Several Drafts of an Intended Preface to the Commercium Epistolicum

Author: Isaac Newton

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① Ad lectorem of {Com Epist}

The occasion of publishing this Collection of Letters will be understood by the Letters of M^r Leibnits & M^r Keil in the end thereof. M^r Leibnitz taking offence at a passage in a discours of M^r Keil published in the transactions about years ago, wrote a Letter to the Secretary of the R. Society complaining thereof as a calumny, desiring a remedy from the R. Society, & suggesting that he beleived they would judge it equal that M^r Keil should make a publick acknowledgment of his fault. M^r Keil chose rather to return an answer in writing, wherein he explained his meaning in that passage & justified that meaning. M^r Leibnitz not meeting with that satisfaction he desired wrote a second Letter to the Secretary of the R. S. wherein he complained still of M^r Keill, representing him a young man not acquainted with what was done before his time nor authorized by the person concerned; And appealed to the equity of the Society to checque the unjust clamours of this person. And Vpon this repeated appeale the R. Society appointed a Committee to examin the ancient Letters & Records relating to this matter, & ordered the Report of the Committee to be published with so much of the Letters & papers as related to this matter.

The question is about the invention of the method called by M^r Leibnitz the Differential method by M^r Newton the method of fluxions. M^r Leibnitz contends that he found it by himself without the assistance of any other person, & in his last letter allows the like to Sir Isaac Newton, but yet claims the title of Inventor & justifies a Paper printed in the Acta Leipsica whereby he makes himself the first inventor. The first inventor is the inventor & whether the second Inventor found it by himself or not is a question of no moment. He blames M^r Keill for medling with this matter without authority from Sir I. Newton but the R. Society have not found fault with him on that account, it being every mans right to repell injuries from his neighbour, & a right very necessary to be preserved among learned men least this dilemma be put upon Inventors, that they must either lose their inventions to pretenders or spend their time in controversies & run the hazzard of being censured for valuing themselves upon things of that nature.

The method of fluxions being described in a letter of M^r Newton to M^r Collins A.C. 1672 concerning the method of Tangents ascribed to Slusius, it was thought fit to print in the following papers that Letter with two or three others relating to it. And it may not be amiss to observe that what was said in those letters concerning that method of tangents as known to Slusius before he printed his Mesolabium, was grownded upon <539v> a mistake of M^r Oldenburg, for rectifying of which it may not be amiss to reprint here a Letter of M^r Collins to M^r Newton dated 18th Iune 1673 & printed by D^r Wallis in the third volume of his works. The Letter runs thus.

Quod ad Slusij methodum . . . . ab ipso propediem expectar{e} debeant. And this is that method of Tangents which Slusius when he published his Mesolabium forbore to write of least he should prevent his friend Riccius & which Riccius afterwards declining mathematical studies desired Slusius to publish, & Slusius thereupon promised to send to Mr Oldenberg to be published in the Transactions. Vpon this notice Mr Oldenburgh & Mr Collins became of opinion that the Method of Tangents which by the leave of Riccius they expected from Slusius was known to Slusius when he published his Mesolabium But Slusius sent them another method very different from that Riccius & not derivable from his principles And Mr Oldenburgh & Mr Collins not understanding the difference applied to this new method what Slusius had written to Mr Oldenburgh concerning the method which Riccius had given him leave to publish & accordingly wrote to Mr Newton that it was known to Slusius before he published his Mesolabium. How Slusius came to send Mr Oldenberg another Method of tangents then that which Riccius gave him leave to publish, doth not appeare: but its observable that when Slusius was to demonstrate this new method, he proposed three Lemmas representing that it flowed from them without further explication, but Mathematicians have not yet been able to tell us how it flows from them. Let Mathematicians therefore consider whether this method of Tangents can be demonstrated by the said three Lemmas or whether Slusius had a better demonstration.

Neither doth it appeare that Slusius could demonstrate the method which he sent. For in the end of his letter in which he described it, he said Addo tantum me Regulæ meæ Demonstrationem habere facilem et quæ solis constet Lemmatibus; quod mirum Tibi forte videbitur. And a little after he sent the following three Lemmas

Tractabant Barrovius, Gregorius & Fermatius rem tangentium per differentias Ordinatarum Et idem fecit Slusius ut ex tribus ejus Lemmatibus, in quibus methodum suam fundavit, manifestum est. Nam per ejus Lemmata duo prima, differentia dignitatum homologarum applicata ad differentiam laterum infinite parvam producit factum ex dignitate et indice suo applicatum ad latus. Per differentias laterum et dignitatum Slusius hic exponit differentias abscissarum et ordinatarum. Vtrum Leibnitius

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1. Mr Leibnitz was in England in the beginning of the year 1673 & in the end of the year 1676 & in the intervall in france & all that time kept a correspondence with Mr Iohn Collins by means of Mr Oldenberg & what he learnt from the English by that correspondence is the main Question, & Mr Oldenburg Mr Collins being long since dead for deciding it the R. Society found it necessary to search out & publish what could be met with of that correspondence in writing relating to the things in Question.

2 By the Analysis published in the beginning the reader will find that Mr Newton was acquainted with the method of infinite series Anno 1669 & by the help of the method of fluent quantities & their incrementa momentanea or moments had then applied that method very generally to the solution of Problemes And it appears not by any that Mr Leibnits was aquainted that method of moments or differences as he calls them before the year 1677. 2B And by the Letters next following them Analysis That Mr Gregory in them end of the year 1670 fell into them same method but did not claim inventoris jura, because he had received one of Mr Newtons series from Mr Collins with notice that Mr Newton had a general method for finding such series at pleasure That Mr Collins was thenceforward very free in communicating the series which he had received from Mr Newton & Mr Collins, that Mr Leibnitz soon after his being in London began to talk of his having two such series both of them found by one & the same method, that both these series could not be found by the Method of transmutations, & that Mr Leibnits in those days pretended to no other method, & therefore had only the series without a method of finding them.

3 You will find also that in April or May 1675 Mr Leibnitz received from Mr Collins by the letters of Mr O. eight or nine series & knew none of them to be his own: that before them end of the year Mr Gregory died & Mr Leibnitz communicated the last of those series as his own to his friends at Paris without letting them know that he had received it from Mr Collins 4 That two others of those series being sent by Mr Collins to Paris by one Mohr a Dane, Mr Leibnitz in May 1676 desired Mr O. to procure him the demonstration of those two series therefore & had not yet the method of finding them, [& that one of those two series was for finding the

arch by the sine, (as was one of the two which he wrote of soon after his going from London to Paris) & the other for finding the sine by the arch]. ⊙ < insertion from the bottom of the page > ⊙ That he then represented his own meditations or inventions (of which had writ to M^r O. some years before) very different from these series, but never produced them: that he then admired these two series & especially the second for its elegance, but knew not how to derive <540v> it from the first, For he wrote the next year to M^r Oldenburg that he found in his old papers a method of M^r Newtons for such purposes, but had neglected it for some time for want of an elegant instance of its use. When he had series which would have given him elegant examples of this method he had forgot the method & when he had the method he wanted series to give him an elegant example of it.] His own series which he had in the years 1674; 1675 & 1676 afforded him no elegant examples & therefore were different from those communicated to him by M^r Oldenburgh & M^r Mohr. And these gave him no elegant examples tho they were all of them elegant examples of such a method & therefore he had forgot the Method before he received M^r Oldenburghs Letter of April 1675. And therefore the method is old enough to make M^r L. the first inventor. And hence it follows also that the series which he had before that time are not yet published: for those hitherto published by M^r Leibnitz give elegant examples. < text from f 540r resumes > 6 That when M^r Newton at the request of M^r O & M^r C sent his method of series by M^r O. to M^r Leibnitz he sent back the above mentioned series as his own which he had received of M^r O. & [communicated at Paris as his own] & claimed also some other seiries as his own which by some small alterations he had derived from the series sent him by M^r Newton, & which were of the same kind with those of which he wanted the Demonstration & therefore could not be found out by him before he received M^r Newtons Letter.

