson has attestested publick ly & D^{r} Halley still attests

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2 Of the Method of fluxions & moments

And M^{r} Leibnitz in his Letter dated 21 Iune 1677 drew tangents by putting the symbols dy & dx for the letters a & e of D^{r} Barrow. , or $d\,y^3 = 3yy\,d\,y$ & in general the Lemma became $d\,y^n = ny^{n-1}\,d\,y$; & the convers of this is the first Rule of the Analysis per Æquationes numero terminorum infinitas]

When M^{r} Newton in his Letter of 10 Decem. 1672 had described his method of Tangents, he added. Hoc est unum particulare vel Corollarium potius Methodi generalis quæ extendit se citra molestum ullum calculum non modo ad ducendum Tangentes ad quasvis Curvas sive Geometricas sive mechanicas vel quomodo cunque rectas lineas aliasve Curvas respicientes verum etiam ad resolvendum alia abstrusi <148v> ora Problematum genera de Curvitatibus Areis Longitudinibus, centris gravitatis Curvarum &c. Neque (quemadmodum Huddenij methodus de maximis et minimis) ad solae restringitur æquationes illas quæ surdis quantitatibus sunt immunes. And in his Letter dated 13 Iune 1676, after he had described his method of series he added: Ex his videre est quantum fines Analyseos per hujus modi Æquationes infinitas ampliantur: quippe quæ earum beneficio ad omnia pene dixerim problemata, si numeralia Diophanti et similia excipias, sese extendit: non tamen omnino universalis evadit nisi per ulteriores quasdam methodos eliciendi series infinitas   Sed quomodo in istis casibus procedendum sit non vacat dicere: ut neque alia quædam tradere quæ circa reductionem serierum infinitarum in finitas ubi rei natura tulerit, excogitavi. And in his Letter dated 24 Octob. 1676, he represented that a Tract which he wrote five years before upon the method of Series, was for the most part taken up with other things.That there was in it the method of Slusius built upon another foundationwhich gave that method readily, even without a particular demonstration, & made it more general so as not to stick at surdes; the Tangent not withstanding surdes being speedily drawn without any reduction of the Equation which would often render the work immense. And that the same manner of working held in Questions de Maximis & Minimis & some others which in that Letter he forbore to speak of. And that upon the same foundation the Quadrature of Curves became more easy, an example of which he gave in a series which brake off & became finite when the Quadrature might be done by a finite equation. And that this method extended to inverse problemes of Tangents & others more difficult. But the foundation of this method he concealed in sentences set down enigmatically, the first of which was this: Data æquatione quotcunque fluentes quantitates involvente, fluxiones invenire, & vice versa

<149r> An example of this you have in his Demonstration of the Construction of the first Proposition of his Book of Quadratures. vizt Data æquatione fluentes quotcunque quantitates involvente invenire fluxiones And another example you have in his demonstration of the first of the three Lemmas upon which he grownded his Treatise intituled Analysis per æquationes numero terminorum infinitas. [But when he is only investigating any truth or the solution of any Problem he supposes the moment of time to be infinitely little in the sense of Philosophers, & works in figures or schemes infinitely small & uses any approximations, which he conceives will make no error in the conclusion as by putting the arc chord, sine & tangent equal to one another, & for the greater dispatch he neglects to write down the moment o.

In the Analysis above mentioned which Dr Barrow sent to Mr Collins in the year 1669, his principall designe was to describe the method of series: but that method being inseparably conjoyned with the method of fluxions, When he described the three Lemmas upon which he founded this Analysis in squaring of Curves, he touched upon the method of fluxions in the following manner. Et hæc de areis Curvarum investigandis

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[This Analysis is the first piece printed in the Commercium. It was sent to Mr Collins in Iuly 1669 as appears by the dates of three of Dr Barrows Letters still extant. Mr Newton in his Letter dated 24 Octob. 1676 mentions it in this manner. Eo ipso tem <149r> pore quo Liber [Mercatoris] prodijt communicatum est per amicum D. Barrow (hunc matheseos Professorem Cantab) cum D. Collinio, compendi um quoddam Methodi harum serierum, in quo significaveram areas & Longitudines Curvarum omnium & solidorum superficies & con tenta, ex datis Rectis; et vice versa ex his datis Rectas determinary posse: et methodum illustraveram diversis seriebus. Mr Oldenburg in a Letter to Mr Slusius dated 14 Sept. 1669, & entred in the Letterbook of the R. Society, in giving an account of it cites several sentences out of it. And Mr Collins in a Letter to Mr Strode dated 26 Iuly 1672 mention of it. Exemplus ejus [Logarithmotechnia]   si cum meridiana clasitate conferatur.

Now This Analysis or Compendium conteins a general method composed of two methods the one converging series, the other of moments & fluxions.] In this Compendium.

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An Account of the Analysis per Quantitatum Series Fluxiones ac Differentias cum Enumeratione Linearum tertij Ordinis, published by Mr Iones.

11 If an Equation contein two indeterminate quantities suppose x & y; to find either of them when the other is greatest or least. Des Cartes teaches that the quantity to be found suppose x will in that case have two of its roots become equal. Fermat for the difference of those two roots before they become equal, puts the letter o, & thereby has two equations in both which the other quantity y ought to be one & the same. THen by

exterminating that other quantity & reducing the equation & putting the two roots suppose x & x+o equal, that is, by putting the difference o equal to nothing he finds the quantity x, & by this method draws perpendiculars to Curves & resolves other Problemes by maxima & minima.

12 Iames Gregory in the 7th Proposition of his Geometriæ pars Vniversalis published 1678 puts the letter o for the difference of the Ordinates, & that this line may become the Tangent of the Curve, makes the difference o vanish.

13 Barrow in his 10th Lecture published 1669, to find Tangents, puts the letters a & e for the differences of two Abscissas & two Ordinates, conceives a right line to be drawn through the ends of the Ordinates, brings the Probleme to an equation rejects all the terms of the equation in which a & e are either wanting or of more dimensions then one & by the proportion of a to e draws the Tangent.

was communicated by Hudde to Schortem in November 1659 to be kept secret, & Slusius also & Newton fell upon the same method

14 Slusius founded his method of Tangents on three Lemmas the first of which was this. Differentia duarum dignitarum ejusdem gradus applicata ad Differentiam laterum dat partes singulares gradus inferioris ex binomio laterum, ut $\frac{y^3-x^3}{y-x} = yy+yx+xx$. That is, in the language of Mr Leibnitz, $\frac{d\,y^n}{d\,y} = ny^{n-1}$ or $d\,y^n = n\,d\,yy^{n-1}$

15 A month before Slusius sent his Method to Mr Oldenburgh Newton at the request of Mr Collins sent his in a letter dated 10 Decem. 1672, & added Hoc est unum particulare vel Corollarium potius methodi generalis, quæ extendit se citra molestum ullum calculum non modo ad decendum Tangentes ad quasvis Curvas sive Geometricas sive Mechanicas vel quomocunque rectas lineas aliasve Curvas respicientes; verum etiam ad resolvendum alia abstrusiora Problematum genera de Curvitatibus, Areis Longitudinibus, Centris gravitatis Curvarum &c. Neque quemadmodum Huddenij Methodus de Maximis et Minimis ad solas restringitur quantitates illas quæ quantitatibus surdis sunt immunes. And a copy of this Letter was sent to Mr Leibnitz among the extracts of Gregories Letters above mentioned, Iun 26.

16 Mr Leibnitz being then at London in the beginning of the next year 1673 pretended to the invention of the Differential method of Mouton & being reprehended for it by Dr Pell, persisted in making himself coinventor of that method, & it appears not that he had any other Differential method at that time.

When Mr Leibnitz wrote

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28 Mr Leibnitz in a letter to Mr Oldenburg dated 2818 Novem 1676 Methodus Tangentium a**{s}** lusip publicata nondum rei fastigium tenet potest aliquid amplius prœstari in es genere quod maximi foret usus ad omnis generis Problemata: etiam ad meam (sine extractionibus Æquationum ad Series Reductionem. Nimirum posset brevis calculari quæ calculari circa Tangentes Tabula, eouque continuanda donec progressio Tabulæ apparet, ut eam scilicet quisque, quousque libuerit, sine calculo continuare possit.

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3. Of the Method of Fluxions & Moments

But as for the Method of fluxions it was certainly known to Mr Newton when he wrote his Letter to Mr Oldenburg dated 24 Octob. 1676. For in that Letter he comprehended it in these sentences. Data æquatione quotcunque fluentes quantitates involvente fluxiones invenire et vice versa. And said that this was the foundation of the method upon which in conjunction with the method of Series he had writ a treatise five years before vizt in the year 1671

In the mean time it remains to be considered whether M^r^ Leibnitz after M^r^ Newton had in three Letters sent to him described the characters of his method & told him that in the year 1671 he had wrote a treatise of it

In the mean time, when M^r^ Newton had told M^r^ Leibnitz that in the year 1671 he had wrote a treatise of the the method of Series & of another method together, when he had in three Letters which came to the hands of M^r^ Leibitz described the characters & universality of this other method so far as to enable M^r^ Leibnitz to compare it with the Differential & see that they did not ab invicem abludere but were similes, when he had concealed the foundation of it in an Ænigma to hide it not from honest men but from plagiaries

Now after M^r^ Newton had told M^r^ Leibnitz that in the year 1671 he wrote a treatise of the method of Series & of another method together, after he had in three Letters which came to the hands of M^r^ Leibnitz described the characters & universality of this other method so far as to enable M^r^ Leibnitz to compare it with the differrential method & say that they did not abinvicem abluder but were similes, after M^r^ Newton had concealed the foundation of it in an Ænigma to hid it not from honest men but from plagiaries: the Question is, whether M^r^ Leibnitz should not rather have invited & encouraged M^r^ Newton to have published his method then have rivalled him & claimed the method from him by saying &

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My computation of the time between the burning of the first Temple & building of the 2^d^ is this.

