Bernouilli's problem in the Acta Eruditorum for October 1698

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In Actis Eruditorum pro mense Octobri Anni 1698, pag. 471, D. Iohannes Bernoullius hac scripsit. **{illeg}** Methodum quam optaveram generalem secandi [curvas] ordinatim positione datas sive algebraicas sive trascendentales in angulo recto sive obliquo, invariabili sive data Lege variabili, tandem ex voto erui, cui Leibnitio approbatore, ne γρυ addi posset ad ulteriorem perfectionem, et vel ideo tantum quod perpetuo ad æquationem deducat in qua si interdum indeterminatæ sunt inseparabiles, Methodus non ideo imperfectior est, Non enim hujus sed alius est Methodi indeterminatas separare Rogamus igitur fratrem ut velit suas quoque vires exercere in re tanti Momenti. Suscepti Laboris Non pœnitebit si felix successus fructu jucundo compensaverit. Scio **{r}**elicturum suum quem nunc fovet modum, qui in paucissimis tantum exemplis adhiberi potest.

Hi tres viri celeberrimi sese jam ab annis quatuor vel quinque circiter in solvendis hujusmodi Problematibus exercuerant. Absque spiritu divinandi eandem solutionem cum Bernoulliana tradere difficile fuerit. Sufficit quod solutio sequens sit generalis, et ad æquationem semper deducat.

Problema

Quæritur Methodus generalis inveniendi seriem Curvarum quæ Curvat in serie alia quacumque data constitutas, ad angulum vel datum, vel data Lege variabilem secabunt.

Solutio.

Natura Curvarum secandarum dat tangentes earundem <369v> ad intersectionum puncta quæcumque, et anguli intersectionum dant perpendicula Curvarum secantium, et perpendicula duo coeuntia, per concursum suum ultimum, dant centrum curvaminis Curvæ secantis ad punetum intersectionis cujuscumque. Ducatur Abscissa in situ quocumque commodo, et sit ejus fluxio unitas, et positio perpendiculi dabit fluxionem primam Ordinatæ ad Curvam quæsitam pertinentis, et curvamen hujus Curvæ dabit fluxionem secundam ejusdem Ordinatæ. Et sic Problema semper deducetur ad æquationes. Quod erat faciendum.

Scholium.

Non hujus, sed alius est methodi æquationes reducere, et in series convergentes, ubi opus est convertere. Problema hocce, cum Nullius fere sit usus, in Actis Eruditorum annos plures Neglectum, et insolutum Mansit. Et eadem de causa solutionem ejus Non ulterius prosequor.

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Sir

M^{r} Iohn Bernoulli in the Acta Eruditorum for October 1698. pag. 471 wrote in this manner. Methodum quam optaveram generalem secandi [curvas] ordinatim positione datas sive algebraicas sive trascendentes, in angulo recto, sive obliquo, invariabili, sive data lege variabili, tandem ex voto erui, cui Leibnitio approbatore, ne γρυ addi posset ad ulteriorem perfectionem, et vel ideo tantum quod perpetuo ad æquationem deducat, in qua si interdum indeterminatæ sunt inseparabiles, methodus non ideo imperfectior est, non enim hujus sed alius est methodi indeterminatas separare. Rogamus itque fratrem ut velit suas quoque vires exercere in re tanti momenti. Suscepti laboris non penitebit si fœlix successus fructu jucundo compensaverit. Scio relicturum suum quem nunc fovet modum qui in paucissimus tantum exemplis adhibere potest. These Gentlemen had been four or five years about Problemes of this kind, & to give the very same solution with that here mentioned might require a spirit of divination. But the Probleme may be generally solved after the following manner.

The Probleme

A series of Curves being given of one & the same kind, succeeding one another (in forme & position) in an uniform manner according to any general Rule find another Curve which shall cut all the Curves in the said Series, in any angle right, or oblique, invariable, or variable according to any Rule assigned.

The method of Solution

Let BD be any one of the Curves in the Series, CD the Curve which is to cut it, D the point of intersection AE the common Abscissa of the two Curves & ED the common ordinate of : & the two Rules will give [the angle of intersection] the perpendicular to the Curve CD & the radius of its curvity at the point D, & the position of the perpendicular will give the first fluxion & the curvity the second fluxion of the ordinate of the same curve D. And so the Probleme will be reduced to equations involving fluxions & by Separating or extracting the fluents will be resolved.

Let the Ordinate of the Curve desired be represented by the area of <371v> another curve upon the same Abscissa & the first fluxions will be represented by the Ordinate of this other Curve, & the second fluxion by the proportion of the Ordinate to the subtangent. And so the Probleme is reduced to the property of a Tangent

and the first Rule will give the tangent of the Curve BD at the point D, & the second Rule will give the angle of intersection & tangent of the other Curve CD at the same point D, & both the Rules together will give the Radius of the curvity of the other Curve CD at the same point D. Let the Abscissa AE flow uniformly & its fluxion be called 1, & the position of the Tangent of the Curve CD will give the first fluxion of its Ordinate ED, & the Curvity of the same Curve CD at the point D will give the second fluxion of the same Ordinate And so the Problem will be reduced to equations involving the first & second fluxions of the Ordinate of the Curve desired, & by reducing the Equations & extracting or separating the fluent (which is not the business of this method) will be resolved.

There may be some Art in chusing the Abscissa & Ordinate or other Fluents to which the invention of the Curve CD may be best referred & in reducing the Equations after the best manner. But nothing more is here desired then a general method of resolving the Probleme without entring into particular cases.

The curvity of the intersecting Curve CD at the point D is found by taking in the tangent of that Curve CD another point d infinitely near to the point D, & finding the tangent at the point d of the Curve in the series which passes through that point, d & also the tangent of the Curve intersecting it at the same point d; & upon the two intersecting curves at the points of intersection D & d erecting perpendiculars. For these perpendiculars shall intersect one another at the Center of the curvity of the intersecting Curves.