5 That in his answer to M^r Newtons Letter he sent his method of transmutations to M^r Newton as a general method of Series , which was not a general method before M^r Newton made it so by communicating his method of extracting roots, & that this method of series is tedious & useless, it being easier to find series by M^r Newtons methods alone.

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$$\frac{1}{x+cxx} = y.$$

$$\frac{1}{x+cxx+o+2cox+coo} = \frac{1}{x+cxx} - \frac{o+2cox+coo}{x+cxx \times x+cxx} + \frac{co+4coox+2cooxx+2co^3+4c^2o^3x}{\overline{x+cxx}\big|^3} - \frac{o^3+{}^6co^3x+12cco^3xx+8c^3o^3x^3}{\overline{x+cxx}\big|^4} + \&c\ .$$

$$\frac{1}{a+b} = \frac{1}{a} - \frac{b}{aa} + \frac{bb}{a^3} - \frac{b^3}{a^4}$$

$$y + \dot{y} = \frac{1}{x+cxx} - \frac{o+2cox}{\overline{x+cxx}\big|^2} + \frac{oo+3coox+3ccooxx}{\overline{x+cxx}\big|^3} - \frac{o^3+4cxo^3+6ccxxo+4c^3x^3o^3}{\overline{x+cxx}\big|^4}$$

$$\text{Area} = \frac{1}{x+cxx}o - \frac{1+2cx}{2\times\ \overline{x+cxx}\big|^2}o^2 + \frac{c+3cx+3ccxx}{3\times\ \overline{x+cxx}\big|^3}o^3 - \&c$$

$$eS = R\sqrt{1+QQ}$$

$$\frac{\sqrt{1+xx+2cx^3+ccx^4}}{\overline{x+cxx}\big|^2} \text{ in } \frac{1+2cx}{2\times\ \overline{x+cxx}\big|^2} = \frac{ec+3ecx+3eccxx}{3\times\ \overline{x+cxx}\big|^3}$$

$$mn + 3 \times 2m^2n^2 + 6mn + 42m^3n^3 + \underset{6}{6}mmnn + \underset{18}{4}mn \;{}_{+12}$$

$$\overline{x^m + cx^{m+1}}\Big|^n = x^{mn} + ncx^{mn+1} + \frac{mn-n}{2} \times c^2x^{mn+2} + \frac{n^3-3nn+2n}{6}c^3x^{mn+3}\ .$$

$$\text{Ord} = \frac{1}{mn+1}x^{mn+1} + \frac{nc}{mmnn+3mn+2}x^{mn+2} + \frac{mnnc^3-nnc^3}{2m^3n^3+12mmnn+22mn+12}x^{mn+3}$$

$$\sqrt{1} + \frac{1}{mmnn+2mn+1} = \frac{\sqrt{mmnn+2mn+1} \text{ in } x^{2mn+2}+ee}{mn+1} \times 2\frac{mnnc^3-nnc^3}{mn+2\times mn+2\times mn+3} = \frac{\frac{4}{3}nncc}{mn+1\times nn+1}$$

Cum Resistentia sit ad gravitatem ut $3S\sqrt{1+QQ}$ ad 4RR: Sit hac resistentia ut Medij densitas et Velocitatis V potestas V^n: & Medij densitas erit ut resistentia directe & velocitatis potestas V^n inverse, id est ut $\frac{3S\sqrt{1+QQ}}{4RRV^n}$ pro V scribatur $\sqrt{\frac{1+QQ}{R}}$, et Densitas erit ut $\frac{3S\sqrt{1+QQ}}{4RR\times\frac{1+QQ}{R}^{\frac{n}{2}}} = \frac{SR^{\frac{n-4}{2}}}{1+QQ^{\frac{n-1}{2}}} = \frac{SR^{\frac{n-4}{2}}}{1+QQ^{\frac{n-1}{2}}}$. Sit $n=1$, et Densitas erit ut $\frac{S}{R^{\frac{3}{2}}}$. Sit $n=3$ et Densitas erit ut $\frac{S}{\overline{1+QQ}\text{ in }R^{\frac{1}{2}}}$. Sit $n=5$ et Densitas erit ut $\frac{SR^{\frac{1}{2}}}{\overline{1+QQ}\rceil^{2}}$ Sit $n=4$ et densitas Medij erit ut $\frac{S}{\overline{1+QQ}\rceil^{\frac{3}{2}}}$. Sit $n=2$ et Dens. erit ut $\frac{S}{R\sqrt{1+QQ}}$. Sit $n=0,1,2,3,4,5,6\ \ 7$ et Densitas Medij erit ut $\frac{S\sqrt{1+QQ}}{RR}$, $\frac{S}{R^{\frac{3}{2}}}$, $\frac{S}{R\sqrt{1+QQ}}$, $\frac{S}{R^{\frac{1}{2}}\times\overline{1+QQ}}$

$\frac{S}{R^{\frac{1}{2}},\overline{1+QQ}\rceil^{1}}$, $\frac{S}{R^0\times\overline{1+QQ}\rceil^{\frac{3}{2}}}$, $\frac{S}{R^{-\frac{1}{2}}\times\overline{1+QQ}\rceil^{2}}$, $\frac{S}{S^{-1}\times\overline{1+QQ}\rceil^{\frac{5}{2}}}$, $\frac{S}{R^{\frac{-3}{2}}\times\overline{1+QQ}\rceil^{3}}$, &c . Et universaliter, Velocitas ut $\overline{1+QQ}\rceil^{\frac{1}{2}}\times R^{-\frac{1}{2}}$, Resistentia ut $\overline{1+QQ}\rceil^{\frac{1}{2}}\times S\times R^{-2}$, Densitas ut $S\times R^{\frac{n-4}{2}}\times\overline{1+QQ}\rceil^{\frac{1-n}{2}}$.

Newtonus autem anno 1672 methodum suam tangentium describendo quam Slusius etiam mox communicavit, scripsit hanc esse unum particulare vel corollarium tantum methodi suæ generalis. Et Literæ Newtoni cum Leibnitio communicatæ sunt ut supra.

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Ad Lectore{m}. {I. Can. Epis}

1 The occasion of publishing this Collection of Letters will be understood by the Letters of M^r^ Leibnitz & M^r^ Keil in the end thereof. M^r^ Leibnitz taking offence at a passage in a discourse of M^r^ Keil published in the Transactions A C. 1708, wrote a Letter to the Secretary of the R. Society complaining thereof as a calumny, desiring a remedy from the Society & suggesting that he beleived they would judg it equal that M^r^ Keil should make a publick acknowledgment of his fault. M^r^ Keil chose rather to return an answer in writing wherein he explained his meaning in that passage & defended it. M^r^ Leibnitz not meeting with that satisfaction he desired wrote a second Letter to the Secretary of the R. Society, wherein he still complained of M^r^ Keil, representing him a young man not acquainted with what was done before his time nor authorized by the person concerned, & appealed to the equity of the Society to cheque his unjust clamours.