Pharoah Nechoh reigned over Phenicia & Syrea as far as Euphrates the three first years of Iehojakim 2 King **{**3**}**3

In the year Nebuchadnezzar (having newly conquered Assyria) came against him & besieged Ierusalem & in the 4^th^ year beat him at Euphrates & took from him all Syria & Phœnicia from the river of Egypt to the river Euphrates & reigned in his stead, the first year of Nebuchadnezzar over those countries being the 4^th^ of Iehojakim. But whether he took Ierusalem in the thrid or fourth year doth not appear. Dan. 1,1. Ier 46, 2

Nebuchadnezzar continued with his army in those parts to conquer the nations round about (Ier 25.9,11) & to recover to Babylon whatever had lately belonged to Assyria & settle his new conquests untill he heard of the death of his father which was in the fift or sixt year of Iehojakim, & then hasted to Babylon to succeed his father in the whole kingdom leaving his captains to follow with the captives, & in the 43^th^ year of his reign counted from the death of his father he the died **{**Eupolamug &**}** Berosus apud Euseb l.9 c.39,40 By the Canon of Ptolomy he succeded his father A. Nabonass 144 &died A. Nabonass 187. But the Iews recconed not by the years of Nabonassar

Iehojakim reigne eleven years & Iehojakim three months being captivated in the end of the eleventh year or beginning of the 12^th^ (2 Chrom 36. 10.) And the 12^th^ year was the first year of his successor Zedekiah & the first year of the captivity of Iehojakin, the years of this captivity beginned with the reigh of Zedekiah. The Iews recconed by Lumisolar years the Babylonian Astronomers by the years of Nibonassar.

In the 37^th^ year of this captivity, that is in the 45^th^ year of the reign of Nebuchadnezzar recconned by Lumisolar years from the 4^th^ year of Iehojakim inclusively, Nebuchadnezzar died, & was succeeded by his son Evilmerodach. And in the 25^th^ day of the 12^th^ month of the year brought his friend Iehojakim out of prison 2 King. 25. 27 Ier. 52. 31. It is not likely that after he came to the throne he would lett his great friend Iehojakim stay long in prison & therefore it's reasonable to beleive that Nebuchadnezzar died in the end of winter neare the end of the 45^th^ year of his reign & by consequence in the beginning of the year of Nabonassar 187 Iehojakim therefore began his reign in the year of Nabonassar 139, & Nebuchadnezzar in the year year of Nabonassar 142 according to the Iewish account, & one or two years after that he succeded his father at Babylon & reigned 43 years form the death of his father, according to the recconing of the Chaldeans, & Canon of Ptolomy, & 44 years & some months according to the recconing of the Iews.

The Iews had fasted in the 5^t^ month for the burning of the City & in the 7th month for the death of Gedaliah just 70 years befoer the 9th month of the 4th year of Darius Hystaspis 2 King. 25. 1 Zech 7. 1 And in the eleventh month of the 2d year of Darius Gods indignation against the cities of Iudah had lasted just 70 years, the indignation beginning in the tenth month of the ninth year of Zedekiah See Ier 34. 7 & Zech 1.12, And these two recconingagrees exactly with the computation here set down

The Iews were to serve the King of Babylon 70 years. & after 70 years were accomplished at Babylon they were to return to their own land (Ier 25. 12 & 29. 10). They began to serve him in the first year of his reign over them which was the fourth year of Iehojakim A. Nabonass. 142. They remained in captivity until the reign of the kingdom of Persia, & in the first year of Cyrus king of Persia (A. Nabonass. 212) they returned home (2 Chron. 36. 20,21,22) But Dr Alex places the 4th year of Iehojakim & 1st of Nebuchadnezzar two years later.

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My computation is therefore favored by 4 arguments taken from scripture 1st the death of Nebuchadnezzar in the 37th year of Iehojakins captivity. 2dly the fasting 70 years. 3dly the indignation upon the cities of Iudah 70 years & 4thly the serving the king of Babylon 70 years.

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Ex mente Newtoni, primus annus Nebuchadnezzaris juxta Iudœos fuit 4tus Iehojakimi. juxta Iudæos et non secundus sed sextus junta Chaldæos. Et regnavit ille annos 48 a morte Patris, annos vero 44 & menses aliquot a quarto Iehojakim inclusive. Obijt enim & filio Euilm erodacho regnum reliquit anno 37mo captivitatis Iehojachin, hoc est, anno 37 regni Zedekiæ ed est. anno 48vo regni Iehojakim, et propterea anno 45to regni proprij quod capit anno 4to Iehojakimi inclusive.

Et intra annum unum generationes quator eatitisseo (sc Regem Galliæ scilicet & tres Delphinos) proba**{**biliest**}** est & quam aliquos in vivis esse qui Regem nostrum Henricum Octavum de facie norant.

Observations upon the Notes of the Reverend Dr Alix.

The Dr in the 2d Paragaph supposes that I place the 1st year of Nebuchadnezzar according to the Chaldees, upon the 2d of Iehojakim, disputes against this opinion & in the 6t Paragraph concludes that I erred in placing the destruction of Ierusalem upon the 17th year of Nebuchadn. according to the Chaldees. But if I placed the taking of Ierusalem upon the 17th year of Nebuchadn. dated from the death of his father according to the Chaldees, I placed the first year of his reign not upon the 2d but uopn the sixt year of Iehojakim.

Evilmerodach succeeded his father Nbuchadnezzar in the 37th year of Iehojakins captivity (2 King 25. 27) The years of this captivity & the years of the reign of Zedekiah have the same epocha. Add the eleven years of Iehojakim & the death of Nebuchnezzar will fall upon the 48th year of Iehojakim recconed from the first year of his reign inclusively. Take away the three first years of Iehojakins &the death of Nebuchadnezzar will fall upon the 45th year of his reign recconed from the 4th year of Iehojakim inclusively. Whereas Nebuchadnezzar reigned but 43 years from the death of his father according to the Canon.

During the three first years of Iehojakim, the king of Egypt reigned over Palestine & Cœlosyria. Nebuchadnezzar came against him in the 3d year of Iehojakim & captivated some of the Iews & the next year beat him at Carchemish & took from him all Syria & Palest**{**ine**}** from Euphrates to the borders of Egypt (2 King. 24.7) & reigned in his stead. And when the war was fully ended Nebuchadnezzar heard of the death of his father & returned in hast to Babylon to succeed him, leaving his Captains to follow with his army & the captives according to Berosus.

His reign therefore might have a double beginning, the first when he succeeded Pharaoh Nechoh in Syria & Palestine & the neighbouring coasts of Arabia & Hamath, the second when he succeeded his father at Babylon the first commonly used by the people of Syria & Phenicia, the second by these in Balonia.

Lin 4. lige, annis tardius scil. a 6to Iehojakimi (vide lin 28) Lin 9, lege A. Nabonass 160 lin: 11 An Ezekiel in Chaldœa inter captivos usus sit annis Nabonassari? Lin 13 Annis 10 dantur regno Iehojakimi, 12 regno Zedekiœ. Nam regnum Zedeckiæ cœpit cum annis captivitatis Iehojakin. Lin 14 lije 1 37 ad finem vergente ab ipsius deportatione, A ultimo Nebuchadnezzaris quo Evilmerodach regnare cœpit. Lin 6. Nebuchadnezzar anno 3^{o} Iehojakimi Hieroslyma obsedisse dicitur, eodem anno cœpissa non dicitur. L. 22. Suppano, Pharaonem cœsum anno quarto Iehojakimi & Nebuchadnezzarum eidem in regno Syria et Palestine tune successisse, patri vero mortua successisse post bellum finitum. Lin 50 Quatuor fuerunt generationes eodem tempore in Gallia Rex ipse et tres Delphini.

In the first year of Cyrus Iddo might be as old as the present king of France & Z solary as oldn then his Grandson the King of Spain. But could

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An account of the Analysis per quantitatum series fluxiones ac differentias cum enumeratione linearum tertij Ordinis published by M^{r} Iones

Leibnits in the year 1677, for the letters a & e, substituted the symbols dx & dy, & drew tangents after the same manner with Barrow. This method he afterwards published in the Acta Eruditorum mensis Octobris 1684, & in the conclusion added. Et hæc quidem initia sunt tantum Geometriæ cujusdam multo sublimioris ad difficillima & pulcherrima quæque etiam mistæ matheseos Problemata, quæ sine calculo nostro differentiali, aut simili, non temere quisquam pari facilitate tractabit. By the words aut simili he means Newtons method as is evident by his letter of 21 Iune 1677. For he had notice of this method by three of M^{r} Newtons letters dated 10 Decem. 1673. 13 Iune 1676 & 24 Octob 1676 & communicated to him by M^{r} Oldenburgh. Whe he changed the letters a & e into dx & dy he tells us two years after in the Acta Eruditorum mensis Iunij 1686 Malo autem, saith he, dx et similia ad hibere quam literas pro illis quia istud dx est modifactio quædam ipsius x, & ita ope ejus fit ut sola quando id fieri opus est litera x, cum suis scilicet potestatibus & differentijs calculum ingrediatur & relationes transcendentes inter x et aliud exprimantur. Qua ratione etiam lineas transcendentes æquatione explicare licet.

<154v> quantity or particle of time & proceeds in finite quantities & finite figures by Euclides Geometry to the end of the calculation without any approximation or error, & then supposes the indefinitely small quantitys to decrease in finitum & vanish or become nothing. But when he is only investigating a truth or the solution of a Probleme he supposes the moment o to be infinitely little & proceeds in the calculation by such approximations as he thinks will create no error in the conclusion, as by putting arches & their chords sines & tangents for one another. And for making dispatch he forbeares to write down the letter o putting the symbol of the fluxion alone for the moment of the fluxion, but understanding that symbol to be multiplied by the moment o to make it infinitely little whenever it signifies a moment. For fluxions or velocities are finite quantities but moments are infinitely little. In the book of Quadratures & some other papers M^{r}

This Tract of Analysis is founded on three Rules, The two first of which are equipollent . . . . .