2 M^r^ Leibnitz was in England in the beginning of the year 1673 & again in the end of the year 1676 & in France all the time between & kept a correspondence all that time with M^r^ Oldenburgh & by his means with M^r^ Iohn Collins , & what he learnt from the English by means of that correspondence is the main Question. M^r^ Leibnitz appeals from young men to old ones who know what passed in those days, refuses to let any man meddle without authority from Sir Isaac Newton & desires that Sir Isaac would give judment. Sir Isaac lived then at Cambridge & knew only his own correspondence printed by D^r^ Wallis M^r^ Oldenbergh & M^r^ Collins are long since dead & so are D^r^ Barrow M^r^ Gregory & D^r^ Wallis who corresponded with M^r^ Collins. The R. Society therefore being twice pressed by M^r^ Leibnitz & having no other means of enquiring into this matter, ordered the antient Letters Letter books & papers left by M^r^ Oldenburg in the hands of the R. S. & those found amongst the papers of M^r^ Collins relating to the matters in dispute between M^r^ Leibnitz & M^r^ Keil to be searched out & examined & appointed a Committe to do it & ordered the Report of the Committe to be published together with the extracts of the Letters & Papers presented to them by the Committee. And the substance of the Letters & Papers is as follows.

3 By the Analysis published in the beginning of this Collection the Reader will find that M^r^ Newton was acquainted in the year 1669 with the method of infinite series & by the help of the method of fluent quantities & their incrementa momentanea or moments had then applied it very generally to the solution of problemes, & by the method of series had demonstrated the method of fluents, in which demonstration he considers the first term of the series as the fluent, the rest as the augmentum of the fluent & the second alone as the

augmentum momentaneum or moment & the moment as the exponent of the velocity of increase or fluxion. Whence it is that in this & a following tract he joyned these two methods together as two parts of one general method for solving of Problems: which method proceed by resolving finite æquations into series when it is necessary & applying both finite & infinite equations to the solution of Problemes by the method of fluents.

<541v> M^r Newton being desired by M^r Leibnits to relate the original of his Theoreme for turning the dignities of binomials into infinite series, related in his letter of Octob 24 1676 that he found it by interpoling a series of D^r Wallis a little before the plague which raged in England in the years 1665 & 1666 & in the year 1671 at the request of some friends wrote a Treatise of the methods of infinite series & fluent quantities together. This was written to M^r Leibnitz long before the present disputes arose & no dispute has hitherto risen about it. As M^r Newton conjoyned the two methods in his Analysis, as one general method so he conjoyned them afterwards in this Tract written more at large upon the same subject.

4: The Analysis being sent to M^r Collins in the year 1669, he sent one of the series to M^r I. Gregory with notice that M^r Newton had a general method of solving problemes by such series, & M^r Gregory after much search found out the method in the end of the year 1670 & the next year sent several series to M^r Collins (one of which was that claimed afterwards by M^r Leibnitz for the sector of the circle & Hyperbola), & gave leave to M^r Collins to communicate his series freely but left it to M^r Newton to publish the Method as the first inventor.

5 In December 1672 M^r Newton at the request of M^r Collins sent him his method of Tangents & represented that it was but one particular or rather a Corollary of general & easy method of solving difficulter Problems without sticking at radical quantities meaning the method of fluent quantities & their moments. And about a month after Mon Slusius sent his method of Tangents to M^r Oldenburgh which proved to be the same with M^r Newtons but not so perfect For it stuck at radicals & was not applicable to mechanical curves. M^r Oldenburgh represented to M^r Collins & both to M^r < insertion from f 542r > Newton < text from f 541v resumes > that Slusius found it first & so it goes under the name of Slusius's method.

6 In the beginning of the next year he | M^r Leibnits was at London & pretended to Moutons differential method in such a manner as makes it evident that he knew then of no other differential method. The next year A.C. 1674 being at Paris he began to write to M^r Oldenburgh of two series found by one & the same method But it doth not appear that he had the method. For one of the series was for finding the arc by the sine, & sometime after he wrote to M^r Oldenburgh for the method of finding two series, one of which was for the arc by the sine the other for the sine by the arc, & his method of transmutations (the only method which he then pretended to be his own) doth not extend to the invention of either of these series without the help of M^r Newtons method of extracting roots at that time not known to M^r Leibnitz.

7 In May 1675 M^r Oldenburgh sent him from M^r Collins four of M^r Newtons series & four or five of M^r Gregories, & he owned the receipt of them & knew none of them to be his own. For he promised to compare them with his own. But M^r Gregory died before the end of the year & M^r Leibnitz communicated the last of them in writing to his friends at Paris as his own series & the next year sent it back to M^r Oldenburgh & M^r Newton as his own. <542r>

9 In the winter between 1675 & 1676 he received two series from one M^r Mohr which M^r Mohr had received from M^r Collins & as if he had forgotten that he had received them before with several others from M^r Oldenburg & taken time to consider them he wrote to M^r Oldenburgh to procure from M^r Collins the Demonstration of those two series shewed him by M^r Mohr, that is the method of finding them. And thereupon M^r Oldenburgh & M^r Collins least he should also forget the receipt of the Demonstration wrote pressingly to M^r Newton to describe his own Method himself. & M^r Leibnits receiving M^r Newtons method with some examples of series, endeavoured by small alterations to lay claim to some of those series tho he had no method for finding them before he received M^r Newtons Letter. And at the same time he sent M^r

Newton a method of his own for finding series by transmutation of figures representing it a general method tho it was far from being general till Mr Newton made it so by communicating his method of extracting roots,

10 Mr Gregory had but one series sent him with notice that it was the result of a general method & therby within the space of a year found out the method, but left it to Mr Newton to publish the Method as the first inventor. Mr Leibnitz had eight series sent him with the same notice, but this method being altioris indaginis he could not find it out but after a years consideration was forced to desire Mr Oldenburg to procure him the method, & yet he continues to this day to number himself among the inventors of the methods of infinite series

11 Mr Newton in his first Letter had said that the Analysis of the Moderns, by the help of infinite æqu. it self extended to almost all Problemes, except numeral ones like those of Diophantus meaning here by these æquations the method of series together with the consequence thereof the method of moments. For in his next Letter he explained himself in this manner by extending this method to inverse problems of tangents & to the resolution of Equations involving fluxions. Mr Leibnitz in his answer disputed this assertion & said that there were many Problemes, & amongst others the Problemes of the inverse method of Tangents, which depended neither on Equations nor on Quadratures. By which answer it appears that he knew nothing yet of the method of fluents & their moments which he has since called the differential method. And the same thing is concluded also from an Opusculum which Mr Leibnits composed in a vulgar method concerning the series of Mr Gregory above mentioned & communicated to his friends at Paris before the end of the year 1675 & continued to adorn & polish in the year 1676 in order to send it to Mr Oldenburg in recompence for the Demonstration of the two series of Mr Mohr. but left of to polish after he fell into other business & after the invention of his new Analysis or Differential method did not think worth publishing.

8 After the death of Mr I. Gregory was known to the Mathematicians at Paris, Mr Leibnitz & some others there desired that his letters & papers might be <542v> collected into a body & preserved. The collection was made by Mr Collins at the pressing desire of Mr Oldenburgh & sent by him to Paris to be communicated to Mr Leibnitz & returned back to London. And in this collection there is an Epistle of Mr Gregory dated 15 Feb. 1671 conteining several series one of which is that above mentioned which Mr Leibnitz received from Mr O. & communicated at Paris as his own. [But Mr Leibnitz soon forgot that he had seen either Mr Oldenburghs Letter or this collection & having sent the series back to Mr Oldenburgh & MrNewton as his own series published six years after as his own in the Acta Lipsica, & continues to this day to claim it for his own. In the same Collection was] And another is Mr Newtons Letter to MrCollins dated 15 Dec. 1672 concerning his method of Tangents to Curves whether Geometrical or Mechanical or howsoever related to right lines & concerning his general method **{of}** solving Problemes whereof the method of Tangents was but a particular or Corollary.

12. But after Mr Leibnits had seen the said Collection & was there informed that the Method of Tangents described in Mr Newton's aforesaid Letter was but one particular or rather a Corollary of a general & easy method of solving Problems which stuck not at surd quantities & knew that this method was the same with that of Mr Slusius but more general method of And after he had also received Mr Newtons Letter of 13 Iune 1676 wherein he was told that the method of series extended to the solution of almost all Problemes except numeral ones like those of Diophantus: He considered the method of Slusius & how to improve it as appears by his Letter of 18 Novem 1676 & the beginning of his Letter of 21 Iune 1677, & tried to improve it by considering the differences of the Ordinates, that is by considering the second termes of the series representing when the Ordinates are turned into series. For this consideration would presently lead into a more general method of Tangents extending even to mechanical curves & not sticking at surd quantities. And if he had not such a method before the receipt of Mr Newtons second Letter Mr Newton said enough in that Letter of his general methods: altho to avoid enlarging upon it he concealed the fundamental Propositions by ænigmas as Galilæo & Hugens in other cases had done before. And now Mr Leibnitz in his answer described the differential method & its use in drawing of tangents as in the method of Slusius but without sticking at surds, & in squaring of Curves, & allowed that in these things it resembled the method which Mr Newton

endeavoured to conceal, & that it extended also to the solution of inverted Problems of tangents by Equations & Quadratures. But he forgot to acknowledge that <543r> he had invented this method since his Letter of 27 Aug. 1676 wherein he reprimanded Mr Newton for making his method too general & objected that may Problemes were so intricate & particularly the inverse probleme of Tangents as not to depend upon æquations & quadratures. He was convinced now that Mr Newtons methods were very general & understood that Mr Newton had writ a treatise of these methods five years before & it becomes not men of candor & modesty to interrupt one anothers proceedings & snatch away one anothers inventions.

When Mr Leibnitz could not procure the Method of series from Mr O. & Mr C. without the knowledge of Mr Newton he desired Mr Newton to communicate his method of deducing reciprocal series from one another & Mr Newton sent it. Mr Leibnits understood it with difficulty but so soon as he understood it he wrote back to Mr O. that he had found it long before as he perceived by his old papers but not meeting with a good example of its use had neglected it. It seems he had found it & forgot it again before he had the series of Mr Gregory which would have helped him to a very good example.

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At length Mr Leibnitz published the differential method in the Acta Leipsica A.C. 1684, but made no mention of Mr Newtons having found the like method long before. For he then forgot that Mr Newton had a general method of that kind, & wrote afterwards that in the year A.C. 1684 when he published the elements of his calculus he knew nothing more of Mr Newtons inventions of this kind then what he had signified in his Letters namely that he could draw Tangents without taking away irrational quantities. But when he saw Mr Newtons Principles he perceived his method was of much larger extent but knew not that it was so like the differential method before Dr Wallis's works came abroad, & still contends that the Differential method was the original, representing soemtimes that Mr Newton substituted fluxions for differences soemtimes allowing that he found out the method of fluents by himself. Mr Keil on the contrary represents that the method of fluents was the original. This is the state of the matter is drawn up from the following papers, as you will find by reading them.

12 After Mr Leibnitz was told by Mr Newton in his Letter of 13 Iune 1676 that his method of applying series to the solution of Problems was very general, & he had also seen the aforesaid Collection of Mr Gregories Letters, amongst which was Mr Newtons Letter to Mr Collins dated 15 Decemb. 1672, & therby understood that Mr Newton had a general method of solving Problems whereof his method of Tangents was but a particular or Corollary & that it extended to Mechanical Curves & by consequence proceeded upon the augmenta momentanea of the lines by which Mechanical Curves are defined & determined, (for there is no other way yet known of drawing tangents to mechanical Curves;) & after he understood also that Mr Slusiuss Method of Tangents so far as it extended was the same with Mr Newtons & by consequence was capable of being improved into a general method: his mind ran upon improving the method of Tangents by the incrementa momentanea of lines, & particularly the method of Slusius by the incrementa momentanea of the Ordinates as is manifest by his Letters of 18 Nov 1676 & the beginning of his Letter of 21 Iune 1677. These incrementa momentanea of quantities Slusius in the first of **{** the**}** three Lemmas on which he founded his method called differences & Mr Leibnitz retained the name**{illeg}**.**{**[]**}** And while he had these things under consideration he received further light into the method by Mr Newtos Letter of 24 Octob. 1676. And falling into Mr Newton's method described it in his answer, Dated 21 Iune 1677 & with Slusius gave the name of Differences to the incrementa momentanea of quantities For Slusius called them Differences in the first of the three Lemmas upon which he founded his method of Tangents. And now Mr Leibnitz acknowledged the extent of Mr Newtons general method for drawing Tangents without Sticking at surds, squaring of fig**{**ures**}** <544r> & solving of inverse Problemes of Tangents.

The Question is Who was the first author of the method called by Mr Leibnits the differential method. Mr Leibnitz claims Inventoris jura, & allows that Sir Isaac Newton might also find it apart. Mr Keill asserts Sir Isaac to be the first Inventor & the Report of the Committe is on his side.

The papers & Letters till the year 1677 are to shew that Sir Isaac had a general method of solving Problemes by resolving finite equations into infinite ones when it was necessary, & deducing fluents & their moments from one anoth**{er}** by the help of Equations finite or infinite. The Analysis printed in the beginning of this Collection shews that he had such a method in the year 1669, & that it was very general And by the Letters which follow it appears that in the year 1671 he composed a larger treatise of this method & that it was very general & easy without sticking at surds & extended to problems of tangents direct & inverse & to finding the areas lengths, centres of gravity & curvature of curves & to other more difficult problems & that in mechanical curves as well as others, & by consequence was founded upon the consideration of the indefinitely small particles of quantities called indivisibles by Cavallerius, Augmenta momentanea & moments by M^r Newton & differences by Slusius & Leibnitz. For there are no other ways of drawing tangents to mechanical curves or of squaring any curves then by considering the particles of Quantities. That this method was so general as to extend to the solution of almost all Problemes except the numeral ones of Diophantus & M^r Newton gave examples of this his method in drawing tangents squaring of Curves & solving inverse Problemes of Tangents, all which was communicated to M^r Leibnits & at the request of M^r Leibnitz M^r N. communicated to him also that part of the method which consisted in the reduction of finite equations to infinite series, & thereby enabled him to find the Ordinates of Mechanical curves with their Differences which are the second terms of the series expressing the Ordinate. M^r I. Gregory by having one of M^r Newtons series with notice that it was the result of a general method found out the method in the space of a year. M^r Leibnitz had eight series sent him by M^r Oldenburg besides two other which he pretended to have a year or two before: but this method being altioris indaginis M^r Leibnits could not find it out but at length wrote to M^r Oldenburg to procure him the Method from M^r Colling. But M^r Leibnitz having forgot the receipt of the eight series they forbore to send him the method without M^r Newton's knowledge, & desired M^r Newton to describe his own method himself, which he did at their importun**{ing}** And M^r Leibnits not yet fully understanding the method him desired M^r Newton to explain it further & how he derived reciprocal series from one another.