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2 Off the Differential Method

D^{r} Barrow in his 10th Lecture published 1670, to find Tangents to Curves, puts the Letters a & e for the indefinitely small Differences of the Abscissas & Ordinates draws a right line through the ends of the Ordinates, by the conditions of the Curve brings the Probleme to an Equation, rejects all the terms of the Equation in which a & e are either wanting or of more dimensions then one & by the proportion of a to e draws the Tangent. And this method readily gives the method of Slusius.

Mr Newton at the request of Mr Collins sent him his method of Tangents in a Letter dated 10 Decem. 1672. It proved to be the same with the method of Slusius, but was sent as a Corollary of a general method of solving Problems: which method in drawing of Tangents agrees with those of Gregory & Barrow.

Slusius sent his method to Mr Oldenburg in Ianuary 1673. It was founded on three Lemmas, the first of which was this. Differentia duarum dignitatum ejusdem gradus applicata ad Differentiam Laterum dat partes singulares gradus inferioris ex binomio laterum; ut $\frac{y^3-x^3}{y-x} = yy+yx+xx$

Mr Leibnitz for $\frac{y^3-x^3}{y-x} = yy+yx+xx$, wrote $\frac{d\,y^3}{d\,y} = 3yy$, putting dy & dx for the a & e of Dr Barrow. For in his Letter of 21 Iune 1677 in which he first proposed his method of Tangents, he wrote thus Clarissimi Slusij methodum Tangentium nondum esse absolutam celeberrimo Newtono assentior: et jam a multo temporerem Tangentium longe generalius tractavi, scilicet per differentias Ordinatarum. And a little after he added. Hinc nominando dy differentiam duarum proximarum y & dx differentiam duarum proximarum x; patet $d\,y^2$ esse $2y\,d\,y$, & $d\,y^3$ esse $3y^2$ et ita porro. Which is the first Lemma of Slusius. Then putting y for the Abscissa & x for the Ordinate of a Curve he assumes an equation expressing the relation between them, & to find the Tangent to this Curve he substitutes in this equation the Abscissa y+ dy for y & the Ordinate $x+d\,x$ for x, & in doing this he writes down first those terms in which dy & dx are not, & draws a line under them. Then under that line he writes down those terms in which dy & dx are but of one dimension, & draws a line under them. And under that line he writes down those terms in which dy & dx are either severally or joyntly of more dimensions then one. And then he adds: Vbi abjectis illis quæ sunt supra primam lineam, quippe nihilo æqualibus per æquationem primam; et abjectis illis quæ sunt infra secundam quia <155v> in illis duæ infinitæ parvæ in se invicem ducuntur, restabit tantum quicquid reperitur inter lineam primam et secundam. Then after he had shewn by what remained between the lines to draw the tangent, he added: Quod coincidit cum Regula Slusiana, ostenditque eam statim occurrere hanc methodum intelligenti. By hanc methodum therefore he did not understand the method of Slusius but another method which readily gave the method of Slusius; & this was the method of Dr Barrow. For Dr Barrow proposes to compute an equation from any conditions of the Curve & in doing this prescribes these Rules : Primo inter computandum omnes abjicio terminos saith he in quibus ipsarum a vel e potestas habetur, vel in quibus ipsæ ducuntur in se. Etenim isti termini nihil valebunt. Secundo post æquationem constitutam omnes abjicio terminos literis constantes quantitates notas seu determinatas designantibus, aut in quibus non habentur a vel e. Etenim illi termini semper ad unam æquationis partem adducti nibilum adæquabunt. These were Dr Barrows Rules & these Rules are followed by Mr Leibnitz who sets between two lines the terms that are to be retained & above the upper line & below the lower those two sorts of terms that by Dr Barrows Rules are to be rejected, & rejects them accordingly.

And that this was the original of Mr Leibnitz method of Tangents is further confirmed by what he wrote in the Acta Eruditorum mensis Iunij 1686 pag 299. Malo autem, saith he, dx et similia adhibere quam literas pro illis quia istud dx est modificatio quædam ipsius x et ita ope ejus fit ut sola quando id fieri opus est litera x cum suis scilicet potestatibus & differentialibus calculum ingrediatur & relationes transcendentes æquatione explicare licet. Dr Barrow used the letters a & e. Mr Leibnitz allows that he might have used Letters but tells us that for certain reasons he chose rather to use the symbols dx & dy. But he should have told us whence he had the method. He should have acknowledged that he used Dr Barrows method of Tangents, excepting that for certain reasons he had changed the letters a & e used by Dr Barrow, into the symbols dx & dy. For he had seen Dr Barrows Lectures.

Mr Leibnitz first published his method of Tangents in the Acta Eruditorum mensis Octobris An. 1684, pag.467, with this Title Nova Methodus pro maximis et minimis itemque tangentibus quæ nec fractas nec irrationales quantitates moratur, & singulare pro illis calculi genus, per G.G.L. And in the end of it he added Et hœc quidem initia sunt tantum Geometriæ cujusdam multo sublimioris ad difficillima et pulcherrima quæque etiam mistæ matheseos problemata pertingentis, quæ sine calculo nostro differentiali, aut simili, non temere quisquam pari facilitate tractabit. It remains that we enquire how Mr Leibnitz came to know that this method of tangents stuck not at fractions or surds & that it was the principles of a far more sublimer Geometry reaching to all the most difficult & curious Problems in Mathematicks & what was the Calculus similis here hinted at.

At the request of M^{r} Collins M^{r} Newton sent to him his Method of Tangents in a Letter dated 10 Decem. 1672. It proved to be the same with that which Slusius about five weeks after sent to M^{r} Oldenburg but was derived from more general Principles. For when M^{r} Newton had described it, he subjoyned in the same Letter Hoc est unum particulare vel Corollarium potius Methodi generalis quæ extendit se citra molestum ullum calculum non modo ad ducendum Tangentes ad quasvis Curvas sive Geometricas sive Mechanicas, vel quomodocunque rectas lineas aliasve Curvas respicientes; verum etiam ad resolvendum alia abstrusiora Problematum generade curvitatibus, Areis, Longitudinibus centris gravitatis Curvarum &c Neque (quemadmodum Huddenij methodus de maximis & minimis) ad solas restringitus æquationes illas quæ surdis quantitatibus sunt immunes. And a Copy of this Letter was sent by M^{r} Oldenburg Iune 26th 1676, to M^{r} Leibnitz at Paris amongst the extracts of M^{r} Gregories Letters collected by M^{r} Collins as above.

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And M^{r} Newton in his Letter dated 13 Iune 1676 & sent by M^{r} Oldenburg to Paris Iune 26, taught how to resolve any dignity of a binomium into a converging series, the second terme of which Series by the method of D^{r} Barrow readily gives the first Lemma of Slusius together with his wole method. And after he had in that Letter described his method of Series, he subjoyned: Ex his videre est quantum fines Analyseos per hujusmodi æquationes infinitas ampliantur: quippe quæ earum beneficio ad omnia pene dixerim problemata, si numeralia Diophanti et similia excipias, sese extendit: non tamen omnino universalis evadit nisi per ulteriores quasdam methodus eliciendi series infinitas . . . . . . Sed quomodo in istis casibus procedendum sit non vacat dicere: ut neque alia quædam tradere quæ circa reductionem serierum infinitarum in finitas ubi rei natura tulerit, excogitabi.

And in his Letter dated 24 Octob. 1676, he represented how the Tract which he wrote five years before upon the method of Series, was for the most part taken up with other things. That there was in it the method of Slusius built upon another foundation which gave that method readily, even without a particular Demonstration, & made it more general so as not to stick at surdes; the tangent, not withstanding surdes, being speedily drawn without any reduction of the Equation which would often render the work immense. And that the same manner of working held in Questions de Maximis & Minimis & some others, which in that Letter he forbore to speak of. And that upon the same foundation the Quadratures of Curves became more easy, example of which he gave in a Series which brake of and became finite when the Quadrature might be done by a finite equation. And that this method extended to inverse problems of Tangents & others more difficult. But the foundation of this method he concealed in sentences set down œnigmatically: the first of which was this. Data æquatione fluentes quotcunque æquationes involvente fluxiones invenire, & vice versa.

Thus M^{r} Newton in these three Letters represented that his method was very universal, that it gave the method of Slusius as an obvious Corollary, & that it proceeded without sticking at surds & facilitted Quadratures. And after all this information M^{r} Leibnitz in his Letter of 27 Iune 1677 proposed his differential calculus in these words Clarissimi Slusij Methodum tangentium nondum esse absolutam Celeberrimo Newtono assentior. Et jam a multa tempore rem Tangentium longe generalius tractavi, scilicet per differentias Ordinatarum. Then he defines his new Notation, saying: Hinc nominando in posterum dy differentiam duarum proximarum y, &c. Then he gives an example of drawing Tangents changing the a & e of D^{r} Barrow into dy & dx, & observes how the method of Slusius follows from it, & how it is to be improved so as not to stick at surds, & then adds Arbitror quæ celare voluit Newtonus ab his non abludere. Quod addit ex hoc eodem fundamento Quadraturas quoque reddi faciliores me in sententia hac confirmat; nimirum semper figuræ illæ sunt quadrabiles quæ sunt ad æquationem differentialem. Thus he concludes that he had now got a method like that of M^{r} Newton, & therefor in the Acta Eruditorum by the calculus similis meant M^{r} Newtons method.

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But he tells us: Et jam a multo tempore rem tangentium longe generalius tractavi, scilicet per differentias Ordinatarum. If he means that he had used Dr Barrows method of Tangents jam a multo tempore, tis nothing to his purpose. But if he means that he had improved it into a general method jam a multo tempore, it lies upon him to prove it. For by the law of all nations, in cases of controversy no man can be a witness for himself. And for any man to insist upon his own candour with a designe to be admitted a witness for himself is a demonstration of his want of candour. If there had been no competition in the case he might have been credited without doing injustice to any man: but he is here putting in his claim to the methods of Dr Barrow & Mr Newton, & therefore by the law of all nations it lies upon him to prove his assertion. In the mean time these Arguments make against him.

In the beginning of the year 1672 he claimed the differential method of Mouton as his own & was reprehended for it by Dr Pell, & yet persisted in maintaining that he had invented it apart & much improved it, but he did not yet pretend to any other Differential method.

In the year 1675 he composed a small work upon the Quadrature of the circle vulgari more because he had not yet found out his new Analysis. For in the Acta Eruditorum mensis Aprilis 1691 pag 178 he wrote thus. Iam anno 1675 compositum habebam opusculum Quadraturæ Arithmeticæ ab amicis ab illo tempore lectum, sed quod materia sub manibus crescente limare ad editionem non vacavit post quam aliæ occupationes supervenere; præsertim cum nunc prolixius exponere vulgari more quæ Analysis nostra nova paucis exhibet non satis operæ pretium videatur. The matter grew under his hands till other affairs came on, that is, till he was called hom to be imployed in publick affairs which happened in October & November 1676,; & after that when he had found his new Analysis which exprest that Quadrature in few words, he did not think it worth his while to o on with his composition vulgari more.

In his Letter to Mr Oldenburg dated 12 May 1676 he wrote that he was polishing the Demonstration of this Quadrature; & he sent it to him in his Letter of 27 August. 1676 composed vulgari more without the help of his new Analysis: & therefore he had not yet found out that method.

In the same Letter of 27 August 1676, when Mr Newton had said that his Analysis by the help of infinite equations extended to the solution of almost all Problems, he replied: Id mihi non videtur. Sunt enim multa usque adeo miro et implexa ut neque ab Æquationibus pendeant neque ex Quadraturis. Qualia sunt (ex multis alijs) Problemata methodi Tangentium inversæ. Which is a Demonstration that he had not yet found out the Differential method

After he had received a copy of Mr Netons Letter of 10 Decem. 1672 whereby he had notice that the method of Tangents published soon after by Slusius was but a branch or Corollary of a general method for solving of Problems; his mind ran upon improving that Method, as appears by his Letter to Mr Oldenburg from Amsterdam dated $\frac{18}{28}$ Novemb. 1676 For there he wrote: Methodus Tangentium a Slusio publicata nondum rei fastigium tenet. Potest aliquid amplius præstari in eo genere quod maximi foret usus ad omnis generis Problemata. Nimirum posset brevis quædam calculari circa Tangentes Tabula, eousque continuanda donec progressio Tabula apparet. Amstelodami cum Huddenio locutus sum. Methodus tangentium a Slusio publicata dudum illi fuit nota. Amplior ejus methodus est quam quæ a Slusio fuit publicata. And these were the improvements of the Method of Slusius which then occured to Mr Leibnitz.

But when he had received Mr Newtons Letter dated 24 Octob. 1676, which gave him further light into the true improvement, he wrote back: Clarissimi Slusij methodum Tangentium nondum esse absolutam Celeberrimo Newtono assentior. And: Hinc nominando in posterum dy differentiam duarum proximarum y &c He had now fixed his Notation & began here to communicate it: And if he would have his method of an earlier date, he is in point of candor & by the law of all nations to prove it.

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Mr Leibnitz for $\frac{y^3-x^3}{x-y} = yy + yx + xx$, wrote $\frac{dy^3}{dy} = 3yy$, or rather finding the method of Dr Barrow to be founder upon clearer & & more general principles changed his a & e into dx & dy.

In the beginning of the year 1673 he claimed the differential method of Mouton as his own & was reprehended for it by D^{r} Pell amp; yet persisted in maintaining that he had invented it apart & much improved it,; but did not yet pretend to any other differential method.

In the year 1675 he composed a small work upon the Quadrature of the Circle vulgari more because he had not yet found out his new Analysis. For in the Acta Eruditorum mensis Aprilis 1691 pag 178, he writes thus. Iam anno 1675 compositum habebam opusculum Quadraturæ Arithmetiticæ ab amicis ab illo tempore lectum, sed quod materia sub manibus crescente limare ad editionem non vacavit postquam aliæ occupationes supervenere; præsertim cum nunc prolixius exponere vulgari more quæ Analysis nostra nova paucis exhibet, non satis operæ pretium videatur. The matter grew under his hands till other affairs, & after that, when he had fonud his new Analysis which exprest it in few words he did not think it any longer worth his while to propose it prolixly vulgari more He returned home by England & Holland in October & November & December 1676 & therefore found the Differential method after that time.

In his Letter to M^{r} Oldenburg dated 12 May 1676 he wrote to M^{r} Oldenburg that he was polishing the Demonstration of this method, & he sent it to him in his Letter of 27 Aug. 1676 composed more vulgari without the help of his Analysis nove: therefore he had not yet found out that method.

In the same Letter of 27 Aug 1676 when M^{r} Newton had said that his Analysis by the help of infinite equations extended to the solution of almost all Problemes he replied Id mihi non non videtur. Sunt enim multa usque adeo mira & implexa ut neque ab Æquationibus pendeant neque ex Quadraturis. Qualia sunt (ex multis alijs) problemata methodi tangentium inversæ. Which is a Demonstration that he had not yet found out the Differential method.

in his letter to M^{r} Oldenburg from Amsterdam dated $\frac{18}{28}$ Novem. 1676, he wrote Methodus tangentium a Slusio publicata nondum rei fastigium tenet. Potest aliquid amplius prœstari in eo genere quod maximi foret usus ad omnis generis Problemata: [etiam ad meam (sine extractionibus) Æquationum ad series reductionem]. Nimirum posset brevis quædam calculari circa Tangentes Tabula eousque continuanda donec progressio Tabulæ apparet. And this was the improvement of the method of Slusius which his mind then ran upon.

The next year upon his arrival at Hannover he fell into public

And M^{r} Newton in his Letter of . . . . taught how to resolve any dignity of a binomium into a converging series the second, term of series by the method of D^{r} Barrow gives the first Lemma of Slusius together with his whole method. And after he had described this method of series he subjoyned: Ex his <u>videre est</u> the reduction of infinite equations into finite ones when it might be,

And whereas he had said in his Letter of 13 Iune 1676 that his method of series became not universal without some other methods, & which he then forbore to describe as also what he had invented concerning he here set down the foundation of those methods in sentences exprest enigmatically & gave a series for squaring of figures which brake off & gave the quadrature in a finite equation which it might be.

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M^{r} Newton at the request of M^{r} Collins sent him his method of Tangents in a Letter dated 10 Decem. 1672. It proved to be the same with that which Slusius sent to M^{r} Oldenburg about five weeks after but founded upon the method of fluxions which in drawing of Tangents agrees with the method of D^{r} Barrow.. [If x be the Abscissa & x^{n}=y the ordinate & the Abscissa be increased by an indefinitely small quantity o so as to become x+o the ordinate will be $\overline{x+o}|^{p}$ which being reduced into an infinite series become $x^{p} + pox^{p-1} + \&c$ as is set down in M^{r} Newton Analysis pag 19. And by D^{r} Barrows rules if all the terms be rejected in which o is either wanting or of more dimensions then one there will remain pox^{p-1} for the increase of the ordinate. Therefore the subtangent is to the Ordinate as o the increase of the abscissa to pox^{p-1} the increase of the Ordinate that is as 1 to px^{p-1}, & by consequence the subtangent is is $\frac{y}{pxp-1}$ or $\frac{yx}{pxp}$

M^{r} Newton by interpolation of series

This method readily gives the method of Slusius]

M^r Newton in his Letter M^r O. stated 24 Novem. 1676. wrote that the had explained his method of Tangents in a Tract written 5 years before viz^t A. 1671 that it was the same with the method communicated by Slusius, but flowed from a fountain which gave it readily without needing a particular Demonstration, & & which in like manner extended to the determining of maxima & minima & some other sorts of Problems & rendred Quadratures of Curves more easy & stuck not at surds & was concealed in this sentence exprest enigmatically: <u>Data æquatione quotcunque fluentes quantitates involvente fluxiones invenire, & vice versa.</u>

Thus M^r Newton in these three Letters represented that his method as very universal, that it gave the method of Slusius as an obvious Corollary, & that it proceeded without sticking at surds & faciliated quadratures. And after all this information M^r Leibnits in his Letter of 21 Iune 1677 proposed his differentiall calculus in these words Clarissimi Slusij Methodum Tangentium nondum esse absolutam Celeberrimo Newtono assentior

In the meane time these arguments make against him.

business, which hindered him from finishing his Arithmetical Quadrature of the circle for the Press until he found his New Analysis which made him not think it worth the while to finish what he had been composing vulgari more.

But after when he had received M^r Newtons Letter dated 24 Octob. 1676 which gave him further light into the true improvement: he wrote back: <u>Clarissimi Slusij methodum tangentium nondum esse absolutam celeberrimo Newtono assentior.</u> And <u>Hinc nominando in posterum dy differentiam duarum</u> proximarum y &c: Here he fixed his notation & began to communicate it. And if he would have his method of an earlier date he is bound in point of candour to prove it.

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as may appear by the following comparison

The calculation by the method of D^r Barrow

$$\left.\begin{array}{r} \underline{\mathrm{a+by+cx+dyx+ey+fx^2+gy^2x+hyx^2\ \&c}} \\ \mathrm{ba+ce+dye+2eya+2fxe+2gxye+2hxye\ \&c} \\ \underline{\mathrm{+dxa\qquad\qquad +gy^2a+hx^2a\ \&c}} \\ \mathrm{+dae+eaa+fee+gxaa+hyee} \\ \mathrm{+2gyae+2hxea\ \&c} \\ \mathrm{+geaa+haee} \end{array}\right\} = 0$$

The calculation by the Method of M^r Leibnitz

$$\begin{array}{c} \underline{\mathrm{a+by+cx+dyx+ey^2+fx^2+gy^2x+hyx^2\&c}} \\ \mathrm{bdy+cdx+dydx+22ydy+2fxdx+2gxydy+} \\ \mathrm{+dxdy+gy^2dx+} \end{array}$$

This put M^r Leibnitz upon considering the method of Slusius & how it might be improved. For in his Letter to M^r Oldenburg dated from Amsterdam $\frac{18}{28}$ Novem 1676 he wrote thus. Methodus tangentium a slusio publicata nondum rei fastigium tenet. Potest aliquid amplius præstari in eo genere quod maximi foret usus ad omnis generis Problemata. Nimirum posset brevis quædam calculari circa Tangentes Tabula, eousque continuanda donec progressio Tabulæ apparet. And a little after: Methodus tangentium a slusio publicata dudum Huddenio fuit nota Amplior ejus methodus est quam quæ a slusio fuit publicata. He was not yet master of the right way of improving it, but this winter or in spring following began to understand it.