Hitherto M^r Leibnitz continued to compose things in the vulgar way of writing, which after he fell into the Differential method he did not think worth publishing. Hitherto he continued of opinion that M^r Newton had made too large a description of the extent of his method & that many problems & particularly the inverse problems of Tangents depended not on **{equ}**ations nor on quadratures: but after these descriptions & examples of M^r Newtons general method had been sent to him & one half of the method at **{illeg}** **{**his**}** request had been described to him he fell into the rest of the method & began to describe it in his letter of 21 **{**Iun**}{**e**}** 16**{**77**}** <544v> & saw now that it extended to the drawing of Tangents without sticking at radica**{**ls**}** & to quadratures & inverse Problemes of Tangents, & that the solution of the inverse problems of Tangents in the Example which M^r Newton ha**{**d**}** sent him to convince him that these Problemes were to be solved by æquations & Quadratures, was practicable & flowed from his Arts as well as from Newtons. But he forgot to acknowledge that his Arts were but just found out. For without acknowledging this, he ught not to have intermedled with M^r Newton method. Candid Men are not to interrupt one anothers proceedings, & snatch away one anothers inventions. D^r Pell reprehended him for pretending to Moutons Method. M^r Collins reprehended him for intermedling with what Gregory & Tschurnhause were about as appears by a letter not yet published. He should not have pretended to two series in the year 1674, when he wanted the method of finding them, He should not have forgot the receipt of the eight series which Oldenburgh sent him nor have published one of them as his own after he knew | without mentioning that he had received it from Oldenburg & knew that Gregory had sent it to Collins in the beginning of the year 1671, He should not have endeavoured to get M^r Newtons method of series from M^r Oldenburgh & M^r Collins without M^r Newtons knowledge when he will not allow M^r Keil to asser**{**t**}** M^r Newtons right without M^r Newtons authority. He should not reccon himself among the inventors of infinite series & of the methods of finding them when he has not produced one series of note invented by himself nor has any general method of finding them besides what he received from M^r Newton. H.

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that is, the method of finding them And thereupon M^r Oldenburg & M^r Collins seing that he wanted the method of finding the two series which he had boasted of & the the eight series which they had sent him & he **{illeg}**te as if he had forgot them, forbore to send him the method without M^r Newtons knowledg**{e}** & wrote very pressingly to M^r Newton to describe his own Method himself. And M^r Leibnitz.

– are not of so large extent. Without the method of fluents they do not amount to a general method for solving of Problems. By the method of series finite equations are to be resolved into infinite ones when there is occasion & both sorts of equations are to be applied to the solution of Problems by the method of fluents to make the general method here spoken of.

1 Scripserat Newtonus se methodum generalem habere solvendi problema, & hanc 2 methodum in re tangentium ad curvas mechanicas extendi (p. 30, 47,   ) quod perinde 3 est ac si dixisset methodum suam generalem in momentis quantitatum fundari. Tangentes 4 enim ad Curvas mechanicas absque consideratione momentorum duci non possunt. Ordinatæ in Curvis mechanicis sunt series infinitæ & serierum 5 momenta sunt earum termini secundi. Scripserat etiam Newtonus methodum suam 7 ad omnia pene problemata præter numeralia Diophanti & similia se 8 extendere (p. 55) etiam ad inversa tangentium problemata & his difficiliora (p. 85, 9 86) & Leibnitius hæc intellexerat de seriebus et $\frac{1}{2}$ momentis in unam methodum generalem conspirantibus. (p 92, 93.) Tractantur utique Problemata per æquationes seu finitas seu infinitas & fluentium momenta conjunctim. Leibnitius igitur de tali methodo admonitus rescribit se quoque rem tangentium generalius tractass scilicet per differentias Ordinatarum. Differentias enim vocat quæ Newtonuus momenta Sed et methodum mox extendit a Quadraturas $15\frac{1}{2}$ Curvarum & inversa tangentium Problemata, Newtoni vestigijs insistendo

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Ad Lectorem of Commercium 3

The occasion of publishing this Collection of Letters & Papers will be understood by the Letters of M^r Leibnitz & M^r Keil in the end thereof. M^r Leibnitz taking offence at a passage in a discourse of M^r Keil published in the Transactions A.C. 1708, wrote a Letter to the Secretary of the R. Society complaining thereof as a calumny, desired a remedy from the Society, & suggested that he beleived they would judge it equal that he should make a publick acknowledgment of his fault. M^r Keil chose rather to return an answer in writing, wherein he explained his meaning in that passage & defended it. M^r Leibnitz not meeting with that satisfaction he desired wrote a second Letter to the Secretary of the R. Society, wherein he still complained of M^r Keil, representing him a young man & not acquainted with what was done before his time, nor authorized by the person concerned, & appealed to the equity of the Society to cheque his unjust clamours.

M^r Leibnitz was in England in the beginning of the year 1673 & again in October 1676, & during the intervall in France & all that time kept a correspondence with M^r Oldenburg, & by his means with M^r Iohn Collins & sometimes with M^r Is. Newton, & what he might learn from the English either in London or by that correspondence is the main Question. M^r Oldenburg & M^r Collins are long since dead & M^r Newton lived then at Cambridge & knew little more then his own correspondence since published by D^r Wallis. M^r Newton can be no witness for M^r Keill nor M^r Leibnitz for himself, & there appears no other living evidence. The R. Society therefore being twice pressed by M^r Leibnitz against M^r Keill, appointed a Committee to search out & examin the Letters Letter-books & papers left by M^r Oldenburg in the hands of the Society & those found among the papers of M^r Iohn Collins relating to the matters in dispute, & to report their opinion thereupon & ordered the Report of the Committee with the extracts of the Letters & Papers to be published.

When M^r Newton wrote the Analysis printed in the beginning of this collection, he had a method of resolving finite equations into infinite ones & of applying both finite & infinite equations to the solution of Problemes by meanes of the proportions of the incrementa momentanea of growing or increasing quantities. These incrementa M^r Newton calls particles & moments & M^r Leibnitz infinitesimals indivisibles & differences. The increasing quantities M^r Newton calls fluents & M^r Leibnitz summs, & the velocities of increase M^r

Newton calls fluxions & exposes these <546r> fluxions by the moments of the flowing quantities. That part of the method which consists in resolving finite equations into infinite ones, was at the request of Mr Leibnitz communicated to him by Mr Newton in his Letters of Iune 13th & Octob 24th 1676. And Mr Newton having so far touched upon the other part as to reckon that it was become sufficiently obvious (p. 72 lin. 1,) to secure it from being taken from him before he should have occasion to explain it, he expressed it in cyphre after the manner used by Galilæo & Hugenius upon other like occasions. Mr Leibnitz claimed the invention of this other part & Mr Keil asserts it to Mr Newton & is favoured in his opinion by the Report of the Committee. But there is nothing in these papers which can affect other persons abroad who have received the method from Mr Leibnitz. They were strangers to the correspondence between Mr Leibnitz & Mr Oldenburg. They found the method useful & are much to be commended for the use & improvements that they have made of it.

Some Notes are added to the Letters to enable such Readers as want leasure, to compare them with more ease & see the sense of them at one reading.

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Ad Lectorem of Comm Epis 4

The occasion of publishing this Collection of Letters & Papers will be understood by the Letters of Mr Leibnitz & Mr Keill in the end thereof. Mr Leibnitz taking offence at a passage in a discourse of Mr Keil published in the Transactions A.C. 1708, wrote a Letter to the Secretary of the R. Society complaining thereof as a calumny, desiring a remedy from the Society & suggesting that he beleived they would judge it equal that he should make a publick acknowledgment of his fault. Mr Keil chose rather to return an answer in writing, wherein he explained his meaning in that passage & defended it. Mr Leibnitz not meeting with that satisfaction he desired wrote a second Letter to the Secretary of the R. Society, wherein he still complained of Mr Keil, representing him a young man not acquainted with what was done before his time nor authorized by the person concerned & appealed to the equity of the Society to cheque his unjust clamours.

Mr Leibnitz was in England in the beginning of the year 1673 & again in October 1676 & during the interval in France, & all that time kept a correspondence with Mr Oldenburg, & by his means with Mr I. Collins, & sometimes with Mr Newton, & what he might learne from the English either in London or by that correspondence is the main Quæstion. Mr Oldenburg & Mr Collins are long since dead, & Mr Newton lived then at Cambridge & knew little more then his own correspondence since published by Dr Wallis. Mr Newton can be no witness for Mr Keill, nor Mr Leibnitz for himself & there appear no other living evidence. The R. Society therefore being twice pressed by Mr Leibnitz against Mr Keil, appointed a Committee to search out & examin the Letters Letter-books & papers left by Mr Oldenburgh in the hands of the Society & those found among the papers of Mr Collins relating to the matters in dispute & report their opinion thereupon & ordered the Report of the Committee with the extracts of the Letters & Papers to be published

✝ The Question is about the first invention of the infinitesimal method, & the opinion of Mr Keil against Mr Leibnitz is favoured by the Committee But there is nothing in these papers that can affect other persons abroad who have received the method from Mr Leibnitz; They were strangers to the correspondence between Mr Leibnitz & Mr Oldenburg, They found the method usefull & are much to be commended for the use & improvements they have made of it

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✝ The Question is about the first invention of the infinitsimal method called by Mr Newton the method of fluents, fluxions & moments & by Mr Leibnitz the method of differences & indivisibles The Analysis printed in the first place shews that Mr Newton had the method in 1669: the first instance of Mr Leibnitz's knowing the method is in his Letter dated 21 Iune 1677. In the times between there are Letters which shew that Mr Newton had the method & Mr Leibnitz had it not, & that Mr Leibnitz received light into it from Mr Newton.

The infinitesimal method & the method of series were in those days considered by M^r Newton as two methods nearly related & subservient to one another & conspiring into one general method for the solution of almost all sorts of problems as appears by the Letters. At the request of M^r Leibnitz M^r Newton communicated to him one half of this general method by his Letter of 13 Iune 1676, & it doth not appear that M^r Leibnitz found out the other half till some time after this communication. When he received the first half of the method he put in for coinventor. When he had light into the second half he put in for coinventor & when M^r Newtons Principia philosophiæ came abroad he put in for coinventor. But the Gentlemen of the Committee upon examining the Letters & Papers think that M^r Keill has done him no wrong in representing M^r Newton the first inventor in the second case as well as in the first & third.

<548v>

$$\frac{1}{1-x} = 1+x+xx+x^3 \quad x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4 \quad \frac{1}{36}x^6$$

$$x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4} \qquad -x^2-\frac{1}{2}x^3-\frac{1}{3}x^4$$

$$x-\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{12}x^4$$

$$\frac{3bb}{a^4}\sqrt{1+QQ}.\frac{4b4}{a6} .:: 3\sqrt{\ } . \frac{4bb}{aa} :: 3aa\sqrt{\ }.4bb :: \frac{GT}{DN}.\frac{4bb}{3aa}$$

Schol. Fingere liceret quod projectile pergeret in arcuum GH, HI, IK chordis, et in solis punctis G, H, I, K, per vim gravitatis & vim resistentiæ agitaretur, perinde ut in Propositione prima libri primi corpus per vim centripetam intermittentem agitabatur; deinde chordas in infinitum diminui ut vires redderentur continuæ. Et solutio Problematis hac ratione facillima redderetur.

p 268. lin. 10, lege [gravitatem ut 3XY ad 2YG

p 269. lin. 8. lege ut ^3S in $\frac{XX}{A}$ ad 4RR id est ut XY ad $\frac{2nn+2n}{n+2}VG$

p 270. lin 9, 14. $\frac{2nn+2n}{n+2}$.

injuriam Newtono illatam repellendo

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Ad Lectorem of Comm Episto 5

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M^r Leibnitz was in England in the beginning of the year 1673 & again in October 1676, & during the intervall in France, & all that time kept a correspondence with M^r Oldenburg, & by his means with M^r Iohn Collins & sometimes with M^r Is. Newton, & what he might learn from the English either in London or by that correspondence is the main question. M^r Oldenburg & M^r Collins are long since dead, & M^r Newton lived then at Cambridge & knew little more then his own correspondence since published by D^r Wallis. M^r Newton can be no witness for M^r Keill nor M^r Leibnitz for himself, & there appears no other living evidence. The R. Society therefore being twice pressed by M^r Leibnitz against M^r Keill, appointed a Committee to search out & examin the Letters Letter-books & papers left by M^r^ Oldenburg in the hands of the Society & those found among the papers of M^r^ Iohn Collins relating to the matters in dispute & report their opinion thereupon & ordered the Report of the Committee with the extracts of the Letters & Papers to be published.

When M^r^ Newton wrote the Analysis printed in the beginning of this Collection, he had a method of resolving finite equations into infinite ones, & of applying both finite & infinite equations to the solution of Problemes by meanes of <549v> the proportions of the augmenta momentanea of growing or increasing quantities. These augmenta M^r^ Newton calls particles & moments, & M^r^ Leibnitz infinitesimals indivisibles & differences. The increasing quantities M^r^ Newton calls fluents & M^r^ Leibnitz summs, & the velocities of increase M^r^ Newton calls fluxions & exposes these fluxions by the moments of the flowing quantities. That part of the method which consists in resolving finite equations into infinite ones, was at the request of M^r^ Leibnitz communicated to him by M^r^ Newton in his Letters of Iune 13^th^ & October 24^th^, 1676. And M^r^ Newton having so far touched upon the other part as to reckon that it was become*a*[1] sufficiently obvious, to secure it from being taken from him before he should have occasion to explain it, he expressed it in cyphre after the manner used by Galilæo and Hugenius upon other like occasions. M^r^ Leibnitz claims the invention of this other part, & M^r^ Keill asserts it to M^r^ Newton & is favoured in his opinion by the Report of the Committee. But there is nothing in these Papers which can affect other persons abroad who have received the method from M^r^ Leibnitz. They were strangers to the correspondence between M^r^ Leibnitz & M^r^ Oldenburg. They found the method usefull & are much to be commended for the use & improvements that they have made of it.

Some Notes are added to the Letters to enable such Readers as want leasure, to compare them with more ease & see the sense of them at one reading.