M^r Newton in his Letter dated — — — others more difficult.

And after all this description of an universal method, which stuck not at surds & whereof the method of Tangents published by Slusius was but a branch or Corollary but which was derived from a more general principle M^r Leibnitz [at length [fell upon the differential method of **{**dawing**}** Tangents & found that it was capable of those improvements mentioned by M^r Newton &] in his Letter of 21 Iune 1677 proposed his differential method in these words Clarissimo Slusij Methodum Tangentium nondum esse absolutam Celeberrimo Newtono assentior. Et jam a multo tempore rem Tangentium longe generalius tractavi, scilicet per differentias Ordinatarum. Then explaining what he meant by these differences he proposes his new notation Hinc nominando in posterum dy differentiam duarum proximarum y &c Then he goes on with D^r Barrows method shewing how Tangents may be drawn, thereby & how the Method of Slusius follows from it & how it is to be improved so as not to stick at surds, & then adds <u>Arbitror quæ celare voluit Newtonus ab his non abludere. Quod addit, ex hoc eodem fundamento Quadraturas quoque reddi faciliores me in sententia hac confirmat, nimirum semper figuræ illæ sunt quadrabiles quæ sunt ad æquationem differentialem.</u> Thus he concludes that he had now got a method like that of M^r Newton & therefore in the Acta Eruditorum by the calculus similis meant M^r Newtons method.

But he tells us: Et jam a multo tempore rem Tangentium longe generalius tractavi scilicet per differentias Ordinatarum. If he means that he had used D^r Barrows method of Tangents jam a multo tempore, tis nothing to his purpose. But if he means that he had improved it into a general method jam a multo tempore: it lies upon to prove it. For by the law of all nations, in cases of controversy no man can be a witness for himself. And for any man to insist upon his own candor with a designe to be a witnes for himsel**{**f**}**, is a demonstration of his want of candor. If there had been no competition in the case, he might have been credited without doing injustice to any body: but he is here putting in his claim to the methods of D^r Barrow & M^r Newton & therefore by the laws of all nations must prove his assertion

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p. 88. pro nota * Idem fecit &c scribe

* Gregorius in Prop. 7 Geometriæ universalis anno 1668 impressæ rem langentium tractavit per differentias ordinatarum. Barrovius in ejus Lect 10 anno 1669 impressa idem fecit, sed, paulo generalius. Slusius methodum suam tangentium fundavit in hoc Lemmate: Differentia duarum Dignitatum ejusdem gradus applicata ad differentiam laterum dat partes singulares gradus inferioris ex binomio laterum, ut $\frac{y^3-x^3}{y-x}=yy+yx+xx$. Et hoc Lemma ad rem Tangentium appliunt per differentias infinite parvas. Newtonus in Epistola 10 Decem. ad Collinium data cujus exemplar inter Collectiones Gregorianas D. Oldenburgus anno superiore ad D. Leibnitium miserat, scripsit methodum Tangentium Slusij esse particulare quodaam vel Corollarium potius Methodi generalis quæ extenderet se citra molestum ullum calculum ad resolvendum alia abstrusiora Problematum genera de Curvitatibus, Areis, Longitudinibus, centris gravitatis curvarum &c et ad quantitates surdas & Curvas Mechanicas minime hæreret. Et in epistola 13 Iunij 1676 ad D. Leibnitium itidem missa scripserat Analysin suam beneficio serierum ad omnia pene Problemata sese extendere. D. Leibnitius respondit id sibi non videri; esse enim multa usque adeo mira & implexa ut neque ab æquationibus pendeant neque ex quadraturis, qualia sunt (ex multis alijs) Problemata methodi tangentium inversæ. Newtonus rescripsit inversa tangentium Problemata esse in potestate aliaque illis difficiliora, Et methodum Tangentium flusij a suis principijs statim consequi idque generalius cum methodus sua quantitates surdas minime moraretur, & eodem modo se rem habere in quæstionibus de maximis et minimis alijsque quibusdam & eodem fundamento quadraturm Curvarum simpliciorem reddi cujus exempla quædam dedit sed fundamentum ipsum literis transpositis celavit hanc sententiam involventibus. <u>Data Æquatione quotcunque fluentes quantitates invol vente fluxiones invenire; et vice versa</u>. Et D. Leibnitius his omnibus admonitus ne a Newtono aliquid didicisse videretur, tandem respondit in hæc verba: <u>Clarissimi Slusij methodum Tangentium nondum esse</u> absolutam <u>Celeberrimo Newtono assentior; et jam a multo tempore rem Tangentium longe generalius tractavi, scilicet per differentias ordinatarum</u>. Et in epistola 29 Decem. 1711 data, addidit, se <u>inventum plusquam nonum in annum pressisse</u>: Quasi habuisset ante mensem Octobrem anni 1675 ideoque a Newtono nil didicisset.

Ad Notam * pag 90 adde. Certe D. Leibnitius similitudinem methodorum non tantum jam intellexerat & sed etiam postea ubi methodum differentialis elementa in lucem emisit sub hoc titulo: Nova methodus pro maximis et minimis, itemque tangentibus, quæ nec fractas nec irrationales moratur, & subjiunxit: Et hæc quidam initia sunt tantum Geometriæ cujusdam multo sublimioris ad difficillima & pulcherrima quæque etiam mistæ Matheseos pertingentis quæ sine calculo nostro differentiali, aut SIMILI, non temere quisquam pari facilitate tractabit. Vide Acta Eruditorum Mensis Octob. pag 467, 473. Conferatur hæc methodi differentialis descriptio cum descriptione consimili methodi fluxionem in Epistolis tribus Newtoni ad D. Leibnitium missis, ut similitudinem methodorum Leibnitio cognitam videas.

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The History of the Method of Moments called Differences by Mr Leibnitz.

In the Introduction to Book of Quadratures published A.C. 1704 wrote that found the Method of fluxions gradually in the years 1665 & 1666, & tho this was not so much as Dr Wallis said nine years before in the Preface to the first volume of his works without being then contradicted, yet in the Acta Eruditorum for Ianuary 1705, in giving an Account of this Book Mr Leibnitz is called the Inventor; & from thence is deduced this conclusion: Pro differentijs igitur Leibnitianis Newtonus adhibet semperque [pro ijsdem] adhibuit fluxiones — iijsque tum in Principijs Naturæ Mathematicis tum in alijs postea editis [pro differentijs illis] eleganter est usus quemadmodum et Honoratus Fabrius in sua Synopsi Geometrica motuum progressus Cavallerianæ methodo substituit. Dr Wallis was homo vetus & informed himself of these matters from the beginning, being very inquisitive in Mathematicall affairs, & having received from Mr Oldenburg cópies of my two Letters of 13 Iune & 24 Octob. 1676 when they were newly written. and in the said Preface he said that in those Letters I had explained to Mr Leibnitz the Method found by hime ten years before or above; meaning that I had found the Method above ten years before Mr Leibnitz, & that I had so far discovered it to him by those Letters, as to leave it easy to find out the rest. And even before Mr Leibnitz had the Method, I said in one of those Letters (that of 24 Octob 1676) that the foundation of the Method was obvious, & therefore, since I had not then leasure to describe it at large, I would conceale it in an Ænigma. This I did, not to make a mystery of it, but to prevent its being taken from me because it was obvious. And in that Ænigma I set down the first Proposition of the book of Quadratures in the very words of the Proposition; & therefore had this Proposition with the Method founded upon it where I wrote that Enigma: or rather, because the very words of the Proposition are copied in the Ænigma, it argues that the Book of Quadratures was then before me, & so was written before Mr Leibnitz had the Method. This Book was extracted out of older papers. In the said Letter of 24 Octob. 1676 I set down a series for squaring of figures which in some cases breaks off & becomes finite & illustrated it with examples & said that I found this & some other Theorems of the same kind by the method whose foundation was comprehended in that Ænigma, that is, by the method of fluxions. And how I found them I explained in the first six Propositions of the Book of Quadratures, & do not know any other method by which they could be found: & therefore when I wrote that Letter I had the Method of fluxions so far as it is conteined in those six Propositions. After I had finished the Book & while the 7th 8th 9th & 10th Propositions were fresh in memory I wrote upon them to Mr I. Collins that Letter which was dated 8 Novem 1676 & being found amongst his Papers was published by Mr Iones. The <160r> Theoremes at the end of the tenth Proposition for comparing curvilinear figures with the Conic sections are mentined in my said Letter of 24 Octob 1676, & all the Ordinates of the figures in the second part of the Table are there copied from the Book. And therefore the Book was then before me. To understand the two Letters of Octob 24 & Novem 8 1676 & how to perform the things & find the Theorems mentioned in them requires skill in the Method of fluxions so far as it is comprehended in all the first ten Propositions of the Book. And the eleventh & last Proposition depends upon a series of first second third & fourth fluxions.

The first Proposition of this Book & its solution illustrated with examples in first & second fluxions &c was at the request of Dr Wallis sent to him almost verbatim in a Letter dated 27 Aug. 1692, & printed by him that year in the second Volume of his works, which came abroad the next year, A.C. 1693. And thus the Rule for finding second third & fourth fluxions there set down was published some years before the Rule for finding second third & fourth differences & was at least seventeen years in manuscript before it was published. In the Introduction to this Book the method of fluxions is taught without the use of prickt letters; for I seldom used prickt Letters when I considered only first fluxions: but when I considered also second third & fourth fluxions I distinguished them by the number of pricks. And this notation is not only the oldest but is also the most expedite, tho it was not known to the Marquess de l'Hospital when he recommended the differential Notation.