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6 Ad Lectorem of the Commercium

The occasion of publishing this Collection of Letters will be understood by the Letters of M^r^ Leibnitz & M^r^ Keil in the end thereof. M^r^ Leibnitz taking offence at a passage in a discourse of M^r^ Keil published in the Transactions A.C. 1708, wrote a Letter to the Secretary of the R. Society complaining thereof as a calumny, desiring a remedy from the Society & suggesting that he beleived they would judge it equal that M^r^ Keil should make a publick acknowledgment of his fault. M^r^ Keil chose rather to return an answer in writing, wherein he explained his meaning in that passage & defended it. M^r^ Leibnitz not meeting with that satisfaction he desired wrote a second Letter to the Secretary of the R. Society, wherein he still complained of M^r^ Keil, representing him a young man not acquainted with what was done before his time nor authorized by the person concerned & appealed to the equity of the Society to cheque his unjust clamours.

M^r^ Leibnitz was in England in the beginning of the year 1673 & again in October 1676 & during the interval in France, & all that time kept a correspondence with M^r^ Oldenburgh & by his means with M^r^ Iohn Collins, & what he learnt from the English by that correspondence is the main Question. M^r^ Leibnitz appeals from young men to old ones who knew what passed in those days, refuses to let any man be heard against him without authority from Sir Isaac Newton & desires that Sir Isaac himself would give judgment. Sir Isaac lived then at Cambridge, & knew only his own correspondence printed by D^r^ Wallis. M^r^ Oldenburgh & M^r^ Collins are long since dead & so are D^r^ Barrow M^r^ Gregory & D^r^ Wallis who corresponded with M^r^ Collins, & D^r^ Wallis gave judgment against M^r^ Leibnitz in a Letter dated Apr 10^th^ 1695 & not yet printed. The R. Society therefore being twice pressed by M^r^ Leibnitz & having no means of enquiring into this matter by living evidence, appointed a Committee to search out & examin the Letters Letter-books & papers left by M^r^ Oldenburgh in the hands of the Society & those found among the papers of M^r^ Collins relating to the matters in dispute, & ordered the Report of the Committee with the extracts of the Letters & Papers to be published.

The Question is, Who was the first author of the method called by Mr Leibnitz the infinitesimal method the Analysis of indivisibles & infinites, & the Differential & summatory method & by Mr Newton the Method of fluents fluxions & moments. Mr Leibnitz claims inventoris jura, & sometimes allows that Mr Newton might also find it apart. Mr Keil asserts Mr Newton to be the first inventor & the Committe is on the same opinion.

The Letters & Papers till the year 1676 inclusively shew that Mr Newton had a general method of solving Problemes by reducing them to equations finite or infinite whether those equations include moments (the exponents of fluxions) or do not include them, & by deducing fluents & their moments from one another by means of those equations.

The Analysis printed in the beginning of this Collection shews that he had such a general method in the year 1669. And by the Letters & Papers which follow, it appears that in the year 1671, at the desire of his friends he composed a larger Treatise upon this method (p. 27. l. 10, 27 & p. 71. l. 4, 26) that it was very general & easy without sticking at surds or mechanical curves & extended to the finding tangents areas lengths centers of gravity & curvatures of Curves &c (p. 27, 30, 85) that in Problems reducible to Quadratures it proceeded by the Propositions since printed in the book of Quadratures which Propositions are there founded upon the method of fluents (p. 72, 74, 76) that it extended to the extracting of fluents out of æquations involving their fluxions & proceeded in difficulter cases by assuming the terms of a series & determining them by the conditions of the Probleme (p. 86) that it determined the curve by the length thereof p 24 & extended to inverse Problems of tangents & others more difficult & was so general as to reach almost all Problemes except nume <551v> ral ones like those of Diophantus (p. 55, 85, 86) And all this was known to Mr Newton before Mr Leibnitz understood any thing of the method as appears by the dates of their Letters.

For in the year 1673 he was upon another differential method p. 32. in May 1676 he desired Mr Oldenburgh to procure him the method of infinite series (p. 45) & in his Letter of 27 Aug 1676 he wrote that he did not beleive Mr Newton's method to be so general as Mr Newton had described it. For, said he, there are many Problemes & particularly the inverse Problems of Tangents which cannot be reduced to æquations or Quadratures (p. 65) which words make it evident that Mr Leibnitz had not yet the method of differential equations. And in the year 1675 he communicated to his friends at Paris a tract written in a vulgar manner about a series which he received from Mr Oldenburgh, & continued to polish in the year 1676 with intention to print it; but it swelling in bulk he left off polishing it after other business came upon him, & afterwards finding the Differential Analysis he did not think it worth publishing because written in a vulgar manner. p. 42, 45. In all these Letters & Papers there appears nothing of his knowing the Differential method before the year 1677: It is first mentioned by him in his Letter of 21 Iune 1677, & there he began the description of it with these words Hinc nominando IN POSTERVM dy differentiam duarum proximarum y &c. p. 88.

Mr Newton was therefore the first inventor, & whether Mr Leibnitz invented it proprio Marte afterwards or not, is a question of no consequence. The first inventor is the inventor, & inventoris jura are due to him alone. He has the sole right till another finds it out, & then to take his right from him without his consent & share it with another would be an Act of injustice, & an endless encouragement to pretenders. But however, there are great reasons to beleive that Mr Leibnitz did not invent it proprio marte, but received some light from Mr Newton.

For it is to be observed that wherever Mr Newton in his Letters spake of his general method, he understands his method of series & fluents taken together as two parts of one general method. In his Analysis the series are applied to the solution of problemes by the method of fluents & thereby give new series, & the method of fluents is demonstrated by the method of series (p. 14, 15, 18, 19) & in the year 1671 he wrote of both together, p. 71. The series in his book of Quadratures are derived from the method of fluents & were derived from it before the year 1676, p. 72. The method of extracting fluents out of equations involving their fluxions comprehends both together, & the method of assuming the terms of a series & determining them by the conditions of the probleme proceeds by means of the method of fluents, p 86. When Mr Newton represented that his method of series extended to the solution of almost all Problemes except numeral ones like those of Diophantus, he included inverse problemes of Tangents (p. 55, 56, 85) & those problems are not tractable without the method of fluents. And sometimes series are considered as fluents & their second terms as moments. And M^r^ Newton sometimes derives his method of fluents from the series into which the power of a binomium is resolved, p 19. lin. 19, 20.

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In the next place it must be observed that M^r^ Newton at the request of M^r^ Leibnitz communicated to him freely & plainly one half of this general method, namely the method of series p. 45, 49. M^r^ Gregory by the help of but one series, with notice that it was the result of a general method, found out the method within the space of a year, p. 22, 23, 24. M^r^ Leibnitz pretended to have two series in the year 1674, & had eight others sent him by M^r^ Oldenburg in April 1675 as the result of a general method & took a years time to consider them p. 38, 40, 41, 42: but this method being altioris indaginis he could not find it out but at length requested M^r^ Oldenburg to procure it from M^r^ Collins (p. 45) & at the request of M^r^ Oldenburg & M^r^ Collins, M^r^ Newton sent it to him. And when he had it he understood it with difficulty & desired M^r^ Newton to explain some things further p. 49, 63.