In my Analysis per æquationes numero terminorum infinitas communicated by D^r^ Barrow to M^r^ Collins in Iuly 1669, I said that my Method by series gave the areas of Curvilinear figures exactly when it might be, that is, by the Series breaking off & becoming finite: & thence it appears that when I wrote that Analysis, I had the Method of fluxions so far, at least, as it is conteined in the first six Propositions of the Book of Quadratures; tho those Propositions were not then drawn up in the very words of the Book. In that Tract of Analysis I represented time by a line increasing or flowing uniformly & a moment of time by a particle of the line generated in the moment of time, & thence I called the particle a moment of the line; & the particles of all other quantities generated in the same moment of this time I called the moments of those quantities; & the fact under the rectangular Ordinate & a moment of the Abscissa I considered as the moment of the cursa (rectilinear or curvilinear) described by that Ordinate moving uniformly upon the Abscissa. For a moment of time I put the letter o, & thence computed the moments of the other quantities generated in the moment of time, & for those moments put any other symbols. And for the Area of a figure I sometimes put the Ordinate included in a square. And by considering how to deduce moments from increasing quantities & quantities from their moments, I deduced Ordinates of figures from their Areas & the Areas from the Ordinates: which is the same thing with deducing <161r> fluxions from fluents & fluents from fluxions. And in the end of the Book I demonstrated by this sort of calculus the first of the three Rules set down in the beginning thereof. And in this Rule for the index of a Dignity I put an indefinite quantity affirmative or negative integer or fract, & thereby introduced indefinite indices of Dignities into Analysis. And applying this Method of Moments not only to finite equations but also to equations involving converging series I gave this Tract the name of Analysis per æquationes numero terminorum infinitas.

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The history of the Differential Method.

M^r^ Collins having received from me & M^r^ Iames Gregory several series for squaring the Circle & Conic Sections, was very free in communicating them to the Mathematicians both at home & abroad in the years 1670, 1671 & 1672 & M^r^ Leibnitz was in London in the beginning of the year 1673 & went from thence to Paris in the end of February carrying Mercators Logarithmotechnia along with him & kept a correspondence with M^r^ Oldenburg till Iune following about Arithmetical Questions, being not yet acquainted with the higher Geometry. Then he intermitted his correspondence till Iuly 1674 & in the mean time studied the higher Geometry beginning with the Horologium oscillatorium of M^r^ Huygens which came abroad in April 1673. His following correspondence was about converging series till spring 1676. And then upon the news of M^r^ Iames Gregories death, he wrote for a collection of Gregories Papers, & the Demonstration of my Series, meaning my Method of finding them & promised M^r^ Oldenburg a reward for my Method & told him that M^r^ Collins could help him to it. I suppose he meant my Analysis per Series numero terminorum infinitas. For that was the only Paper in which I had sent my Method of Series to M^r^ Collins.

Thereupon M^r^ Collins drew up Extracts out of Gregories Letters, & the Collection was sent to Paris In Iune following to be returned & it is now in the Archives of the R. Society; but instead of sending a copy of my Analysis, he & M^r^ Oldenburg jointly sollicited me to send the Method which M^r^ Leibnitz desired, & thereupon I wrote my Letter of 13 Iune 1676, & this was sent to him at the same time with the Collection. In this Collection was a copy of a Letter of M^r^ Iames Gregory to M^r^ Collins dated 15 Feb. $16\frac{70}{71}$ which conteins several Series one of which was that famous one for finding the Arc whose tangent is given: which series had been also sent by M^r^ Oldenburg to M^r^ Leibniz the year before & the receipt thereof acknowledged. There was also in the same Collection a copy of a Letter of M^r^ Gregory to M^r^ Collins dated Sept 5. 1670 in which Gregory wrote that by improving the method of Tangents of Barrow he had found a method of Tangents without calculation. There was also in the same Collection a copy of a Letter which I had writ to M^r^ Collins above thre years before. The Letter was dated 10 Decem 1672, & is as follows. Ex animo gaudeo – – me grave ducas. And copies of these two last Letters were communicated also by M^r Oldenburg to M^r Tschunhause in Iune 1675.

In my letter of 13 Iune 1676 I had said (with relation to the Method described in my Analysis per æquationes numero terminorum infintas,) Ex his videre est quantum fines Analyseos per hujusmodi infinitas æquationes ampliantur: quippe quæ earum beneficio, ad omnia pene dixerim problema (si numeralia Diophanti & similia excipias) sese extendit. And M^r Leibnitz in his Answer dated 27 Aug. 1676, replied: Quod dicere videmini plerasque difficultates (exceptis Problematibus Diophænteis) ad series Infinitas reduci; id mihi non videtur. Sunt enim multa usque adeo mira et implexa ut neque <164r> ab Æquationibus pendeant, neque ex quadraturis. Qualia sunt (ex multis alijs) Problemata methodi Tangentium inversæ. In the same Letter he placed the perfection of Analysis not in the Differential method but in another method composed of Analytical Tables of tangents & the Combinatory Art. Nihil est, said he, quod norim in Analysi momenti majoris. And a little after: Ea vers nihil differt ab Analysi illa SVPREMA, ad eujus intima Cartesius non pervenit. Est enim ad eam constituendam opus Alphabeto cogitationum humanarum. This was the top of his skill at that time & therefore he had not yet found out the differential method nor had hitherto used fluxions for **{h}**is differences.

In October 1676 he came to London a second time & there met with D^r Barrows Lectures, & saw my Letter of Octob. 24. 1676 , & therein had fresh notice of my Compendium of Series or Analysis communicated by D^r Barrow to M^r Collins, & consulting M^r Collins saw in his hands many of mine & Gregories Letters, especially those relating to series & in his way home from London was meditating how to improve the method of Tangents of Slusius as appears by his Letter to M^r Oldenburgh dated from Amsterdam $\frac{18}{28}$ Novem. 1676. And the next year in a letter to M^r Oldenburgh dated 21 Iune he sent hither his new Method with this Introduction. Clarissimi Slusij Methodum Tangentium nondum esse absolutam celeberrimo Newtono assentior. And in describing this Method he abbreviated D^r Barrows method of Tangents by new symbols & shewed how it might be improved so as to the Method of Slusius (which was the same with that of Gregory) & to proceed in equations involving surds; & then subjoyned: Arbitror quæ celare voluit Newtonus de tangentibus ducendis ab his non abludere. Quod addit, ex eodem fundamento Quadraturas reddi faciliores me in sententia hac confirmat. This was the first time that he began to communicate his differential Method & therefore I had not hitherto used fluxions for his Differences; nor can the assertion be true Pro differentijs Leibnitianis Newtonus semper adhibuit fluxiones.

M^r Iames Bernoulli in the Acta eruditorum for December 1691 pag. 14 said that the Calculus of M^r Leibnitz was founded in that of D^r Barrow & differed not from it except in the notation of differentials & some compendium of operation. And the Marquess de l'Hospital in the Preface to his Analysis of infinite petits published A.C. 1696, represented that where D^r Barrow left off M^r Leibnitz proceeded, & that the improvement which he made to the Doctors Analysis consisted in excluding fractions & surds: but the Marquiss did not then know that M^r Leibnitz had notice of this improvement from me by two Letters above mentioned, dated 10 Decemb. 1672 & 24 Octob. 1676, a copy of the first being sent to h im in Iune 1676. After he had notice that such an improvement was to be made, he might find it proprio Marte, but by that notice knew that I had it before him. And in his Letter of 21 Iune 1677 he confessed that he had such notice.

In the Acta Eruditorum for October 1684 M^r Leibnitz published the Elements of the differential Method as his own without mentioning the correspondence which he had formerly had with me <164v> about these matters. He mentioned indeed a Methodus similis; but whose that Method was or what he knew of it he did not say, as he should have done. And this his silence put me upon a necessity of writing the Scholium upon the second Lemma of the second Book of Principles, least it should be thought that I borrowed that Lemma from M^r Leibnitz

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— but **{s}**aid **{illeg}** that I had interwoven it with the method of infinite series & that being tyred with these speculations I had absteined from them five years, that & therefore had this method above five years before that is before the year 1671. And that it was so general as to reach almost all Problems except perhaps some numeral ones like those of Diophantus. And

& therein he was again told that my Method of working gave me the method of tangets of Slusius directly & immediately so as to need no demonstration thereof & stuck not at equations affected with radicals involving one or both indeterminate quantities, & proceeded in the same manner in questions about maxima & minima & in some others which I did not there mention, & gave me general Theremes for Quadratures, one of which I there set down & illustratd with examples & that I wrote a tract upon this method & the method of series together five years before with a designe to print it together with a tract about light & colours.

In my Letter of Octob 24 I

After things M^{r} Leibnitz in the year 1684 published the elements of this method as his own without making any mention of the foregoing correspondence

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The first Proposition of the Book of Quadratures is certainly the foundation of the method of fluxions. This Proposition was comprehended verbatim in the ænigma by which in my Letter of Octob. 24 1676 I concealed the foundation of the Method there spoken of & therefore that method was the method of fluxions. In that Letter I said that I had written a Tract on this Method & the Method of series together five years before but did not finish it nor meddle any more with these things till the year 1676 being tyred with them. And in my Letter of Iune 13 I wrote to the same purpose. And this is the Method which I described in my Letter of Decem 10th 1672. In my Analysis per Æquationes numero terminorum infinitas I said of the method there described: Denique ad Analyticam merito pertinere censeatur, cujus beneficio Curvarum areæ & longitudines &c (id modo fiat) exacte & Geometrice determinentur. Sed ista narrandi non est locus. This was effected by series which in some cases break off & become finite, as you may understand by the Letter of M^{r} Collins to M^{r} Strode Iuly 26, 1672. And therefore before D^{r} Barrow sent that Analysis to M^{r} Collins, that is before Iuly 1669 I had the method of fluxions so far at the least as it is conteined in the first six Propositions of the Book of Quadratures.

I sent him also the method of extracting fluents out of equations involving fluxions, which to the best of my memory was composed in the year 1671.

M^{r} Leibnitz never was in quiet possession abroad nor I out of possession in England. He has been told again & again that he was not the first Inventor & never would answer directly to this point, but answered indirectly by pretending that he found it apart, & that gave it to him in the Scholium upon the 2^{d} Lemma of the 2^{d} Book of Principles

< insertion from the bottom of the page > In a Paper dated 29 Iuly 1713 & written by one who used the Leibnitian phrase illaudabil**{i}** laudis amore & knew what M^{r} Leibnitz did at Paris 40 years before I was singled out & treated very reproachfully. And this was to make me appear. And in his Postscript to Mr. l'Abbe Cont written in Autumn. 1715 singled out & treated in a very provoking manner. to make me appear. And when I was prevailed with to return an Answer to this Postscript he declared in his Letter of Apr. 9th 1716 to Mr. l' Abbé Conti that he would by no means enter the lists with my forlorn hop**{e}** (meaning M^{r} Keill & M^{r} Coles &c) but since I was willing to appear my self he would give me satisfaction. Nothing could content him but to make me appear & declare my opinion, & now I have declared it, I leave every body to his own opinion, out of a desire to be quiet. < text from f 164v resumes >

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The History of the Method of Moments Fluxions & approaching Series.

In the Introduction to Book of Quadratures published A.C. 1704 I wrote that I invented the Method of fluxions gradually in the years 1665 & 1666, & tho this was not so much as D^{r} Wallis said nine years before in the Preface to the first volume of his works without being then contradicted, yet in the Acta Eruditorum for

Ianuary 1705 in giving an Account of this Book M^{r} Leibnitz is called the Inventor, meaning the first Inventor. And from thence is deduced this conclusion. Pro differentijs igitur Leibnitianis Newtonus adhibet semperque [pro ijsdem] adhibuit fluxiones — iijsque tum in Principijs Naturæ Mathematicis, tum in alijs postea editis [pro differentijs illis] eleganter est usus, quemadmodum et Honoratus Fabrius in sua Synopsi Geometrica, motuum progressus Cavallerianæ methodo substituit. D^{r} Wallis was homo vetus & rerum anteactarum peritissimus being inquisitive in these matters & having received from M^{r} Oldenburg copies of my two Letters of 13 Iune & 24 Octob. 1676 when they were newly written. And he said that in those Letters I had explained to M^{r} Leibnitz the method found by me ten years before or above, meaning that I had it above ten years before him & had so far discovered it to him by those Letters as to leave it easy to find out the rest. And in one of those Letter that of 24 Octob 1676,) I said that the foundation of the Method was obvious & therefore (since I had not leasure to describe it at large) I would conceale it in an Ænigma. This I did not to make a mystery of it but to prevent its being taken from me because it was obvious. And in That Ænigma I set down the first Proposition of the book of Quadratur**{**es**}** in the very words of the Proposition ‖ < insertion from the left margin > ‖ & therefore had this Proposition with the method founded upon it when I wrote the said Letter. Or rather; because the very words of the Proposition are copied in the Ænigma, it argues that the Book of Quadratures was then before me. < text from f 165r resumes > For this Book was then before me, being newly written. It was extracted for the most part out of a Tract which I wrote in the year 1671 but left unfinished & out of some other older papers. In the said Letter of 24 Octob. 1676 I set down a series for squaring of figures which in some cases breaks off & becomes finite, & illustrated it with examples & said that I found this & some other Theorems of the same kind by the method whose foundation was comprehended in that Ænigma, that is, by the Method of fluxions. And how I found them I explained in the first six Propositions of the Book of Quadratures: & I do not know any other method by which they could be found: & therefore when I wrote that Letter I had the Method of fluxions so far as it is conteined in those six Propositions. After I had finished the Book & the 7th 8th 9th & 10 Propositions were fresh in memory I wrote upon them to M^{r} Iohn Collins that Letter which was dated 8 Novem. 1676, & being found amongst his Papers was published by M^{r} Iones. The Theorems at the end of the tenth Proposition for comparing curvilinear figures with the Conic sections were known to me also before I wrote the said Letter of 24 Octob 1676, they being there mentioned & all the Ordinates of the Figures being there copied from the Book in due order. [Those Theoremes were copied from the Tract which I wrote in the year 1671.] To understand these two Letters & how to performe the things & find the Theorems conteined in them requires skill in the Method of fluxions so far as it is comprehended in all the first ten Propositions of the Book. And the eleventh & last depends upon a series of first second third & fourth fluxions.

The first Proposition of this Book and its solution illustrated with examples was at the request of D^{r} Wallis sent to him in a Letter dated 27 Aug. 1692 & printed by him almost verbatim that year in the second Volume of his works which came abroad the next year, A.C. 1693. And thus the Rule for finding second third & fourth fluxions there set down was published some years before the Rule for finding second third & fourth differences, & was at least seventeen years in manuscript before it was published . In the Introduction to this Book the method of fluxions is taught without the use of prickt letters: for I seldome used prickt letters when I considered only first fluxions. But when I considered also second third & fourth fluxions I distinguished them by the number of pricks. And this notation is not only <166r> the oldest but is also the most expedite, tho it was not known to the Marque**{**ss**}** de l'Hospital when he recommended the differential Notation.

And applying this method of moments not only to finite equations but also to equations involving converging series, I gave this Tract the name of Analysis per æquationes numero terminorum infinitas. And ✝ < insertion from f 165v > And shewing also thereby to find the Ordinates & Areas of Mechanical Curves &c [that their lengths & tangets may be found by the same Method;] I said: Nec quicquam hujusmodi scio ad quod hæc methodus idque varijs modis sese noli extendit. Imo . . . & quiquid vulgaris Analysis per æquationes ex finito terminorum numero constantes (quando id sit possibile) perficit, hæc per æquationes infinitas semper perficiet. And in my Letter of 13 Iune 1676 I said of this Analysis: Ex his &c < text from f 166r resumes > And in my Letter of 13 Iune 1676 I said of this Analysis: Ex his videre est quantum fines Analyseos per hujusmodi infinitas æquationes ampliantur: Quippe quæ earum beneficio ad omnia pene dixerim Problemata (si numeralia Diophanti et similia excipias) sese eatendit. And M^{r} Leibnitz in his Letter of 27 Aug. 1676 replied that he did not believe that ⊡ < insertion from f 165v > ⊡ my method was so general the inverse Problems of Tangents & many others not being reducible to Equations or Quadratures. And in the same

Letter he placed the perfection of Analysis not in this Method but in a Method composed of Analytical Tables of Tangents & the Combinatory Art, saying of the one; Nihil est quod norim in tota Analysi momenti majoris: & of the other; Ea vero nihil differt ab Analysi illa suprema, ad cujus intima, quantum judicare possum, Cartesius non pervenit. Est enim ad eam constituendam opus Alphabeto cogitionum humanarum. This was the top of his Analytical skill at that time. < text from f 166r resumes >

Mr Collins in a Letter to Mr Strode dated 26 Iuly 1672 gave this account of these Methods. Mense Septembri 1668 Mercator Logarithmotechniam edidit suam, quæ specimen hujus methodi (i.e. Serierum infinitarum) in unica tantum figura, nempe quadraturam Hyperbolæ, continet. Haud multo postquam prodierat liber, exemplar ejus Cl. Wallisio Oxonium misi, qui suum de eo judicium in Actis Philosophicis statim fecit: aliumque Barrovio Cantabrigiam, qui quasdam Newtoni Chartas – – – extemplo remisit: E quibus et ex alijs, quæ olim ab Auctore cum Barrovio communicata fuerant, patet illam Methodum a dicto Newtono aliquot annis antea excogitatam et modo universali applicatam fuisse: ita ut ejus ope in quavis figura Curvilinea proposita quæ una vel pluribus proprietatibus definitur, Quadratura vel area dictæ figuræ, accurata si possibile sit, sin minus, infinite vero propinqua; evolutio vel longitudo lineæ curvæ; Centrum gravitatis figuræ; solida ejus rotatione genita, et eorum superficies, idque non obstantibus radicalibus, quæ eam non morantur, obtineri queant. ✝ < insertion from f 165v > ✝ < text from f 166r resumes > So then by the testimony of Dr Barrow grounded upon papers which I had communicated to him from time to time I had the method here described some years before the Doctor sent my Analysis to MrCollins, that is, some years before Iuly 1669. And this is sufficient to justify what I said in the Introduction to the Book of Quadratures, vizt that I found the Method gradually in the years 1665 & 1666. Dr Barrow then read his Lectures about motion, & that might put me upon taking these things into consideration. I found the Method of Series in the beginning of the year 1665 the method of Moments soon after * < insertion from f 165v > * If it be asked why I did not publish this method sooner, it was for the same reason that I did not publish the Theory of colours sooner. I found the method of series in the beginning of the year 1665 & the method of fluents & moments soon after & the theory of colours in the beginning of the year 1666, & in the year 1671 was about < text from f 166r resumes > the Theory of colours in the beginning of the year 1666, & in the year 1671 was about to publish them all : but for a reason mentioned in my Letter of 24 Octob. 1676, I desisted till the year 1704, excepting that some of my Letters were published before by Dr Wallis & Mr Oldenburg.]

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The first line that Mr Leibnitz began to communicate the differential Method was his Letter of 27 Iune 1677, & therefore the Editors of the Acta eruditorum accused me falsly in saying Pro differentijs Leibnit**{oanis}** Newtonus adhibet semperque adhibuit fluxiones: And they & their adherents ought to have proved the accusation.

In one of those Letters (that of 24 Octob. 1676)

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Mr Newton in his Letter dated 24 Octob. 1676 comprehended the method of Fluxions is this sentence Data æquatione quotcunque fluentes quantitates involvente invenire fluxiones & vice versa. And the fift Proposition of the book of Quadratures was set down at length in the same Letter & said to be deduced from the same method of Fluxions & called the first Theorem in a series of such Propositions. Whence the sixt Proposition of the book of Quadratures was also then known to Mr Newton it being the second of the series. And how these two Propositions were found out by the method of fluxions is fully set down in the first <167v> six Propositions of the book of Quadratures. And therefore These six Propositions were known to Mr Newton in the year 1676; & so were some others that follow in the same book as may be gathered from what is cited out of them in the same Letter & in another Letter written by Mr Newton to Mr Collins Nov. 8. 1676 & publis**{hed}** by Mr Iones. And by so many things quoted out of the book of Quadratures in the year 1676, it may be concluded that the boo**{k}** of Quadratures (except the Introduction & last Scholium) was written before that year. For indeed the designe of the book was to make a step towards the inverse method of fluxions.