And as for the other half of the method, M^r^ Leibnitz had a general description of it in M^r^ Newton's Letters of 10 Decemb. 1672, 13 Iune 1676 & 24 Octob 1676, with examples in drawing of Tangents (p. 30, 47) squaring of curves p 42 & solving of inverse problemes of Tangents p. 86. & understanding by the same Letters that the method of tangents printed by Slusius was a branch & Corollary of M^r^ Newtons general method (p. 30) he set his mind upon improving his method of Tangents so as to bring it to a general method of solving problemes. For in his journey home from Paris by London & Amsterdam, he was upon a project of extending it to the solution of all sorts of problemes by calculating a certain Table of Tangents as the most easy & useful thing he could then think of, for a calculator which he wanted & wrote of this designe to M^r^ Oldenburg in a Letter dated at Amsterdam 28 Novemb. 1676 (pag. 87.) & therefore he had not then improved the method of Slusius into a general method of solving all sorts of problems but was endeavouring to do it. Now M^r^ Newton had told him that his method extended to Tangents of mechanical Curves & to Quadratures centers of gravity & Curvatures of Curves in general & to inverse problemes of Tangents, & he was thereby sufficiently informed that this method was founded upon the consideration of the small particles of quantity called augmenta momentanea & moments by M^r^ Newton, & infinitesimals indivisibles & differences by M^r^ Leibnitz. For there is no other way of resolving any of those sorts of Problemes, then by considering these particles of quantity. This consideration therefore might make him think upon the methods of Fermat Gregory Barrow & Slusius, who drew tangents by the proportion of the particles of lines. For he tells us that he found out the Differential method by considering how to draw Tangents by the differences of the Ordinates & how thereby to render the method of Slusius more general (p. 88) & considered that as the summs of the Ordinates gave the area so their differences gave the tangents, & thence received the first light into the differential method (p. 104) And with Slusius he gave the name of differences to the moments of dignities. (Transact. Philosoph Num. 95.) And when he had found the Method, he saw that it answered <552v> to the description which M^r^ Newton had given of his method in drawing of Tangents, in rendring Problems of Quadratures more easy, & in bringing inverse Problems of Tangents to Equations & Quadratures, which in his letter of 27^th^ August 1676 he had represented impossible. p. 65, 88, 89, 90, 91, 93.

But when he sent his method to M^r^ Newton he forgot to acknowledge that he had but newly found it, & that the want of it had made him of opinion the year before that inverse problemes of Tangents & such like could not be reduced to Equations & quadratures. He forgot to acknowledge that by means of this invention he now perceived that M^r^ Newtons method which extended to such Problems was much more general then he could beleive the year before. He forgot to acknowledge that in the collection of Gregories Letters & Papers which at his own request were sent to him at Paris by M^r^ Oldenburg & M^r^ Collins, he found the copy of M^r^ Newtons Letter of Decem. 10^th^ 1672, conteining his method of Tangents & representing it a branch or corollary of a general method of solving all sorts of problems & that the agreement of this method of Tangents with that of Slusius, put him upon considering how to enlarge the method of Slusius, by the Differences of the Ordinates. He forgot to acknowledge that M^r^ Newtons Letters of 13 Iune & 24 Octob. 1676, gave him any light into the method. And those things are now so far out of his memory, that he has told the world that when he published the elements of his differential method he knew nothing more of M^r^

Newtons inventions of this sort then what M^{r} Newton had formerly signified in his Letters, namely that he could draw tangents without taking away irrationals: which Hugenius had signified that he could also do before he understood the infinitesimal method. p. 104, 107.

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When M^{r} Newton was desired by M^{r} Leibnitz to tell him the original of his Theoreme for reducing binomials into series, he gave him an historical account of the invention but tooke care in the same Letter that the narrative should not prejudice M^{r} Mercator. M^{r} Leibnitz is in like manner to forbeare expecting that his own testimony for himself will be taken in evidence to the prejudice of M^{r} Keil. If he would have it beleived that he found the differential method before the winter between the years 1676 & 1677 he must bring better evidence then there is in these Letters & papers to the contrary, & forbeare to call those men imprudent & unjust that will not take his bare word for it.

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that he found the method above nine years before that is before October 1675, & that he & his friends allowed M^{r} Newton to have invented the like principles by himself meaning perhap**{s}** before the writing of his Letter of Octob 24. 1676. And but before M^{r} Leibnitz. And here his memory seems to have faild him again. For in the years 95 & 96 he was polishing his opusculum & when he wrote his letter of Aug 27 1696 (which was but a month or six weeks before he left Paris) he was of opinion that inverse problems of tangents were not reducible to æquations or quadratures. When M^{r} Newton was desired by M^{r} Leibnitz to tell him the original of his Theoreme for reducing Binomials into series, he gave him an historical account of the invention but tooke car at the same letter that Mercator should not be prejudiced by the narration: M^{r} Leibnitz is in like manner to

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When M^{r} Newton wrote the Analysis printed in the beginning of this Collection, he had a method of resolving finite equations into infinite ones & of applying both finite & infinite æquations to the solution of Problemes by means of the proportions of the incrementa momentanea of growing or increasing quantities. These incrementa M^{r} Newton calls particles & moments, & M^{r} Leibnitz infinitesimals indivisibles & differences. The increasing quantities M^{r} Newton calls fluents & M^{r} Leibnitz summs, & the velocities of increase M^{r} Newton calls fluxions & exposes these fluxions by the moments of the flowing quantities. That part of the method which consists in resolving finite æquations into infinite ones M^{r} Newton at the request of M^{r} Leibnitz communicated plainly to him in his Letters of 13 Iune & 24 Octob. 1676. ✝ < insertion from the bottom of the page > ✝ And having so far touched upon the second other part so as to reckon that the invention thereof was become sufficiently obvious, (pag 72 lin. 1) to secure it from being taken from him till there should **{be}** a occasion of explaining it , he expressed it in cyphre after the manner used by Galilæus & Hugenius upon other like occasions. M^{r} Leibnitz the next year put in for the invention of this other part & now claims it, & M^{r}Keil asserts it to M^{r} Newton & is favoured in his opinion by the Committe. But there is nothing &c And having so far touched upon the other part as to reccon that it was become sufficiently obvious (p. 72 lin 1;) to secure it from being taken from him before there occasion should be offered of explaining it, he expressed it in cyphre after the manner used by Galilæus & Huygenius < text from f 555r resumes > M^{r} Leibnitz contends for the first invention of the other part of the method which he calls the differential method: M^{r} Keil that M^{r} Newton was the first inventor thereof, & the opinion of M^{r} Keil is favoured by the Committee. But there is nothing in these Papers which can affect other persons abroad, who have received the method from M^{r} Leibnitz. They were strangers to the correspondence between M^{r} Leibnitz & M^{r} Oldenburg. They found the method usefull, & are much to be commended for the use & improvements that they have made of it.

Some notes are added to the Letters to enable such Readers as want leasure, to compare them with more ease & see the sense of them at one reading.

p. 119 l 11. ad verbam [non properavi] In epistola Aug. 27. 1676 properavit se coinventorem methodi serierum asserere. In epistola 21 Iunij 1677 properavit methodum **{**ca**}**pere de qua Newtonus tractatum ante annos quinque scripserat. In schedis tribus anno 1689 impressis properavit Principia Philosophiæ deflorare.

ib ad verba [plusquam nonum] Probandum est.

Pag. 119 lin 11. Ad verba [non properavi] notetur. In Epistola Aug. 27 1676 properavit se coinventorem methodi serierum proponere. In Epistola Iunij 21 1677 properavit methodum ut suam describere de qua Newtonus tractatum ante annos quinque scripserat. In schedis tribus anno 1689 impressis properavit Propositiones principales Principiorum Philosophiæ ad calculum differentialem revocatas in lucem edere ut in Inventoris jura veniret.

Ib. Ad verba [plusquam nonum] notetur. Probandum est.

[1] *a* pag. 72. lin. 1