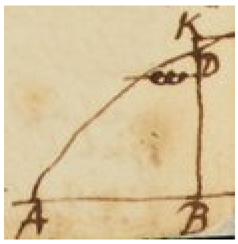
Another step was made by the solution of this Probleme, Ex æquatione fluxionem radicis involvente radicem extrahere. The solution whereof was sent to D^{r} Wallis by M^{r} Newton in his Letter dated Sept 17th 1692 & published by the Doctor in the second volume of his work pag. 394. And at the end of the example there set down M^{r} Newto**{n}** added that by the same method the roots of equations involving the second third fourth & other fluxions might be extracted. And this method was known to M^{r} Newton in the year 1676 as appears by his Letter then dated 24 Octob. where he saith Inversa de Tangentibus Problemata sunt in potestate aliaque illis diffilicilora. Ad quæ solvenda usus sum duplici methodo; una concinniori altera generaliori. Vna methodus consistit in extractione fluentis quantiatis ex æquatione simul involvente fluxionem ejus: altera &c.

And by these things its manifest that M^{r} Newton in the year 1676 understood the method of fluxions direct & inverse & had then extended it to the 2^{d} 3^{d} 4th & other fluxions.

And tho he sometimes uses prickt letters for fluxions yet he doth not confine himself to those symbols. In the first Proposition of his book of Quadratures he uses such letters: in the Introduction to the book he desembles the method & gives examples of solving Problemes by it without making any use of such Letters.

M^{r} Newton in his Letter dated 24 Octob 1676 represented that five years before, viz^{t} A.C. 1671, he wrote a treatise of the method of infinite series & of another method which readily gave the method of Tangents of Slusius & stuck not at surds & which was founded in this sentence Data æquatione quotcunque fluentes quantitates involvente invenire fluxiones & vice versa. And in a Letter to M^{r} Collins dated 10 Decem. 1672 (which was some weeks before Slusius sent his method of Tangets into England) he described the same method of Tangets as a Corollary or branch of his general metho**{d}** which stuck not at surds & which was therefore his method of fluxions. And he said of this method that it extended to the abstruser sort of Problems about the Curvatures areas, lengths, centers of gravities of Curves &c & succeeded in Mechanical Curves as well as others. So then he understood the method of fluxions in those days, & by his applying it to Problems about the Curvature of Curves you may know that he had then extended it to the consider**{a}**tion of the second fluxions. The sentence ♁ < insertion from f 168r > ♁ < text from f 167v resumes >

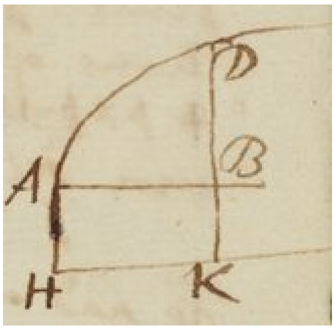
this Analysis per Æquationes numero terminorum infinitas was founded upon three Rules. The first Rule was this. Let a be a given quantity, x the ab**{illeg}** of a Curve, y the Ordinate, & $\frac{m}{n}$ the index of a dignity whole or broken affirmative or negative. And if the Ordinate be $ax^{\frac{m}{n}} = y$ the area of the Curve will be $\frac{an}{m+n}x^{\frac{m+n}{n}}$. That is (as M^{r} Newton afterwards explains) if the fluxion be $ax^{\frac{m}{n}}$ the fluent will be $\frac{na}{m+n}x^{m+n}$ And this Rule he demonstrates in the end of his Analysis by the method of fluxions after the very same manner that **{i}{illeg}** the Introduction to his book of Quadratues he found the inverse thereof by that method viz^{t} that if the fluent be x^n the fluxion will be nx^{n-1}. His Demonstration is this. Let the Abscissa of any Curve ADδ be $AB = x$, the perpendicular ordinate $BD = y$ the area $ABD = z$, the moment of the Abscissa $B\beta = o$, the moment <170r> of the Area $B\beta\delta D = o\upsilon = BD \times BK = \text{rectangulo}\, B\beta HK$, the side of this rectangle BK being called υ. And the Abscissa AB will be $x + o$ & the Area ABδwill be $= 2 + o\upsilon$. Now since $\frac{n}{m+n} \times ax^{\frac{m+n}{n}} = z$, or putting $\frac{xa}{m+n} = c$ & $m + n = p$, since $cx^{\frac{p}{n}} = z$, or $c^n x^p = z^n$, if $x + o$ be written for x & $z + o\upsilon$ be written for z you will have cn in $x^p + pox^{p-1} + \&c = z^n + no\upsilon z^{n-1} + \&c$ the following terms of these two series being omitted because they would vanish in the conclusion b diminishing o in infinitum & making it vanish. Now if you reject the equal quantities c^n in x^p & z^n & divide the remainder by o you will have $c^n px^{p+1} = n\upsilon z^{n-1} \left(= \frac{nyz^n}{z} = \frac{nyc^n x^p}{z}\right)$ & dividing both parts of the equation by $c^n x^p$ you will have $px^{-1} = \frac{ny}{z}$ or $pcx^{\frac{p-n}{n}} = ny$. And by restoring $\frac{aa}{m+n}$ for c & $m + n$ for p that is m for $p - n$ & na for pc you have $ax^{\frac{m}{n}} = y$. Whence on the contrary if $ax^{\frac{m}{n}}$ be $= y$ then $\frac{n}{m+n} ax^{\frac{m+n}{n}}$ will be $= z$. Q. E. D.



If this calculation be compared with that in the Introduction to the book of Quadratures you will find them perfectly of the same kind They both procced by resolving the dignity of a Binomium into a converging series & shewing that when the second term of the binomium is the moment of the first, the second term of the series is the moment of the dignity. But third term of the series is double to the moment of the second & the fourth is triple to the moment of the third.

The second Rule upon which M^r^ Newton founds his Analysis is that if the area of a Curve be described by an Ordinate composed of several parts the whole area shall be composed by all the Areas described by those parts. And the third Rule is to reduce the Ordinates of Curves into parts which describe such areas as may be found by the first Rule. And after several examples of squaring Curves by this method, he described the method of fluxions for finding the Areas, Lengths, convex surfaces, solid contents & centers of gravity of curvelines & curvilinear figures.

Sit ABD Curva quævis et AHKB rectangulum — dum utuntur methodis Indivisibilium.



Here several things are to be observed, as first that when he takes a point for the moment of a line & a line for the moment of a superficies, he takes a point for an infinitely short line & a line for an infinitely narrow superficies as in the method of Cavallerius. And therefore in pulling the ordinates BD & BK for the moments of the areas ABD & ABKH, he takes the Ordinates for infinitely narrow parallelogramms whose bases are the infinitely short parts or moments of the Abscissa AB. Let the moment of the Abscissa (x) be called o & the moments of the Areas ABKH & ABD will be $1 \times o$ & $y \times o$, or o & yo And this M^r^ Newton expresses in the end of his Analysis in demonstrating the first Rule. But where he is only investigating the solutions of Problems he neglects to write down the moment o as was said above.

Its to be further observed that in the sense here described he considers a superficies as the moment of a solid, a line as the moment of a superficies & a point as the moment of a line: which consideration gives the notion of first second & third moments, a point being the moment of a line & the moment of the moment of a superficies & the moment of the moment of the moment of a solid, that is the first moment of a line, the second moment of a superficies & the third moment of a solid.

Its to be observed also that in this Analysis M^r^ Newton teaches how to find <170v> as many Curves as you please which may be squared. And this is the second Proposition in the book of Quadratures, & is performed by assuming any equation expressing the relation between the Abscissa & area of the Curve & finding the relation of their fluxions by the first Proposition, & putting the fluxion of the Area for the Ordinate, the fluxion of the Abscissa being an unit. And this makes it appear that the two first Propositions of the book of Quadratures were known to M^r^ Newton in or before the year 1669 when he wrote his Analysis.

It is to be observed also that M^r^ Newton in the same Analysis affirms of this Method that Ejus beneficio Curvarum areæ & longitudines &c (id modo fiat) exacte et Geometricæ determinantur. Sed ista narrandi non est locus. And this referrs to the fift & sixt Propositions of the book of Quadratures & shews that the first six Propositions of that book were known to M^r^ Newton in or before the year 1669. For the fift & sixt Propositions depend upon the third & fourth & those upon the first & second.

It is still further to be observed that in the same Analysis M^r^ Newton uses the symbol $\boxed{\frac{aa}{64x}}$ to express the area of a Curve whose Ordinate is $\frac{aa}{64x}$ & M^r^ Leibnits expresses the same thing by the symbol $\int \frac{aa}{64x}$, but this symbol is of later date

It is also to be observed that when M^r^ Newton wrote the said Analysis he understood the resolution of any dignity of any Binomium into a converging series. For in the Demonstration of the first Rule he set down the two first terms of the series. Let the dignity be $\overline{x+o}^{p}$ & the series will be $x^p + pox^{p-1} + p \times \frac{p-1}{2} \times oox^{p-2} + px^{\frac{p-1}{2}} \times \frac{p-2}{3} \times o^3x^{p-3} + \&c$ the two first terms of which series were set down in the Analysis.

And further it is to be noted that when M^r^ Newton wrote that Analysis he had observed the great relation which the method of converging series & that of fluxions have to one another. For he every where applys the method of series to the solution of Problems by the method of fluxions & by the method of Series demonstrates the first Rule which is the foundation of the method of fluxions, & by reason of this relation between the methods he had then composed one general method of them both.

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The History of the Differential Method, written by Sir Isaac Newton.
